$$A = \begin{bmatrix} 3 & 5 & -5 \\ -5 & -7 & 5 \\ -5 & -5 & 3 \end{bmatrix}$$

$$\begin{aligned}
\text{Det}(A) &= 3(-21+25) - 5(-15+25) + (-5)(25-35) \\
&= 3(4) - 5(10) - 5(-10) \\
&= 12 - 50 + 50 \\
&= \underline{12.}
\end{aligned}$$

$$\det\begin{pmatrix} 3-\lambda & 5 & -5 \\ -5 & -7-\lambda & 5 \\ -5 & -5 & 3-\lambda \end{pmatrix} = 0.$$

$$\begin{aligned}
&= (3-\lambda)(-21+7\lambda-3\lambda+\lambda^2+25) - 5(-15+5\lambda+25) + (-5)(25-(35+5\lambda)) = 0 \\
&= (3-\lambda)(\lambda^2+4\lambda+4) - 5(5\lambda+10) - 5(-10-5\lambda) = 0. \\
&= 3\lambda^2+12\lambda+12-\lambda^3-4\lambda^2-4\lambda - 25\lambda - 50 + 25\lambda + 50 = 0 \\
&= 3\lambda^2+12\lambda+12-\lambda^3-4\lambda^2-4\lambda) - 25\lambda - 50 + 25\lambda + 50 = 0 \\
&= -\lambda^3 - \lambda^2 + 8\lambda + 12 = 0 \\
&= \lambda^3 + \lambda^2 - 8\lambda - 12 = 0.
\end{aligned}$$

$$\underline{\lambda = -2, 3}$$

Showing v_1, v_2, v_3 are eigenvectors of A:

$$A = \begin{bmatrix} 3 & 5 & -5 \\ -5 & -7 & 5 \\ -5 & -5 & 3 \end{bmatrix} \qquad P = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{ccc} v_1 & v_2 & v_3 \end{array}.$$

$$Ax = \lambda x$$

v_1:

$$\begin{bmatrix} 3 & 5 & -5 \\ -5 & -7 & 5 \\ -5 & -5 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \lambda\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -3+5-5 \\ 5-7+5 \\ 5-5+3 \end{bmatrix} = \lambda\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix} = \lambda\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

λ is a scalar, 3.

v_2:

$$\begin{bmatrix} 3 & 5 & -5 \\ -5 & -7 & 5 \\ -5 & -5 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

λ is a scalar, -2

v_3:

$$\begin{bmatrix} 3 & 5 & -5 \\ -5 & -7 & 5 \\ -5 & -5 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \lambda\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} = \lambda\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

λ is a scalar, -2

eigenvalues: $3, -2, -2$

$$C = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)(x_i-\mu)^T.$$

we need to show $u^TCu \geq 0$ (definition of semi-definite.)

$$= u^T\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)(x_i-\mu)^T\right)u$$

$$= \frac{1}{n}\sum_{i=1}^{n} u^T(x_i-\mu)(x_i-\mu)^T u$$

remember $v^Tw = v \cdot w$ where $v, w \in \mathbb{R}$ vectors.

$$= \frac{1}{n}\sum_{i=1}^{n}\left((x_i-\mu)^T u\right)^2 \geq 0.$$

A squared value will always be ≥ 0 if they are $\in \mathbb{R}$.