Computational Finance

Series 11

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In [1]:
```
import numpy as np
import pandas as pd
import scipy.optimize as optimize
import matplotlib.pyplot as plt
```

In [2]:
```
#bond with 3 years maturity, face value of 100$, and coupon of 0.10 paid semi-
annually

def YTM(price, par, T, coup, freq, step):
    periods = T*freq
    coupon = coup/100.*par/freq
    dt = [(i+1)/freq for i in range(int(periods))]
    ytm_ = lambda y: sum([coupon/(1+y/freq)**(freq*t) for t in dt]) + par/(1+y
/freq)**(freq*T) - price

    return optimize.newton(ytm_, step)

print("Required yield to sell at par (100$): ", YTM(100, 100, 3, 10, 2, 0.05)*
100, "%")
```

```
Required yield to sell at par (100$):  9.999999999999737 %
```

In [3]:
```
principal = [100,100,100,100,100]
maturity = [1/12,2/12,3/12,6/12,12/12]
coupon = [0,0,0,6,8]
price = [99.80,99.60,99.4,100.27,101.57]

data = pd.DataFrame({"principal":principal, "maturity":maturity, "coupon":coup
on, "price":price})
data
```

Out[3]:

	principal	maturity	coupon	price
0	100	0.083333	0	99.80
1	100	0.166667	0	99.60
2	100	0.250000	0	99.40
3	100	0.500000	6	100.27
4	100	1.000000	8	101.57

In [4]:
```
def spot_rate(par, price, maturity):
    total_interest = par/price
    spot = ((1/maturity)*np.log(total_interest))
    total_interest = total_interest-1
    return spot, total_interest
```

In [5]:
```
data['spot_rate'] = data.apply(lambda x : spot_rate(x['principal'],x['price'],
x['maturity'])[0], axis=1)
```

In [6]:
```
data['freq'] = [2,2,2,2,2]
data
```

Out[6]:

	principal	maturity	coupon	price	spot_rate	freq
0	100	0.083333	0	99.80	0.024024	2
1	100	0.166667	0	99.60	0.024048	2
2	100	0.250000	0	99.40	0.024072	2
3	100	0.500000	6	100.27	-0.005393	2
4	100	1.000000	8	101.57	-0.015578	2

But we have to take into account coupon payments

In [7]:
```
def coupon_payment(coupon, freq, periods, data):
    coupon_amount = coupon/freq
    coupon_pay = 0

    for i in range(1, periods):
        period = (i/freq)
        period_spot = data.loc[data['maturity'] == period, 'spot_rate'].values
        discounted_value = coupon_amount / np.exp(period_spot*period)
        try:
            coupon_pay += discounted_value[0]
        except:
            coupon_pay += 0

    return coupon_pay
```

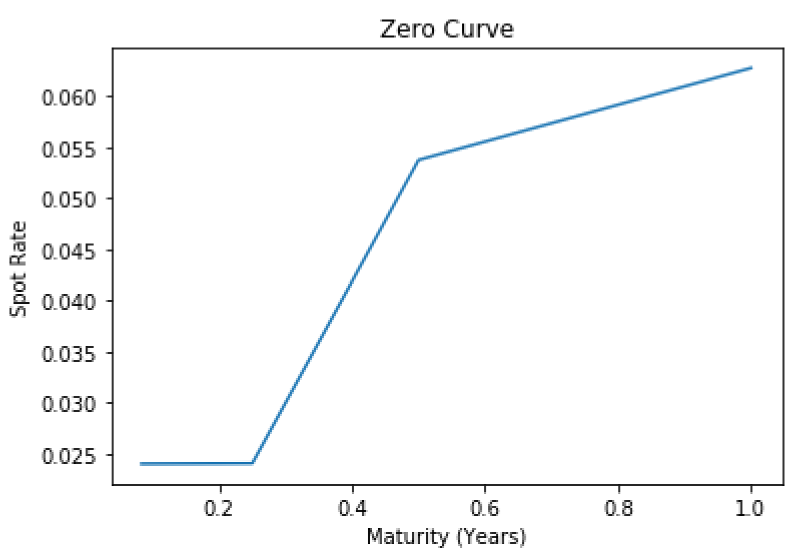
In [8]:
```
def spot_rate(par, price, maturity, coupon, freq, data):
    coupon_pay = 0

    if coupon != 0:
        periods = np.int(maturity*freq)
        coupon_pay = coupon_payment(coupon, freq, periods, data)

    final_flow = par + coupon/freq if maturity%(1/freq) == 0 else par

    final_price = price - coupon_pay

    total_interest = final_flow/final_price

    spot = ((1/maturity)*np.log(total_interest))

    total_interest = total_interest-1

    return spot, total_interest
```

In [9]:
```
for k, v in data.iterrows():
    data.set_value(k, 'spot_rate', spot_rate(v['principal'],v['price'],v['matu
rity'],v['coupon'],v['freq'], data)[0])
```

```
/home/kense/.local/lib/python3.6/site-packages/ipykernel_launcher.py:2: Futur
eWarning: set_value is deprecated and will be removed in a future release. Pl
ease use .at[] or .iat[] accessors instead
```

In [10]:
```
plt.plot(data['maturity'],data['spot_rate'])
plt.xlabel("Maturity (Years)")
plt.ylabel("Spot Rate")
plt.title("Zero Curve")
```

Out[10]: Text(0.5, 1.0, 'Zero Curve')



In [11]:
```python
principal = [100,100,100,100,100,100]
maturity = [2,3,4,5,7,10]
coupon = [4,4,4,4,0,0]
price = [103.21,104.85,106.36,107.77,84.48,77.72]

data = pd.DataFrame({"principal":principal, "maturity":maturity, "coupon":coup
on, "price":price})
data
```

Out[11]:

	principal	maturity	coupon	price
0	100	2	4	103.21
1	100	3	4	104.85
2	100	4	4	106.36
3	100	5	4	107.77
4	100	7	0	84.48
5	100	10	0	77.72

In [12]:
```python
def spot_rate(par, price, maturity):
    total_interest = par/price
    spot = ((1/maturity)*np.log(total_interest))
    total_interest = total_interest-1
    return spot, total_interest
```

In [13]:
```python
data['spot_rate'] = data.apply(lambda x : spot_rate(x['principal'],x['price'],
x['maturity'])[0], axis=1)
```

In [14]:
```python
data['freq'] = [1,1,1,1,1,1]
data
```

Out[14]:

	principal	maturity	coupon	price	spot_rate	freq
0	100	2	4	103.21	-0.015798	1
1	100	3	4	104.85	-0.015787	1
2	100	4	4	106.36	-0.015415	1
3	100	5	4	107.77	-0.014966	1
4	100	7	0	84.48	0.024094	1
5	100	10	0	77.72	0.025206	1

But we have to take into account coupon payments

In [15]:
```
def coupon_payment(coupon, freq, periods, data):
    coupon_amount = coupon/freq
    coupon_pay = 0

    for i in range(1, periods):
        period = (i/freq)
        period_spot = data.loc[data['maturity'] == period, 'spot_rate'].values
        discounted_value = coupon_amount / np.exp(period_spot*period)
        try:
            coupon_pay += discounted_value[0]
        except:
            coupon_pay += 0

    return coupon_pay
```

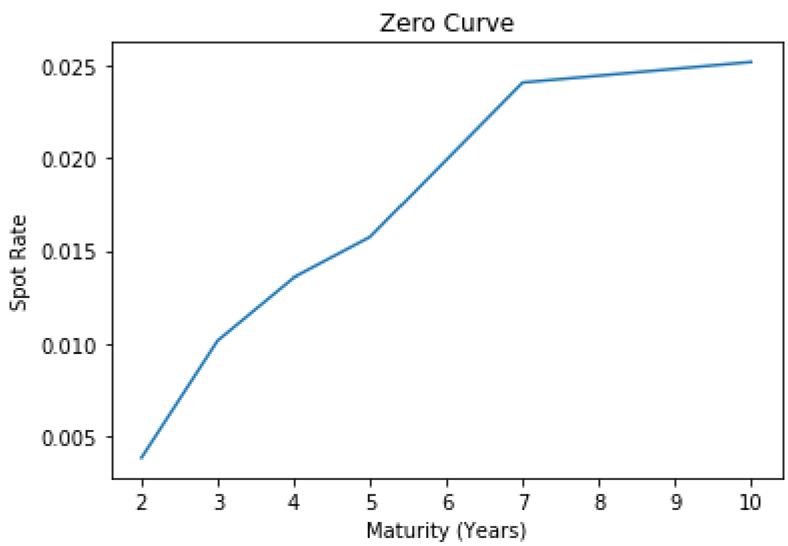
In [16]:
```
def spot_rate(par, price, maturity, coupon, freq, data):
    coupon_pay = 0

    if coupon != 0:
        periods = np.int(maturity*freq)
        coupon_pay = coupon_payment(coupon, freq, periods, data)

    final_flow = par + coupon/freq if maturity%(1/freq) == 0 else par

    final_price = price - coupon_pay

    total_interest = final_flow/final_price

    spot = ((1/maturity)*np.log(total_interest))

    total_interest = total_interest-1

    return spot, total_interest
```

In [17]:
```
for k, v in data.iterrows():
    data.set_value(k, 'spot_rate', spot_rate(v['principal'],v['price'],v['matu
rity'],v['coupon'],v['freq'], data)[0])
```

```
/home/kense/.local/lib/python3.6/site-packages/ipykernel_launcher.py:2: Futur
eWarning: set_value is deprecated and will be removed in a future release. Pl
ease use .at[] or .iat[] accessors instead
```

In [18]:
```
plt.plot(data['maturity'],data['spot_rate'])
plt.xlabel("Maturity (Years)")
plt.ylabel("Spot Rate")
plt.title("Zero Curve")
```

Out[18]: `Text(0.5, 1.0, 'Zero Curve')`



The graphs show the zero coupon rate of the bonds as a function of maturity (in years). As we can see, it is an increase, meaning that the bonds with higher maturity time has a higher spot rate, which as per the price and coupon payments data given, makes sense because the bonds without coupon payments' yield is based on its maturity payment. Thus We can see that the bonds with zero coupon payments tend to have a lower price, since investors would not stand to be making money from coupons holding the bonds. Thus the zero coupon interest rate acts as a sort of comparison factor so investors can determine whether or not these bonds without coupons are worth their investment at the current price compared to other bonds, or even other investments.