Computational Finance

Series 6

Tientso Ning

```python
In [1]: import numpy as np
        import pandas as pd
        import time
        import matplotlib.pyplot as plt
```

```python
In [2]: #read data
        data = pd.read_table("./data.dat", header=None, names=["timestamp", "bid", "ask"])
        data["timestamp"] = data["timestamp"].apply(lambda x : time.strftime("%Y-%m-%d %H:%M:%S", time.gmtime(x)))

        #trim data
        data = data.head(np.around(len(data)*0.3).astype(int))

        #pre-processing removal
        data = data[data["timestamp"] > "2012-01-01 00:00:00"]

        #process data
        data.timestamp = pd.DatetimeIndex(data.timestamp)
        data.set_index('timestamp', inplace=True)
```

```python
In [3]: #15 minute intervals
        data = data.resample('15min').mean()
```

```python
In [4]: #TWAP is just the values we have at each 15 minute interval
        data['TWAP'] = data['bid']

        #VWAP calculations
        data['volume'] = data.index.value_counts()
        data['cumulative.volume'] = data['volume'].cumsum()
        data['tp.v'] = data['bid']*data['volume']
        data['cumulative.tp.v'] = data['tp.v'].cumsum()
        data['VWAP'] = data['cumulative.tp.v']/data['cumulative.volume']
```

```python
In [178]: def ABPE (ts, delta):

              L = len(ts)
              direction = 1
              x = 0
              dc = 1
              pos = []
              neg = []

              for i in range (1, L): #cycle through the ts

                  if np.abs(ts[x]-ts[i]) > delta*ts[x]: #scaling law

                      #calculate directional change/overshoot
                      if np.sign(ts[x]-ts[i]) != np.sign(direction):
                          direction = direction * -1 #change direction
                          dc += 1
                          if np.sign(ts[x]-ts[i]) == 1:
                              neg.append(ts[i])
                          elif np.sign(ts[x]-ts[i]) == -1:
                              pos.append(ts[i])
                      else:
                          #overshoot
                          continue

                      x = i #set new

              return dc, pos, neg
```

```python
In [209]: #sample 20 different time blocks
          for i in range(20):
              x = np.random.randint(0,len(data))
              print(data[x:x+12][['TWAP','VWAP']])

              print("---")

              twap_price = np.sum(data[x:x+12][['TWAP']]*1000000)[0]
              twap_dec_price = (data.iloc[x][['TWAP']]*12000000)[0]
              print("TWAP Price: ", twap_price)
              print("Decision Price: ", twap_dec_price)
              print("Difference: ", twap_price-twap_dec_price)

              print("---")

              vwap_price = np.sum(data[x:x+12][['VWAP']]*1000000)[0]
              vwap_dec_price = (data.iloc[x][['VWAP']]*12000000)[0]
              print("VWAP Price: ", vwap_price)
              print("TWAP - VWAP: ", twap_price-vwap_price)

              print("---")

              dcs, pos_prices, neg_prices = ABPE(data.TWAP.values, 0.0001)
              abpe_price = np.sum(np.multiply(neg_prices[0:12],1000000))
              print("ABPE Price: ", abpe_price)
              print("ABPE - TWAP: ", abpe_price-twap_price)
              print("ABPE - VWAP: ", abpe_price-vwap_price)

              print("===")
```

```
                             TWAP      VWAP
timestamp
2012-01-10 03:00:00  1.277091  1.009339
2012-01-10 03:15:00  1.277213  1.009670
2012-01-10 03:30:00  1.277662  1.010000
2012-01-10 03:45:00  1.277791  1.010329
2012-01-10 04:00:00  1.278843  1.010659
2012-01-10 04:15:00  1.278769  1.010988
2012-01-10 04:30:00  1.278842  1.011316
2012-01-10 04:45:00  1.278670  1.011643
2012-01-10 05:00:00  1.279054  1.011970
2012-01-10 05:15:00  1.279073  1.012296
2012-01-10 05:30:00  1.279284  1.012622
2012-01-10 05:45:00  1.278878  1.012946
---
TWAP Price:  15341171.095654506
Decision Price:  15325088.971962603
Difference:  16082.123691903427
---
VWAP Price:  12133779.856376154
TWAP - VWAP:  3207391.239278352
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  173511.2492343802
ABPE - VWAP:  3380902.488512732
===
                             TWAP      VWAP
timestamp
2012-01-12 17:30:00  1.282547  1.071995
2012-01-12 17:45:00  1.283117  1.072194
2012-01-12 18:00:00  1.282889  1.072392
2012-01-12 18:15:00  1.282561  1.072590
2012-01-12 18:30:00  1.282657  1.072787
2012-01-12 18:45:00  1.282838  1.072984
2012-01-12 19:00:00  1.282562  1.073181
2012-01-12 19:15:00  1.282800  1.073377
2012-01-12 19:30:00  1.282607  1.073573
2012-01-12 19:45:00  1.282840  1.073769
2012-01-12 20:00:00  1.282759  1.073964
2012-01-12 20:15:00  1.282264  1.074159
---
TWAP Price:  15392440.068140764
Decision Price:  15390562.199999996
Difference:  1877.8681407682598
---
VWAP Price:  12876966.345042178
TWAP - VWAP:  2515473.7230985854
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  122242.27674812265
ABPE - VWAP:  2637715.999846708
===
                             TWAP      VWAP
timestamp
2012-01-02 21:15:00  1.293371  1.293451
2012-01-02 21:30:00  1.293332  1.293450
2012-01-02 21:45:00  1.293342  1.293449
```

```
2012-01-02 22:00:00  1.293413  1.293449
2012-01-02 22:15:00  1.293717  1.293451
2012-01-02 22:30:00  1.293750  1.293453
2012-01-02 22:45:00  1.293696  1.293455
2012-01-02 23:00:00  1.293830  1.293458
2012-01-02 23:15:00  1.294688  1.293468
2012-01-02 23:30:00  1.295513  1.293485
2012-01-02 23:45:00  1.295265  1.293499
2012-01-03 00:00:00  1.295498  1.293515
---
TWAP Price:  15529416.702054204
Decision Price:  15520448.0
Difference:  8968.70205420442
---
VWAP Price:  15521583.227059996
TWAP - VWAP:  7833.474994208664
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  -14734.357165317982
ABPE - VWAP:  -6900.882171109319
===
                        TWAP      VWAP
timestamp
2012-01-03 14:45:00  1.304704  1.295707
2012-01-03 15:00:00  1.305626  1.295760
2012-01-03 15:15:00  1.305687  1.295813
2012-01-03 15:30:00  1.305969  1.295867
2012-01-03 15:45:00  1.305581  1.295919
2012-01-03 16:00:00  1.305560  1.295969
2012-01-03 16:15:00  1.305227  1.296018
2012-01-03 16:30:00  1.304953  1.296065
2012-01-03 16:45:00  1.304929  1.296110
2012-01-03 17:00:00  1.305285  1.296158
2012-01-03 17:15:00  1.305548  1.296206
2012-01-03 17:30:00  1.306012  1.296256
---
TWAP Price:  15665081.152074156
Decision Price:  15656449.792531114
Difference:  8631.359543042257
---
VWAP Price:  15551848.588035384
TWAP - VWAP:  113232.56403877214
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  -150398.8071852699
ABPE - VWAP:  -37166.243146497756
===
                        TWAP      VWAP
timestamp
2012-01-19 04:00:00  1.285663  1.015200
2012-01-19 04:15:00  1.286113  1.015361
2012-01-19 04:30:00  1.285253  1.015522
2012-01-19 04:45:00  1.285175  1.015682
2012-01-19 05:00:00  1.285413  1.015843
2012-01-19 05:15:00  1.285690  1.016003
2012-01-19 05:30:00  1.286061  1.016163
2012-01-19 05:45:00  1.286238  1.016324
```

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2012-01-19 06:00:00  1.285807  1.016484
2012-01-19 06:15:00  1.285584  1.016643
2012-01-19 06:30:00  1.285190  1.016802
2012-01-19 06:45:00  1.285310  1.016961
---
TWAP Price:  15427496.797057366
Decision Price:  15427950.252100838
Difference:  -453.45504347234964
---
VWAP Price:  12192988.657779275
TWAP - VWAP:  3234508.1392780915
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  87185.54783152044
ABPE - VWAP:  3321693.687109612
===
                        TWAP   VWAP
timestamp
2012-01-14 23:15:00    NaN    NaN
2012-01-14 23:30:00    NaN    NaN
2012-01-14 23:45:00    NaN    NaN
2012-01-15 00:00:00    NaN    NaN
2012-01-15 00:15:00    NaN    NaN
2012-01-15 00:30:00    NaN    NaN
2012-01-15 00:45:00    NaN    NaN
2012-01-15 01:00:00    NaN    NaN
2012-01-15 01:15:00    NaN    NaN
2012-01-15 01:30:00    NaN    NaN
2012-01-15 01:45:00    NaN    NaN
2012-01-15 02:00:00    NaN    NaN
---
TWAP Price:  0.0
Decision Price:  nan
Difference:  nan
---
VWAP Price:  0.0
TWAP - VWAP:  0.0
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  15514682.344888886
ABPE - VWAP:  15514682.344888886
===
                        TWAP       VWAP
timestamp
2012-01-04 11:30:00  1.298345  1.298299
2012-01-04 11:45:00  1.297943  1.298298
2012-01-04 12:00:00  1.295551  1.298288
2012-01-04 12:15:00  1.295377  1.298277
2012-01-04 12:30:00  1.295925  1.298269
2012-01-04 12:45:00  1.294678  1.298255
2012-01-04 13:00:00  1.293749  1.298239
2012-01-04 13:15:00  1.294331  1.298225
2012-01-04 13:30:00  1.295082  1.298213
2012-01-04 13:45:00  1.295362  1.298203
2012-01-04 14:00:00  1.294602  1.298190
2012-01-04 14:15:00  1.292642  1.298170
---
```

```
TWAP Price:   15543587.981920917
Decision Price:   15580138.815789485
Difference:   -36550.83386856876
---
VWAP Price:   15578926.74760523
TWAP - VWAP:   -35338.76568431407
---
ABPE Price:   15514682.344888886
ABPE - TWAP:   -28905.63703203015
ABPE - VWAP:   -64244.40271634422
===
                          TWAP        VWAP
timestamp
2012-01-11 13:45:00   1.269433    1.048454
2012-01-11 14:00:00   1.267563    1.048684
2012-01-11 14:15:00   1.268197    1.048915
2012-01-11 14:30:00   1.268441    1.049146
2012-01-11 14:45:00   1.269061    1.049377
2012-01-11 15:00:00   1.268378    1.049606
2012-01-11 15:15:00   1.267655    1.049835
2012-01-11 15:30:00   1.267665    1.050062
2012-01-11 15:45:00   1.268541    1.050291
2012-01-11 16:00:00   1.268567    1.050519
2012-01-11 16:15:00   1.268666    1.050746
2012-01-11 16:30:00   1.268733    1.050973
---
TWAP Price:   15220900.18842506
Decision Price:   15233200.355699265
Difference:   -12300.167274205014
---
VWAP Price:   12596607.36801549
TWAP - VWAP:   2624292.82040957
---
ABPE Price:   15514682.344888886
ABPE - TWAP:   293782.1564638261
ABPE - VWAP:   2918074.976873396
===
                          TWAP        VWAP
timestamp
2012-01-16 04:00:00   1.264263    0.961347
2012-01-16 04:15:00   1.264344    0.961565
2012-01-16 04:30:00   1.263957    0.961782
2012-01-16 04:45:00   1.263642    0.961999
2012-01-16 05:00:00   1.263790    0.962216
2012-01-16 05:15:00   1.264490    0.962432
2012-01-16 05:30:00   1.264562    0.962649
2012-01-16 05:45:00   1.264740    0.962865
2012-01-16 06:00:00   1.264552    0.963081
2012-01-16 06:15:00   1.265312    0.963297
2012-01-16 06:30:00   1.265255    0.963512
2012-01-16 06:45:00   1.266172    0.963728
---
TWAP Price:   15175079.013748173
Decision Price:   15171159.48387096
Difference:   3919.5298772118986
---
VWAP Price:   11550473.832769293
```

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TWAP - VWAP:  3624605.1809788793
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  339603.33114071377
ABPE - VWAP:  3964208.512119593
===
                          TWAP      VWAP
timestamp
2012-01-10 09:15:00   1.279762  1.017385
2012-01-10 09:30:00   1.279333  1.017699
2012-01-10 09:45:00   1.280126  1.018012
2012-01-10 10:00:00   1.279957  1.018325
2012-01-10 10:15:00   1.279161  1.018636
2012-01-10 10:30:00   1.278771  1.018945
2012-01-10 10:45:00   1.277123  1.019252
2012-01-10 11:00:00   1.277089  1.019558
2012-01-10 11:15:00   1.277419  1.019864
2012-01-10 11:30:00   1.278853  1.020171
2012-01-10 11:45:00   1.279446  1.020478
2012-01-10 12:00:00   1.281027  1.020786
---
TWAP Price:  15348067.462248016
Decision Price:  15357140.837209292
Difference:  -9073.374961275607
---
VWAP Price:  12229111.188790211
TWAP - VWAP:  3118956.2734578047
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  166614.8826408703
ABPE - VWAP:  3285571.156098675
===
                          TWAP      VWAP
timestamp
2012-01-08 14:00:00        NaN       NaN
2012-01-08 14:15:00        NaN       NaN
2012-01-08 14:30:00        NaN       NaN
2012-01-08 14:45:00        NaN       NaN
2012-01-08 15:00:00        NaN       NaN
2012-01-08 15:15:00        NaN       NaN
2012-01-08 15:30:00        NaN       NaN
2012-01-08 15:45:00        NaN       NaN
2012-01-08 16:00:00        NaN       NaN
2012-01-08 16:15:00        NaN       NaN
2012-01-08 16:30:00        NaN       NaN
2012-01-08 16:45:00   1.269967  0.955637
---
TWAP Price:  1269966.5384615383
Decision Price:  nan
Difference:  nan
---
VWAP Price:  955637.1664912545
TWAP - VWAP:  314329.3719702838
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  14244715.806427348
ABPE - VWAP:  14559045.178397631
```

```
===
                              TWAP        VWAP
timestamp
2012-01-18 13:00:00   1.280995   1.005198
2012-01-18 13:15:00   1.281777   1.005368
2012-01-18 13:30:00   1.282219   1.005539
2012-01-18 13:45:00   1.283309   1.005711
2012-01-18 14:00:00   1.283033   1.005882
2012-01-18 14:15:00   1.283168   1.006053
2012-01-18 14:30:00   1.283385   1.006223
2012-01-18 14:45:00   1.281505   1.006393
2012-01-18 15:00:00   1.282297   1.006562
2012-01-18 15:15:00   1.281819   1.006732
2012-01-18 15:30:00   1.281807   1.006901
2012-01-18 15:45:00   1.281827   1.007069
---
TWAP Price:  15387139.2903555
Decision Price:  15371945.230769224
Difference:  15194.059586275369
---
VWAP Price:  12073629.856491791
TWAP - VWAP:  3313509.4338637087
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  127543.0545333866
ABPE - VWAP:  3441052.4883970954
===
                          TWAP   VWAP
timestamp
2012-01-08 07:15:00    NaN    NaN
2012-01-08 07:30:00    NaN    NaN
2012-01-08 07:45:00    NaN    NaN
2012-01-08 08:00:00    NaN    NaN
2012-01-08 08:15:00    NaN    NaN
2012-01-08 08:30:00    NaN    NaN
2012-01-08 08:45:00    NaN    NaN
2012-01-08 09:00:00    NaN    NaN
2012-01-08 09:15:00    NaN    NaN
2012-01-08 09:30:00    NaN    NaN
2012-01-08 09:45:00    NaN    NaN
2012-01-08 10:00:00    NaN    NaN
---
TWAP Price:  0.0
Decision Price:  nan
Difference:  nan
---
VWAP Price:  0.0
TWAP - VWAP:  0.0
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  15514682.344888886
ABPE - VWAP:  15514682.344888886
===
                              TWAP        VWAP
timestamp
2012-01-13 08:15:00   1.283237   1.083120
2012-01-13 08:30:00   1.282275   1.083298
```

```
2012-01-13 08:45:00  1.282374  1.083475
2012-01-13 09:00:00  1.283132  1.083653
2012-01-13 09:15:00  1.281571  1.083830
2012-01-13 09:30:00  1.280035  1.084004
2012-01-13 09:45:00  1.278490  1.084177
2012-01-13 10:00:00  1.279175  1.084350
2012-01-13 10:15:00  1.279895  1.084524
2012-01-13 10:30:00  1.279230  1.084696
2012-01-13 10:45:00  1.278406  1.084868
2012-01-13 11:00:00  1.277544  1.085038
---
TWAP Price:  15365364.61091268
Decision Price:  15398843.146747353
Difference:  -33478.53583467193
---
VWAP Price:  13009034.611750506
TWAP - VWAP:  2356329.9991621748
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  149317.7339762058
ABPE - VWAP:  2505647.7331383806
===
                        TWAP      VWAP
timestamp
2012-01-16 08:15:00  1.267214  0.965021
2012-01-16 08:30:00  1.266980  0.965236
2012-01-16 08:45:00  1.266275  0.965449
2012-01-16 09:00:00  1.264976  0.965662
2012-01-16 09:15:00  1.265630  0.965874
2012-01-16 09:30:00  1.265217  0.966086
2012-01-16 09:45:00  1.266224  0.966299
2012-01-16 10:00:00  1.266409  0.966511
2012-01-16 10:15:00  1.266925  0.966723
2012-01-16 10:30:00  1.267419  0.966936
2012-01-16 10:45:00  1.266564  0.967147
2012-01-16 11:00:00  1.266590  0.967358
---
TWAP Price:  15196421.595575154
Decision Price:  15206564.957746482
Difference:  -10143.362171327695
---
VWAP Price:  11594301.294235531
TWAP - VWAP:  3602120.3013396226
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  318260.7493137326
ABPE - VWAP:  3920381.050653355
===
                        TWAP      VWAP
timestamp
2012-01-11 09:15:00  1.276508  1.044146
2012-01-11 09:30:00  1.275741  1.044395
2012-01-11 09:45:00  1.275815  1.044643
2012-01-11 10:00:00  1.275581  1.044890
2012-01-11 10:15:00  1.271872  1.045133
2012-01-11 10:30:00  1.270297  1.045373
2012-01-11 10:45:00  1.270201  1.045613
```

```
2012-01-11 11:00:00   1.269086   1.045851
2012-01-11 11:15:00   1.270340   1.046091
2012-01-11 11:30:00   1.270449   1.046329
2012-01-11 11:45:00   1.271298   1.046568
2012-01-11 12:00:00   1.272120   1.046808
---
TWAP Price:  15269306.088006658
Decision Price:  15318092.934782587
Difference:  -48786.84677592851
---
VWAP Price:  12545839.77198018
TWAP - VWAP:  2723466.316026479
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  245376.25688222796
ABPE - VWAP:  2968842.572908707
===
                       TWAP       VWAP
timestamp
2012-01-09 00:00:00   1.269228   0.968557
2012-01-09 00:15:00   1.269305   0.968985
2012-01-09 00:30:00   1.268960   0.969411
2012-01-09 00:45:00   1.268933   0.969836
2012-01-09 01:00:00   1.268894   0.970259
2012-01-09 01:15:00   1.268735   0.970681
2012-01-09 01:30:00   1.269018   0.971103
2012-01-09 01:45:00   1.269237   0.971523
2012-01-09 02:00:00   1.269224   0.971943
2012-01-09 02:15:00   1.268852   0.972360
2012-01-09 02:30:00   1.268890   0.972777
2012-01-09 02:45:00   1.268708   0.973192
---
TWAP Price:  15227983.35269151
Decision Price:  15230740.68292683
Difference:  -2757.330235319212
---
VWAP Price:  11650626.043015614
TWAP - VWAP:  3577357.3096758965
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  286698.99219737574
ABPE - VWAP:  3864056.3018732723
===
                       TWAP       VWAP
timestamp
2012-01-17 07:30:00   1.274815   0.983848
2012-01-17 07:45:00   1.275613   0.984042
2012-01-17 08:00:00   1.276097   0.984237
2012-01-17 08:15:00   1.276057   0.984431
2012-01-17 08:30:00   1.276616   0.984625
2012-01-17 08:45:00   1.277277   0.984819
2012-01-17 09:00:00   1.278918   0.985015
2012-01-17 09:15:00   1.279138   0.985210
2012-01-17 09:30:00   1.278213   0.985404
2012-01-17 09:45:00   1.278376   0.985598
2012-01-17 10:00:00   1.277987   0.985792
2012-01-17 10:15:00   1.278242   0.985986
```

```
---
TWAP Price:  15327349.374519398
Decision Price:  15297778.260869574
Difference:  29571.113649824634
---
VWAP Price:  11819006.231148243
TWAP - VWAP:  3508343.1433711555
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  187332.970369488
ABPE - VWAP:  3695676.1137406435
===
                        TWAP        VWAP
timestamp
2012-01-13 06:00:00  1.285964  1.081480
2012-01-13 06:15:00  1.286334  1.081665
2012-01-13 06:30:00  1.286989  1.081849
2012-01-13 06:45:00  1.286029  1.082033
2012-01-13 07:00:00  1.284905  1.082215
2012-01-13 07:15:00  1.286228  1.082398
2012-01-13 07:30:00  1.285453  1.082580
2012-01-13 07:45:00  1.285127  1.082761
2012-01-13 08:00:00  1.283845  1.082941
2012-01-13 08:15:00  1.283237  1.083120
2012-01-13 08:30:00  1.282275  1.083298
2012-01-13 08:45:00  1.282374  1.083475
---
TWAP Price:  15418759.986248605
Decision Price:  15431562.755344423
Difference:  -12802.76909581758
---
VWAP Price:  12989815.73616417
TWAP - VWAP:  2428944.2500844356
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  95922.35864028148
ABPE - VWAP:  2524866.608724717
===
                        TWAP  VWAP
timestamp
2012-01-14 21:30:00   NaN   NaN
2012-01-14 21:45:00   NaN   NaN
2012-01-14 22:00:00   NaN   NaN
2012-01-14 22:15:00   NaN   NaN
2012-01-14 22:30:00   NaN   NaN
2012-01-14 22:45:00   NaN   NaN
2012-01-14 23:00:00   NaN   NaN
2012-01-14 23:15:00   NaN   NaN
2012-01-14 23:30:00   NaN   NaN
2012-01-14 23:45:00   NaN   NaN
2012-01-15 00:00:00   NaN   NaN
2012-01-15 00:15:00   NaN   NaN
---
TWAP Price:  0.0
Decision Price:  nan
Difference:  nan
---
```

```
VWAP Price:  0.0
TWAP - VWAP:  0.0
---
ABPE Price:  15514682.344888886
ABPE - TWAP:  15514682.344888886
ABPE - VWAP:  15514682.344888886
===
```

Here, we can see that for the most part, TWAP trading algorithm performs better than the decision price made in one purchase at time `t` . However, VWAP prices are overall better than TWAP prices, and the Algorithm Based on Price Evolution tends to beat VWAP when it also beats the TWAP prices. This is likey due to the fact that our implementation of ABPE is based on the TWAP prices.