Computational Finance

Series 10

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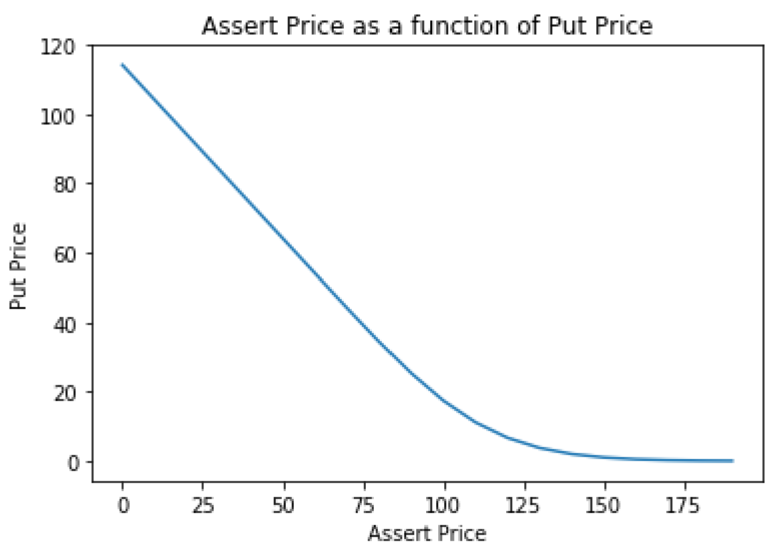
In [2]:
```
import numpy as np
from scipy.stats import norm
import pandas as pd
from scipy.misc import derivative
import matplotlib.pyplot as plt
```

In [35]:
```
#Black Scholes Model from Series 9
def black_scholes (S, K, r, t):
    '''
    Where S is the current stock price
    K is the strike prices
    r is the risk-free interest rate
    t is the time to maturity
    under a Normal distribution
    '''

    v = 0.2 #set volatility at 20% for our problem

    d1 = (np.log(S/K)+((r+(v**2)/2)*t))/(v*np.sqrt(t))
    d2 = d1-(v*np.sqrt(t))
    return K*np.exp(-1*r*t)*norm.cdf(-d2)-S*norm.cdf(-d1)
```

In [37]:
```
#plot the evolution of the put price as a function of the initial assert price S_0
assert_price = []
put_price = []
for i in range(20):
    S = i*10
    assert_price.append(S)
    V = black_scholes(S, 120, 0.05, 1)
    put_price.append(V)

plt.plot(assert_price, put_price)
plt.title("Assert Price as a function of Put Price")
plt.xlabel("Assert Price")
plt.ylabel("Put Price")
```

```
/home/kense/.local/lib/python3.6/site-packages/ipykernel_launcher.py:13: RuntimeWarning: divide by zero encountered in log
  del sys.path[0]
```

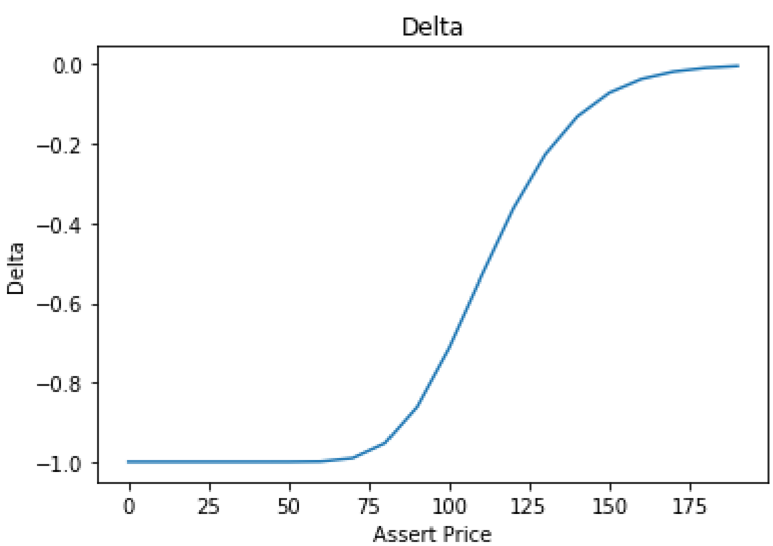
Out[37]:
```
Text(0, 0.5, 'Put Price')
```



In [38]:
```python
#plot the delta for this put
def delta(S, K, r, t):
    '''
    Where S is the current stock price
    K is the strike prices
    r is the risk-free interest rate
    t is the time to maturity
    under a Normal distribution
    '''
    v = 0.2 #set volatility at 20% for our problem

    d1 = (np.log(S/K)+((r+(v**2)/2)*t))/(v*np.sqrt(t))

    return norm.cdf(d1)-1

deltas = []
for i in range(20):
    S = i*10
    D = delta(S, 120, 0.05, 1)
    deltas.append(D)

plt.plot(assert_price, deltas)
plt.title("Delta")
plt.xlabel("Assert Price")
plt.ylabel("Delta")
```

```
/home/kense/.local/lib/python3.6/site-packages/ipykernel_launcher.py:12: Runt
imeWarning: divide by zero encountered in log
  if sys.path[0] == '':
```

Out[38]:
```
Text(0, 0.5, 'Delta')
```

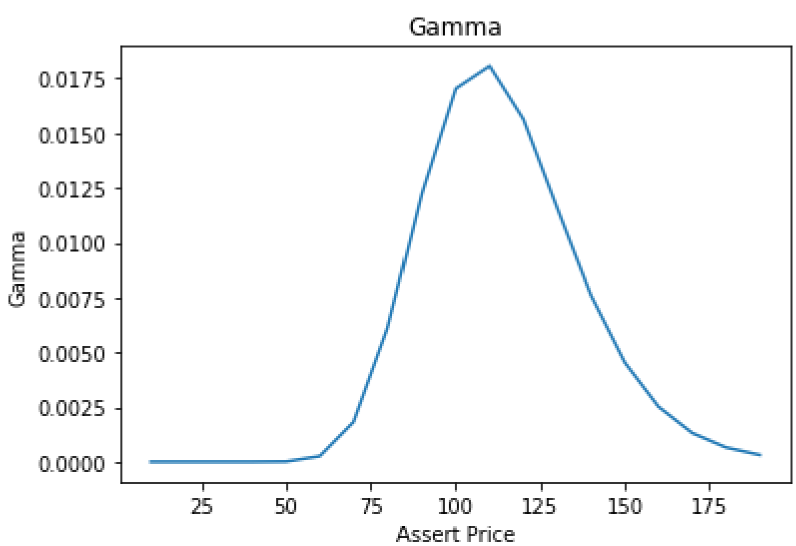


Delta graph shows us that there is a decrease in responsiveness to the price change of the underlying when the price approaches 150~175 range, as the delta is shown to be closer to zero.

In [39]:
```python
#plot the gamma for this put
def gamma(S,K,r,t):
    '''
    Where S is the current stock price
    K is the strike prices
    r is the risk-free interest rate
    t is the time to maturity
    under a Normal distribution
    '''
    v = 0.2 #set volatility at 20% for our problem

    d1 = (np.log(S/K)+((r+(v**2)/2)*t))/(v*np.sqrt(t))

    return norm.pdf(d1)/(S*v*np.sqrt(t))

gammas = []
for i in range(20):
    S = i*10
    G = gamma(S, 120, 0.05, 1)
    gammas.append(G)

plt.plot(assert_price, gammas)
plt.title("Gamma")
plt.xlabel("Assert Price")
plt.ylabel("Gamma")
```

```
/home/kense/.local/lib/python3.6/site-packages/ipykernel_launcher.py:12: Runt
imeWarning: divide by zero encountered in log
  if sys.path[0] == '':
/home/kense/.local/lib/python3.6/site-packages/ipykernel_launcher.py:14: Runt
imeWarning: invalid value encountered in double_scalars
```

Out[39]: `Text(0, 0.5, 'Gamma')`



The gamma graph shows that assert prices from the range 75~150 shows a higher fluctuation of delta values, meaning that the deltas are more prone to change in this range, and thus riskier.

Additional questions

Suppose the asset price is at $S_0 = 100$ and we sell 1000 puts.

- We want to be Δ-neutralized, which position should we have?
- Suppose the price at time $t + \epsilon$ is 105, what is the payoff of this strategy?
- What about 95?

In [24]:
```python
#First determine the delta at price 100, and the gamma
print(delta(100,120,0.05,1))
print(gamma(100,120,0.05,1))
```

```
-0.7128083620948729
0.017036921138505086
```

"We want to be Δ-neutralized, which position should we have?"

- Since the delta is -0.712 and we sell 1000 puts, to hedge this delta we need to sell $1000 * 0.712 = 712$ shares

"Suppose the price goes to 105"

-214.2727395978859

"What about 95?"

-209.5757230608392

In [47]:
```python
712*(100-105) + 1000*(17.3950083566465-14.049281096244385)
```

Out[47]: `-214.2727395978859`

In [48]:
```python
712*(100-95) + 1000*(17.3950083566465-21.16458407970734)
```

Out[48]: `-209.5757230608392`

Additional calculations to be Δ-neutral

- Since the gamma is at 0.017 that means for every dollar the price changes $(105 - 100 = 5)$ the delta changes by 0.017. This insinuates that the delta will change by $5 * 0.017 = 0.085$. That would cause the delta to now be at $-0.7128083620948729 + 0.085 = -0.62780836209$. That means that we would now only need to sell $1000 * 0.627 = 627$ shares, and we can buy back $712 - 627 = 85$ shares.
- Since the gamma is at 0.017, the delta will change by -0.085. That would cause the delta to now be at -0.79780836209. This means we would need to sell $1000 * 0.797 = 797$ shares, and we will need to sell additional $797 - 712 = 85$ shares.