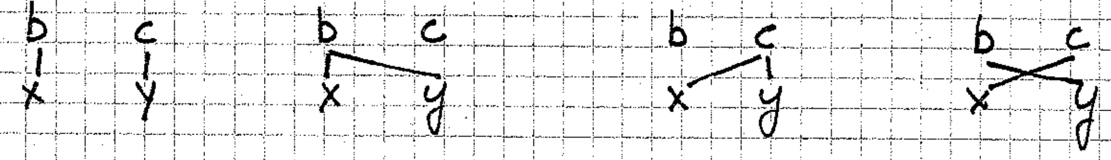
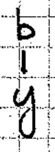
Solutions

Our corpus contains two sentence pairs

(1) b c   (2) b   We don't have
     x y   y   null words

(1a) Enumerate all the possible alignments

(1) 

(2) 

(1b) indicate all the possible translations defined by the alignments

$t(x \mid b)$ $t(x \mid c)$

$t(y \mid b)$ $t(y \mid c)$

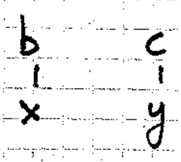
(1c) Assign a uniform distribution to the translations

$t(x \mid b) = 0.5$ $t(x \mid c) = 0.5$

$t(y \mid b) = 0.5$ $t(y \mid c) = 0.5$

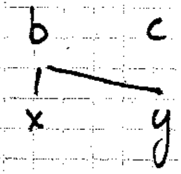
2. PROBABILITY OF ALIGNEMENTS

(2a)



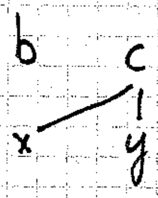
$t(x|b) \times t(y|c)$

$1/2 \cdot \frac{1}{2} = \frac{1}{4}$



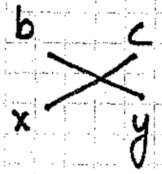
$t(x|b) \cdot t(y|b)$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$



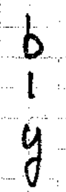
$t(x|c) \cdot t(y|c)$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$



$t(x|c) \cdot t(y|b)$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$



$t(y|b) = \frac{1}{2}$

(2b) Normalise to get the probability of the alignements

$$\begin{array}{cc} b & c \\ | & | \\ x & y \end{array} \qquad \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$$

$$\begin{array}{cc} b & c \\ | & \\ x & y \end{array} \qquad \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$$

$$\begin{array}{cc} b & c \\ & | \\ x & y \end{array} \qquad \frac{1}{4}$$

$$\begin{array}{cc} b & c \\ x & y \end{array} \qquad \frac{1}{4}$$

$$\begin{array}{c} b \\ | \\ y \end{array} \qquad \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

3 PROBABILITY OF TRANSLATION

(3a) Collect fractional counts

$t(x|b) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $t(x|c) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$t(y|b) = \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$ $t(y|c) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(3b) Normaliser

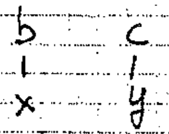
$$t(x|b) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{2}} = \frac{1}{4}$$

$$t(y|b) = \frac{\frac{3}{2}}{\frac{1}{2} + \frac{3}{2}} = \frac{3}{4}$$

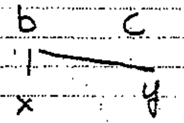
$$t(x|c) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

$$t(y|c) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

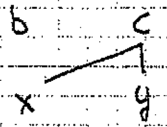
REPEAT STEP 2 COMPUTE PROBABILITY FOR ALL ALIGNEMENTS



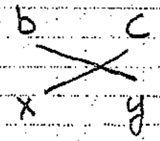
$$t(x|b)\cdot t(y|c) = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$$



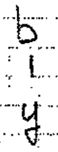
$$= t(x|b)\cdot t(y|b) = \frac{1}{4}\cdot\frac{3}{4} = \frac{3}{16}$$



$$t(x|c)\cdot t(y|c) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

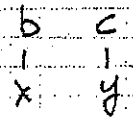


$$= t(x|c)\cdot t(y|b) = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$$

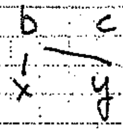


$$= t(y|b) = \frac{3}{4}$$

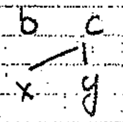
REPEAT 2' NORMALIZE



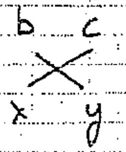
$$= \frac{\frac{1}{8}}{\frac{1}{8}+\frac{3}{16}+\frac{1}{4}+\frac{3}{8}} = \frac{1/8}{2+3+4+6/16} = \frac{2}{15}$$



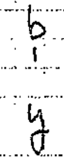
$$= \frac{3/16}{15/16} = \frac{3}{15} = \frac{1}{5}$$



$$= \frac{1/4}{15/16} = \frac{4}{15}$$



$$= \frac{3/8}{15/16} = \frac{6}{15}$$



$$= \frac{3/4}{3/4} = 1$$

REPEAT STEP 3 COLLECT FRACTIONAL COUNTS

$$t(x|b) = \frac{2}{15} + \frac{3}{15} = \frac{5}{15} \left(= \frac{1}{3}\right)$$

$$t(y|b) = \frac{3}{15} + \frac{6}{15} + 1 = \frac{24}{15}$$

$$t(x|c) = \frac{4}{15} + \frac{6}{15} = \frac{10}{15} \left(= \frac{2}{3}\right)$$

$$t(y|c) = \frac{2}{15} + \frac{4}{15} = \frac{6}{15} \left(= \frac{2}{5}\right)$$

REPEAT STEP 3' RENORMALIZE

$$t(x|b) = \frac{5/15}{5/15 + 24/15} = \frac{5}{29}$$

$$t(y|b) = \frac{24/15}{5/15 + 24/15} = \frac{24}{29}$$

$$t(x|c) = \frac{10/15}{10/15 + 6/15} = \frac{10}{16}$$

$$t(y|c) = \frac{6/15}{10/15 + 6/15} = \frac{6}{16}$$

REPEAT STEP 2 FOR ALL ALIGNEMENTS

$\begin{array}{cc} b & c \\ | & | \\ x & y \end{array}$ $\quad t(x|b)\cdot t(y|c)$

$$\frac{1}{3}\cdot\frac{2}{5}=\frac{2}{15}$$

$\begin{array}{cc} b & c \\ x & y \end{array}$ $\quad t(x|b)\cdot t(y|c)$

$$\frac{1}{3}\cdot\frac{24}{29}=\frac{8}{29}$$

$\begin{array}{cc} b & c \\ x & y \end{array}$ $\quad t(x|c)\cdot t(y|c)$

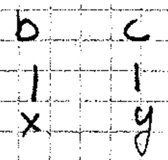
$$\frac{5}{8}\cdot\frac{3}{8}=\frac{15}{64}$$

$\begin{array}{cc} b & c \\ x & y \end{array}$ $\quad t(x|c)\cdot t(y|b)$

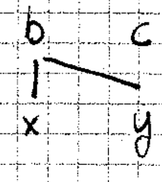
$$\frac{5}{\cancel{8}_{1}}\cdot\frac{\cancel{24}^{3}}{29}=\frac{15}{29}$$

$\begin{array}{c} b \\ | \\ y \end{array}$ $\quad t(y|b)=\frac{24}{29}$

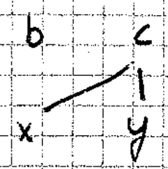
NORMALIZE



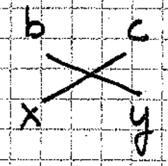
$$\frac{\frac{2}{15}}{\frac{2}{15}+\frac{8}{29}+\frac{15}{64}+\frac{15}{29}} = 0.115$$



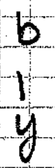
$$\frac{\frac{8}{29}}{\frac{2}{15}+\frac{8}{29}+\frac{15}{64}+\frac{15}{29}} = 0.24$$



$$\frac{15/64}{2/15+8/29+15/64+\frac{15}{29}} = 0.20$$



$$\frac{15/29}{\frac{2}{15}+\frac{8}{29}+\frac{15}{64}+\frac{15}{29}} = 0.46$$



$$\frac{24/29}{24/29} = 1$$

So summing up, if we look at the development of the values of the parameters of the model, this is what we find.

Lexical Probabilities				Alignment Probabilities				
$t(x\|b)$	$t(y\|b)$	$t(x\|c)$	$t(y\|c)$	$\begin{smallmatrix} b & c \\ x & y \end{smallmatrix}$	$\begin{smallmatrix} b & c \\ x & y \end{smallmatrix}$	$\begin{smallmatrix} b & c \\ x & y \end{smallmatrix}$	$\begin{smallmatrix} b & c \\ x & y \end{smallmatrix}$	$\begin{smallmatrix} b \\ y \end{smallmatrix}$
0.5	0.5	0.5	0.5	0.25	0.25	0.25	0.25	1
0.25	0.75	0.5	0.5	0.13	0.20	0.27	0.40	1
0.17	0.83	0.63	0.37	0.115	0.240	0.20	0.46	1

If we continued for a while we would end up with

$t(x|b) = 0.0001$ $t(y|b) = 0.9999$ $t(x|c) = 0.9999$ $t(y|b) = 0.0001$

That is, in the course of time, the lexical translation probabilities of the same source word diverge to a stable state.

What has happened?

The crossing aligments $\begin{smallmatrix} b & c \\ x & y \end{smallmatrix}$ was boosted

by the certain aligmement $\begin{smallmatrix} b \\ y \end{smallmatrix}$.

They both contribute to $t(y|b)$, which is therefore increased. This, in the course of time, decreases $t(y|c)$. But $t(y|c)$ and $t(x|c)$ must sum to 1, so $t(x|c)$ is increased. Most of the probability is assigned to the crossing aligmement in the end.