Ken Sekimoto (Room 839A)
cf. I'm absent during 7-16 nov. for a conference.
Contact: ken.sekimoto@espci.fr

Proposition: MC & DC

Martingale process in *Progressive Quenching*

Background — Martingale

A *stochastic process* $\{m_1, m_2, \ldots, m_n\}$ is *martingale*
with respect to the *stochastic process* $\{X_1, X_2, \ldots, X_n\}$

$\leftrightarrow$ $E[\, m_{t+1} \mid \{X_1, X_2, \ldots, X_t\}\,] = m_t$ $\quad$ $E[x \mid y]$: conditional expectation

An example : $m = x$ = random walk

Martingale is one of the central concepts in the probability theory.
BUT in **physics**, its meaning & importance beyond random walk
was little known until very recently.

Recent major discoveries

1. So-called ***fluctuation theorems*** *(there are many versions) are recognized to be* *exponential martingale* *of the "entropy production".* (Cheterite & Gupta: 2011, Roldan *et al., 2017)*

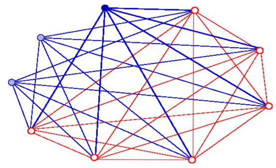
2. *In the protocol of* **progressive quenching** (see right →) non-trivial martingale emerges. (Ventéjou-KS, Etienne-KS: 2018)

More about stochastic processes → **M2 Course in 2nd semester (Ken Sekimoto)**

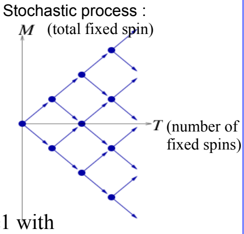
Example (Ventéjou & KS, 2018)

Ising spin model on a *complete lattice* .

$$\beta H_{N,J,h} = -\beta J \sum_{1\le i<j\le N} s_i s_j - \beta h \sum_{1\le i\le N} s_i \qquad \beta = 1, \quad J = \frac{j_0}{N_0}$$



Progressive Quenching:
(t:q)-step: quench *t*-th spin
(t:r)-step: equilibrate (*N–t*) spins
(t+1:q)-step: … repeat until *N* fixed



At each (*t*:q)-step, *t* -th spin s_t takes ± 1 with
$\langle s_t \rangle = m_t$: equilibrium mean spin → **martingale**

Utility of martingale ~ *Stochastic conservation* *(Memory is kept & updated.)*

(1) Statistical ensemble of the *final* quenched magnetization : $M_N = \sum_{t=1}^{N} s_t$
(2) Information : " M_T is common " (1 < *T* < *N*)
(3) Martingale "optional stopping" theorem

→ Inference of *MT* value up to O(1) with ~*N*^2 steps. (not ~exp(c*N*))

Proposition of Master training ("stage")

Object: How is the breaking of *martingale* associated to space/time structure ?

Model studied : 1D spin systems, ***p***-nn spin coupling, ***q***-spin simultaneous quenching
Known fact: Progressive quenching of 1nn & 2nn systems (nn = nearest neighbor)

Exploitation :

New : 1D systems with ***long range interaction*** : $J_{i,j} \sim |i-j|^{(-q)}$ → martingale ? $\quad$ **Method** : Transfer matrix method
New : 1D systems with quenching at ***finite rate*** → martingale ? $\quad$ **Method** : Glauber dynamics

Preparation for DC :
Martingale and other advanced stochastic methods.
Information stochastic thermodynamics.
Network dynamics (cavity, random network, first passage), etc. → **M2 Course in 2nd semester (Ken Sekimoto)**

Proposition of PhD thesis

(Conceptual)
Martingale property as *stochastic conservation in Physics*

— ***Infortmation thermodynamic approach***

* **Progressive Quenching**
= measuring partial information of system
+ measurement-dependent feed-back control

* **Particularity of Progressive Quenching**
→ System size / Degrees of freedom are time dependent

— ***Symmetry principle of stochastic conservation***

(Theoretical+numerical)
Martingale property on networks under Progressive Quenching

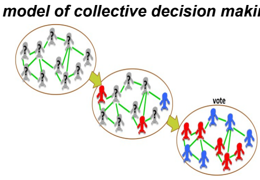
— ***On Bethe tree, factor graph*** **(cavity approx'n)**
→ (Breaking of) martingale vs coordination nb

* **Progressive Quenching**
= isolating branches
+ freezing effective field

— **On complete network +** ***thermalization at finite rate***
→ (Breaking of) martingale /inform'n analysys

(Application)
Progressive Quenching in Sociological / Neurological contextes

— ***model of collective decision making*** **(e.g. Brexit)**



— ***model of knowledge network formation***

B. Ventéjou, K. Sekimoto; Phys. Rev. E **97**, 062150 (2018)
M. Etienne, K. Sekimoto; Acta Physica Polonica, **49**, 883-892 (2018)
Stochastic Energetics, K. Sekimoto (Springer, Lecture Notes in Physics **799** (2010))

Read also, R. Chetrite and S. Gupta, J. Stat. Phys. 143, 543 (2011).
I. Neri, E. Roldán, and F. Jülicher, Phys. Rev. X 7, 011019 (2017).

Proposition MC & DC

Name: Ken Sekimoto (Prof. MSC, Univ Paris Diderot)
Room : 839A (I will be absent for 7th-16th Nov. for a conference)
Contact : ken.sekimoto@espci.fr
Title of project : Martingale process in Progressive Quenching

Background - Martingale in physics

A stochastic process {m_i} (i=1,...) is said to be "martingale" with respect to the stochastic process {X_i} (i=1,...) if the conditional expectation/average satisfies the property: E[*m*_{i+1} | X_1,..., X_n]= *m*_i. An example is *m*=X=unbiased random walk.
The martingale is among the central concepts in the probability theory. However, its meaning and importance in physics beyond the random walk was little known until very recently.

Recent major discoveries

1. So-called fluctuation theorems (there are many versions) are recognized to be the "exponential martingale of entropy production" through the viewpoint of "Radon-Nikodym derivative" (refs. Cheterite and Gupta, J Stat Phys 2011, Roldan et al. Phys Rev X 2017).
2. In the protocol of Progressive Quenching (see below), the non-trivial martingale emerges. (ref. B. Ventéjou and KS, Phys Rev E 2018).
(More about the stochastic processes will be addressed in my M2 course "Stochastic Dynamics and Energetics" in 2nd semester)

Progressive Quench

This is a category of non-equilibrium operation on the system in which the system's grees of freedom are progressively fixed under externally given protocol. For example, in the complete network of spins we quench the spin's state one after another leaving enough time interval so that the unquenched spins can thermalize before quench.

Martingale in Physics as Stochastic Conservation

In physics, the martingale means a kind of "stochastic conservation".
In the above example the mean equilibrium value of unquenched spin is conserved on the average. Unlike the deterministic conservation laws, the conserved value can be updated upon getting the information of the system's actual state.

Utility of martingale

As the deterministic conservation laws play important roles in physics, the stochastic conservation allows the "inference of the past state", with highly reduced algorithmic load (order of *N*^2 calculation instead of ~ exp(*c N*) calculateion, where *N* is the system size).

Project for Master "stage"

Object: "How is the breaking of martingale associated to the space-time structure ?"
Using the 1D spin systems up to p-nearest neighbor interaction and q-spins simultaneous quench, we search the statistical properties of the quenched spin ensemble. (In 1D the martingale is evident ($\langle s_i \rangle=0$) but the correlation matters ($\langle s_i\, s_j \rangle$ vs $|i-j|$)
Especially we exploit the case of (i) Spin chainn interacting with long-range coupling, (J_{i,j}~ |i-j|^(-q)), and of (ii) Spin chain quenched at finite rate (incomplete thermalization).
In the meantime the student is expected to prepare for PhD, studying the theory of martingale as well as general nonequilibrium statistical physics, information theory in relation to thermodynamics, theory of network dynamics (cavity, random network, first passage ...). An important part will be covered in my M2 course (see above).

Project for PhD

1. Conceptual studies

1-i. Establish the information theoretical basis of the stochastic conservation.
The progressive quenching is a combination of "measuring" and "feed-back operation". Unlike the ordinary information physics, the system's degrees of freedom changes during the process.

1-ii. Origin of stochastic conservation.
Like the deterministic conservation laws, we seek for underlying invariance/symmetry.

2. Theoretical and numerical

2-i. Progressive quenching on the network.
Bethe tree, loopy sparse random network. Thermal or abstract (factor graph)

2-ii. On the complete network, with finite rate thermalization.

3. Application

3-i. Model of collective decision making (e.g. Brexit!)

3-ii. Model of knowledge network formation.

References

B. Ventéjou, K. Sekimoto; Phys. Rev. E 97, 062150 (2018)
M. Etienne, K. Sekimoto; Acta Physica Polonica, 49, 883-892 (2018)
K. Sekimoto; *Stochastic Energetics* (book of Springer, Lecture Notes in Physics **799** (2010))

Announcement : **Optional course** of **2nd semester** of Master **PCS**

Course title : ***Stochastic Dynamics and Energetics***

https://physics-complex-systems.fr/stochastic-dynamics-and-energetics.html

Lecturer : Ken Sekimoto (prof. Univ. Paris Diderot and Gulliver, ESPCI — *homepage*)

Place : Jussieu Campus (Room: to be announced)

Dates : Mon. 14-16h, Fri. 11-13h (Start : Mon. 14 Jan*?*, End : Fri. 1st Mar.)

For more information, see the ***Tentative Syllabus*** (separate sheet : Under construction)

homepage: https://www.pct.espci.fr/~sekimoto/sekimoto_hp_espci.html