Tentative Syllabus : ***Stochastic Dynamics and Energetics***

https://physics-complex-systems.fr/stochastic-dynamics-and-energetics.html

Type: Optional course of 2nd semester of Master PCS

Lecturer : Ken Sekimoto (prof. Univ. Paris Diderot and Gulliver, ESPCI - *homepage*)

Place : **Jussieu** Campus (Room: not yet announced)

Dates : Mon. 14-16h & Fri. 11-13h (Start : Mon. 14 Jan (?), End : Fri. 1st Mar.)

Object :
This course addresses advanced concepts and methods of stochastic processes. While we will discuss also some particular hot topics, the main aim is to learn a variety of methods of wide range of applicability.

Features of this Course :
1) We put an emphasis on the analysis of the individual processes/paths relative to the ensemble calculations.
(ex. Temporal coarse graining along single trajctory; First law of thermodynamics along single trajectory.)

2) We also go to-and-fro between mathematician's approaches and physicists' conceptions.
(ex. Conditional average on sigma-albegra vs coarse-graining; Optional stopping theorerm (martingale) vs stochastic conservation.)

3) I will try to make the course more complementary than parallel to Frederic van Wijland's course, which is also recommended !

Course requirements :
— It is better for you to have acquired the basics of the probability, such as the independence, central limit theorem, and preferably the master equation and the Langevin equation, although they will be redefined in the course.

— The participation to the entire course is highly recommended because the small excersices are important part of the course.

How will the course proceed ?
— A small exercise is given at the end of Monday's lecture, for the better comprehension of the course.

— On Friday, the excercise is gathered first and commented. At the end of lecture a weekly summary is handed out.

— The course note will be **Max[** final written exam, (final written exam + participation to homeworks)/2 **]**.

Tentative Syllabus (Under preparation. Apparently I must prune some.)

Warming up lecture:
Posing questions-paradoxes

Mathematician's view and/or physicists' view (... on the shoulder of Kolmogolov)
1. Confidence interval as a random variable of statistical inference.
2. Hidden postulate in the naive error analysis.
3. Conditional average as Mathematician's view of coarse-graining.

Physical kinetic approach - Diagnostic of the heat bath:
1. Disillusion of the Adiabatic piston,
2. Gas-kinetic limit of Gaussian white random force and frictional force. "Protein friction"
3. Frontier of post-Brownian motion in real experiments. "Basset terms" and true equipartition.
4. Exactly solvable model of non-local heat bath.

Stochastic processes with jumps
1. Transition network. — example: Nucleation, Enhanced kinetic sampling, Dynamic "Mpemba".
2. Gillespie algorithm, its relation to the *lossless* reconstruction of empirical probability distribution.
3. *Distribution* of the First Passage Time
4. Stable distributions other than Gaussian one : Lévy's flight.

Advanced concepts in probabilities
1. Sanov's theorem and equi-partition.
2. Implication of *stochastic* Kullback-Leibler divergence / mutual information.
3. Iterative improvement of Beyesian inference - Bialek's works
4. Large Deviation Principle (LDP), generating function(al) and Legendre transformation.
5. Constrained Brownian motions - (i) Brownian bridge. (ii) Fluctuating hydrodynamics as Langevin eq'n.
6. Browian motion and Potential problem. Feynman-Kac formula.

Hierarchy of stochastic processes.
1. Fluctuation-Dissipation relation of 1st kind and 2nd kind. Heat bath as *a system*.
2. Stochastic thermodynamics (heat/entropy, energy, work) at different scales.
3. Overdamping catastroph of Büttiker-Landauer heat engine.
4. Hidden entropy production.
5. Information machine with measurement-feedback module, "Detailed information loss".

Martingale processes in Physics
1. Doob-Mayer decomposition and generalized Itô Stochastic Differential Equation.
2. Grisanov theorem,Wald's martingale, Doob's inequality.
3. Radon-Nikodym derivative and the unified view of Fluctuation Theorems.
4. Martingale as *Stochastic Conservation* - case of Progressive Quenching
5. Physical meaning of optional stopping theorem. Doob's inequality.