

Introduction to Computer Graphics

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Last Time

- Introduction to ray tracing
- Intersection algorithms with a ray
 - Sphere
 - Implicit surface
 - Triangle
- GLSL Sandbox

Last Time

```
for all pixels {  
    ray = generate_camera_ray( pixel )  
    for all objects {  
        hit = intersect( ray, object )  
        if "hit" is closer than "first_hit" {first_hit = hit}  
    }  
    pixel = shade( first_hit )  
}
```

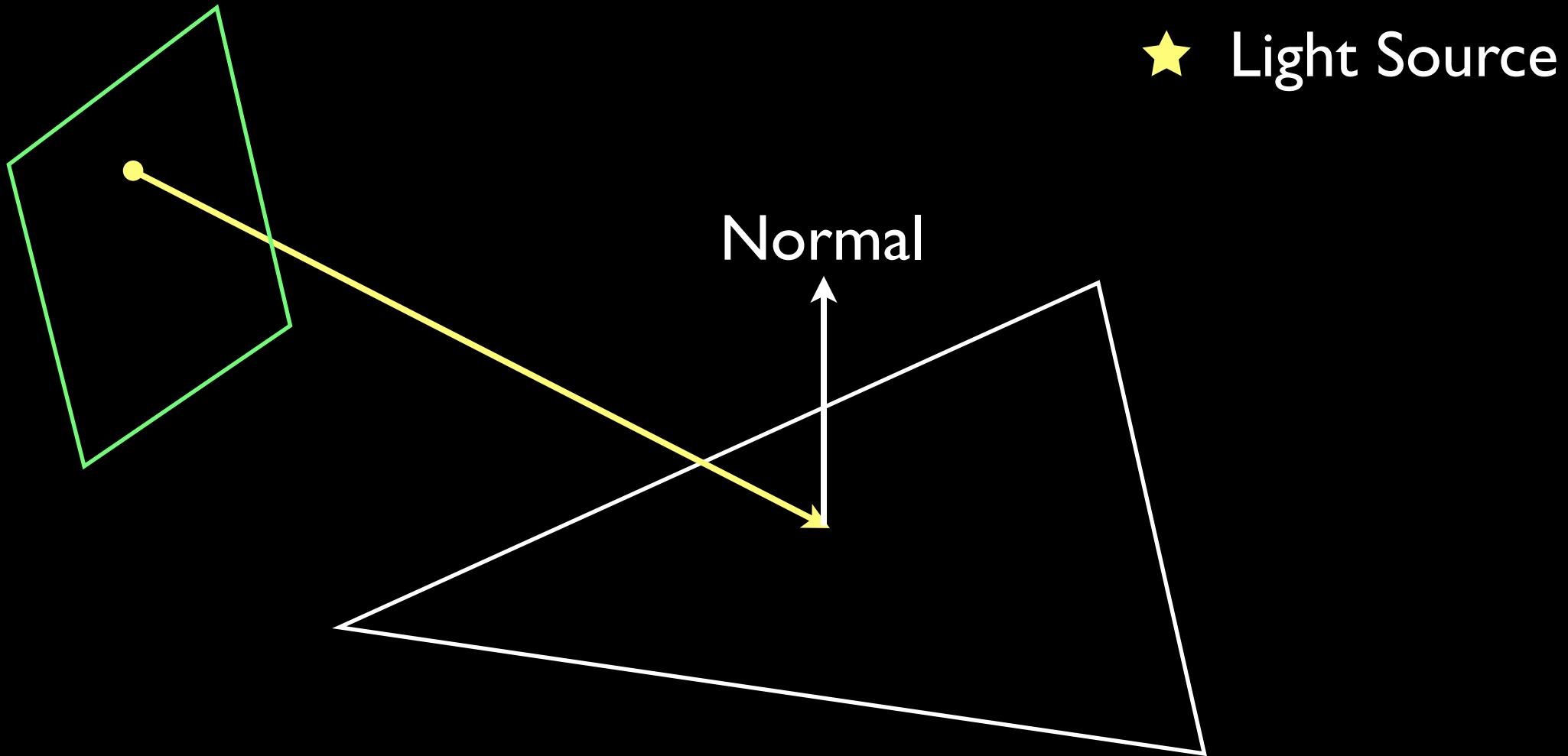
Today

```
for all pixels {  
    ray = generate_camera_ray( pixel )  
    for all objects {  
        hit = intersect( ray, object )  
        if "hit" is closer than "first_hit" {first_hit = hit}  
    }  
    pixel = shade( first_hit )  
}
```

Today

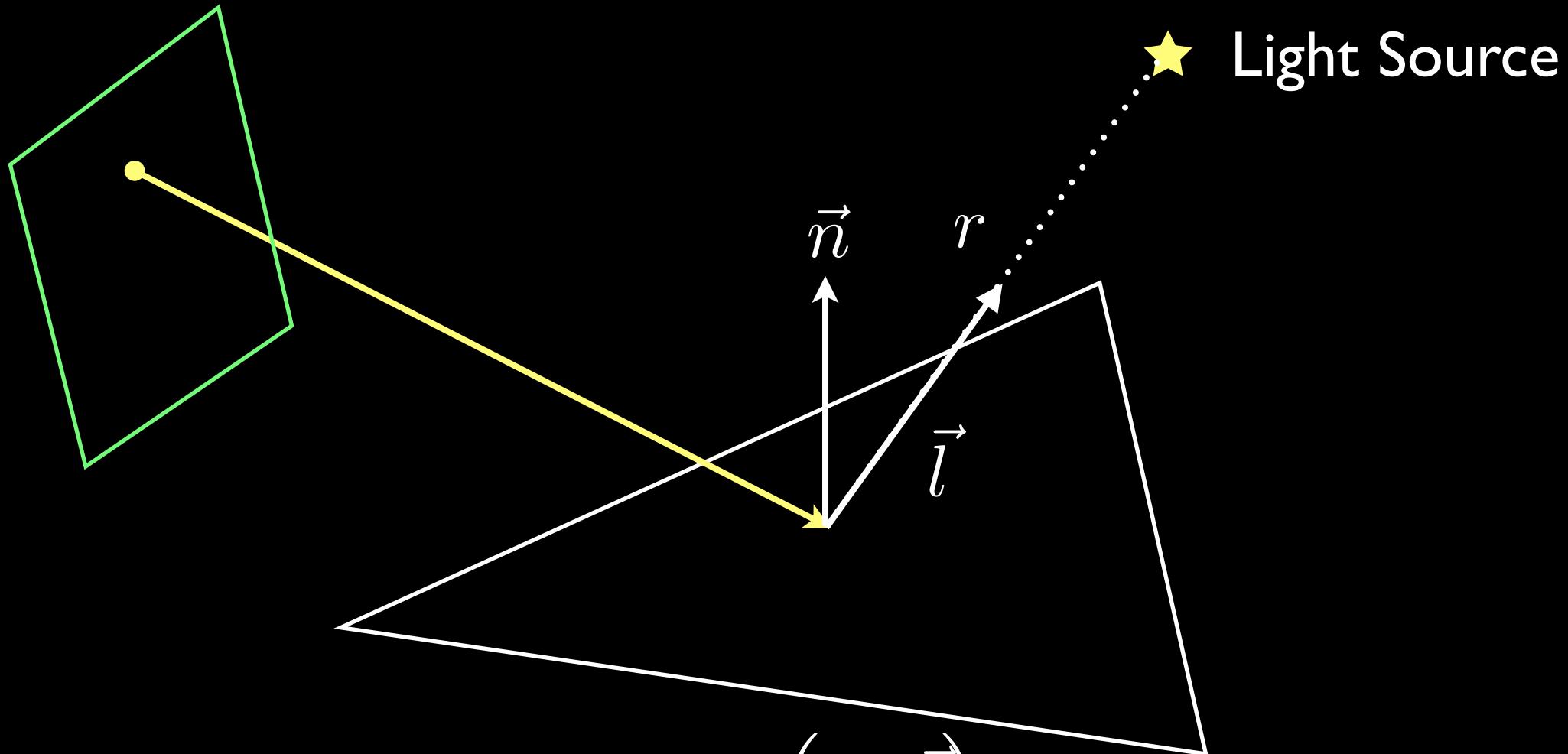
- Shading models
 - BRDF
 - Lambertian
 - Specular
- Simple lighting calculation
- Tone mapping

Basic Shading



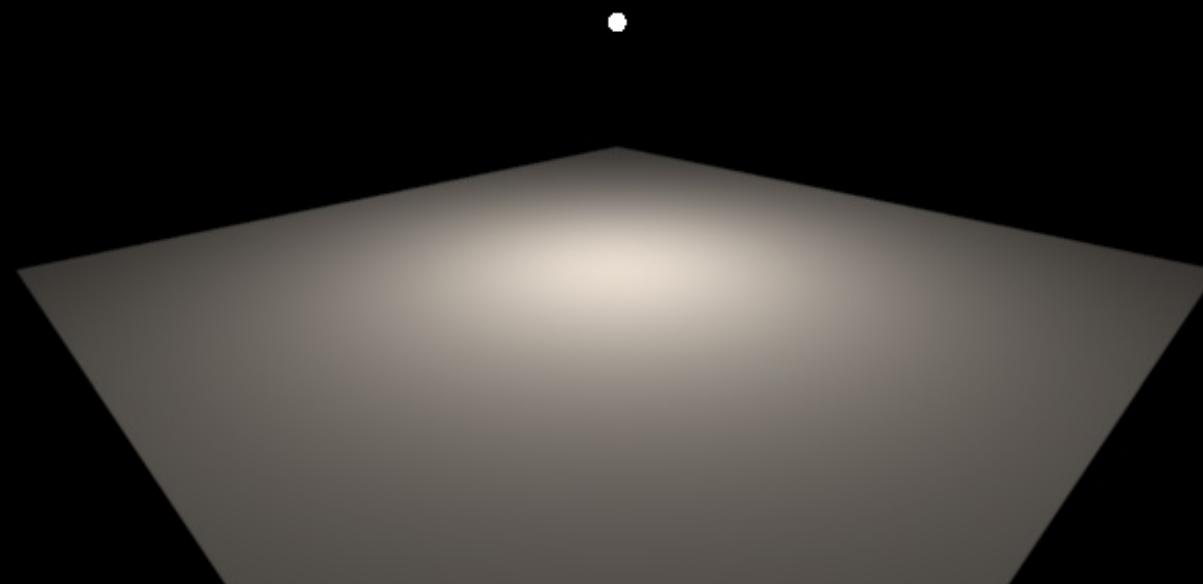
How much light illuminate this point?

Basic Shading



$$\text{Irradiance } E = \frac{\Phi(\vec{n} \cdot \vec{l})}{4\pi r^2}$$

Basic Shading



$$\text{Irradiance } E = \frac{\Phi(\vec{n} \cdot \vec{l})}{4\pi r^2}$$

Surface Appearance

- How is light reflected by
 - Mirror
 - Paper
 - Rough metallic surface



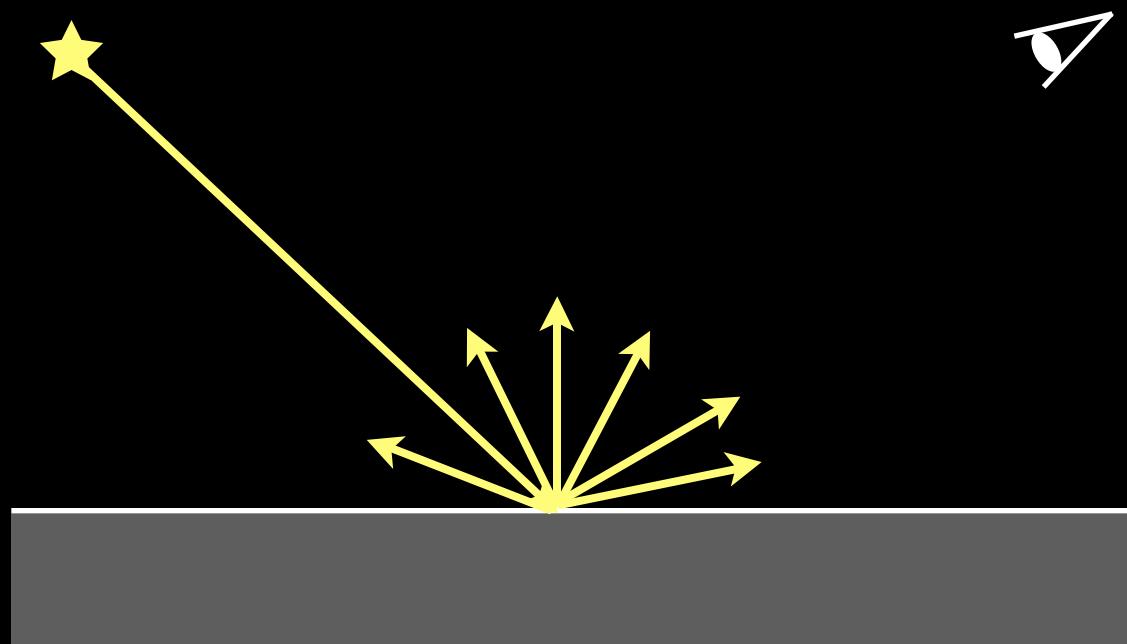
Types of Reflections

- Diffuse: matte paint, paper
- Glossy: plastic, rough metallic surface
- Specular: mirror
- Retro-reflective: the moon



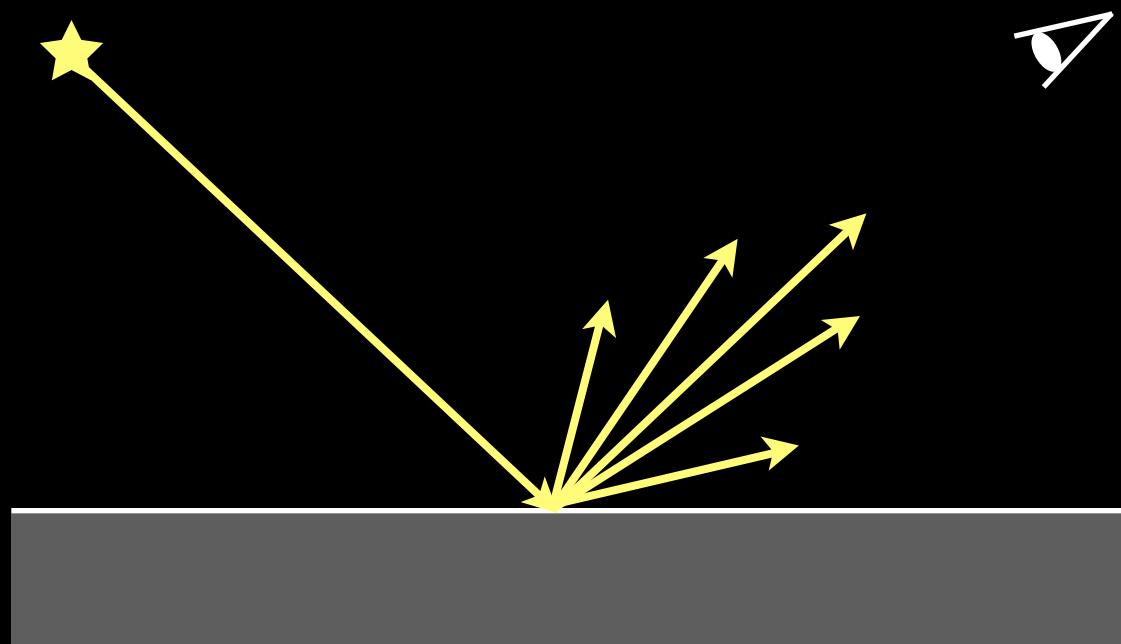
BRDF

- Bidirectional Reflectance Distribution Function
 - How light is reflected off a surface
 - Capture appearance of surface

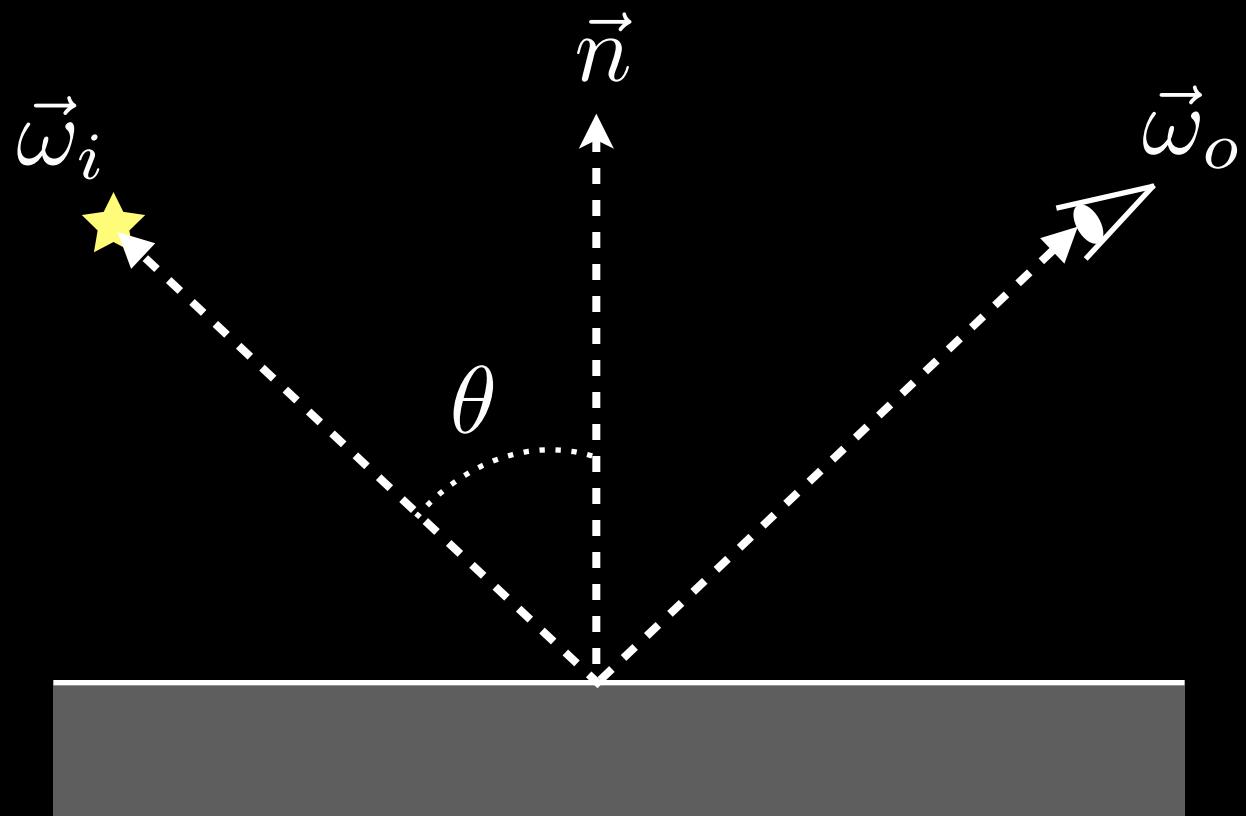


BRDF

- Bidirectional Reflectance Distribution Function
 - How light is reflected off a surface
 - Capture appearance of surface



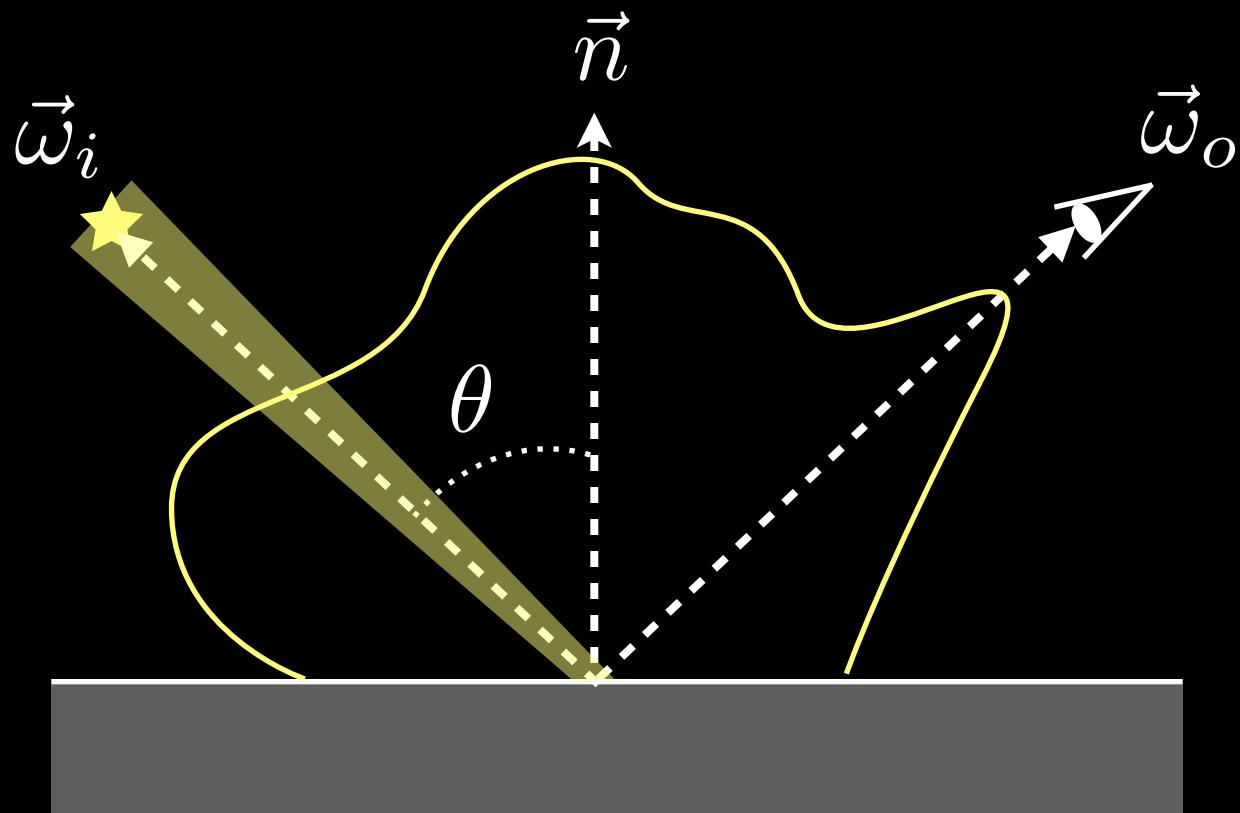
BRDF



BRDF

$$f(x, \vec{\omega}_o, \vec{\omega}_i) = \frac{dL(x, \vec{\omega}_o)}{dE(x, \vec{\omega}_i)}$$

$$dE(x, \vec{\omega}_i) = L(x, \vec{\omega}_i) \cos \theta d\omega_i$$



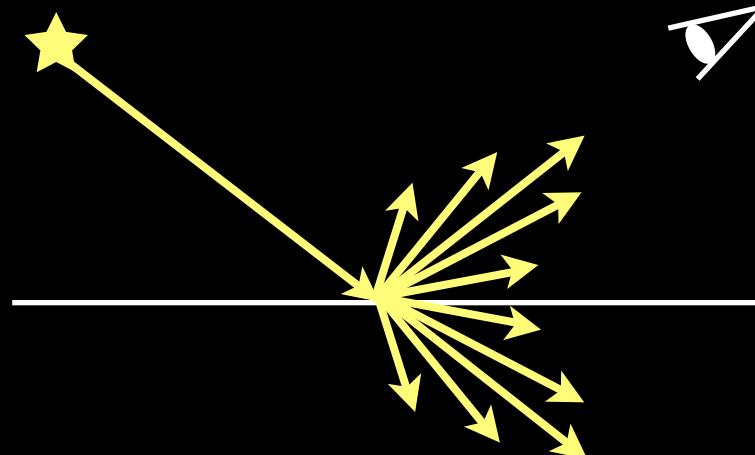
Reflected Radiance

- Sum of contributions from all directions

$$\begin{aligned} L_o(x, \vec{\omega}_o) &= \int_{\Omega} dL_o(x, \vec{\omega}_o) \\ &= \int_{\Omega} f(x, \vec{\omega}_o, \vec{\omega}_i) dE_i(x, \vec{\omega}_i) \\ &= \int_{\Omega} f(x, \vec{\omega}_o, \vec{\omega}_i) L_i(x, \vec{\omega}_i) \cos \theta d\omega_i \end{aligned}$$

BSDF

- Bidirectional Scattering Distribution Function
 - Generalization of BRDF to transparency
 - Defined over all the directions



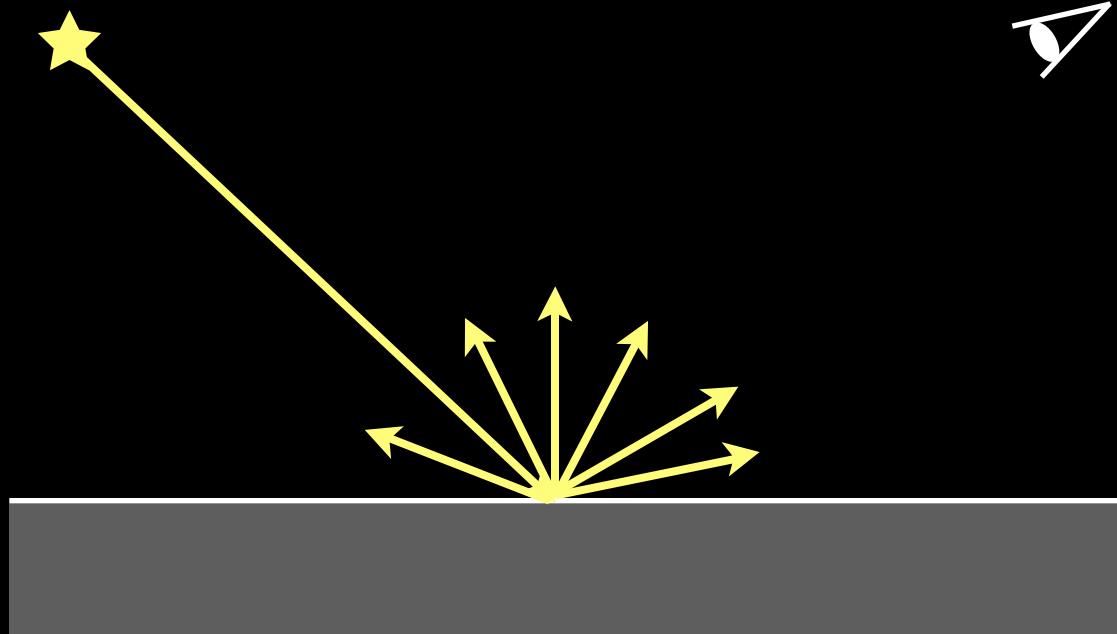
Defining BRDF

- Two approaches
 - Measurement
 - Analytical models
- We learn simple analytical models

Lambertian

- Uniformly reflects light over all directions
- “Matte” surfaces

$$f(\vec{\omega}_o, \vec{\omega}_i) = \frac{K_d}{\pi}$$



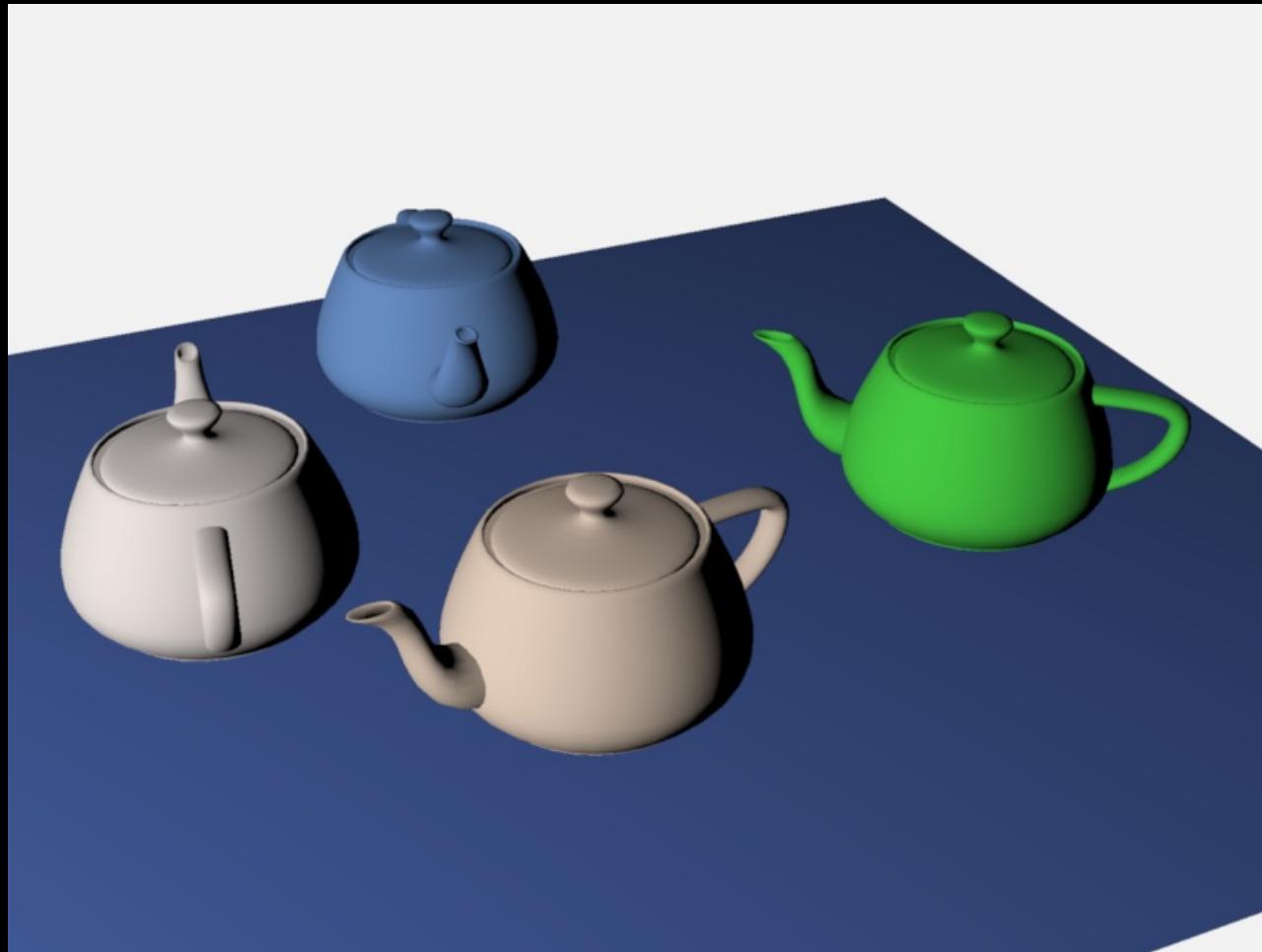
Lambertian

- Irradiance times BRDF = Lambertian shading

$$\begin{aligned} L_o(x, \vec{\omega}_o) &= \int_{\Omega} dL_o(x, \vec{\omega}_o) \\ &= \int_{\Omega} f(x, \vec{\omega}_o, \vec{\omega}_i) dE_i(x, \vec{\omega}_i) \\ &= \frac{K_d}{\pi} \int_{\Omega} dE_i(x, \vec{\omega}_i) = \frac{K_d}{\pi} E_i \end{aligned}$$

```
color shade (hit) {  
    return (Kd / PI) * get_irradiance(hit)  
}
```

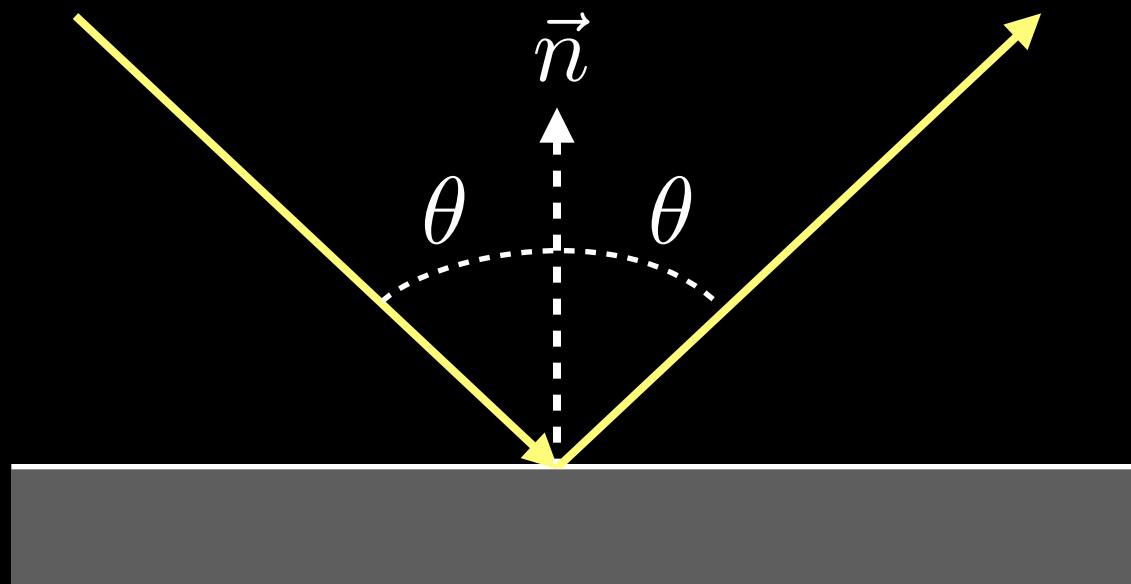
Lambertian - Example



Specular Reflection

- Mirror reflection

$$f(\vec{\omega}_o, \vec{\omega}_i) = \frac{K_s \delta(\vec{\omega}_o, \vec{\omega}_r)}{\cos \theta}$$



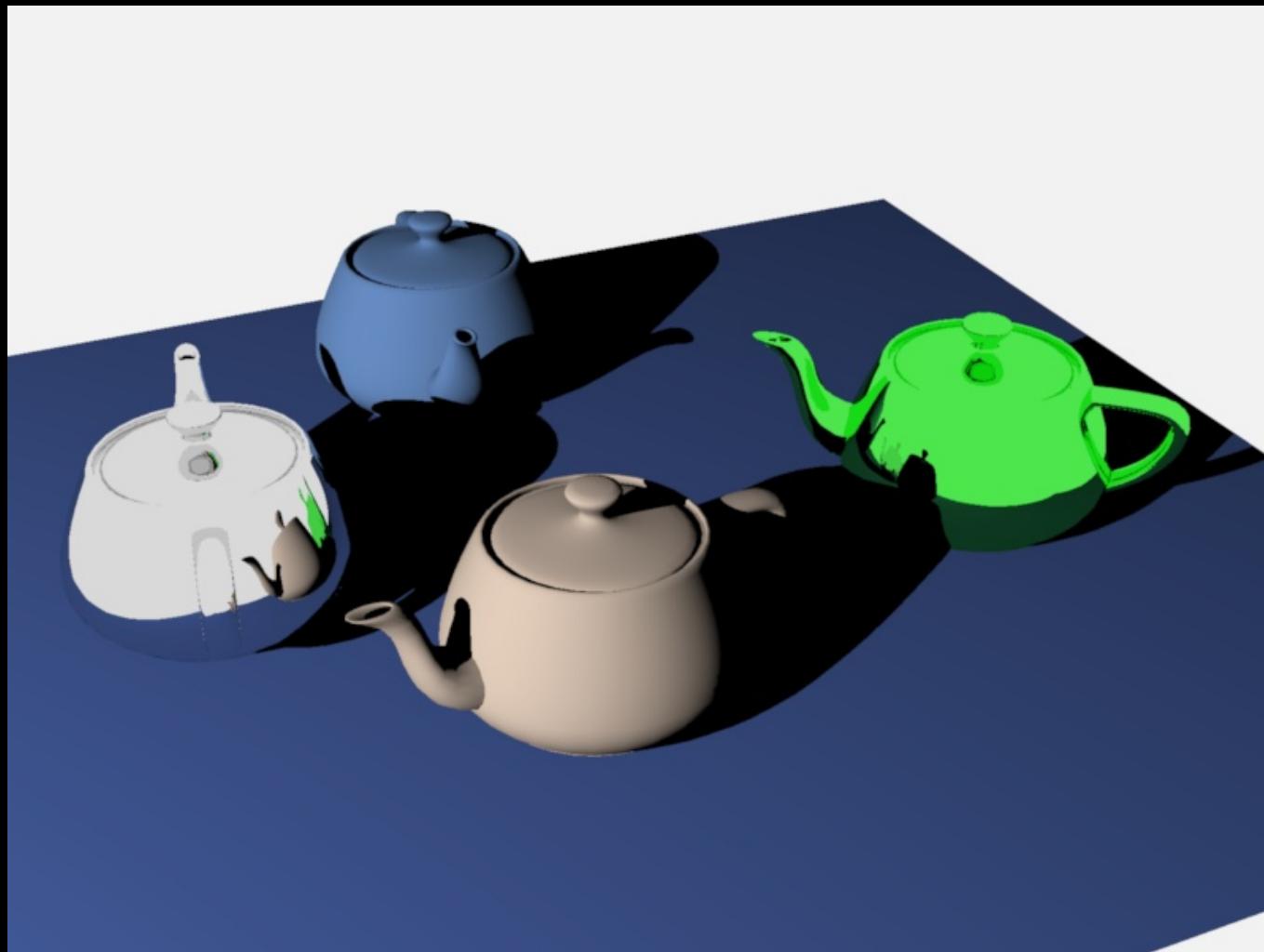
Specular Reflection

- Recursive tracing toward the reflected dir.

$$\vec{\omega}_r = -2(\vec{\omega}_i \cdot \vec{n})\vec{n} + \vec{\omega}_i$$

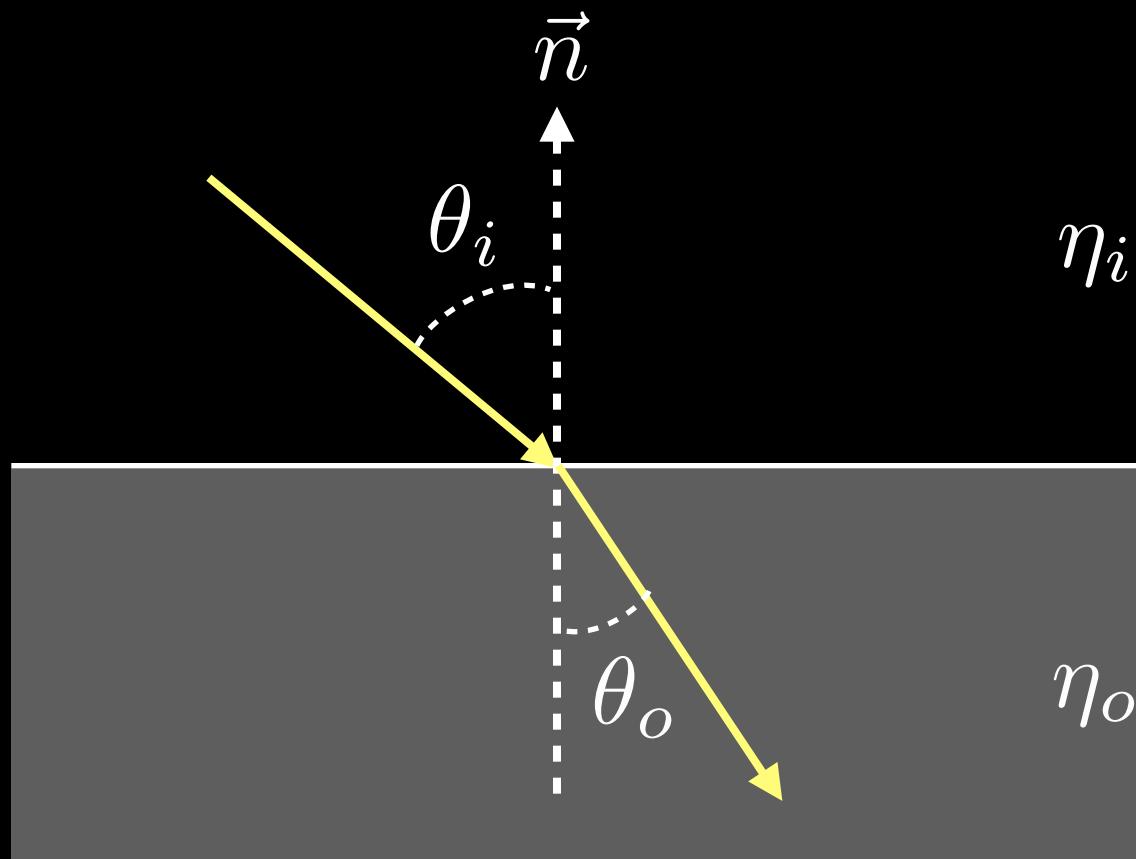
```
color shade (hit) {  
    case hit.material  
        diffuse:  
            return (Kd / PI) * get_irradiance(hit)  
        mirror:  
            return Ks * shade(trace(reflected_ray))  
}
```

Specular Reflection - Example



Specular Refraction

- Glass etc. $f(\vec{\omega}_o, \vec{\omega}_i) = \frac{K_t \delta(\vec{\omega}_o, \vec{\omega}_t)}{\cos \theta_i}$



Specular Refraction

- Same as reflection: recursive tracing

```
color shade (hit) {
```

```
    case hit.material
```

```
        diffuse:
```

```
            return (Kd / PI) * get_irradiance(hit)
```

```
        mirror:
```

```
            return Ks * shade(trace(reflected_ray))
```

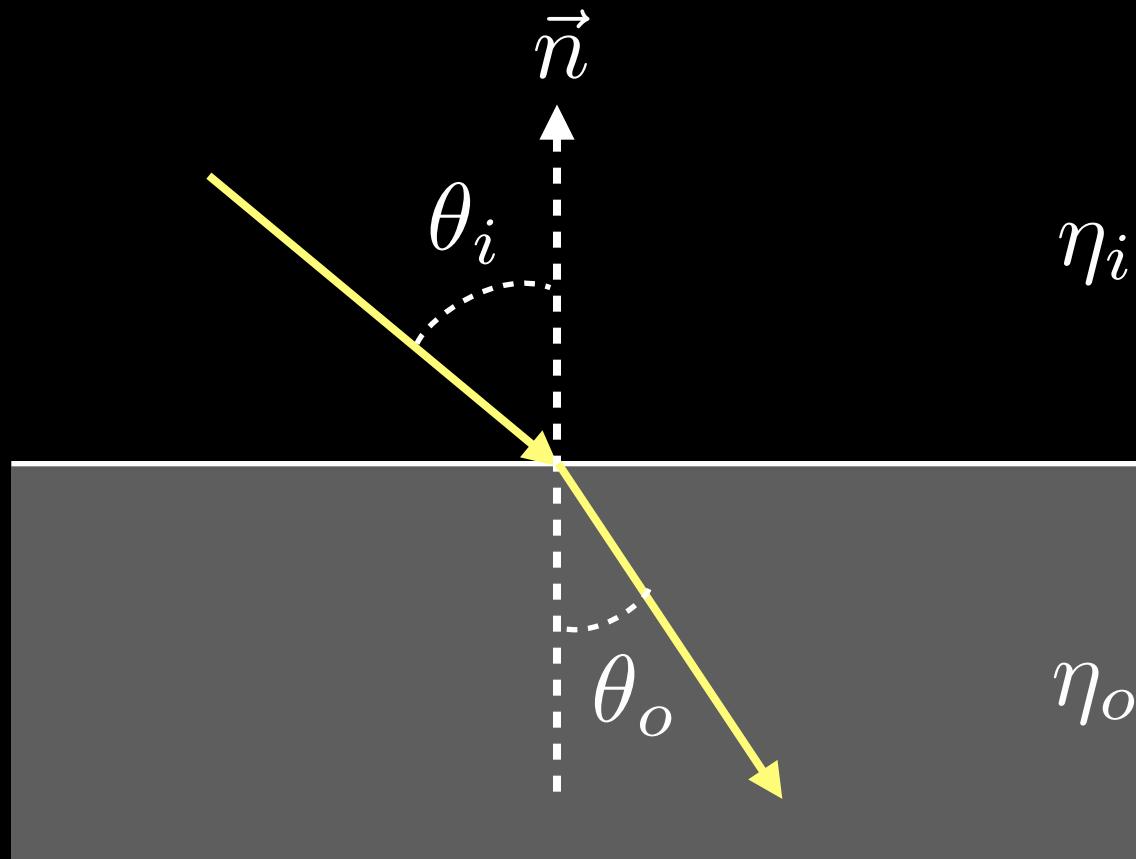
```
        glass:
```

```
            return Kt * shade(trace(refracted_ray))
```

```
}
```

Snell's Law

$$\eta_i \sin \theta_i = \eta_o \sin \theta_o$$

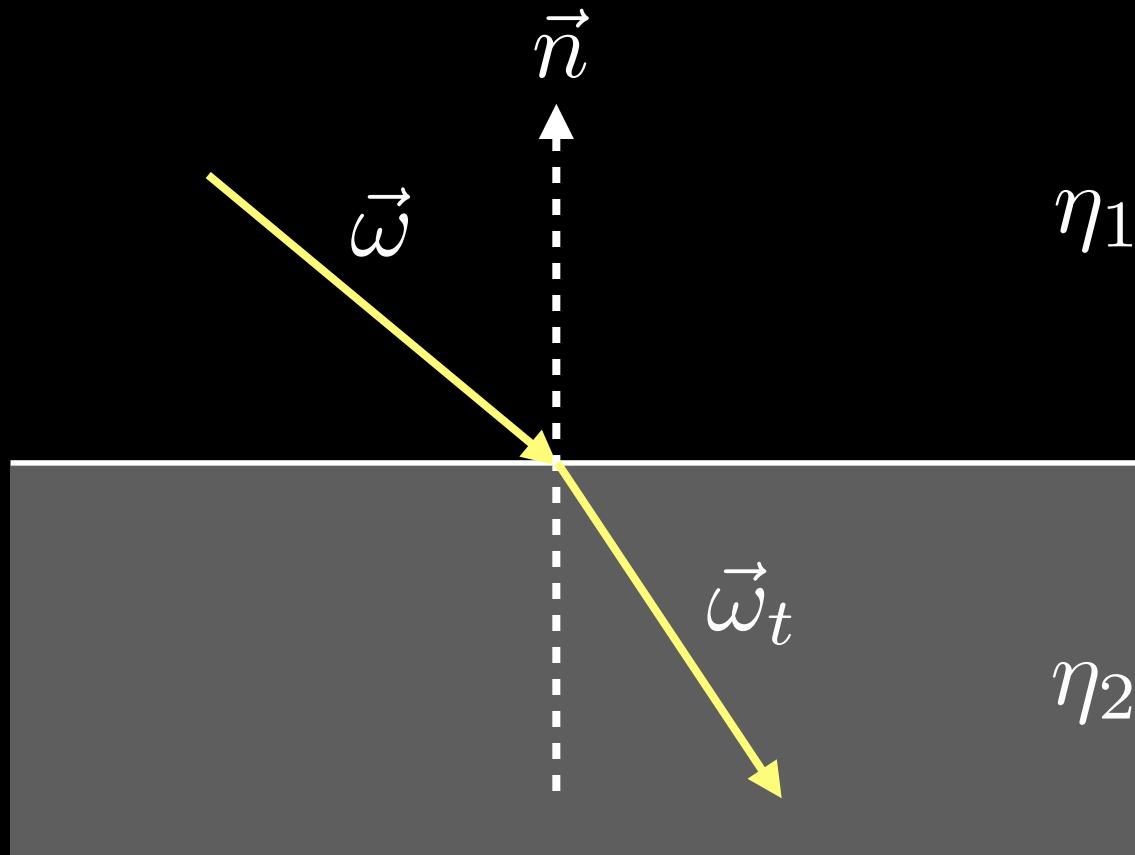


Index of Refraction

- Some values of η

Vacuum	1.0
Air	1.00029
Ice	1.31
Water	1.33
Crown glass	1.52 - 1.65
Diamond	2.417

Refracted Direction



$$\vec{\omega}_t = \frac{\eta_1}{\eta_2} (\vec{\omega} - (\vec{\omega} \cdot \vec{n}) \vec{n}) - \left(\sqrt{1 - \left(\frac{\eta_1}{\eta_2} \right)^2 (1 - (\vec{\omega} \cdot \vec{n})^2)} \right) \vec{n}$$

Refracted Direction

Be aware of what object ray is entering and existing!

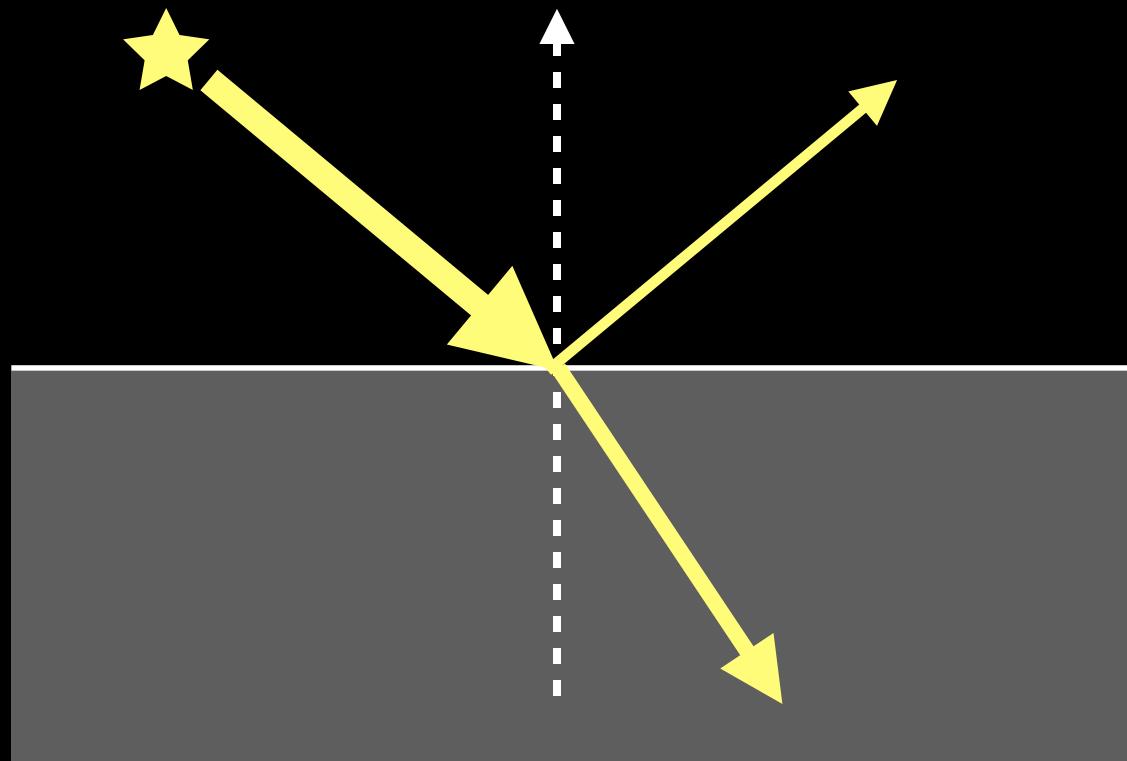


Stack-based solution



Fresnel Reflection

- Both reflection and refraction occur at interface



Fresnel Reflection

- Based on Fresnel equations

$$\rho_s = \frac{\eta_1 \cos \theta_i - \eta_2 \cos \theta_o}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_o}$$

$$\rho_t = \frac{\eta_1 \cos \theta_o - \eta_2 \cos \theta_i}{\eta_1 \cos \theta_o + \eta_2 \cos \theta_i}$$

$$F(\theta_o, \theta_i) = \frac{1}{2} (\rho_s^2 + \rho_t^2)$$

Fresnel Reflection

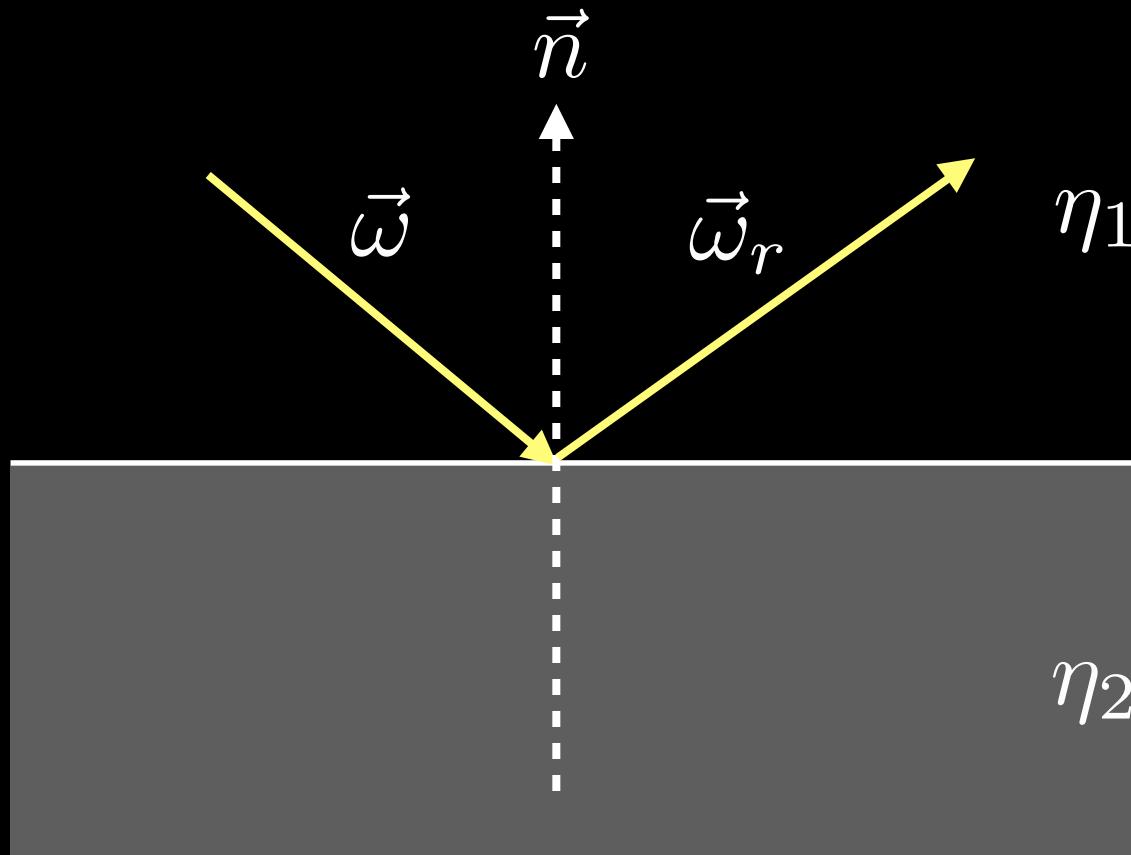
- Recursive call with branch

glass:

```
R = Fresnel(hit, ray);  
return R * shade(trace(reflected_ray))  
+ (1 - R) * shade(trace(refracted_ray));
```



Total Internal Reflection



Trace only reflected ray with $K_s = 1$ if

$$1 - \left(\frac{\eta_1}{\eta_2} \right)^2 (1 - (\vec{\omega} \cdot \vec{n})^2) < 0 \quad (\text{inside of the sqrt})$$

Complete Glass

- Also known as dielectric materials
- Be careful about ray and normal directions

glass:

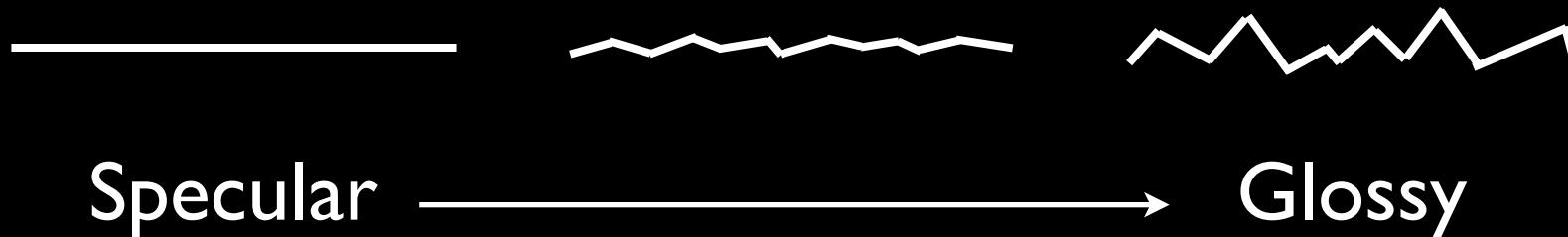
```
if (TotalInternal) return shade(trace(reflected_ray))
```

```
R = Fresnel(hit, ray);
```

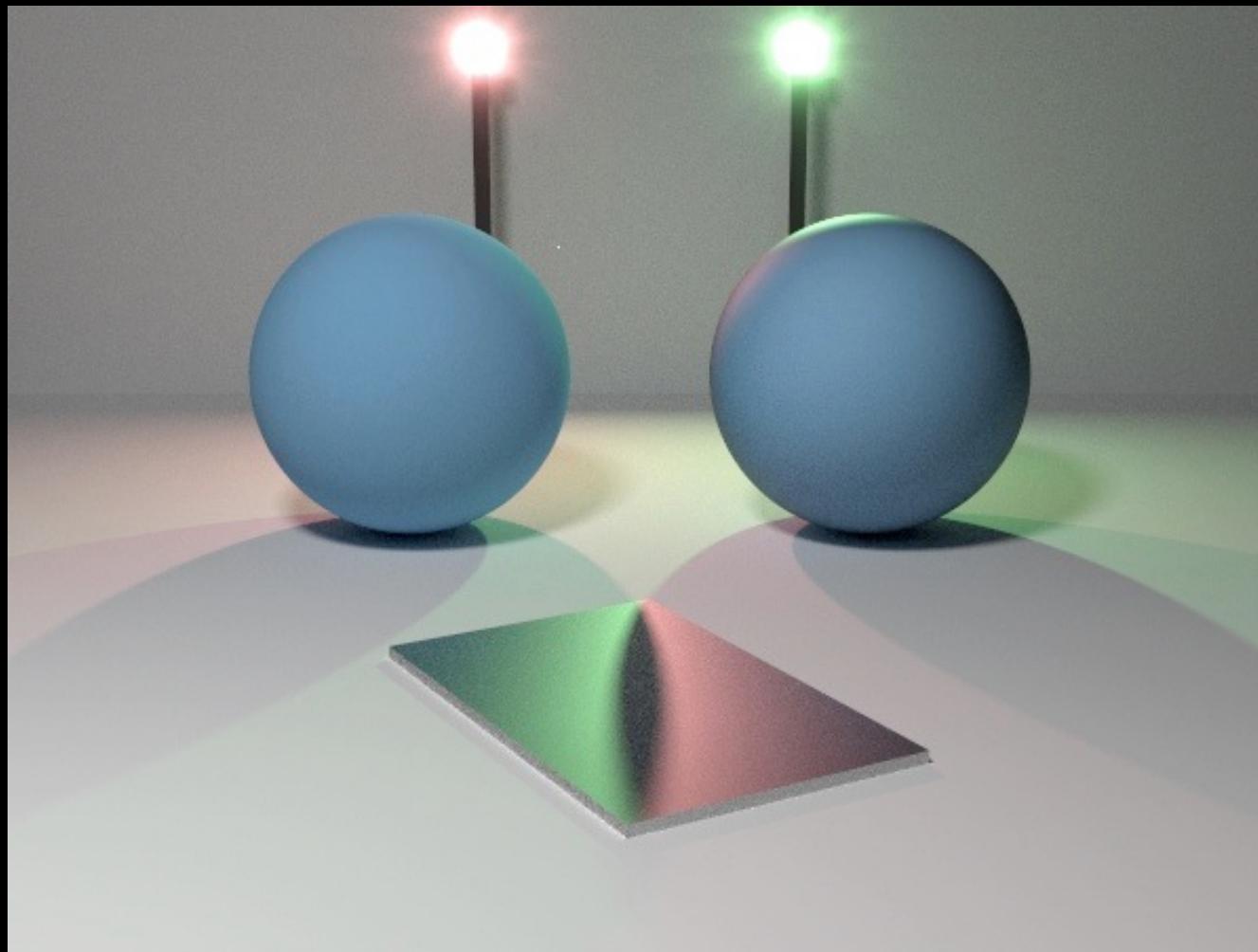
```
return R * shade(trace(reflected_ray))
+ (1 - R) * shade(trace(refracted_ray));
```

Other BRDFs

- Mircofacets model
 - Lots of tiny mirrors at random orientations
 - Distribution of orientations of microfacets decides the sharpness of reflection
 - e.g., Cook-Torrance model



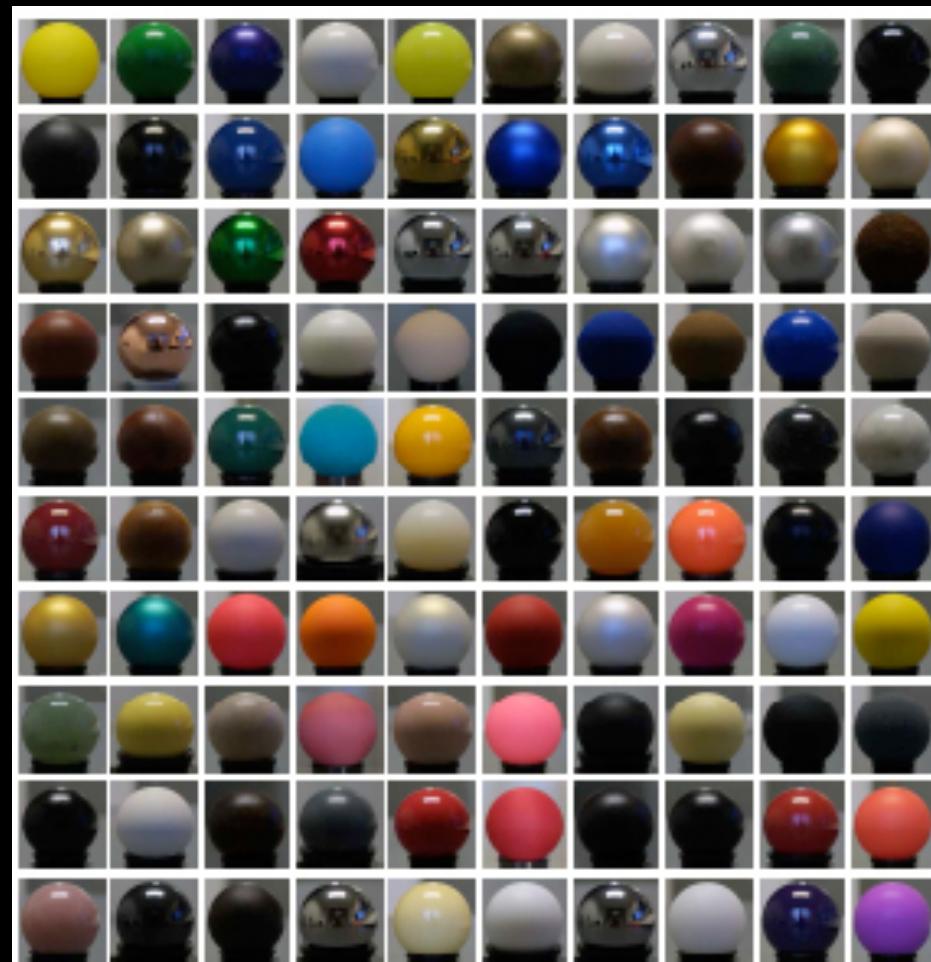
Other BRDFs



[Ashikmin & Shirley]

Other BRDFs

- Measured BRDFs



MERL BRDF Database

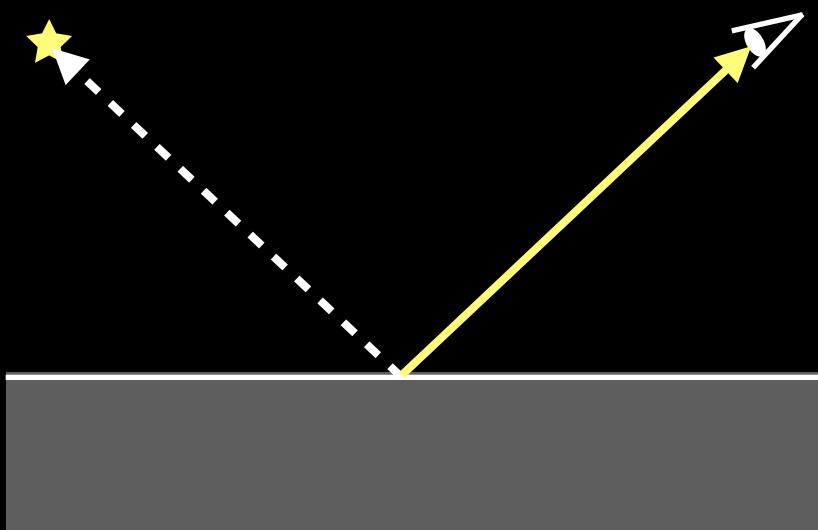


$$f(\vec{\omega}_o, \vec{\omega}_i) = \begin{cases} C_0 & \vec{\omega}_i \cdot \vec{n} \geq T \\ C_1 & \text{otherwise} \end{cases}$$

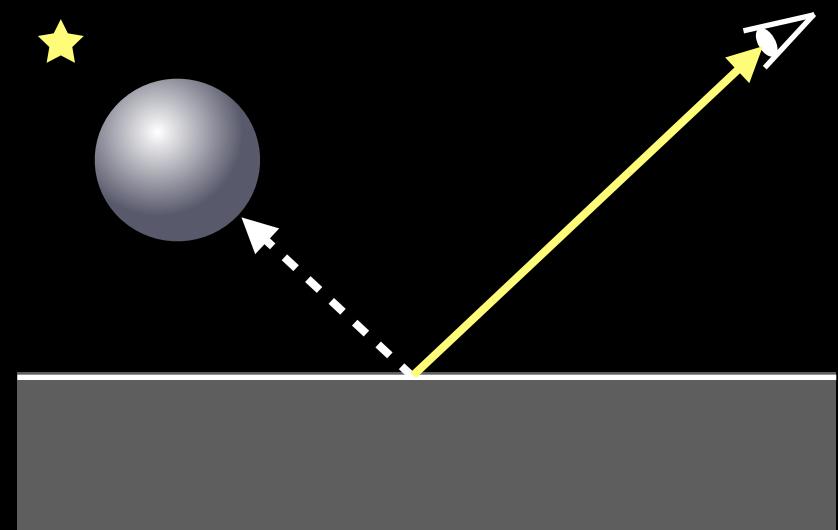
(c) ASCII Media Works, Shirogumi, Tablier Communications inc.

Shadows

- Return irradiance only if the light is visible



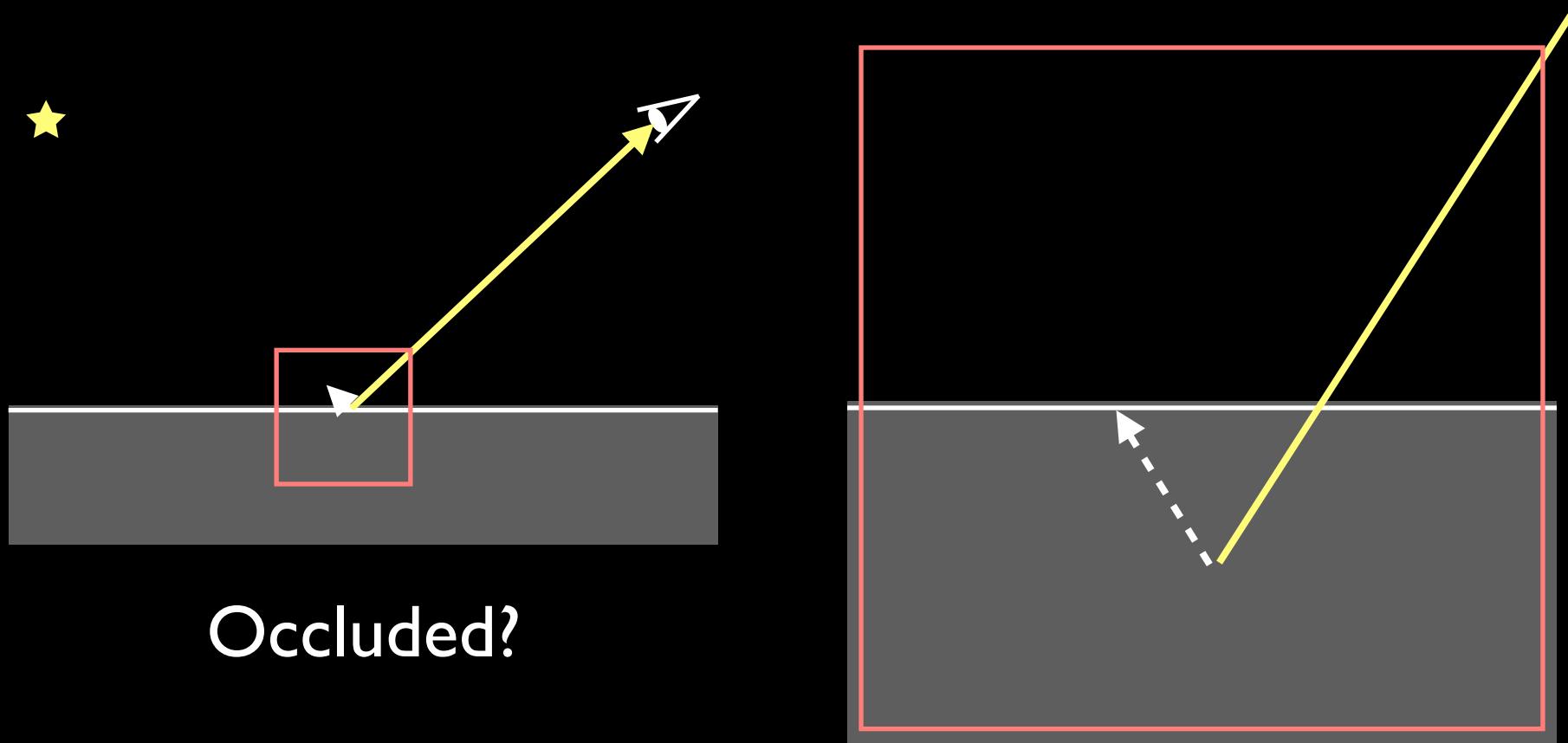
Illuminated



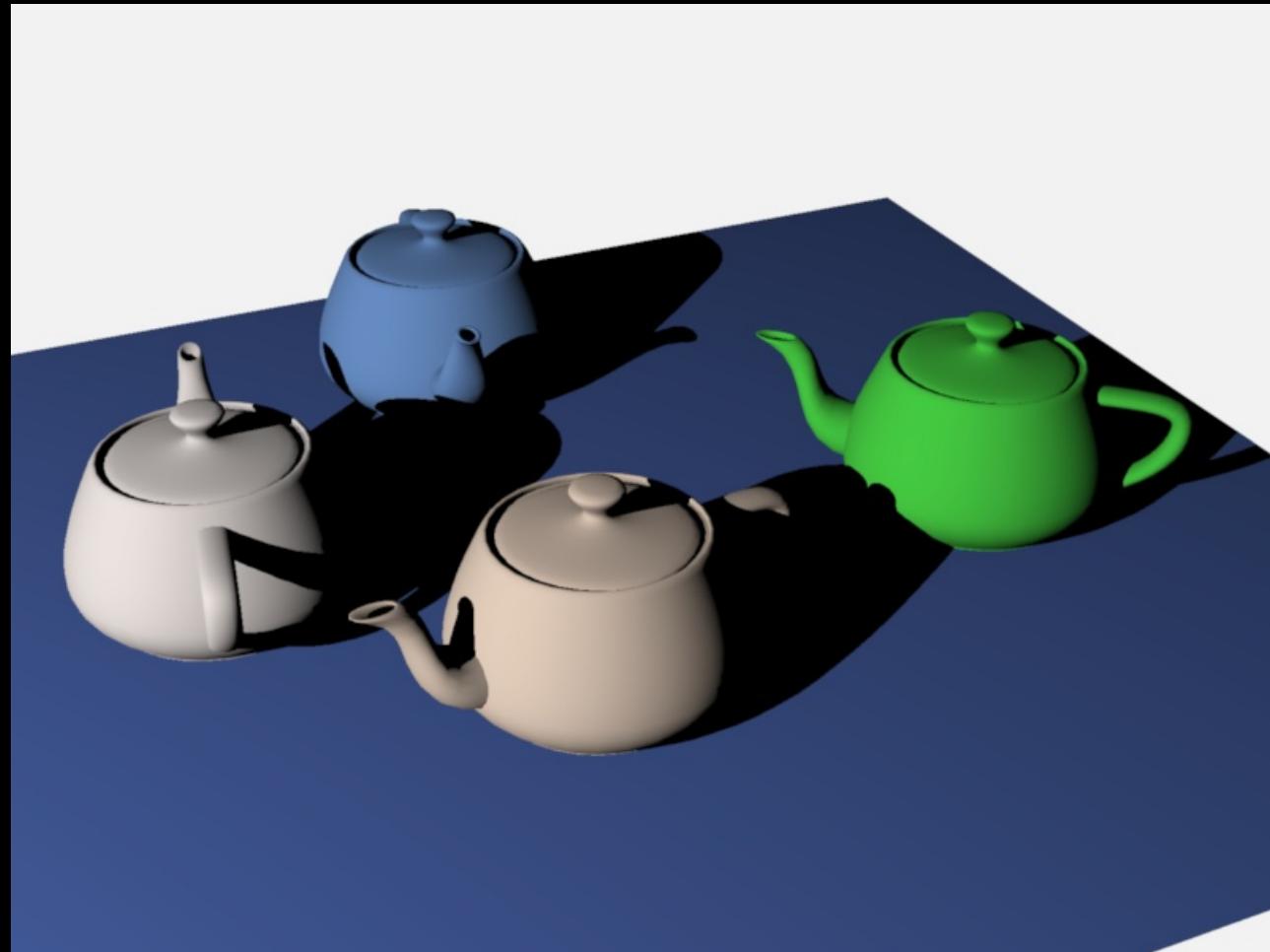
Occluded

Shadows

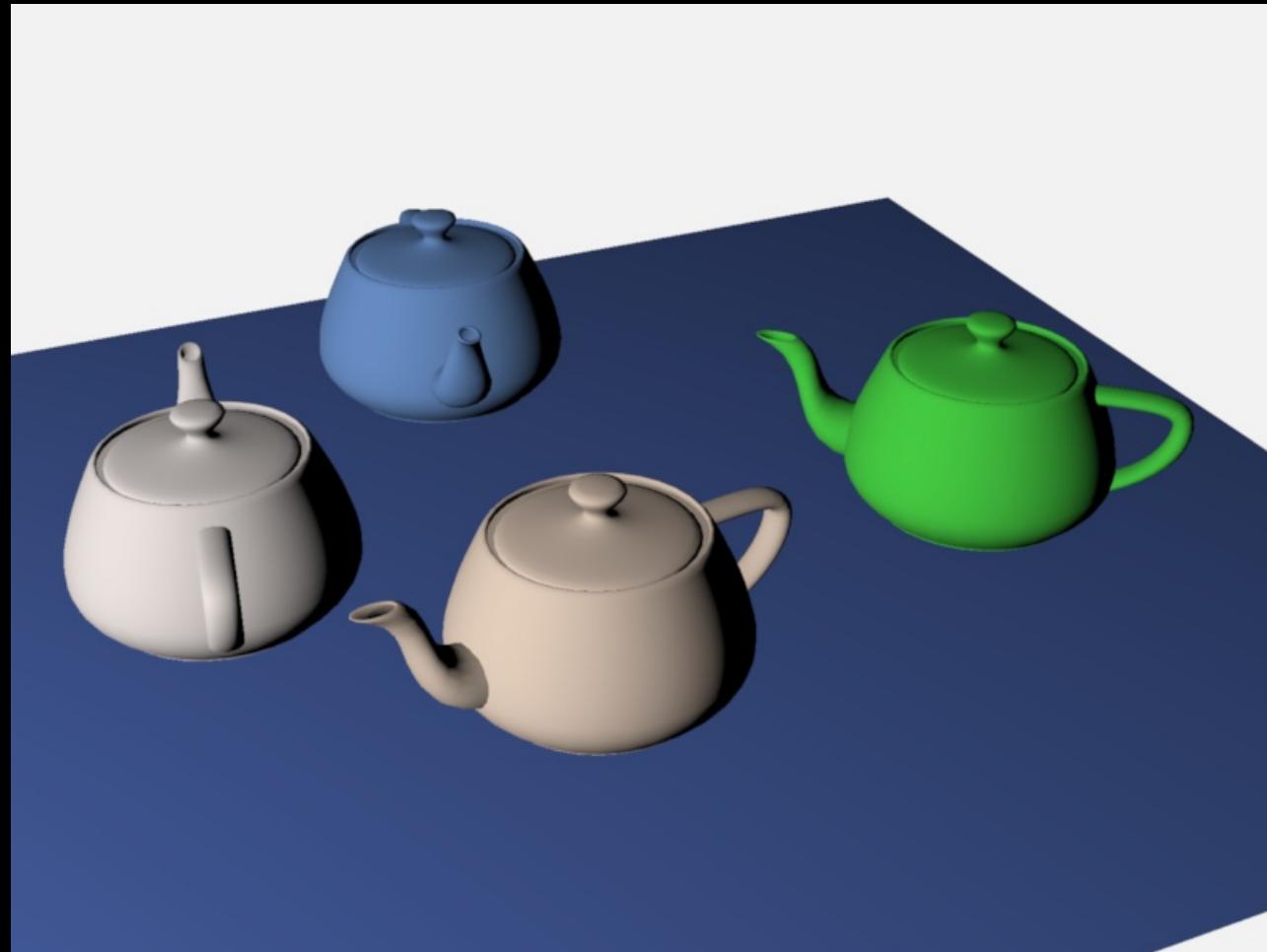
- Avoid self-intersection due to numerical error
 - Ignoring intersections that are too close etc.



With Shadows

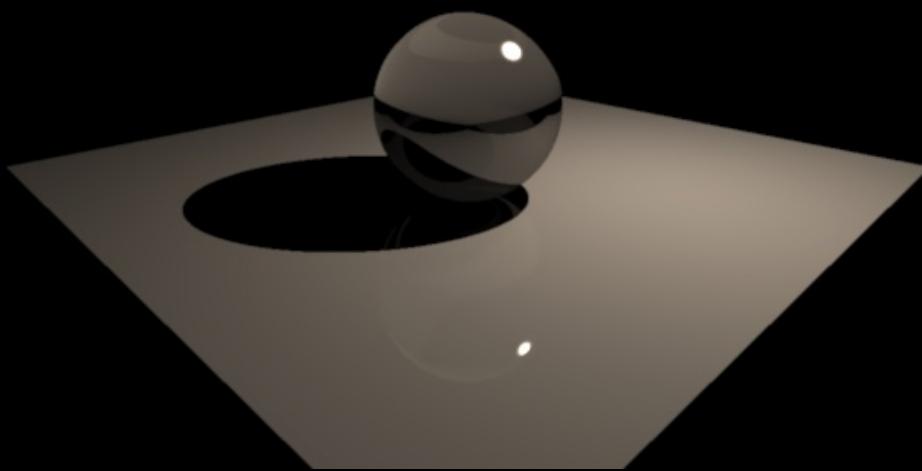


Without Shadows

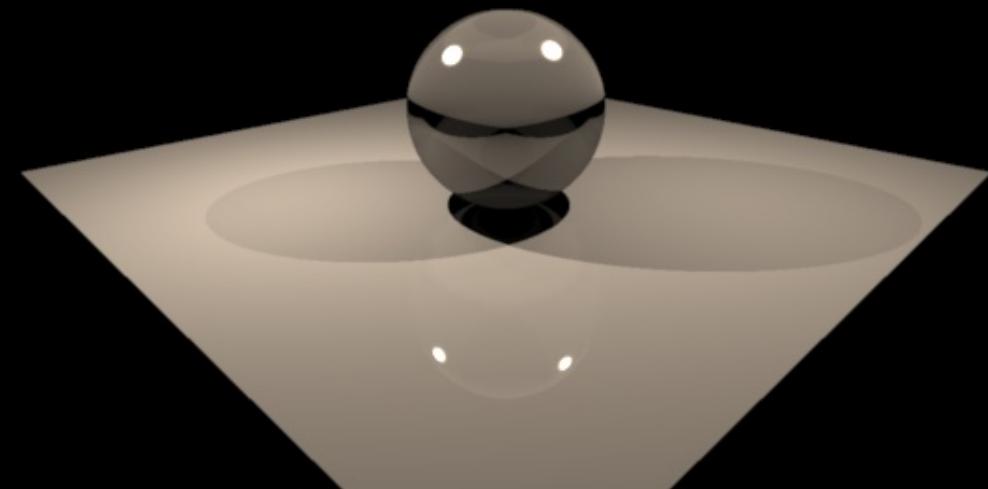


Multiple Light Sources

- Simply add up all the contributions
- Linearity of illumination



One light source



Two light source

Image Based Lighting

- Use of pixels in the image as light sources



Input illumination



Renderings



Image Based Lighting

- If ray hits nothing, return a value from image

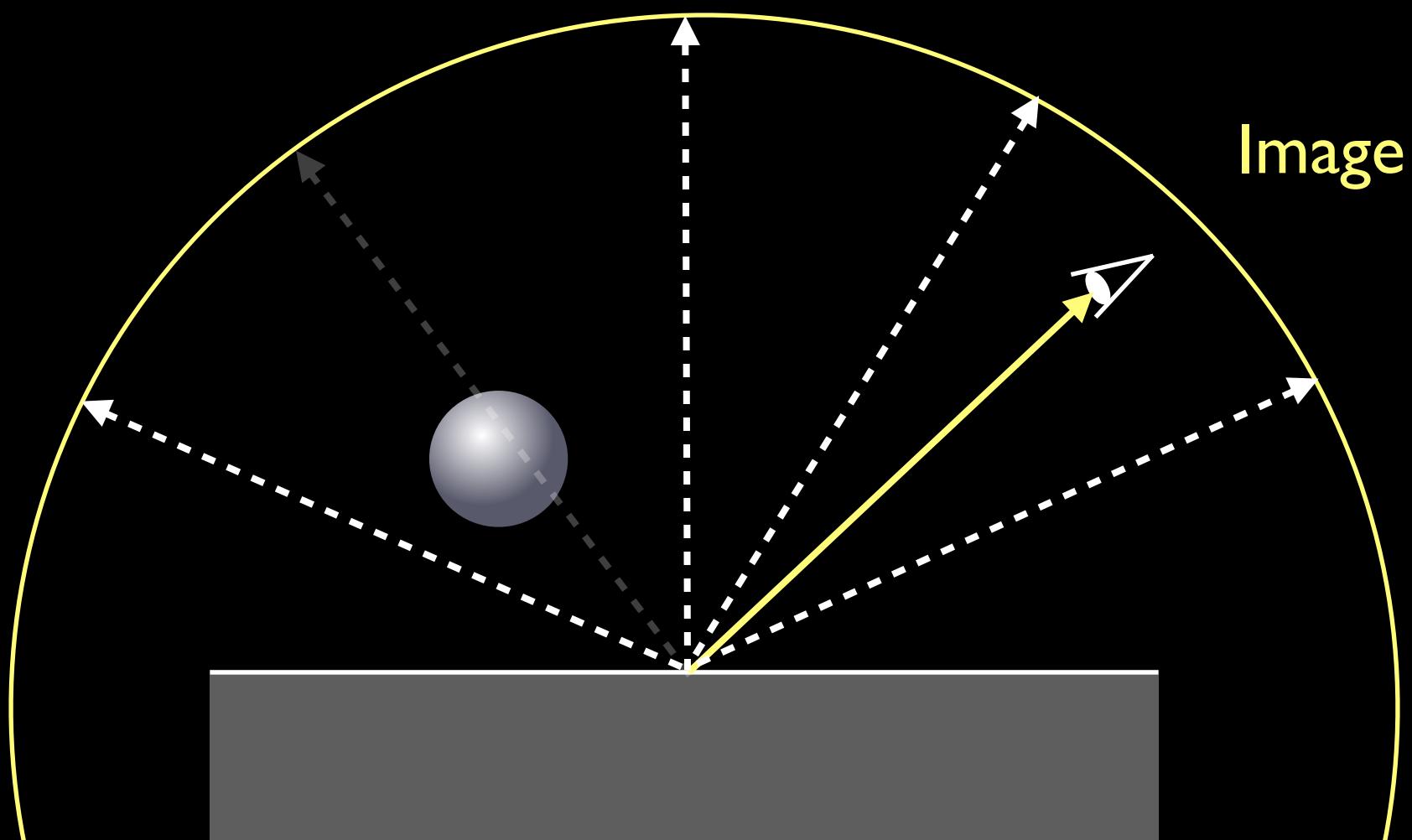


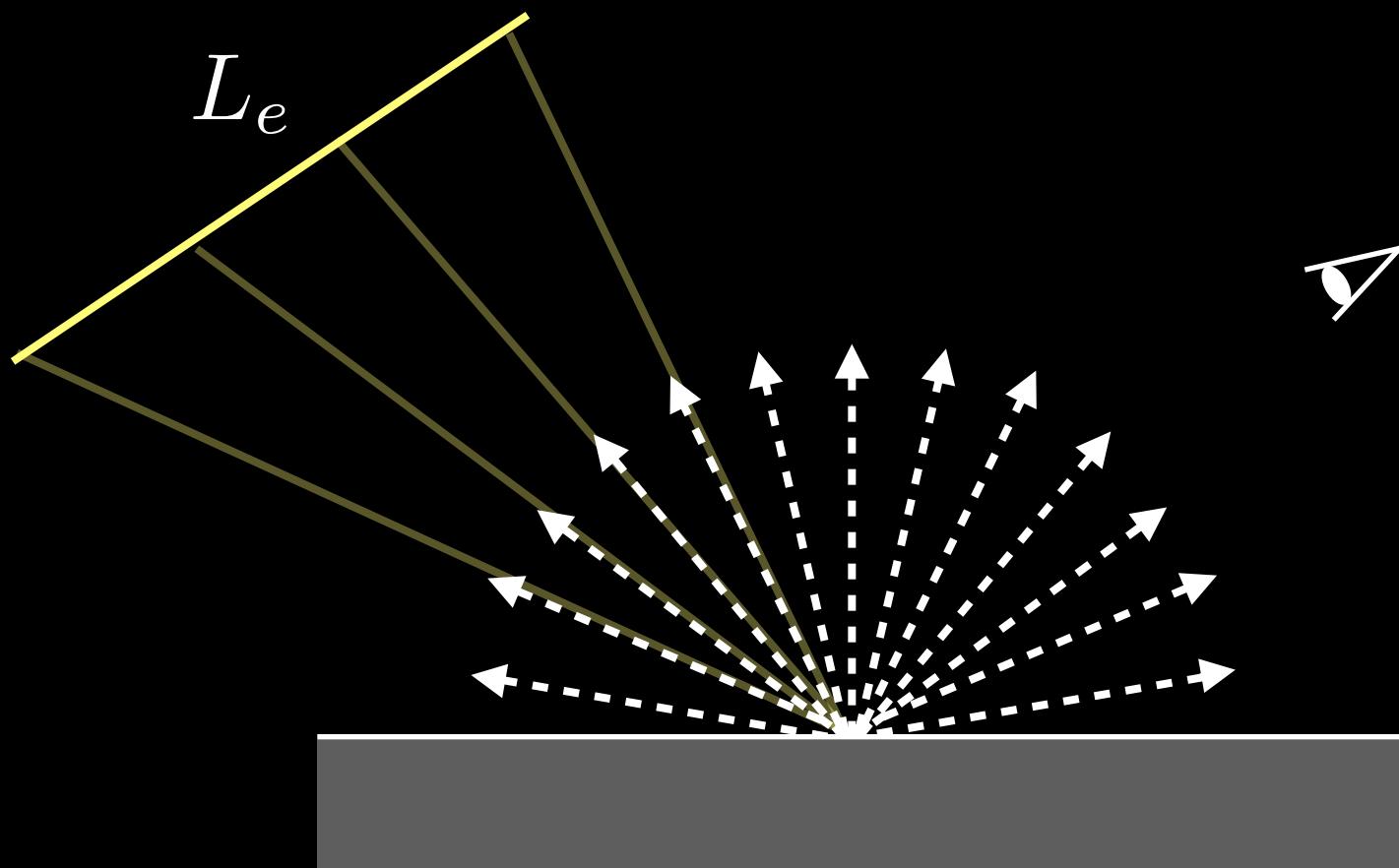
Image Based Lighting

- Trace random ray above the hemisphere

```
color get_irradiance (hit) {  
    Ep = ... // irradiance from point light sources  
    for (i = 1...N) {  
        ray = gen_random_dir(hit.n)  
        if (trace(ray) = no_hit) Ei = Ei + IBL(ray, image)  
    }  
    return Ep + (Ei / N)  
}
```

Area Light Sources

$$L_o(x, \vec{\omega}_o) = \int_{\Omega} f(x, \vec{\omega}_o, \vec{\omega}_i) L_i(x, \vec{\omega}_i) \cos \theta_i d\omega_i$$

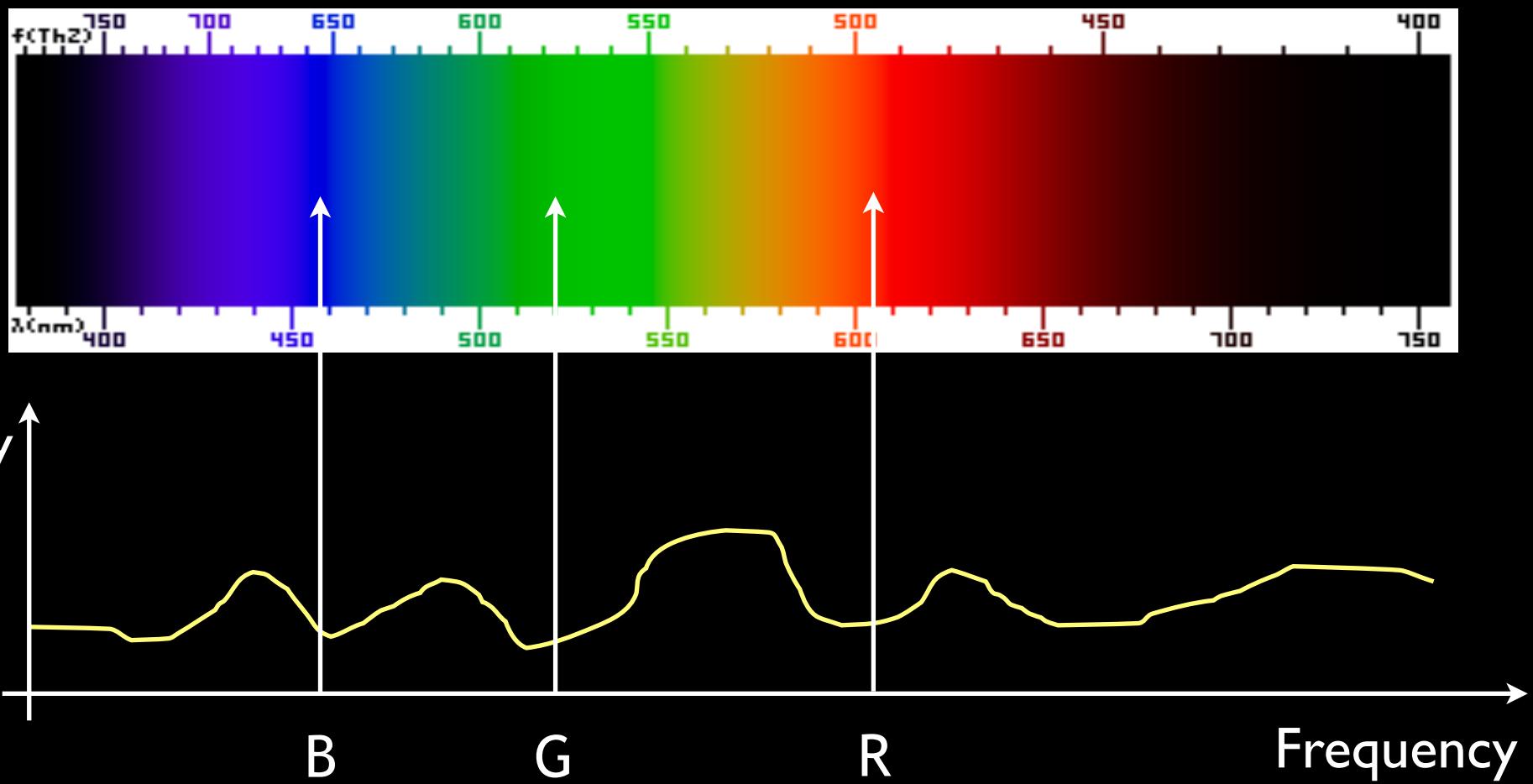


Some Details

- Generating random rays
 - <http://people.cs.kuleuven.be/~philip.dutre/GI/>
 - See Eqn. (34)
- Fetching pixels by rays
 - Depends on parameterization
 - <http://www.pauldebevec.com/Probes/>

Color in Computer Graphics

- Store only three values (Red, Green, Blue)



Why RGB?

- Because we see with RGB
 - Rods: Intensity sensors
 - Cones: Color sensors (typically three types)

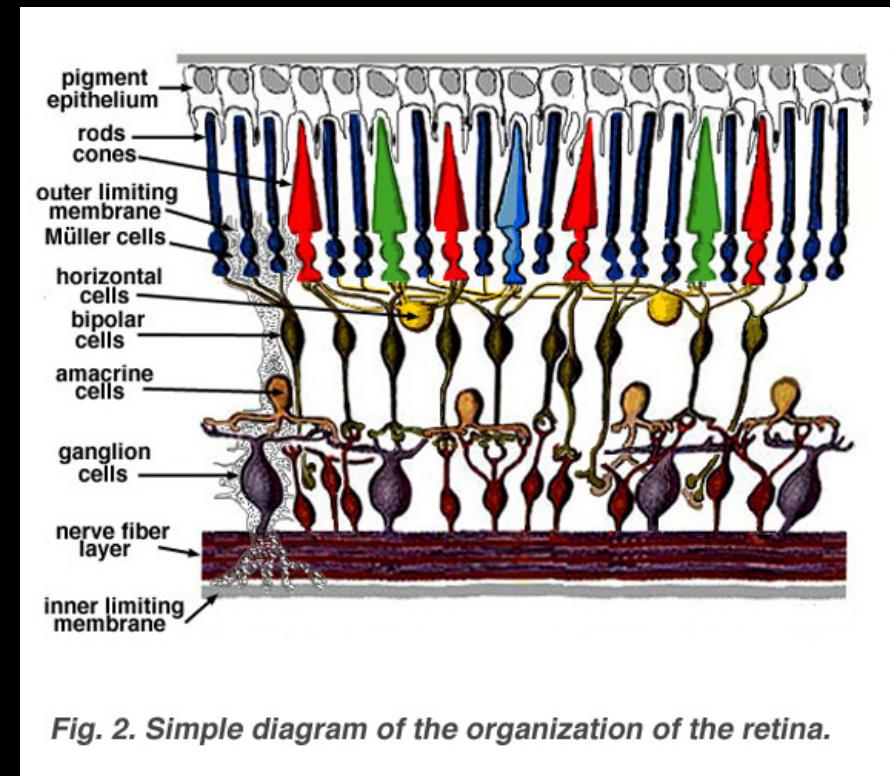
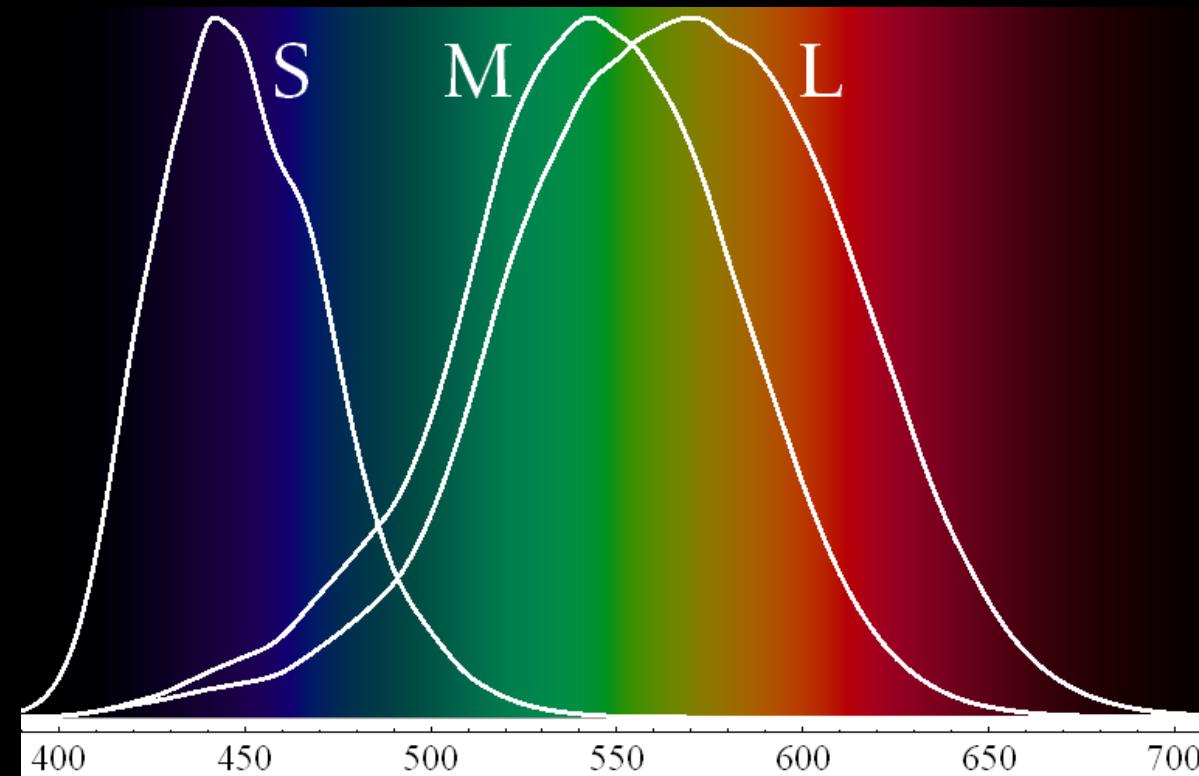
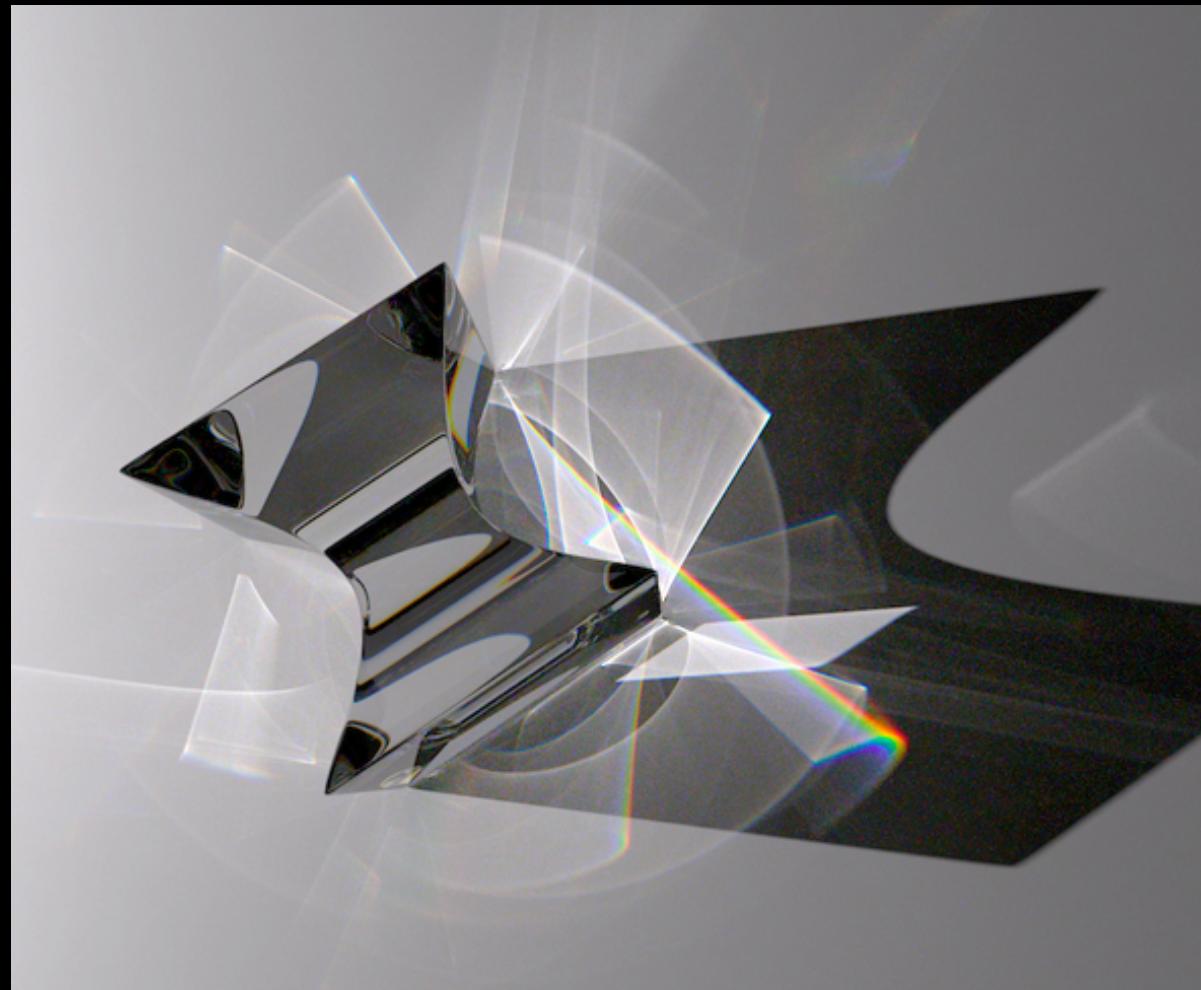


Fig. 2. Simple diagram of the organization of the retina.

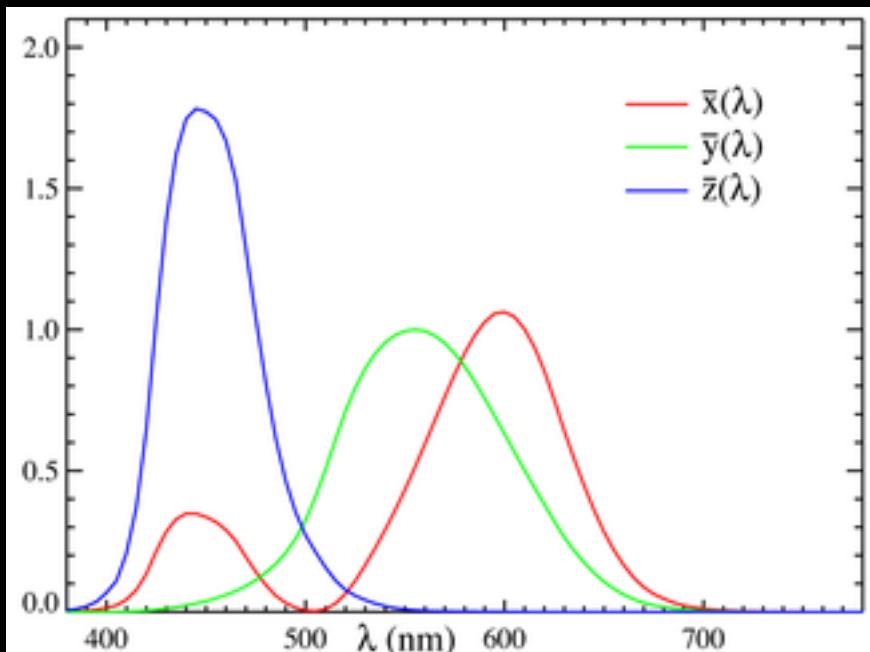
Full Spectrum Rendering

- Simulation at many wavelength \Rightarrow RGB



Spectrum to RGB

- Two steps
 - Compute responses to spectrum as XYZ
 - Convert XYZ to RGB



$$x = \int_{400}^{700} L(\lambda) \bar{x}(\lambda) d\lambda$$

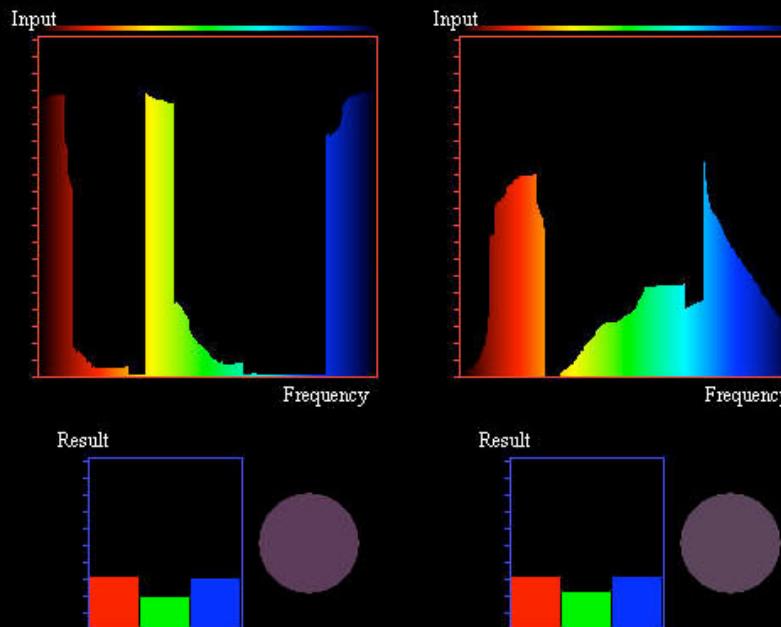
↓

Similar for y & z

$$r = c_0 x + c_1 y + c_2 z$$

RGB to Spectrum

- Ill-conditioned problem
 - Spectrum has much more info than RGB
 - “An RGB to Spectrum Conversion for Reflectances”



High Dynamic Range

- RGB values can go beyond 1.0



W/m^2

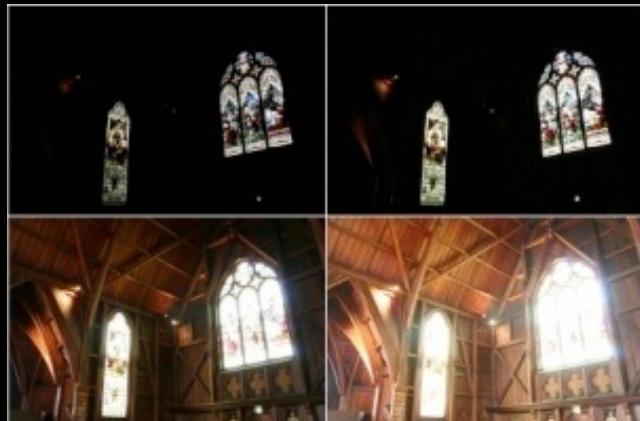


Human Eye vs Monitor

- Human eye
 - $2^{14}:1$ brightness difference
 - 14 bits
- Monitor
 - 255:1 brightness difference
 - 8 bits

Tone Mapping

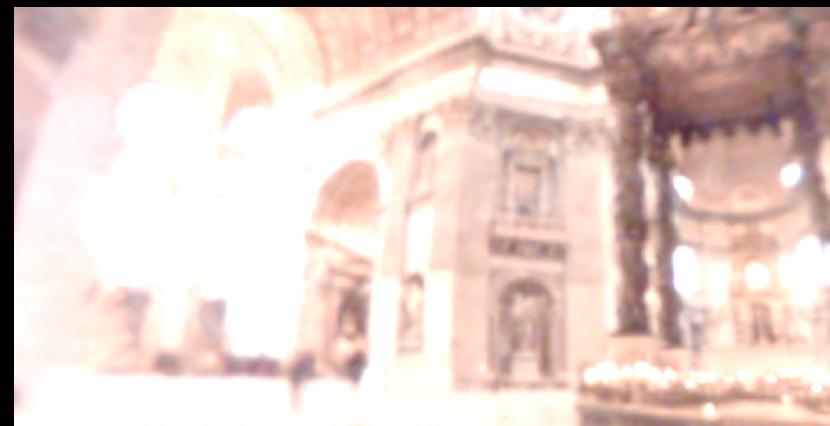
- Convert an HDR value into 0-1 (0-255)



Linear Scaling

- Scale and clamp

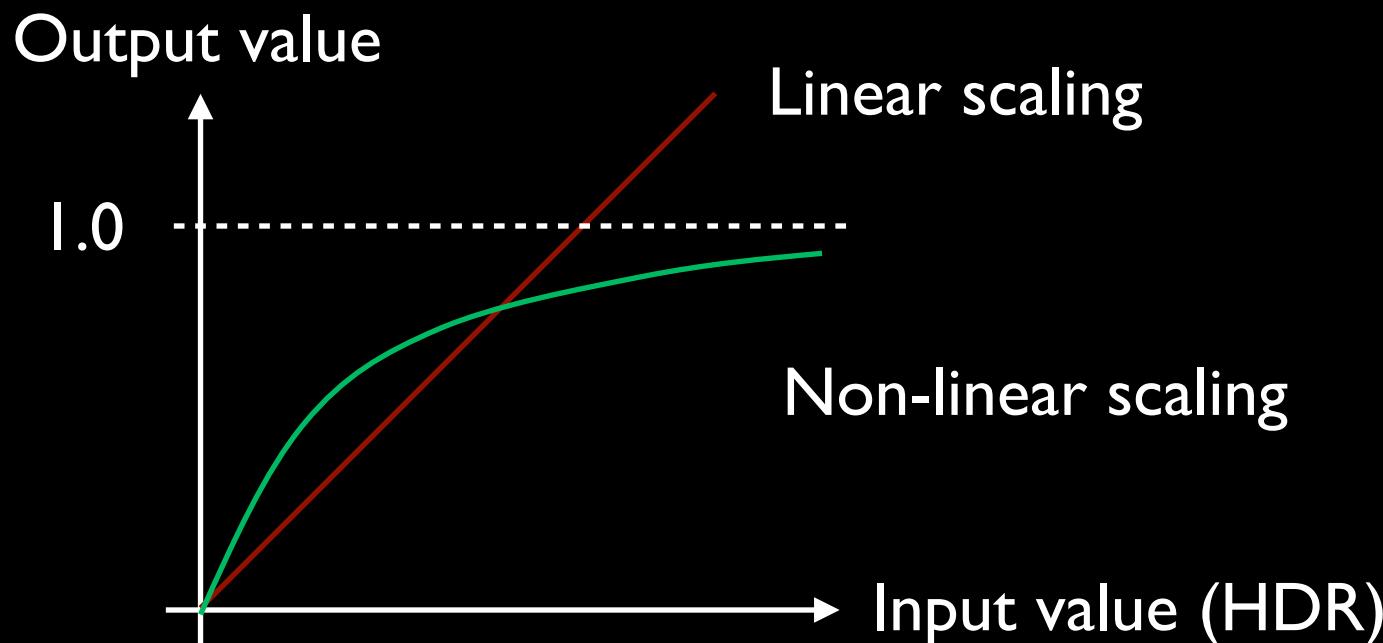
$$l(x, y) = \min(cL(x, y), 1.0)$$



Non-linear Scaling

- Use a function to “compress” HDR into 0-1

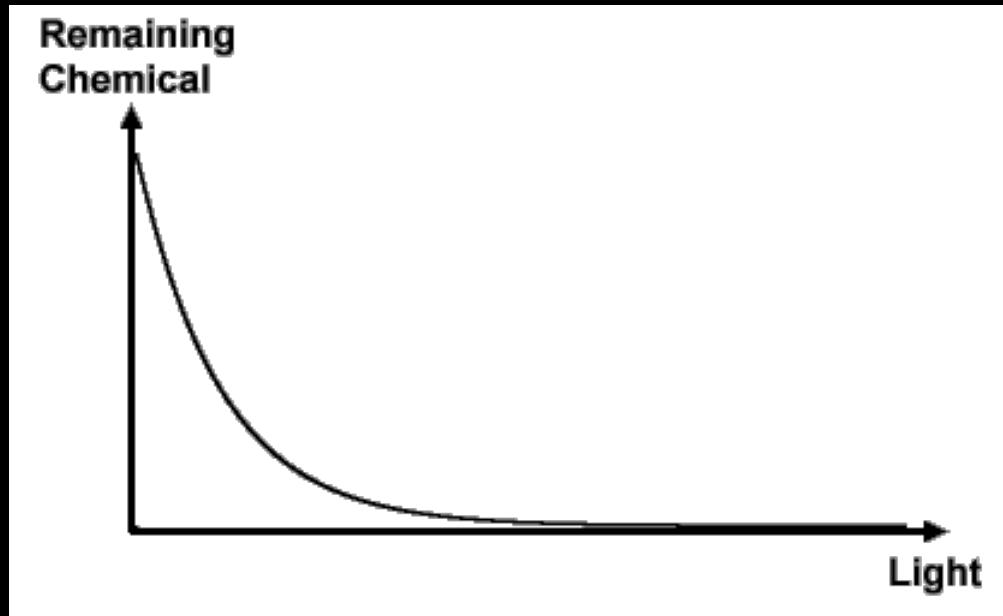
$$l(x, y) = f(L(x, y))$$



Film Response

- Inspired by the response curve of a film
 - Simple and works well often

$$l(x, y) = 1.0 - e^{-cL(x, y)}$$



Spatially Varying Scaling

- Different scaling per pixel
- “A tone mapping algorithm for high contrast images”, Ashikmin, 2002
- Halo problem



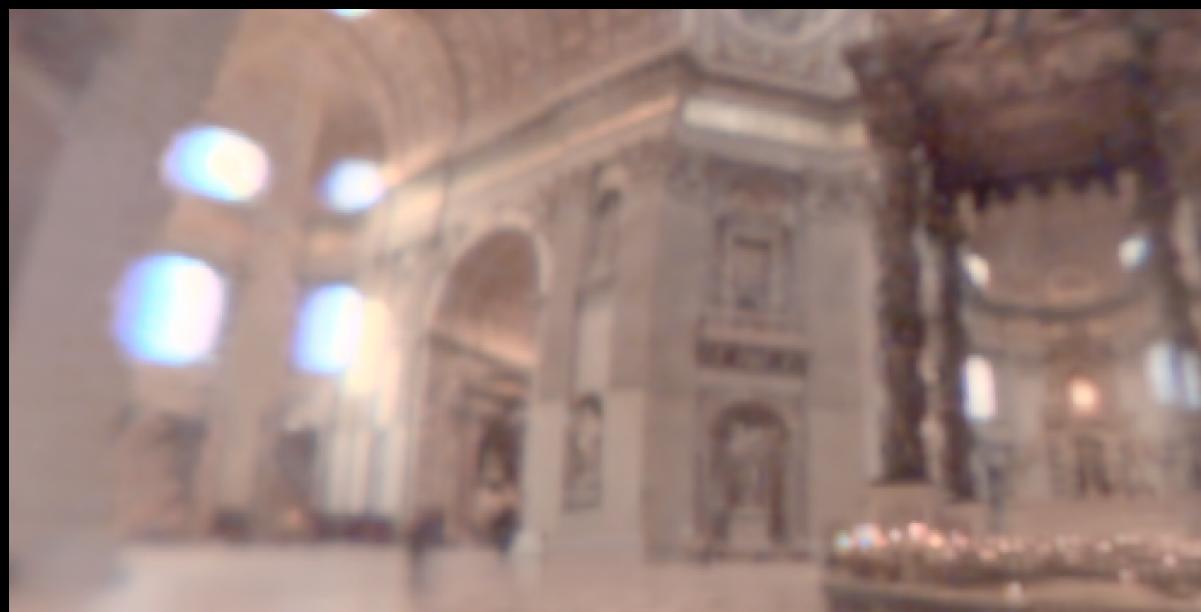
Photographic Tone Reproduction

- Inspired by photography [Reinhard]

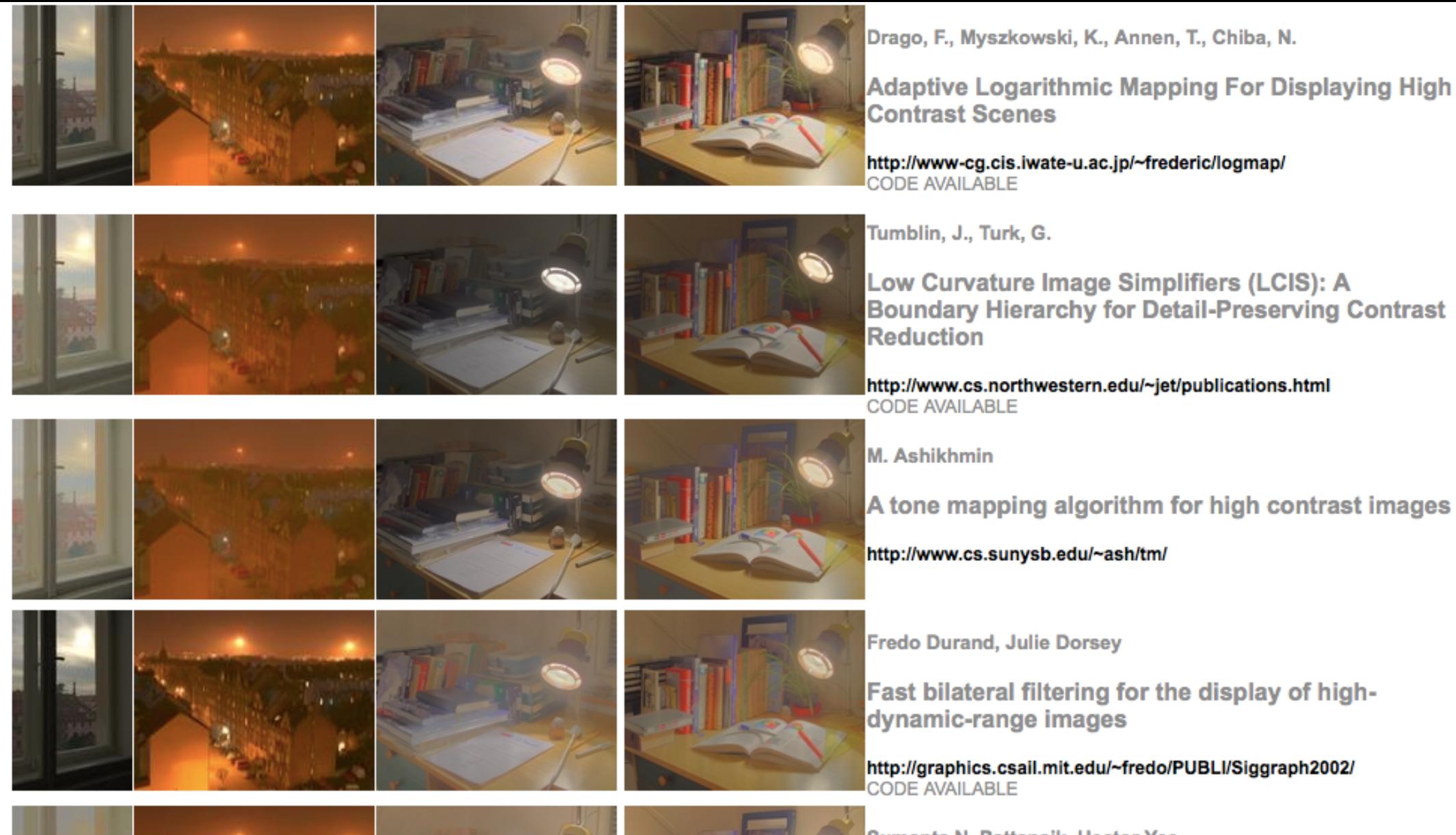
- Often works well

- Desired brightness L_{white}

$$L_d(x, y) = \frac{L(x, y) \left(1 + \frac{L(x, y)}{L_{\text{white}}^2}\right)}{1 + L(x, y)}$$



Comparisons



Gamma Correction

- 0.5 is not necessary displayed as “0.5”
 - Nonlinear mapping on a monitor
 - Correction to input tone-mapped value
 - g is typically 1.7-2.2

$$p(x, y) = l(x, y)^{\frac{1}{g}}$$

Gamma Correction

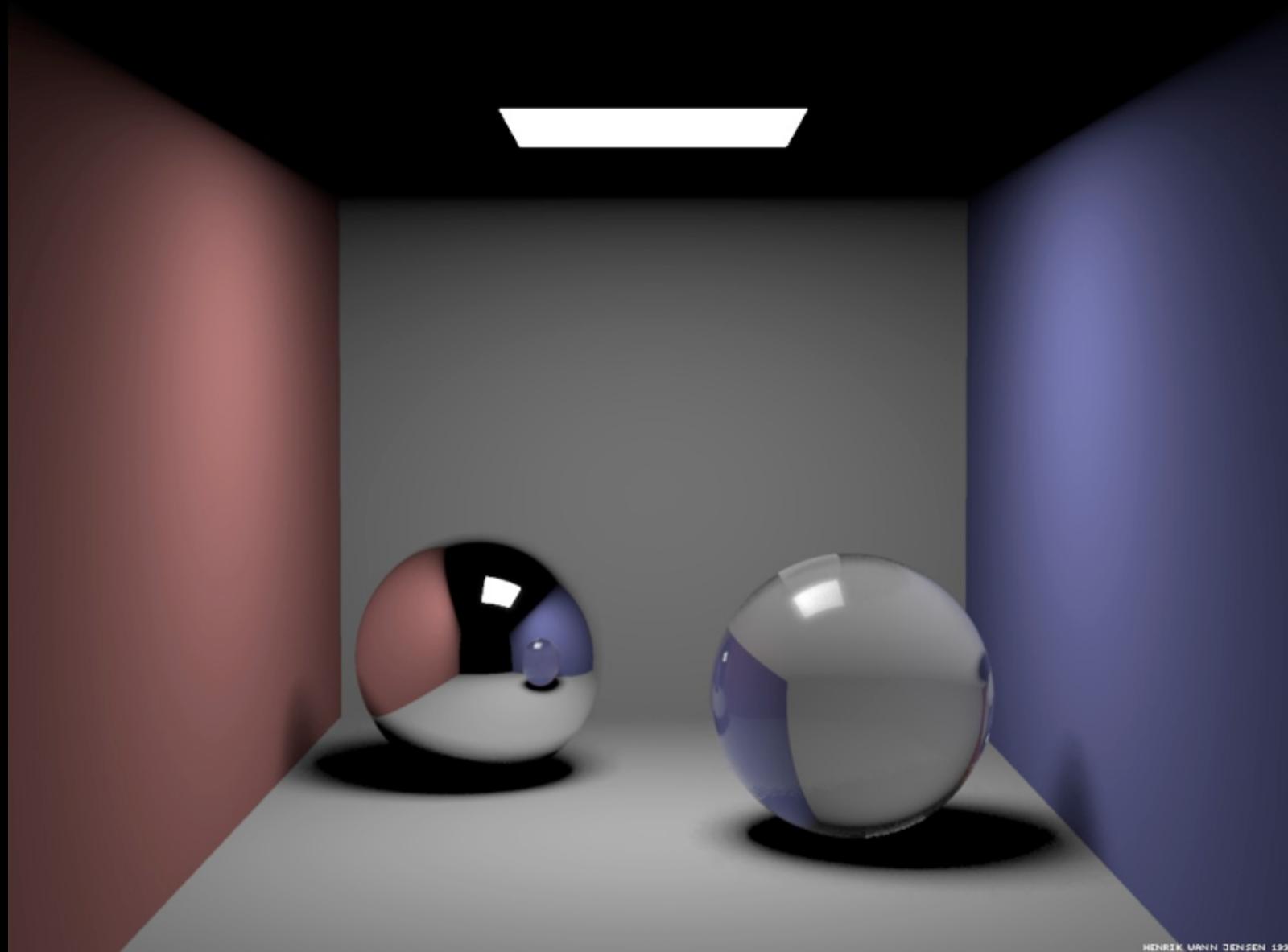


Corrected



No correction

Missing Piece



Missing Piece

