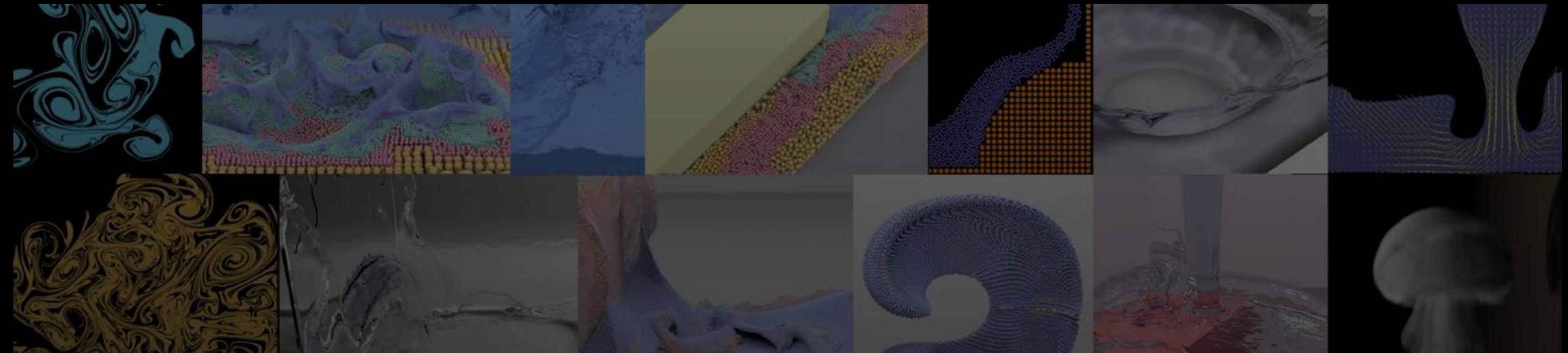


CG物理ベース流体シミュレーション

Physics-based Fluid Simulations for Computer Graphics

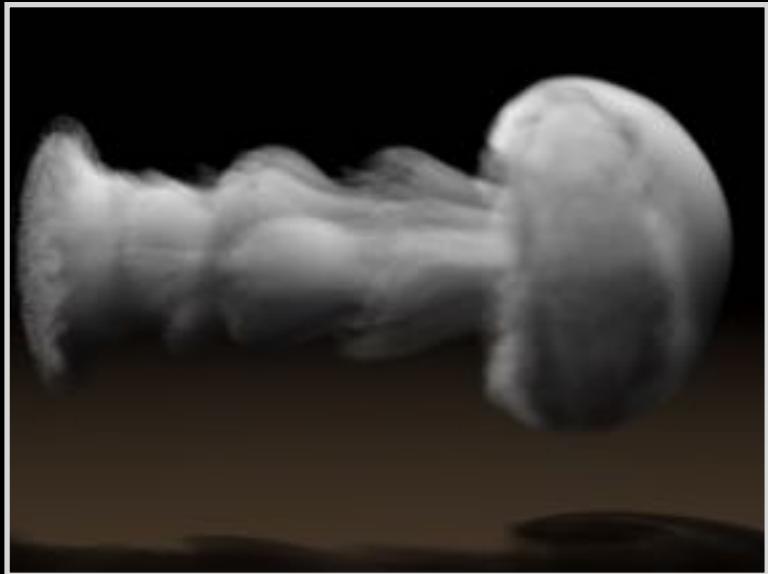


安東 遼一

Ryoichi Ando

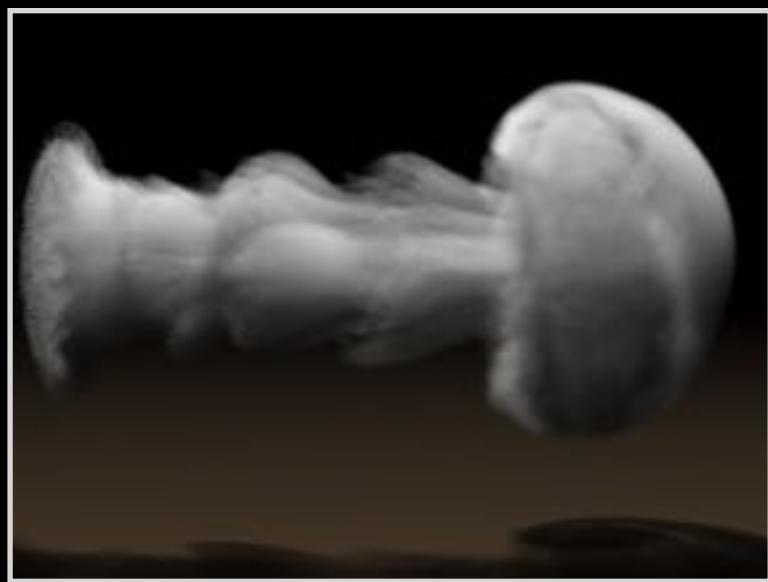
Contents

Contents

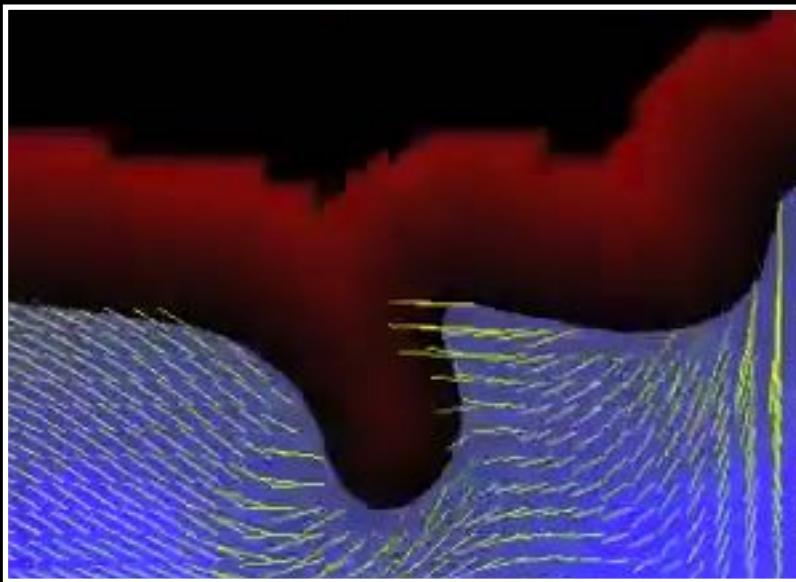


Fluid Solver

Contents

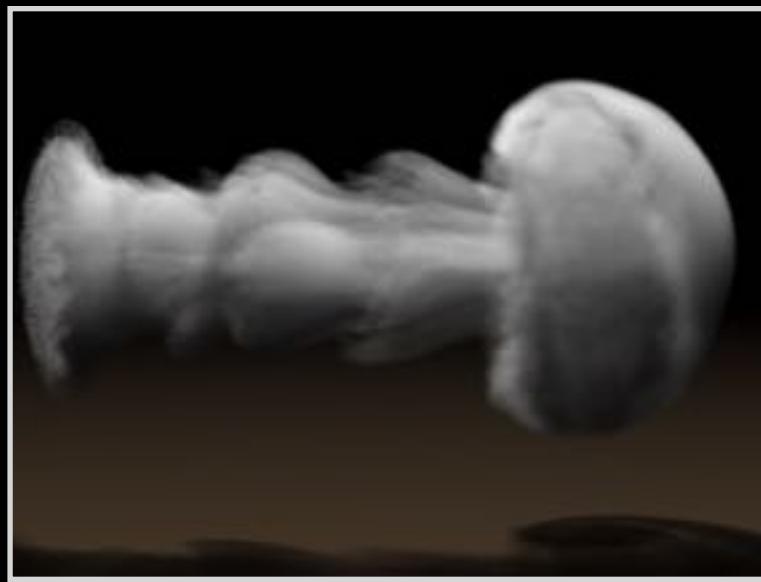


Fluid Solver

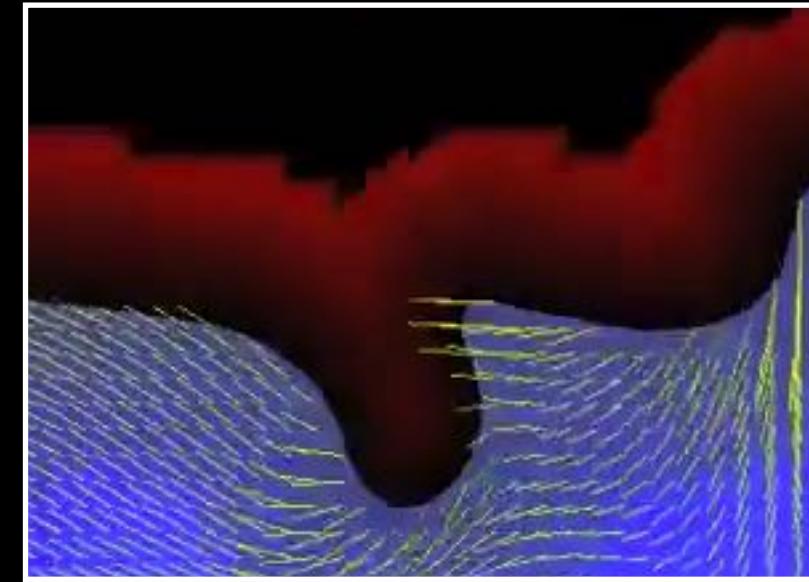


Surface Tracker

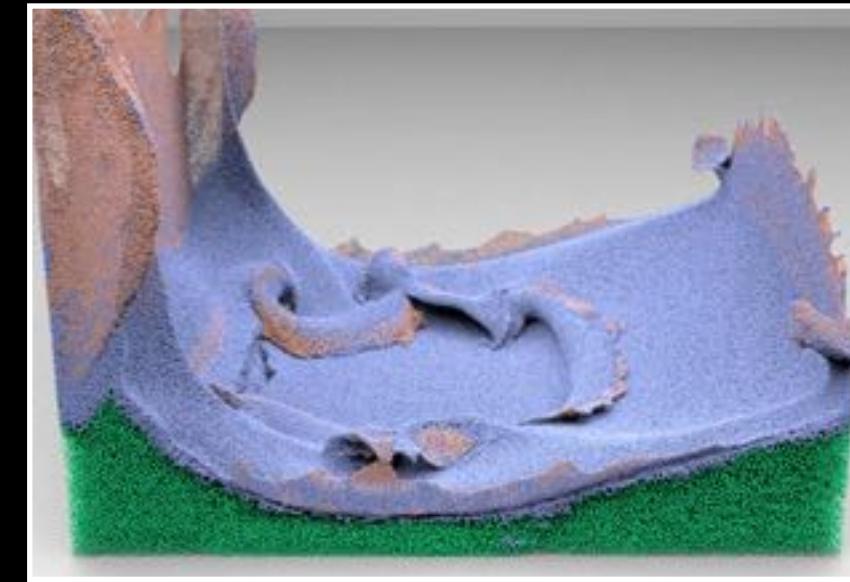
Contents



Fluid Solver



Surface Tracker



FLIP Solver

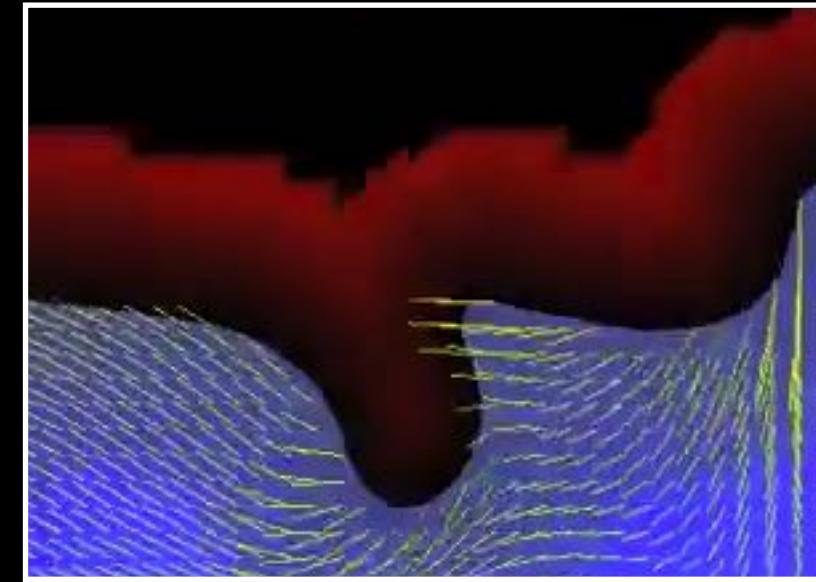
Short Bio



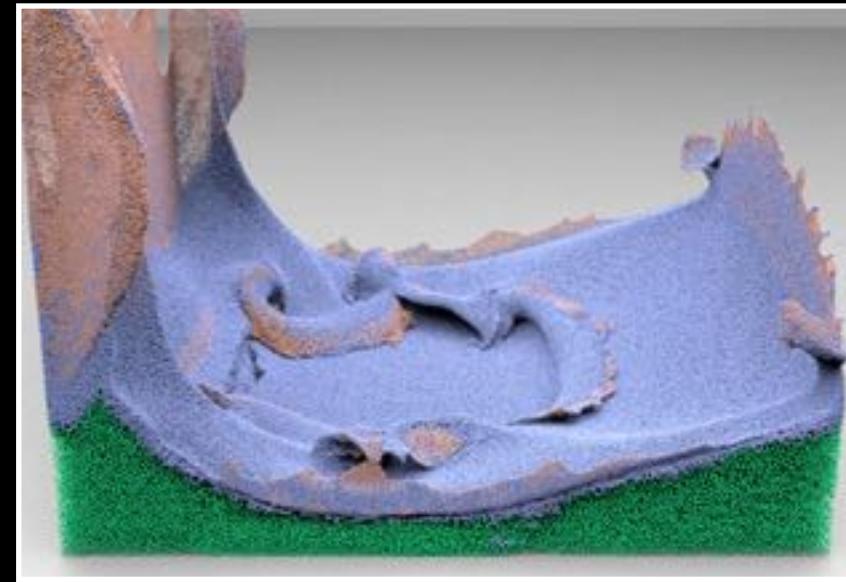
- Ph.D student @ Kyushu Univ
- Visiting Scientist at IST Austria
- Fluid Simulations
- SIGGRAPH 2013 Tech. paper with Chris Wojtan and Nils Thuerey



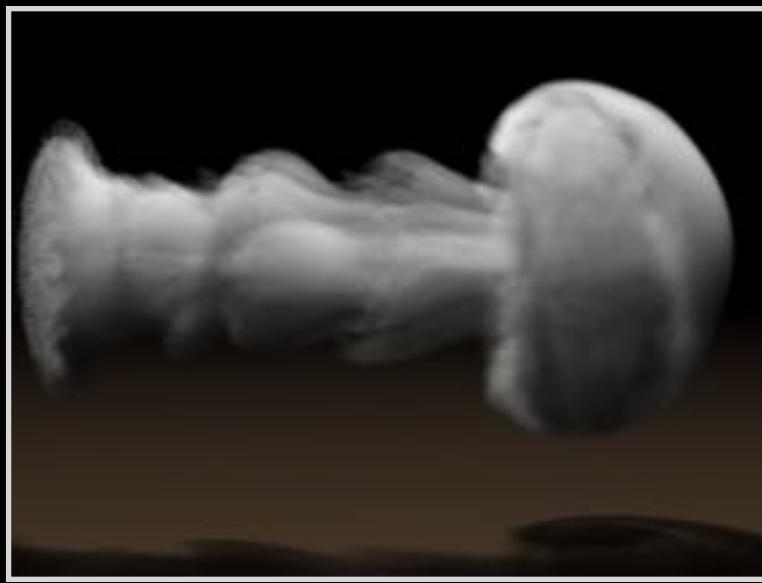
Fluid Solver



Surface Tracker



FLIP Solver



Fluid Solver

Navier-Stokes Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\text{s.t. } \nabla \cdot \mathbf{u} = 0$$

Navier-Stokes Equations

How velocity changes
over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

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Navier-Stokes Equations

How velocity changes
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$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + \boxed{(\mathbf{u} \cdot \nabla) \mathbf{u}} = -\frac{1}{\rho} \nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection term

$$\text{s.t. } \nabla \cdot \mathbf{u} = 0$$

Navier-Stokes Equations

How velocity changes over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u}} = \boxed{-\frac{1}{\rho} \nabla p} + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection term Pressure term

s.t. $\nabla \cdot u = 0$

Navier-Stokes Equations

How velocity changes
over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + \boxed{(\mathbf{u} \cdot \nabla) \mathbf{u}} = -\frac{1}{\rho} \nabla p + \boxed{\mu \nabla^2 \mathbf{u}} + \mathbf{f}$$

Advection term Pressure term Viscosity term

$$\text{s.t. } \nabla \cdot \mathbf{u} = 0$$

Navier-Stokes Equations

How velocity changes
over time

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection term Pressure term Viscosity term External force term

$$\text{s.t. } \nabla \cdot \mathbf{u} = 0$$

Navier-Stokes Equations

How velocity changes
over time

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection term Pressure term Viscosity term External force term

$$\text{s.t. } \nabla \cdot \mathbf{u} = 0$$

Incompressibility Constraint

Navier-Stokes Equations

Navier-Stokes Equations

Large Matrix

Navier-Stokes Equations

Large Matrix

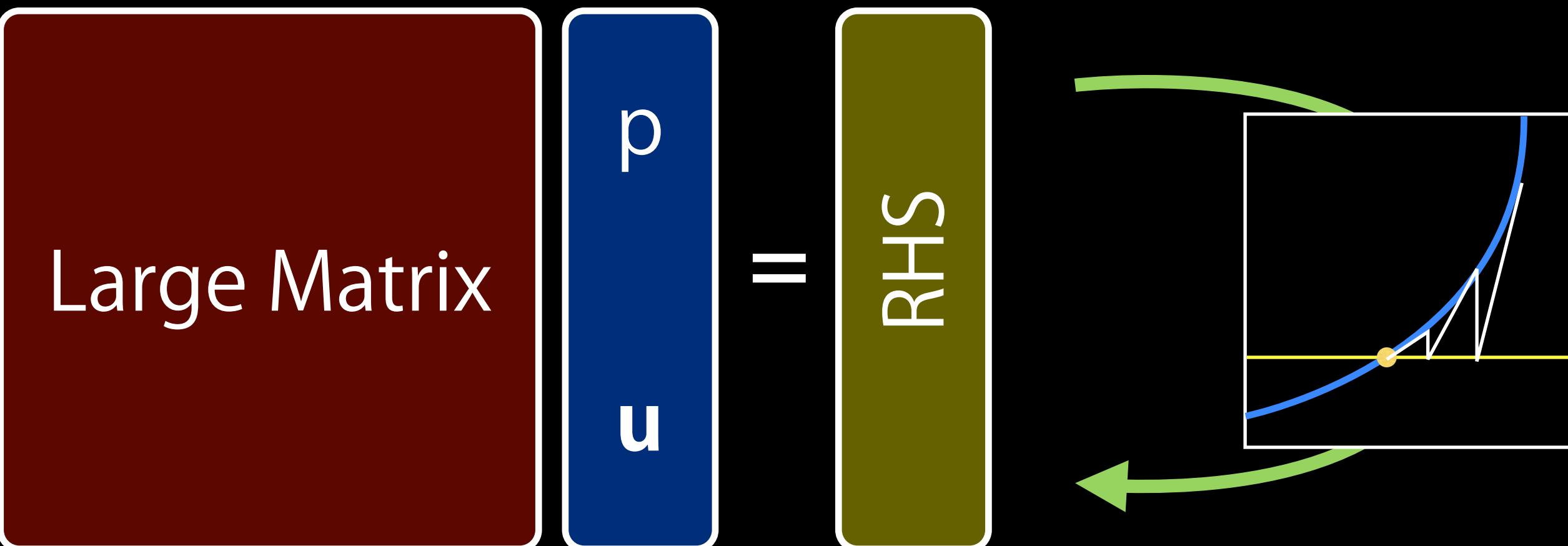
p

u

=

RHS

Navier-Stokes Equations



Navier-Stokes Equations

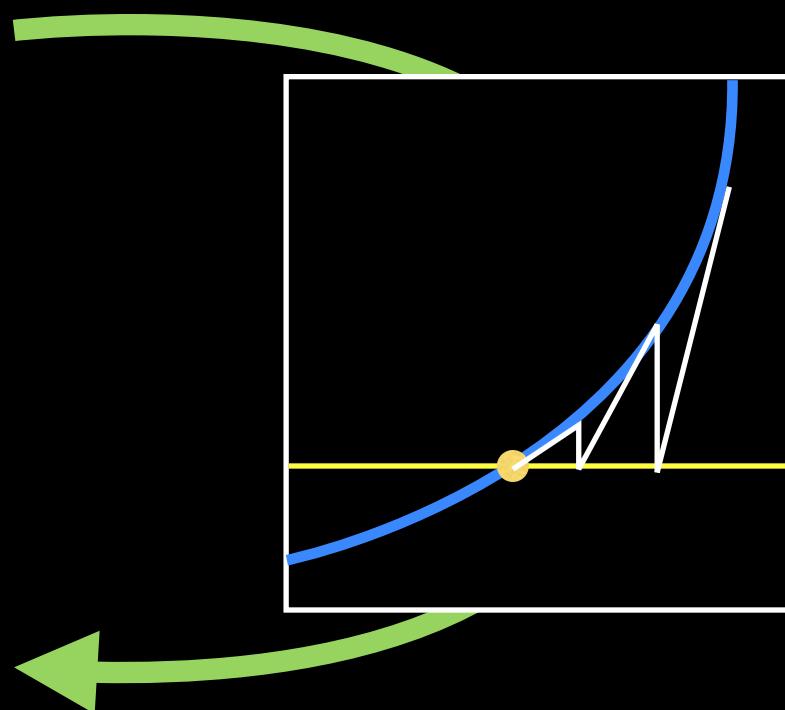


Large Matrix

p
 u

=

RHS



Newton Iterations...

Operator Splitting

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

Operator Splitting

$$\boxed{\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

$$u_{\text{Adv}}^*$$

Operator Splitting

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

$\mathbf{u}_{\text{Adv}}^*$

$\mathbf{u}_{\text{Adv+Visc}}^*$

Operator Splitting

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

$\mathbf{u}_{\text{Adv}}^*$

$\mathbf{u}_{\text{Adv+Visc}}^*$

$\mathbf{u}_{\text{Adv+Visc+Prs}}^*$

Operator Splitting

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

$\mathbf{u}_{\text{Adv}}^*$

$\mathbf{u}_{\text{Adv+Visc}}^*$

$\mathbf{u}_{\text{Adv+Visc+Prs}}^*$

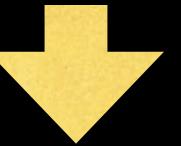
$\mathbf{u}_{t+\Delta t}$

[Stable fluid, Stam99]



First-order integration

$$\frac{\partial u}{\partial t} = f(u, p)$$



$$u_{t+\Delta t} = u_t + \Delta t f(u, p)$$

First-order integration

$$\frac{\partial \mathbf{u}}{\partial t} = f(\mathbf{u}, p)$$



$$\mathbf{u}_{t+\Delta t} = \mathbf{u}_t + \Delta t f(\mathbf{u}, p)$$

e.g, advection $\mathbf{u}_{t+\Delta t} = \mathbf{u}_t + \Delta t \underline{[\mathbf{u} \cdot \nabla \mathbf{u}]}$

First-order integration

$$\frac{\partial u}{\partial t} = f(u, p)$$



First-order accuracy!



$$u_{t+\Delta t} = u_t + \Delta t f(u, p) + O(\Delta t)$$

e.g, advection $\underline{u_{t+\Delta t} = u_t + \Delta t [\mathbf{u} \cdot \nabla \mathbf{u}]}$

Simulation Steps

Simulation Steps

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

Simulation Steps

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

Simulation Steps

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\mu \nabla^2 \mathbf{u}$$

Viscosity term

Simulation Steps

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\mu \nabla^2 \mathbf{u}$$

Viscosity term

$$\mathbf{F}$$

External force
term

Simulation Steps

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

Simulation Steps

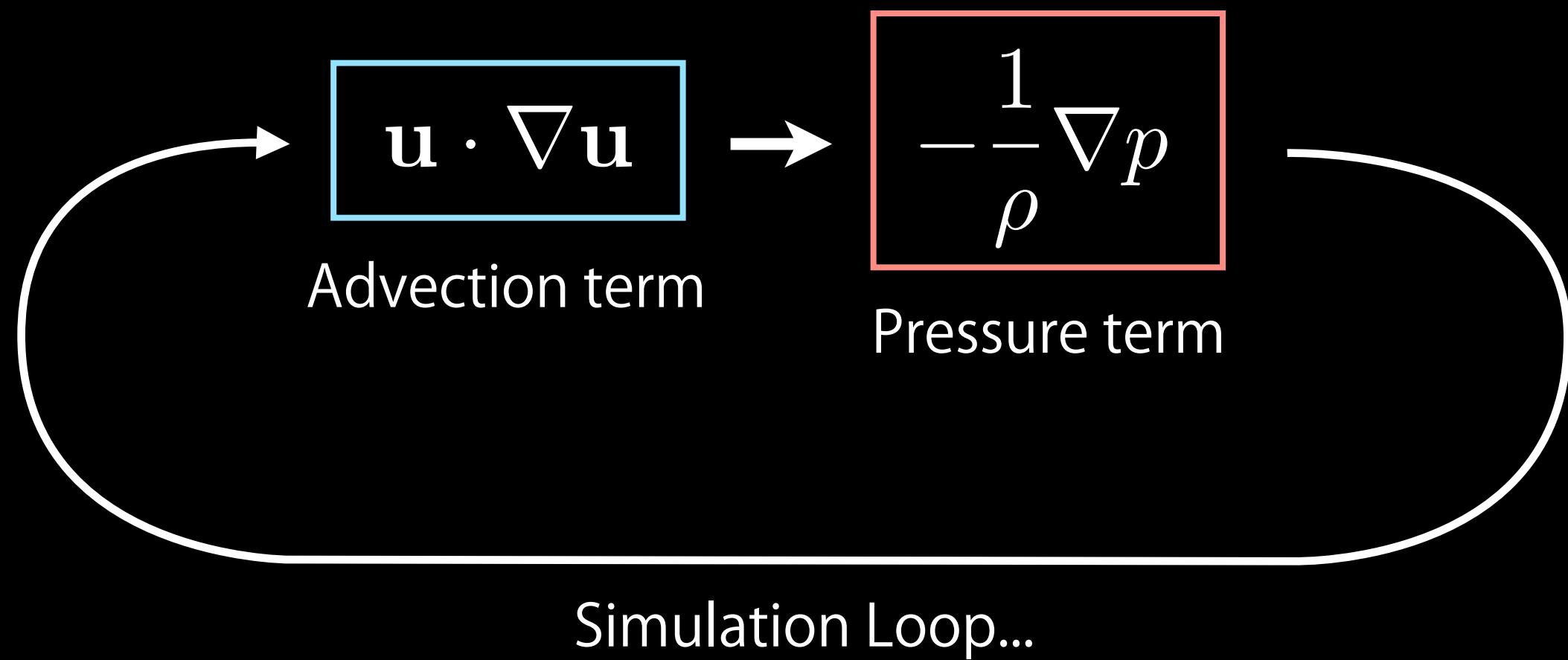
$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

Simulation Steps



$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = 0$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$\frac{\partial \boxed{\phi}}{\partial t} + (\mathbf{u} \cdot \nabla) \boxed{\phi} = 0$$

Scalar

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

Scalar ϕ

$$\frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla) \phi = 0$$

Scalar

↓ Expand...

$$\frac{\partial \phi}{\partial t} + u_x \frac{\partial}{\partial x} \phi + u_y \frac{\partial}{\partial y} \phi + u_z \frac{\partial}{\partial z} \phi = 0$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

Scalar

$$\frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla) \phi = 0$$

Scalar

↓ Expand...

$$\frac{\partial \phi}{\partial t} + u_x \frac{\partial}{\partial x} \phi + u_y \frac{\partial}{\partial y} \phi + \boxed{u_z \frac{\partial}{\partial z} \phi} = 0$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$\frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla) \phi = 0$$

↓ Expand...

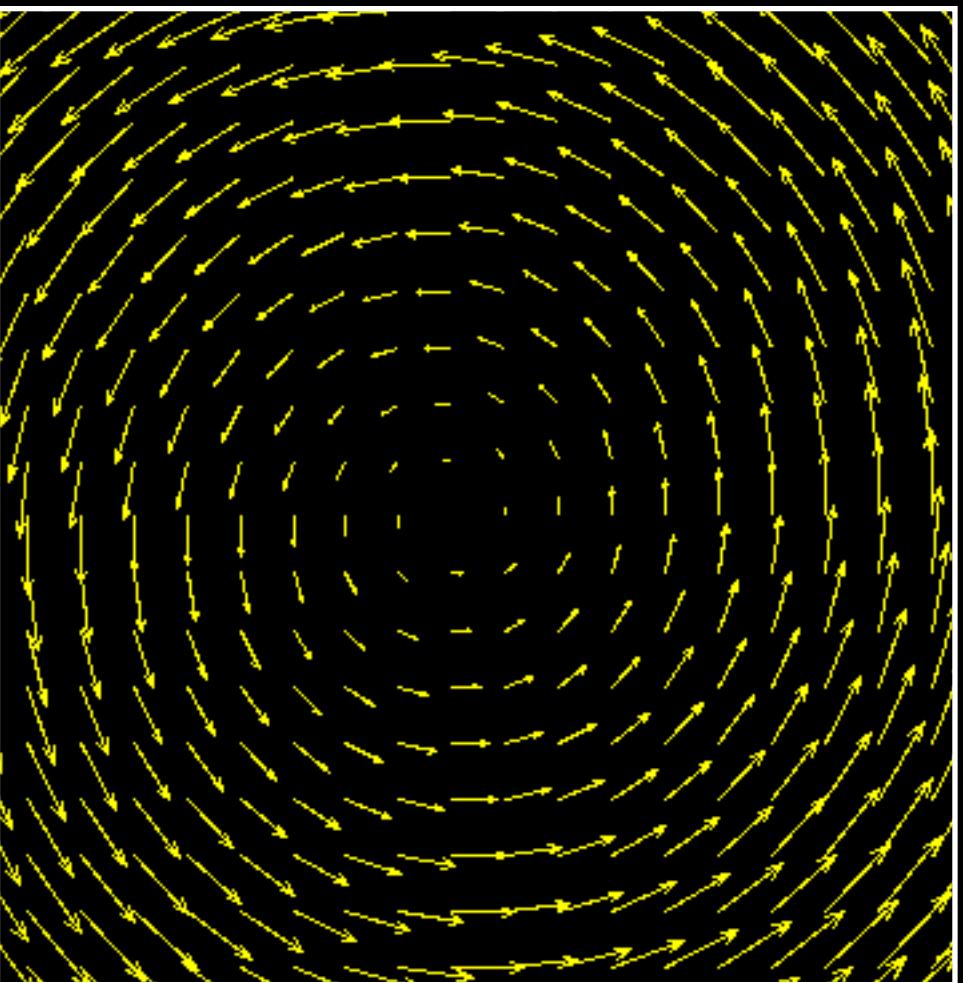
$$\frac{\partial \phi}{\partial t} + \sum u_i \frac{\partial}{\partial x_i} \phi = 0$$

Key component!

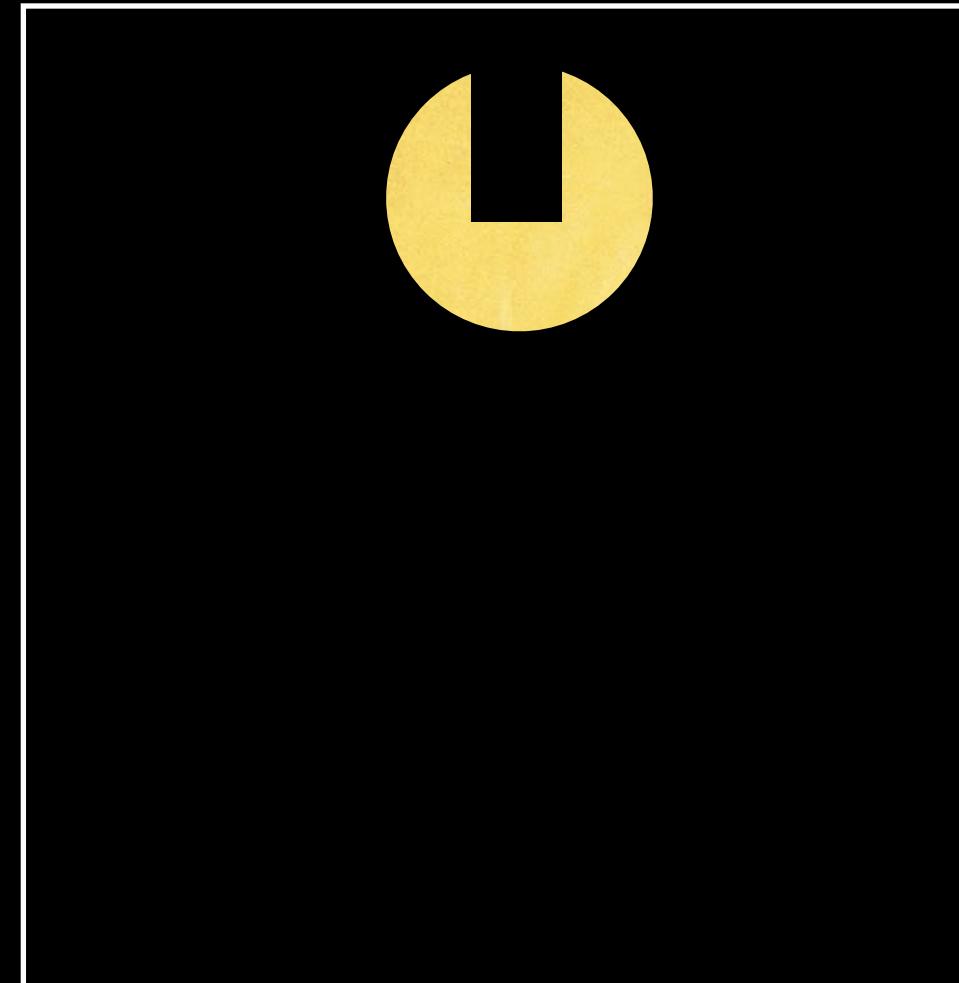
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Velocity field \mathbf{u}

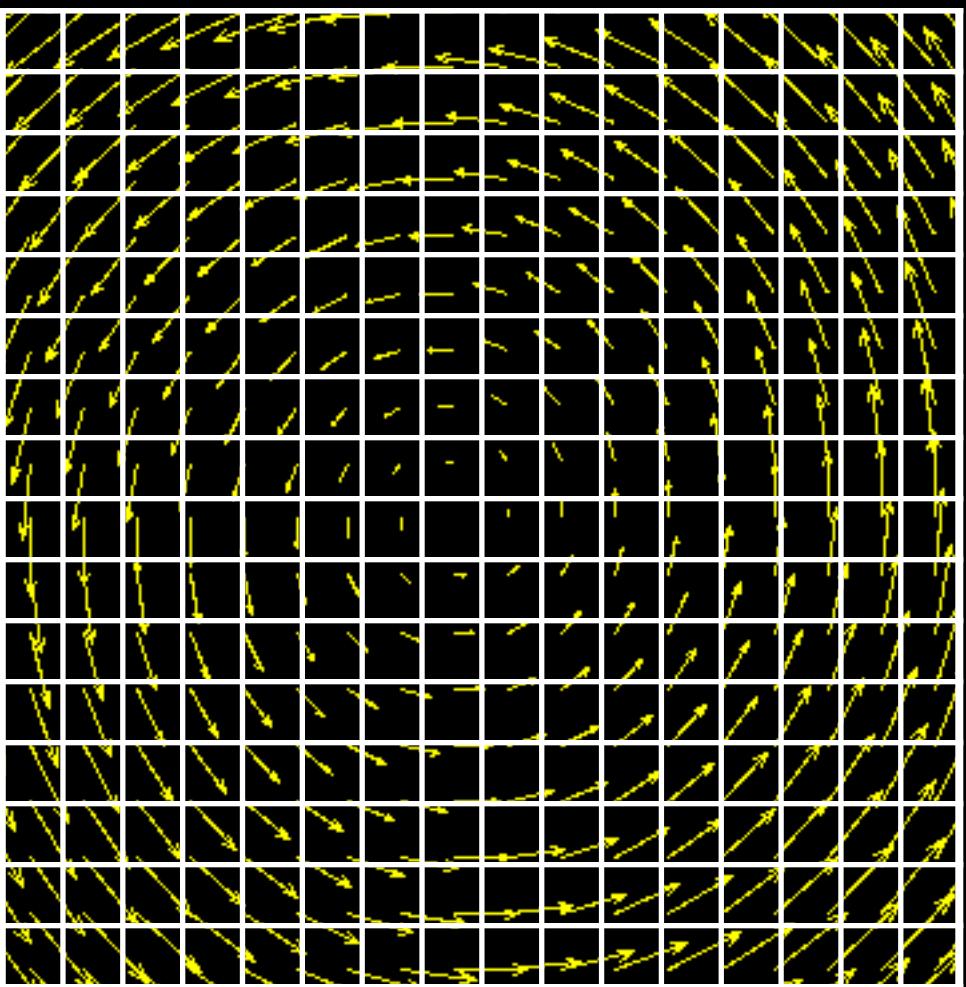


Scalar field ϕ

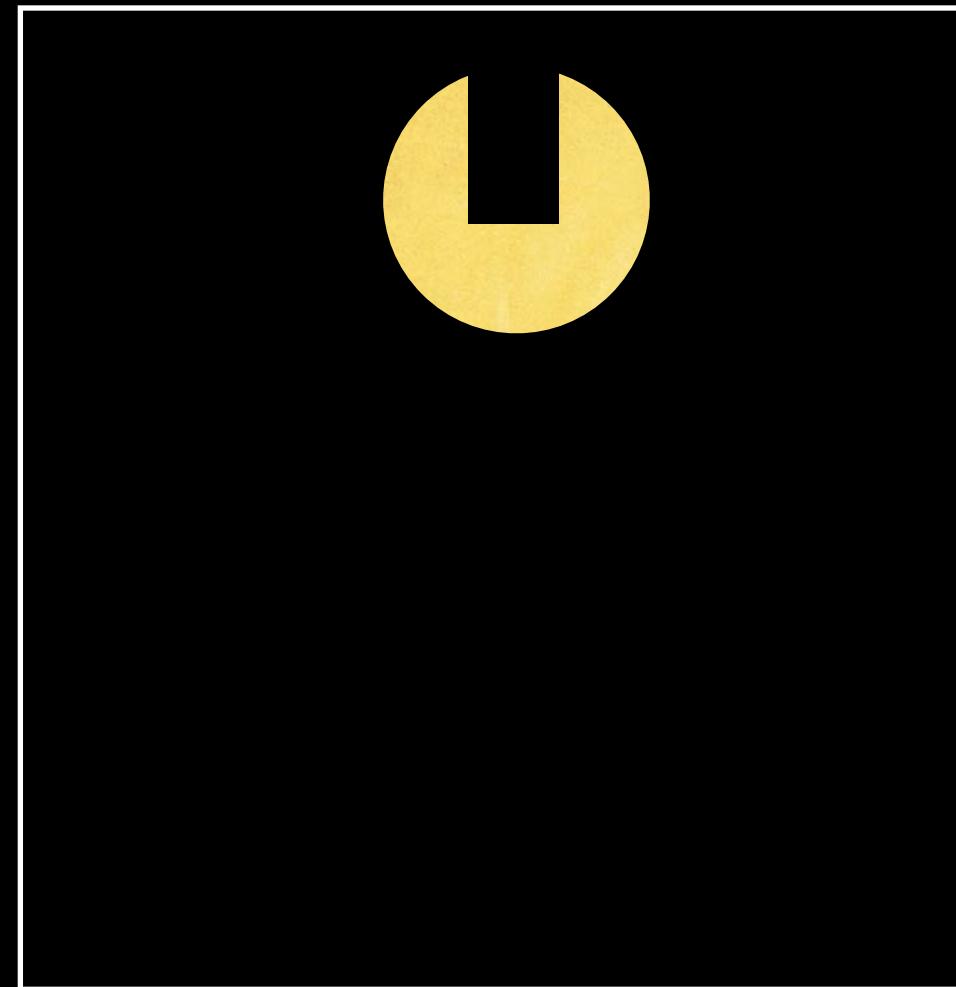
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Velocity field \mathbf{u}

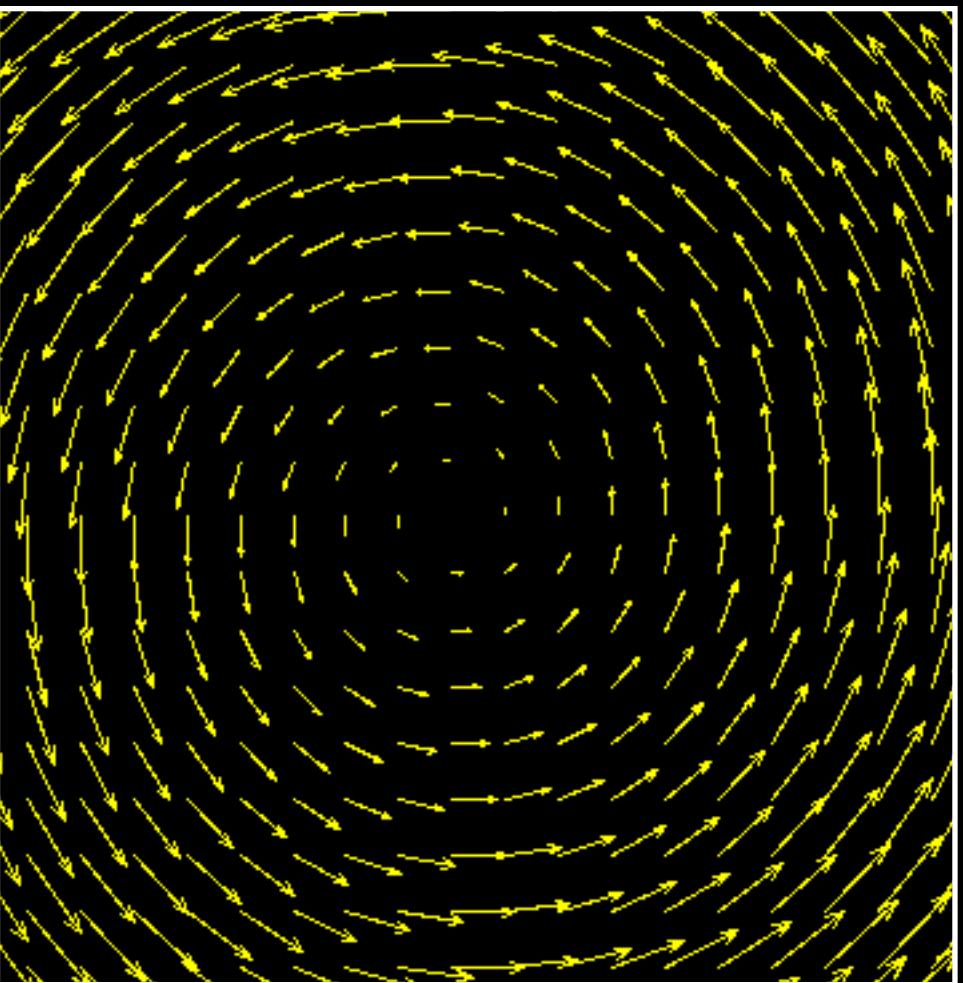


Scalar field ϕ

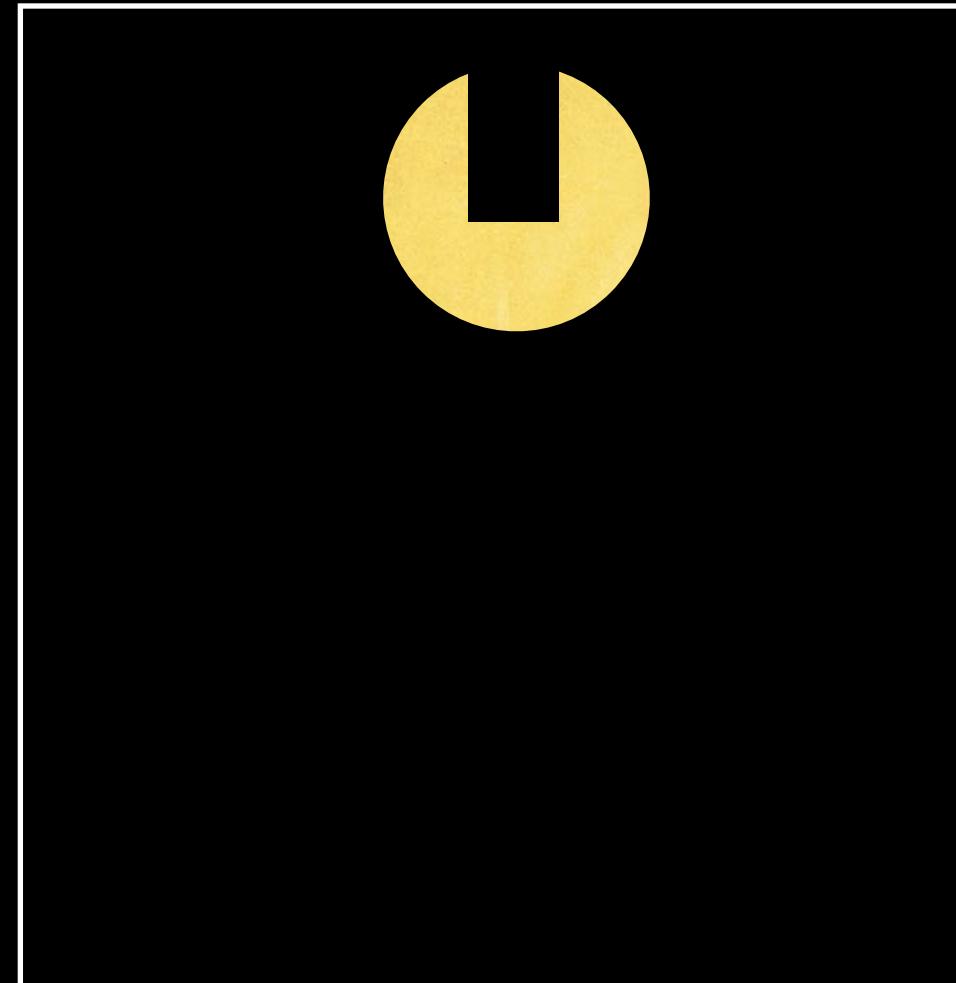
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Velocity field \mathbf{u}

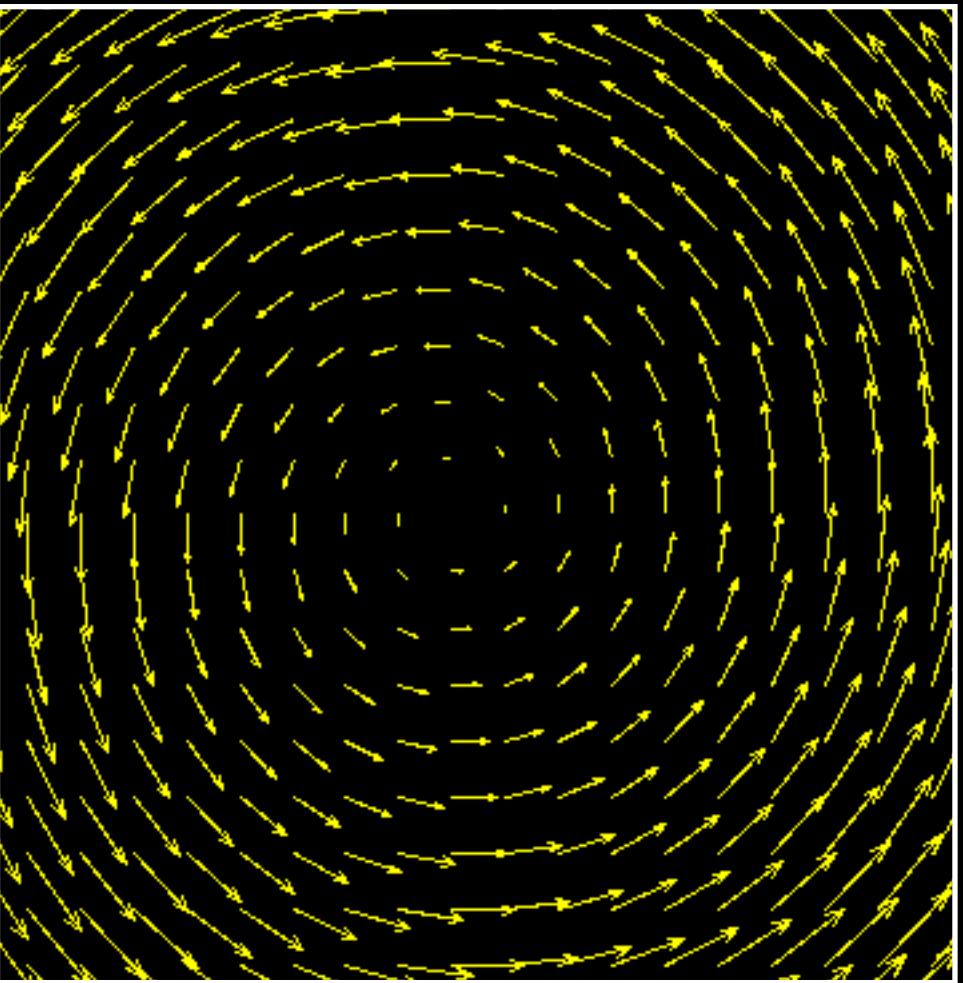


Scalar field ϕ

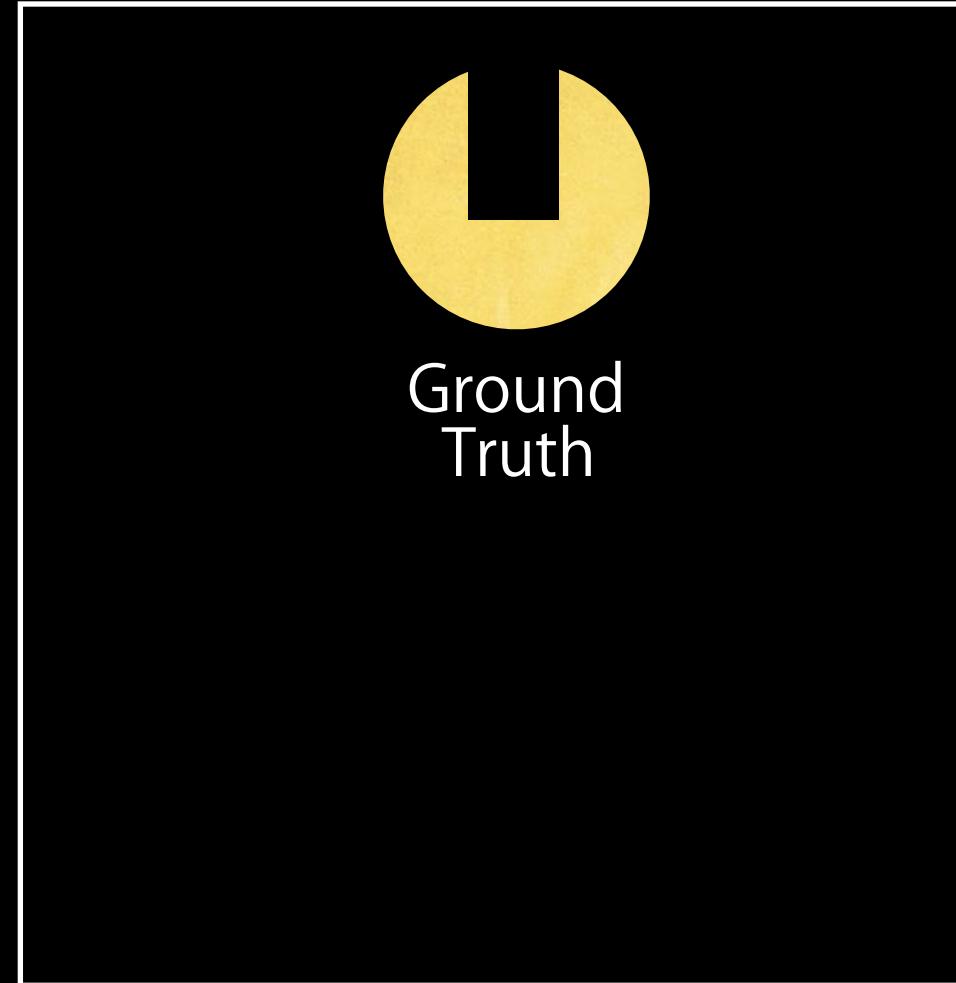
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Velocity field \mathbf{u}



Scalar field ϕ

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

How to discretize $\frac{\partial \phi}{\partial x}$?

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Central Difference

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Central Difference $O(\Delta x^2)$ Accurate

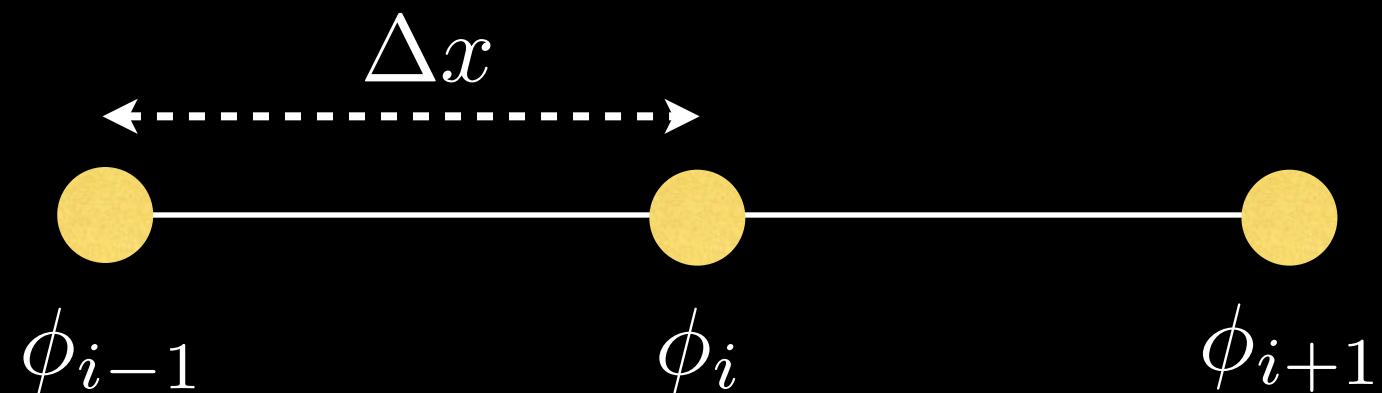
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

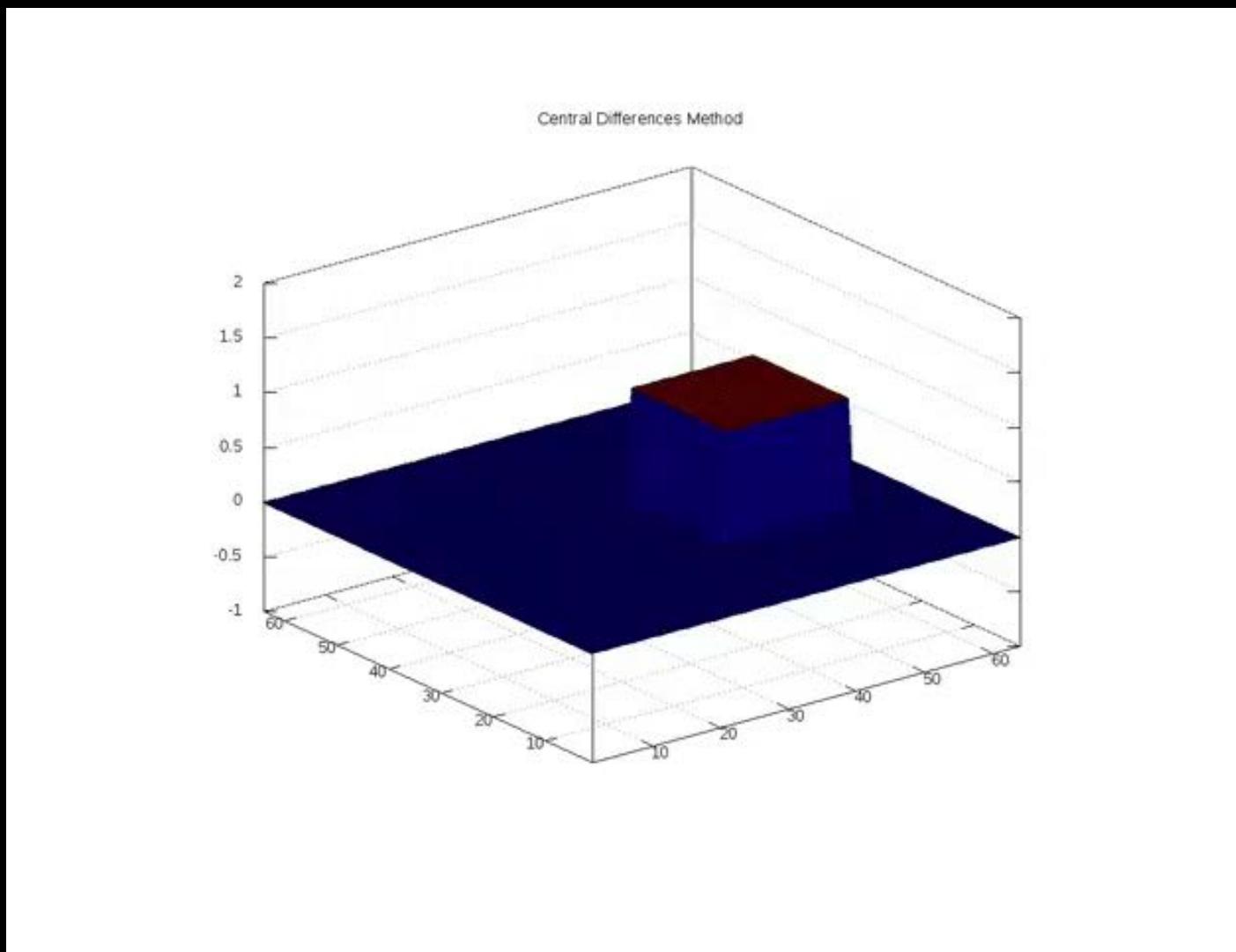
Central Difference $O(\Delta x^2)$ Accurate



Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



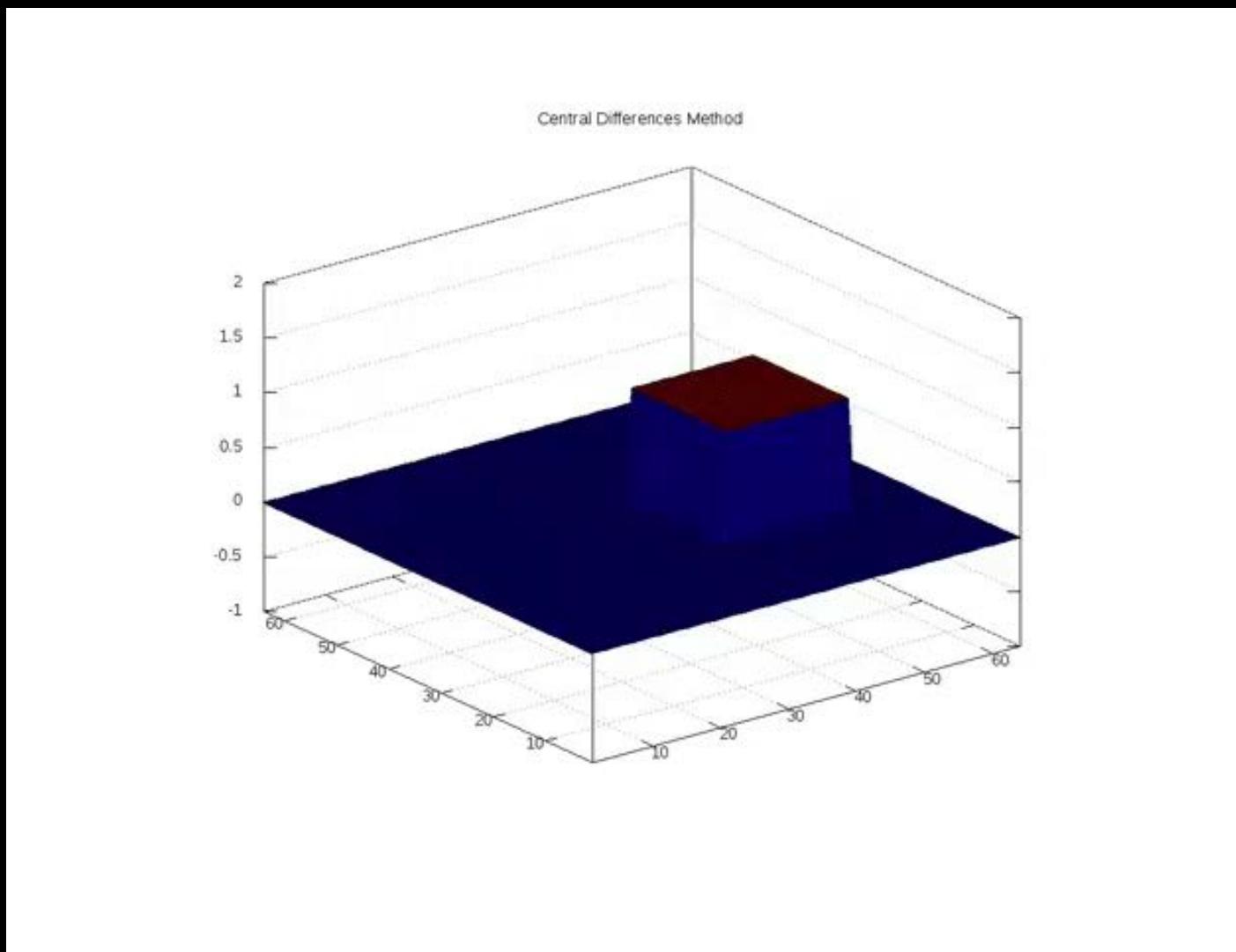
Central Difference

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



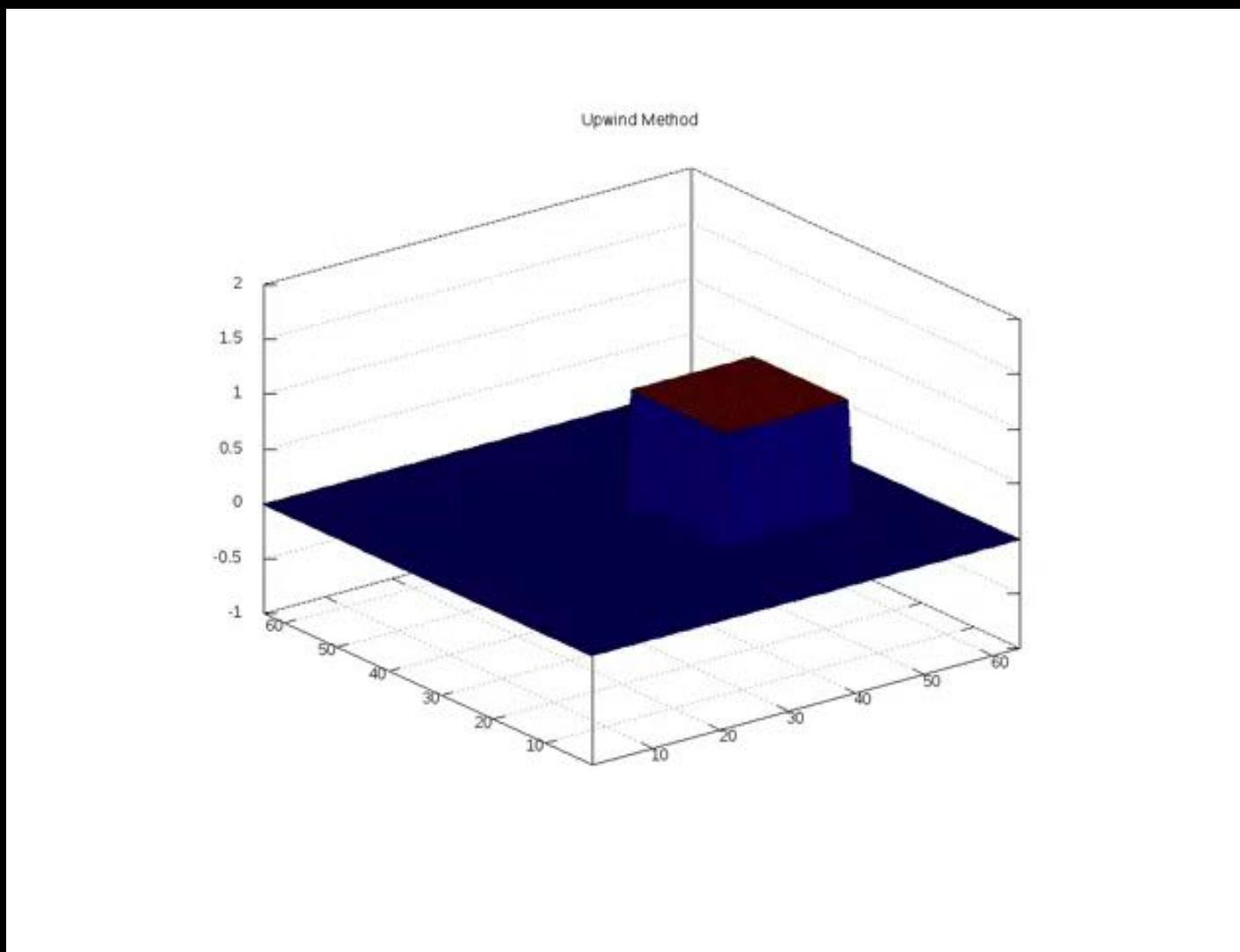
Central Difference

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Upwind Scheme

If $u > 0$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x}$$

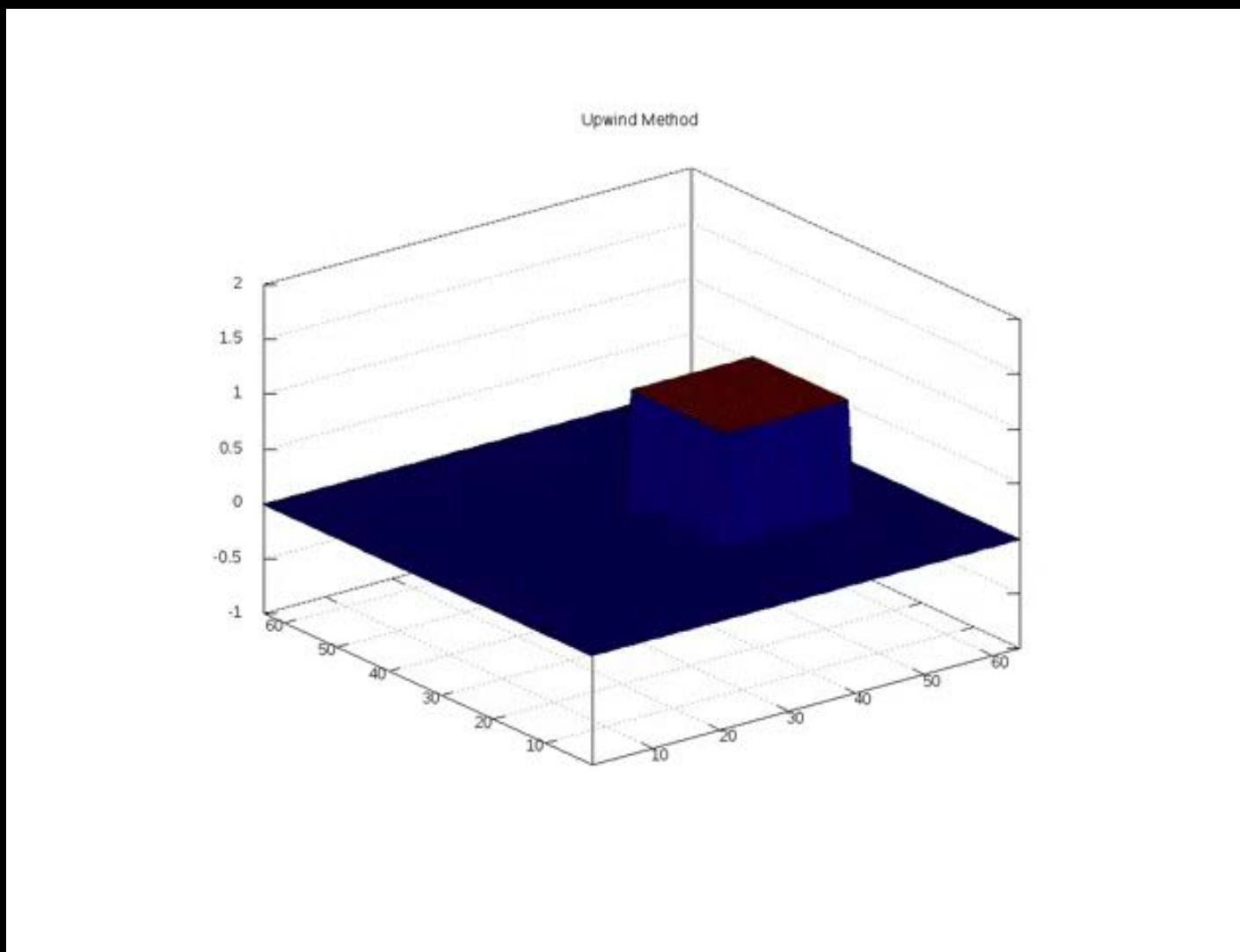
If $u < 0$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Upwind Scheme

If $u > 0$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x}$$

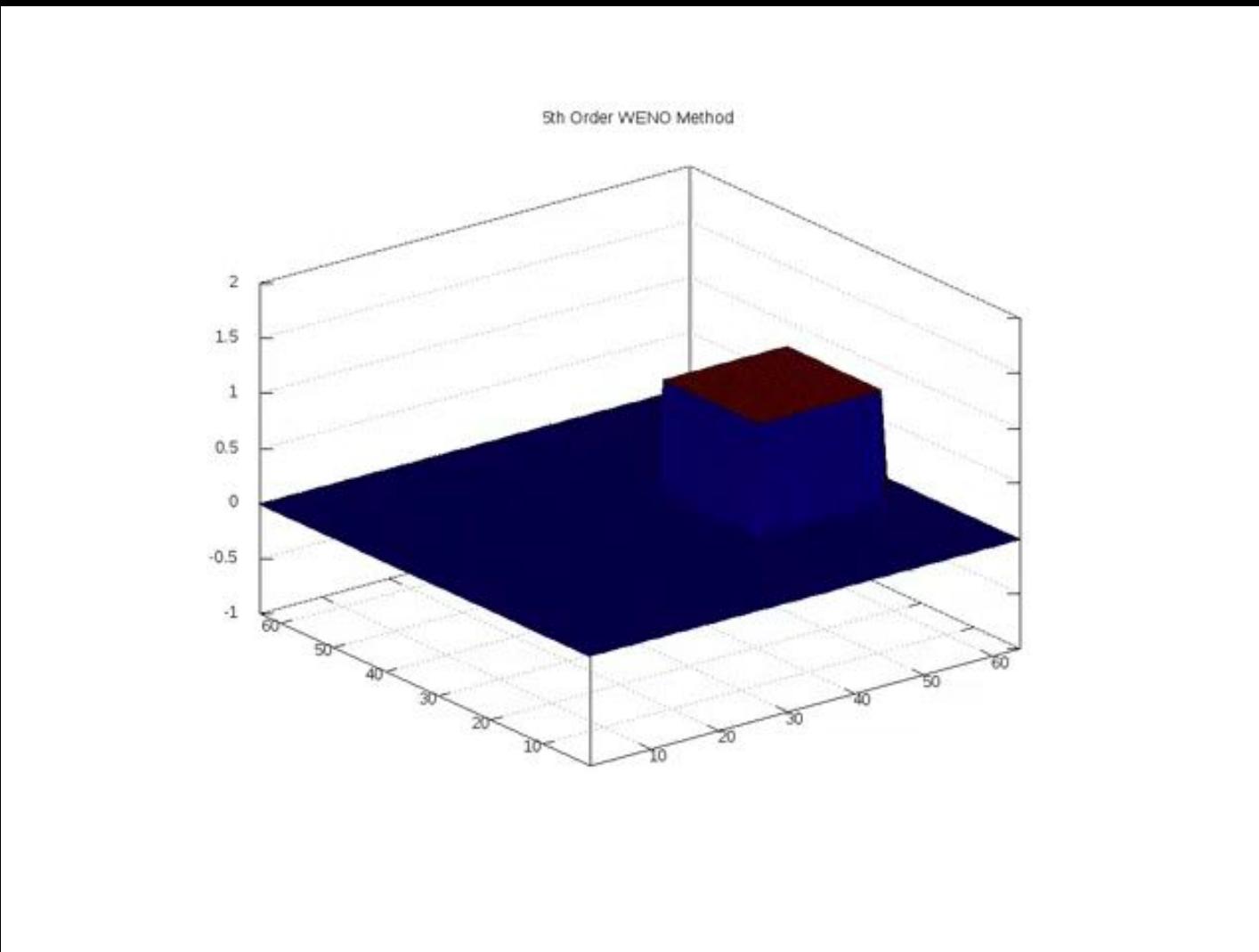
If $u < 0$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



High-order Upwind Scheme
(WENO5)

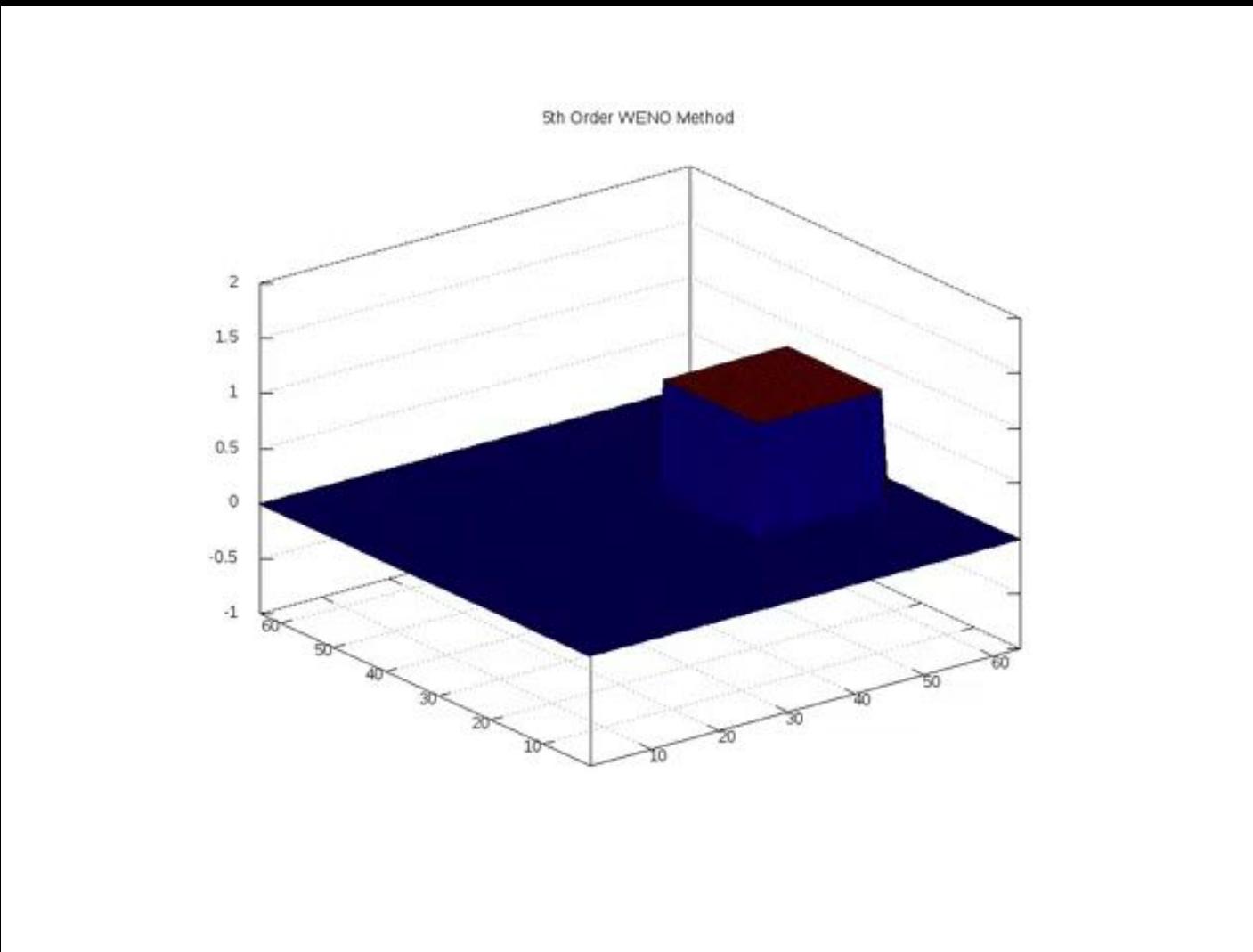
$$\frac{\partial \phi}{\partial x} \approx \frac{1}{\Delta x} [M_{\text{WENO}}]$$

$$\begin{bmatrix} \phi_{i-2} \\ \phi_{i-1} \\ \phi_i \\ \phi_{i+1} \\ \phi_{i+2} \end{bmatrix}$$

Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



High-order Upwind Scheme
(WENO5)

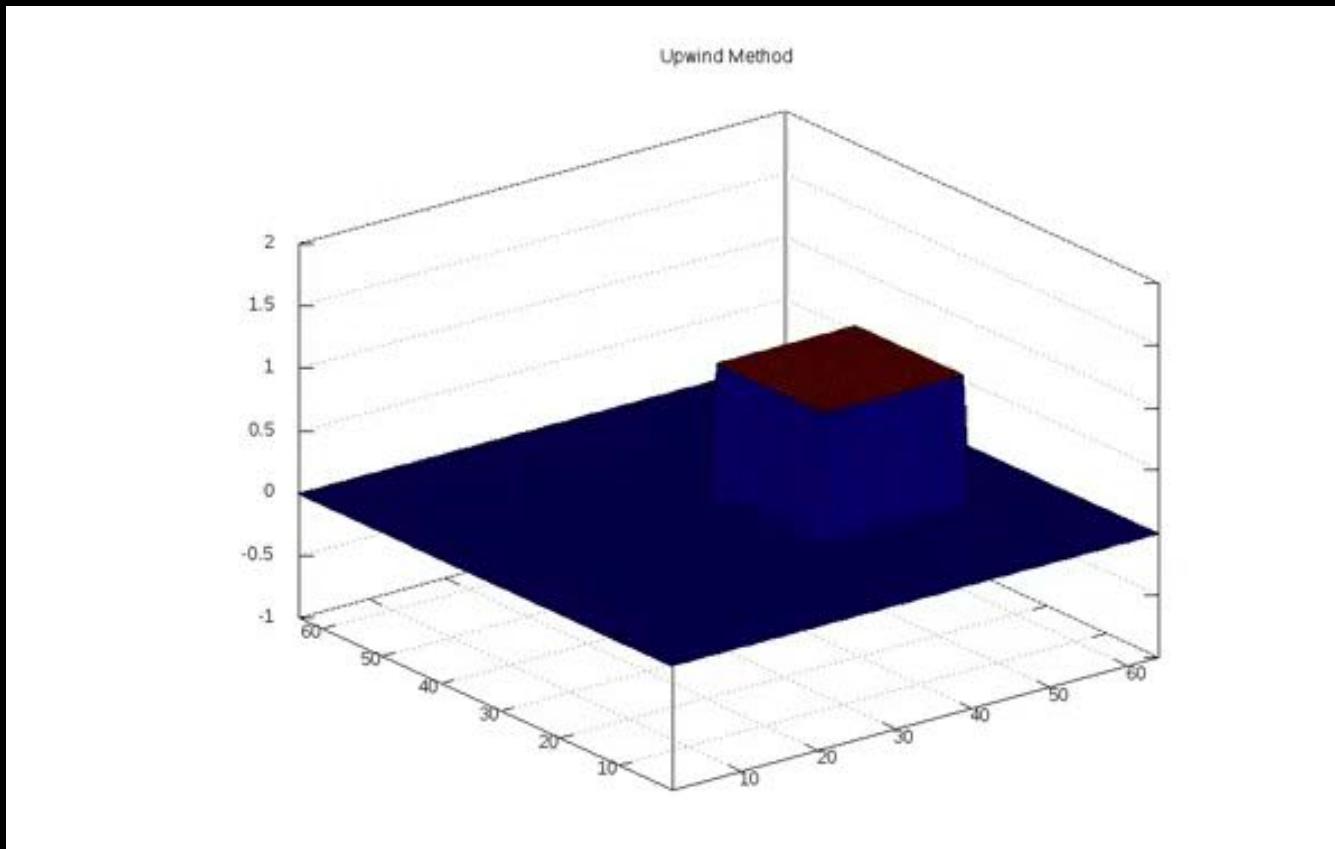
$$\frac{\partial \phi}{\partial x} \approx \frac{1}{\Delta x} [M_{\text{WENO}}]$$

$$\begin{bmatrix} \phi_{i-2} \\ \phi_{i-1} \\ \phi_i \\ \phi_{i+1} \\ \phi_{i+2} \end{bmatrix}$$

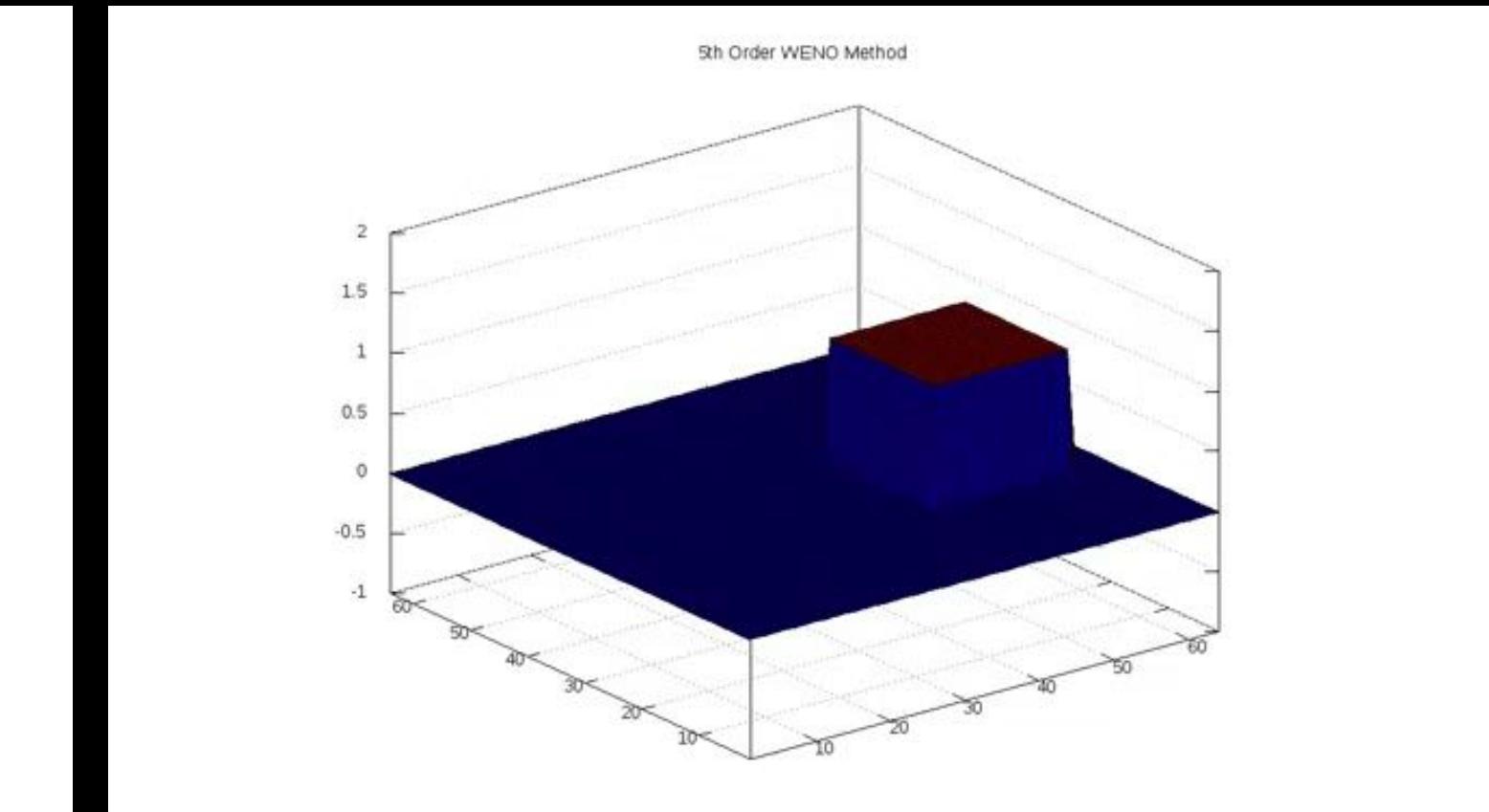
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Upwind Scheme

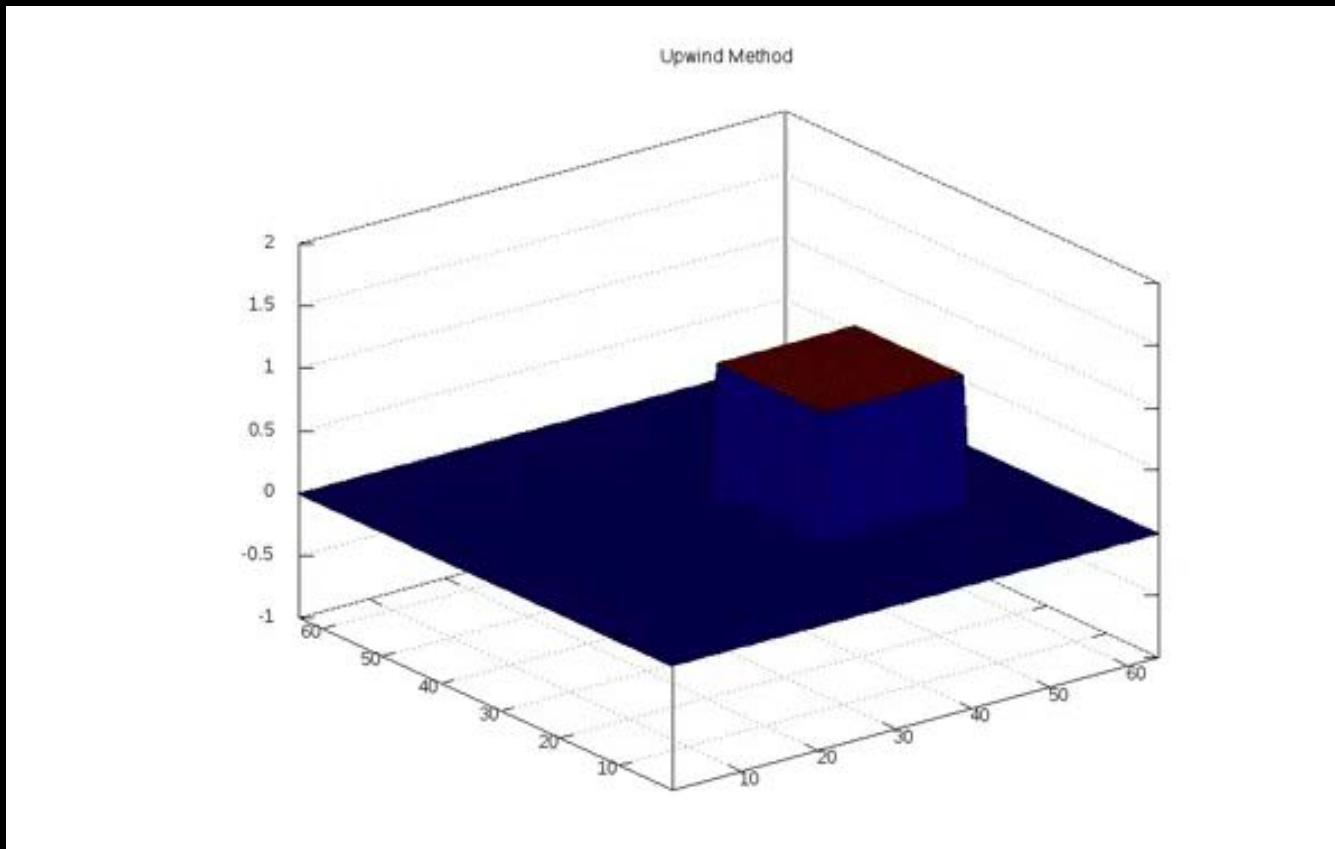


High-order Upwind Scheme
(WENO5)

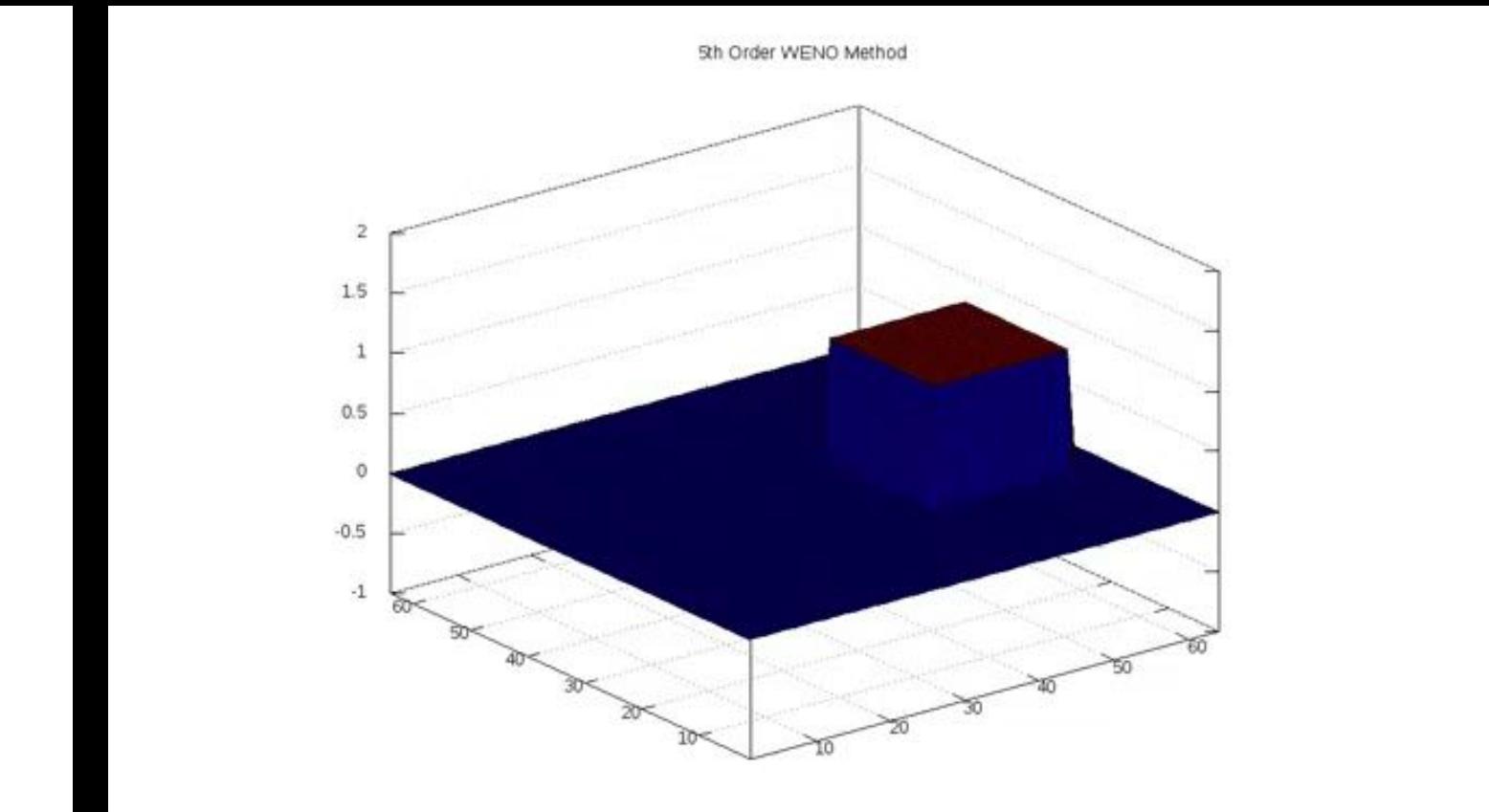
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Upwind Scheme



High-order Upwind Scheme
(WENO5)

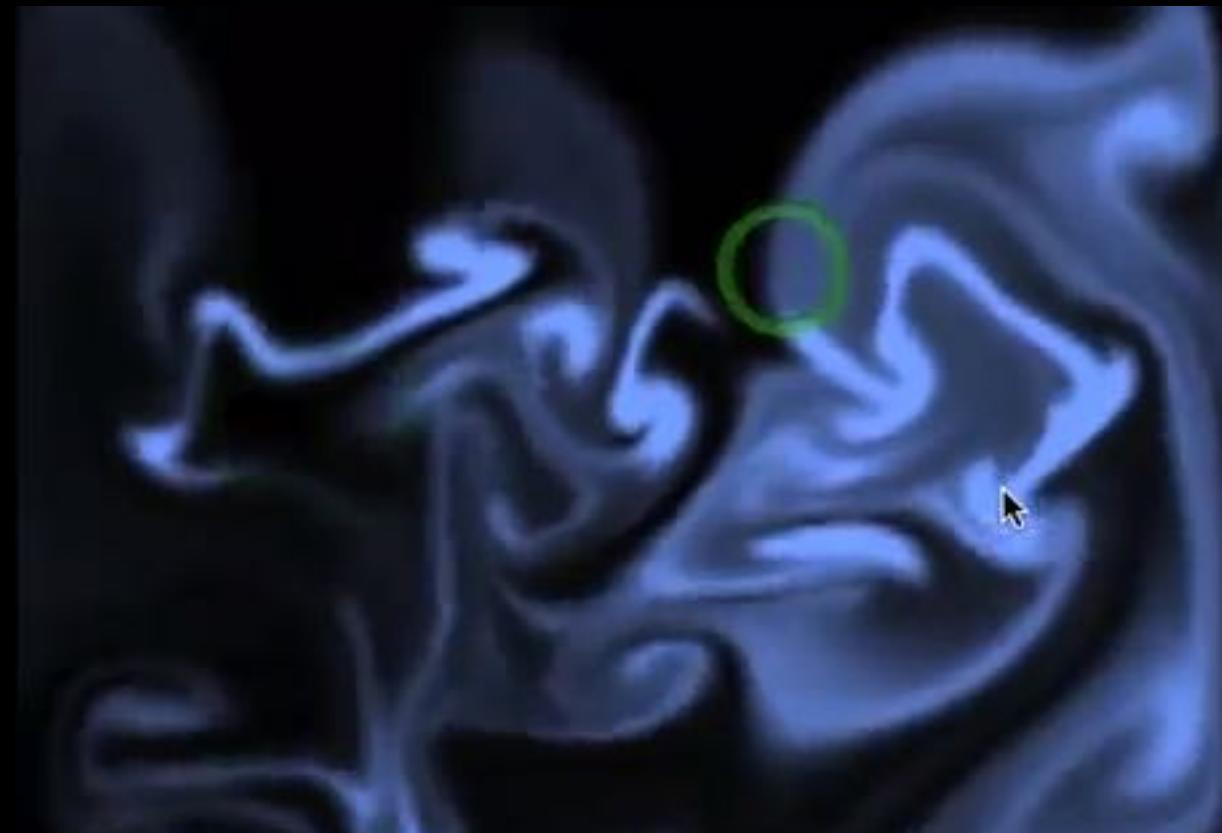
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Upwind Scheme



High-order Upwind Scheme
(WENO5)

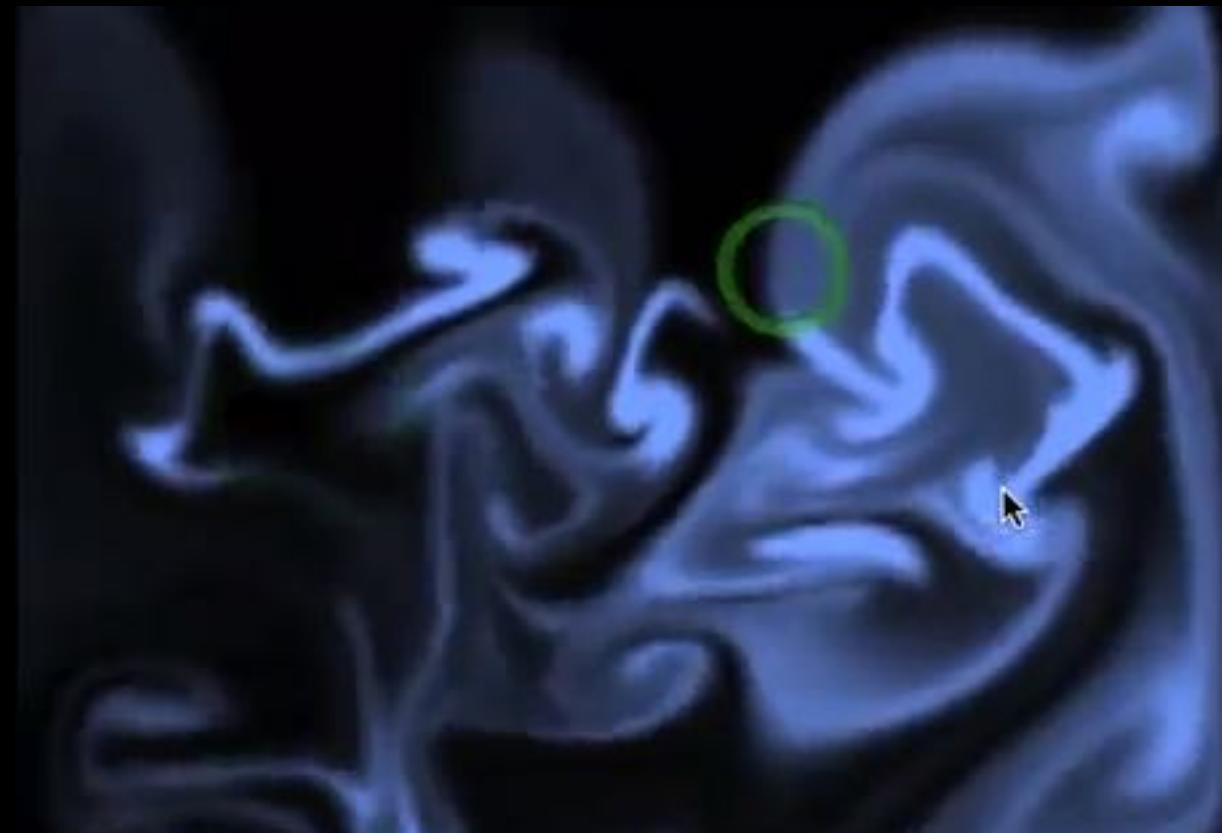
Computing

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term



Upwind Scheme



High-order Upwind Scheme
(WENO5)

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{F}$$

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{F}$$

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p$$

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\left[\frac{\partial \mathbf{u}_{t+\Delta t} - \mathbf{u}^*}{\partial t} + \frac{(\mathbf{u} \cdot \nabla) \mathbf{u}}{\Delta t} \right] = -\frac{1}{\rho} \nabla p$$

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\frac{\mathbf{u}_{t+\Delta t} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho} \nabla p$$

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\nabla \cdot \frac{\mathbf{u}_{t+\Delta t} - \mathbf{u}^*}{\Delta t} = \nabla \cdot -\frac{1}{\rho} \nabla p$$

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\nabla \cdot \frac{\cancel{u_{t+\Delta t} - u^*}}{\Delta t} = \nabla \cdot -\frac{1}{\rho} \nabla p$$

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\nabla \cdot \mathbf{u}^* = \frac{\Delta t}{\rho} \nabla^2 p$$

Pressure Poisson Equation

Computing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\nabla \cdot \mathbf{u}^* = \frac{\Delta t}{\rho} \nabla^2 p$$

Pressure Poisson Equation

Boundary Conditions of

$$-\frac{1}{\rho} \nabla p$$

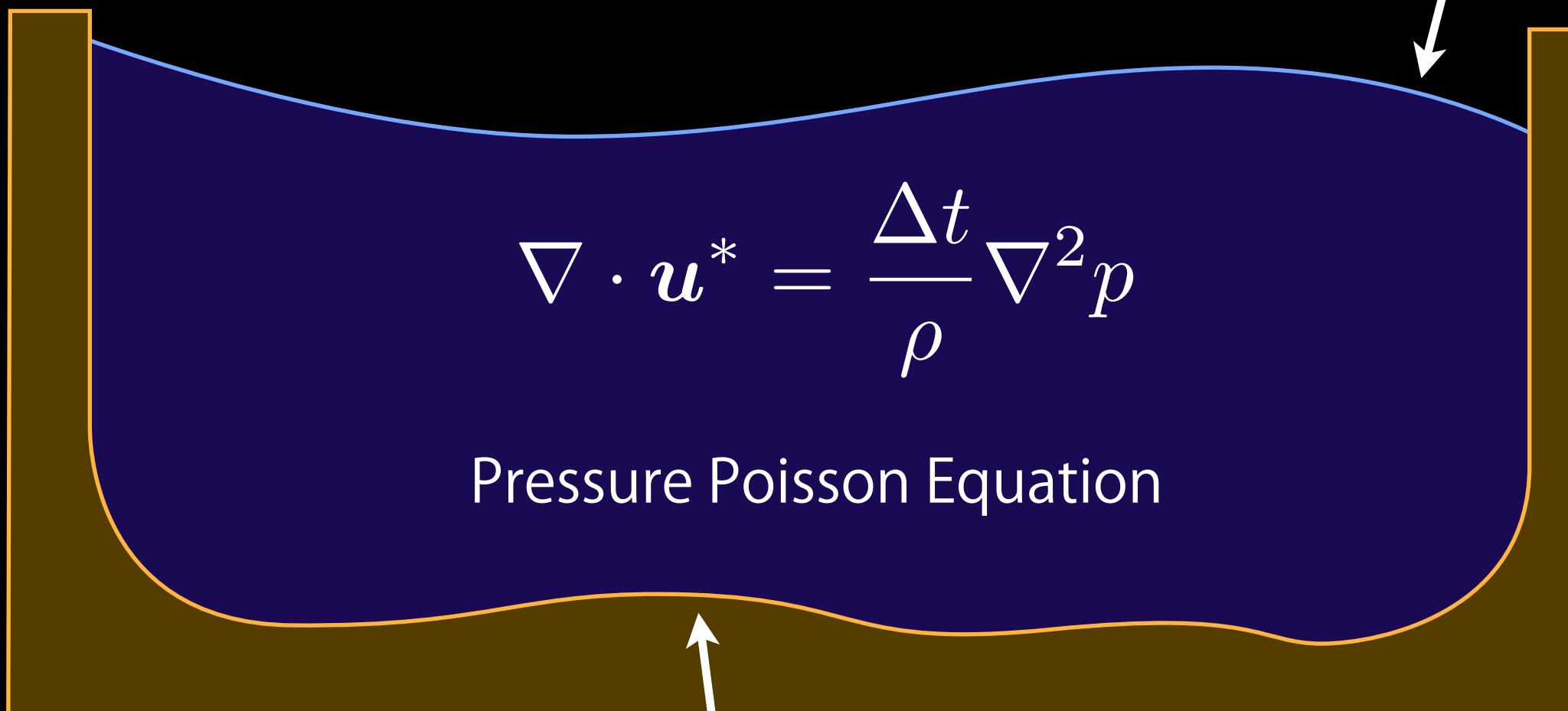
Pressure term

$$p = 0$$

$$\nabla \cdot u^* = \frac{\Delta t}{\rho} \nabla^2 p$$

Pressure Poisson Equation

$$n \cdot \nabla p = 0$$



Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\nabla \cdot u^* = \frac{\Delta t}{\rho} \nabla^2 p$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\int_{\Omega} \nabla \cdot \mathbf{u}^* = \frac{\Delta t}{\rho} \nabla^2 p$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\int_{\Omega} \nabla \cdot u^* dV = \int_{\Omega} \frac{\Delta t}{\rho} \nabla^2 p dV$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\int_{\Omega} \nabla \cdot \mathbf{u}^* dV = \int_{\Omega} \frac{\Delta t}{\rho} \nabla^2 p dV$$

Divergence Theorem 

$$\oint_{\Omega} \mathbf{u}^* \cdot \mathbf{n} = \oint_{\Omega} \frac{\Delta t}{\rho} \nabla p \cdot \mathbf{n}$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\oint_{\Omega} u^* \cdot n = \oint_{\Omega} \frac{\Delta t}{\rho} \nabla p \cdot n$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\oint_{\Omega} \text{blue square} \rightarrow u^* = \oint_{\Omega} \text{yellow square} \rightarrow \nabla p$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\oint_{\Omega} \text{blue square} \rightarrow u^* = \oint_{\Omega} \text{yellow square} \rightarrow \nabla p$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\sum_i^4 \text{blue square} \rightarrow u^* = \sum_i^4 \text{yellow square} \rightarrow \nabla p$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

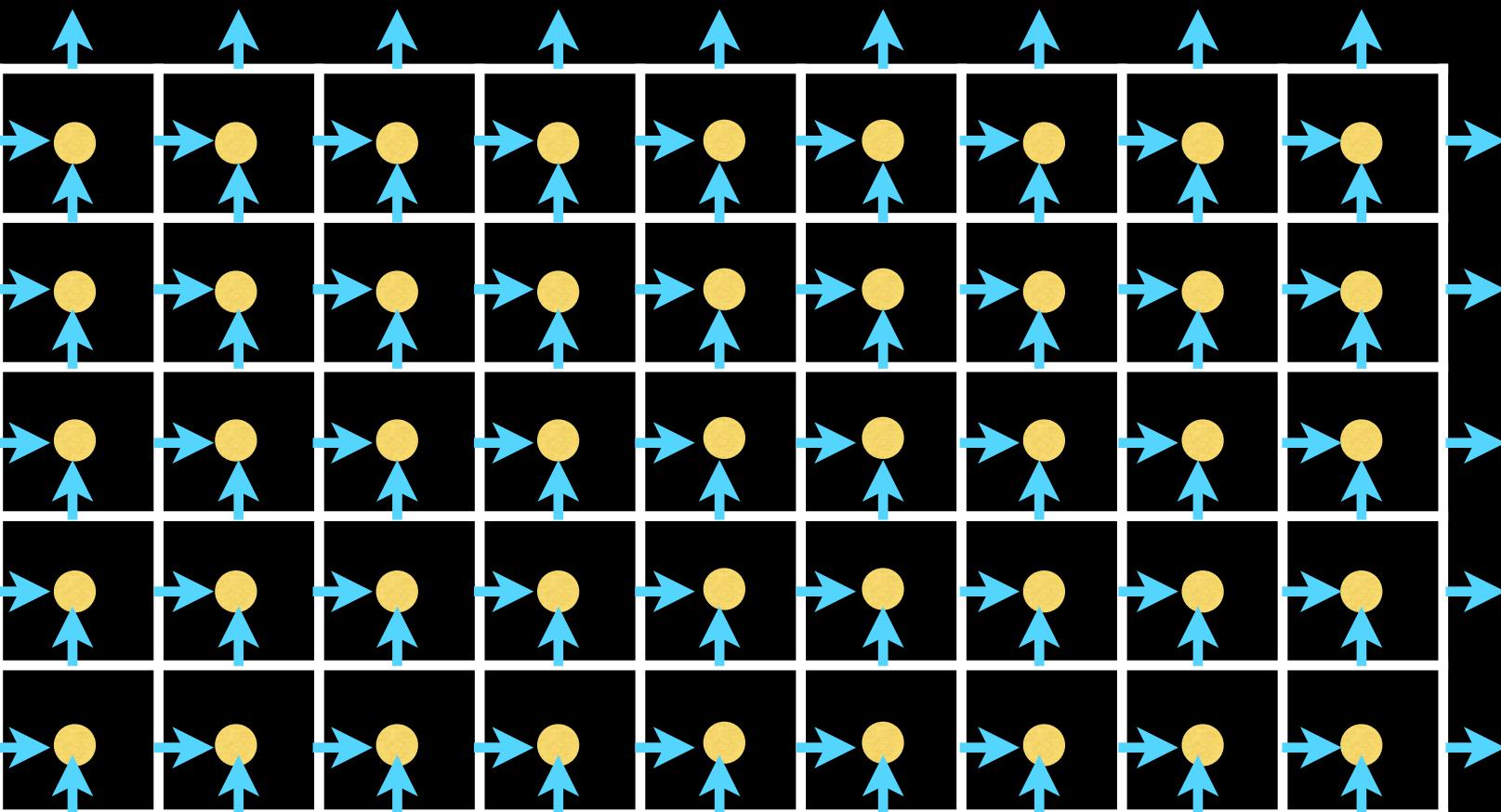
Pressure term

$$\sum_i^4 \text{blue square} \rightarrow u^* = \sum_i^4 \text{yellow square with dashed border}$$
$$\nabla p = \frac{p_{i+1} - p_i}{\Delta x}$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

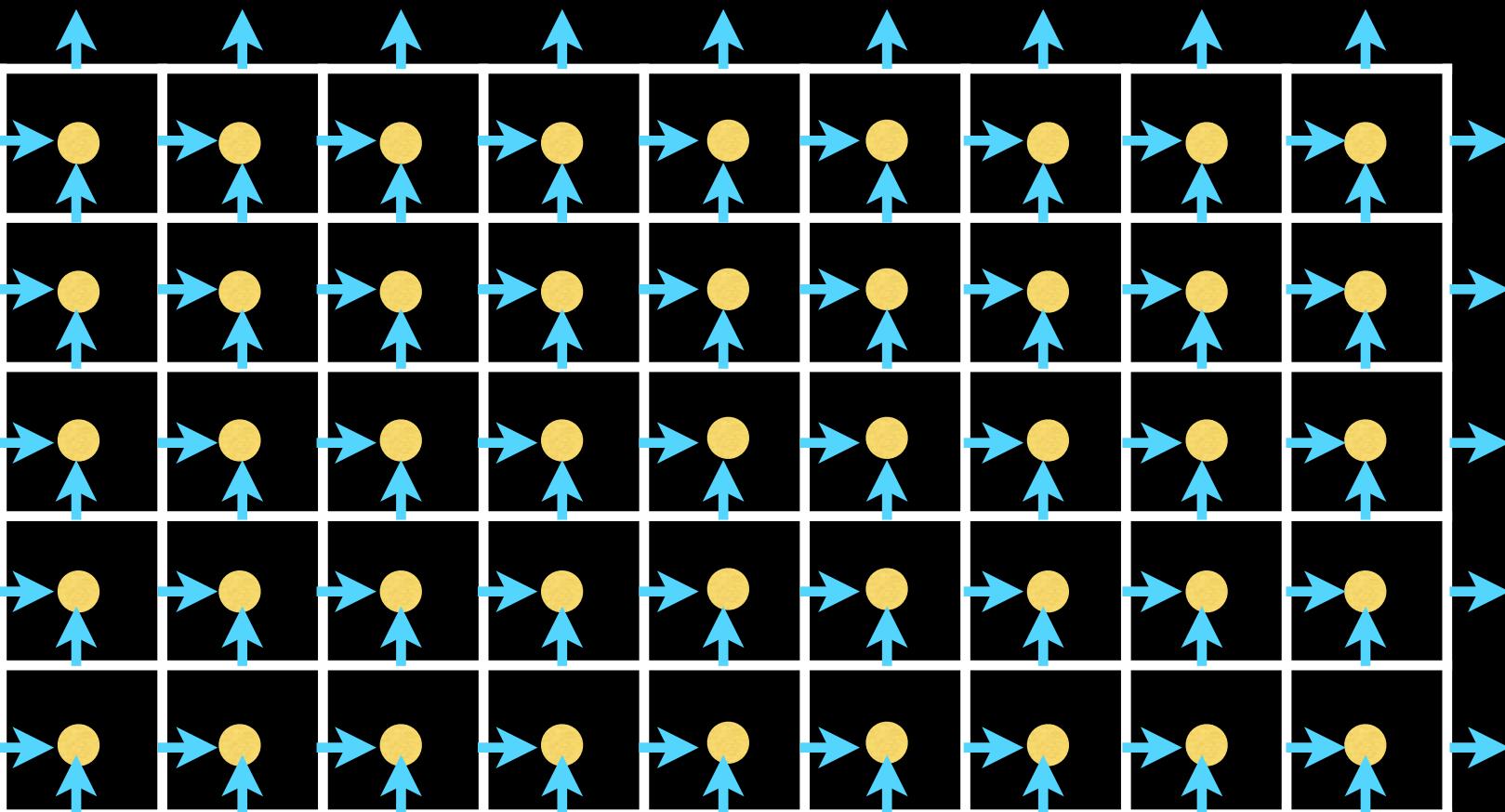
Pressure term



Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

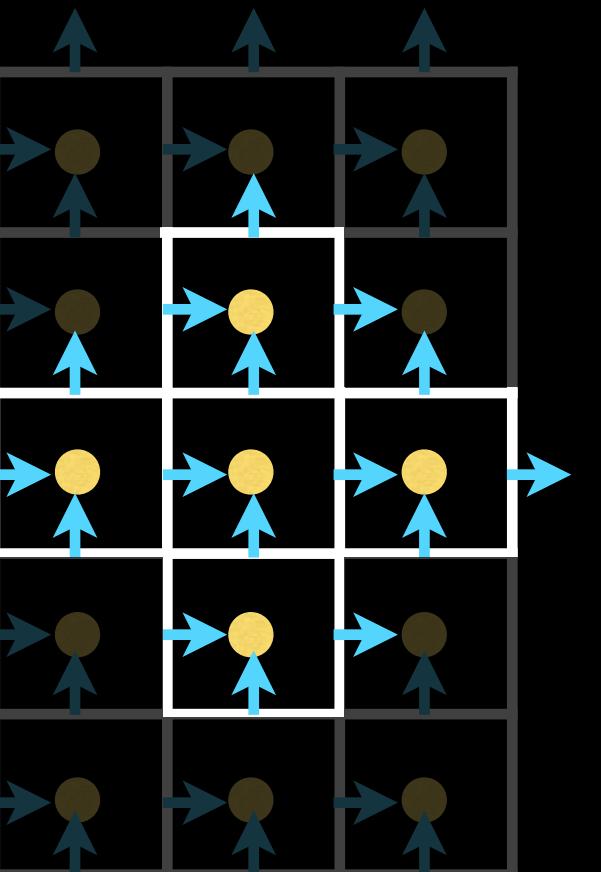


MAC Grid

Discretizing

$$-\frac{1}{\rho} \nabla p$$

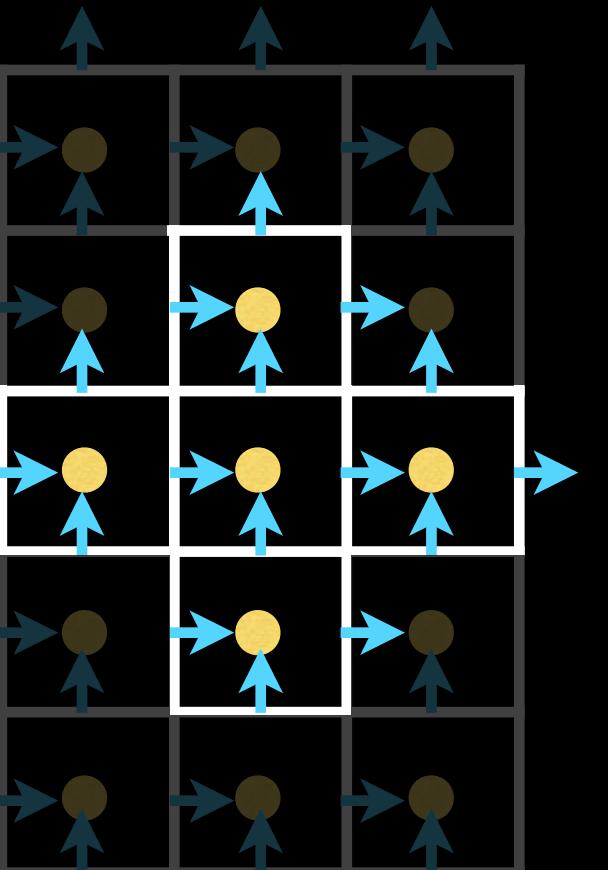
Pressure term



Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

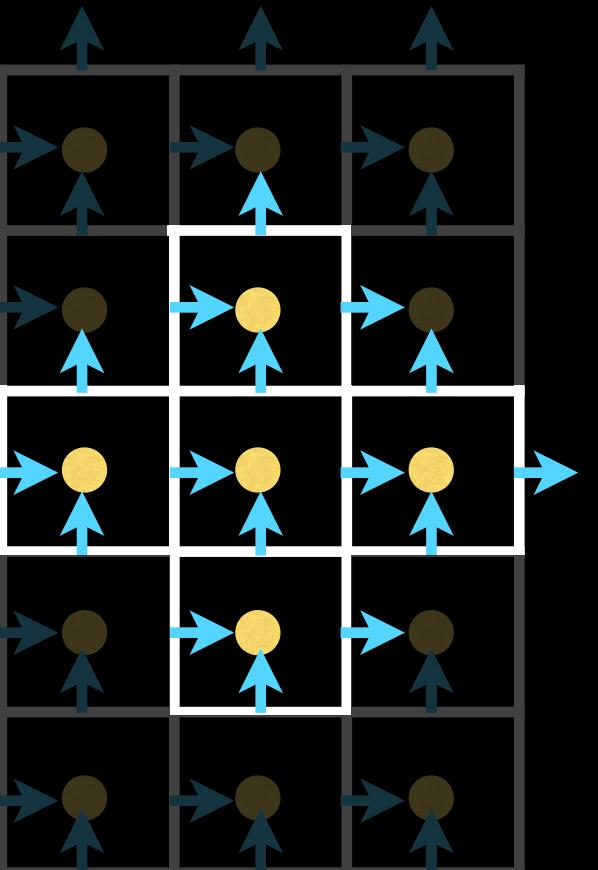


$$\int_{\Omega} \square \rightarrow \nabla p$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

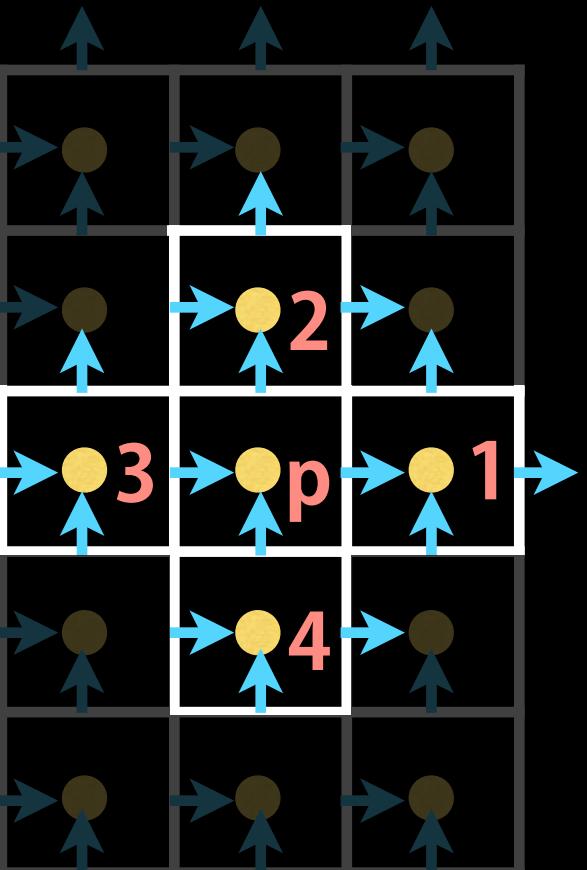


$$\int_{\Omega} \nabla p = \sum_i^4$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

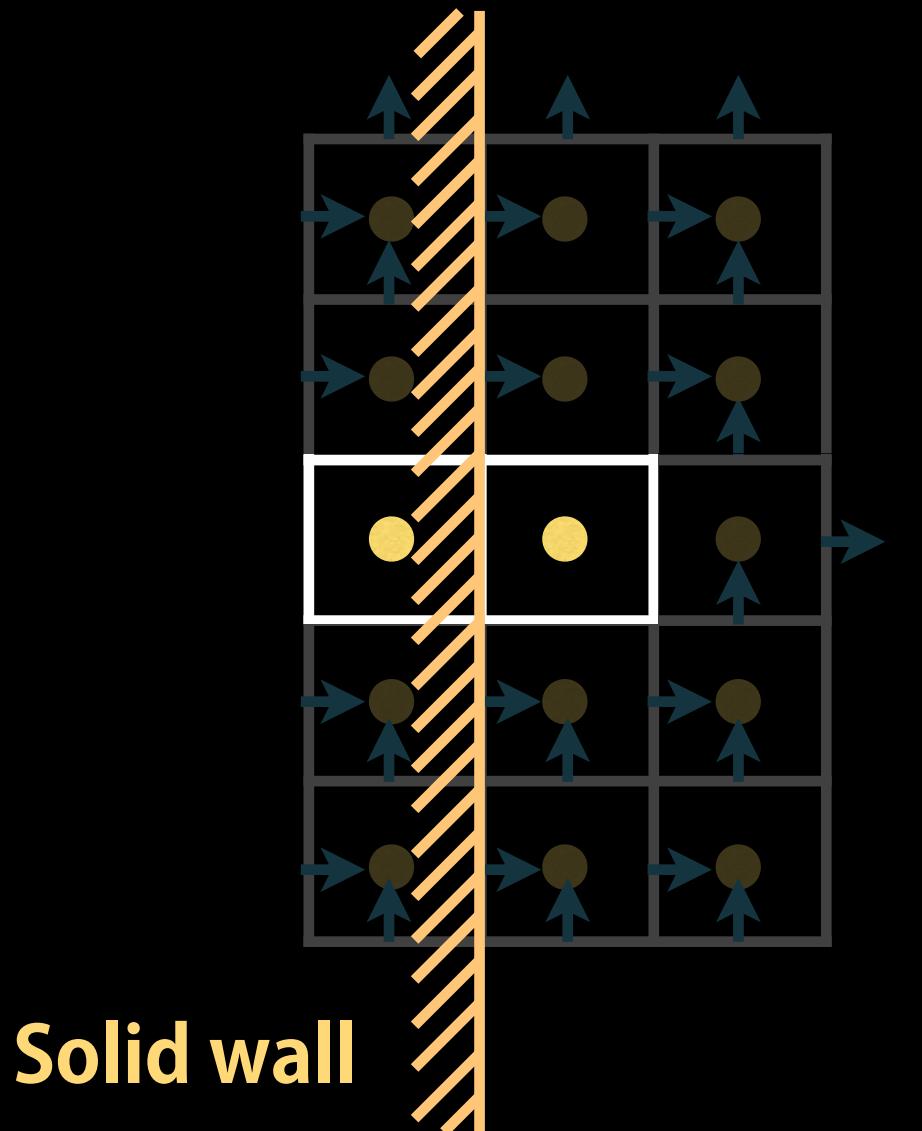


$$\int_{\Omega} \nabla p = \sum_i^4 \frac{4p - p_1 - p_2 - p_3 - p_4}{\Delta x}$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

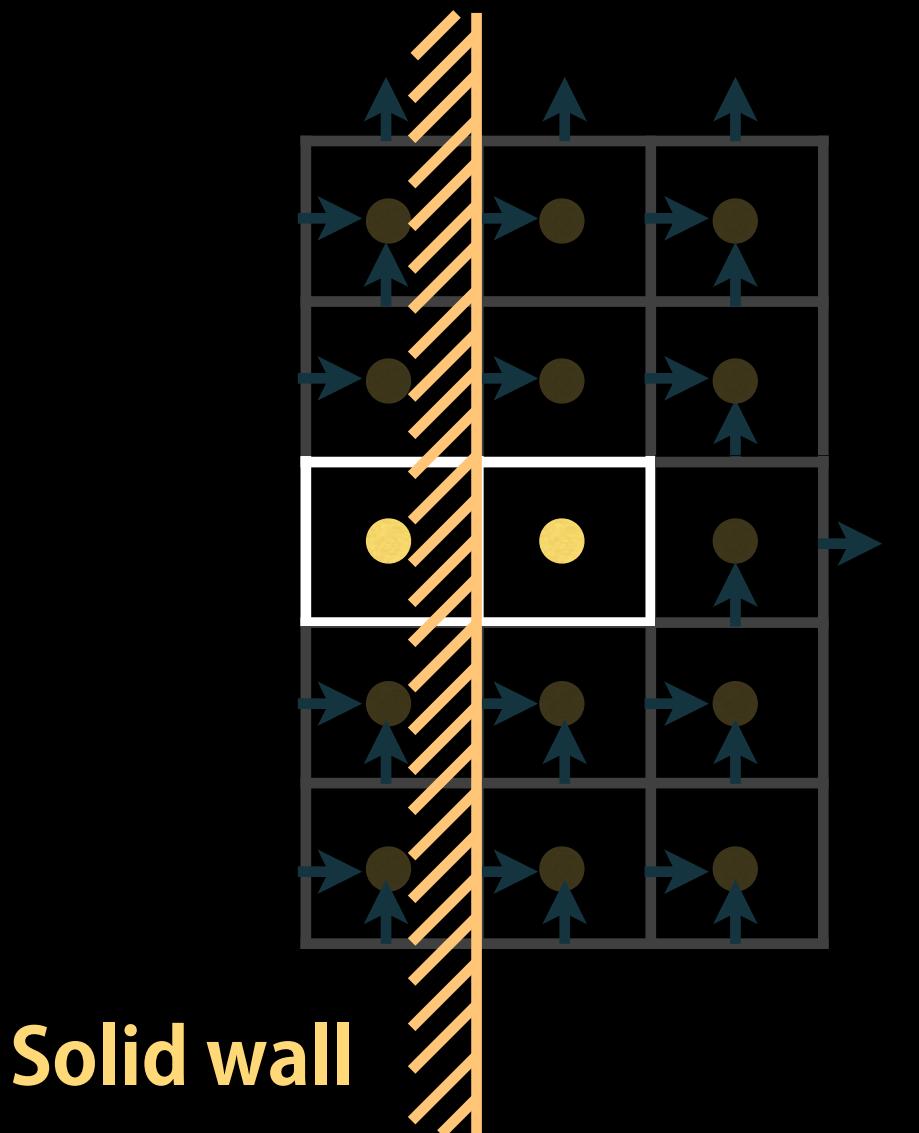


Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

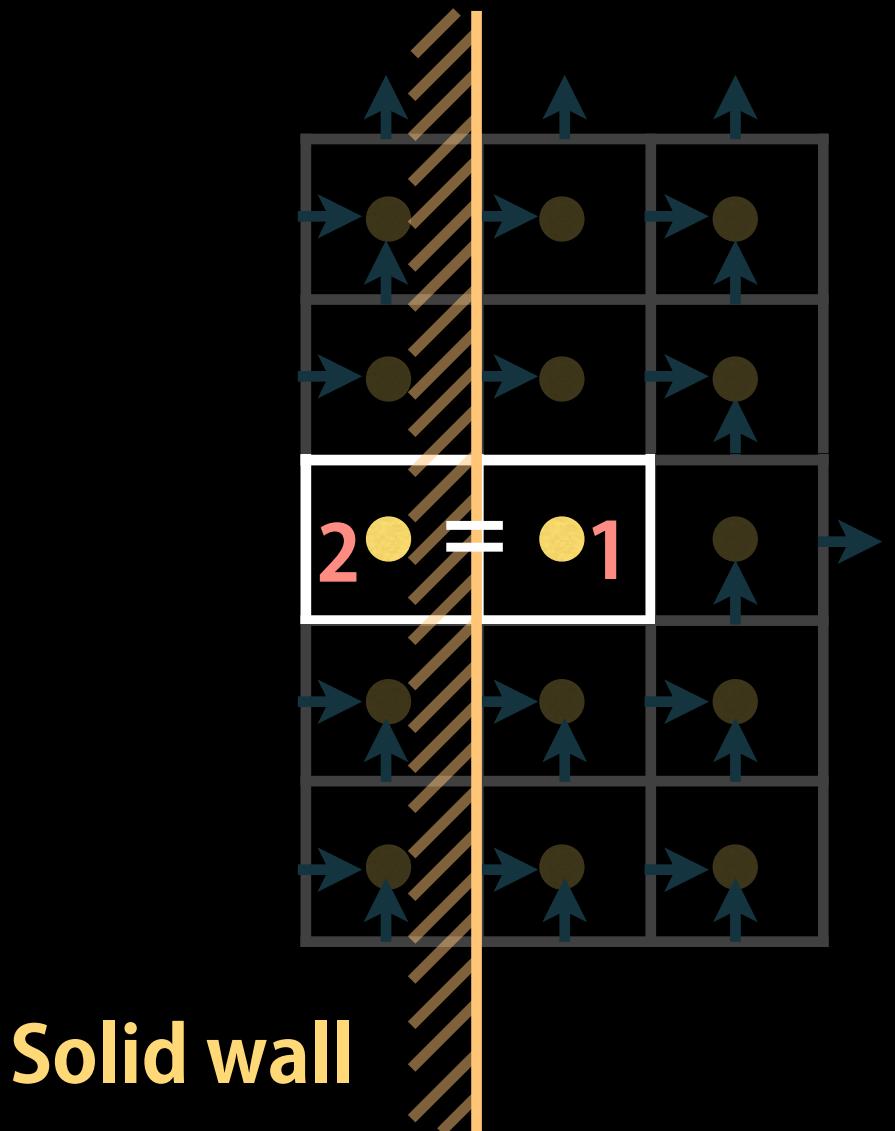
$$\mathbf{n} \cdot \nabla p = 0$$



Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term



$$\mathbf{n} \cdot \nabla p = 0$$



$$p_1 = p_2$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

Empty Matrix

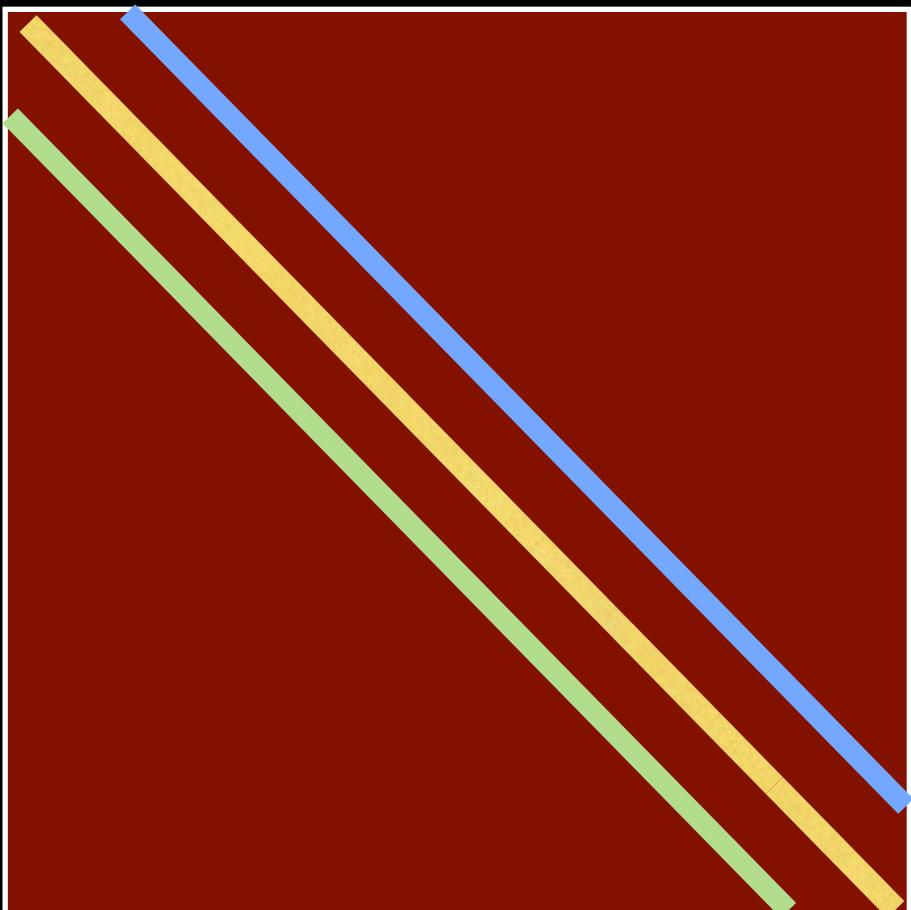
$$\frac{4p - p_1 - p_2 - p_3 - p_4}{\Delta x}$$

$$p_1 = p_2$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

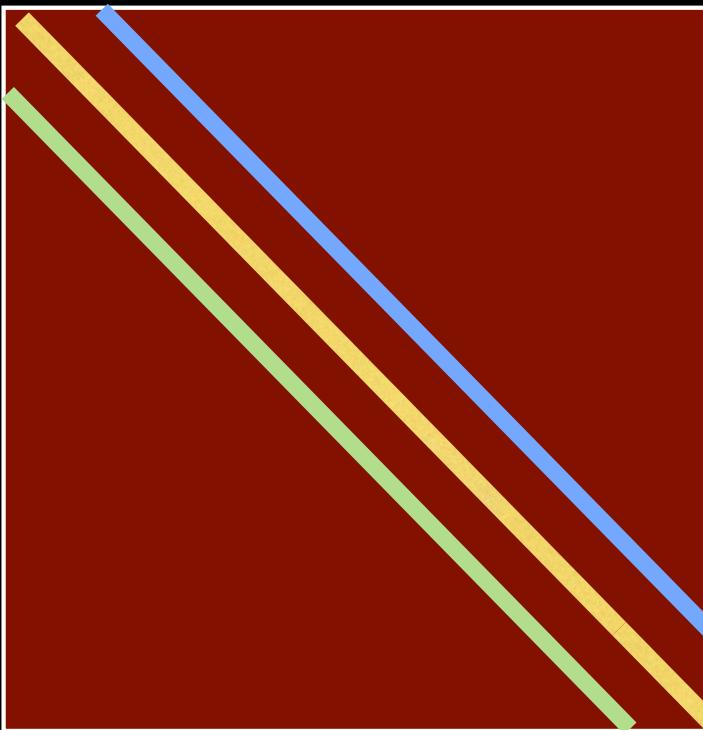


$$p_1 = p_2$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

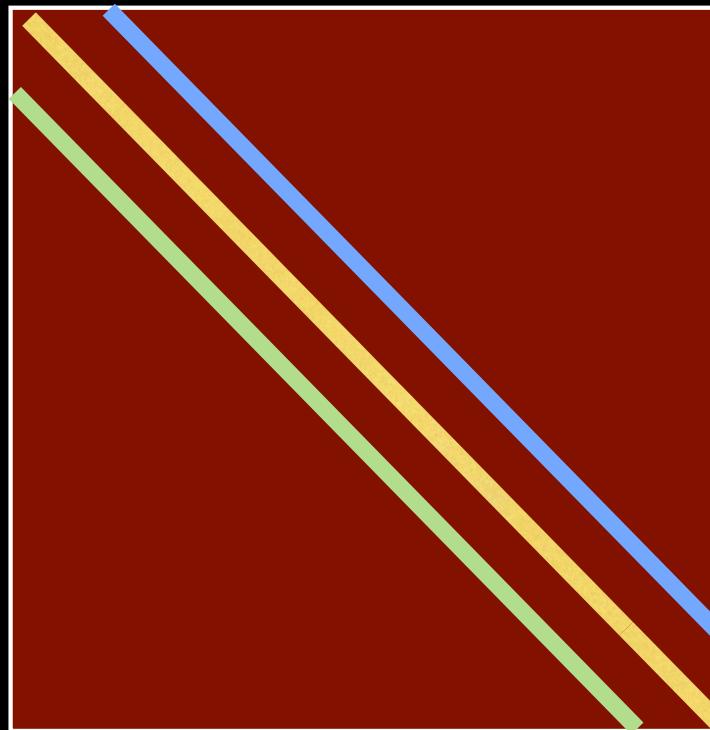
Pressure term



Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

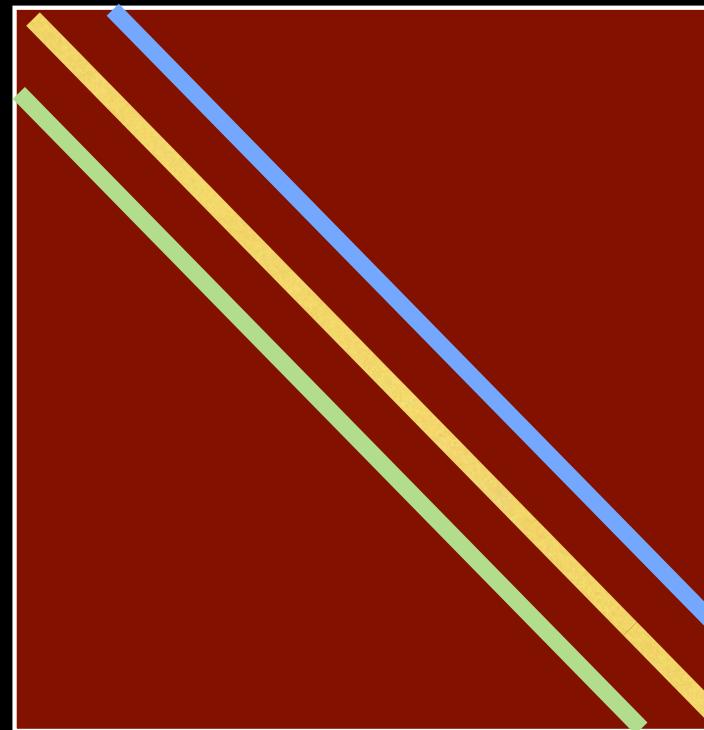


$$\mathbf{p} = \mathbf{u}^*$$

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term



$$\begin{bmatrix} \mathbf{p} \\ \mathbf{u}^* \end{bmatrix} = \begin{bmatrix} \mathbf{u}^* \end{bmatrix}$$

(Semi-)Positive Definite System

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\begin{bmatrix} p \\ \vdots \\ -1 \end{bmatrix} = \begin{bmatrix} \text{blue diagonal} \\ \text{yellow diagonal} \\ \text{green diagonal} \end{bmatrix} \quad \begin{bmatrix} u^* \\ \vdots \end{bmatrix}$$

(Semi-)Positive Definite System

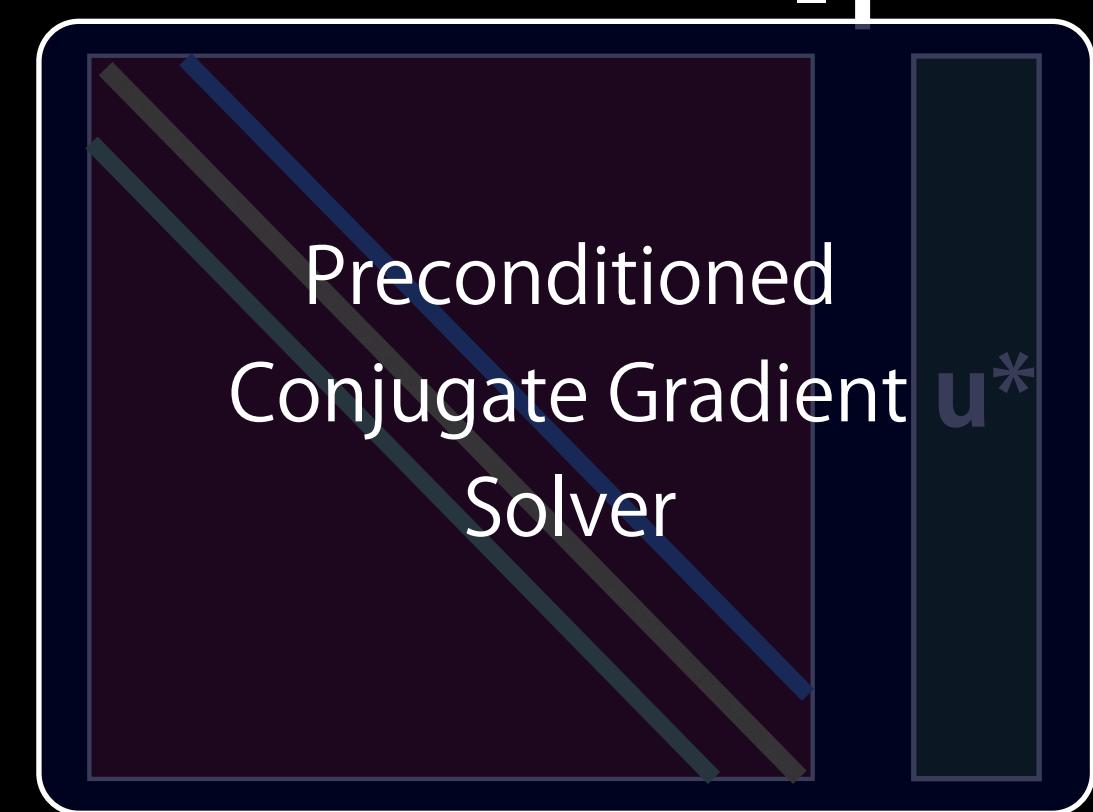
Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\mathbf{p}$$

=



(Semi-)Positive Definite System

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

p

=

Solution

(Semi-)Positive Definite System

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

Solution

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term



Solution

Discretizing

$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$u_{t+\Delta t} = u^* - \frac{\Delta t}{\rho} \nabla$$

Solution

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term



$$-\frac{1}{\rho} \nabla p$$

Pressure term

$$\mathbf{u} \cdot \nabla \mathbf{u}$$

Advection term

$$-\frac{1}{\rho} \nabla p$$

Pressure term

Examples



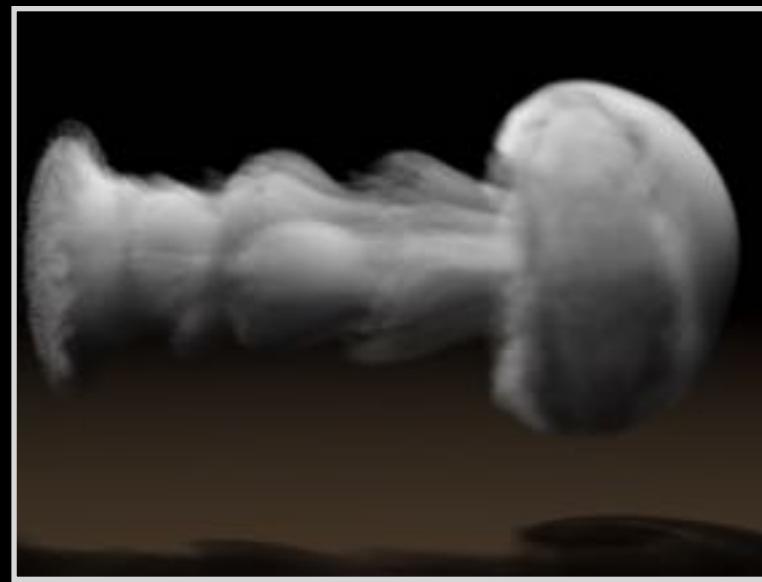
Examples



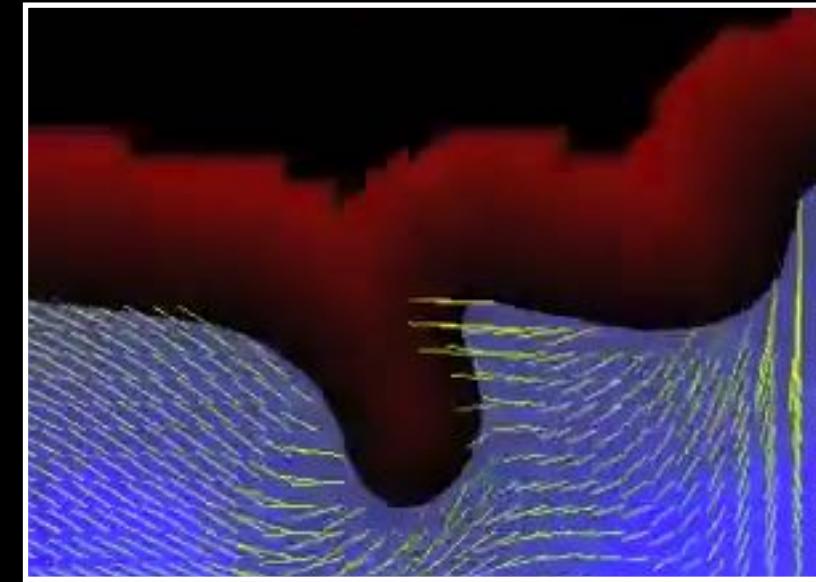
Short Break



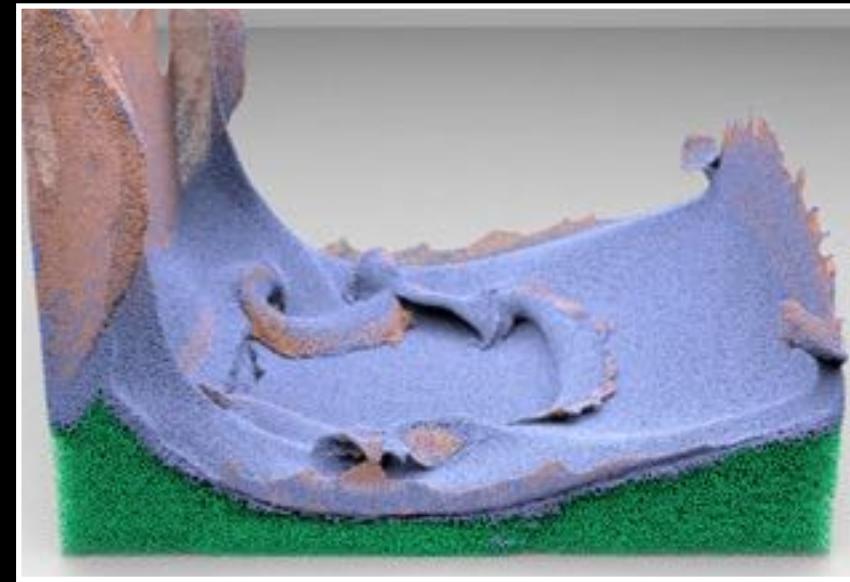
Next: Liquid Simulations...



Fluid Solver



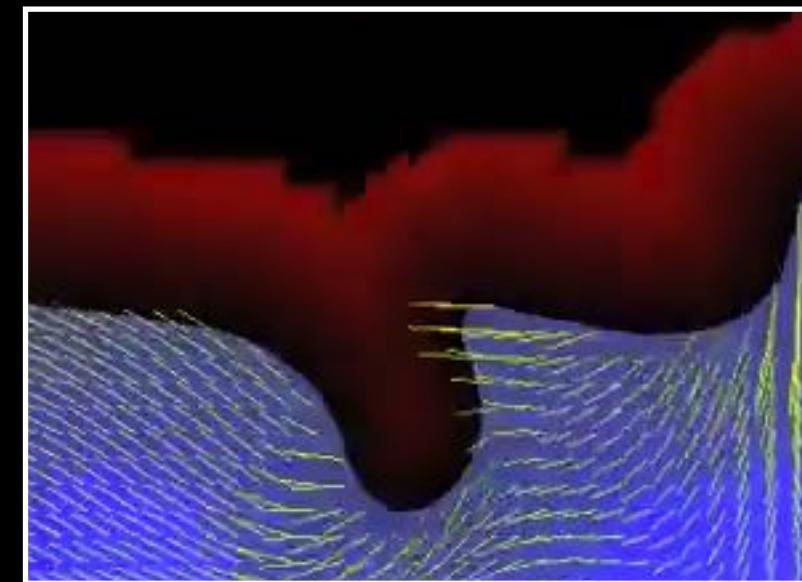
Surface Tracker



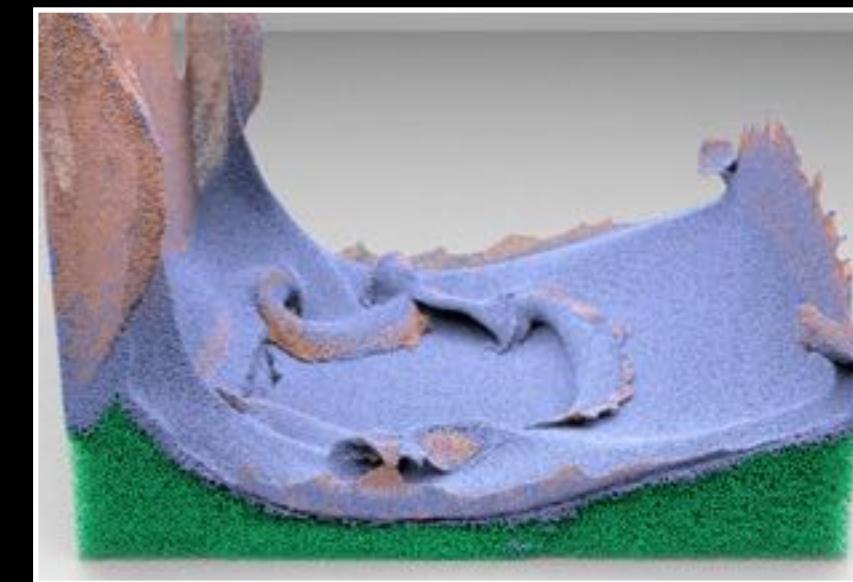
FLIP Solver



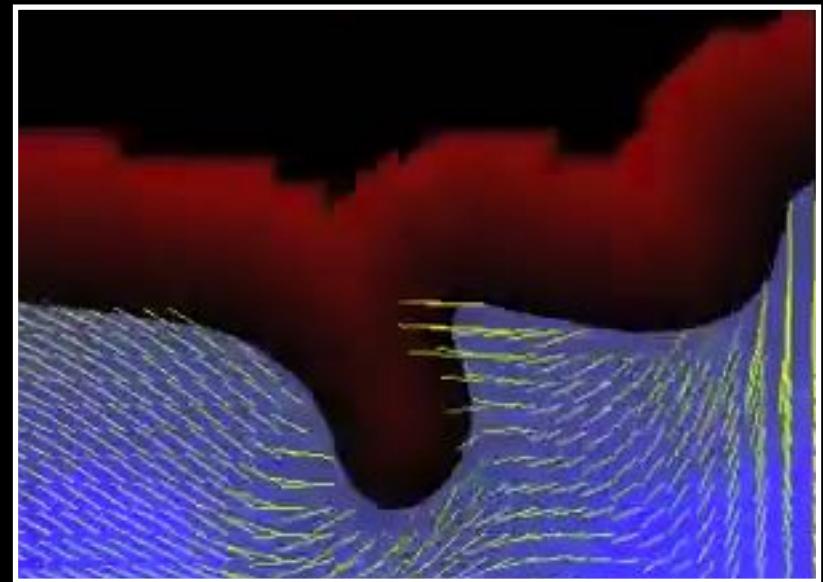
Fluid Solver



Surface Tracker

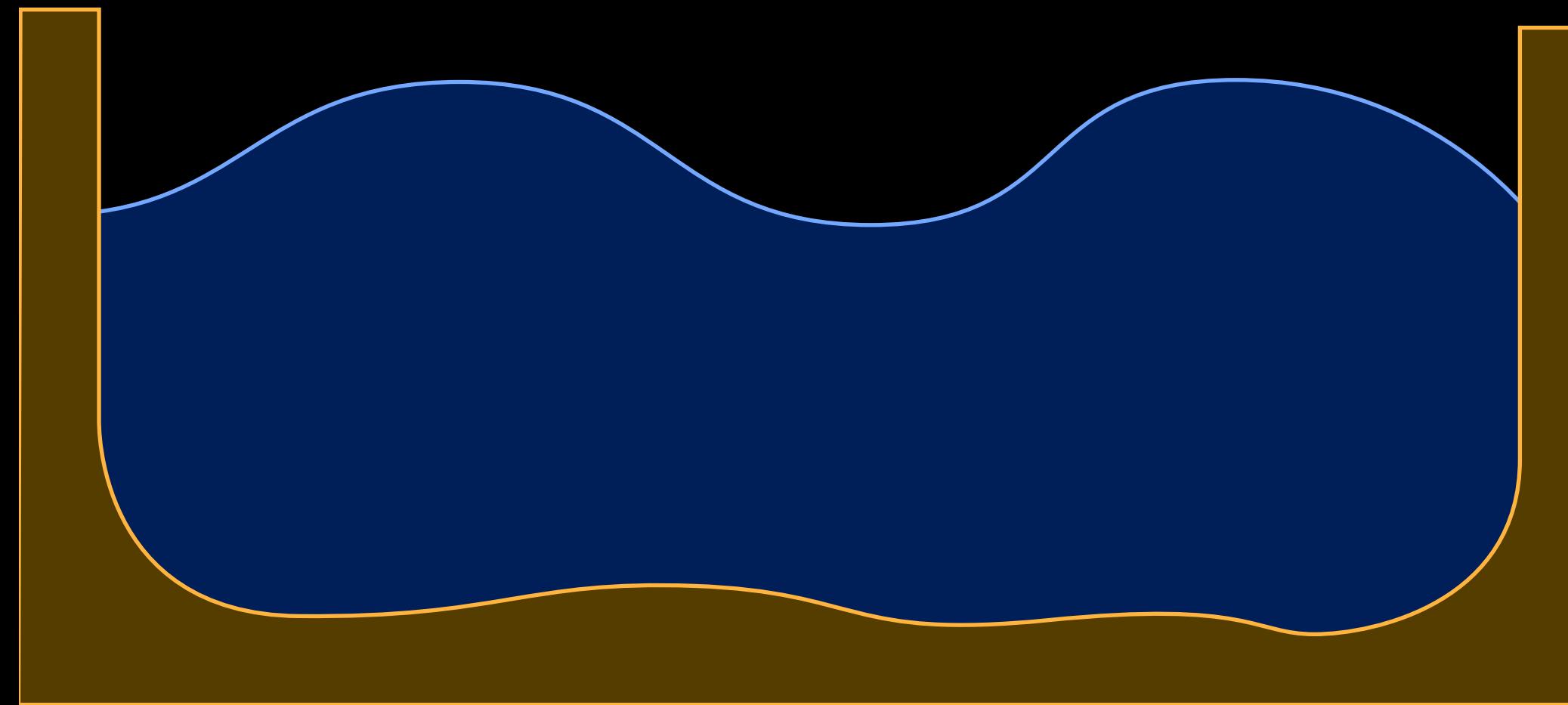


FLIP Solver

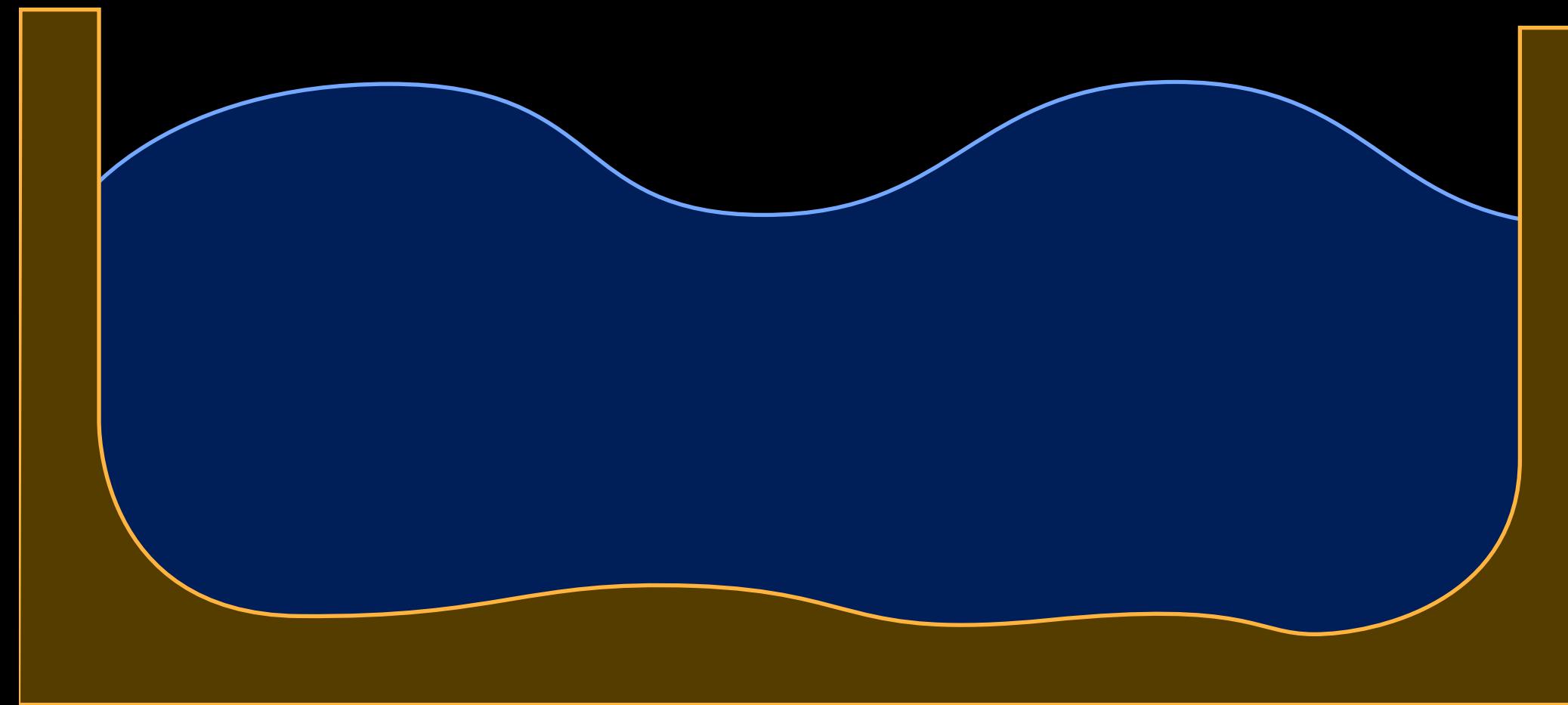


Surface Tracker

Liquid Simulation



Liquid Simulation

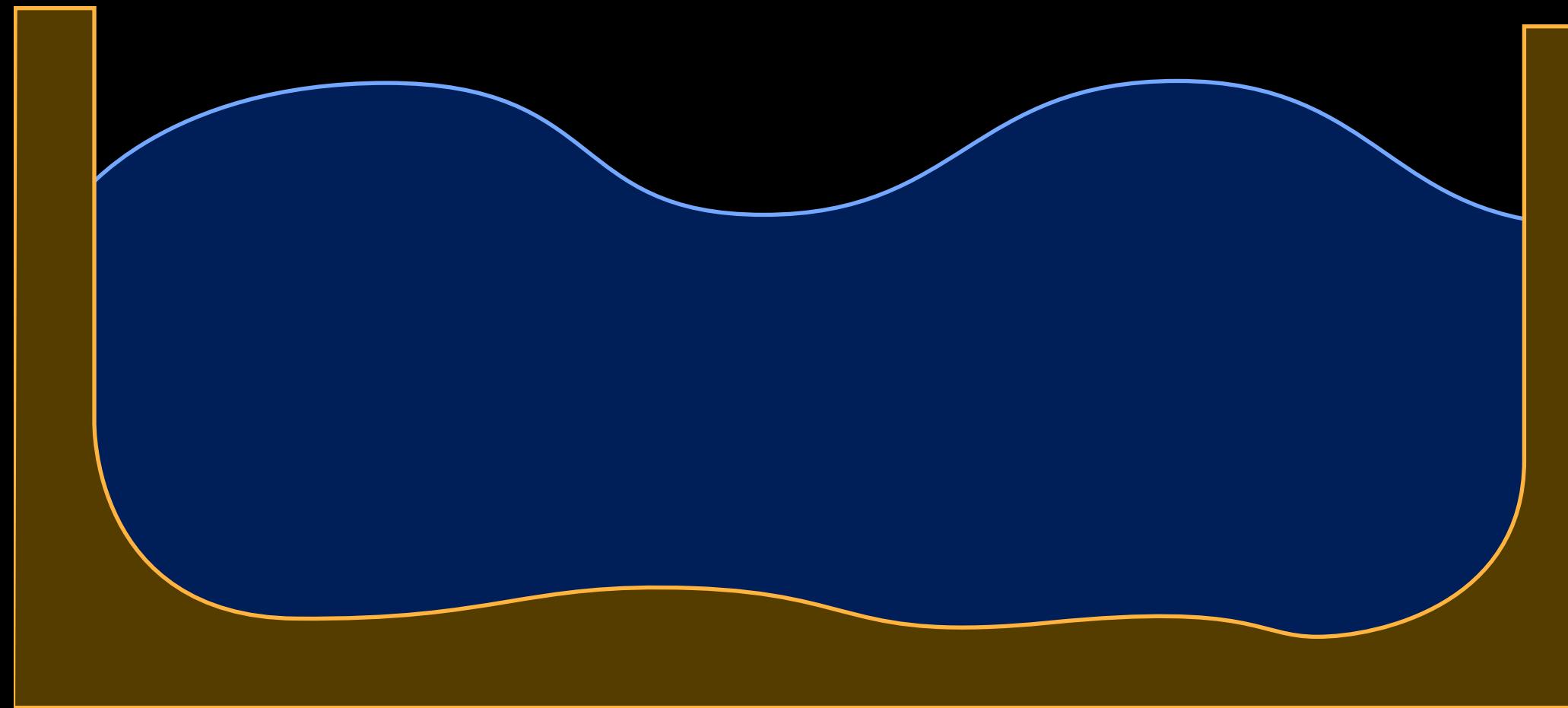


Liquid Simulation

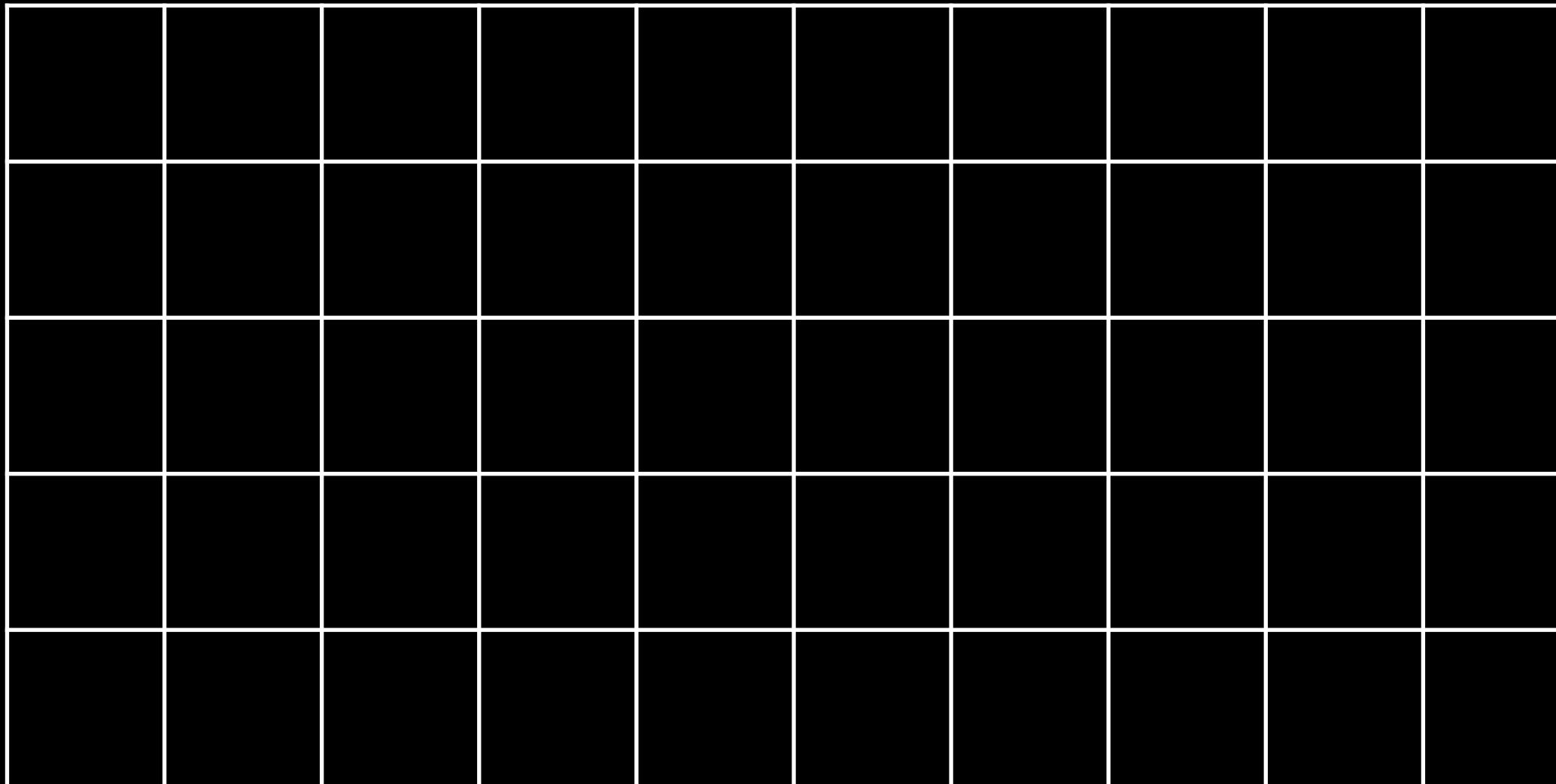


Surface Tracker

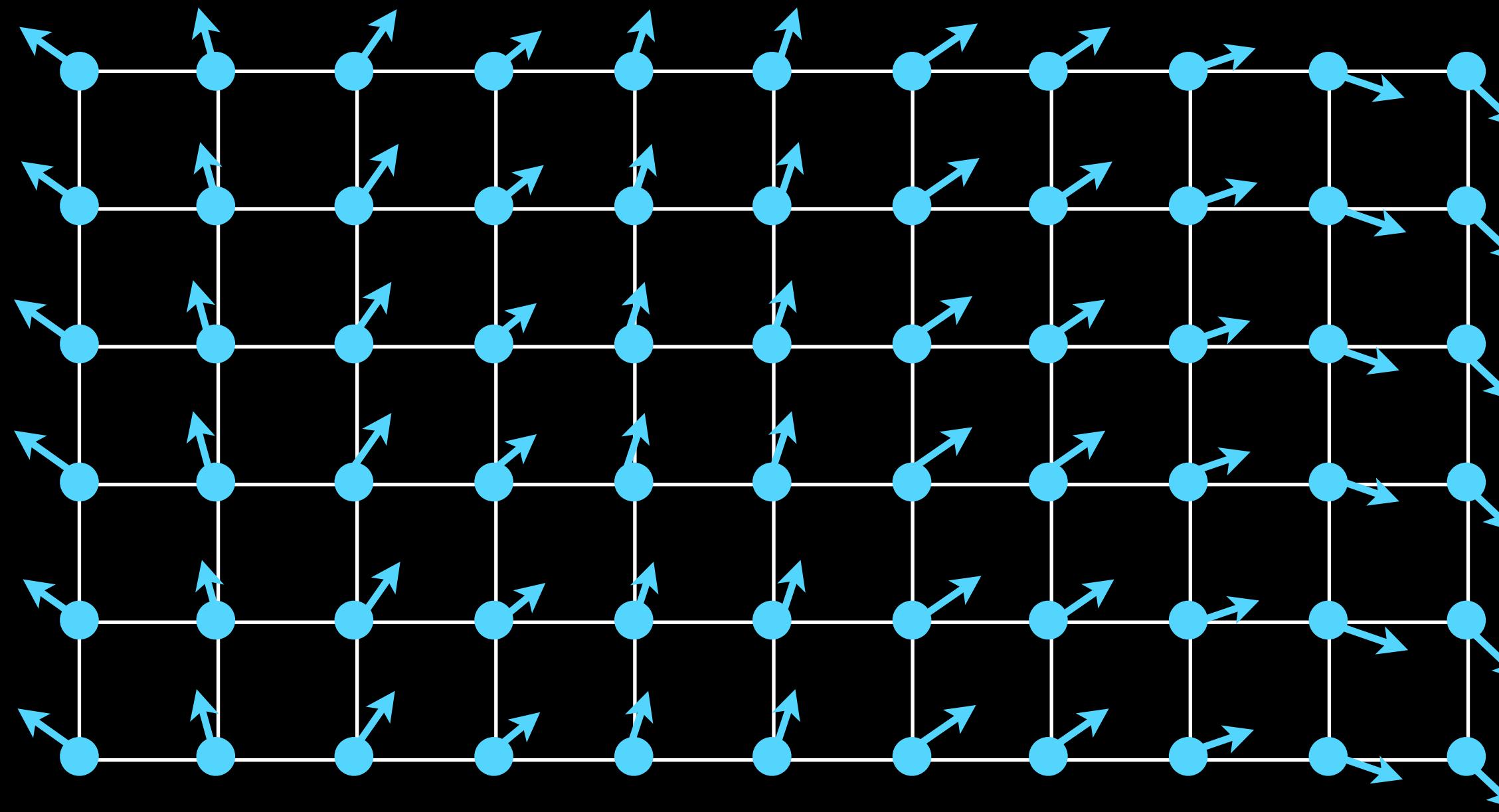
Surface Tracker



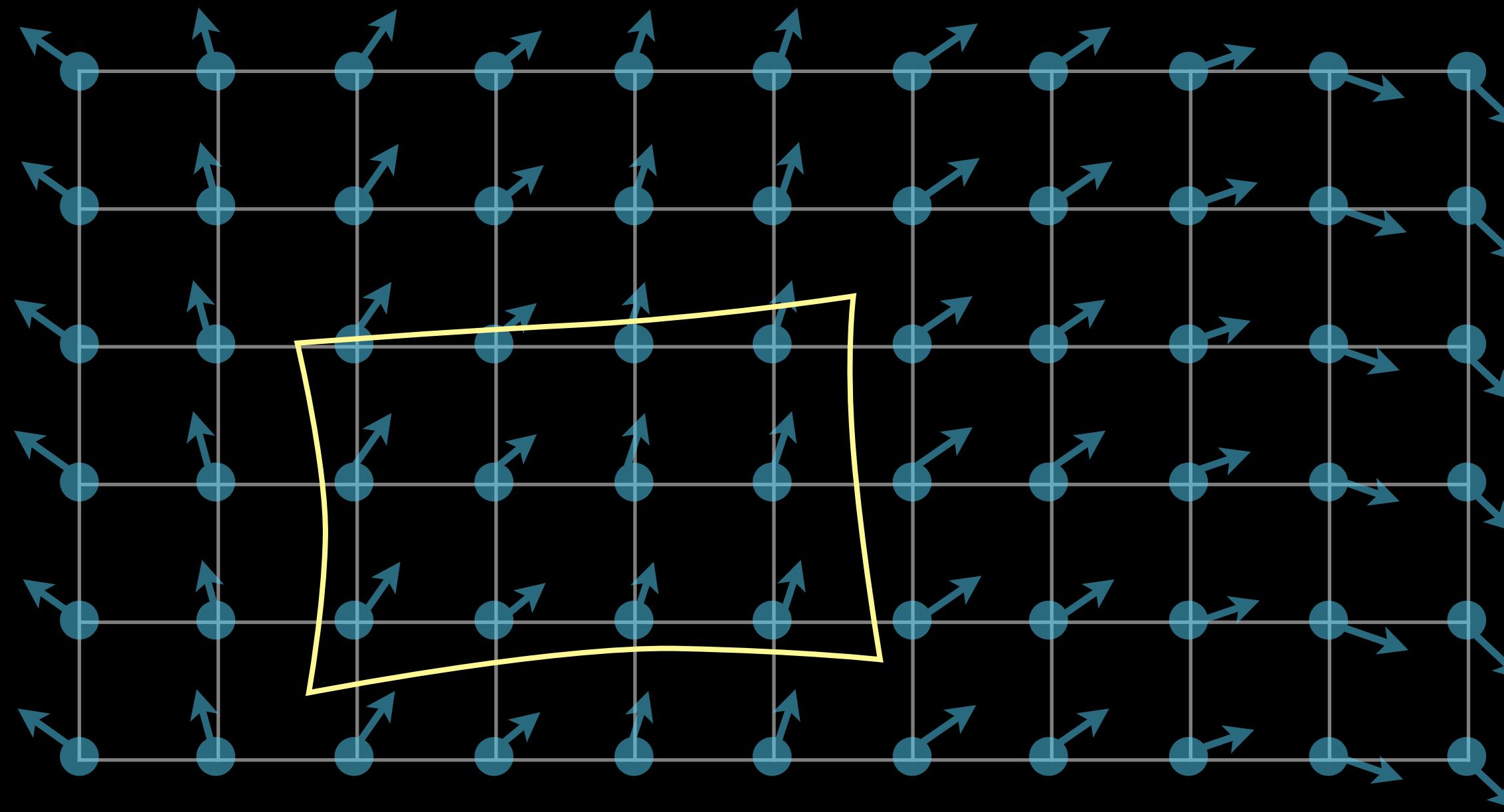
Surface Tracker



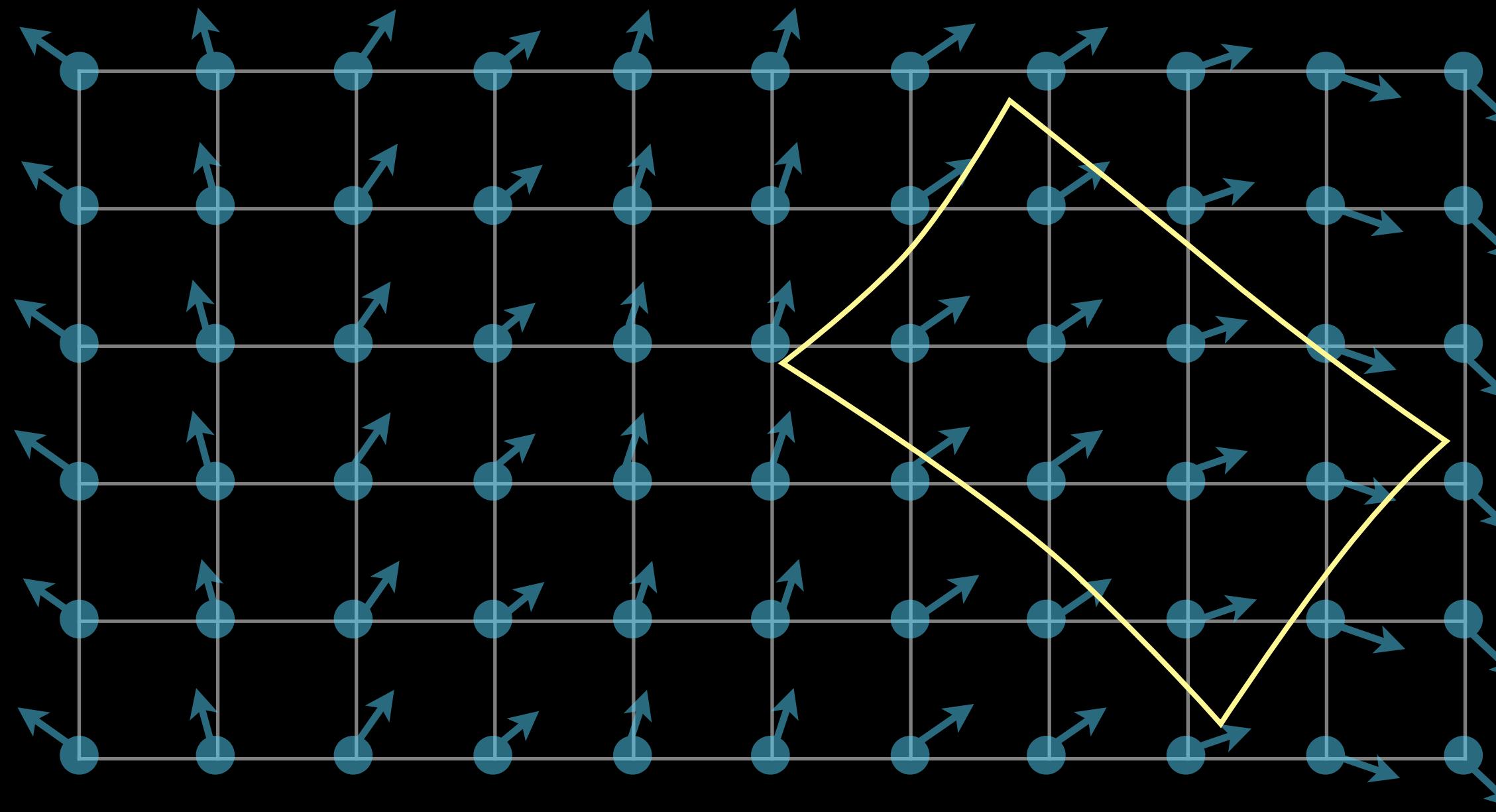
Surface Tracker



Surface Tracker



Surface Tracker

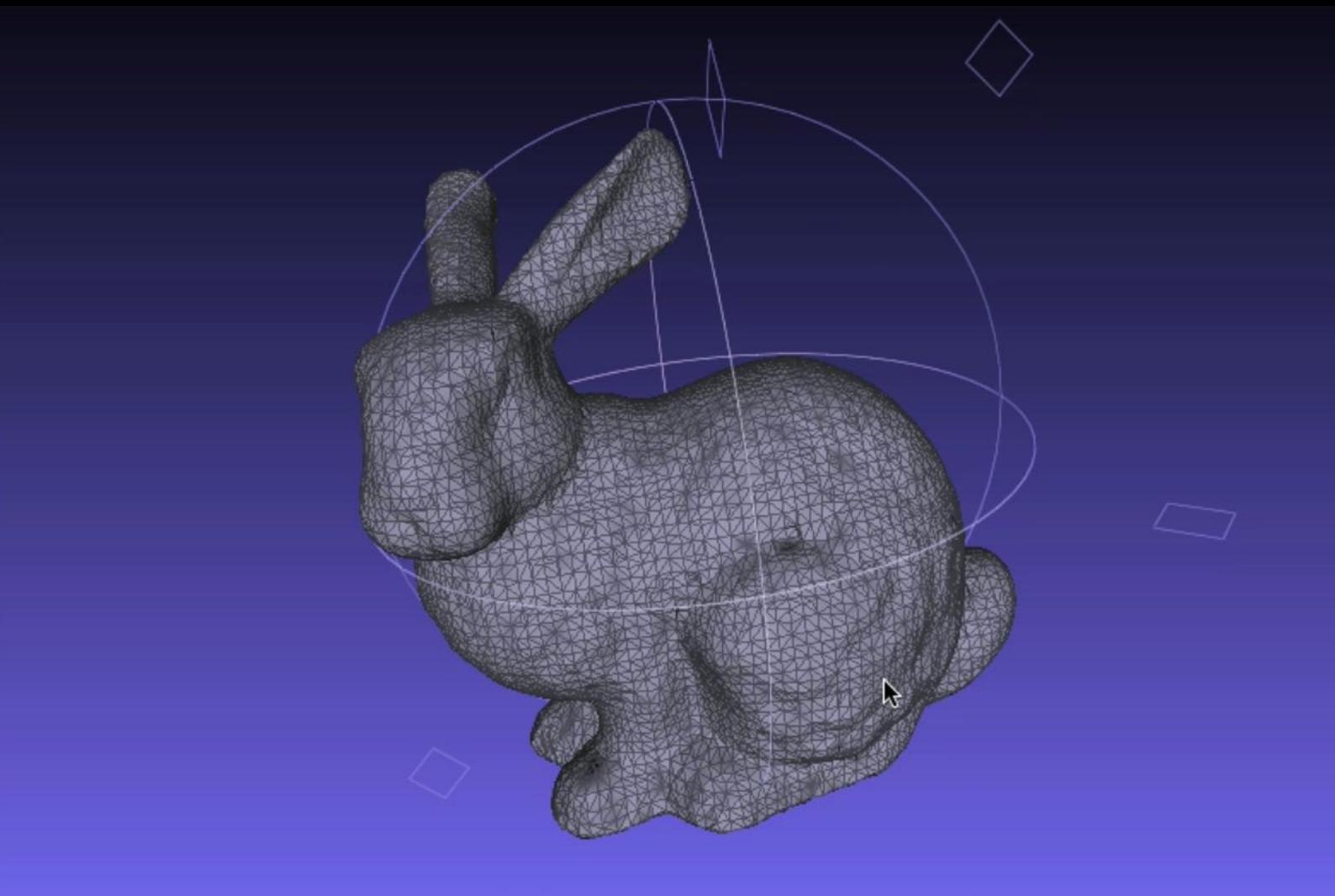


Surface Tracker

- Water tight
- Topology change

Surface Tracker

- Water tight
- Topology change



Surface Tracker

- Water tight
- Topology change



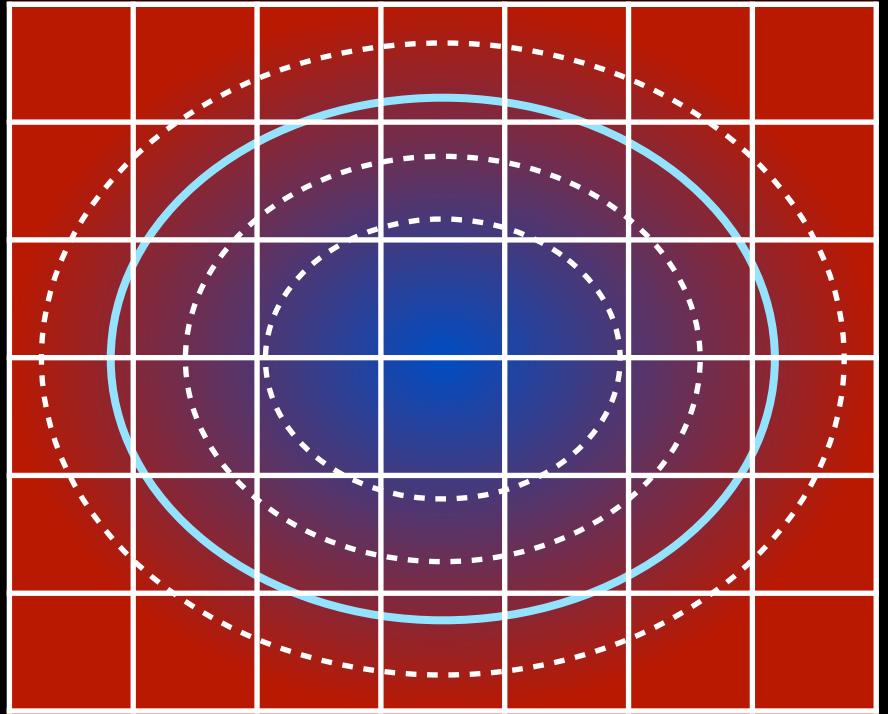
Surface Tracker

- Water tight
- Topology change



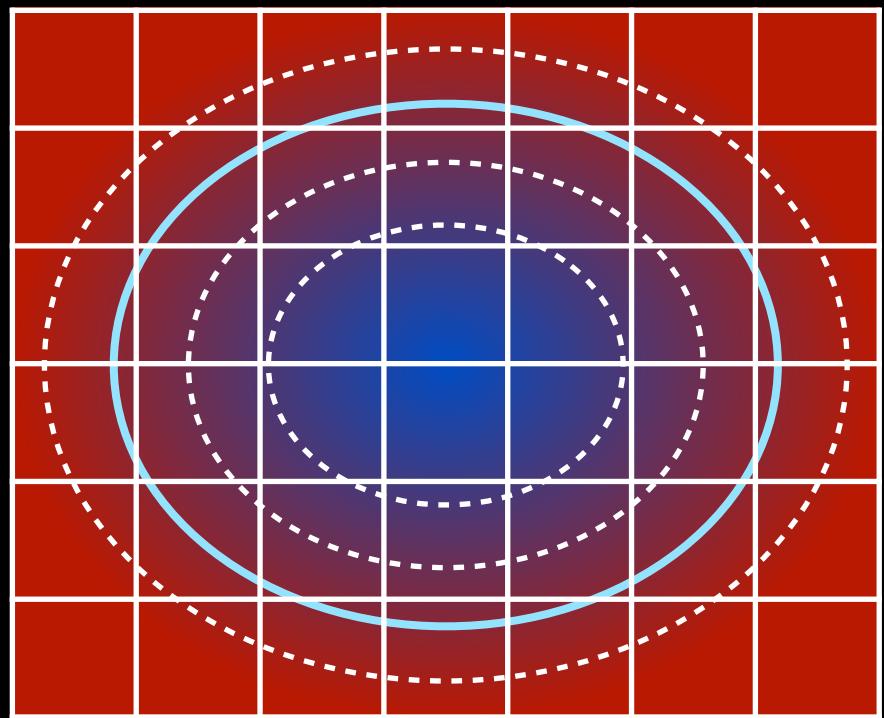
Surface Tracker

Surface Tracker

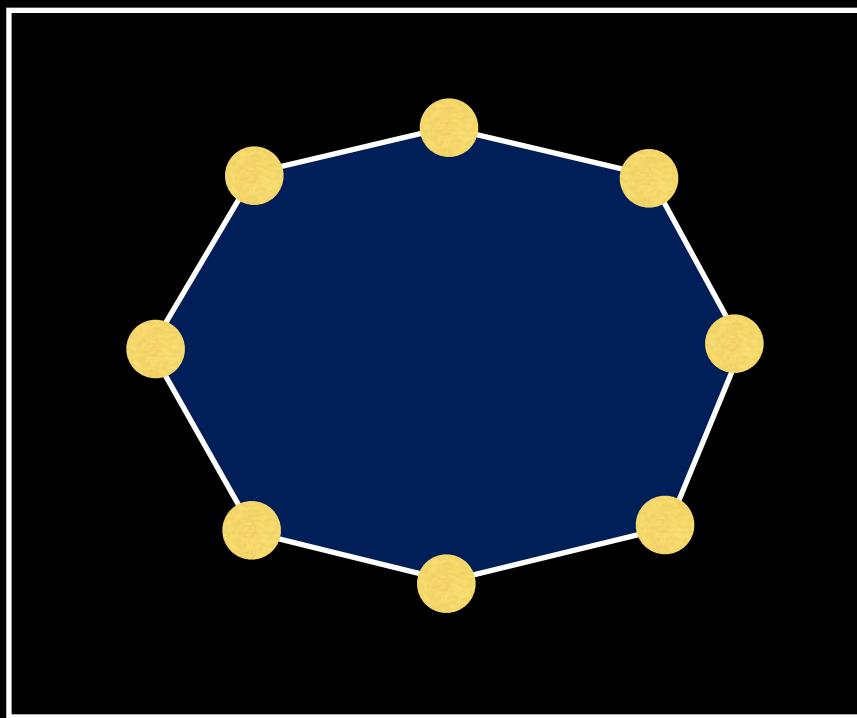


Levelset Method

Surface Tracker

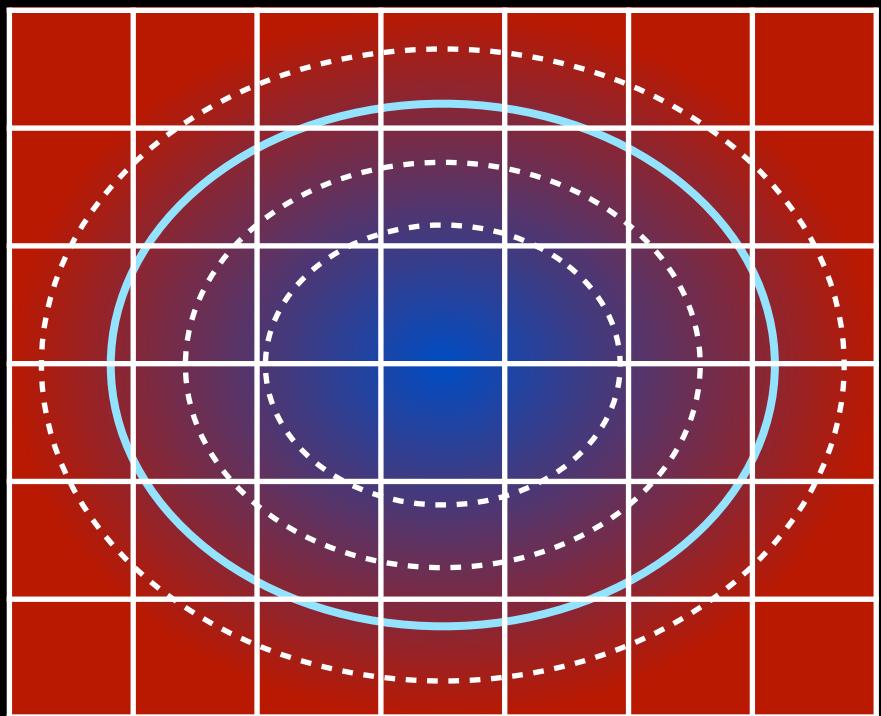


Levelset Method

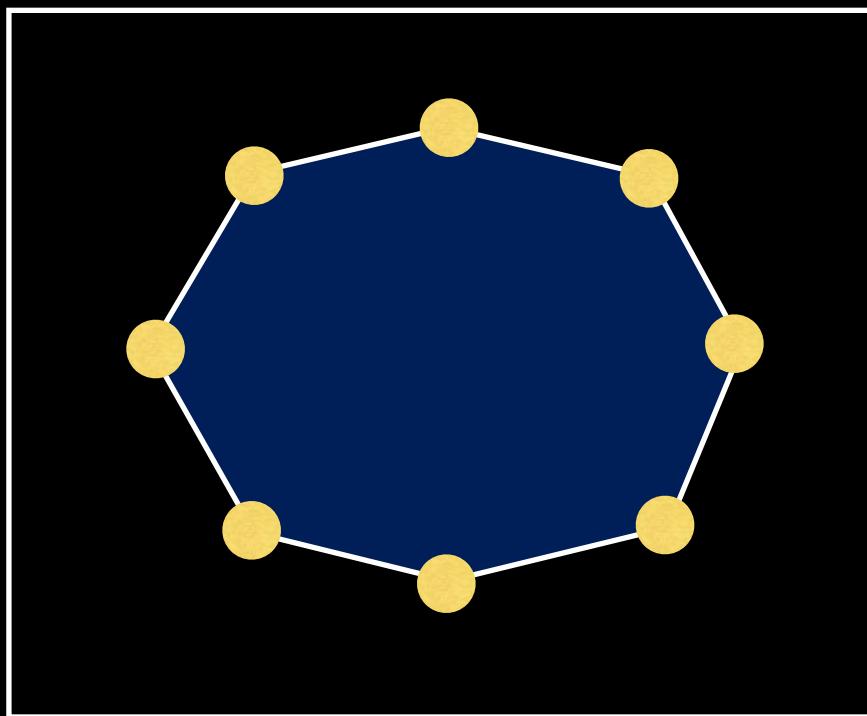


Mesh-based Method

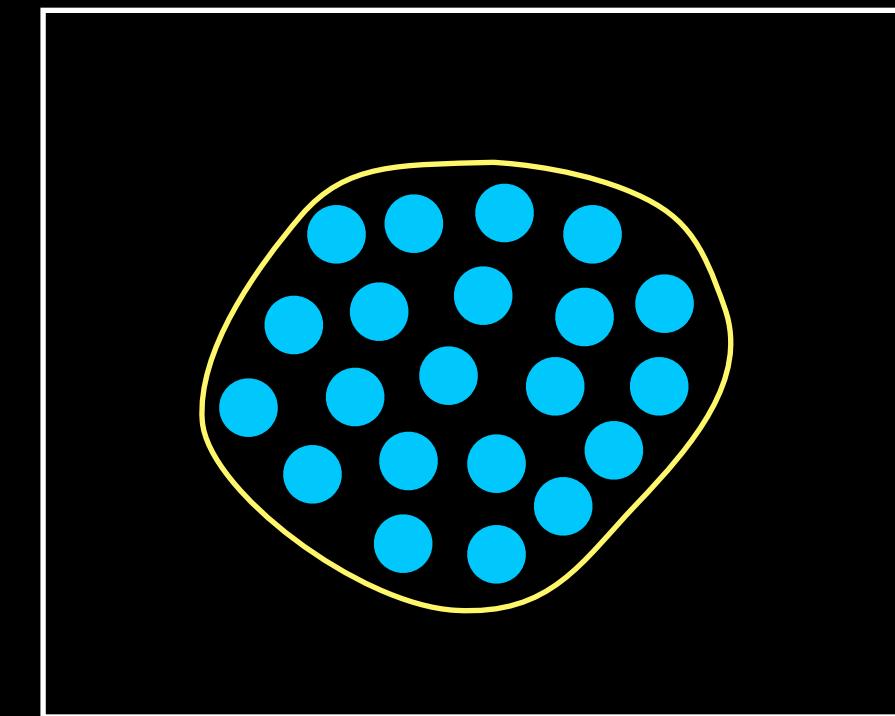
Surface Tracker



Levelset Method

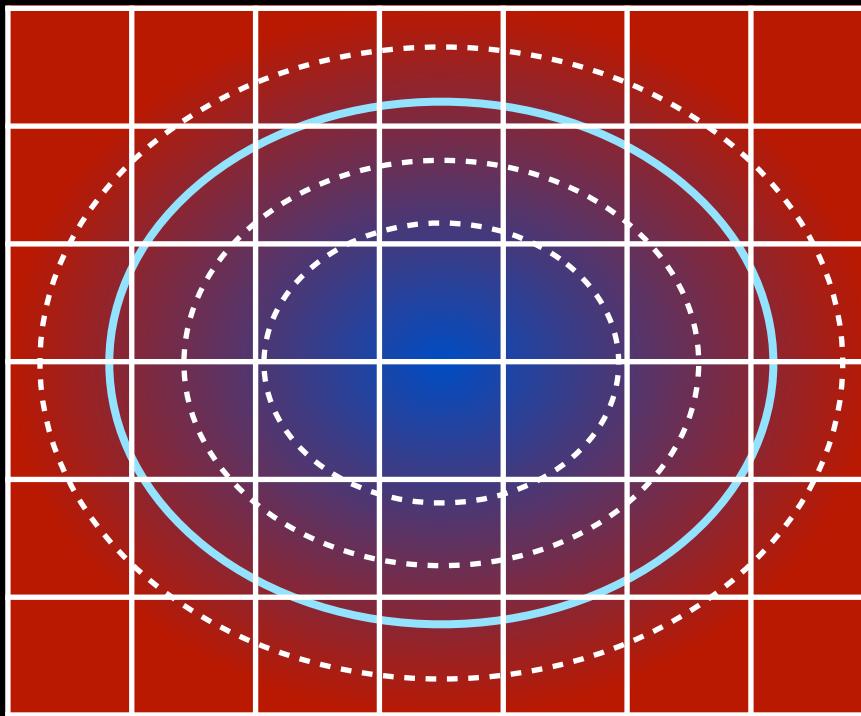


Mesh-based Method



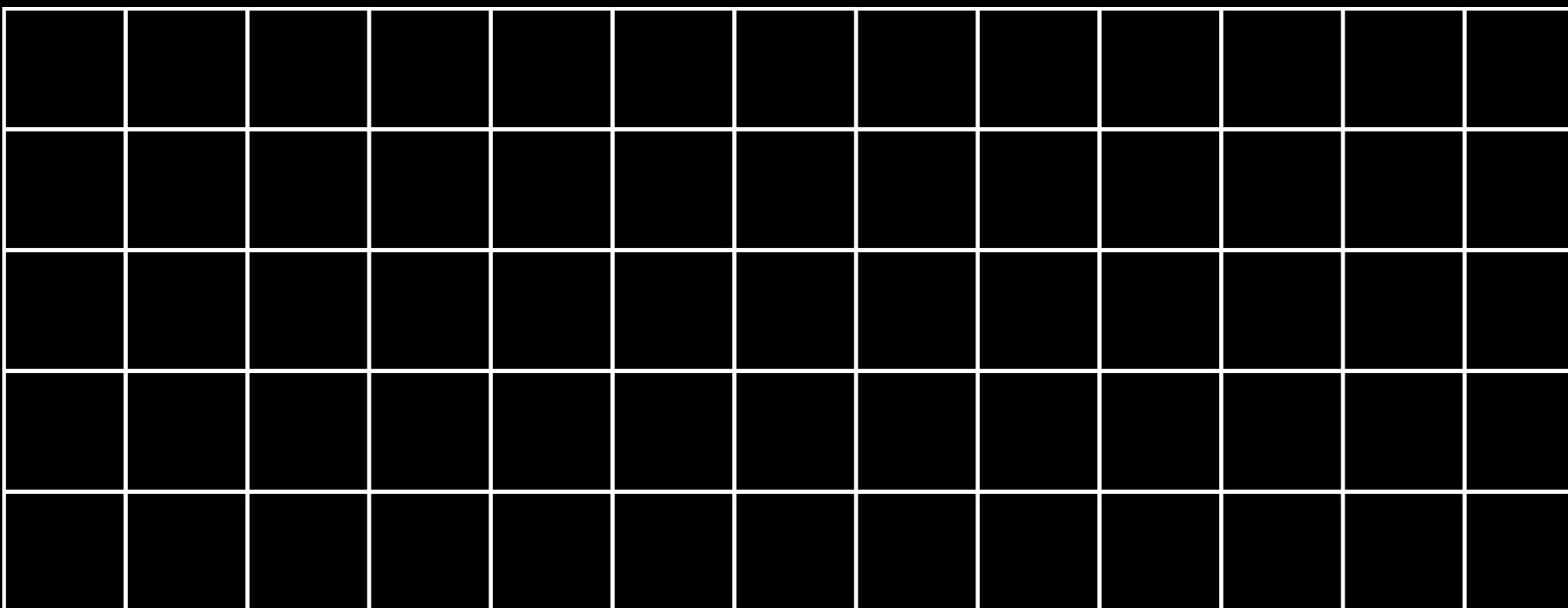
Particle-based Method

Surface Tracker

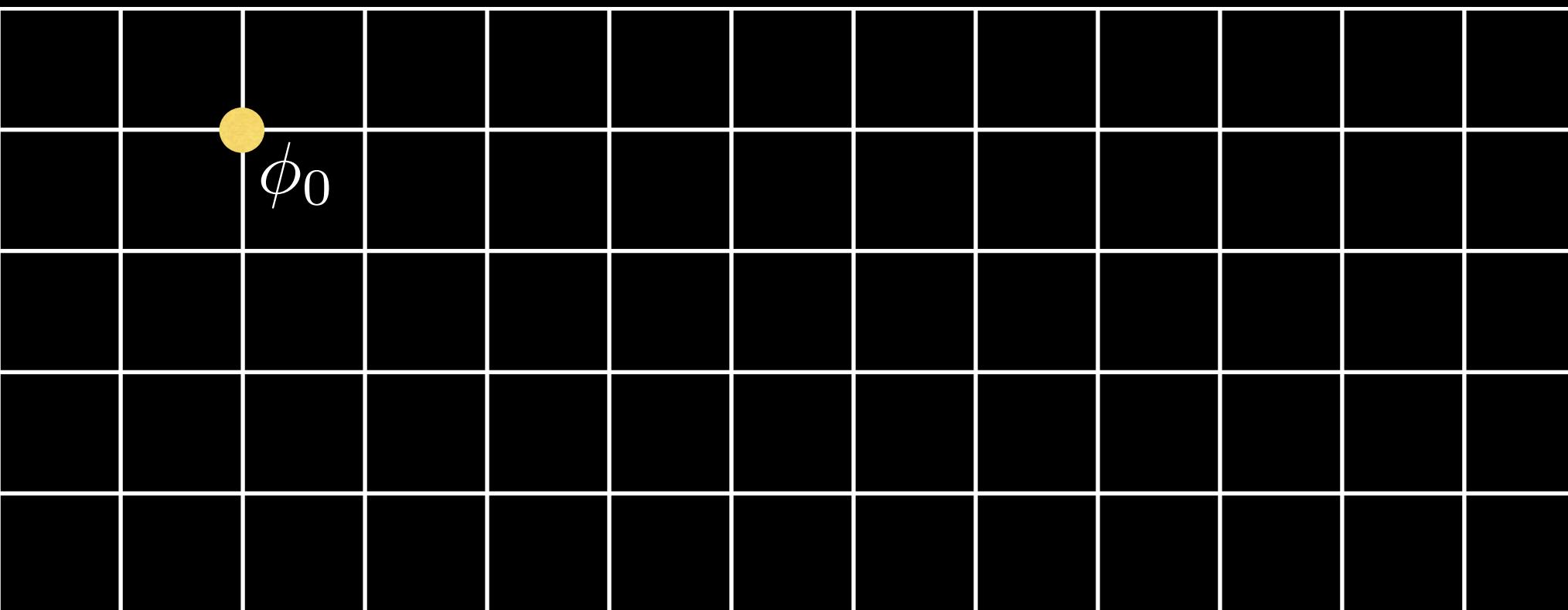


Levelset Method

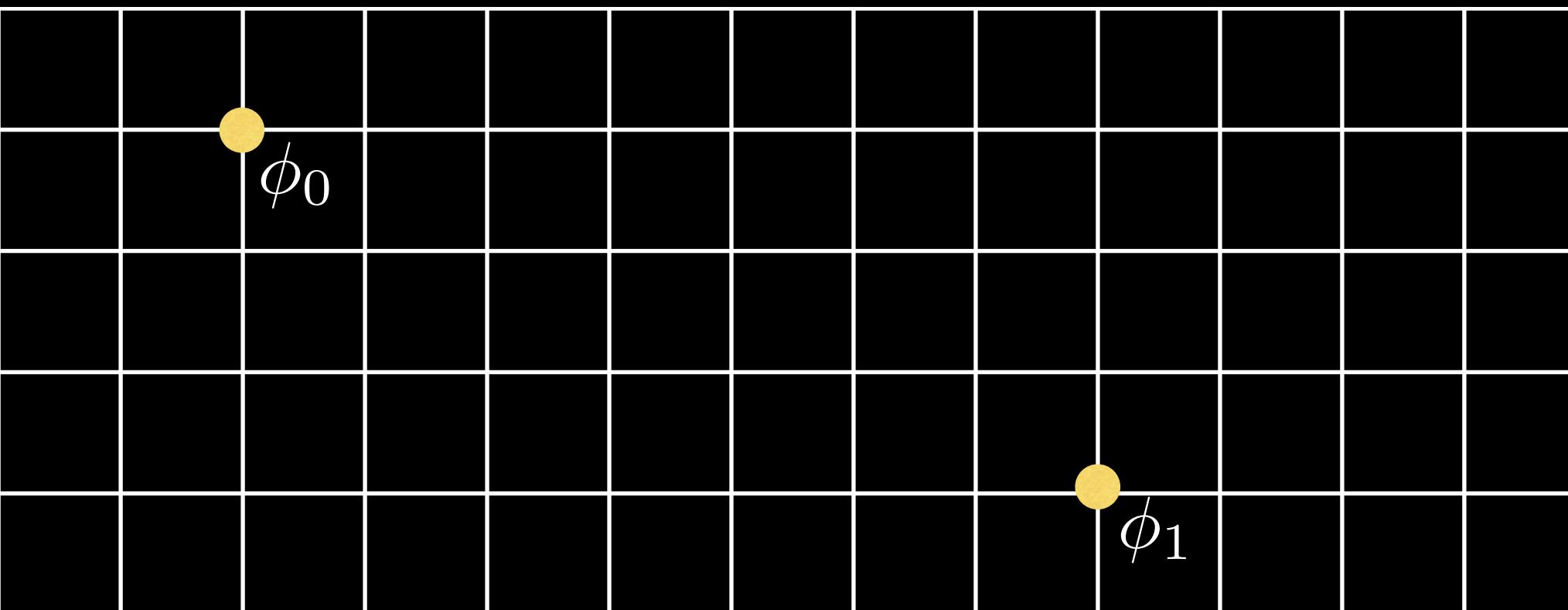
Levelset Method



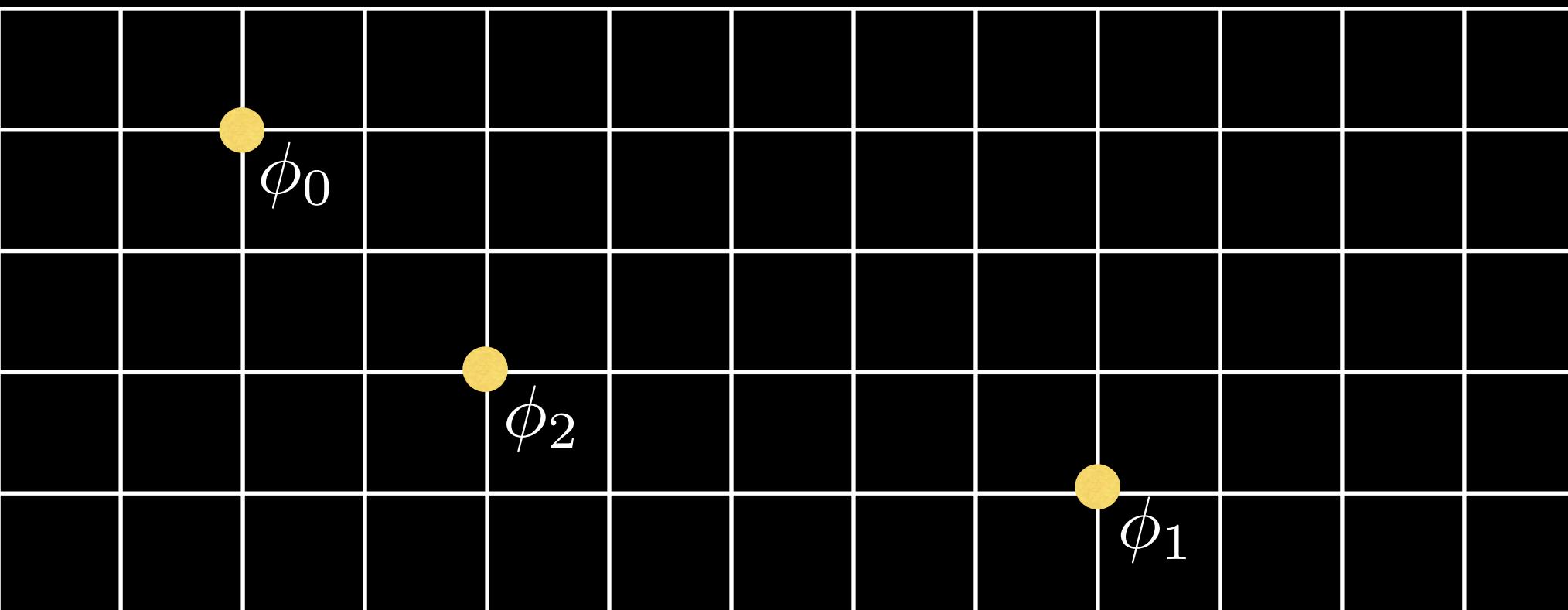
Levelset Method



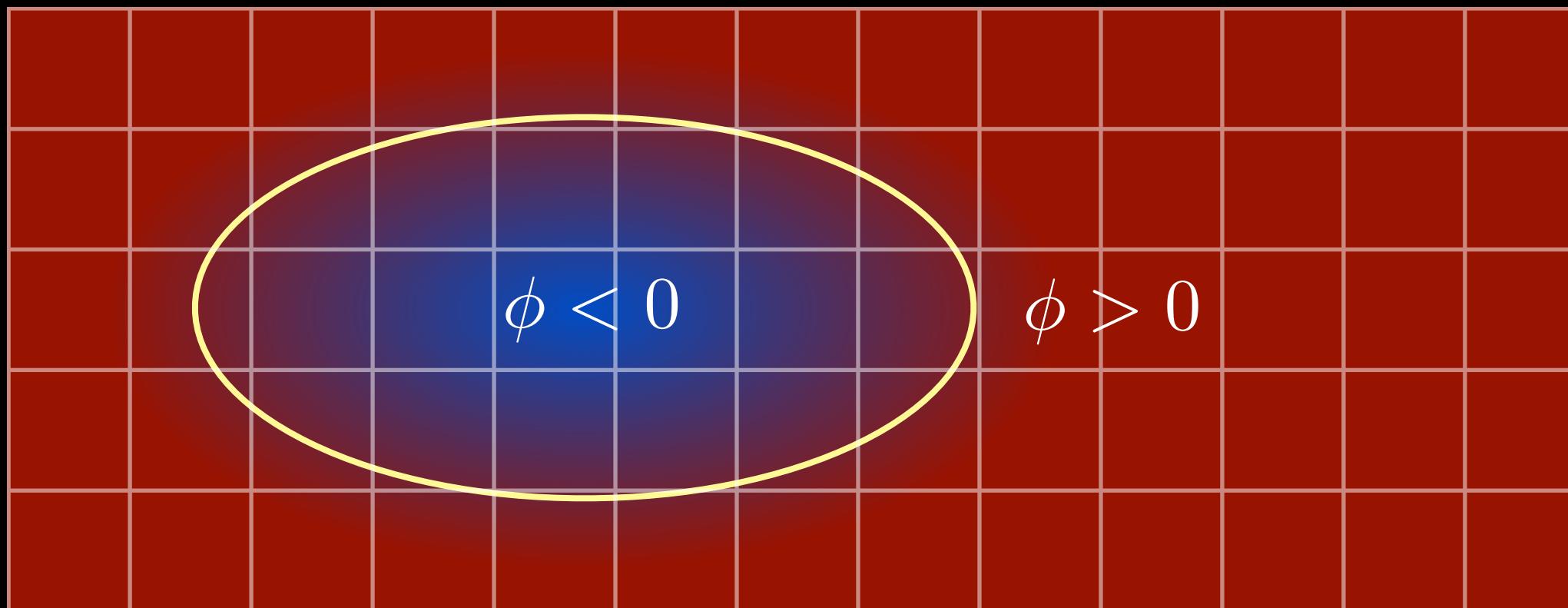
Levelset Method



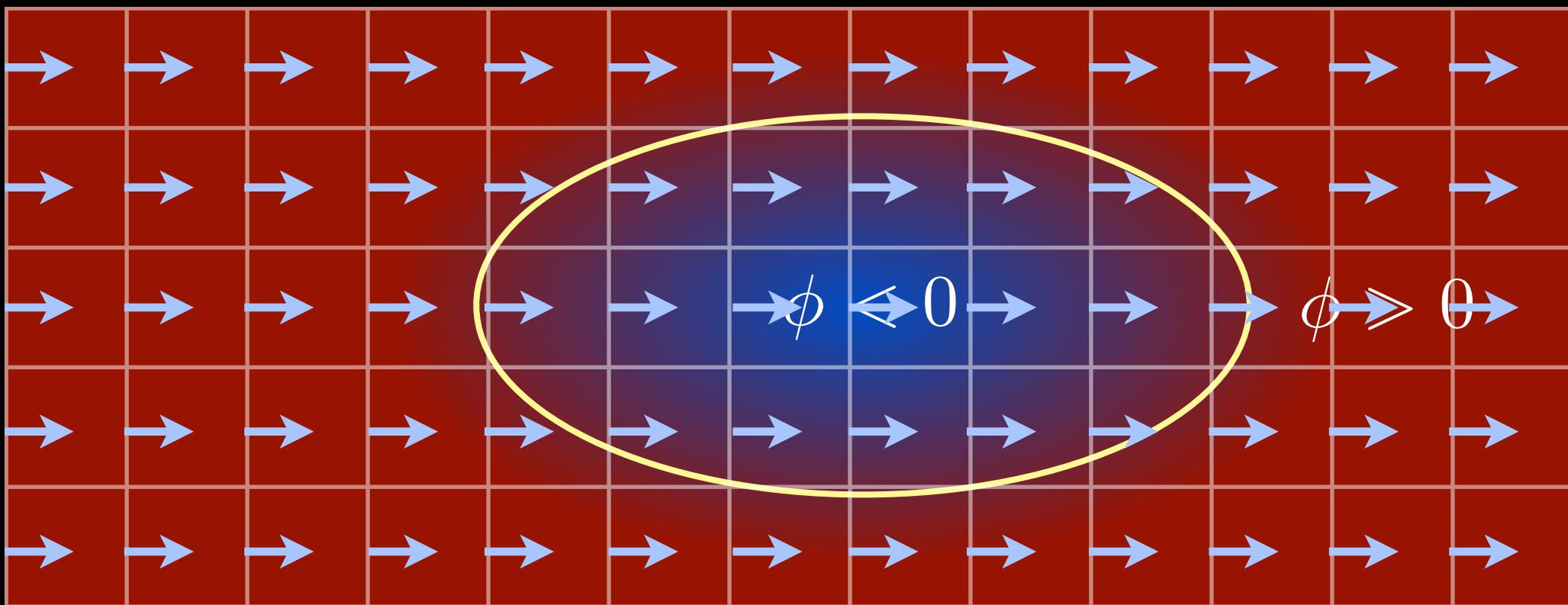
Levelset Method



Levelset Method

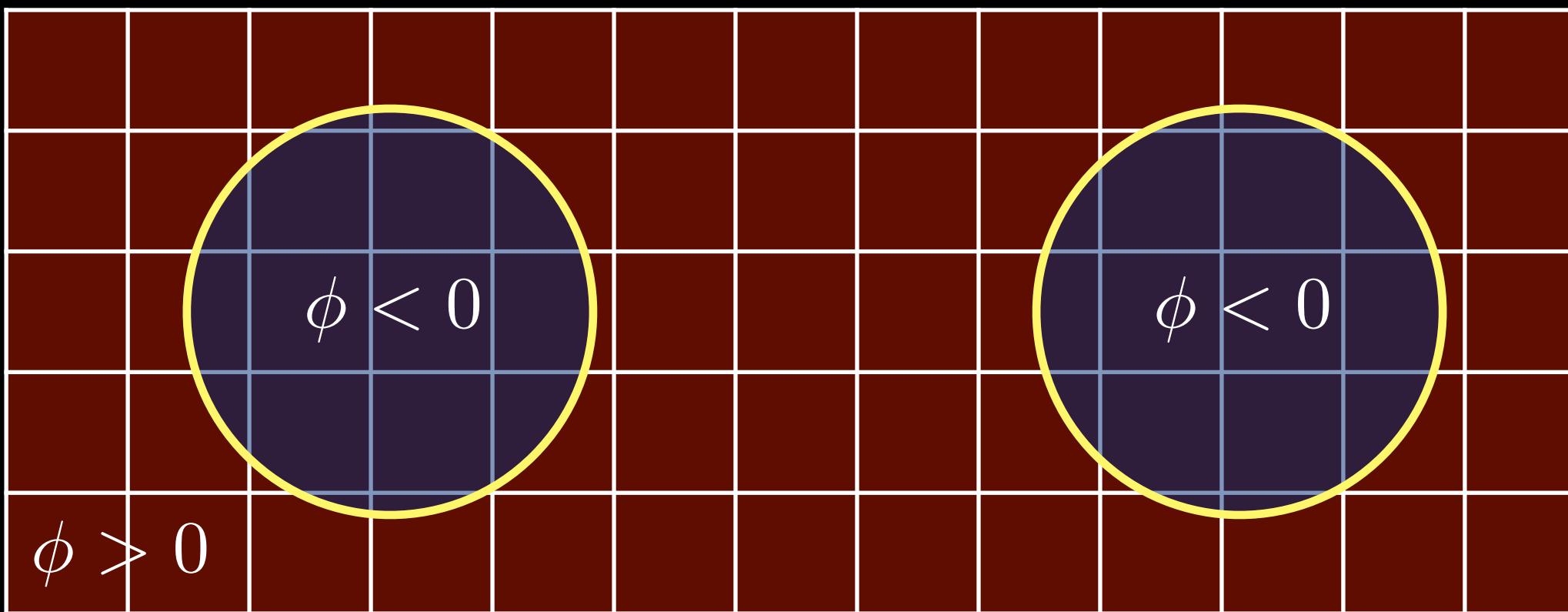


Levelset Method

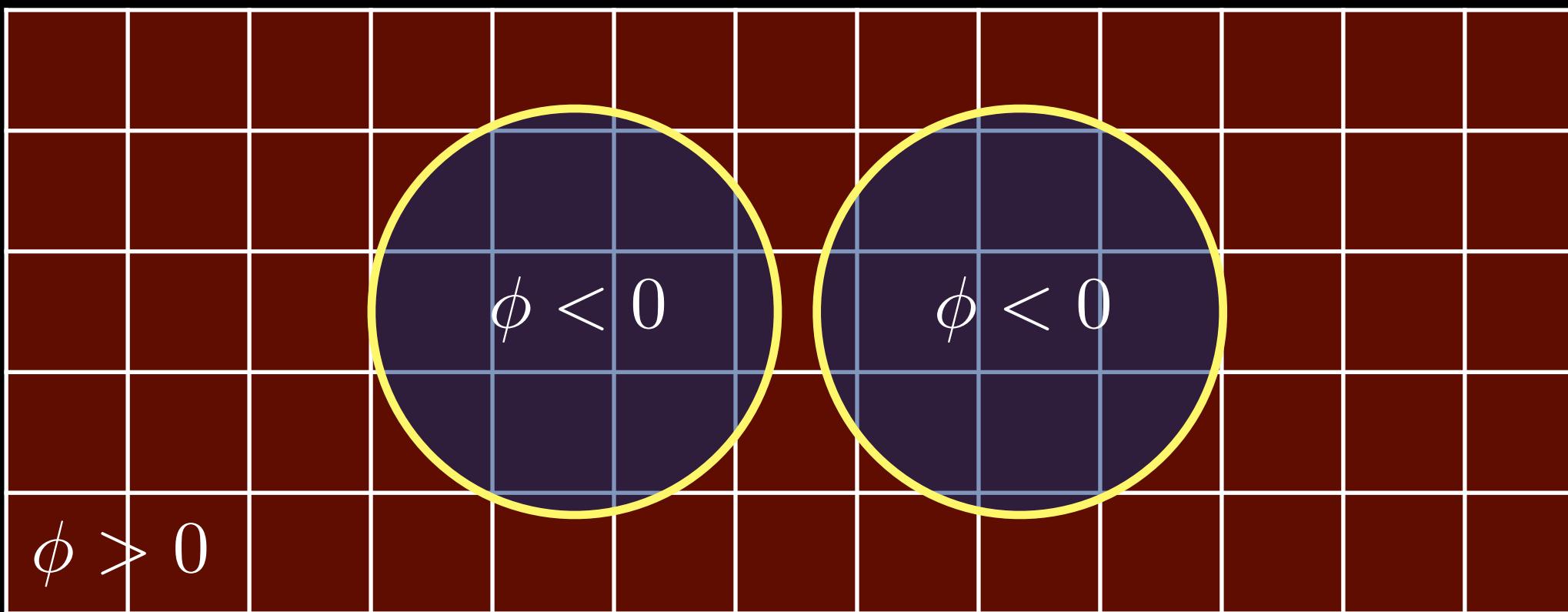


$$\frac{\partial \phi}{\partial t} = -\mathbf{u} \cdot \nabla \phi$$

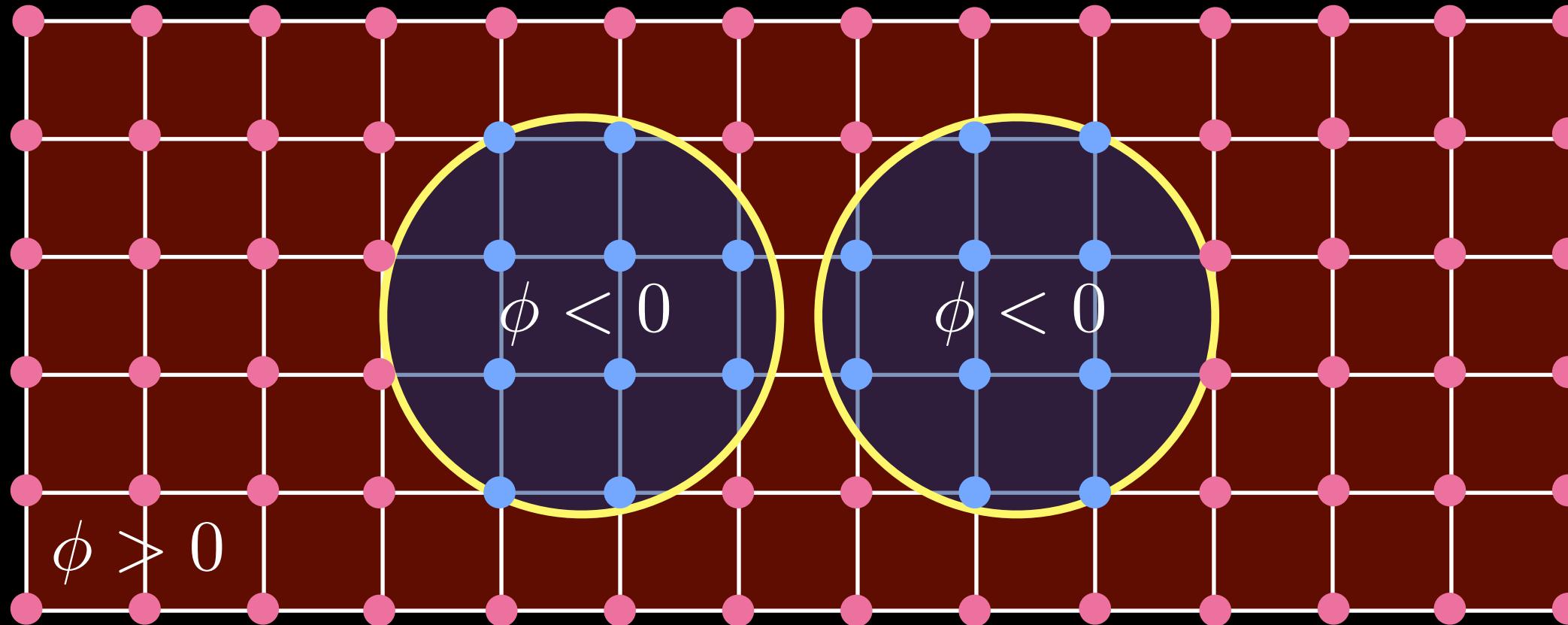
Levelset Method



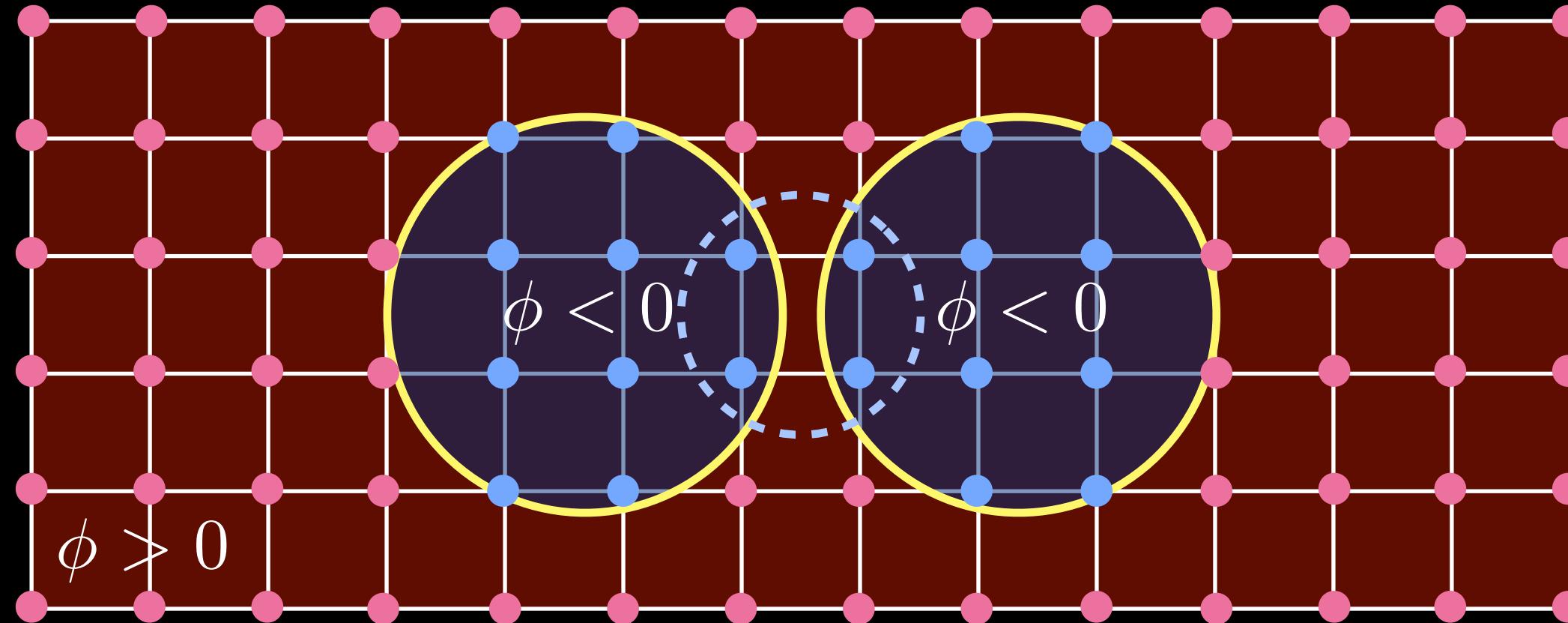
Levelset Method



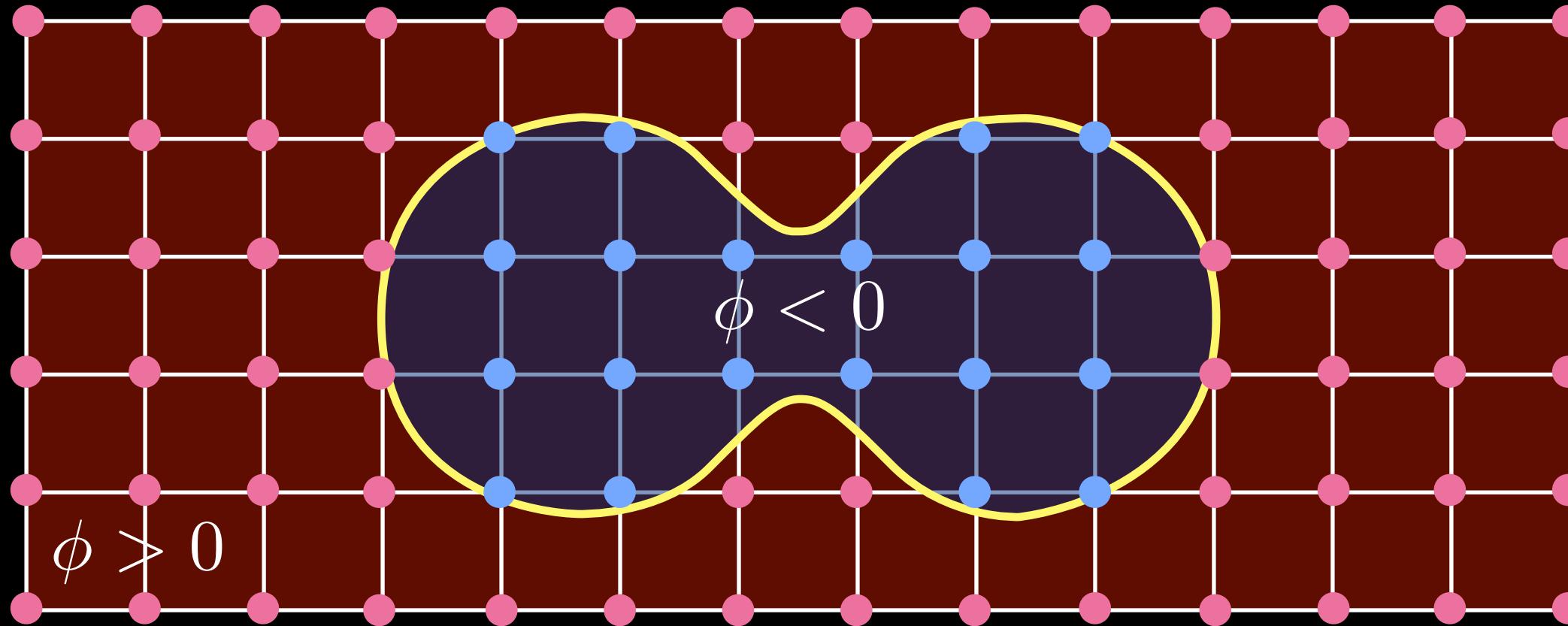
Levelset Method



Levelset Method

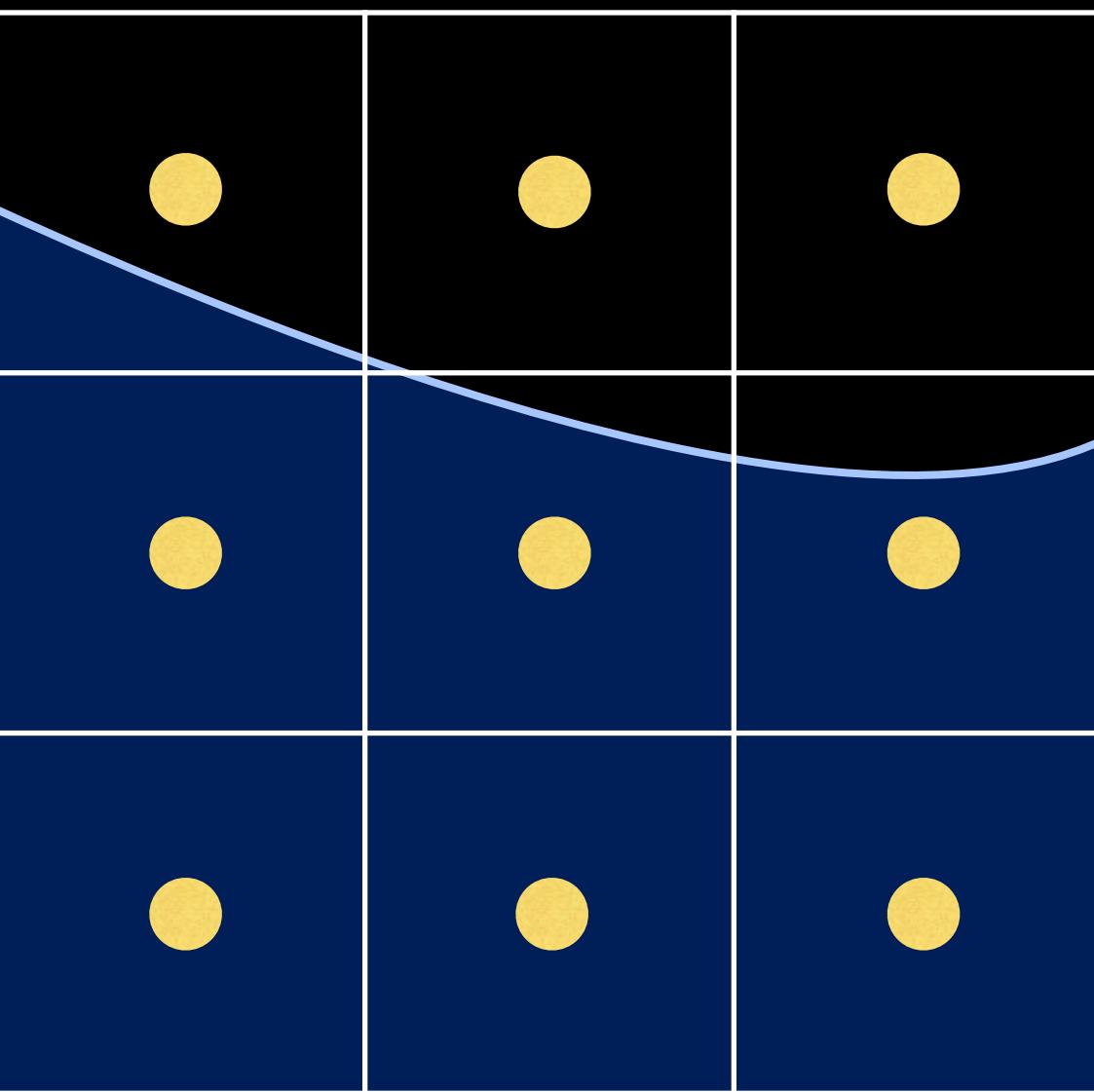


Levelset Method

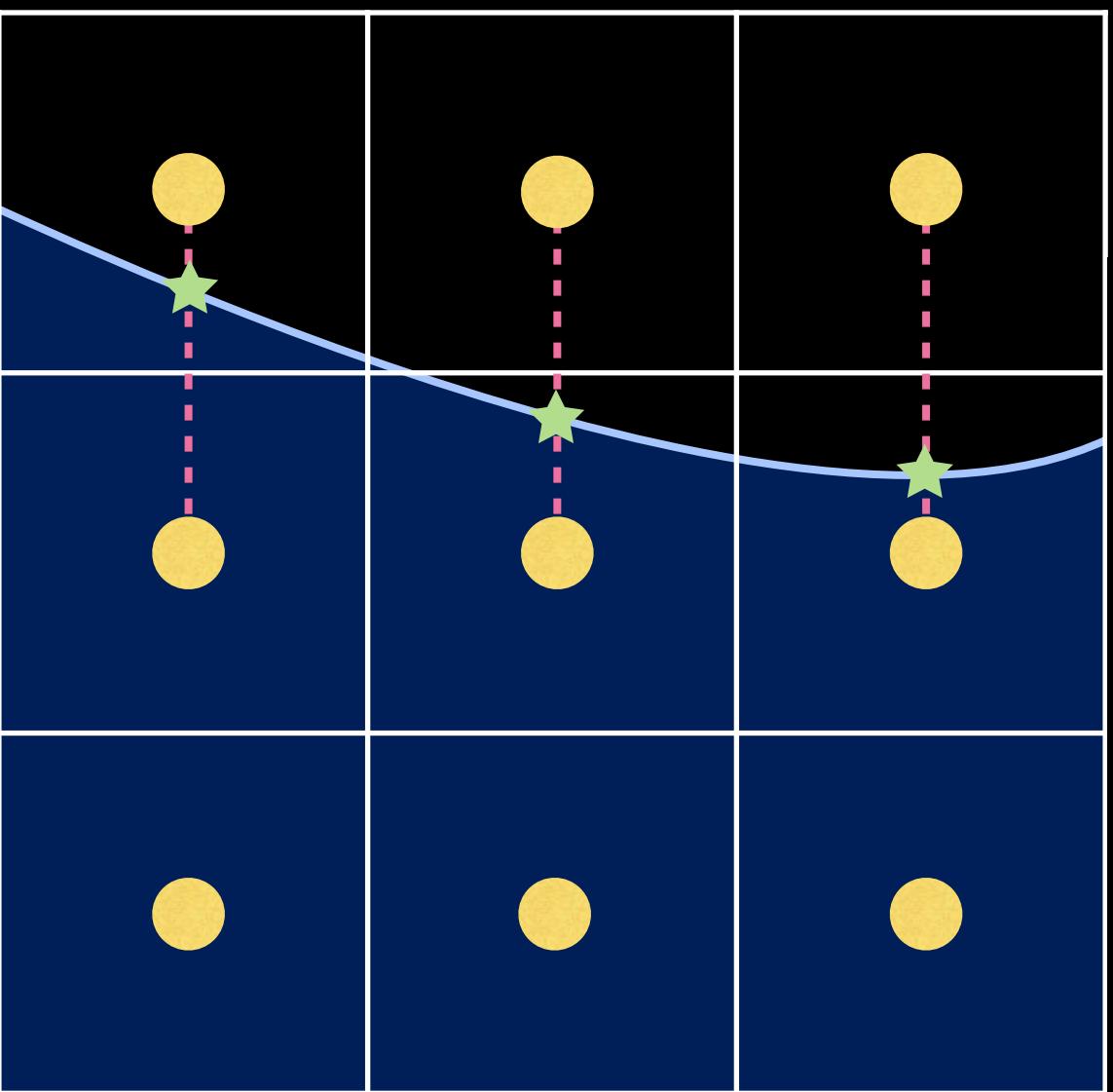


Topology change

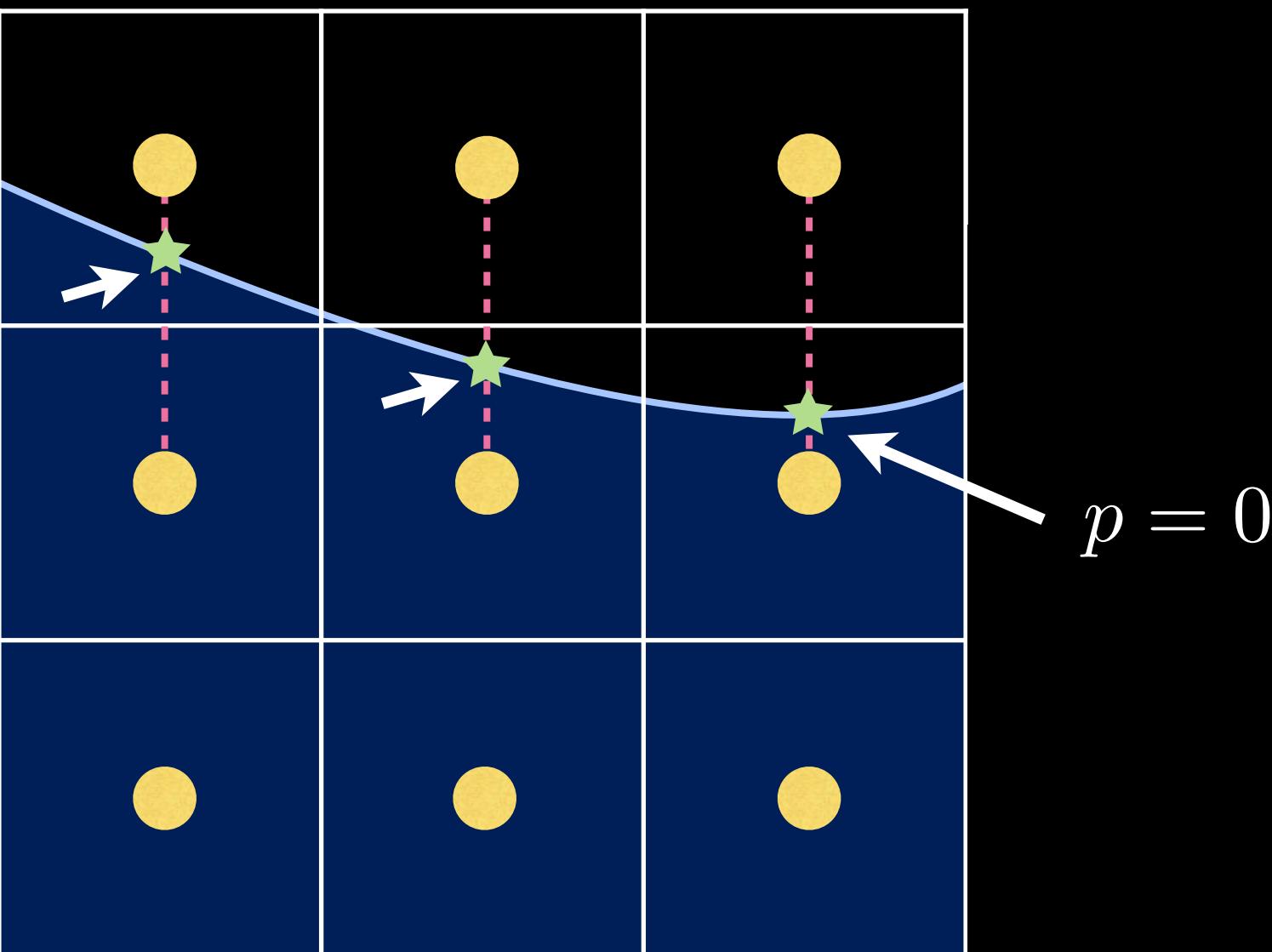
Levelset Method



Levelset Method



Levelset Method



Levelset Method

Push "p" to change the accuracy of boundary projection (current: 2nd order.)

Push "d" to toggle distance field.

Push "f" to toggle liquid field.

Push "g" to toggle grid points.

Push "v" to toggle velocity.

Push "r" to reset.

Push "i" to toggle interpolation method. (current: Catmull-Rom Spline.)

Push "a" to toggle redistance.

Push "c" to toggle volume correction. (current: Enabled.)

Push "s" to toggle pressure solver. (current: MICCG.)

Levelset Method

Push "p" to change the accuracy of boundary projection (current: 2nd order.)

Push "d" to toggle distance field.

Push "f" to toggle liquid field.

Push "g" to toggle grid points.

Push "v" to toggle velocity.

Push "r" to reset.

Push "i" to toggle interpolation method. (current: Catmull-Rom Spline.)

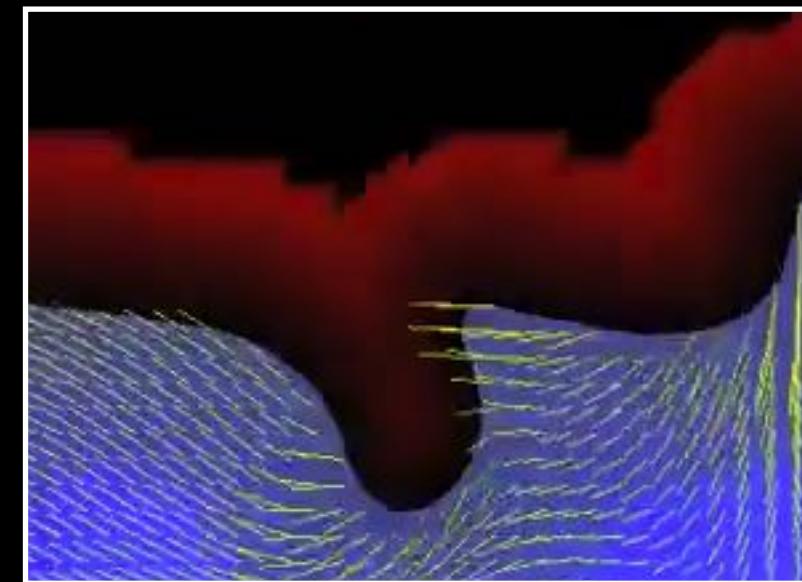
Push "a" to toggle redistance.

Push "c" to toggle volume correction. (current: Enabled.)

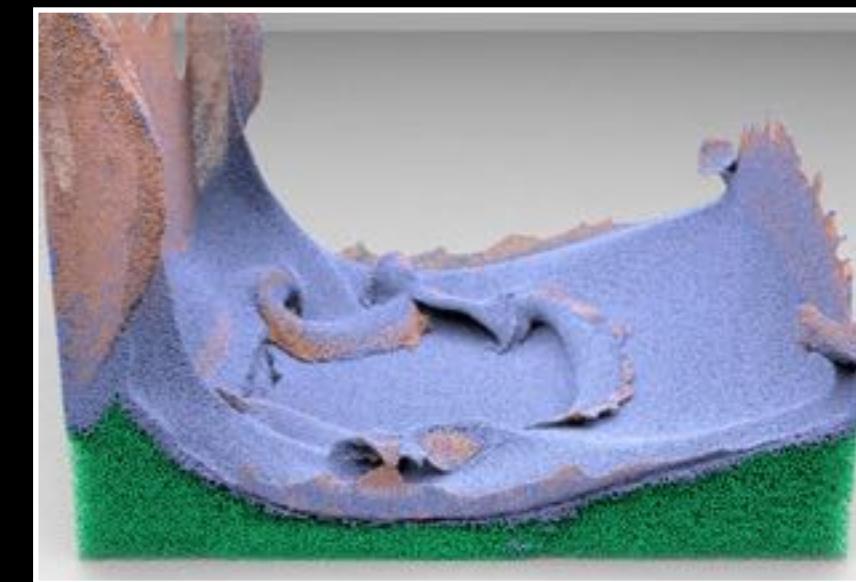
Push "s" to toggle pressure solver. (current: MICCG.)



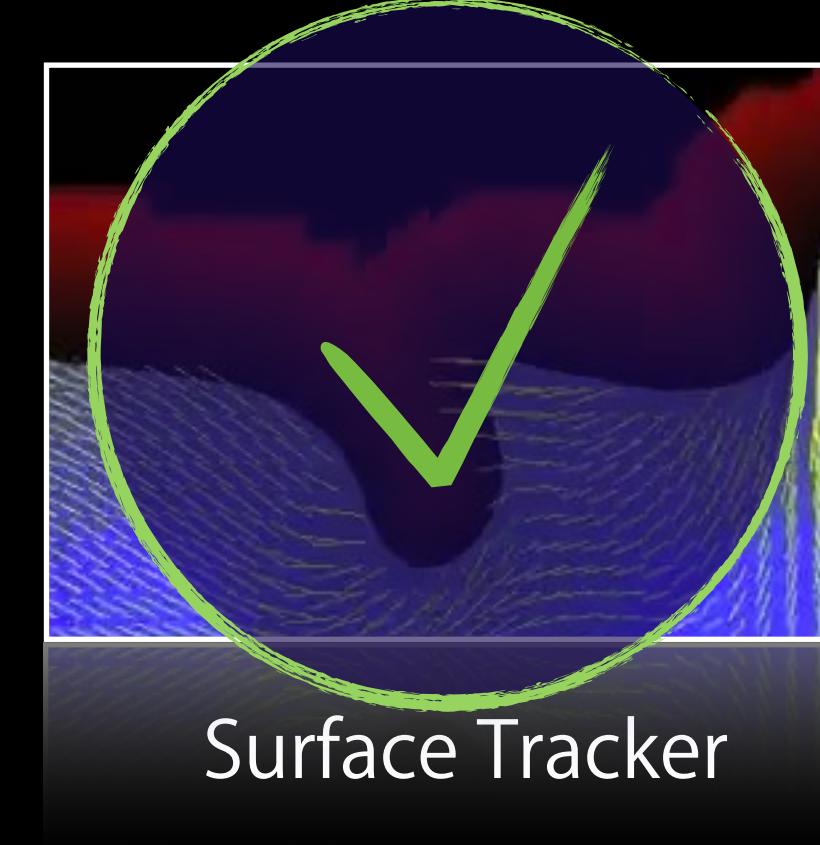
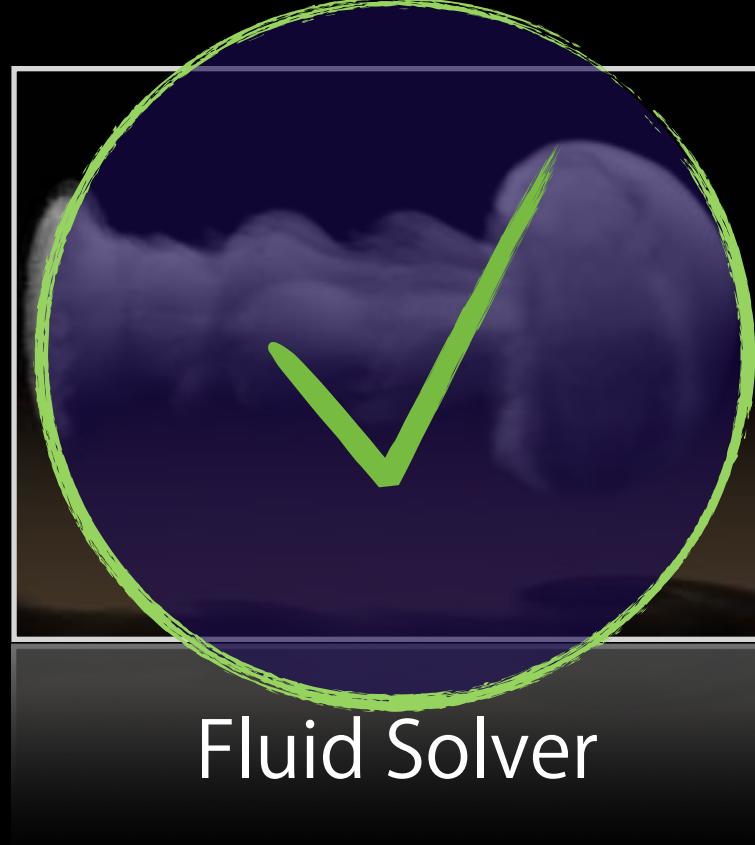
Fluid Solver

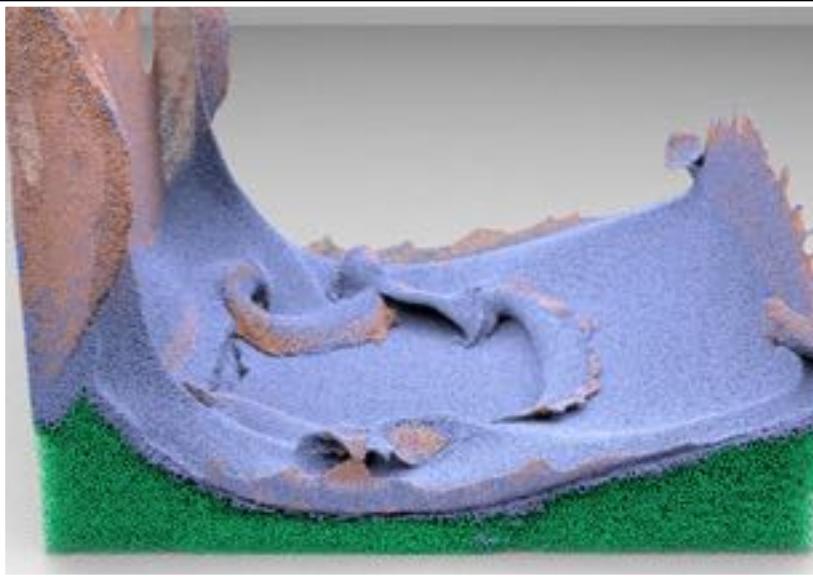


Surface Tracker



FLIP Solver



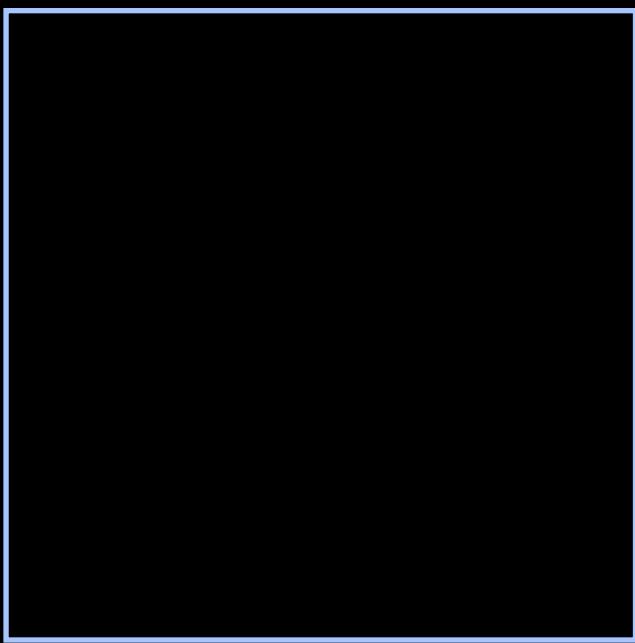


FLIP Solver

FLIP Solver

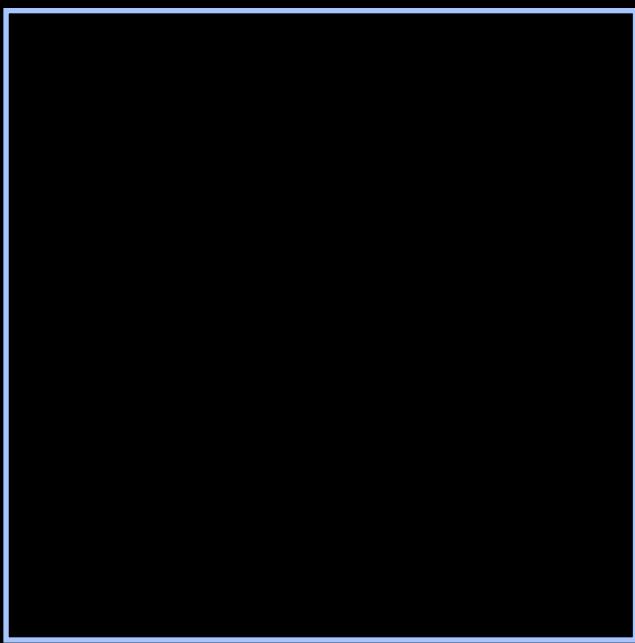
FLIP Solver

FLIP Solver



Particles

FLIP Solver



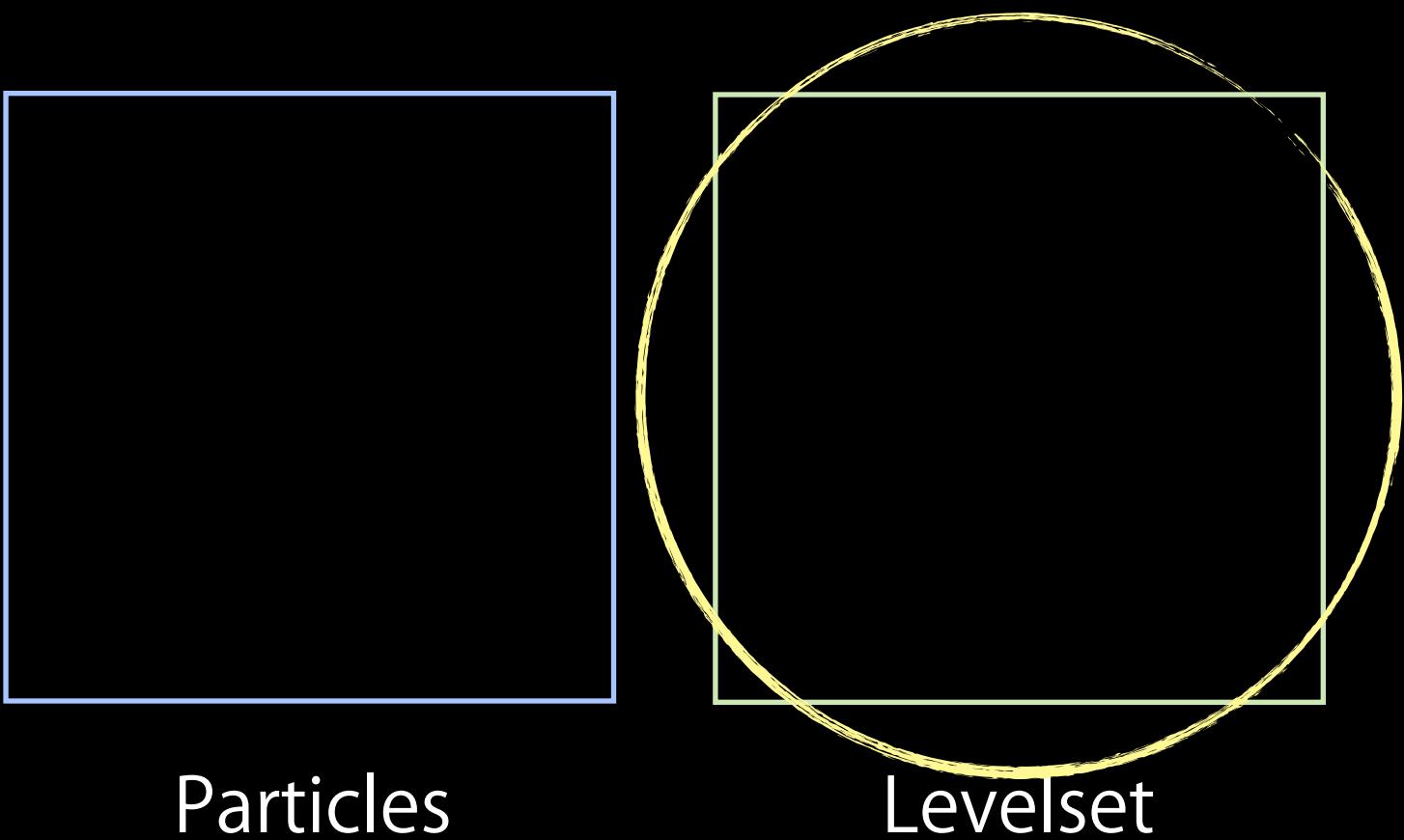
Particles



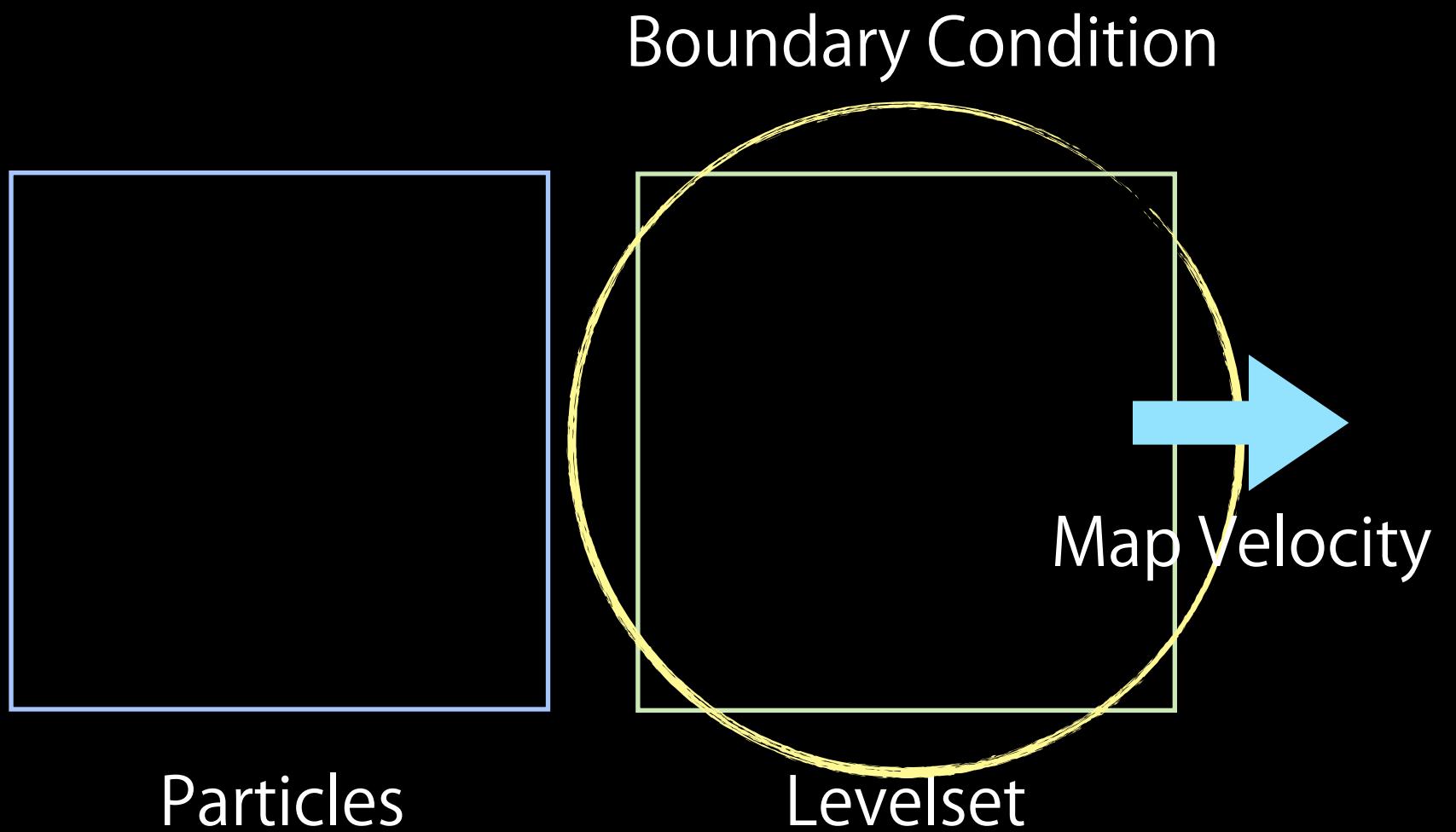
Levelset

FLIP Solver

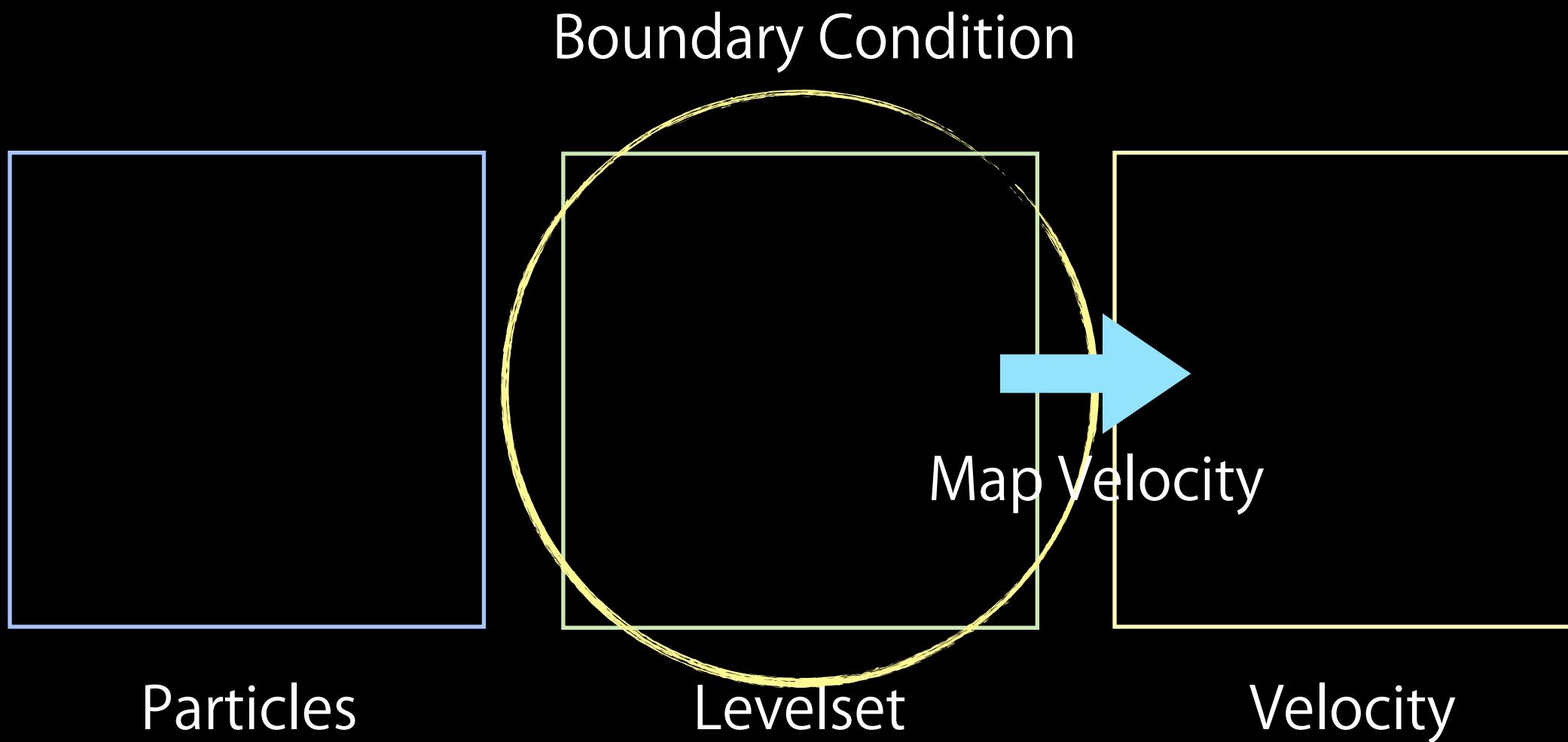
Boundary Condition



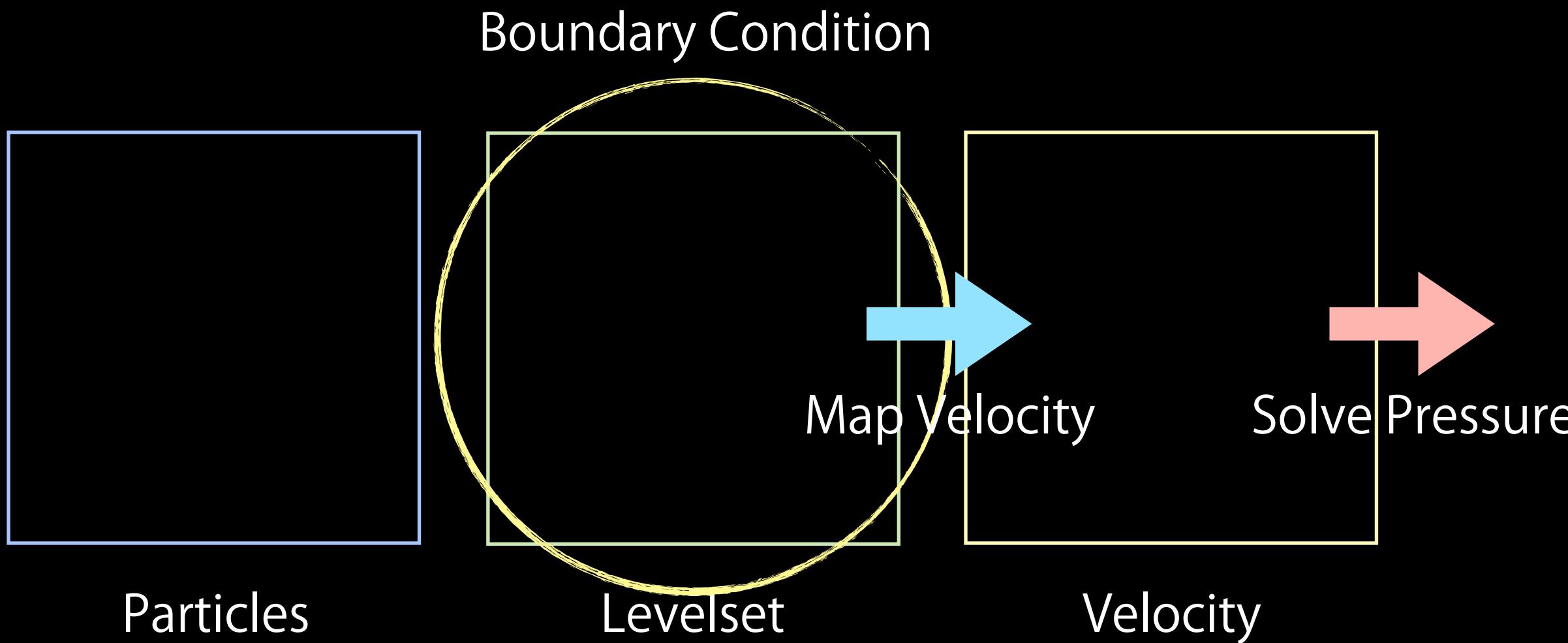
FLIP Solver



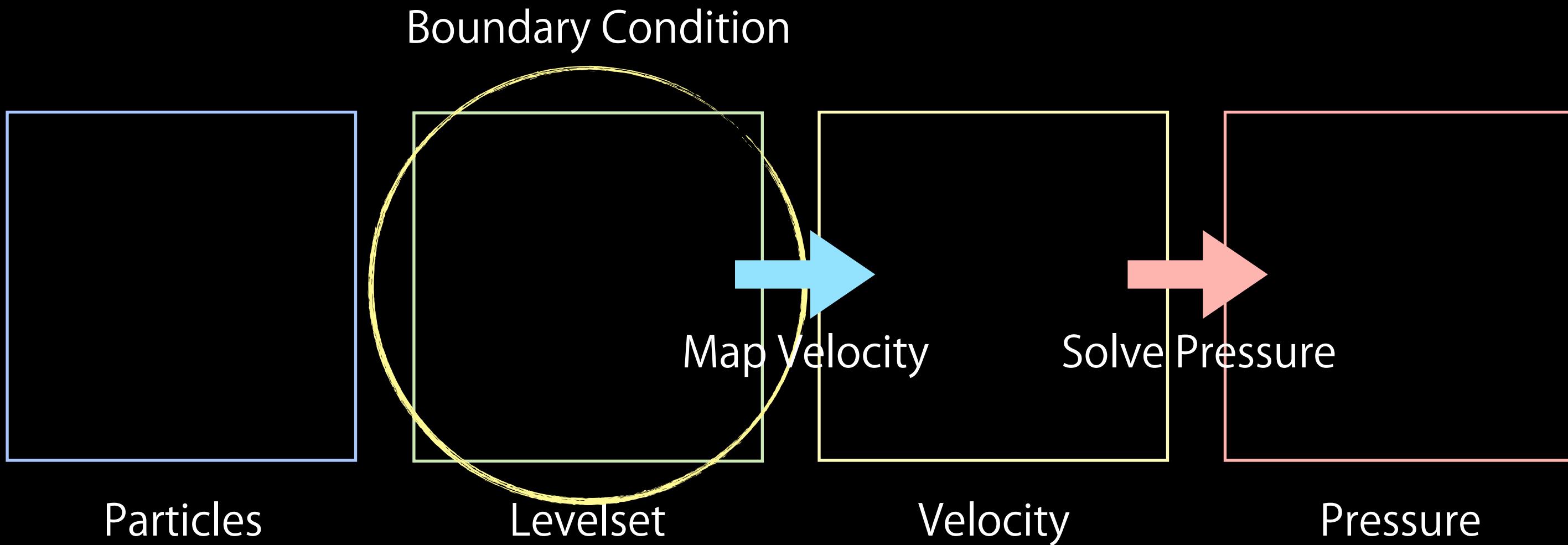
FLIP Solver



FLIP Solver



FLIP Solver



FLIP Solver

FLIP Solver

FLIP Solver

- Easy advection

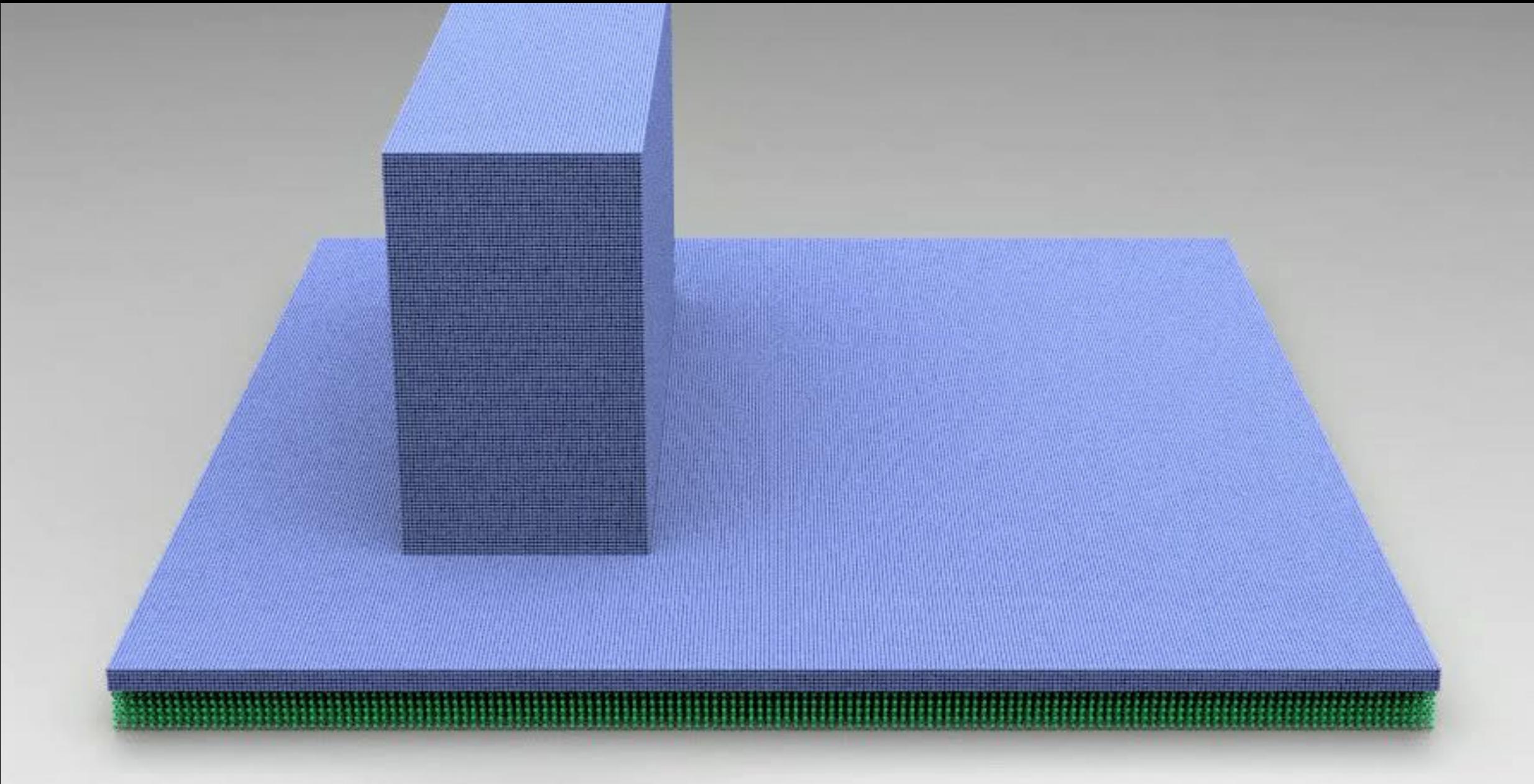
FLIP Solver

- Easy advection
- Fast pressure solver

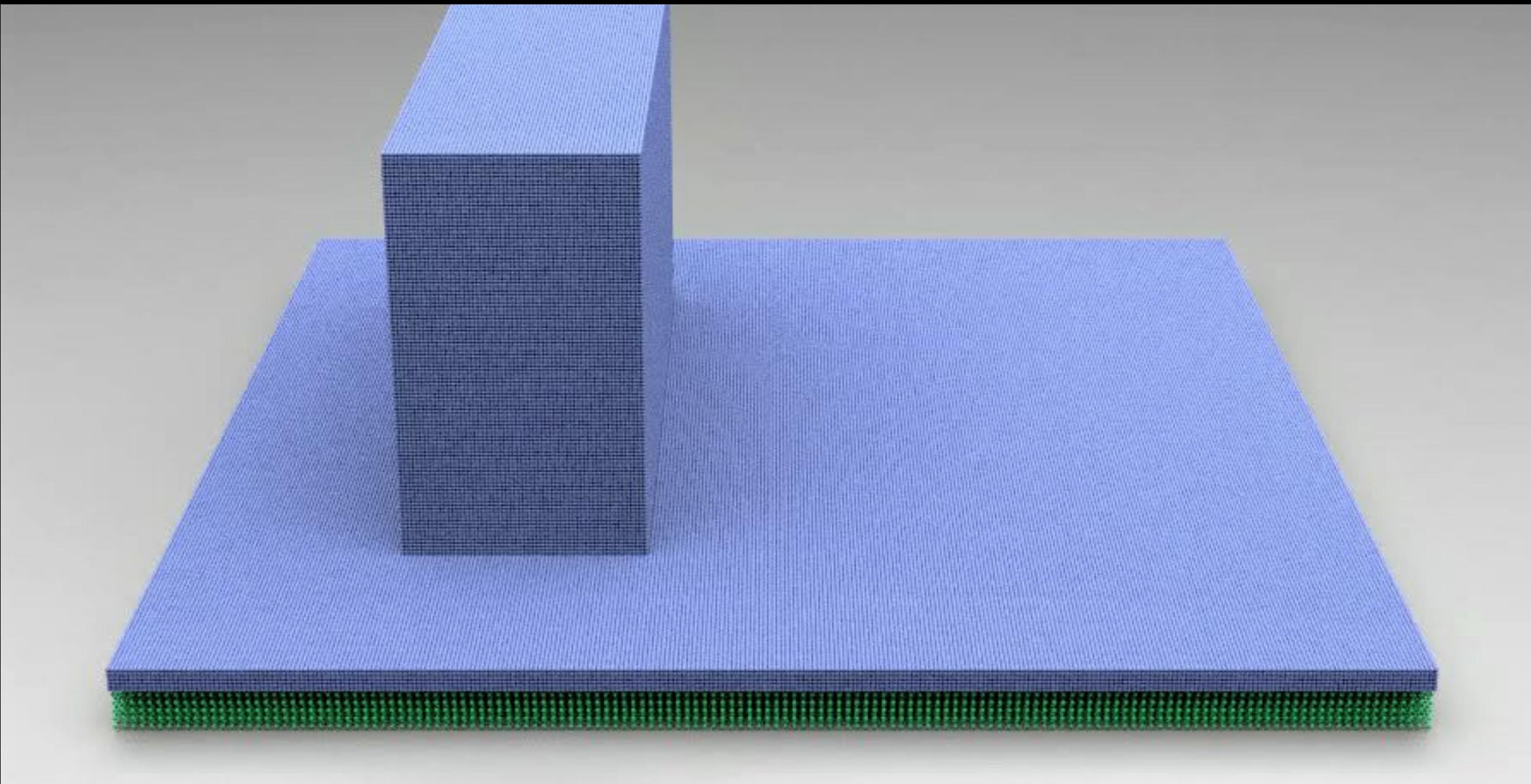
FLIP Solver

- Easy advection
- Fast pressure solver
- Splash !

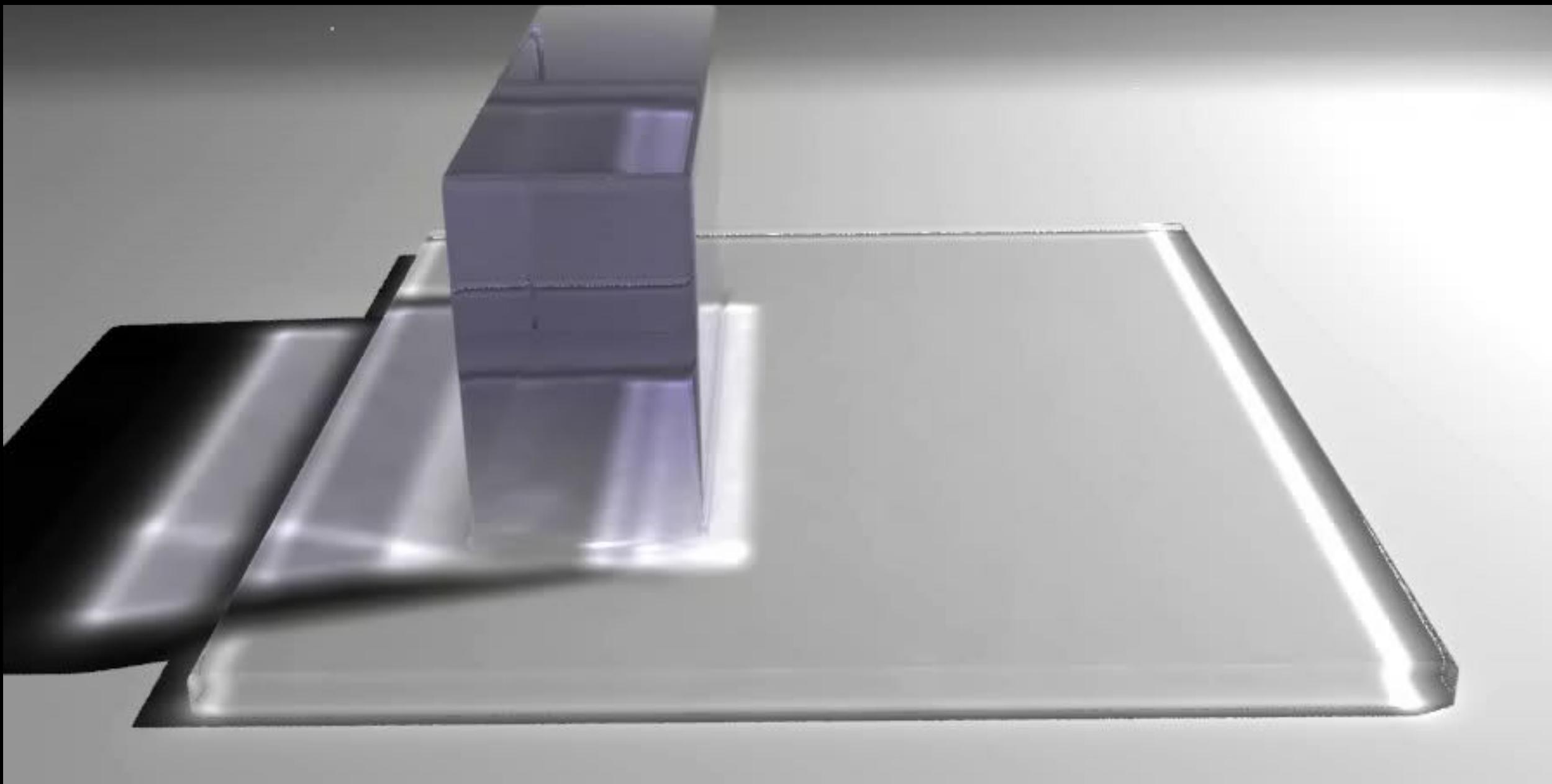
FLIP Solver



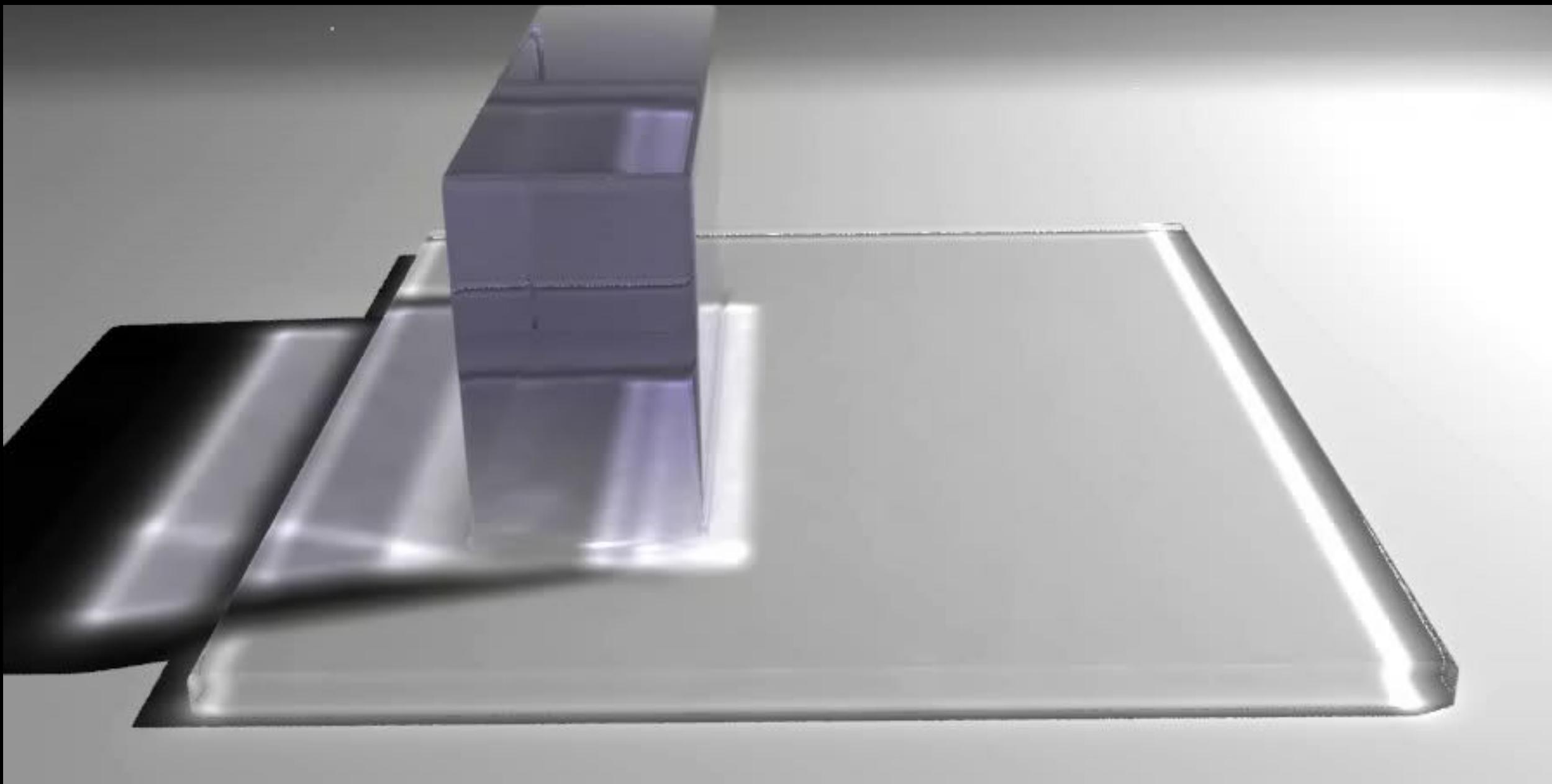
FLIP Solver



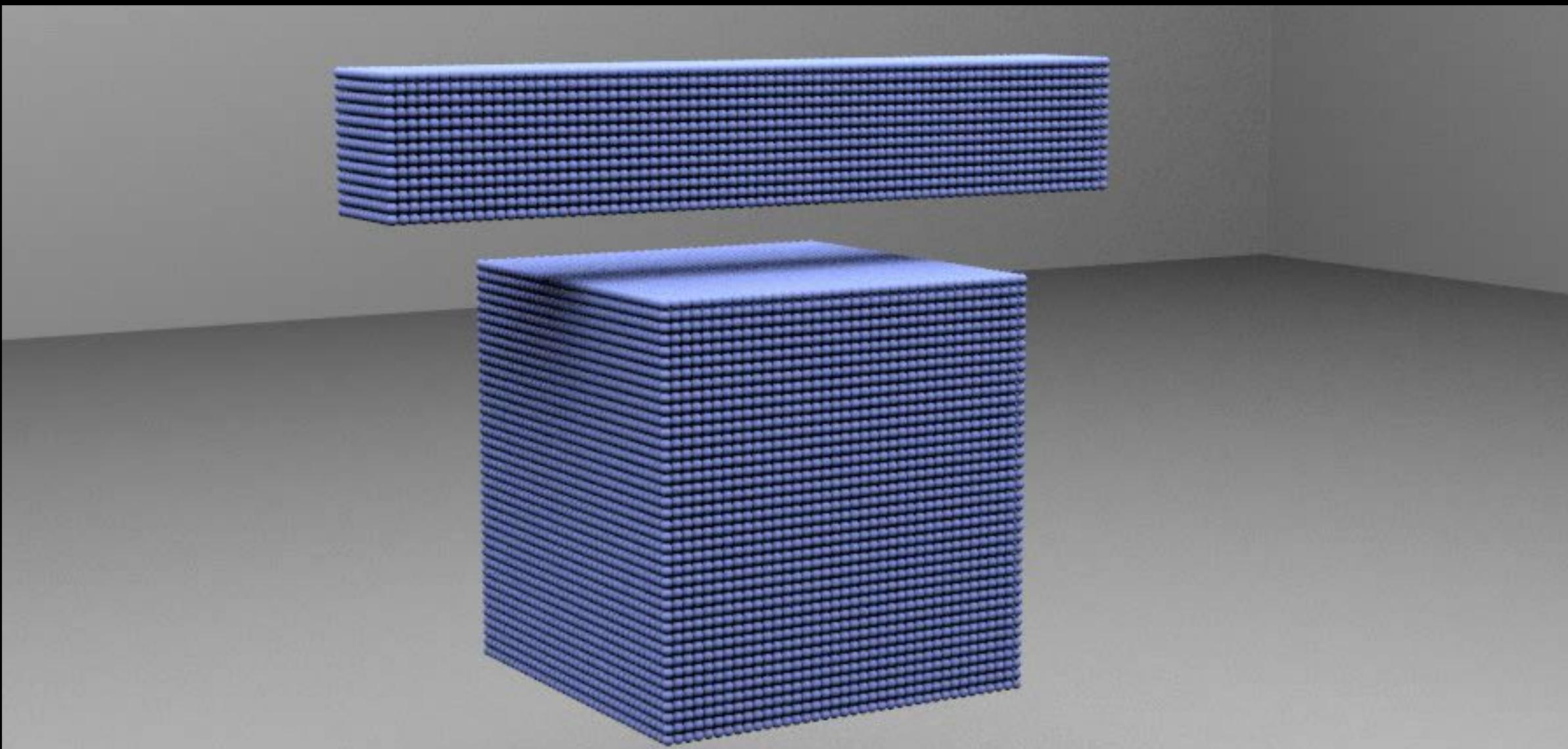
FLIP Solver



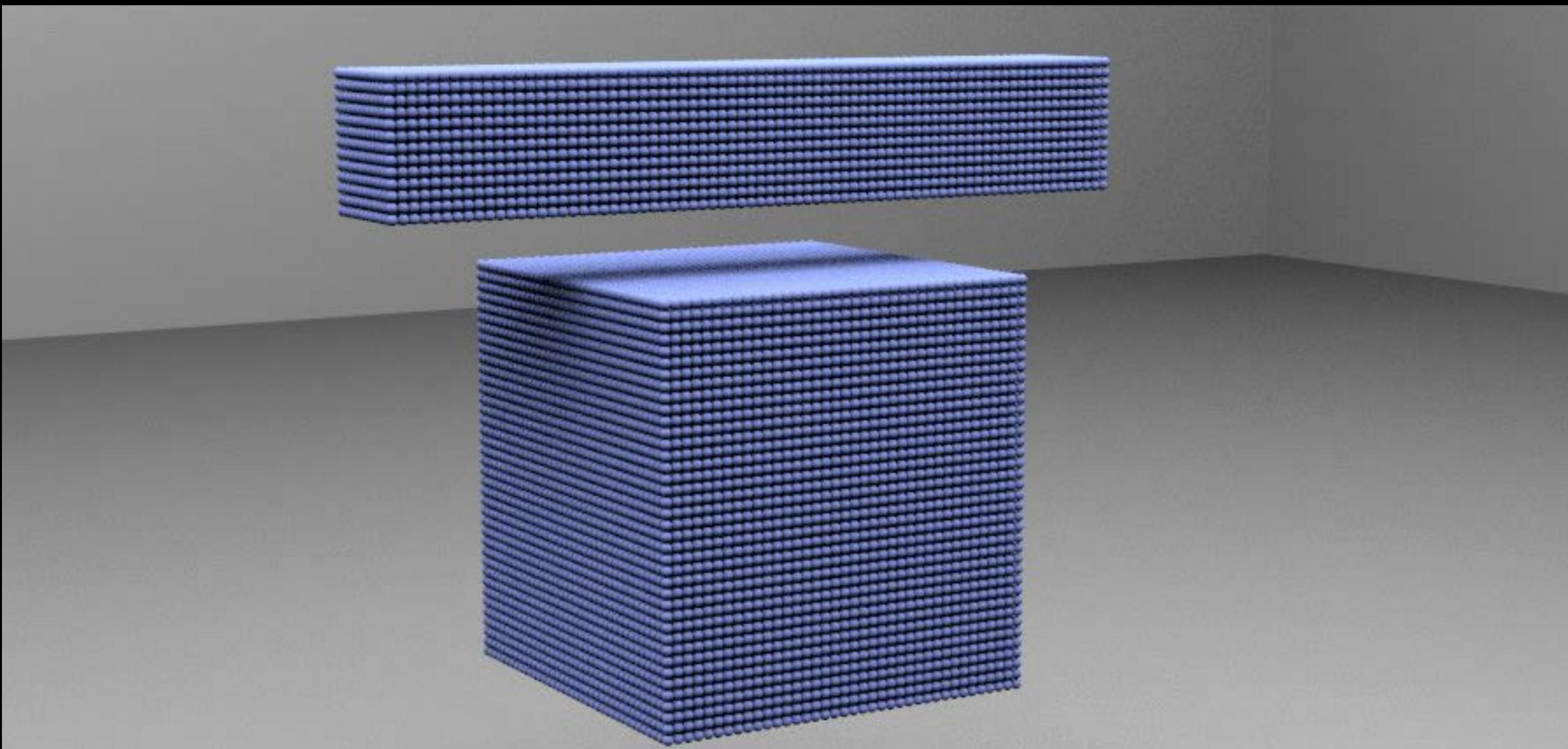
FLIP Solver



FLIP Solver

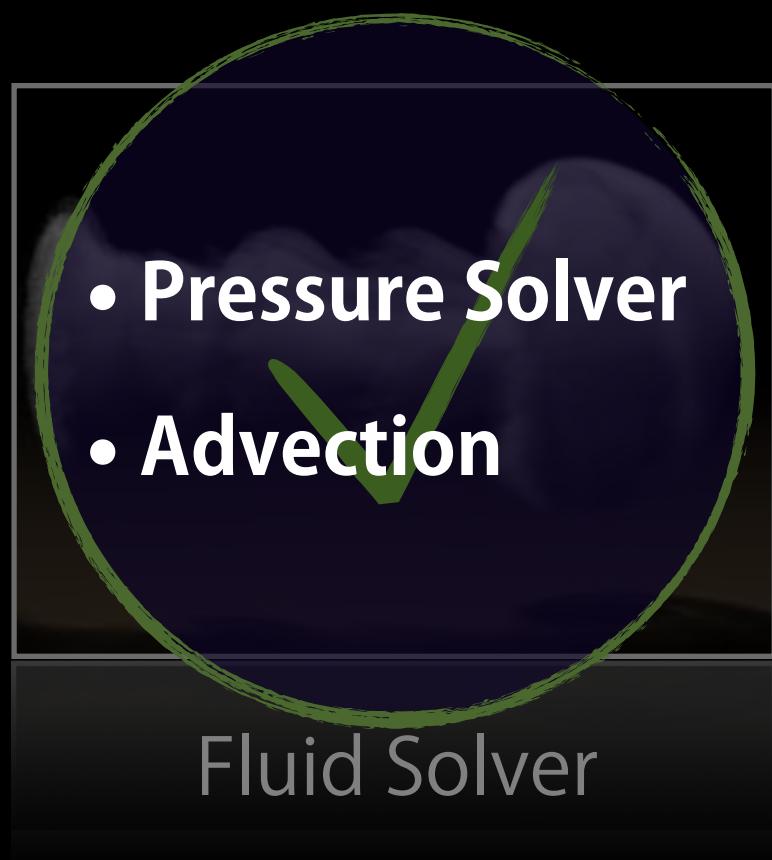


FLIP Solver

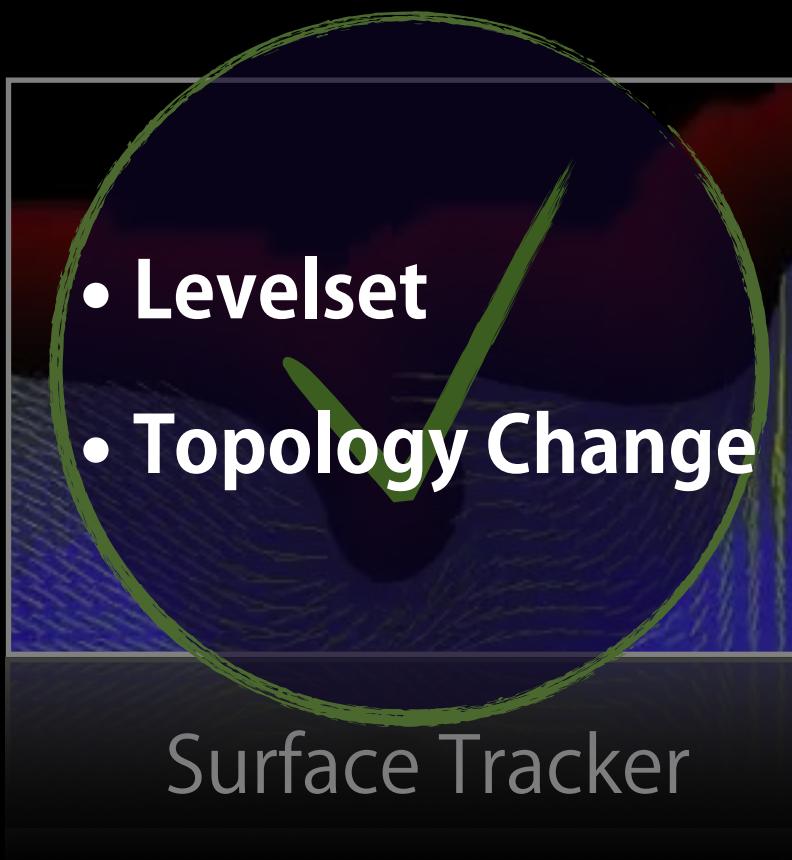
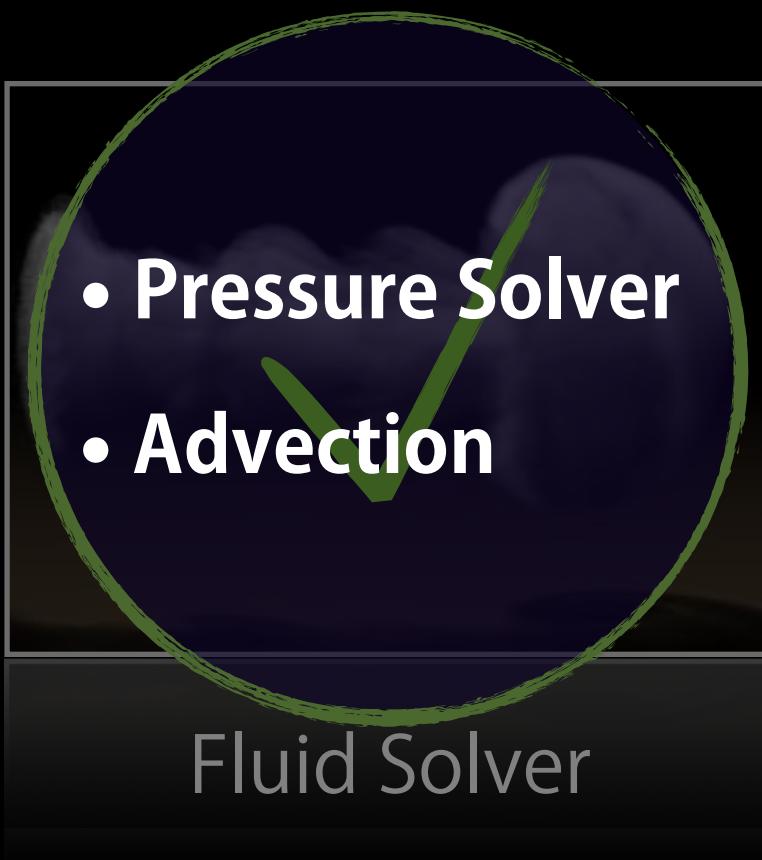


Summary

Summary



Summary



Summary

