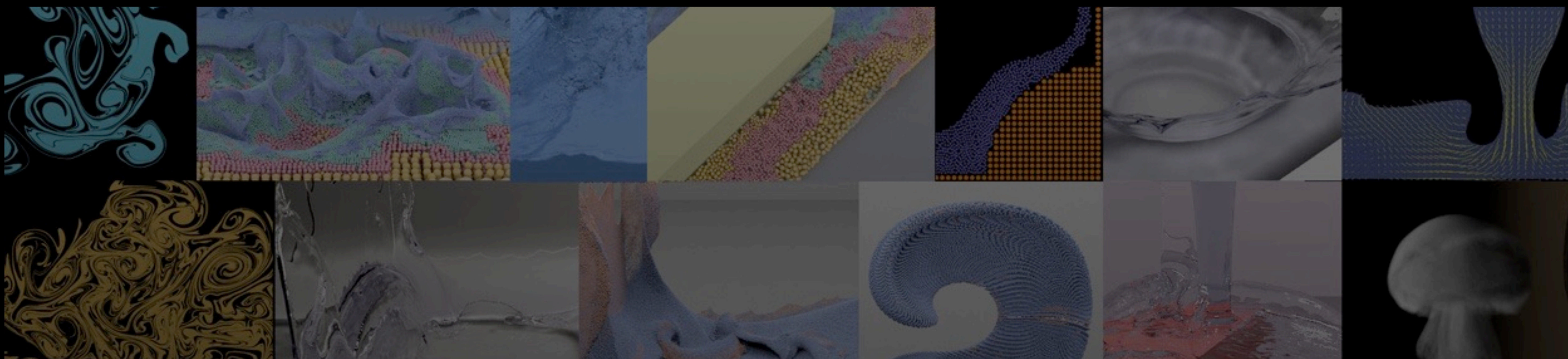
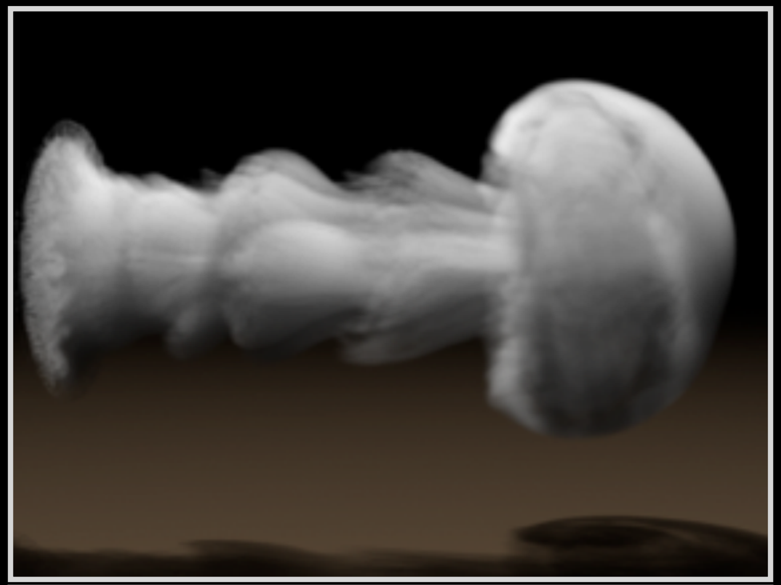
Physics-based Fluid Simulations for Computer Graphics



Ryoichi Ando

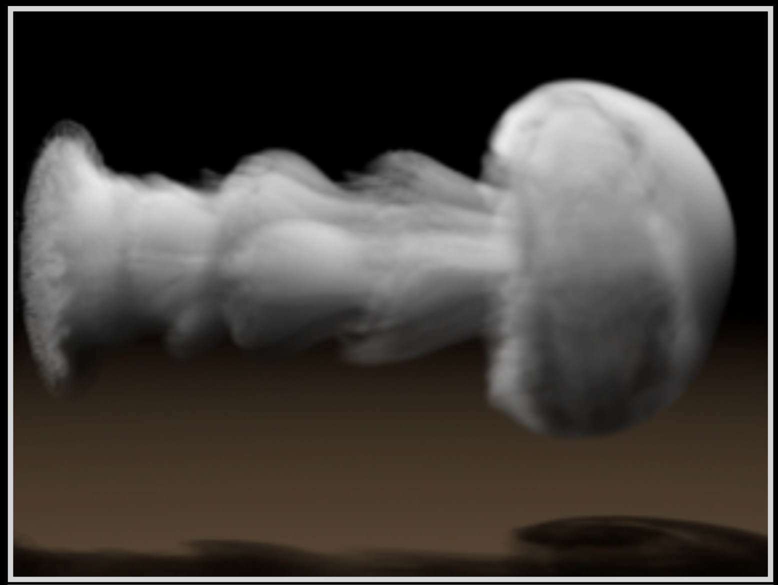
Contents

Contents

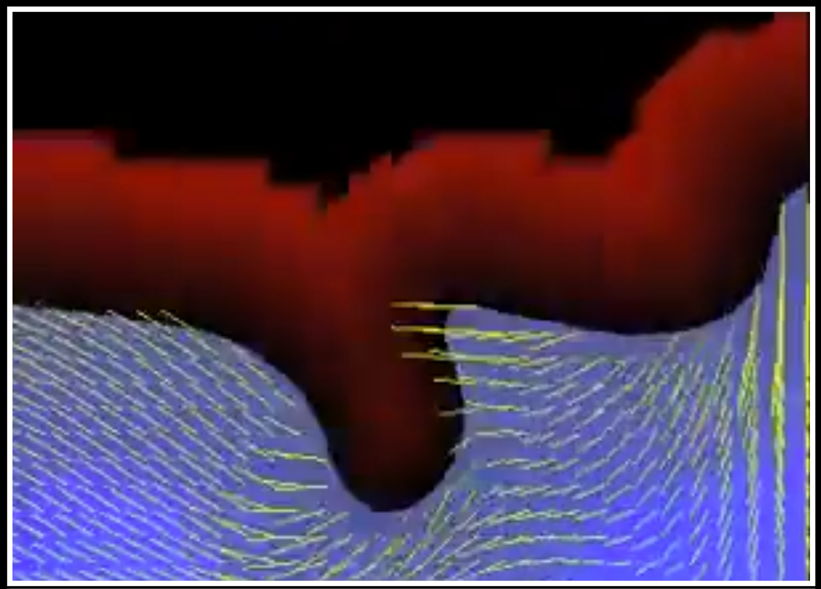


Fluid Solver

Contents

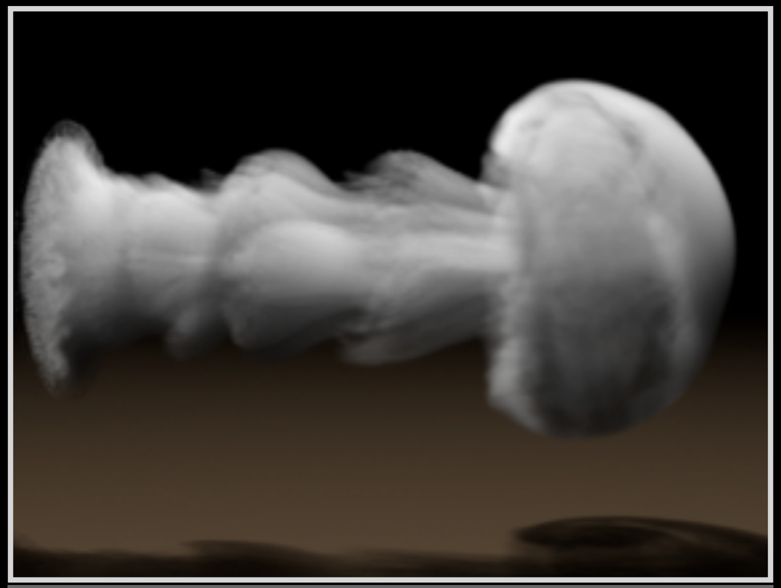


Fluid Solver

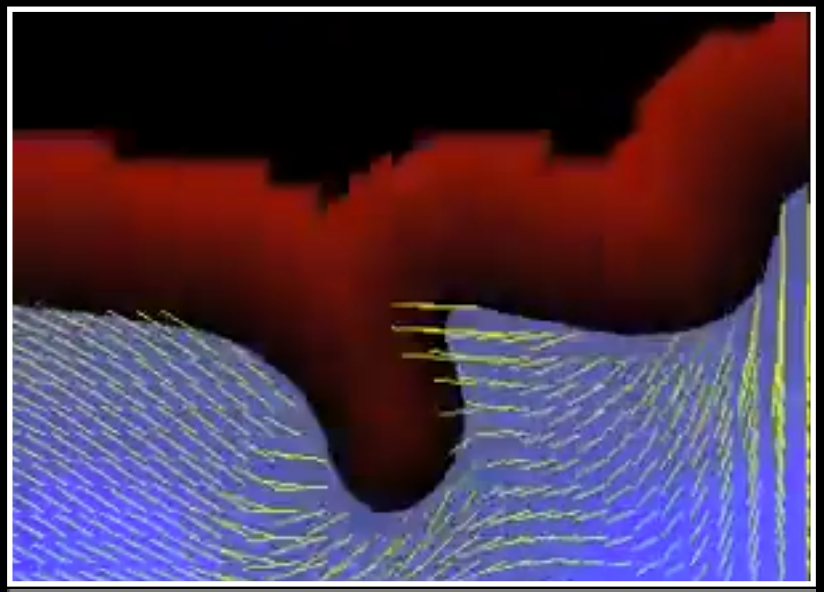


Surface Tracker

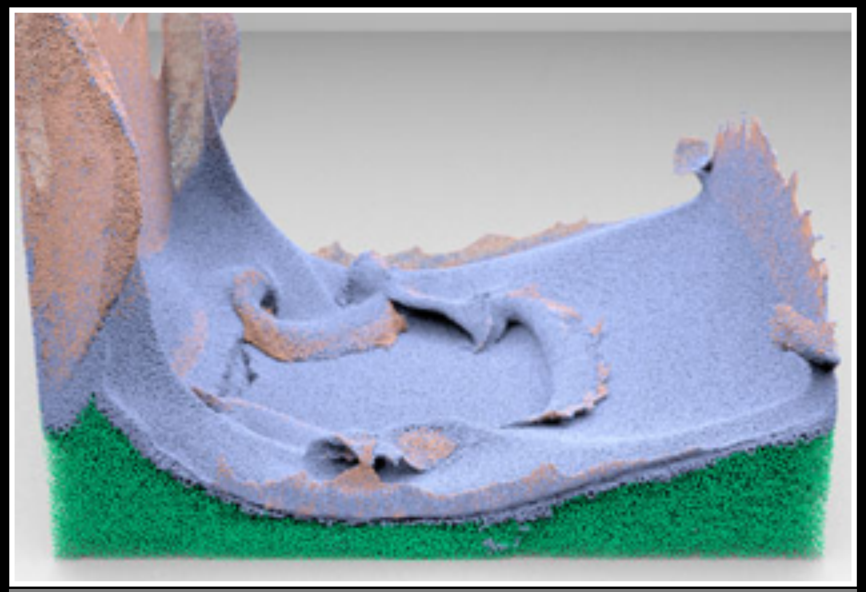
Contents



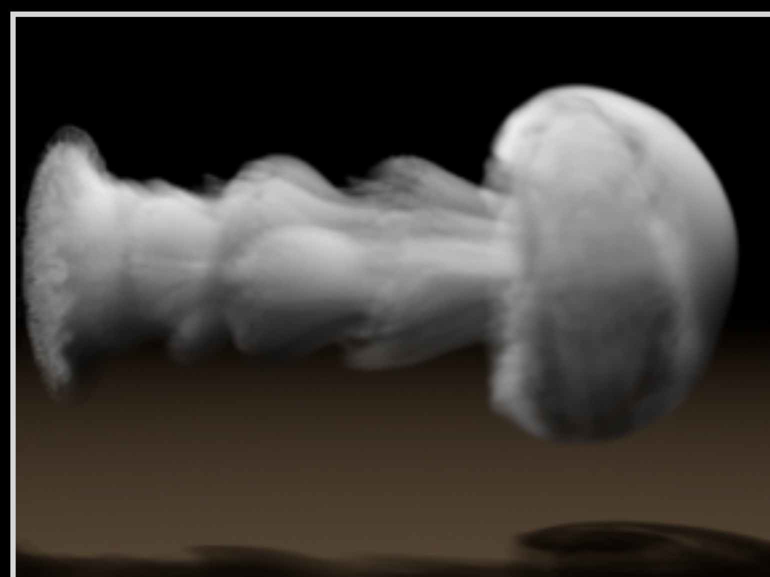
Fluid Solver



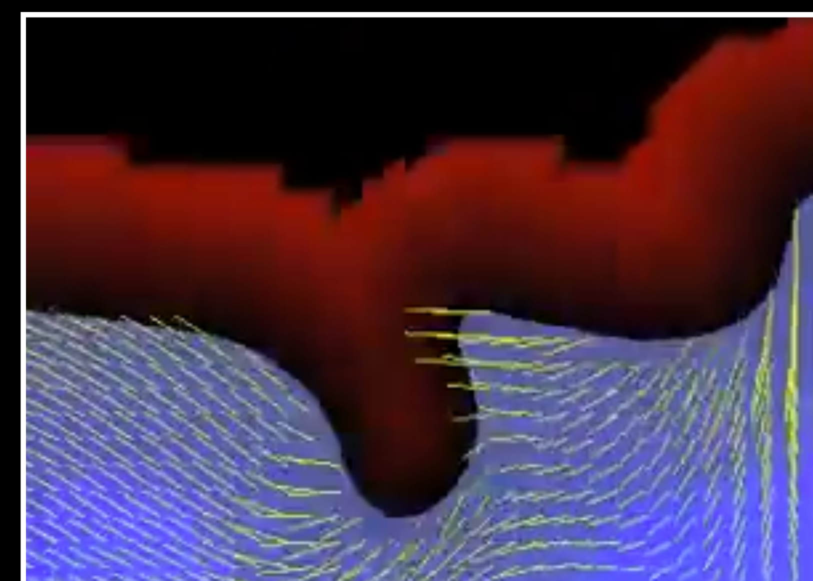
Surface Tracker



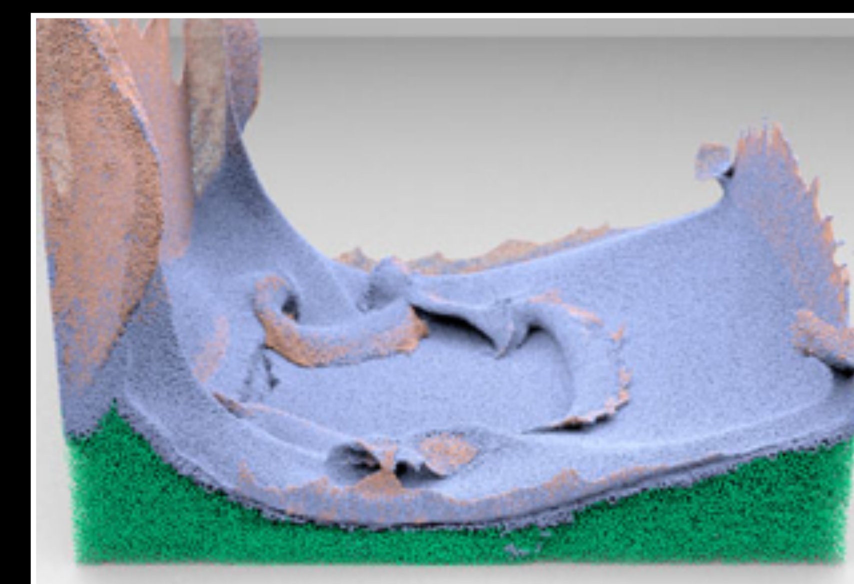
FLIP Solver



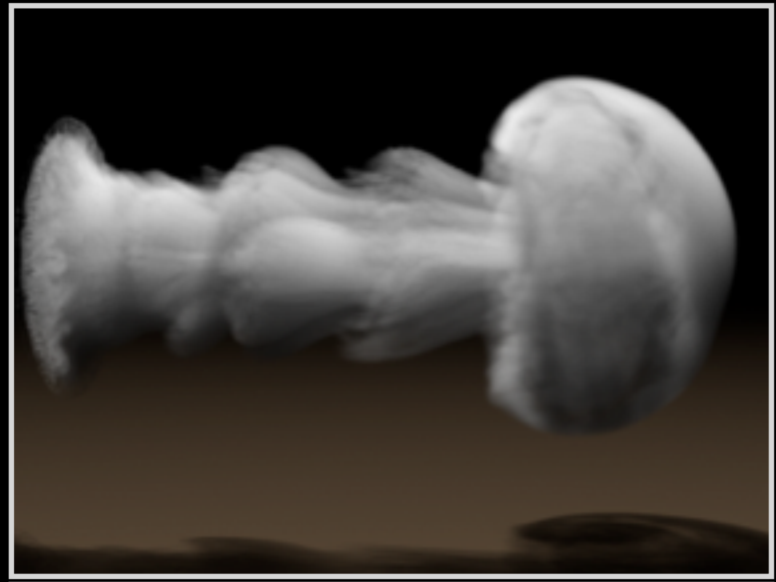
Fluid Solver



Surface Tracker



FLIP Solver



Fluid Solver

Navier-Stokes Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

s.t. $\nabla \cdot \boldsymbol{u} = 0$

Navier-Stokes Equations

How velocity changes
over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

s.t. $\nabla \cdot \boldsymbol{u} = 0$

Navier-Stokes Equations

How velocity changes over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + \boxed{(\mathbf{u} \cdot \nabla)\mathbf{u}} = -\frac{1}{\rho}\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection term

s.t. $\nabla \cdot \boldsymbol{u} = 0$

Navier-Stokes Equations

How velocity changes over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + \boxed{(\mathbf{u} \cdot \nabla)\mathbf{u}} = \boxed{-\frac{1}{\rho}\nabla p} + \mu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection term

Pressure term

s.t. $\nabla \cdot \boldsymbol{u} = 0$

Navier-Stokes Equations

How velocity changes
over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + \underbrace{\boxed{(\mathbf{u} \cdot \nabla)\mathbf{u}}}_{\text{Advection term}} = \underbrace{\boxed{-\frac{1}{\rho}\nabla p}}_{\text{Pressure term}} + \underbrace{\boxed{\mu \nabla^2 \mathbf{u}}}_{\text{Viscosity term}} + \mathbf{f}$$

s.t. $\nabla \cdot \boldsymbol{u} = 0$

Navier-Stokes Equations

How velocity changes over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + \boxed{(\mathbf{u} \cdot \nabla)\mathbf{u}} = \boxed{-\frac{1}{\rho}\nabla p} + \boxed{\mu \nabla^2 \mathbf{u}} + \boxed{\mathbf{f}}$$

Advection term

Pressure term

Viscosity term

External force term

s.t. $\nabla \cdot \boldsymbol{u} = 0$

Navier-Stokes Equations

How velocity changes over time

$$\boxed{\frac{\partial \mathbf{u}}{\partial t}} + \boxed{(\mathbf{u} \cdot \nabla)\mathbf{u}} = \boxed{-\frac{1}{\rho}\nabla p} + \boxed{\mu \nabla^2 \mathbf{u}} + \boxed{\mathbf{f}}$$

Advection term

Pressure term

Viscosity term

External force term

s.t. $\boxed{\nabla \cdot \boldsymbol{u} = 0}$

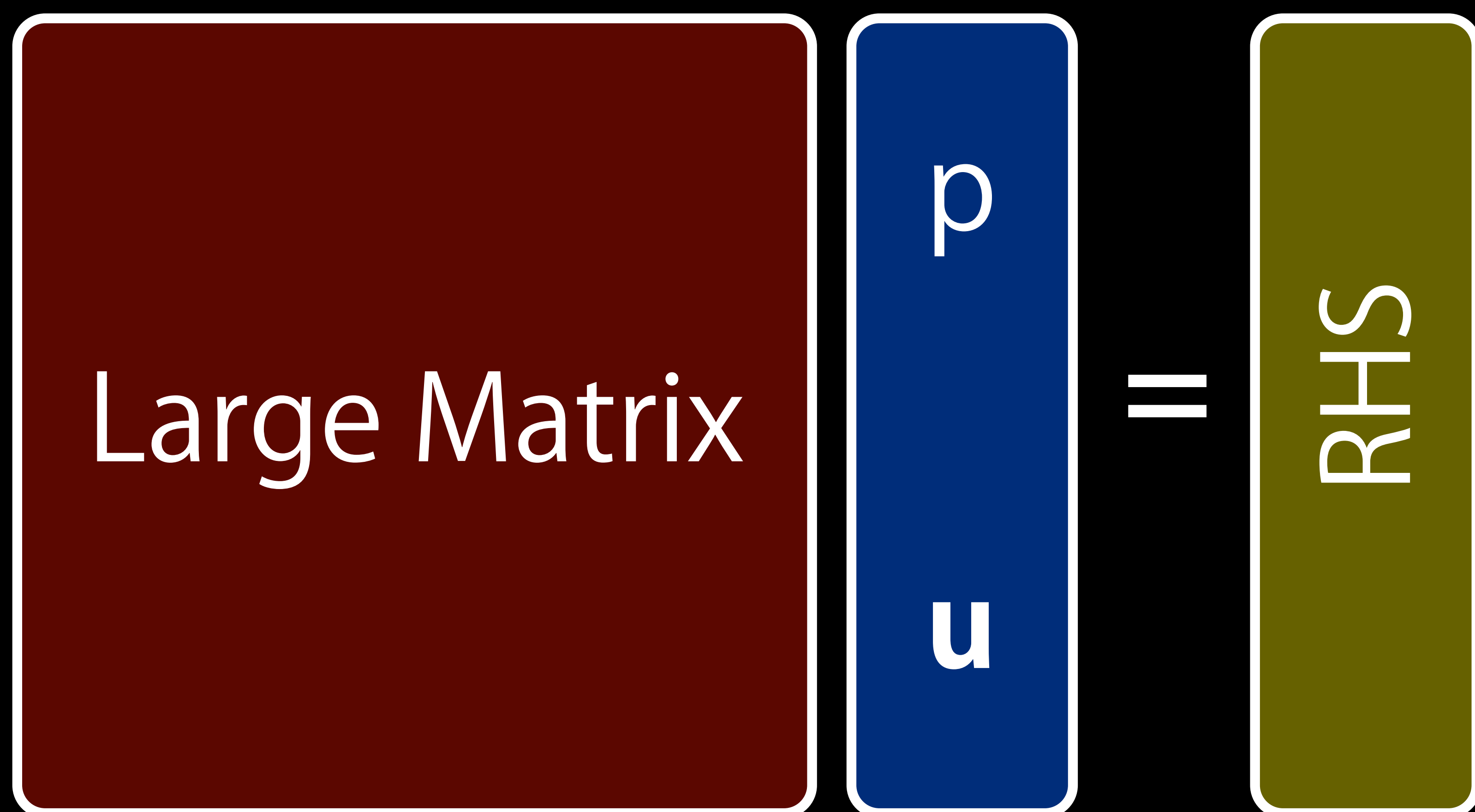
Incompressiblity Constraint

Navier-Stokes Equations

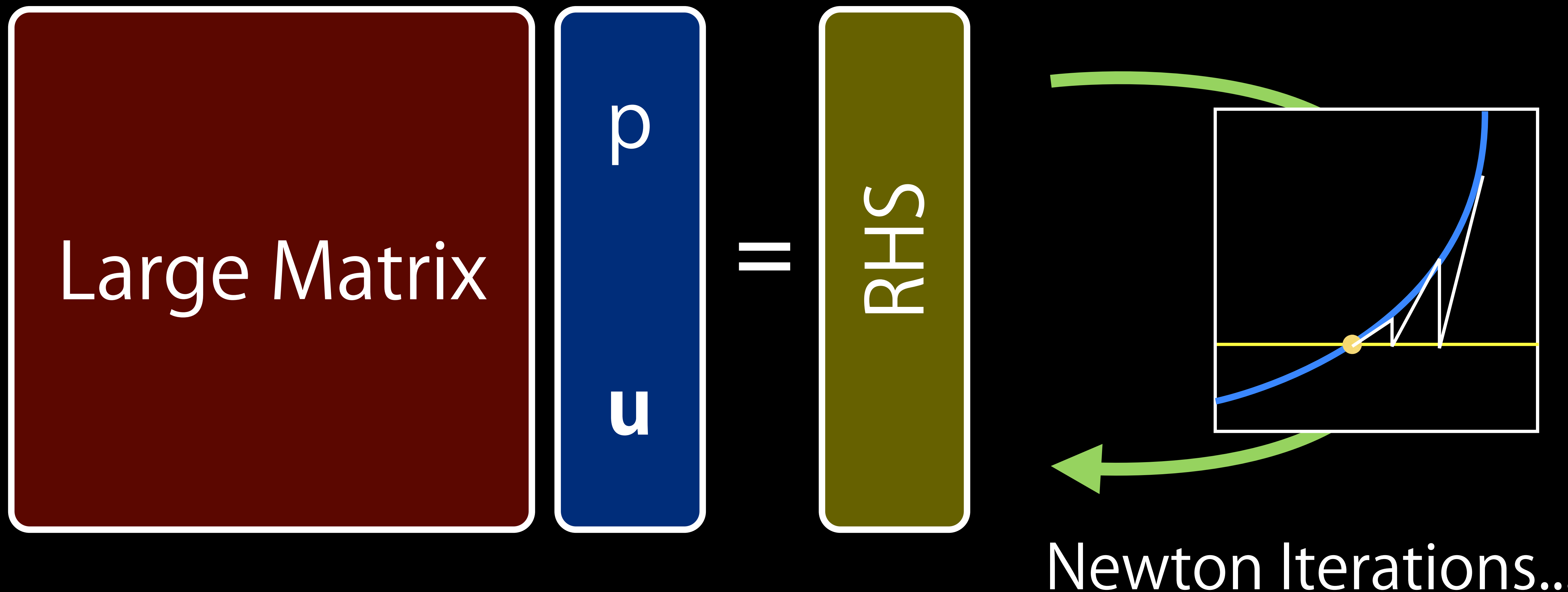
Navier-Stokes Equations

Large Matrix

Navier-Stokes Equations



Navier-Stokes Equations



Navier-Stokes Equations

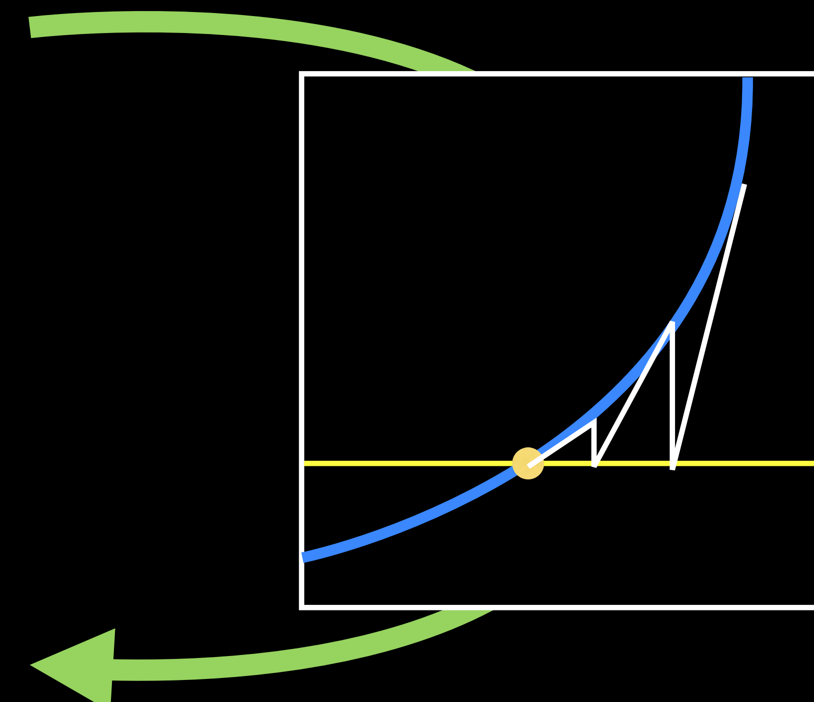
Large Matrix

p

u

=

RHS



Newton Iterations...

Operator Splitting

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

Operator Splitting

$$\underbrace{\boxed{\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}}}_{\boldsymbol{u}^*_{\mathrm{Adv}}} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

Operator Splitting

$$\underbrace{\boxed{\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}}_{\boldsymbol{u}^*_{\mathrm{Adv}}} = \mu \nabla^2 \mathbf{u}}_{\boldsymbol{u}^*_{\mathrm{Adv+Visc}}} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

Operator Splitting

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

$$\boldsymbol{u}^*_{\mathrm{Adv}}$$

$$\boldsymbol{u}^*_{\mathrm{Adv+Visc}}$$

$$\boldsymbol{u}^*_{\mathrm{Adv+Visc+Prs}}$$

Operator Splitting

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mu \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}$$

$\boldsymbol{u}^*_{\mathrm{Adv}}$

$\boldsymbol{u}^*_{\mathrm{Adv+Visc}}$

$\boldsymbol{u}^*_{\mathrm{Adv+Visc+Prs}}$

$\boldsymbol{u}_{t+\Delta t}$

[Stable fluid, Stam99]

First-order integration

$$\frac{\partial \boldsymbol{u}}{\partial t} = f(\boldsymbol{u}, p)$$

$$\Downarrow$$

$$\boldsymbol{u}_{t+\Delta t} = \boldsymbol{u}_t + \Delta t f(\boldsymbol{u}, p)$$

First-order integration

$$\frac{\partial \boldsymbol{u}}{\partial t} = f(\boldsymbol{u}, p)$$

$$\Downarrow$$

$$\boldsymbol{u}_{t+\Delta t} = \boldsymbol{u}_t + \Delta t f(\boldsymbol{u}, p)$$

e.g, advection $\boldsymbol{u}_{t+\Delta t} = \boldsymbol{u}_t + \Delta t \underline{[\mathbf{u} \cdot \nabla \mathbf{u}]}$

First-order integration

$$\frac{\partial \boldsymbol{u}}{\partial t} = f(\boldsymbol{u}, p)$$

First-order accuracy!

$$\boldsymbol{u}_{t+\Delta t} = \boldsymbol{u}_t + \Delta t f(\boldsymbol{u}, p) + O(\Delta t)$$

e.g, advection $\boldsymbol{u}_{t+\Delta t} = \boldsymbol{u}_t + \Delta t \underline{[\mathbf{u} \cdot \nabla \mathbf{u}]}$

Simulation Steps

Simulation Steps

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

Simulation Steps

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho}\nabla p}$$

Pressure term

Simulation Steps

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

$$\boxed{\mu \nabla^2 \mathbf{u}}$$

Viscosity term

Simulation Steps

$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

$\boxed{-\frac{1}{\rho} \nabla p}$

Pressure term

$\boxed{\mu \nabla^2 \mathbf{u}}$

Viscosity term

$\boxed{\boldsymbol{F}}$

External force term

Simulation Steps

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

Simulation Steps

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

Simulation Steps

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}} \rightarrow \boxed{-\frac{1}{\rho} \nabla p}$$

Advection term

Pressure term

Simulation Loop...

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = 0$$

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

Scalar

$$\frac{\partial \boxed{\phi}}{\partial t} + (\boldsymbol{u} \cdot \nabla) \boxed{\phi} = 0$$

Scalar

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

Scalar

$$\frac{\partial \boxed{\phi}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boxed{\phi} = 0$$

Scalar

Expand...

$$\frac{\partial \phi}{\partial t} + u_x \frac{\partial}{\partial x}\phi + u_y \frac{\partial}{\partial y}\phi + u_z \frac{\partial}{\partial z}\phi = 0$$

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

Scalar

$$\frac{\partial \boxed{\phi}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boxed{\phi} = 0$$

Scalar

Expand...

$$\frac{\partial \phi}{\partial t} + u_x \frac{\partial}{\partial x}\phi + u_y \frac{\partial}{\partial y}\phi + \boxed{u_z \frac{\partial}{\partial z}\phi} = 0$$

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

Scalar

$$\frac{\partial \boxed{\phi}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boxed{\phi} = 0$$

Scalar

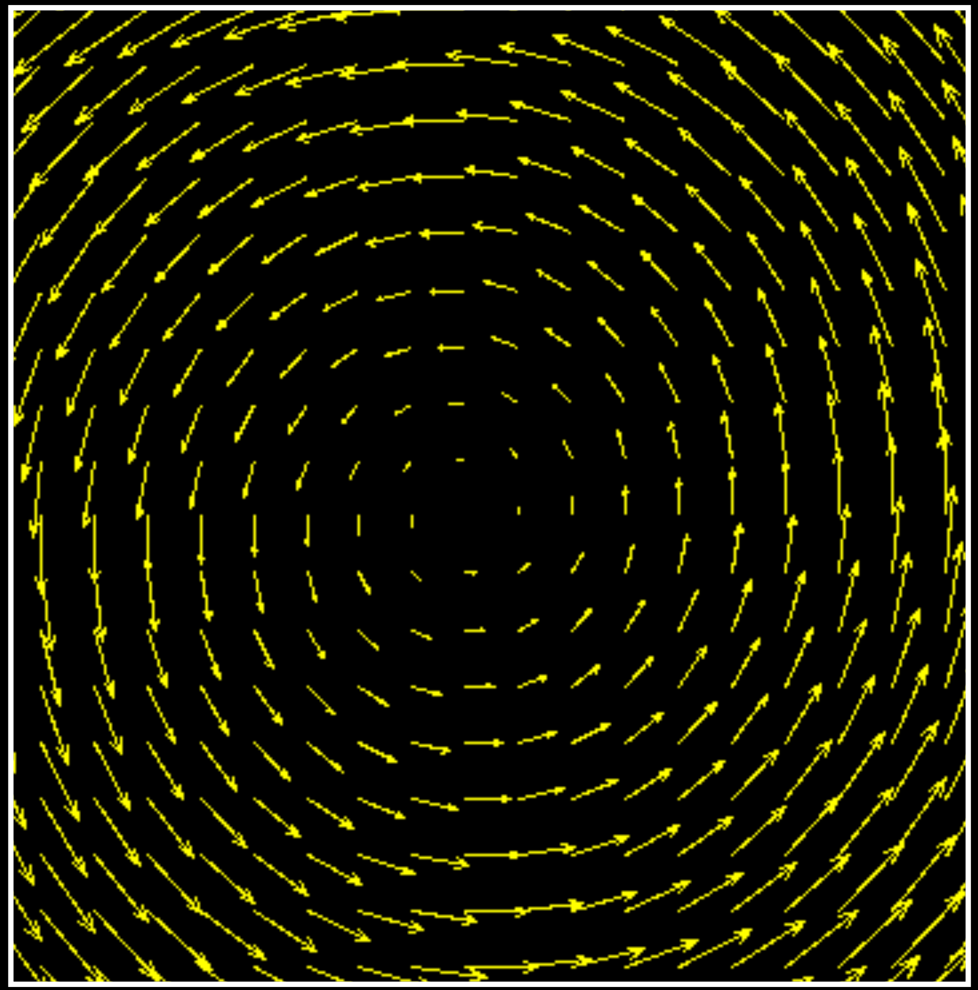
Expand...

$$\frac{\partial \phi}{\partial t} + \sum \boxed{u_i \frac{\partial}{\partial x_i} \phi} = 0$$

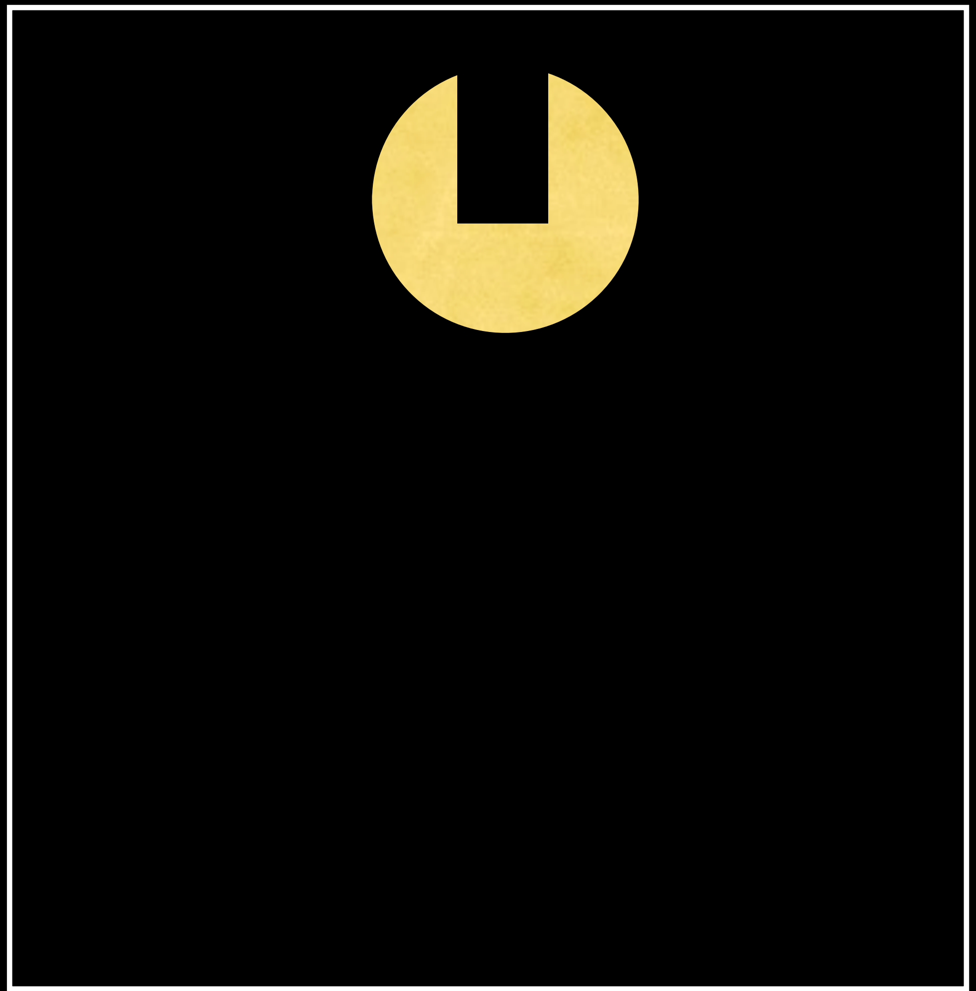
Key component !

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term



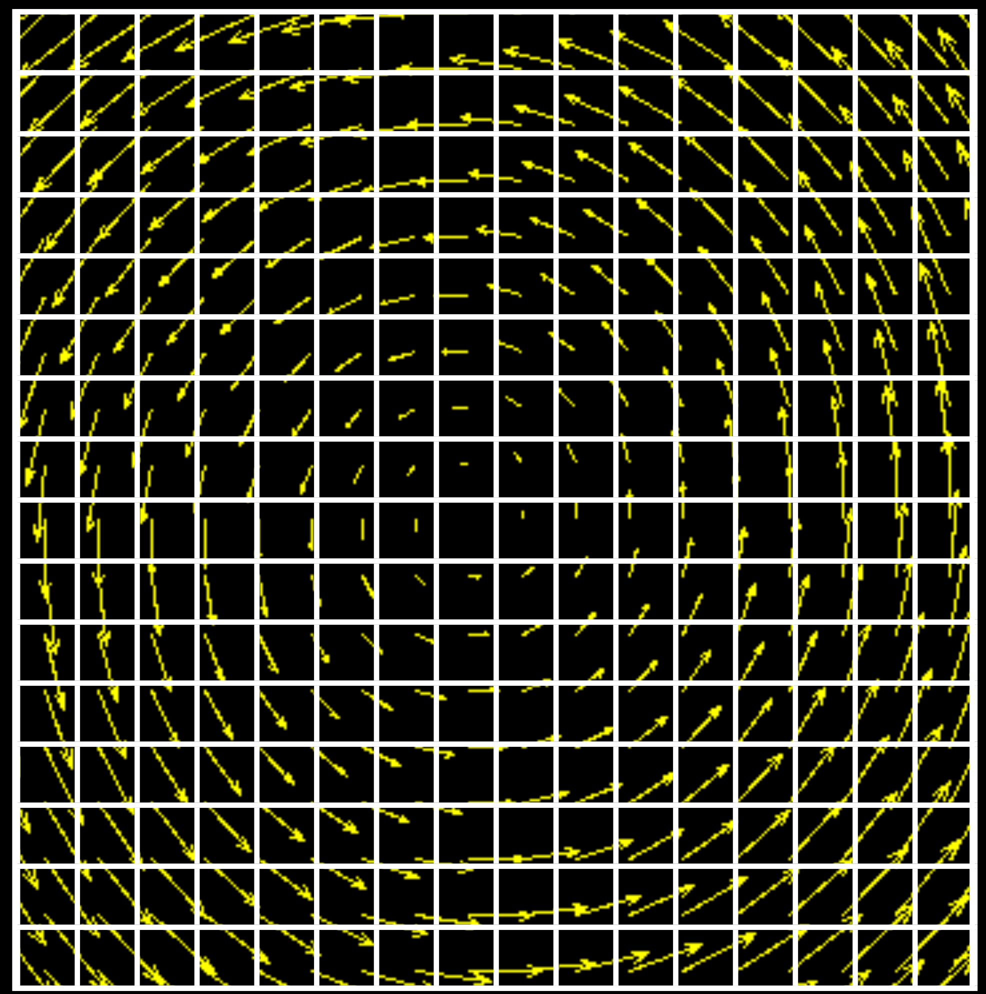
Velocity field $\boldsymbol{u}$



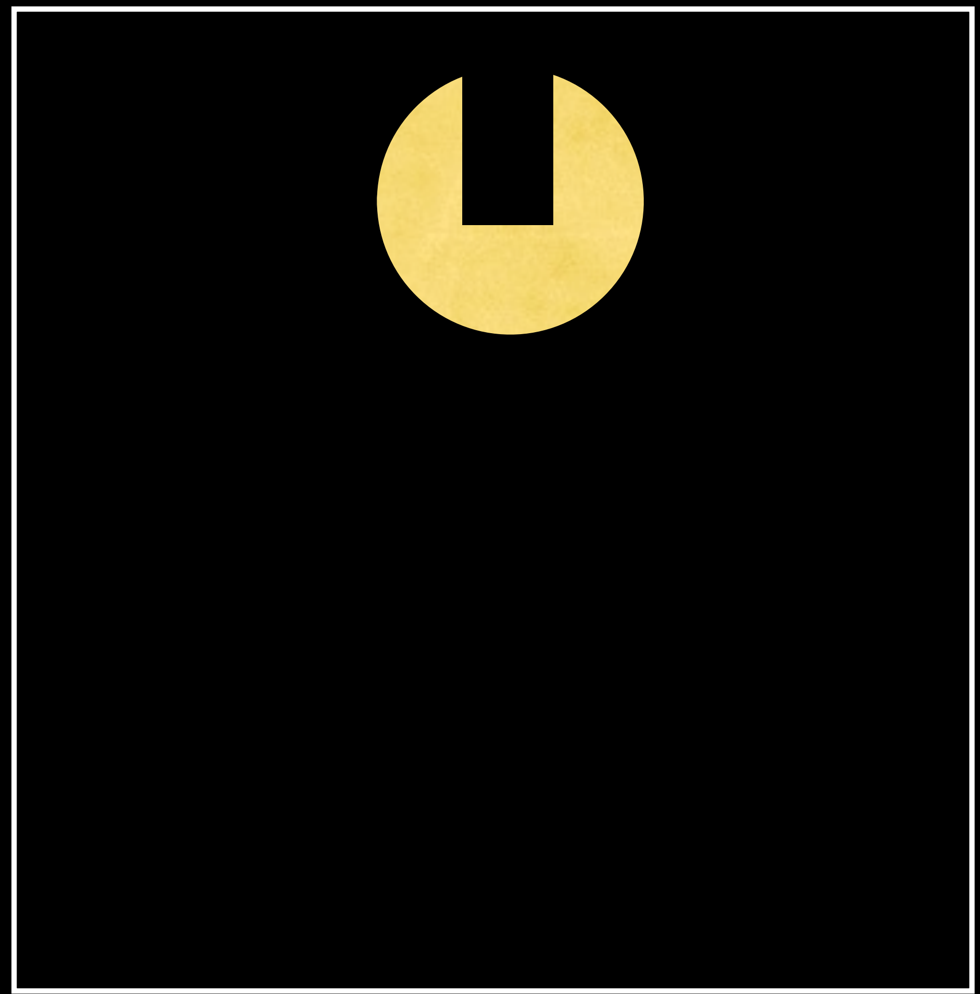
Scalar field ϕ

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term



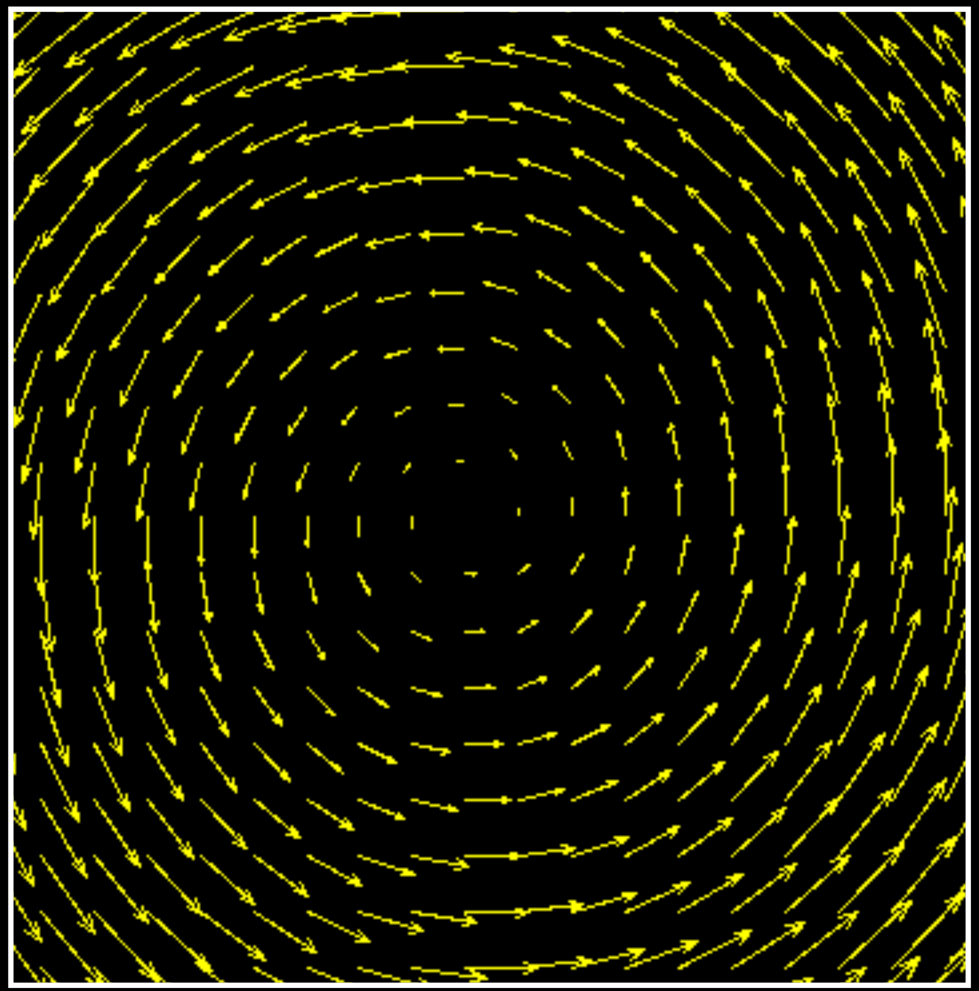
Velocity field $\boldsymbol{u}$



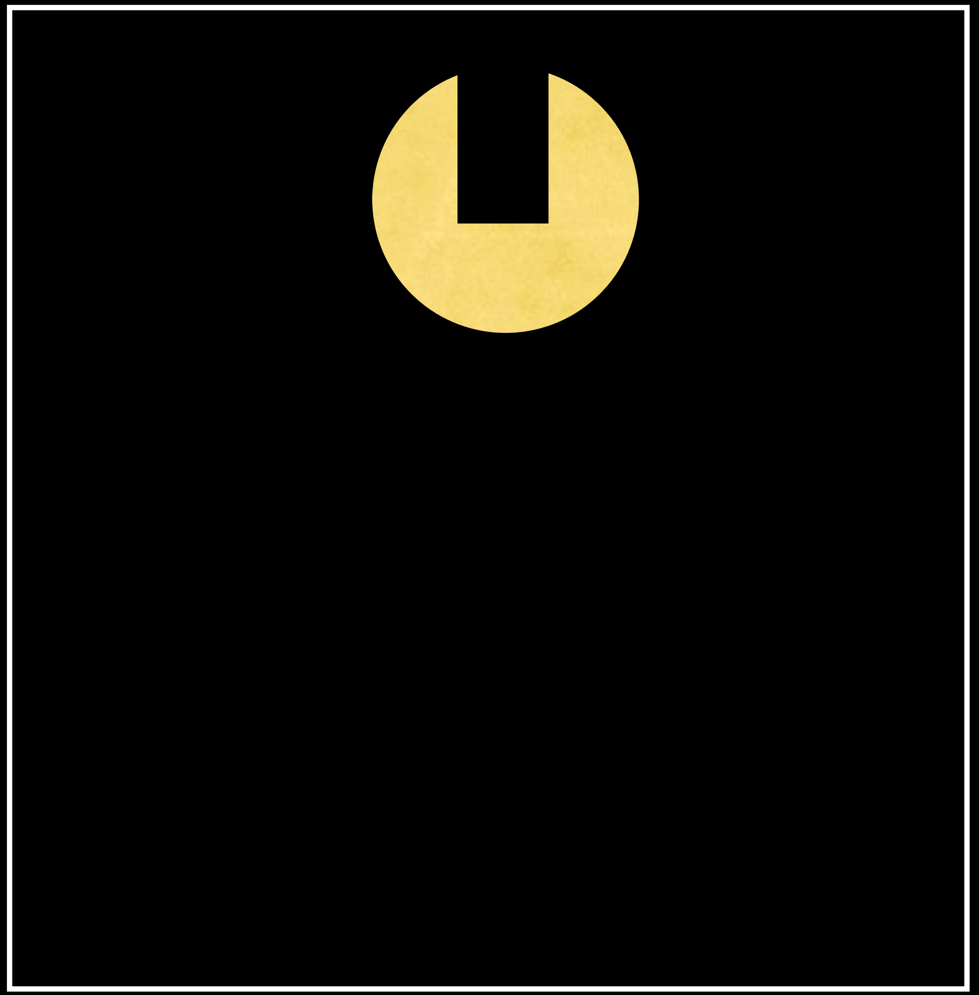
Scalar field ϕ

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term



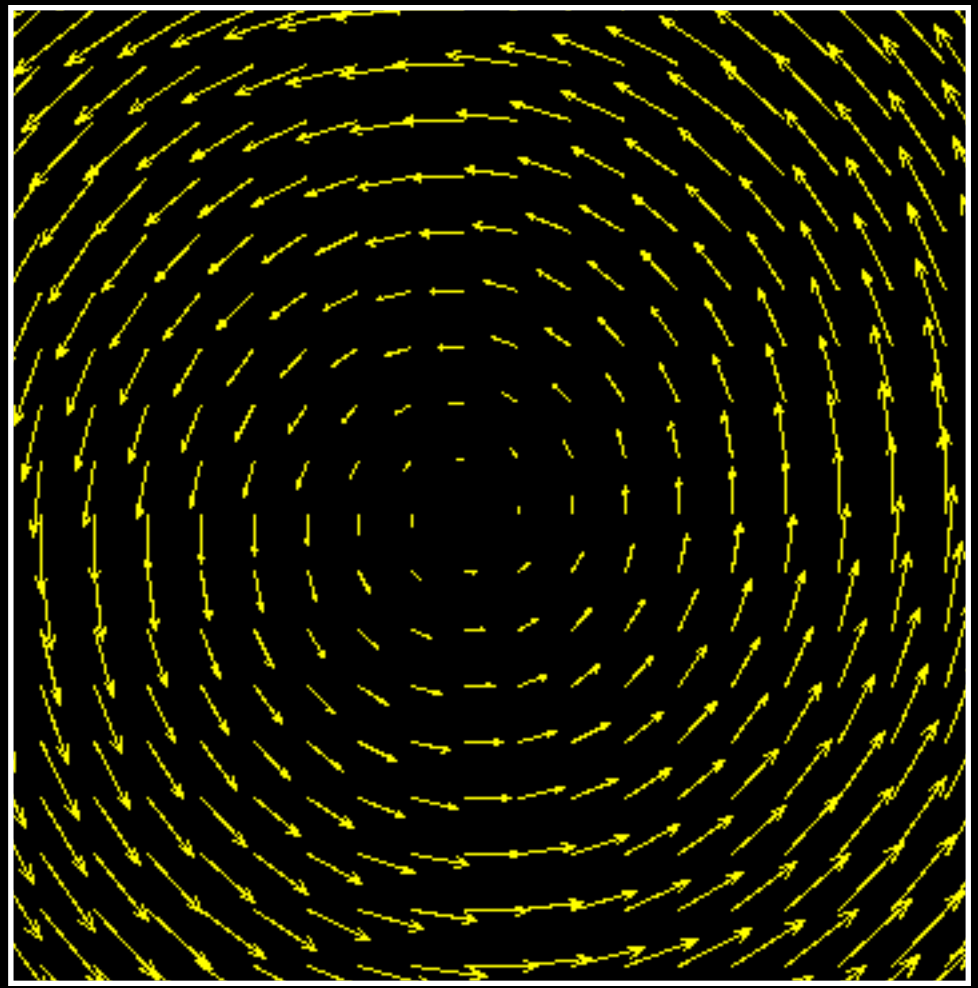
Velocity field $\boldsymbol{u}$



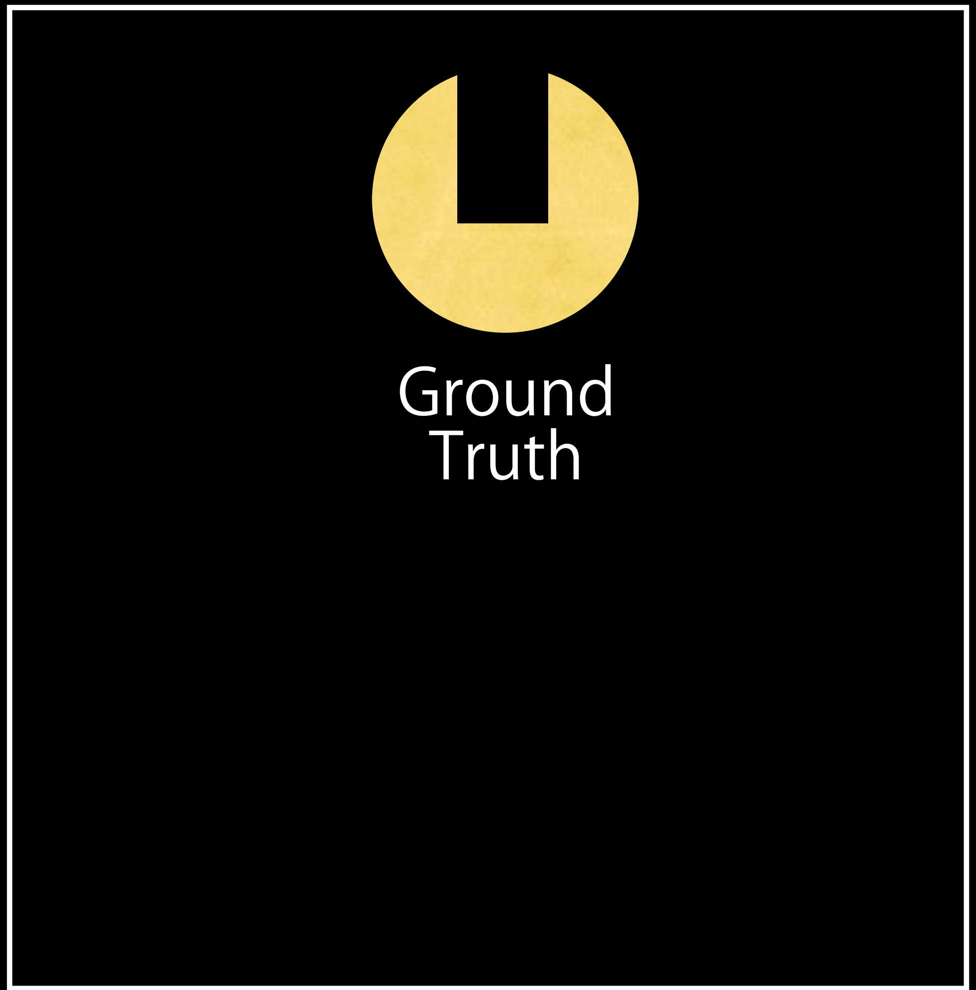
Scalar field ϕ

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term



Velocity field $\boldsymbol{u}$



Scalar field ϕ

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

How to discretize $\dfrac{\partial \phi}{\partial x}$?

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

$$\boxed{\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}}$$

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

$$\boxed{\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}}$$

Central Difference

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

$$\boxed{\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}}$$

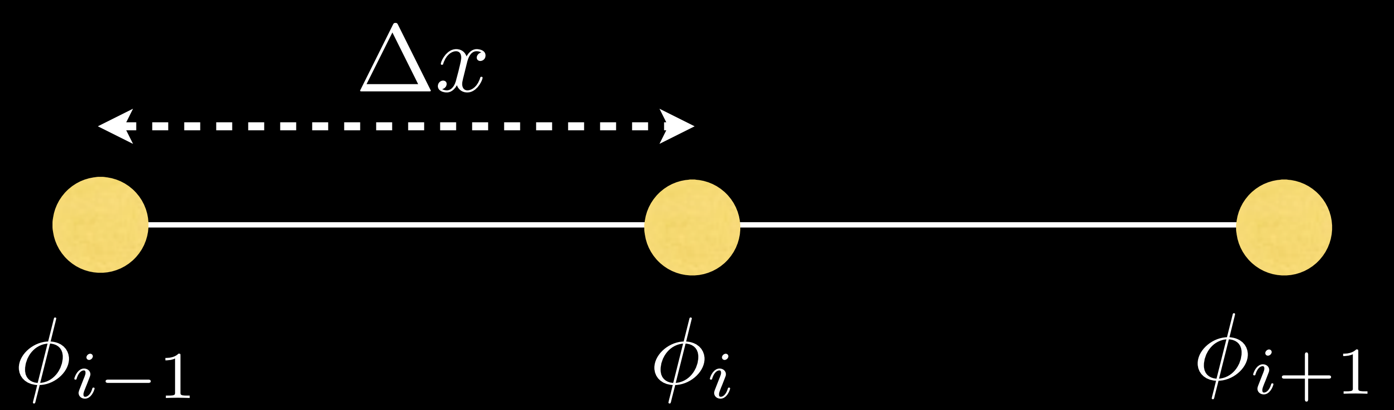
Central Difference $O(\Delta x^2)$ Accurate

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

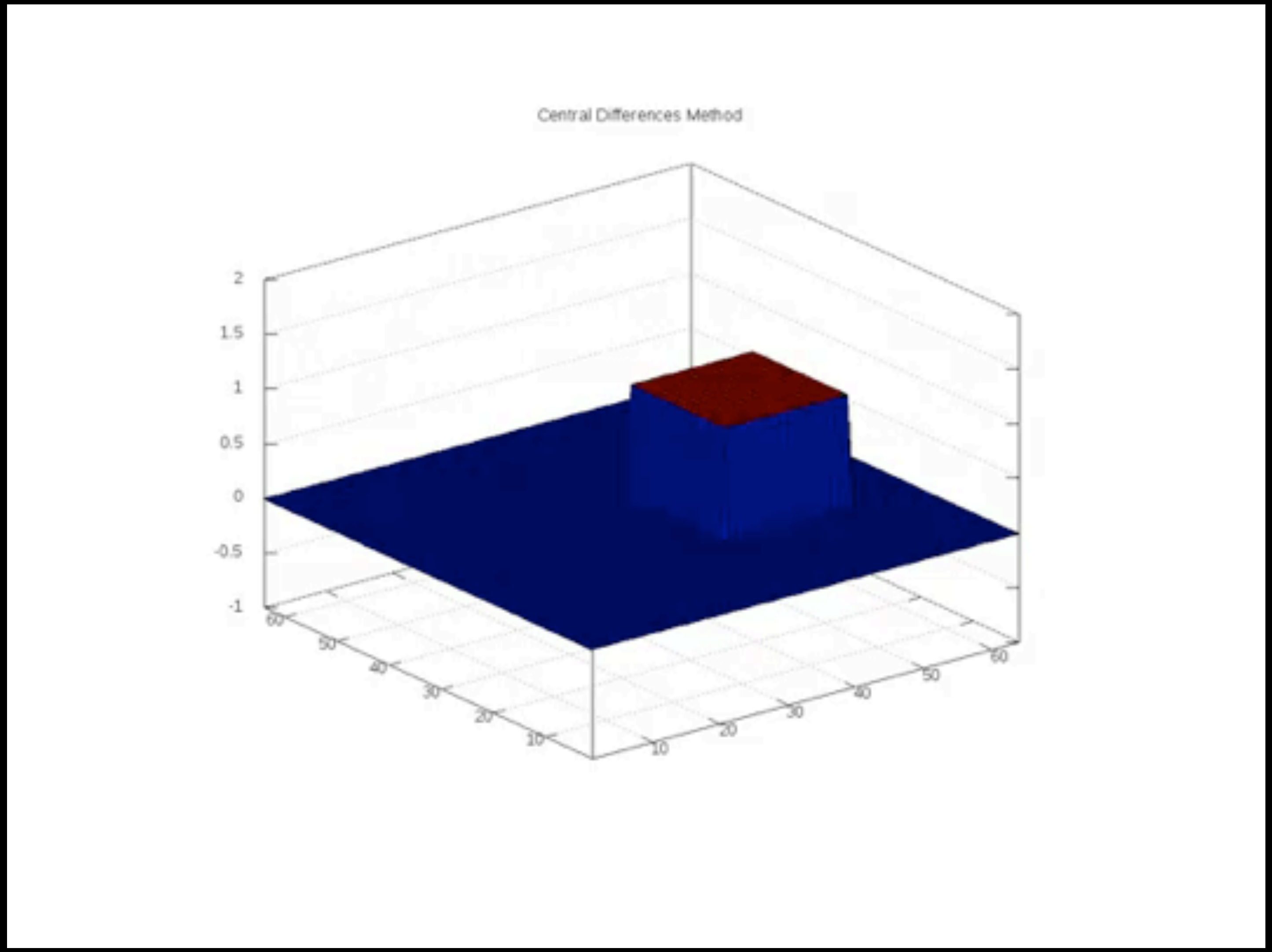
$$\boxed{\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}}$$

Central Difference $O(\Delta x^2)$ Accurate



Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

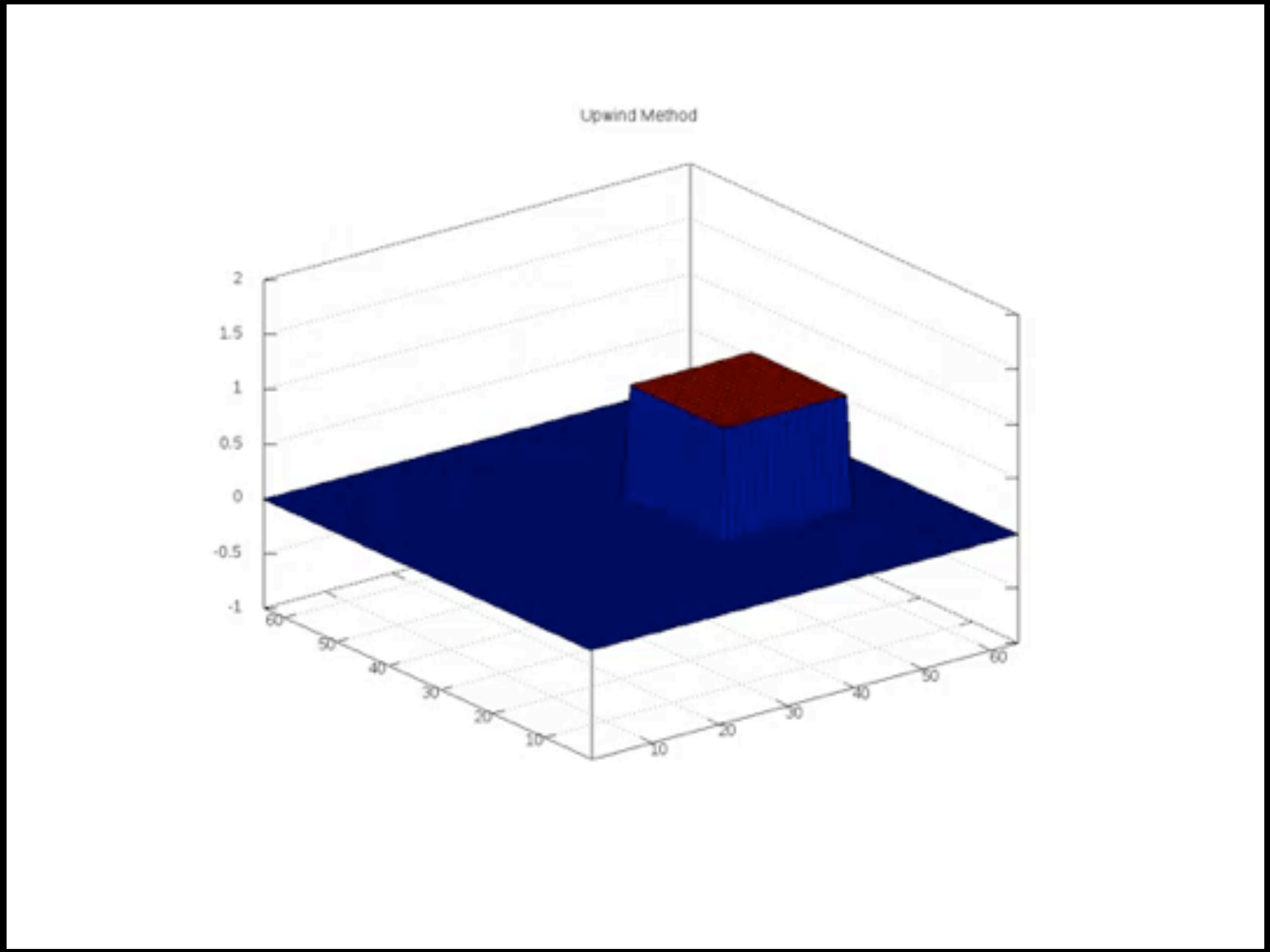


Central Difference

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term



Upwind Scheme

If $u > 0$

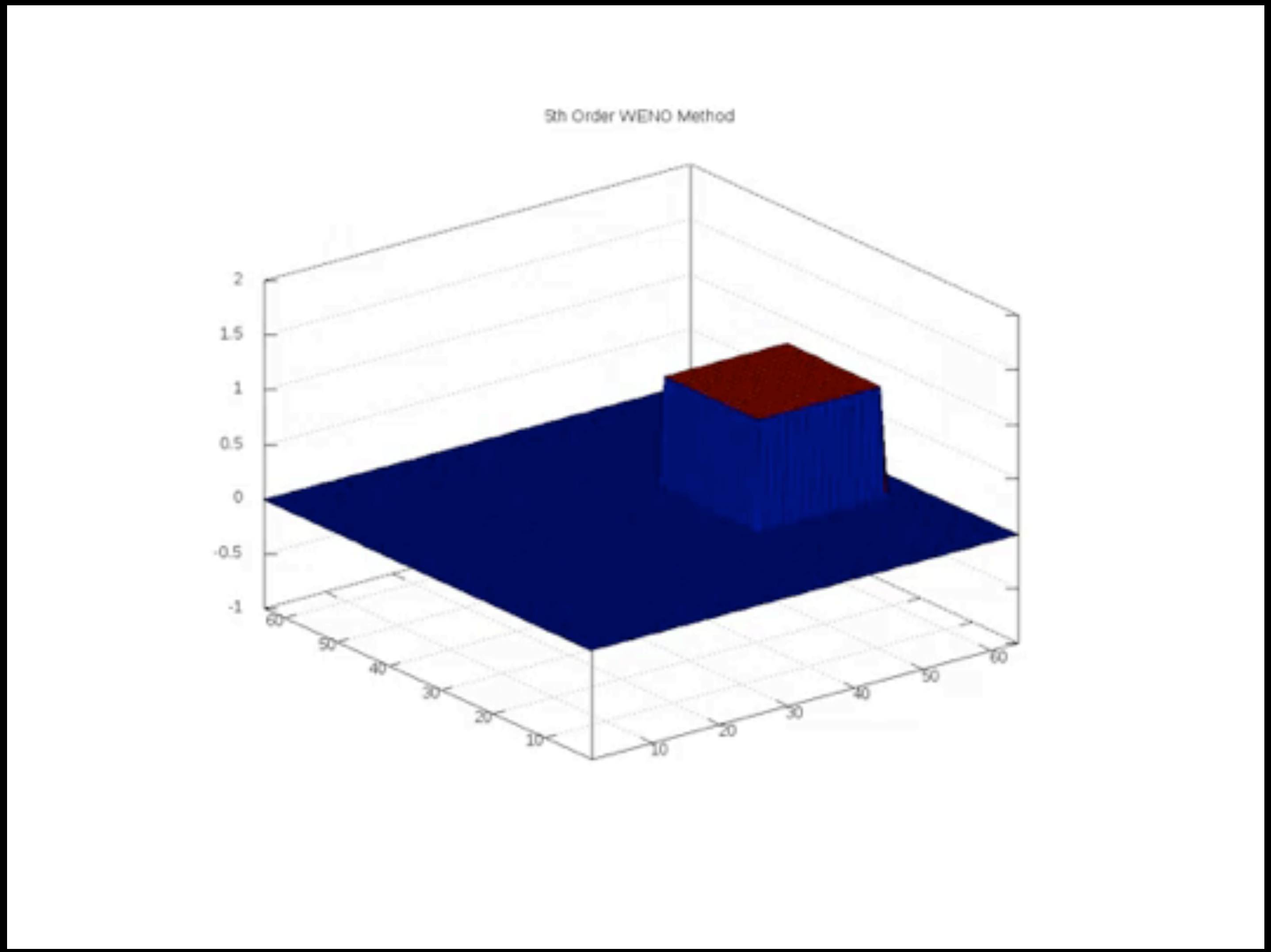
$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x}$$

If $u < 0$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term

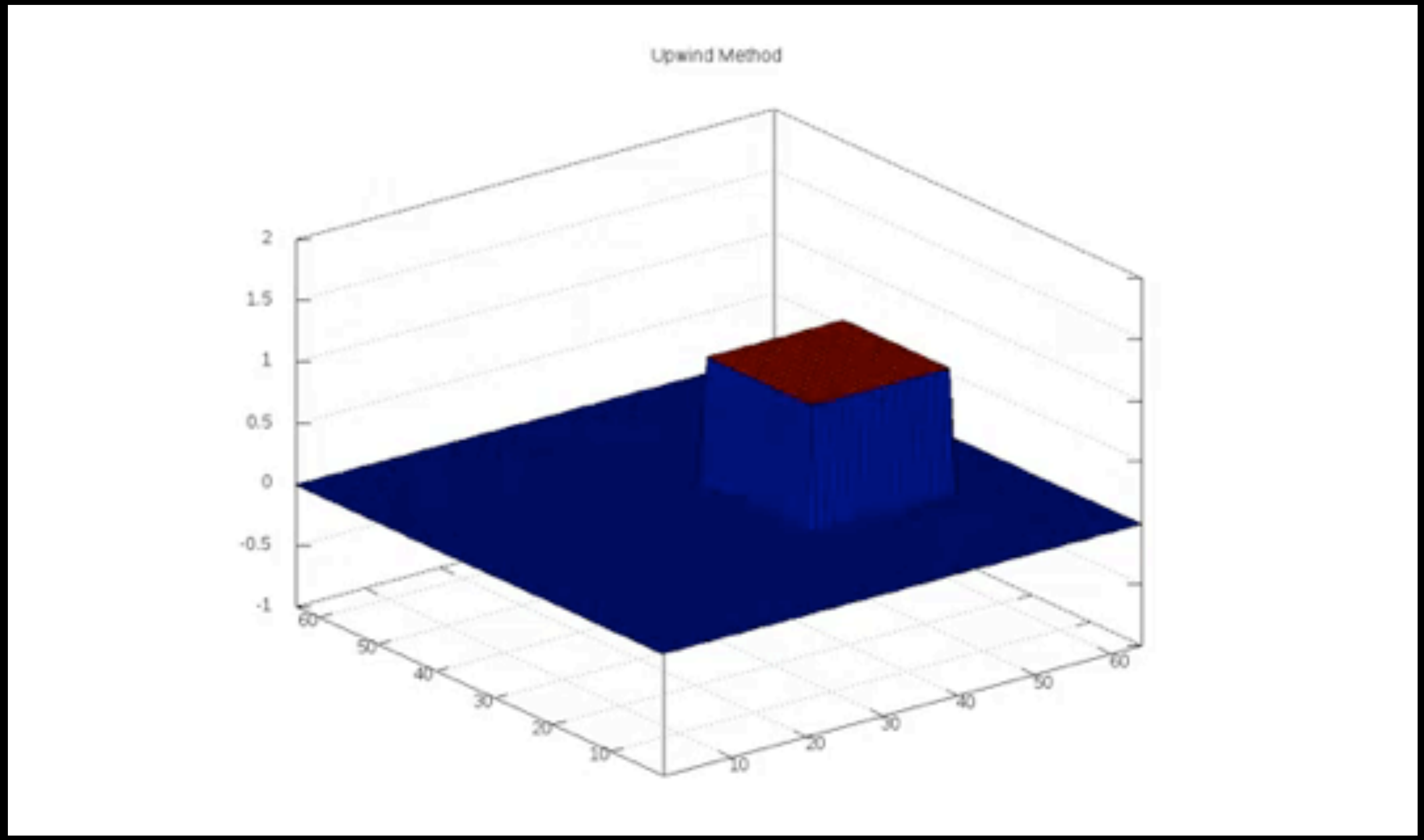


High-order Upwind Scheme
(WENO5)

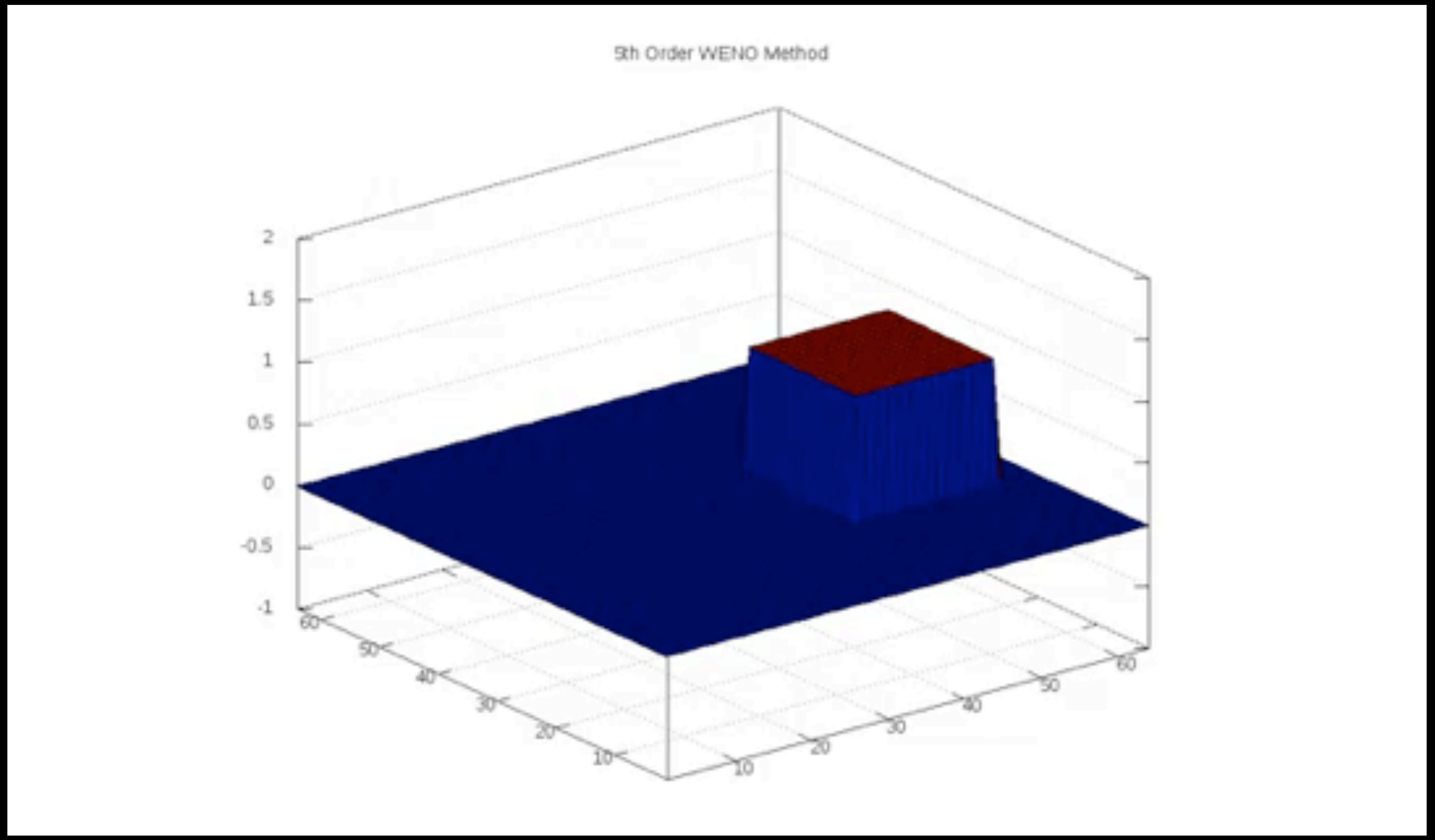
$$\frac{\partial \phi}{\partial x} \approx \frac{1}{\Delta x}[M_{\mathrm{WENO}}] \begin{bmatrix} \phi_{i-2} \\ \phi_{i-1} \\ \phi_i \\ \phi_{i+1} \\ \phi_{i+2} \end{bmatrix}$$

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term



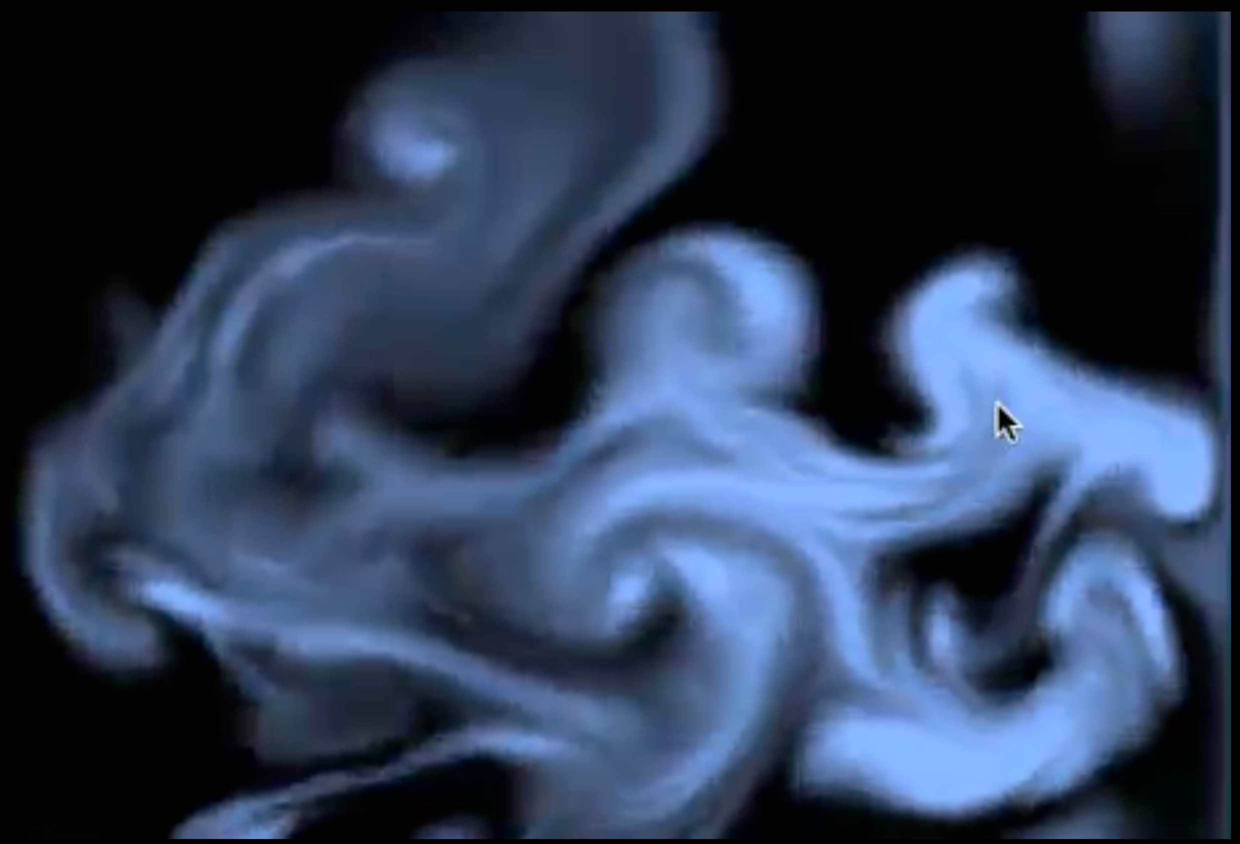
Upwind Scheme



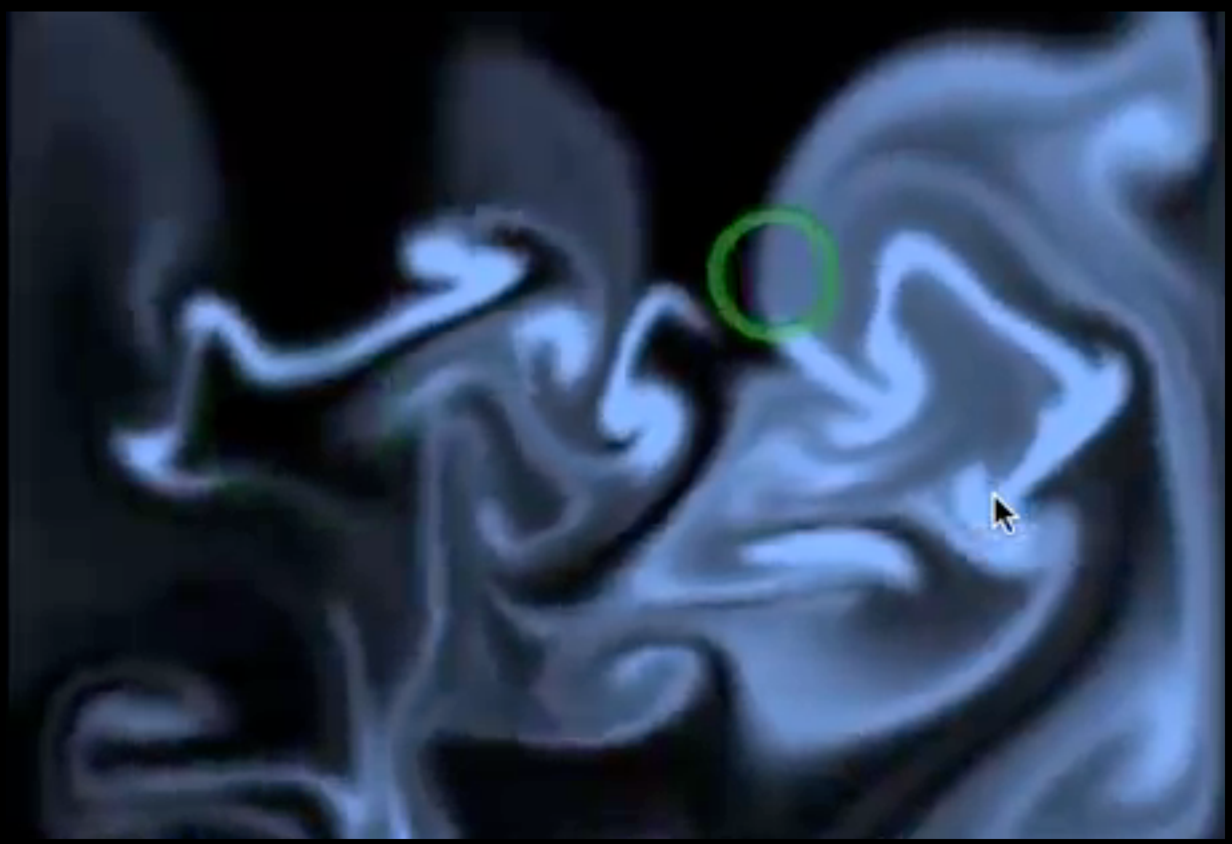
High-order Upwind Scheme
(WENO5)

Computing $\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$

Advection term



Upwind Scheme



High-order Upwind Scheme
(WENO5)

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

$$\boxed{-\frac{1}{\rho}\nabla p}$$

Pressure term

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \mu\nabla^2\boldsymbol{u} - \frac{1}{\rho}\nabla p + \boldsymbol{F}$$

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \cancel{\mu\nabla^2\boldsymbol{u}} - \frac{1}{\rho}\nabla p \cancel{+\boldsymbol{F}}$$

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\frac{1}{\rho}\nabla p$$

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\boxed{\frac{\boldsymbol{u}_{t+\Delta t} - \boldsymbol{u}^*}{\Delta t}} = -\frac{1}{\rho}\nabla p$$

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\frac{\boldsymbol{u}_{t+\Delta t} - \boldsymbol{u}^*}{\Delta t} = -\frac{1}{\rho}\nabla p$$

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\nabla \cdot \frac{\boldsymbol{u}_{t+\Delta t} - \boldsymbol{u}^*}{\Delta t} = \nabla \cdot -\frac{1}{\rho}\nabla p$$

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\nabla \cdot \frac{\cancel{\boldsymbol{u}_{t+\Delta t}} - \boldsymbol{u}^*}{\Delta t} = \nabla \cdot -\frac{1}{\rho}\nabla p$$

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

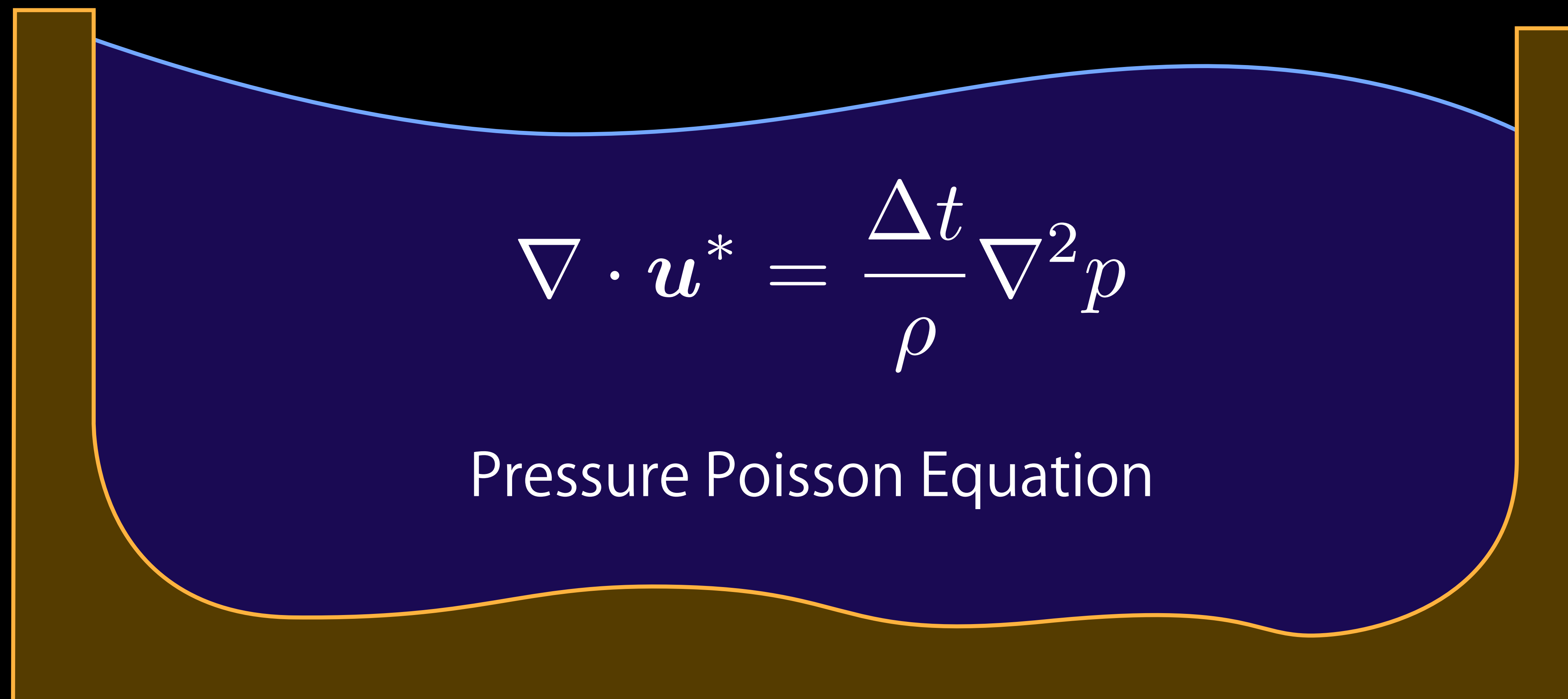
Pressure term

$$\nabla \cdot \boldsymbol{u}^* = \frac{\Delta t}{\rho}\nabla^2 p$$

Pressure Poisson Equation

Computing $\boxed{-\frac{1}{\rho}\nabla p}$

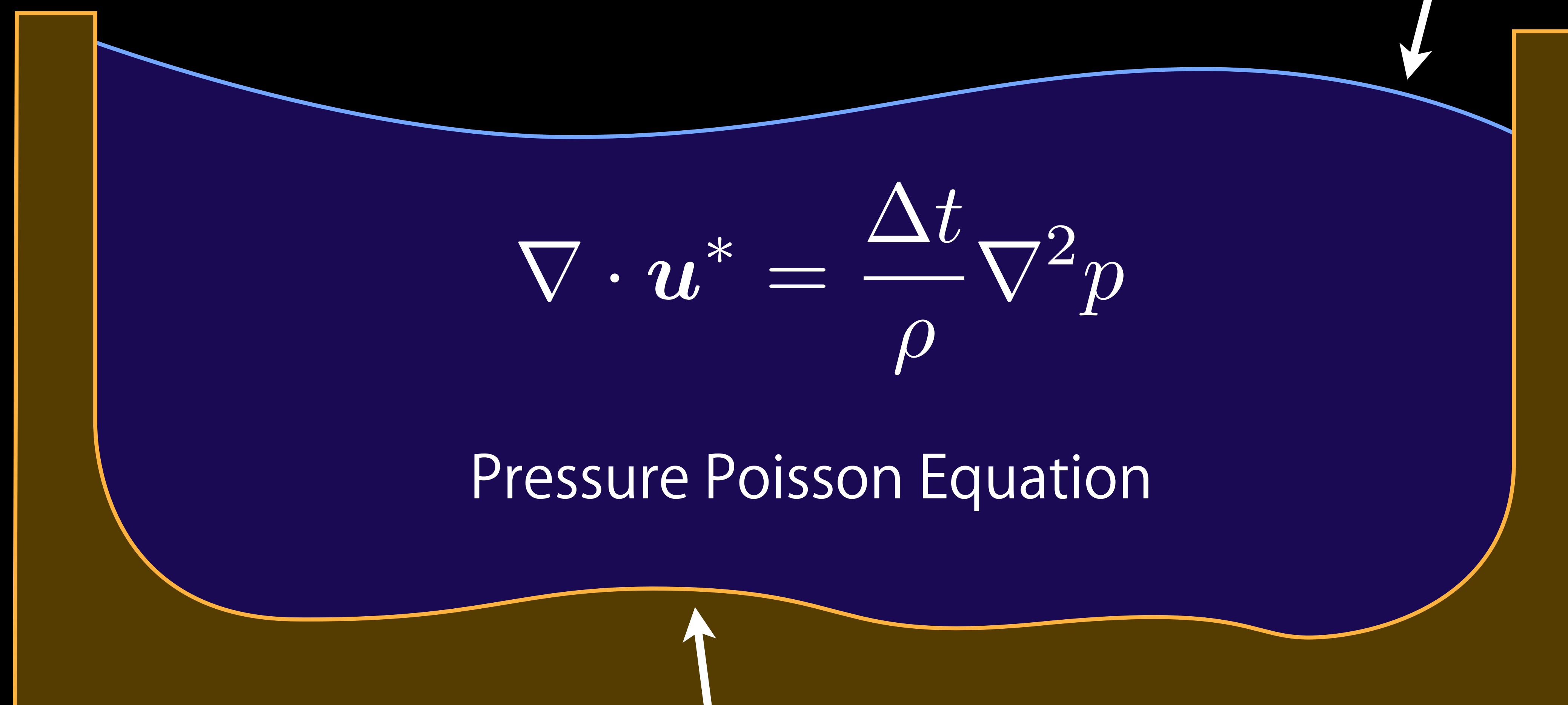
Pressure term



Boundary Conditions of $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$p = 0$



$\boldsymbol{n} \cdot \nabla p = 0$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\nabla \cdot \boldsymbol{u}^* = \frac{\Delta t}{\rho} \nabla^2 p$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\int_\Omega \nabla \cdot \boldsymbol{u}^* = \frac{\Delta t}{\rho}\nabla^2 p$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\int_\Omega \nabla \cdot \boldsymbol{u}^* dV = \int_\Omega \frac{\Delta t}{\rho} \nabla^2 p \, dV$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\int_\Omega \nabla \cdot \boldsymbol{u}^* dV = \int_\Omega \frac{\Delta t}{\rho} \nabla^2 p \, dV$$

Divergence Theorem

$$\oint_\Omega \boldsymbol{u}^* \cdot \boldsymbol{n} = \oint_\Omega \frac{\Delta t}{\rho} \nabla p \cdot \boldsymbol{n}$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\oint_{\Omega} \boldsymbol{u}^* \cdot \boldsymbol{n} = \oint_{\Omega} \frac{\Delta t}{\rho} \nabla p \cdot \boldsymbol{n}$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\oint_\Omega \blacksquare \rightarrow \boldsymbol{u}^* = \oint_\Omega \blacksquare \rightarrow \nabla p$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\oint_\Omega \square \rightarrow \boldsymbol{u}^* = \oint_\Omega \square \rightarrow \nabla p$$

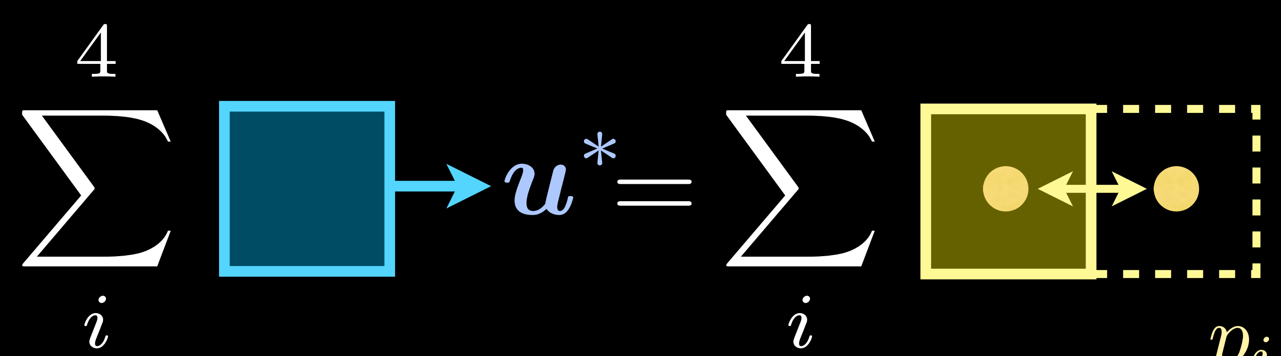
Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\sum_{i}^{4} \square \rightarrow \boldsymbol{u}^{*} = \sum_{i}^{4} \square \rightarrow \nabla p$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

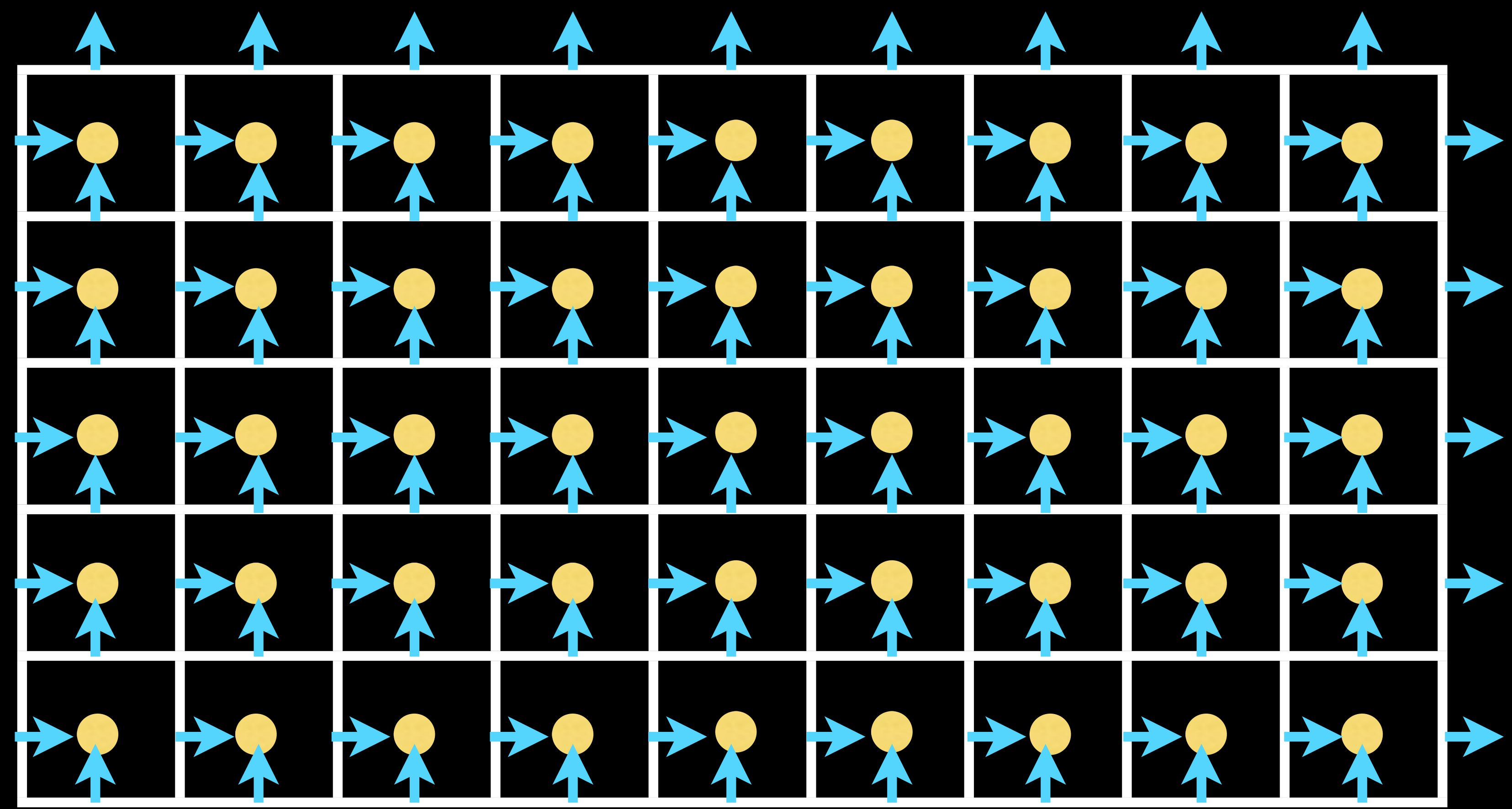
Pressure term



$$\nabla p = \frac{p_{i+1} - p_i}{\Delta x}$$

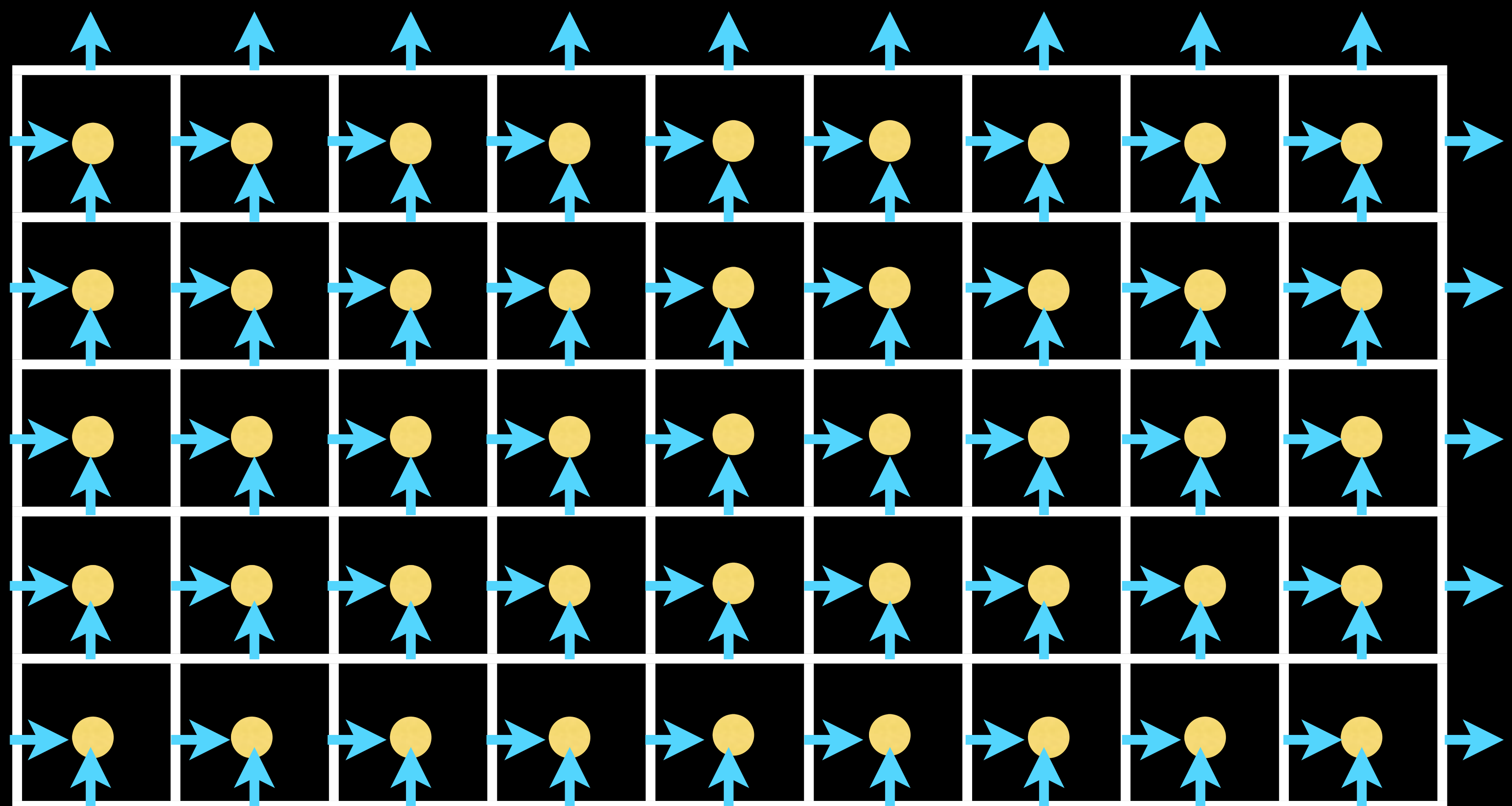
Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term



Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

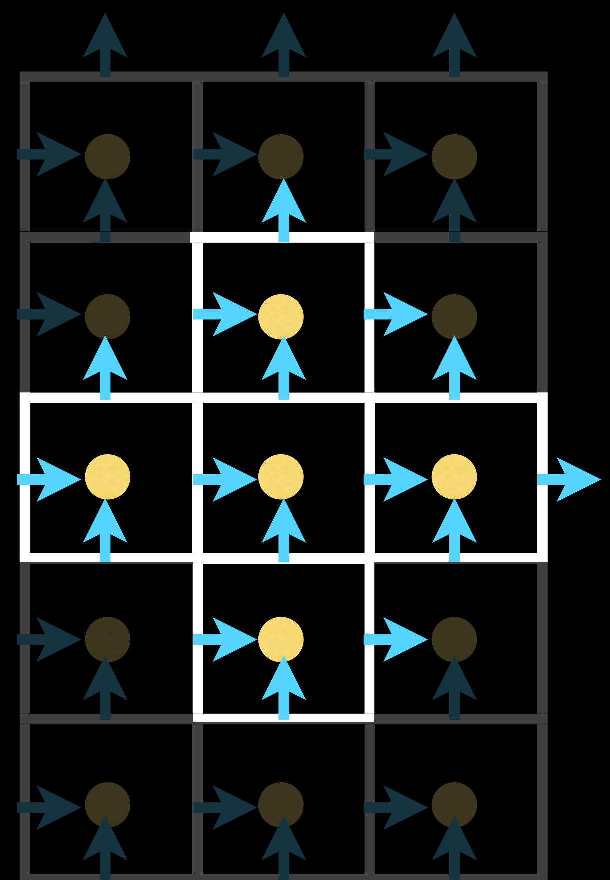
Pressure term



MAC Grid

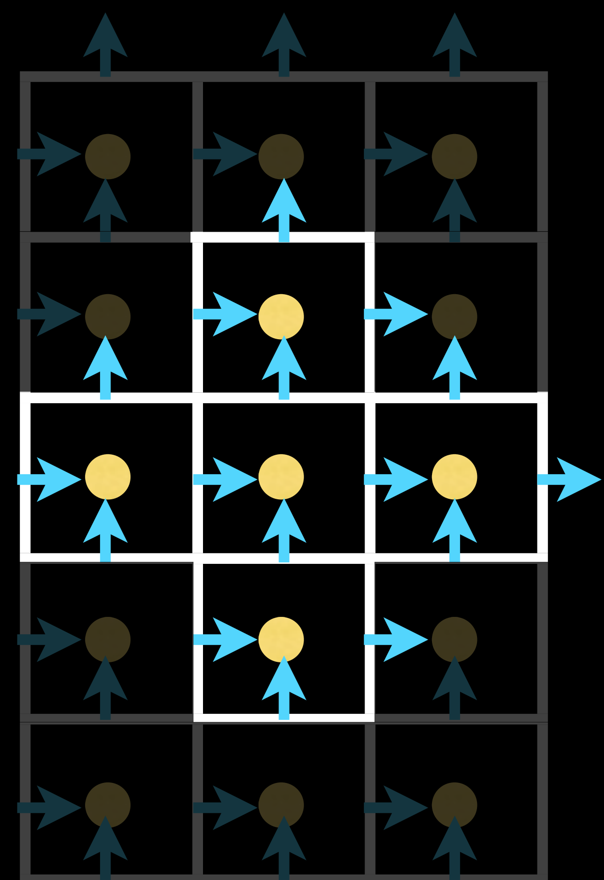
Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

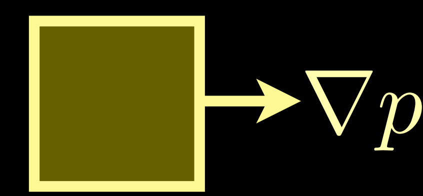


Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

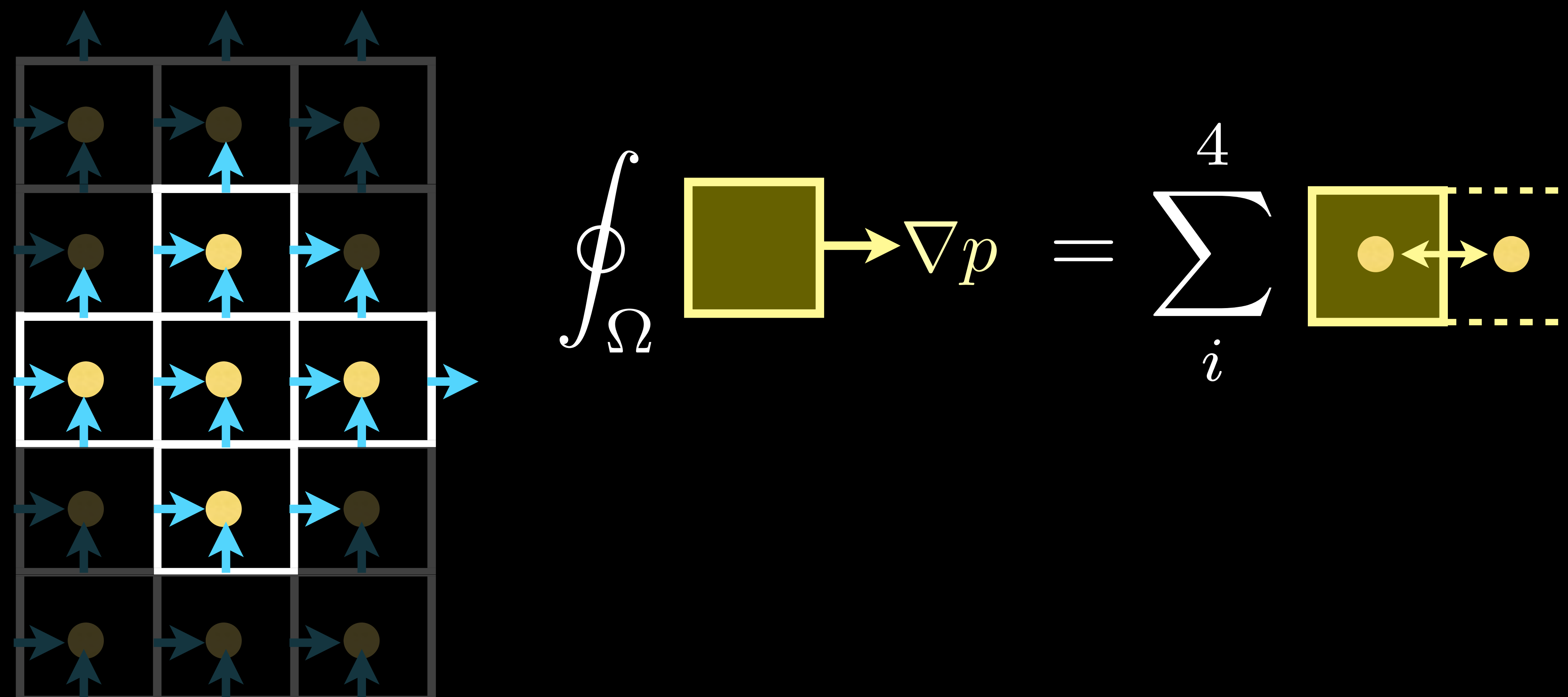


$\oint_{\Omega}$



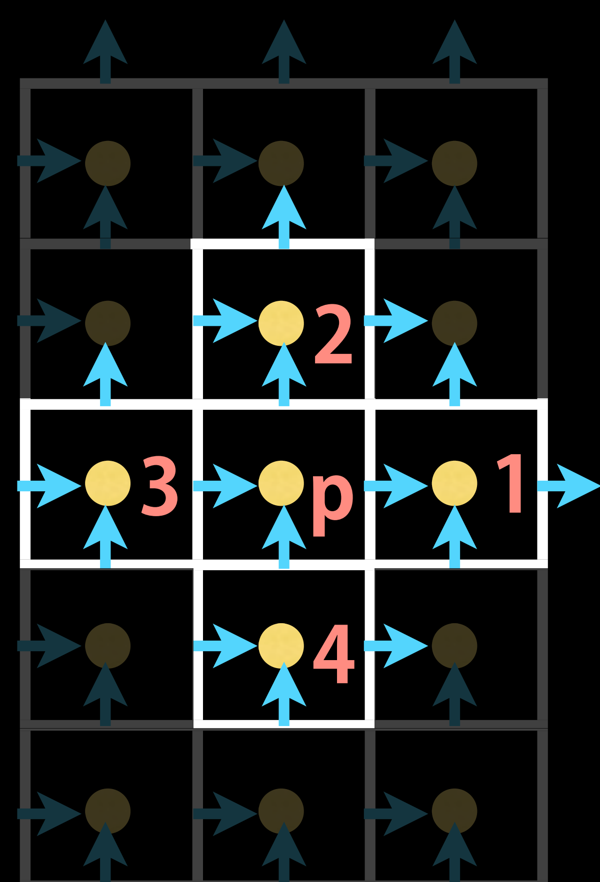
Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

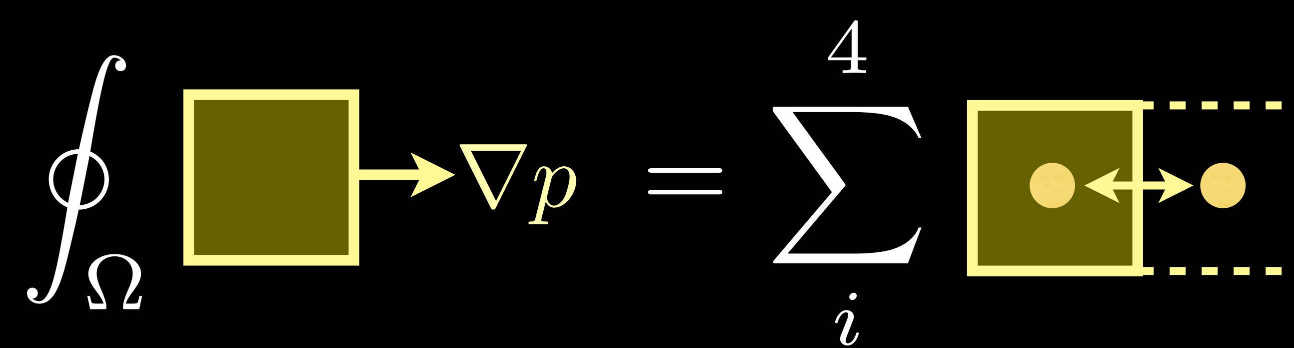
Pressure term



Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

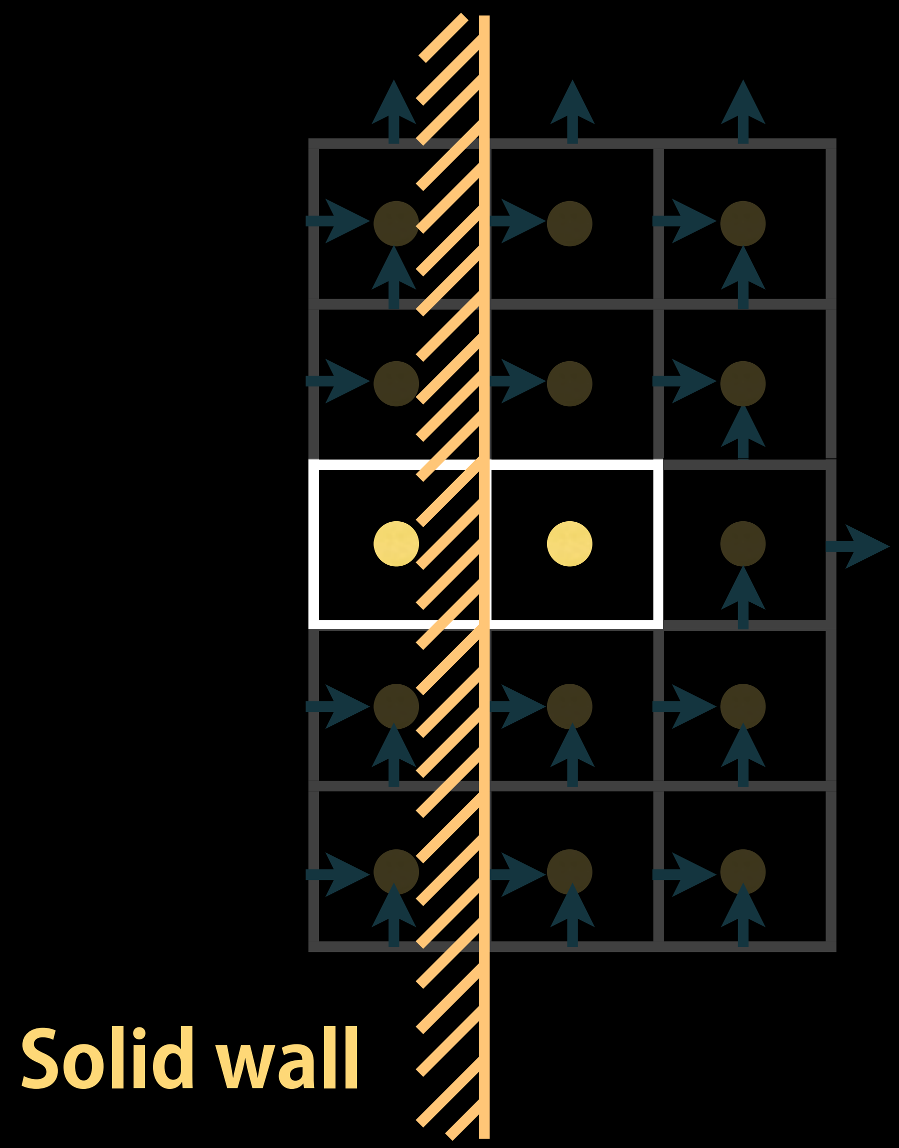




$$= \frac{4p - p_1 - p_2 - p_3 - p_4}{\Delta x}$$

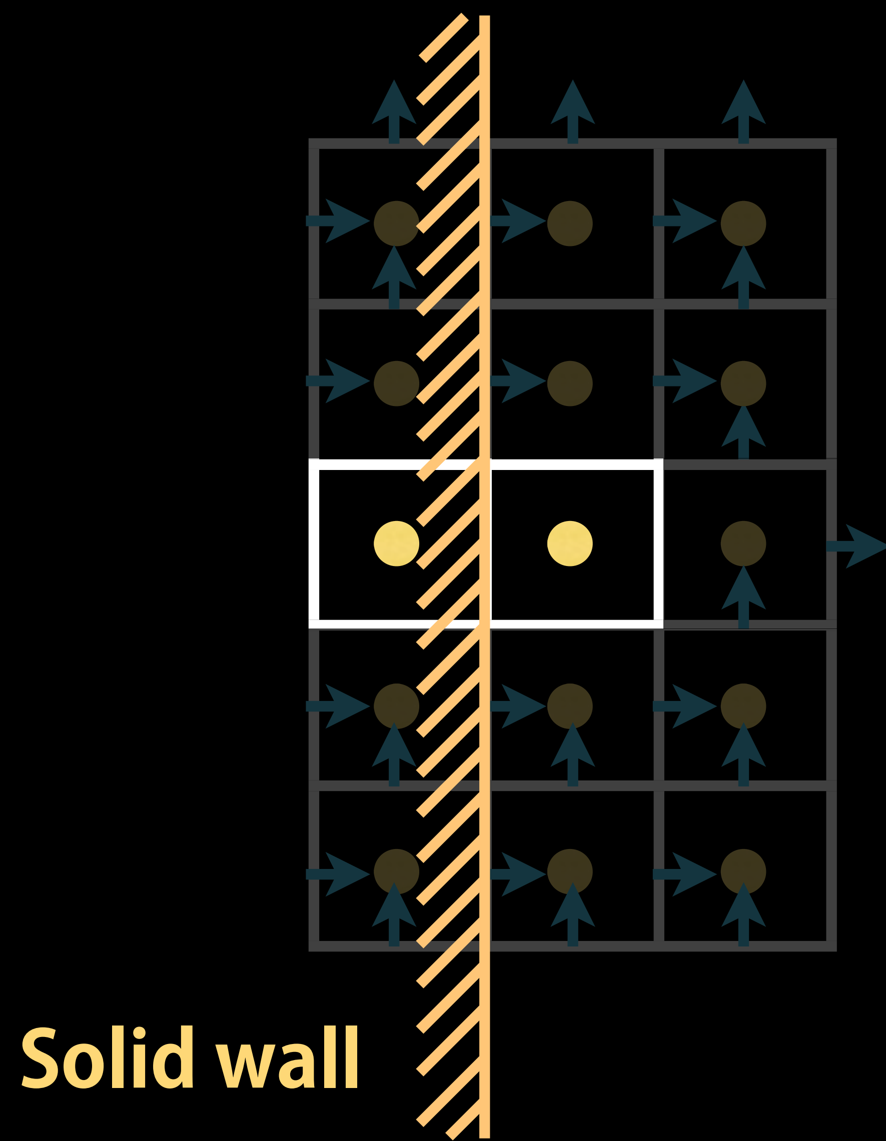
Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term



Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

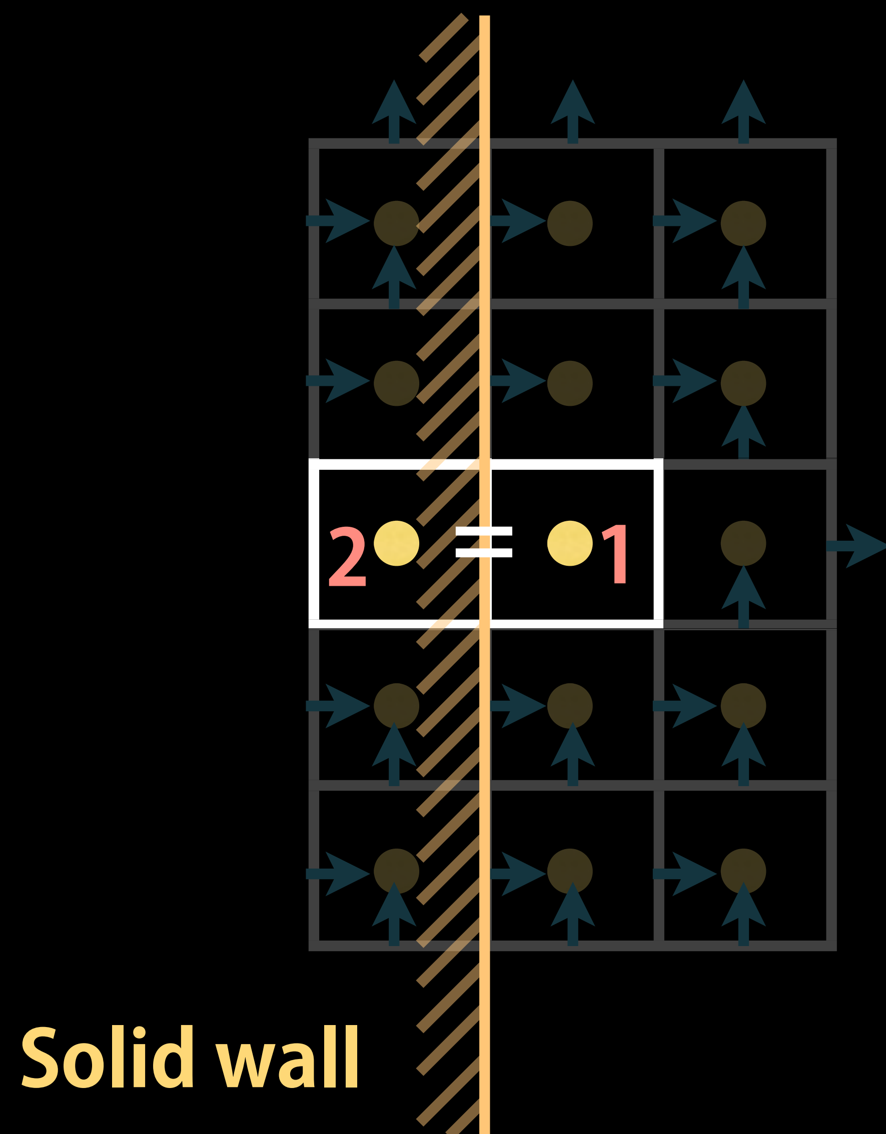
Pressure term



$$\boldsymbol{n} \cdot \nabla p = 0$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term



$$\boldsymbol{n} \cdot \nabla p = 0$$

$$\Downarrow$$

$$p_1 = p_2$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

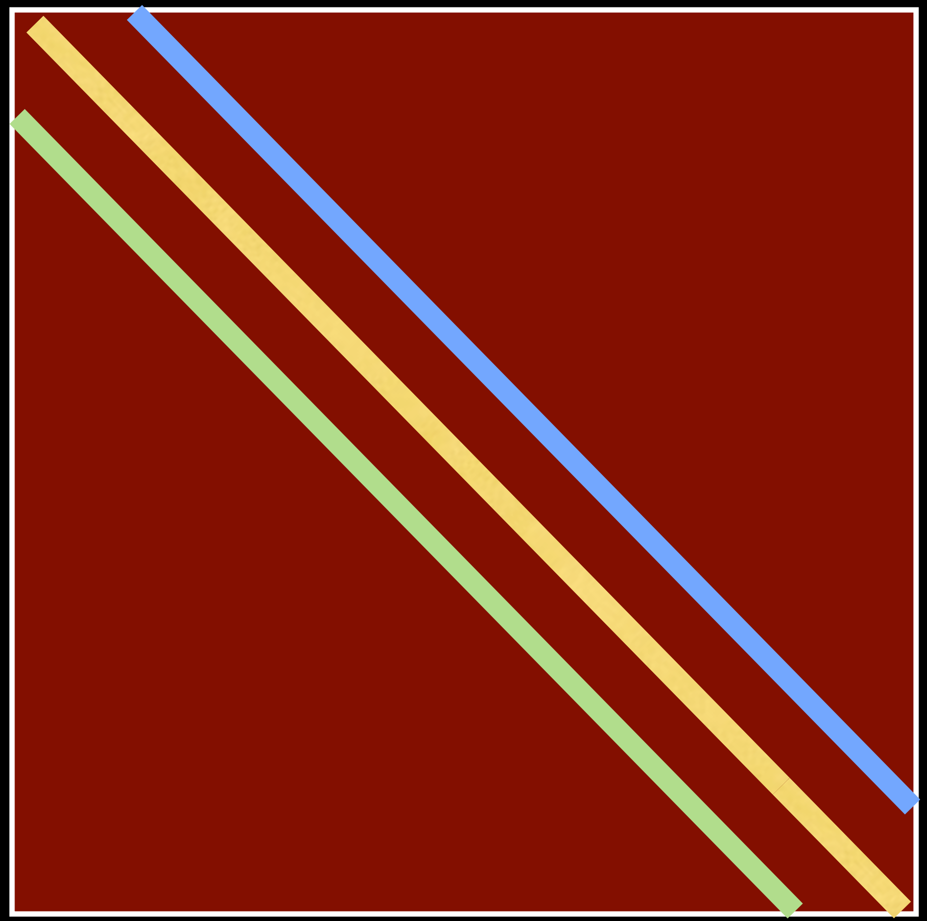
Empty Matrix

$$\frac{4p - p_1 - p_2 - p_3 - p_4}{\Delta x}$$

$$p_1 = p_2$$

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

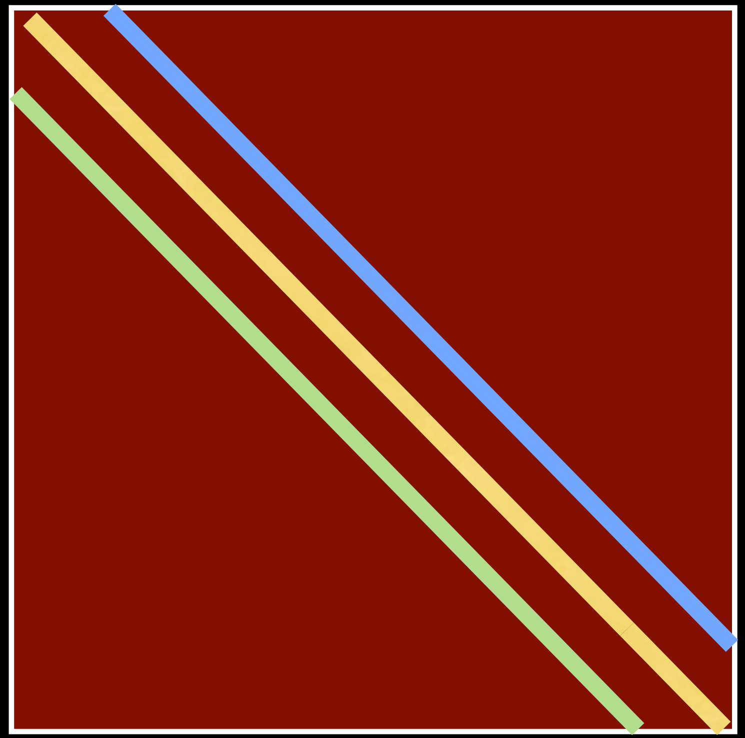
Pressure term



$$p_1 = p_2$$

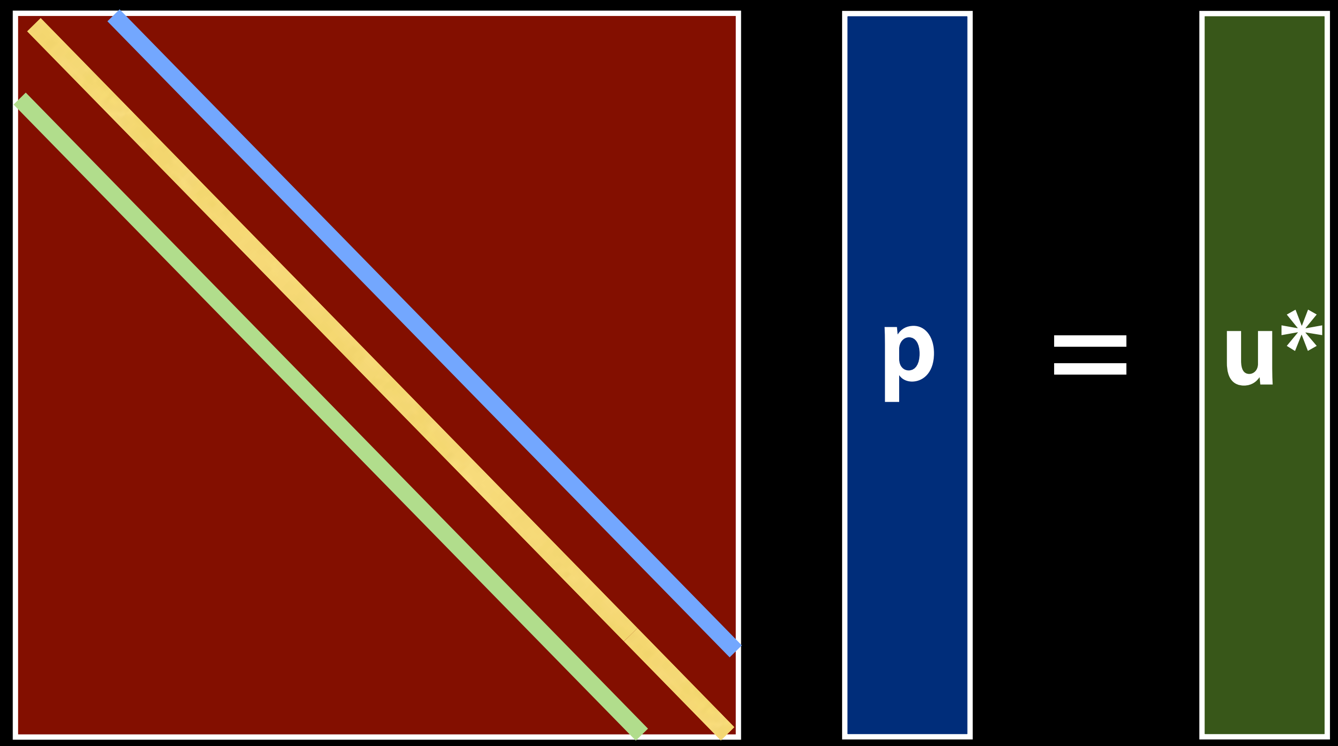
Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term



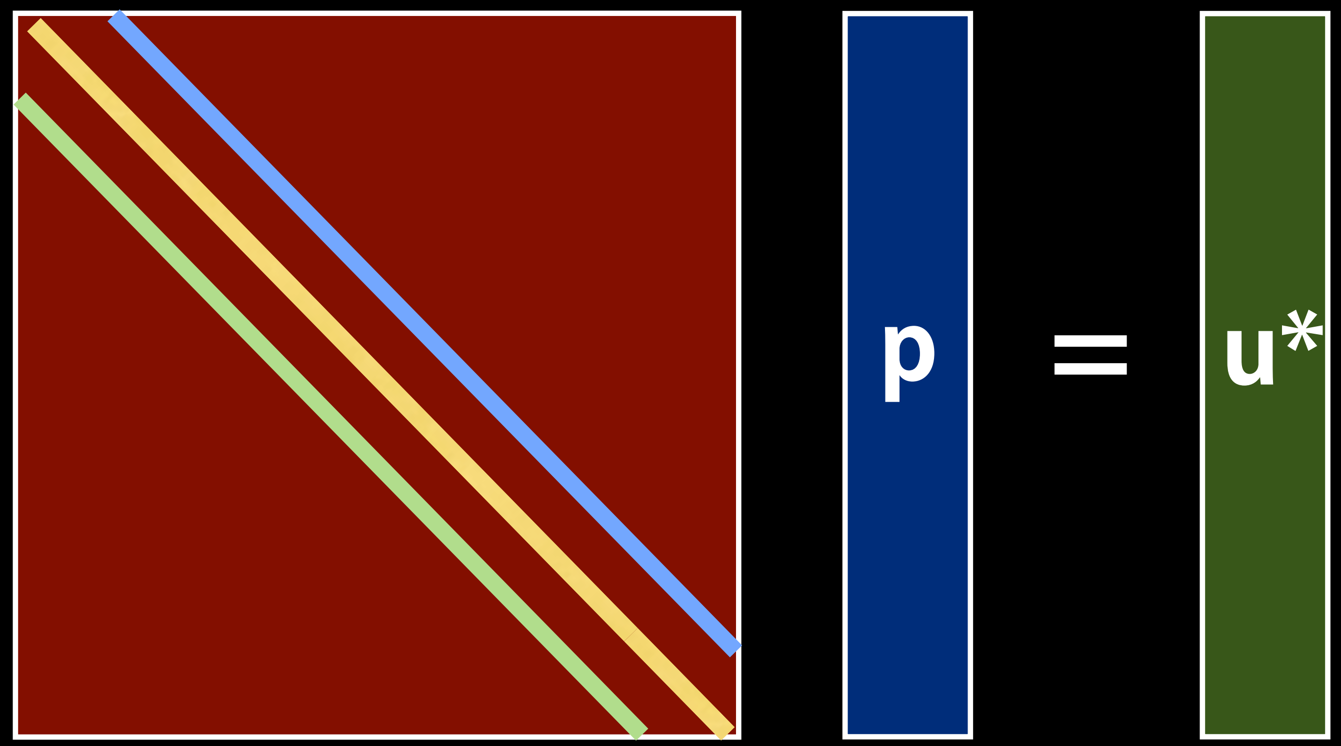
Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term



Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

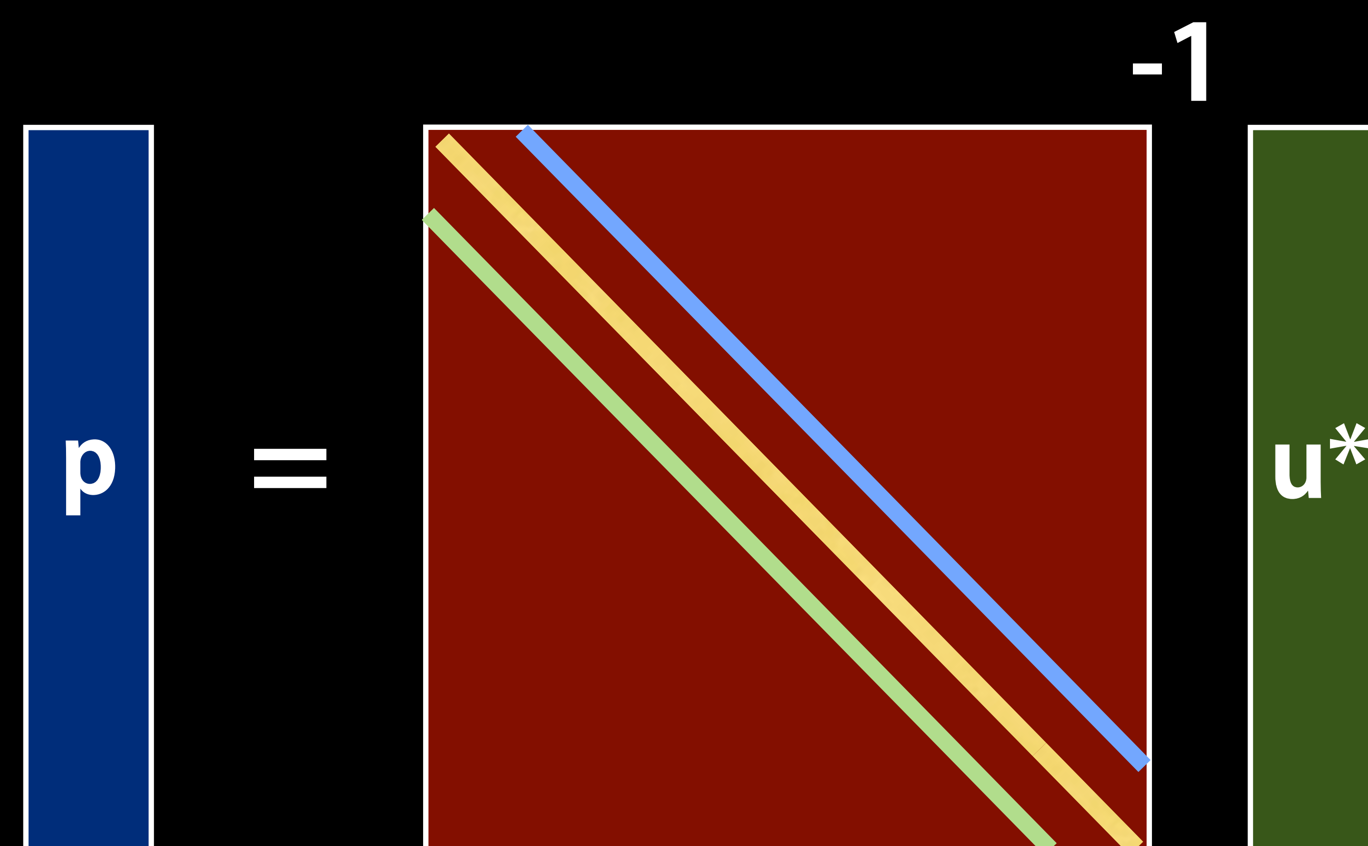
Pressure term



(Semi-)Positive Definite System

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

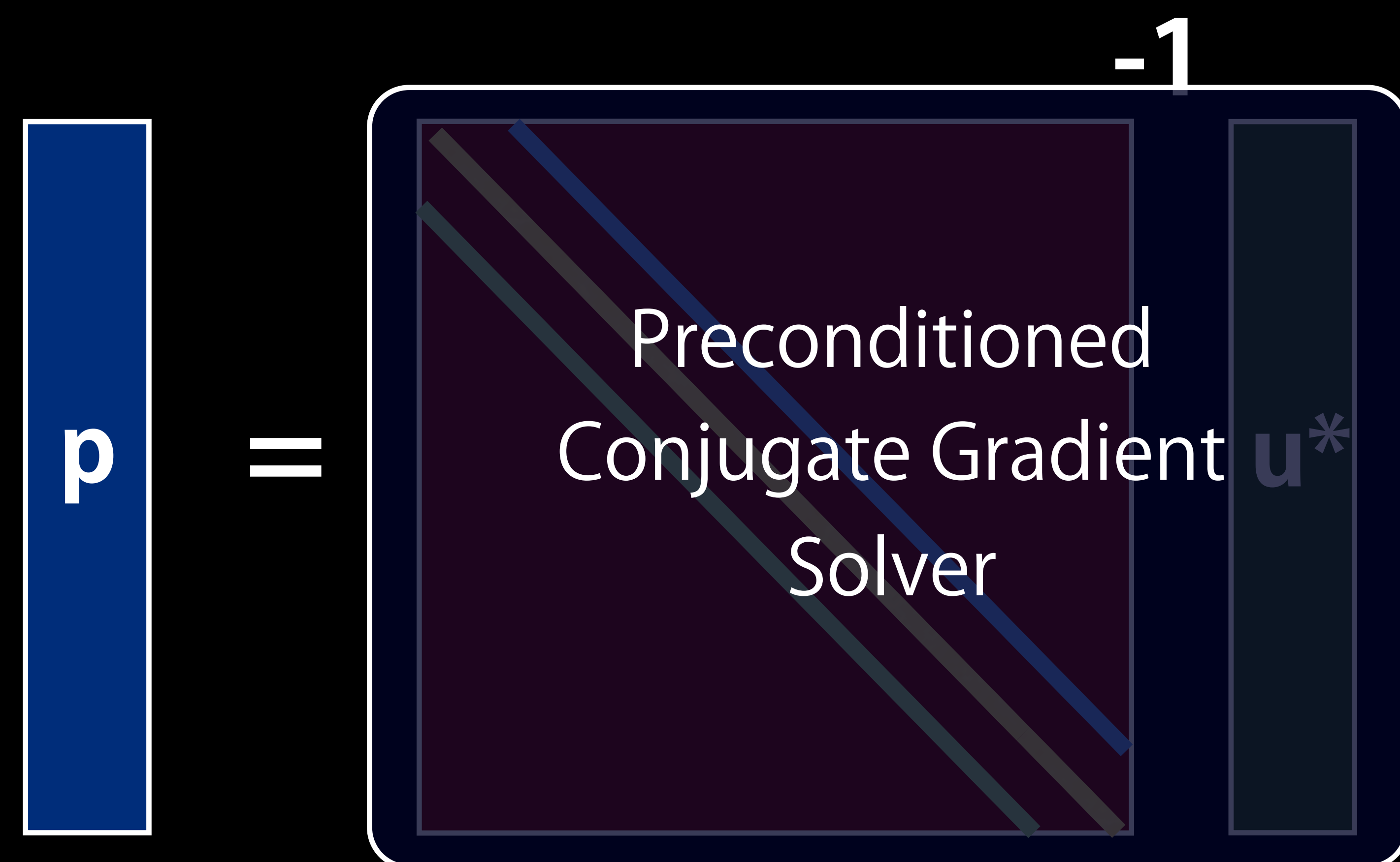
Pressure term



(Semi-)Positive Definite System

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

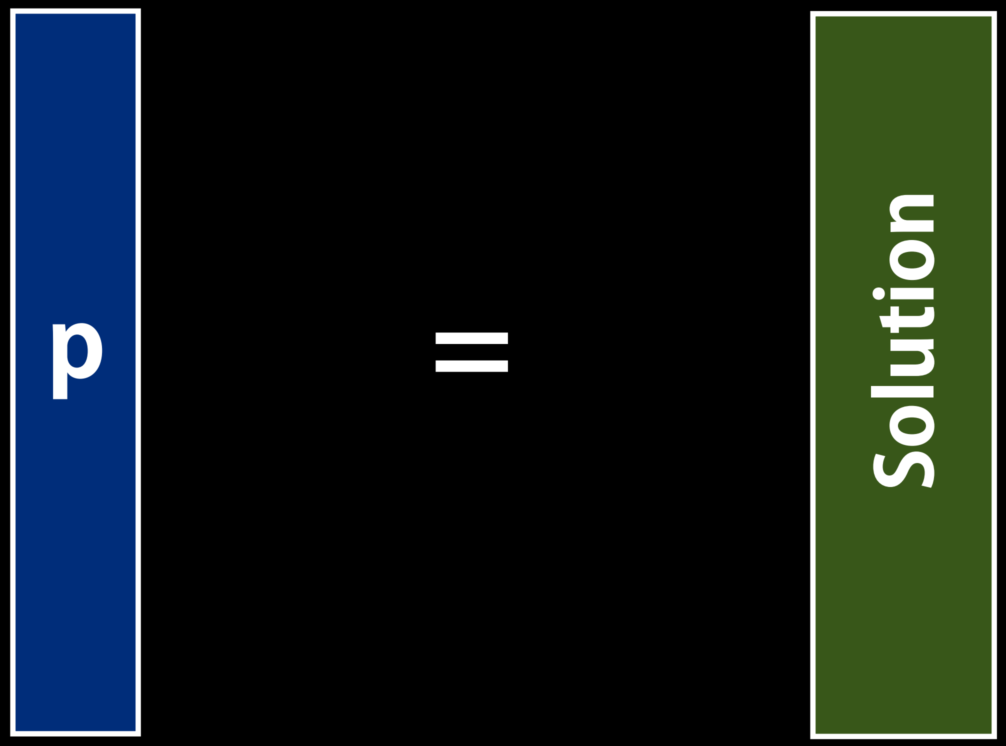
Pressure term



(Semi-)Positive Definite System

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term



(Semi-)Positive Definite System

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

Solution

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

∇ Solution

Discretizing $\boxed{-\frac{1}{\rho}\nabla p}$

Pressure term

$$\boldsymbol{u}_{t+\Delta t} = \boldsymbol{u}^* - \frac{\Delta t}{\rho}\nabla$$

Solution

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

Pressure term

$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho}\nabla p}$$

Pressure term

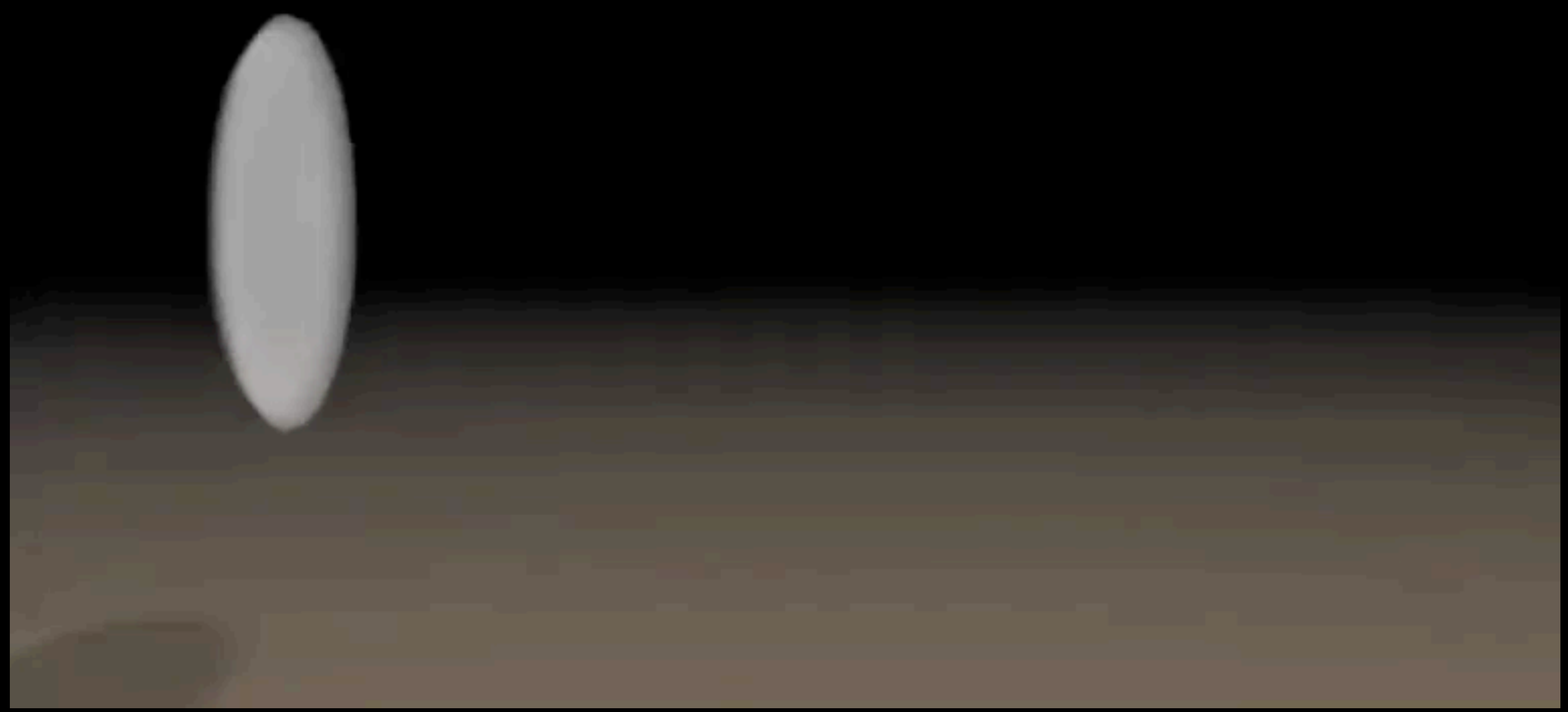
$$\boxed{\mathbf{u} \cdot \nabla \mathbf{u}}$$

Advection term

$$\boxed{-\frac{1}{\rho} \nabla p}$$

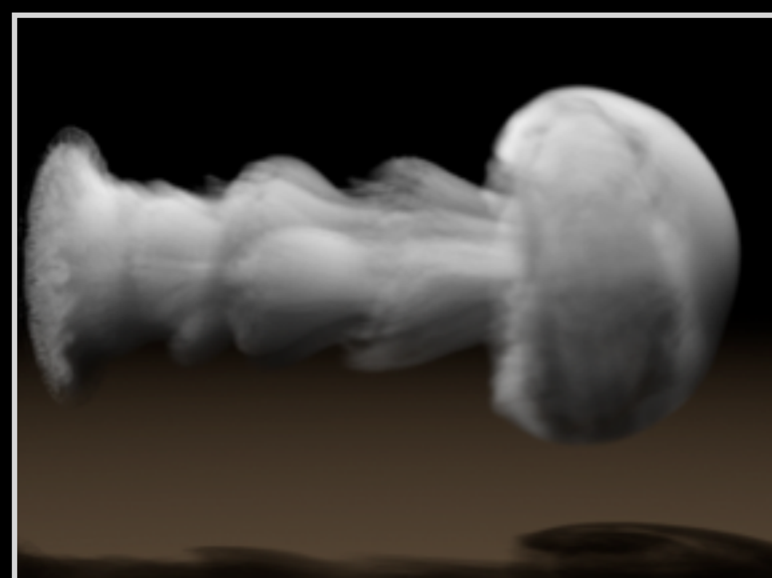
Pressure term

Examples

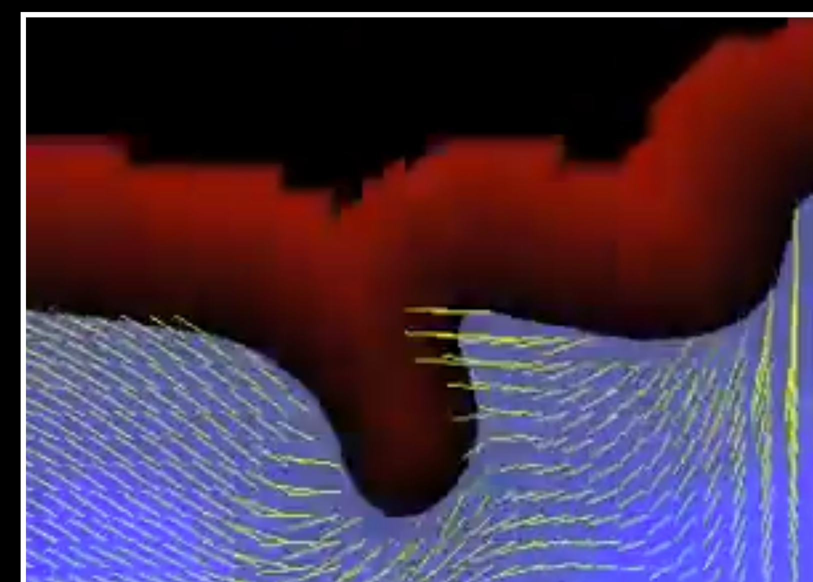


Short Break

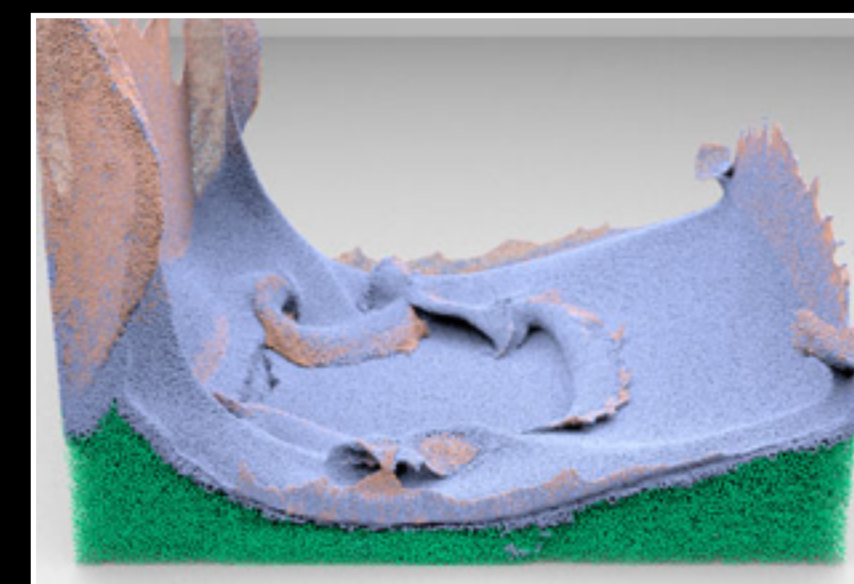
Next: Liquid Simulations...



Fluid Solver

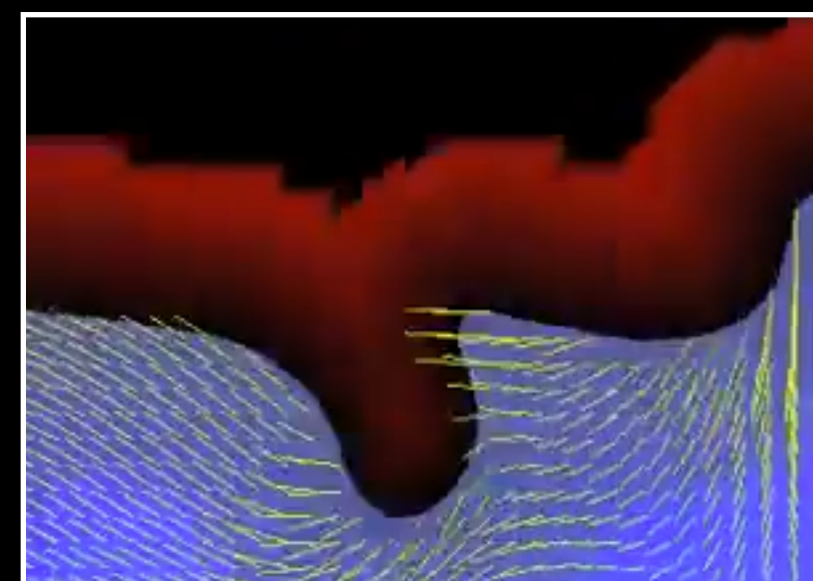


Surface Tracker

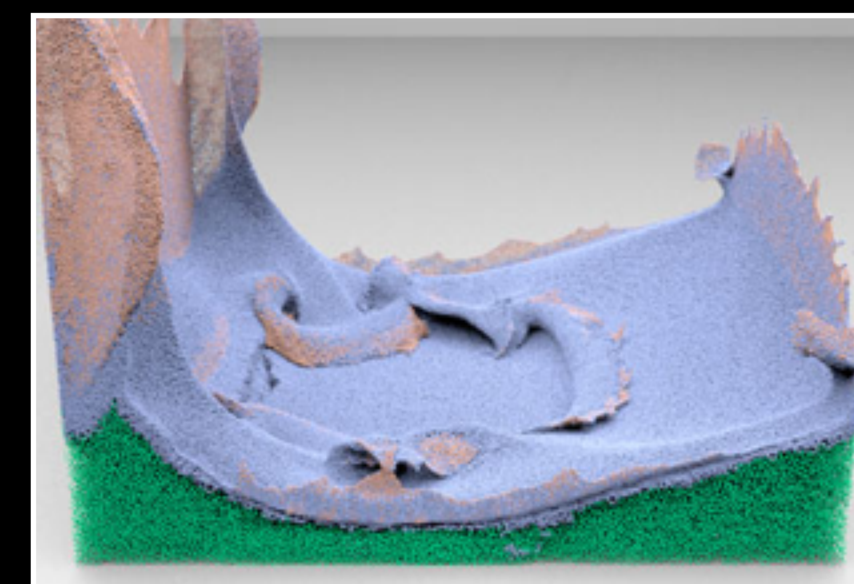


FLIP Solver

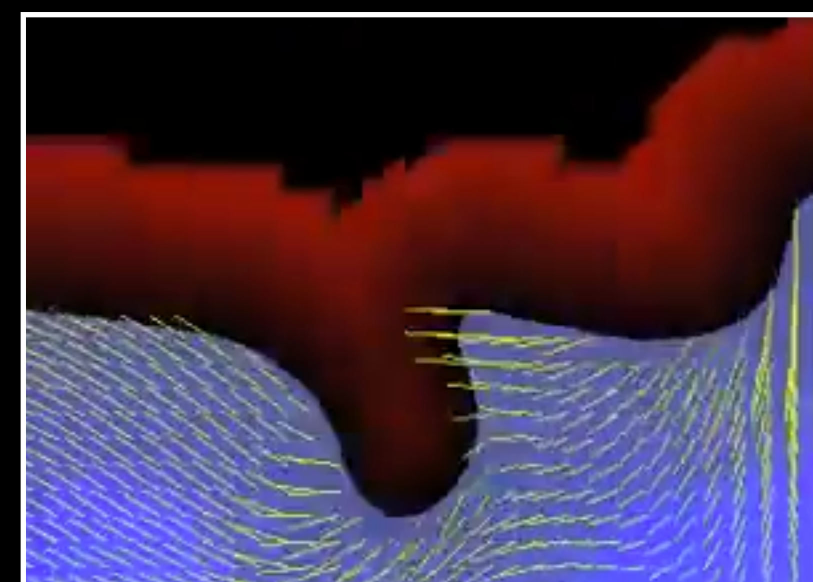
Fluid Solver



Surface Tracker

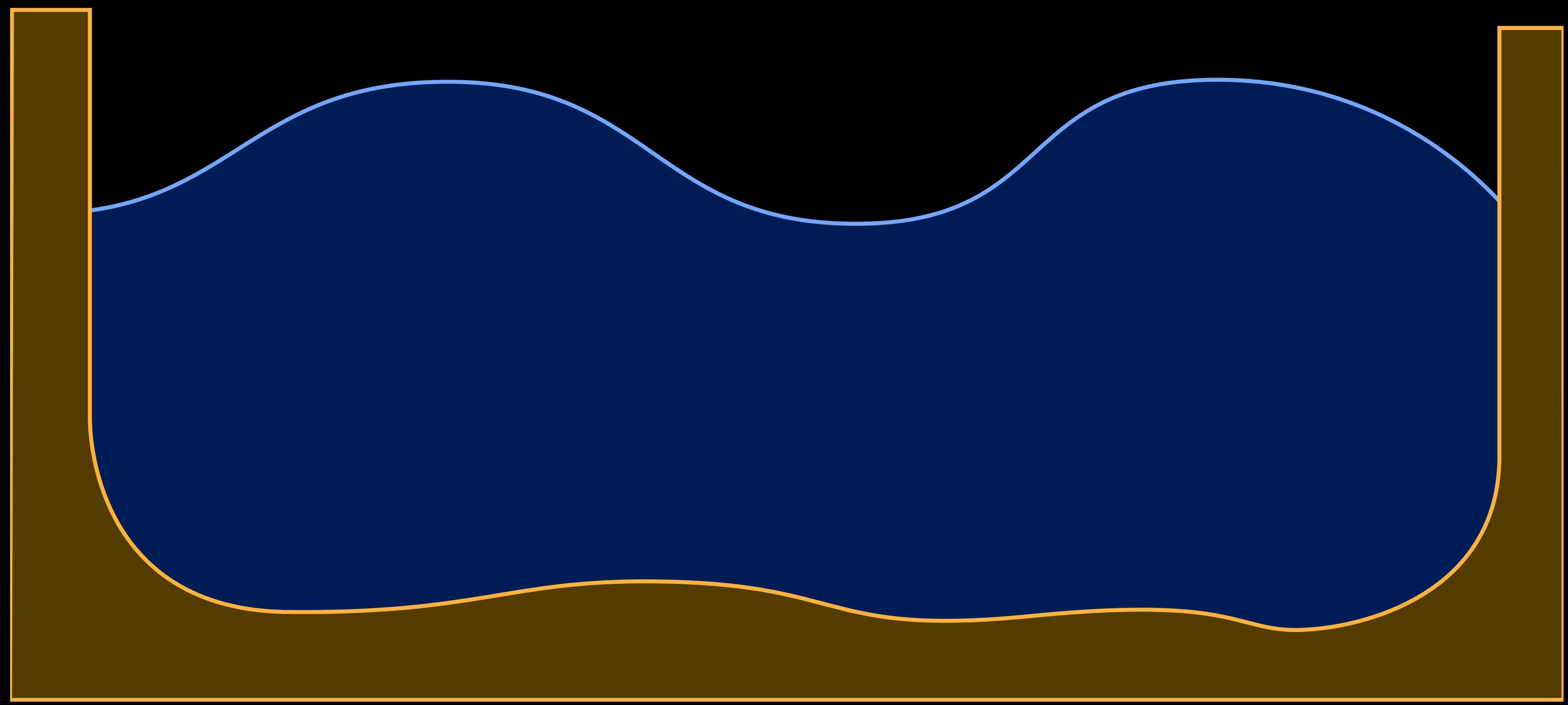


FLIP Solver

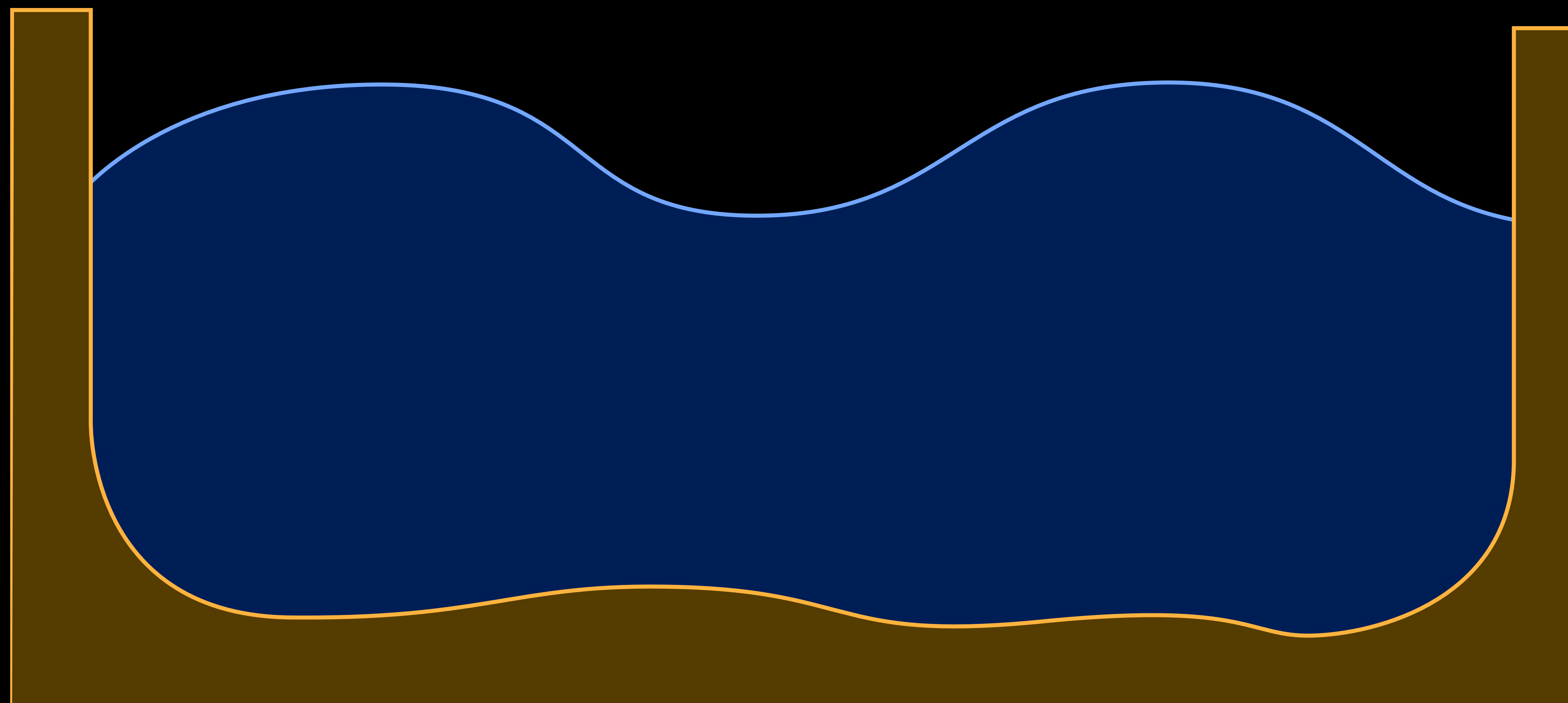


Surface Tracker

Liquid Simulation

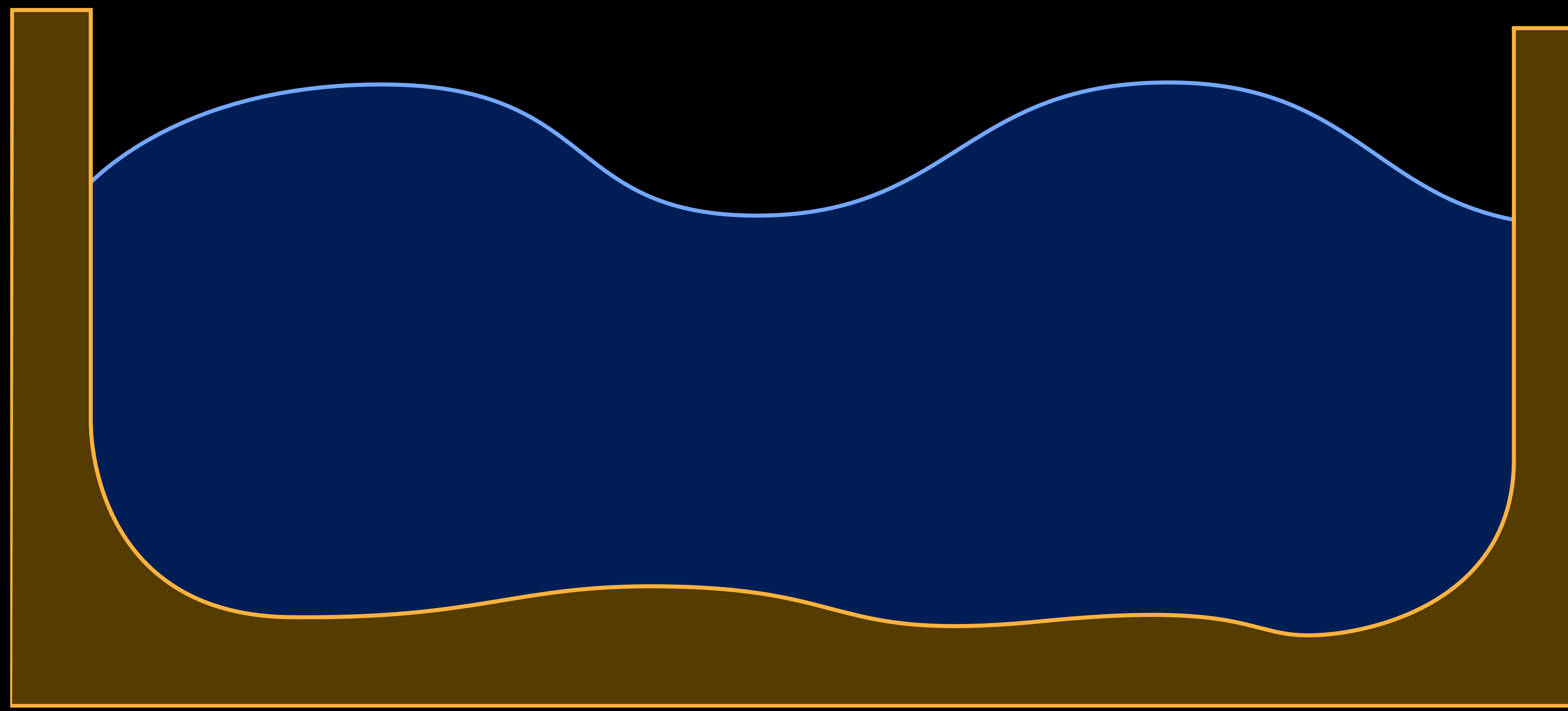


Liquid Simulation

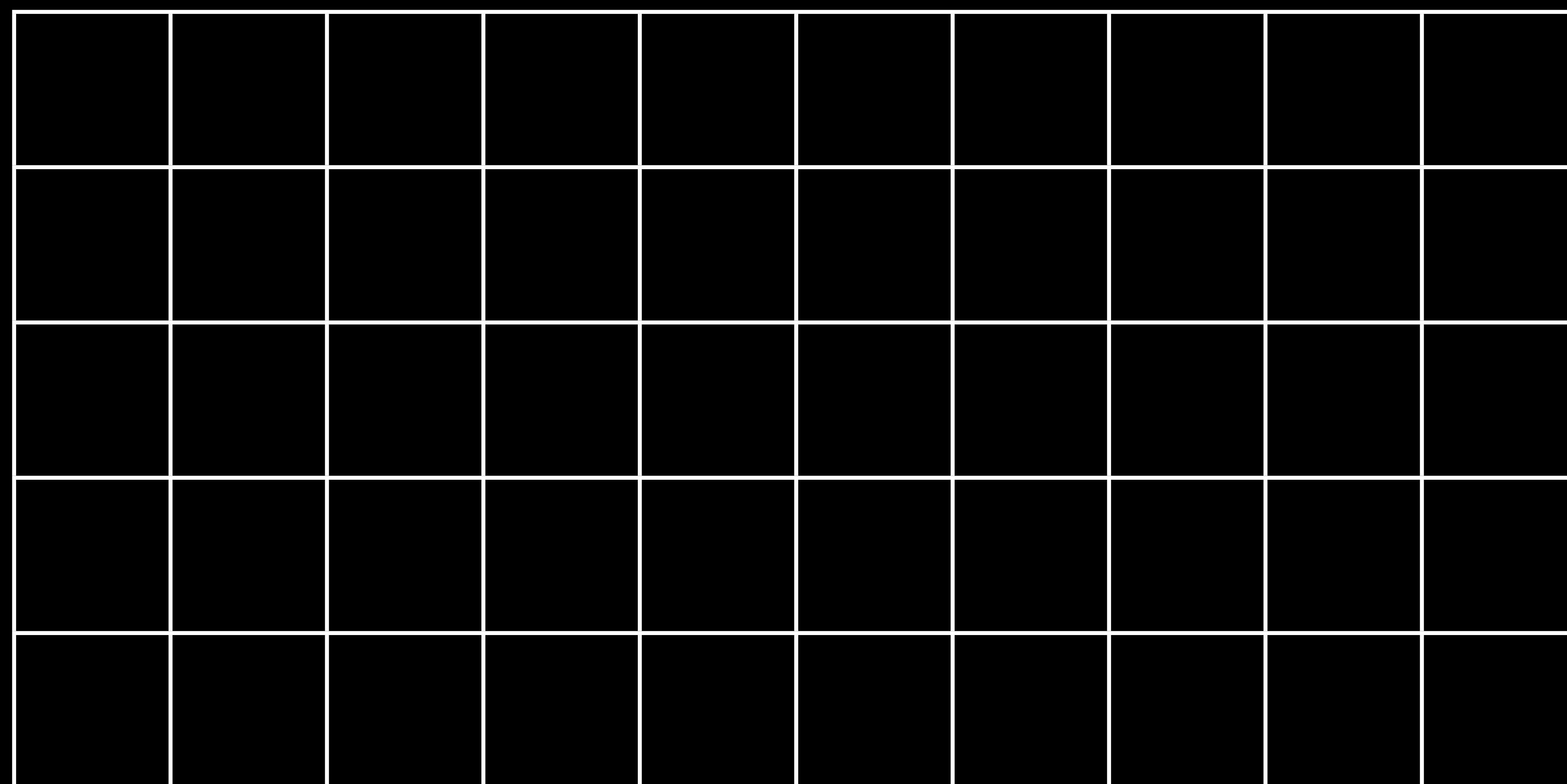


Liquid Simulation

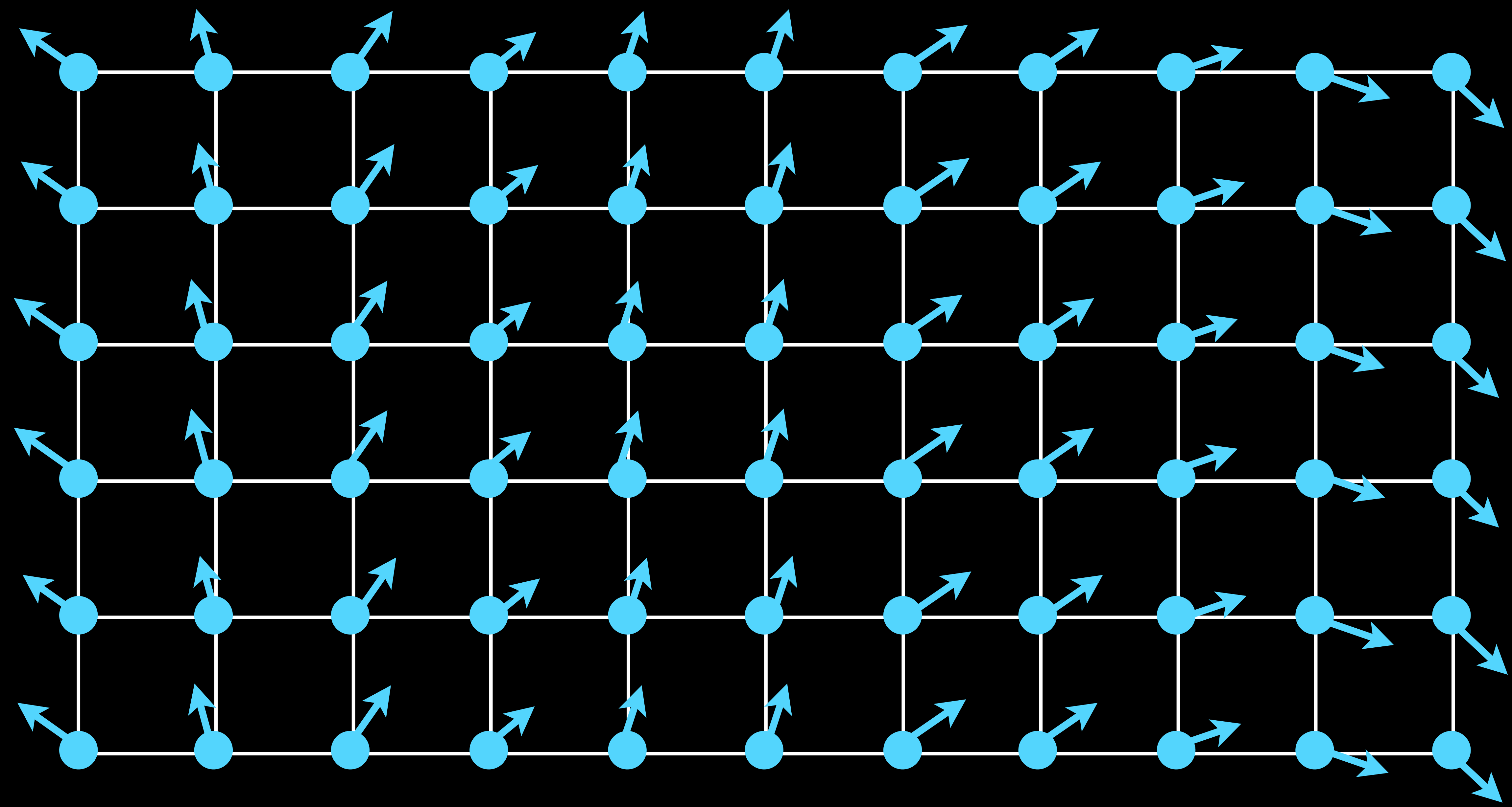
Surface Tracker



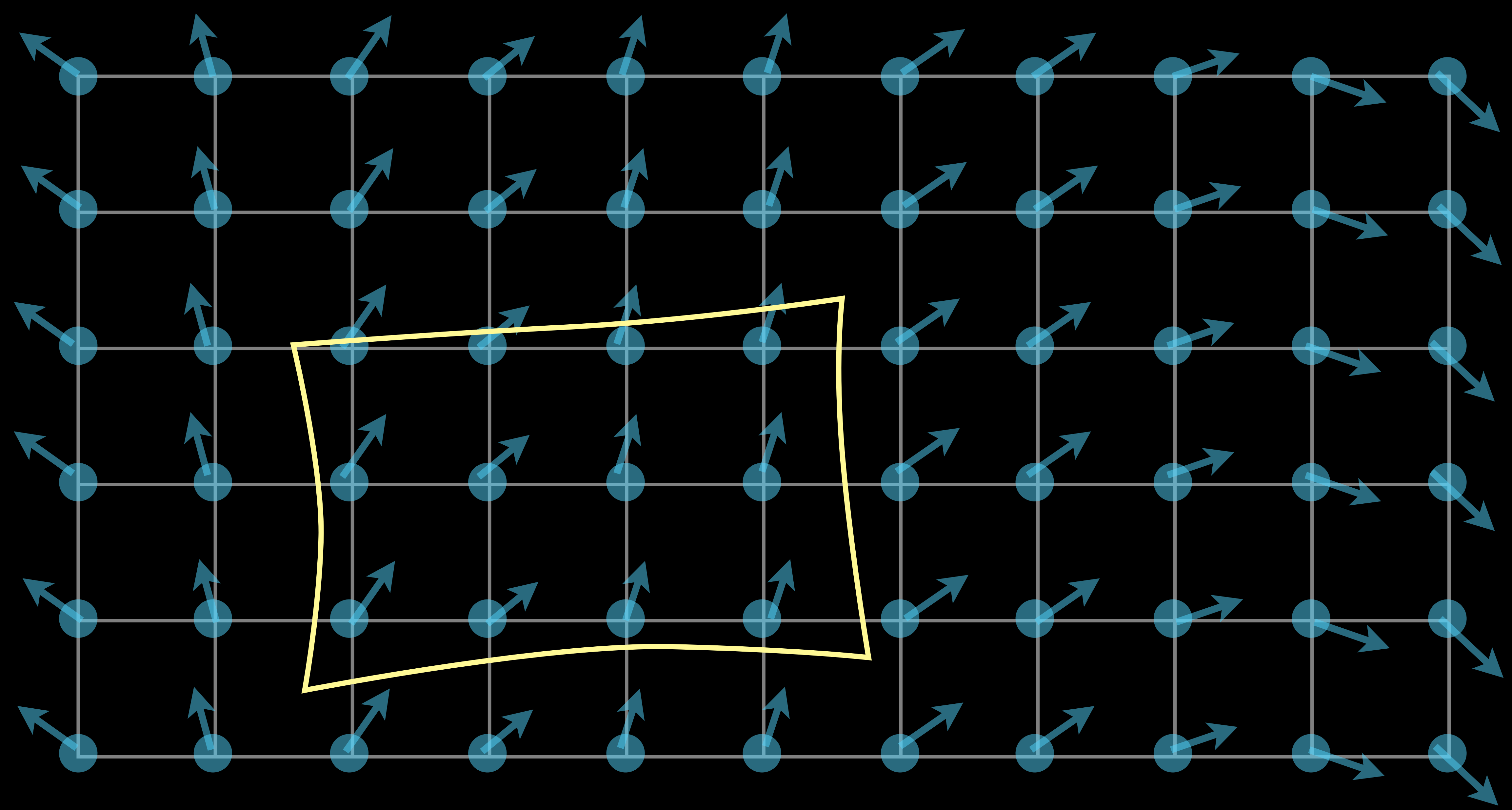
Surface Tracker



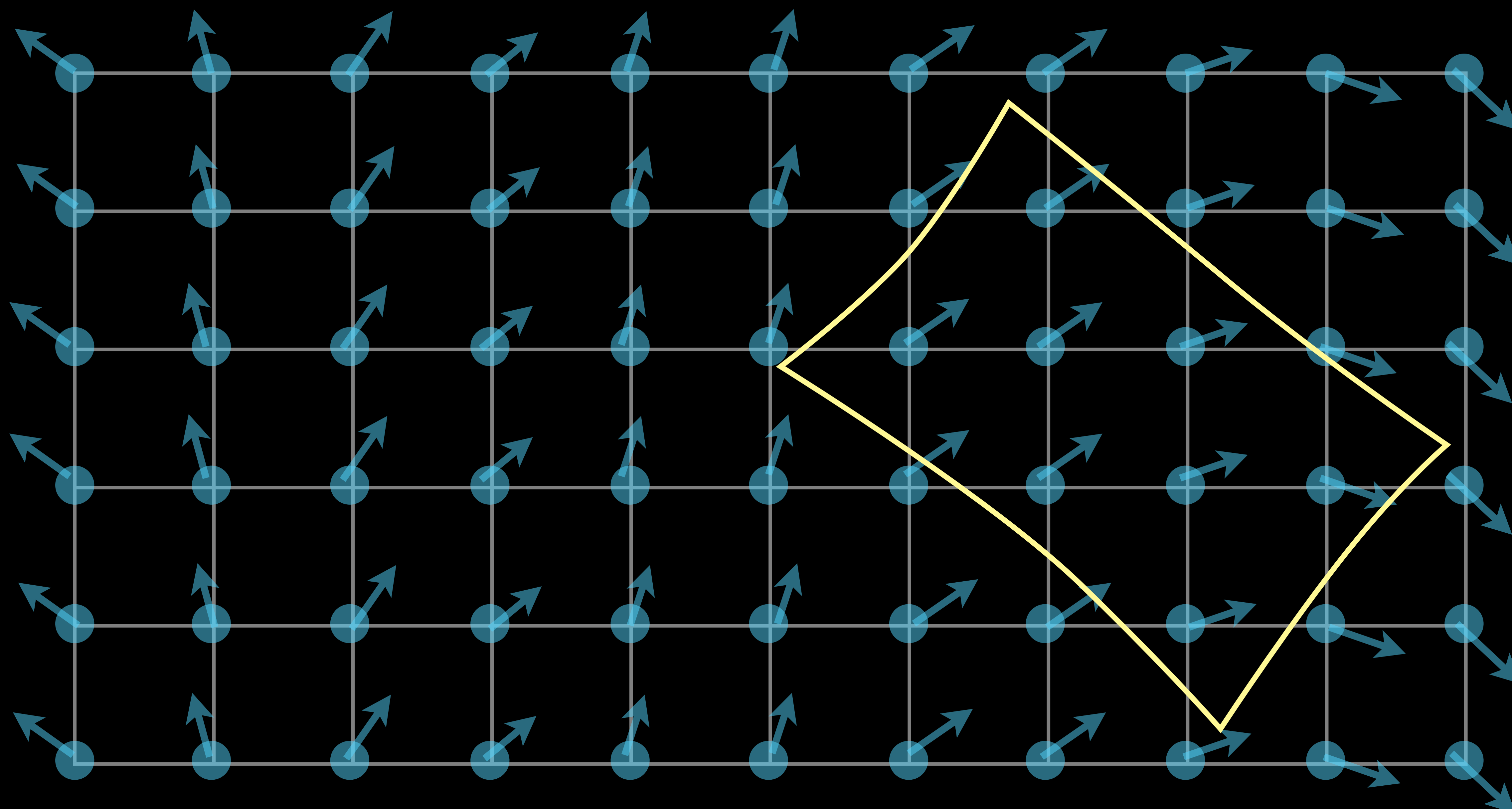
Surface Tracker



Surface Tracker



Surface Tracker

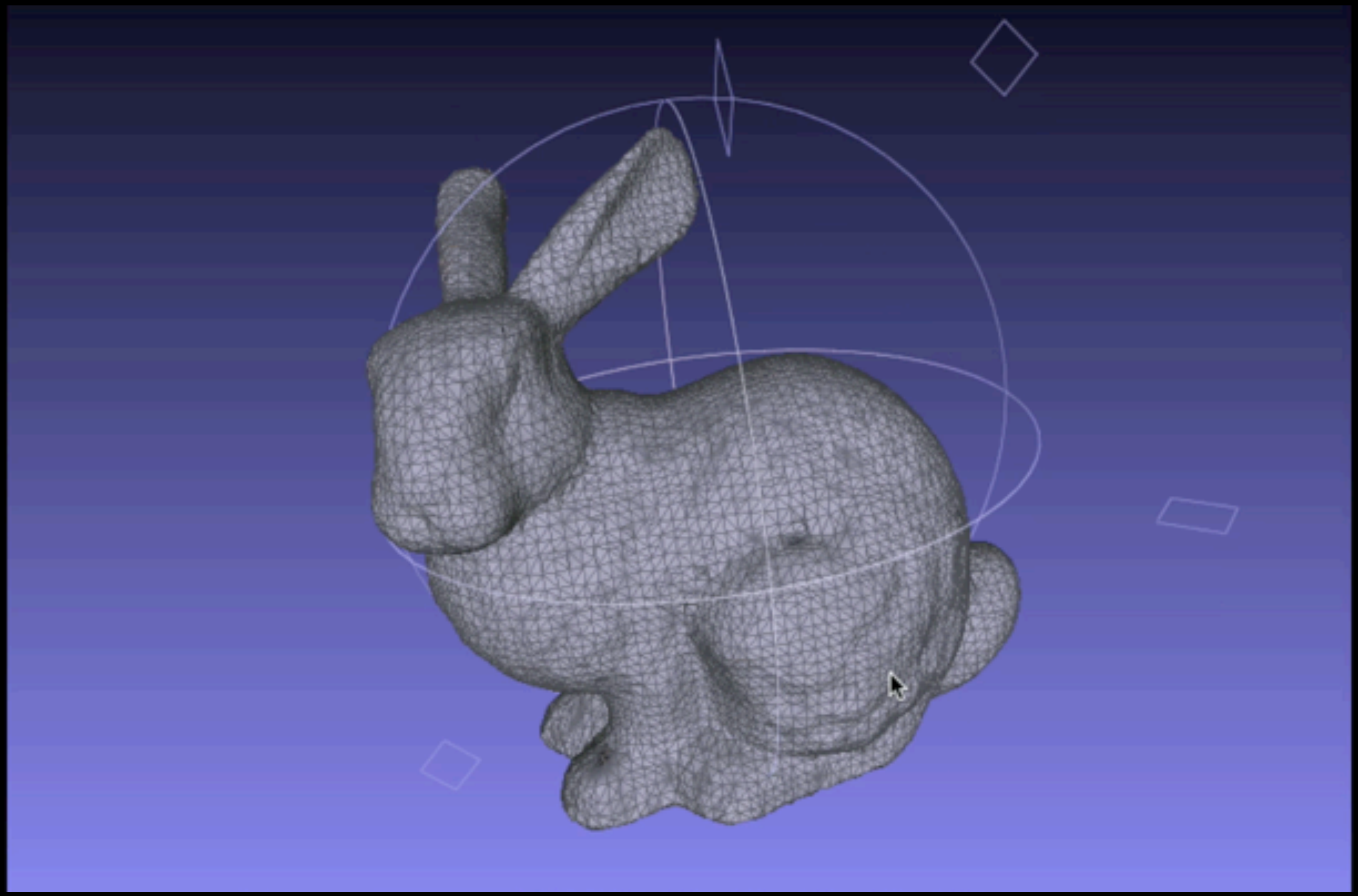


Surface Tracker

- Water tight
- Topology change

Surface Tracker

- Water tight
- Topology change

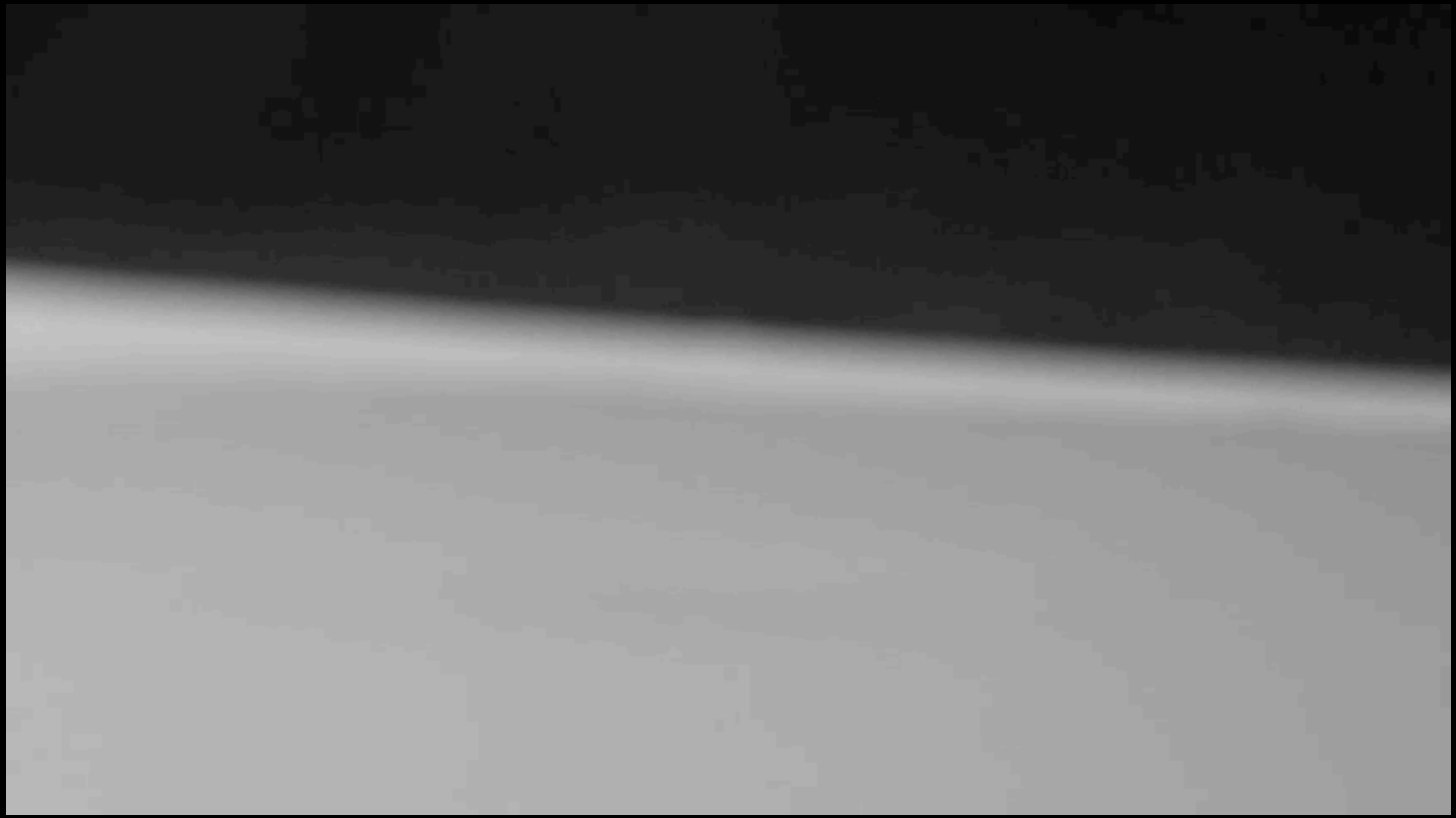


Surface Tracker

- Water tight
- Topology change

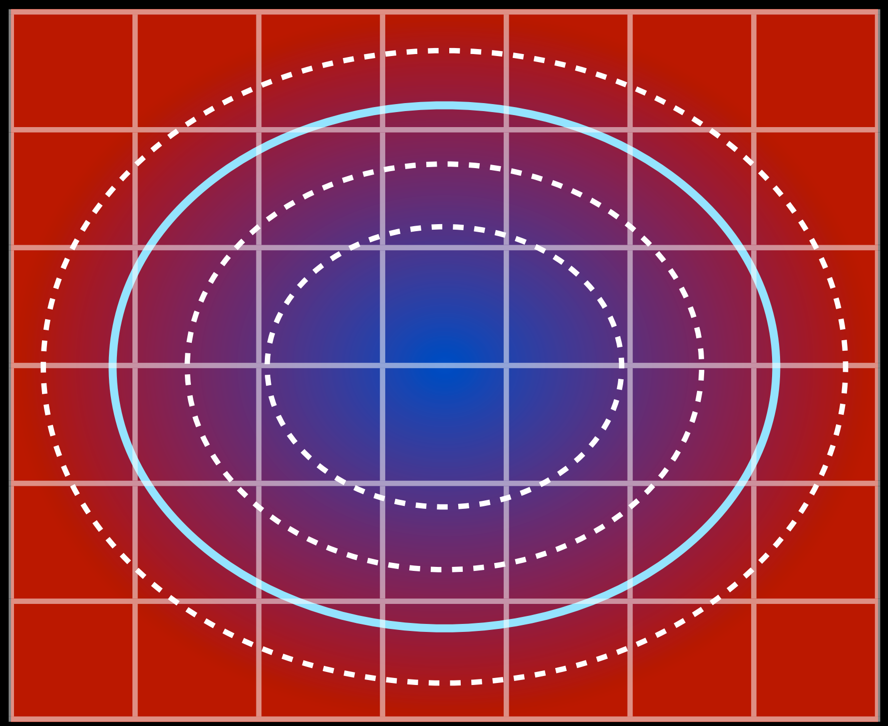
Surface Tracker

- Water tight
- Topology change



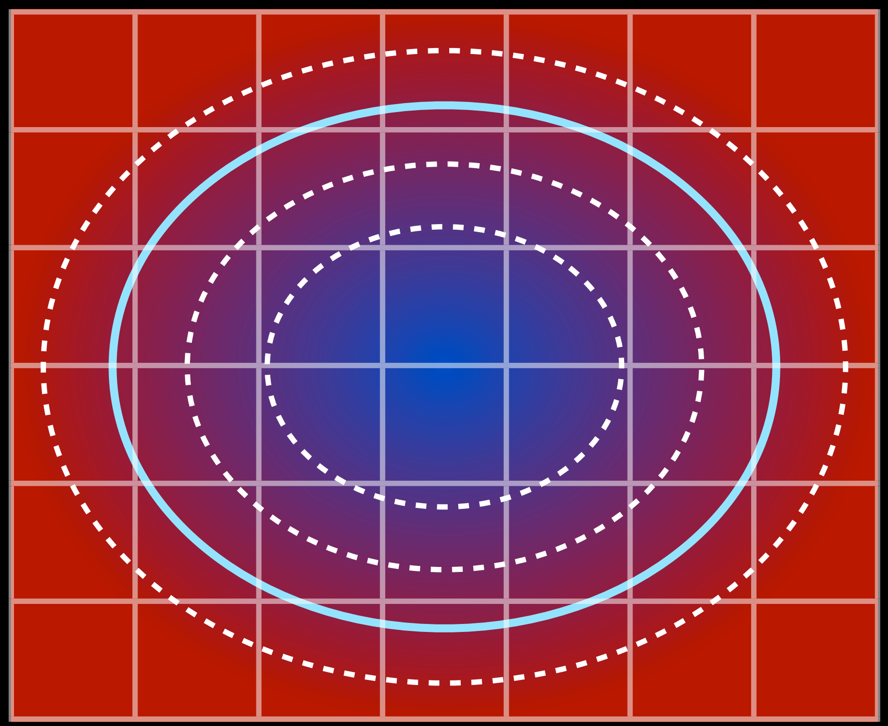
Surface Tracker

Surface Tracker

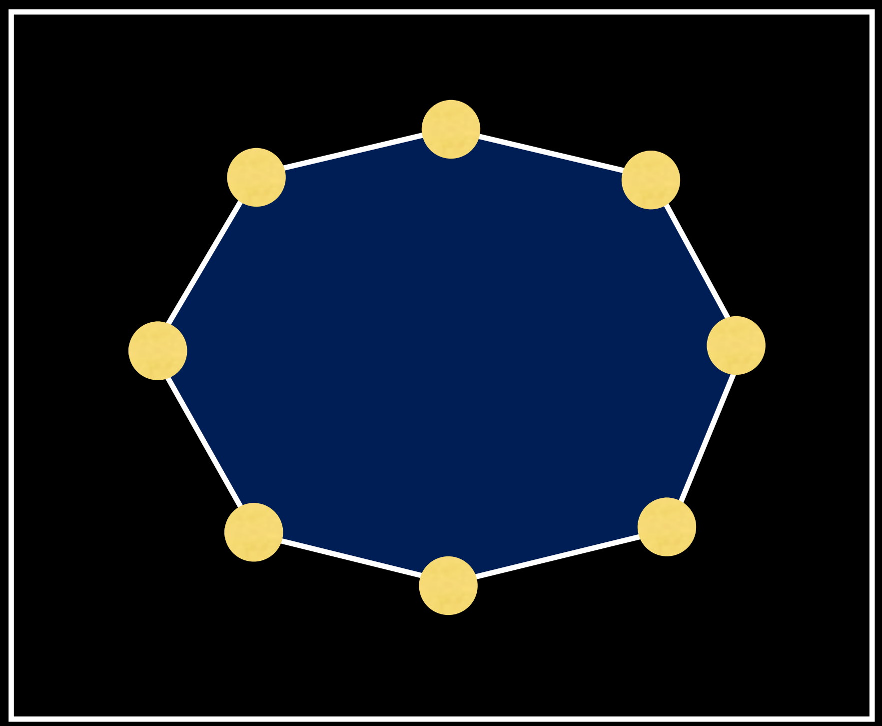


Levelset Method

Surface Tracker

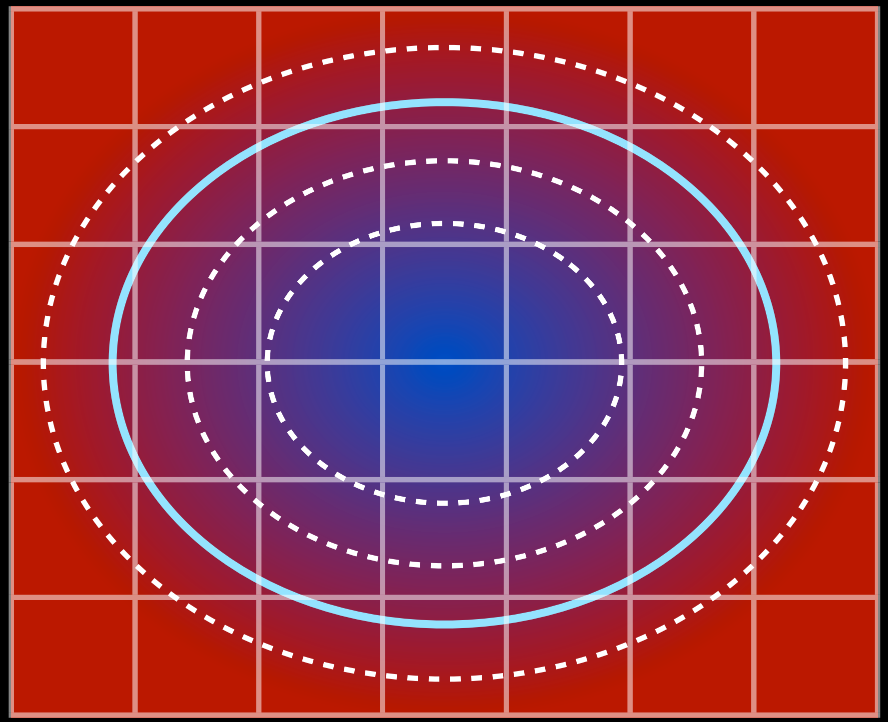


Levelset Method

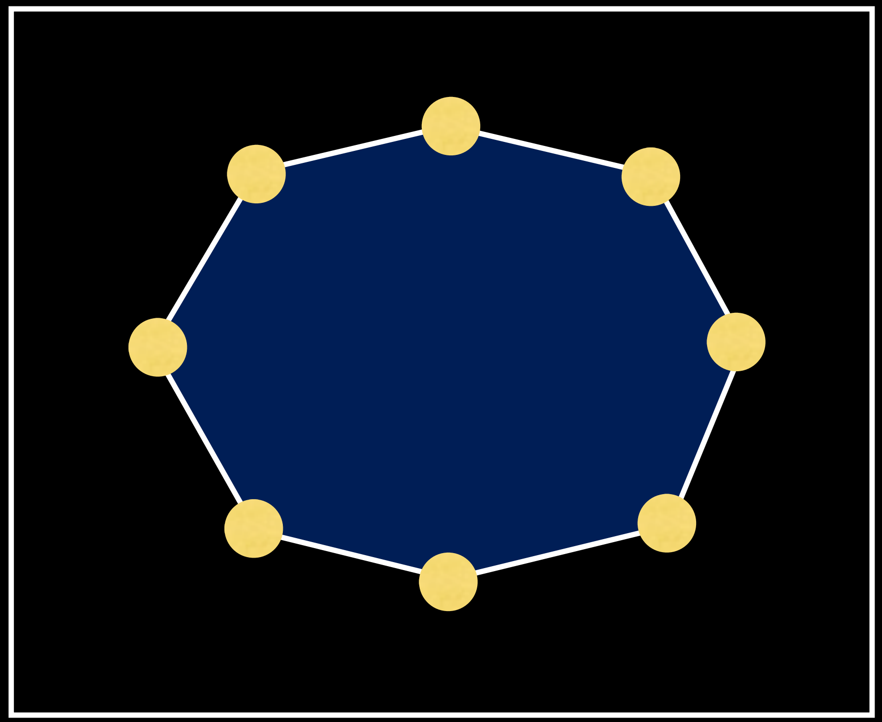


Mesh-based Method

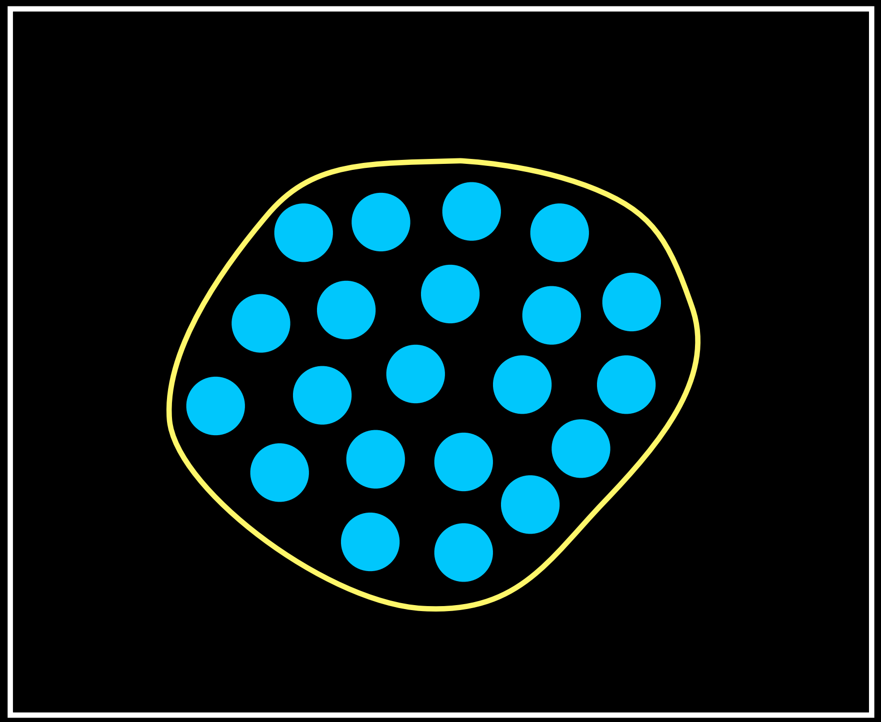
Surface Tracker



Levelset Method

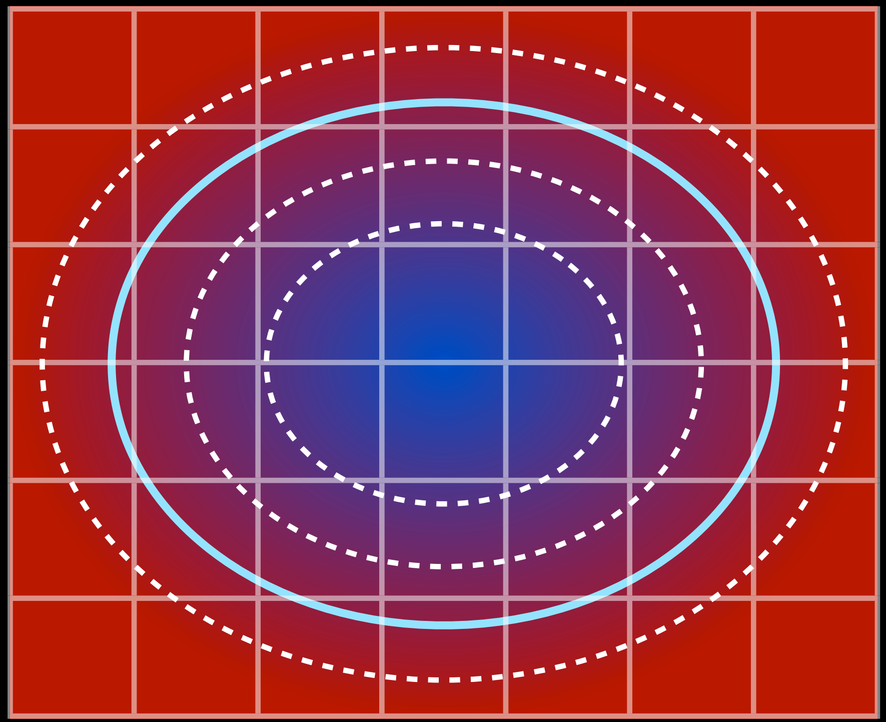


Mesh-based Method



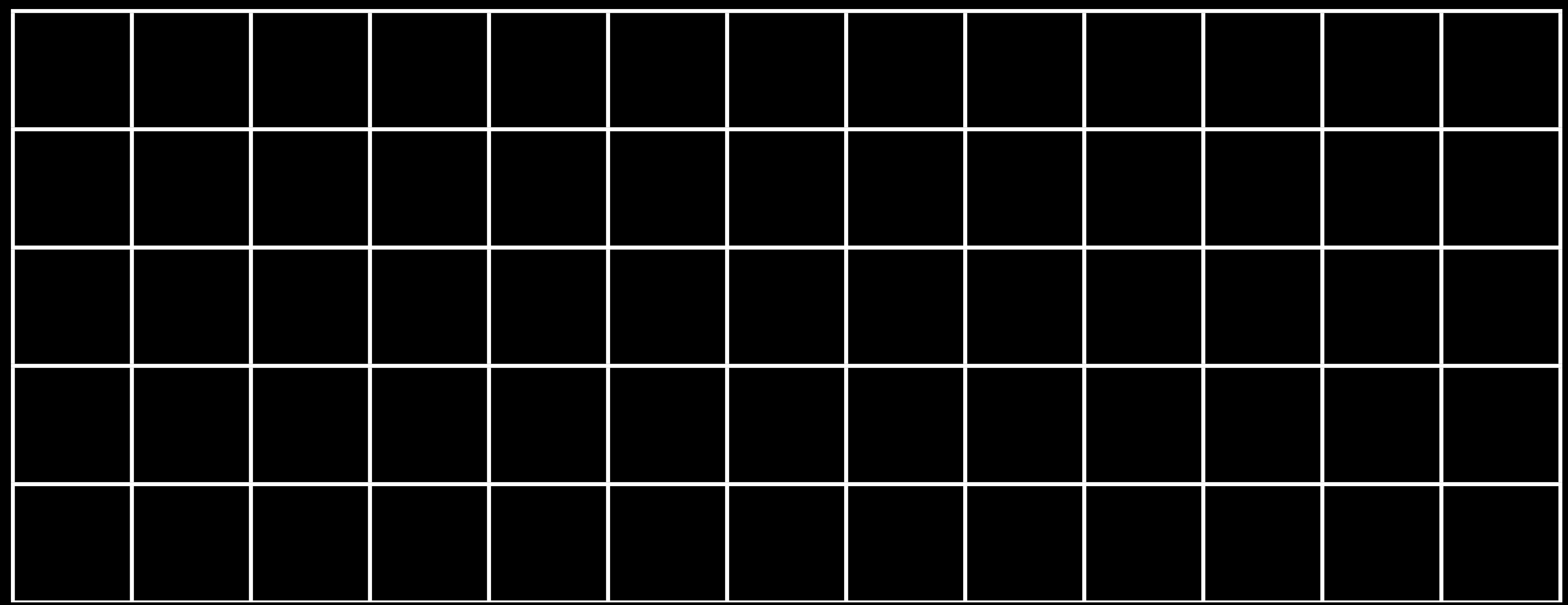
Particle-based Method

Surface Tracker

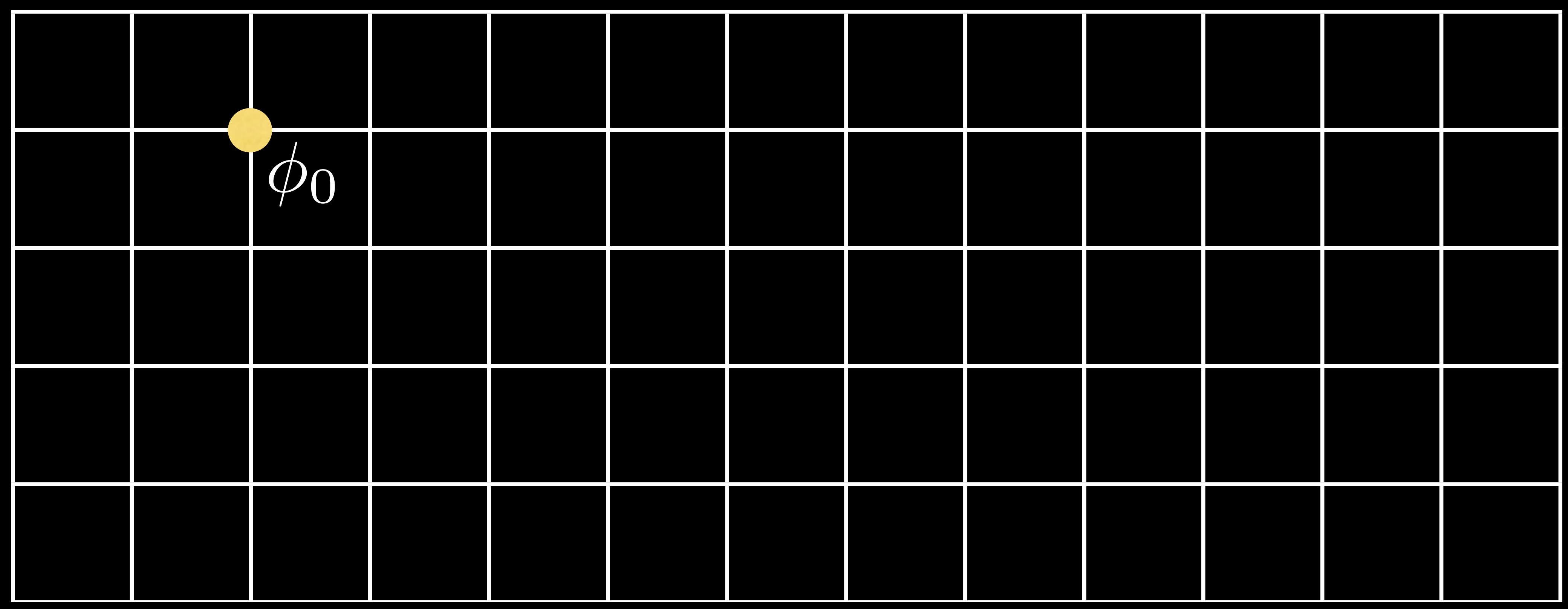


Levelset Method

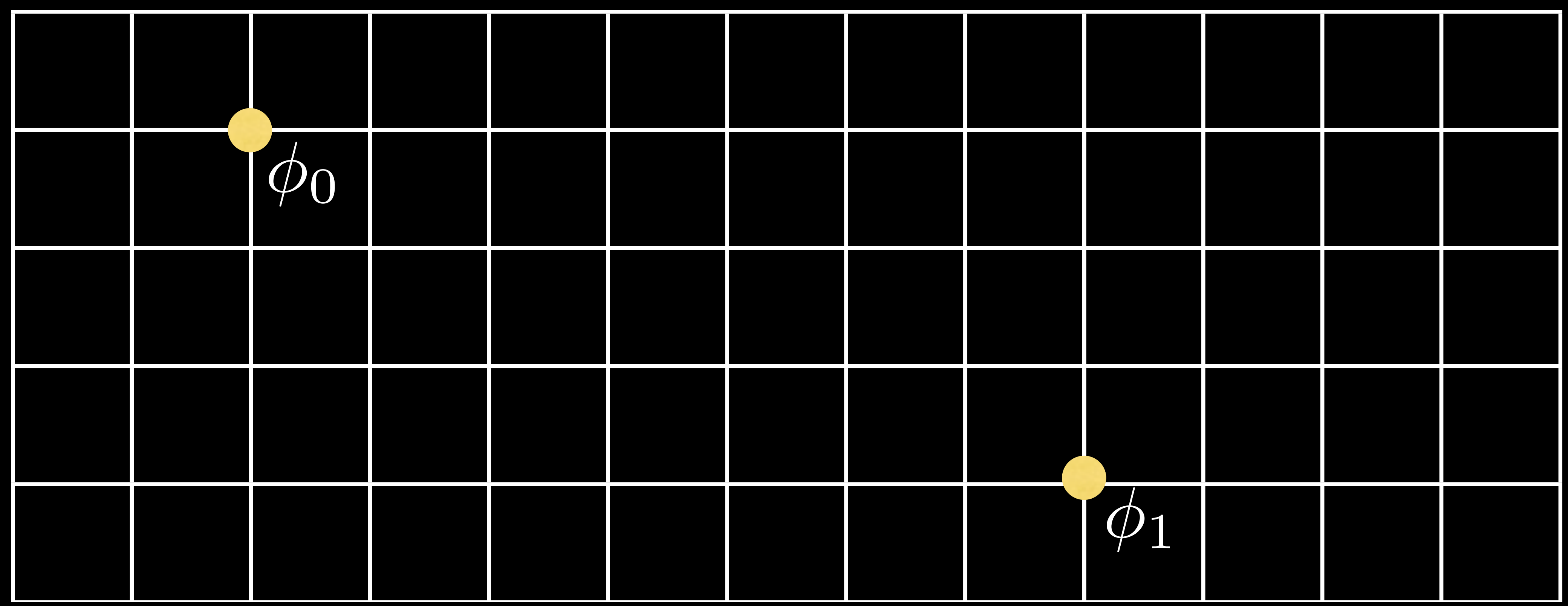
Levelset Method



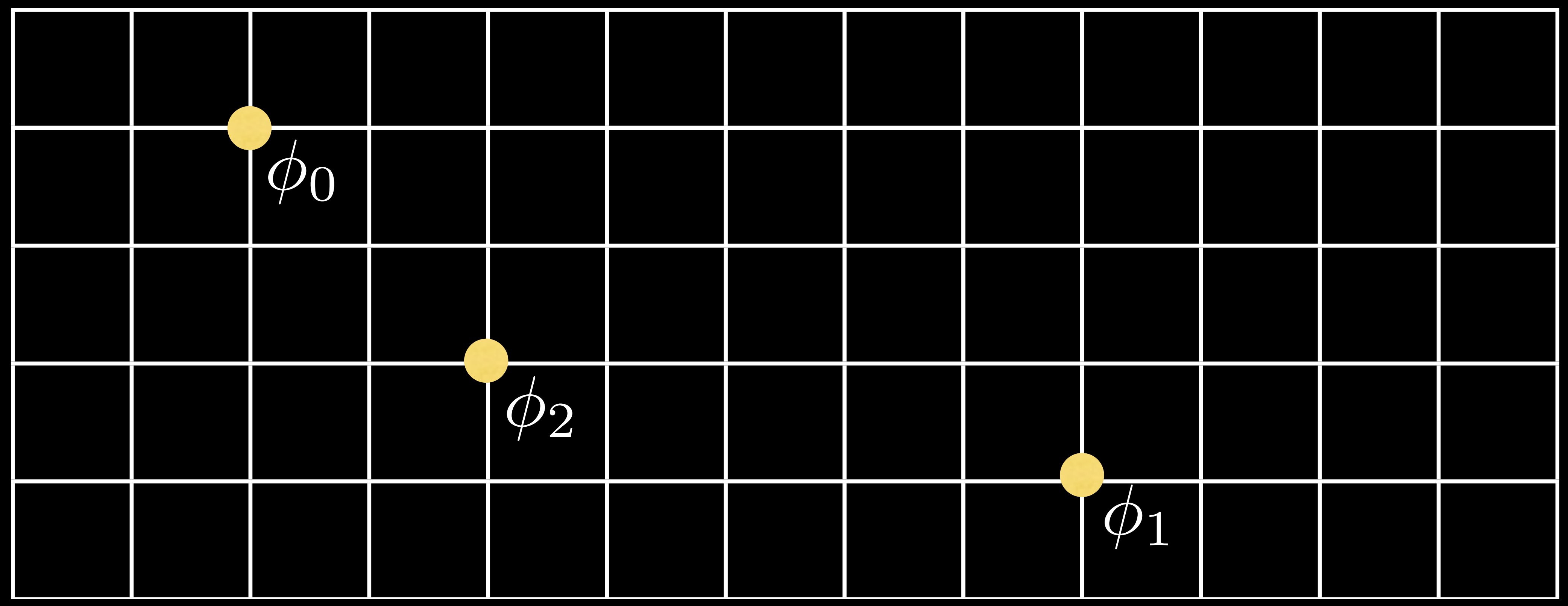
Levelset Method



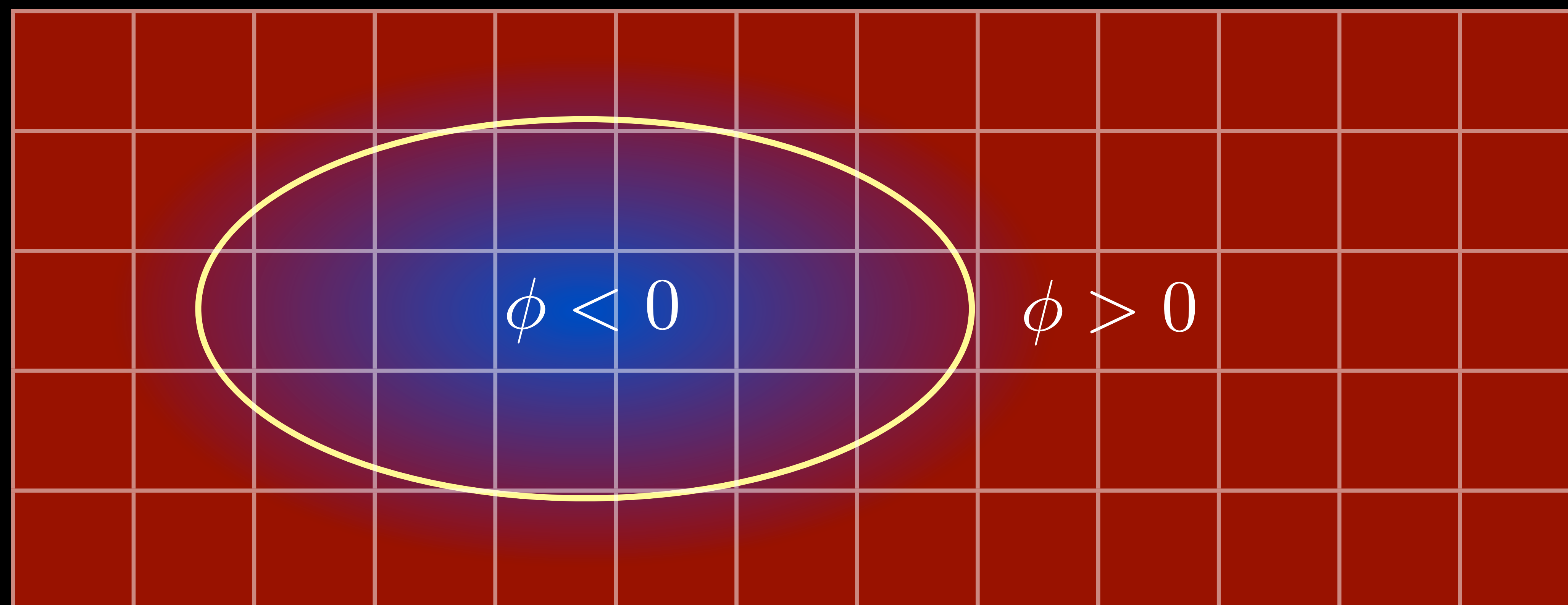
Levelset Method



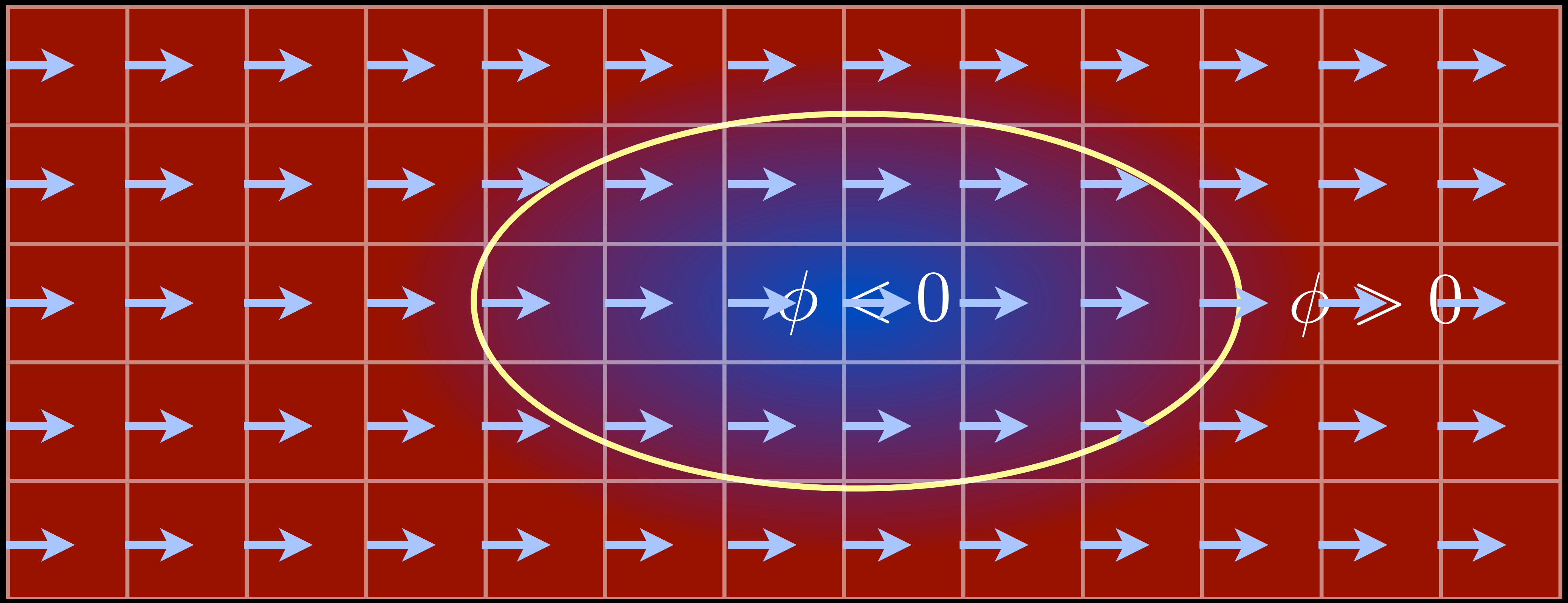
Levelset Method



Levelset Method

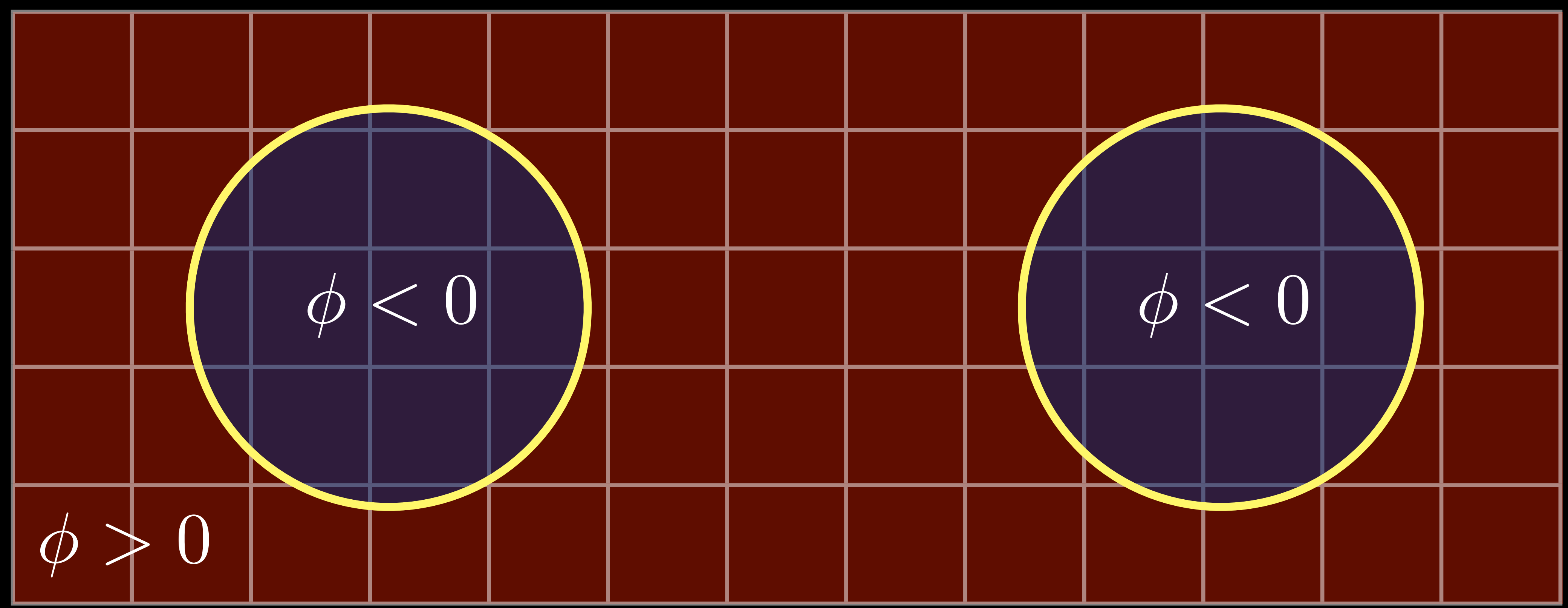


Levelset Method

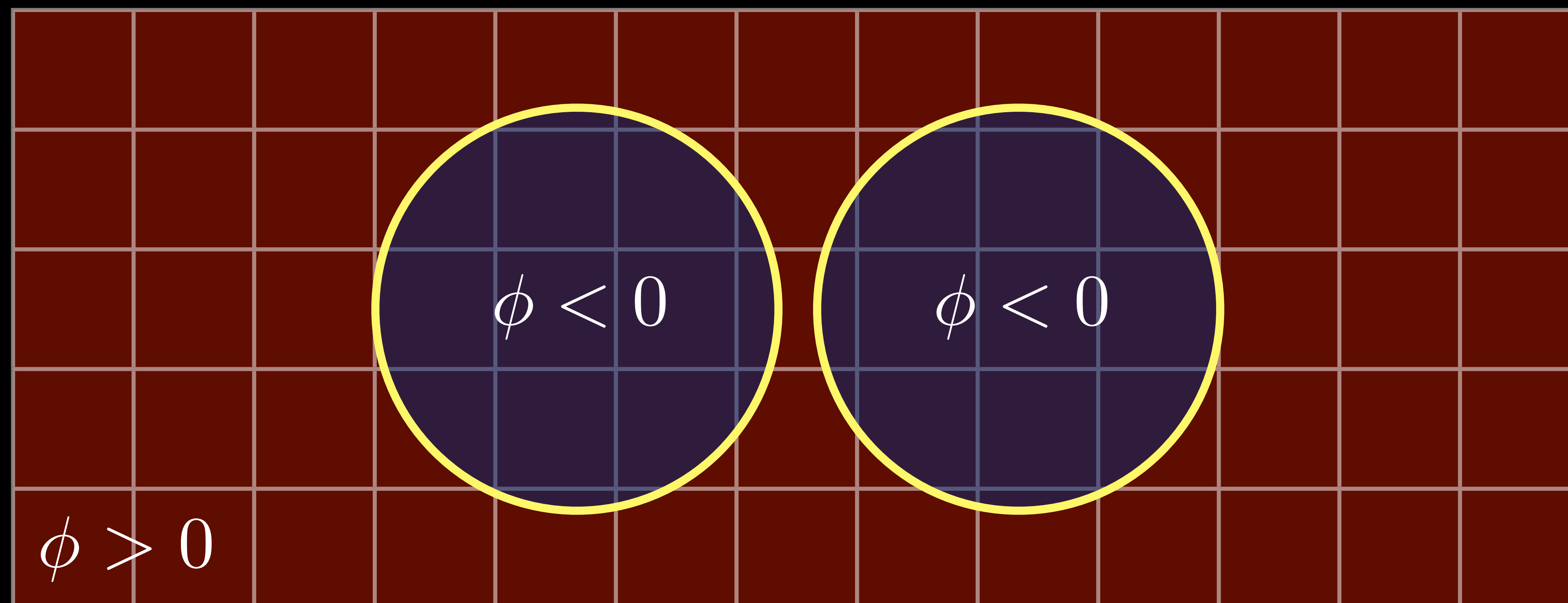


$$\frac{\partial \phi}{\partial t} = -\boldsymbol{u} \cdot \nabla \phi$$

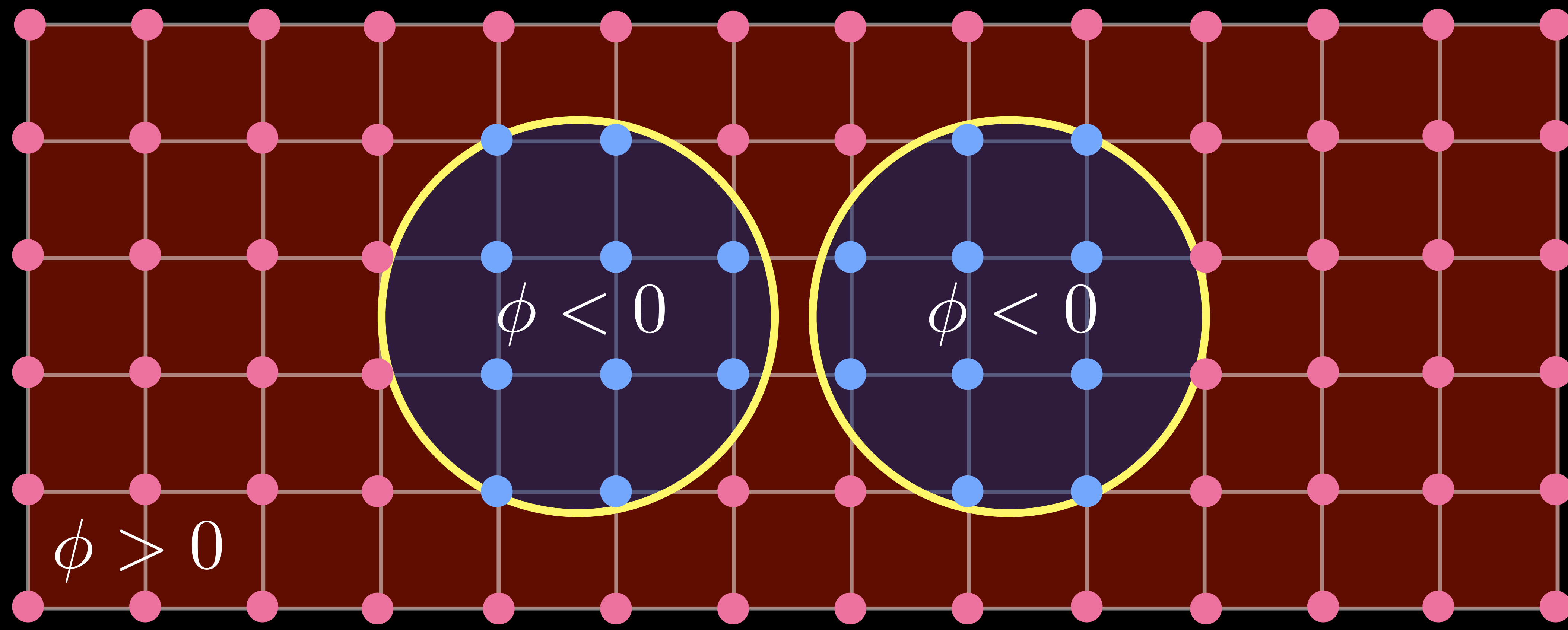
Levelset Method



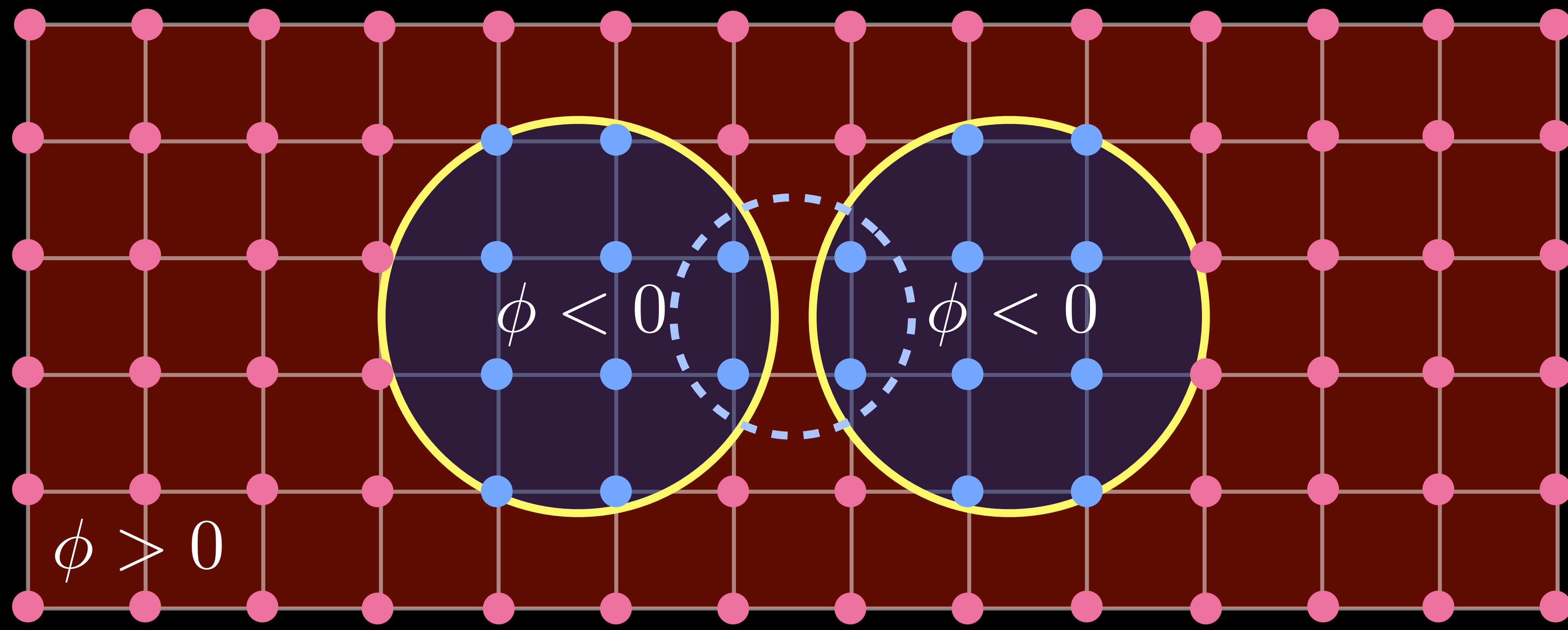
Levelset Method



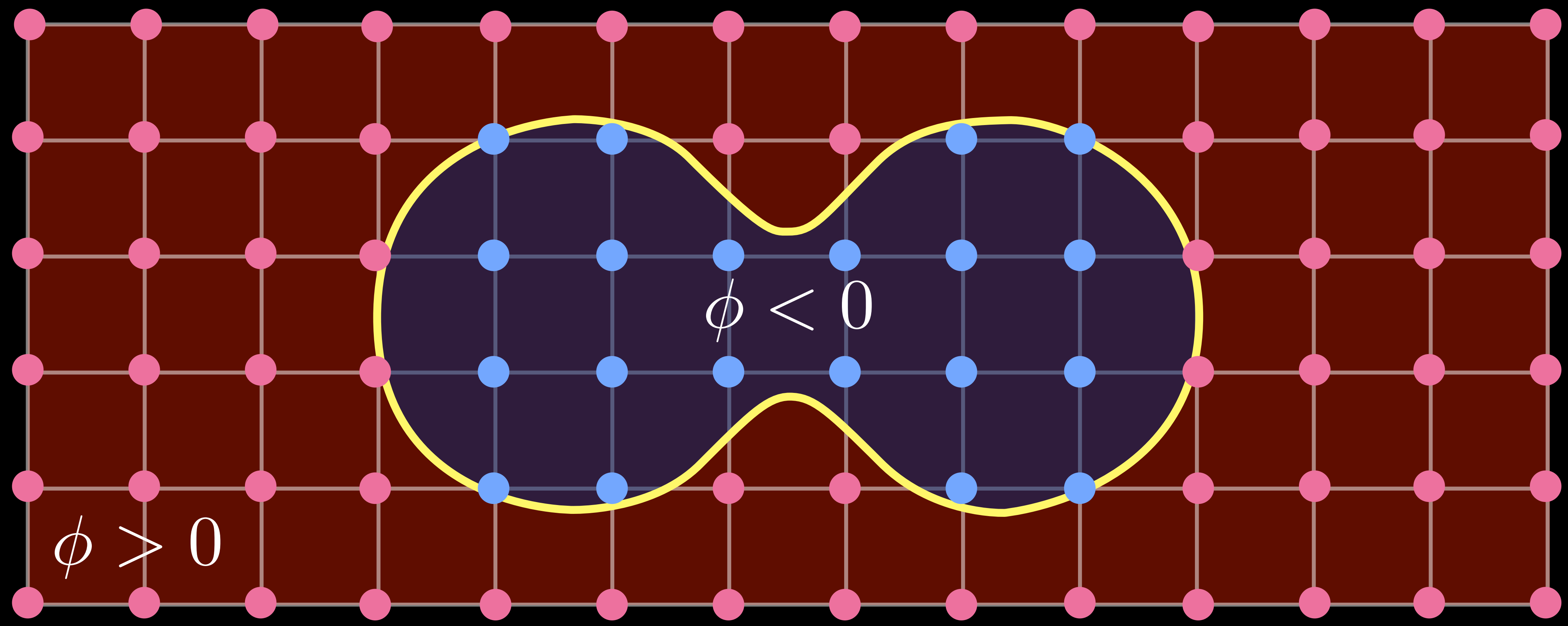
Levelset Method



Levelset Method

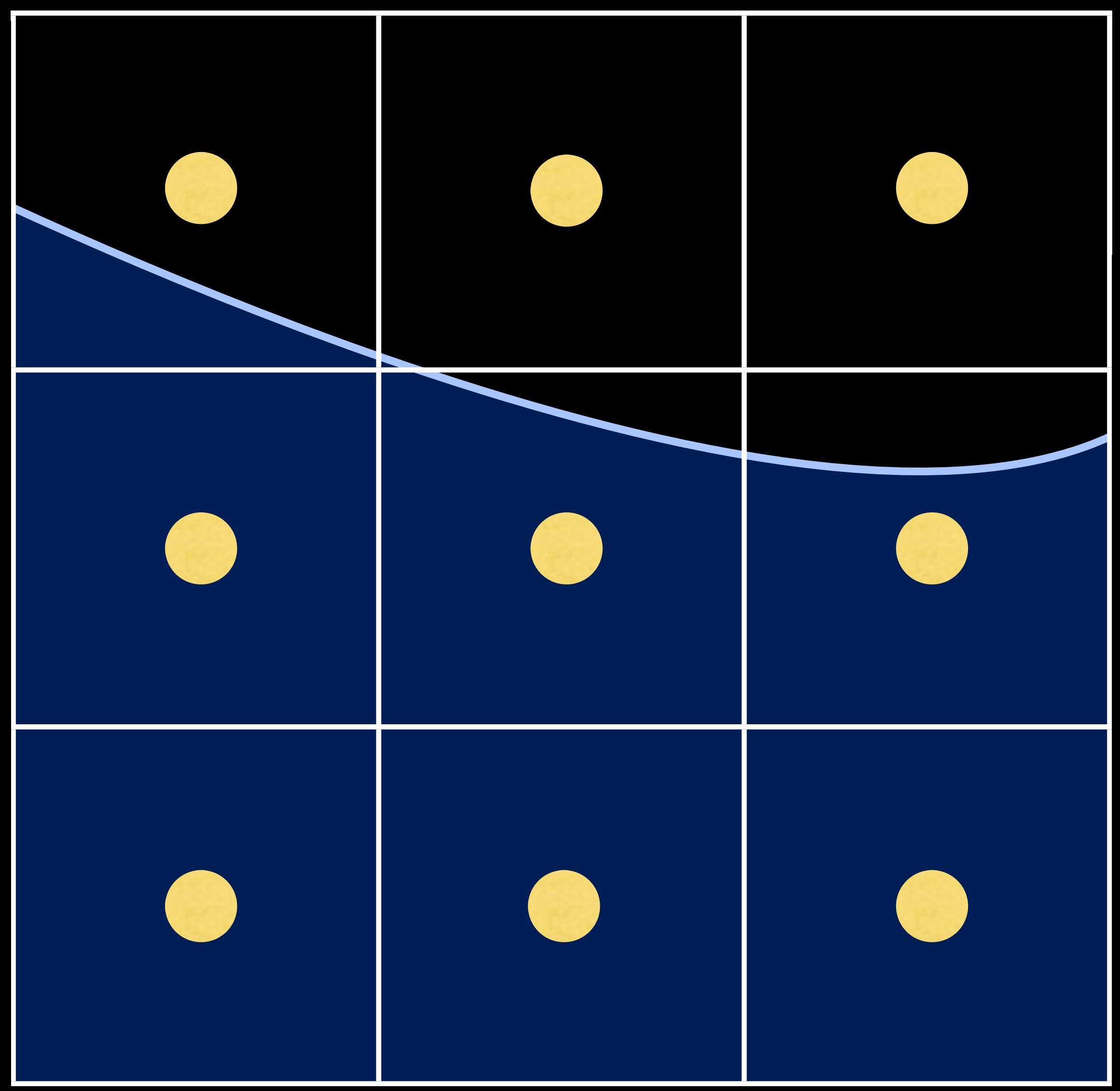


Levelset Method

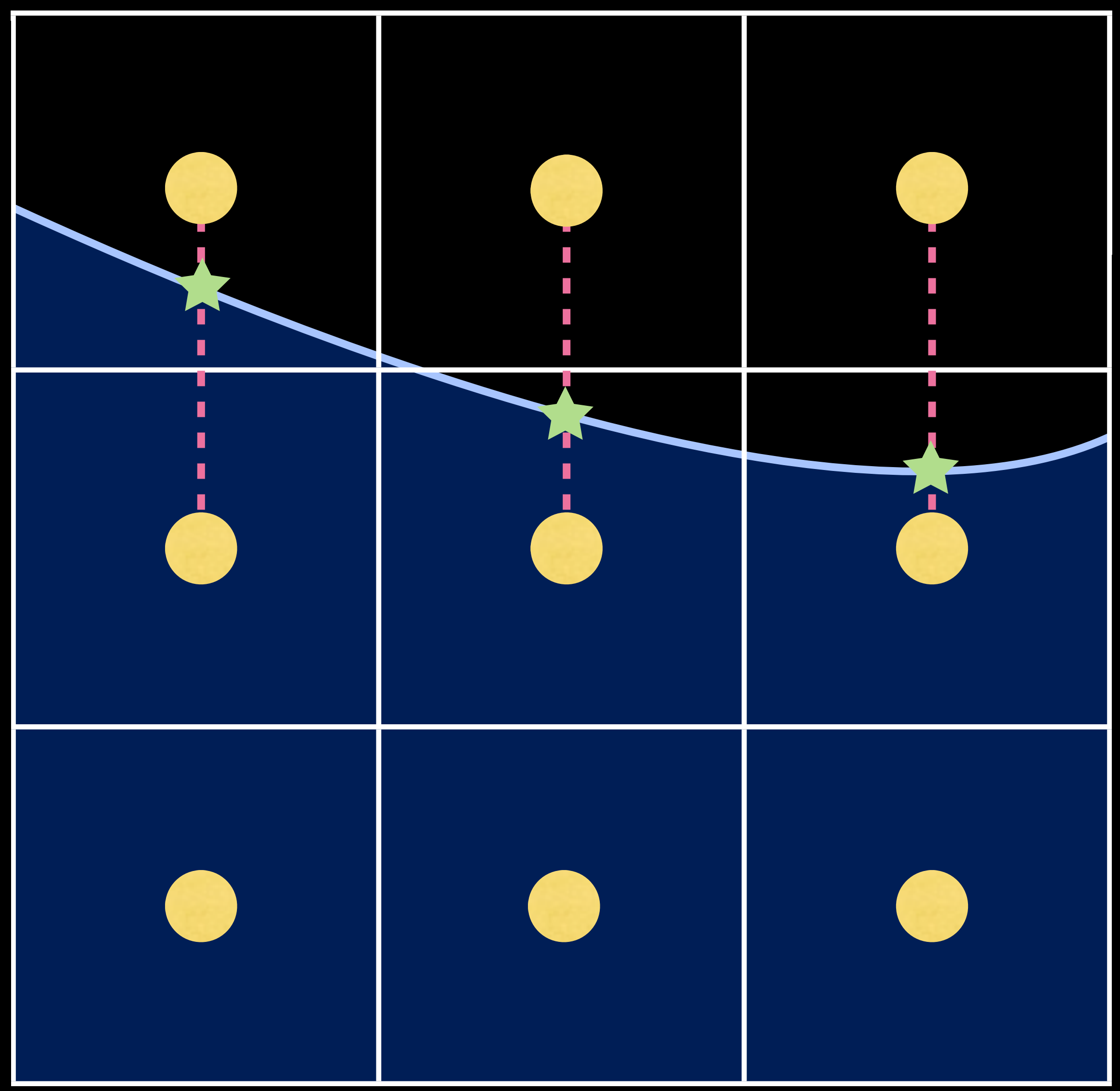


Topology change

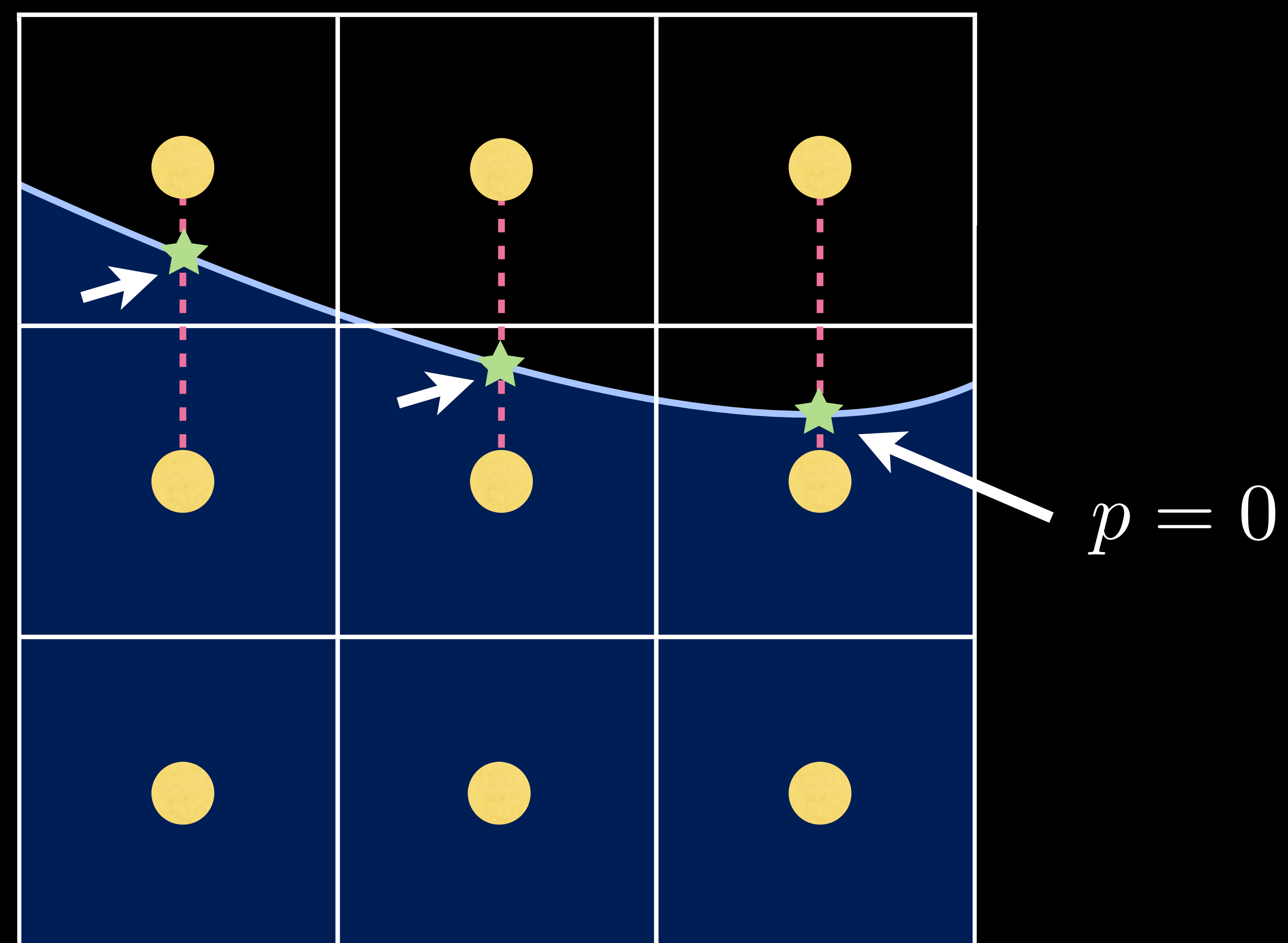
Levelset Method



Levelset Method

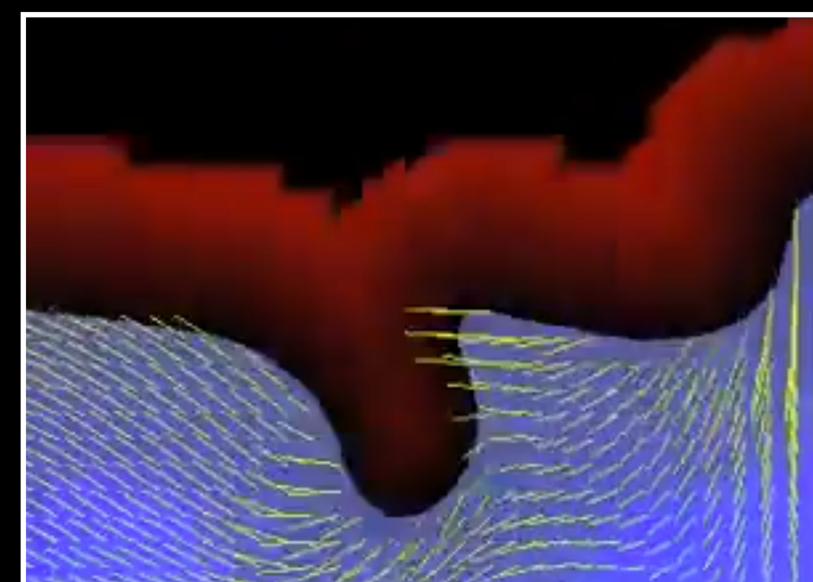


Levelset Method

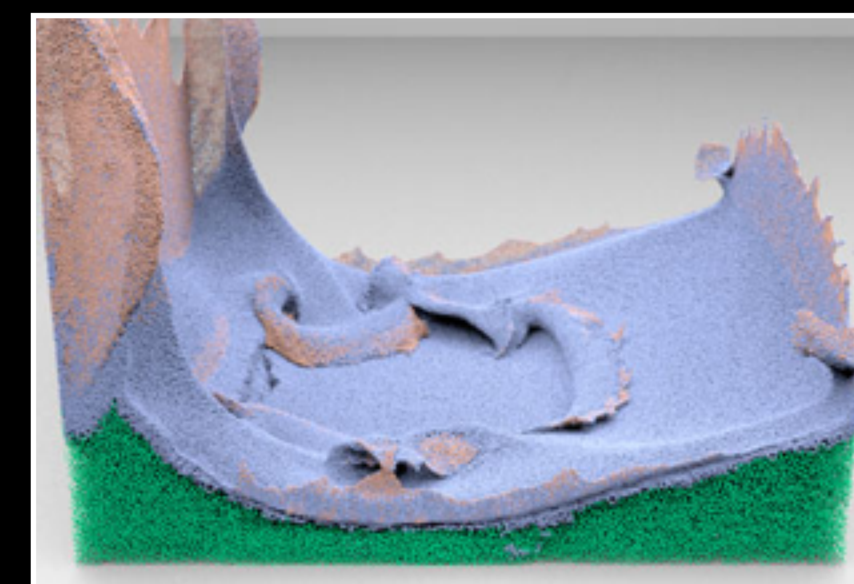


Levelset Method

Fluid Solver

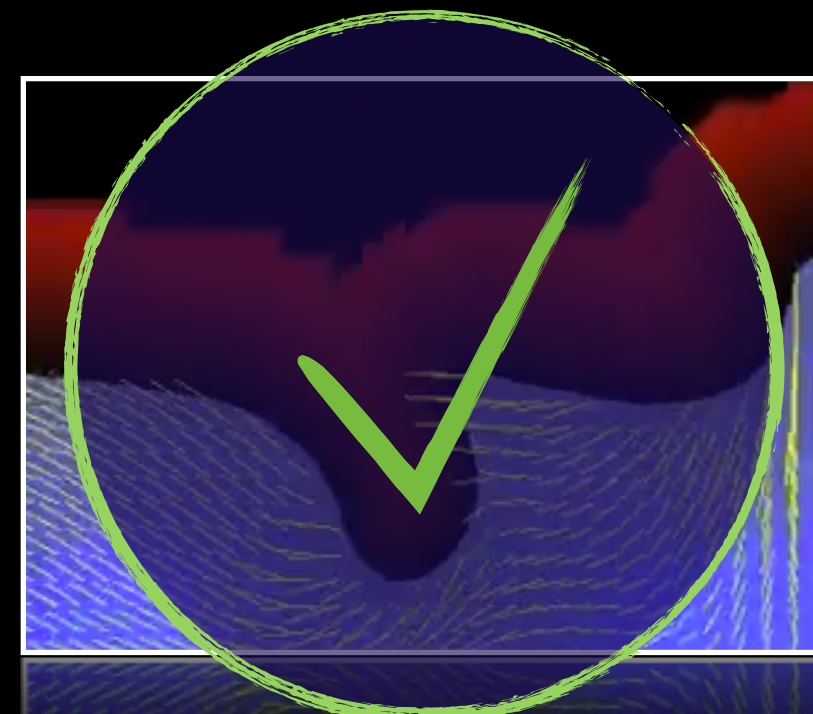


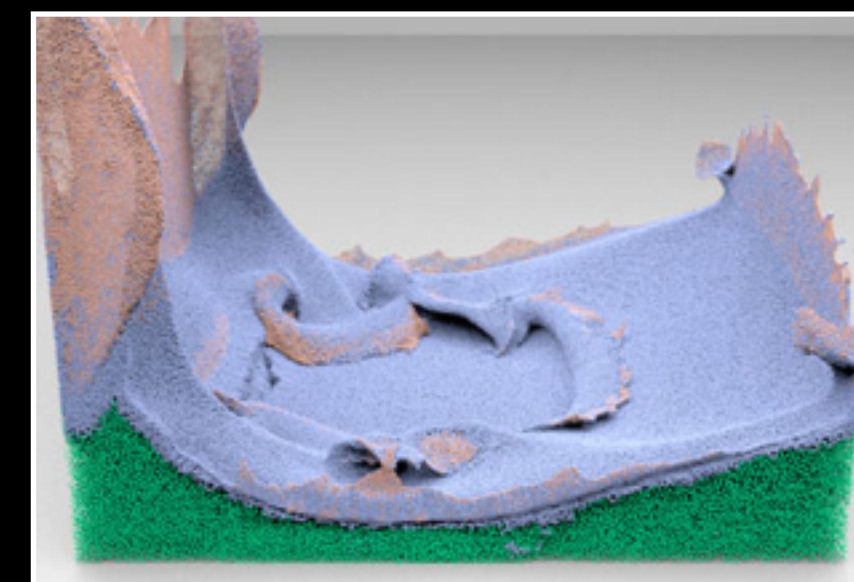
Surface Tracker

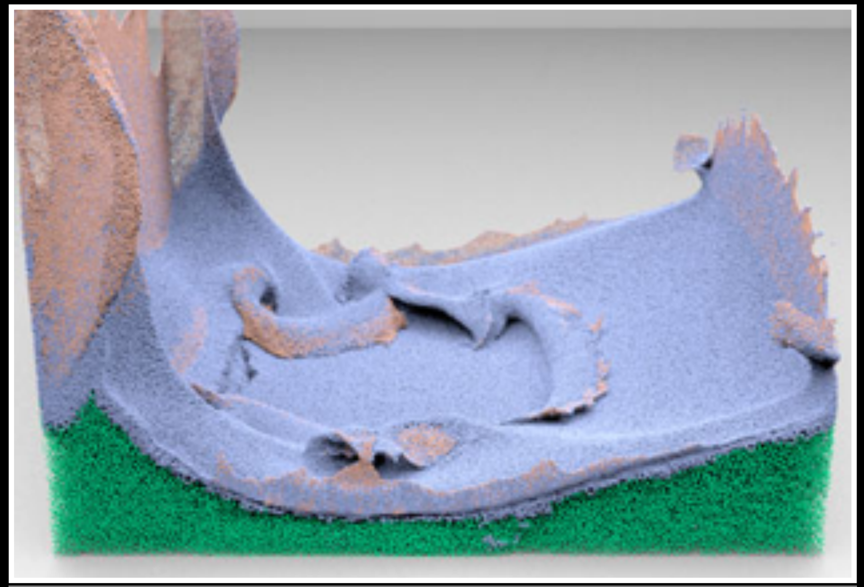


FLIP Solver


Fluid Solver


Surface Tracker


FLIP Solver



FLIP Solver

FLIP Solver

FLIP Solver

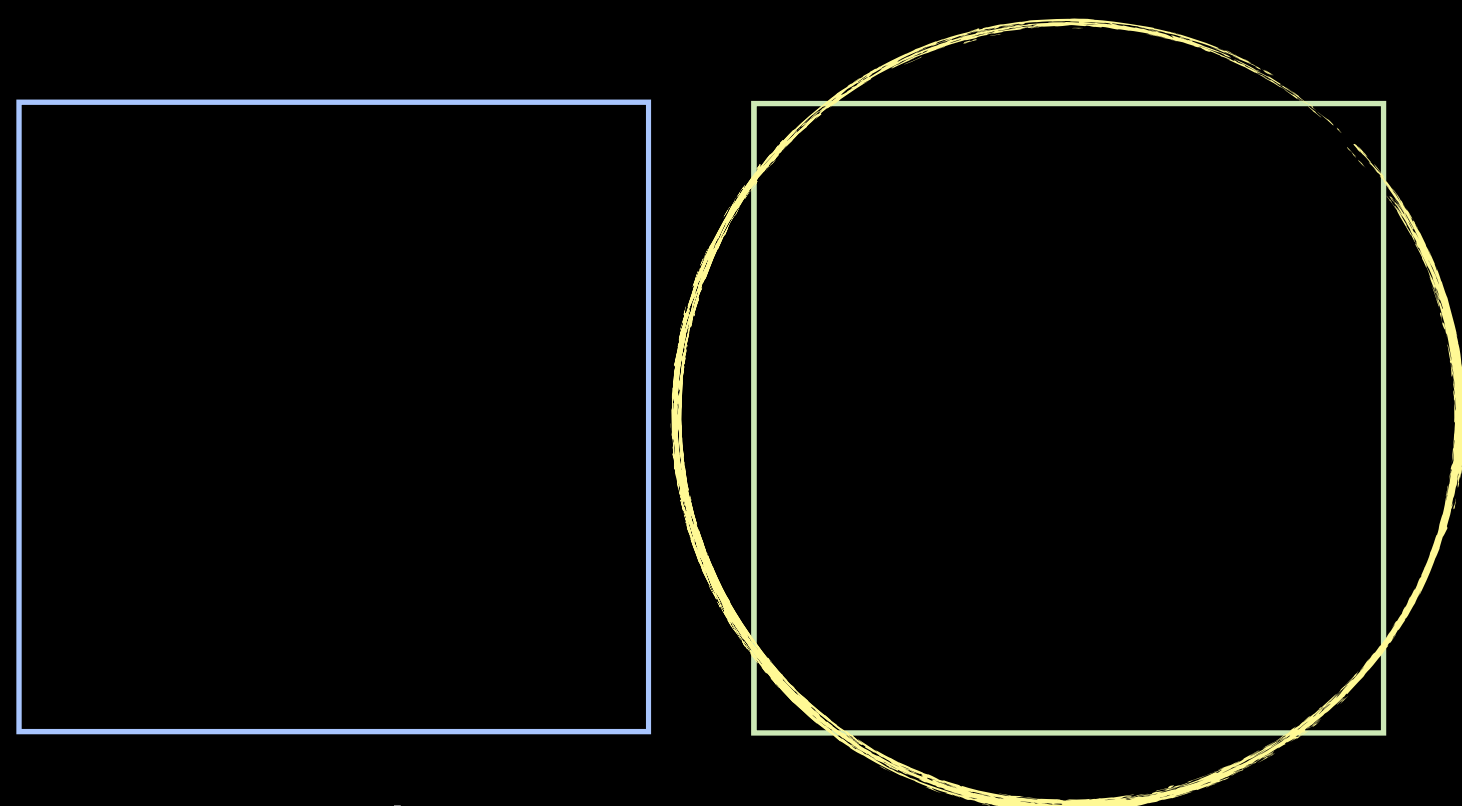
Particles

FLIP Solver

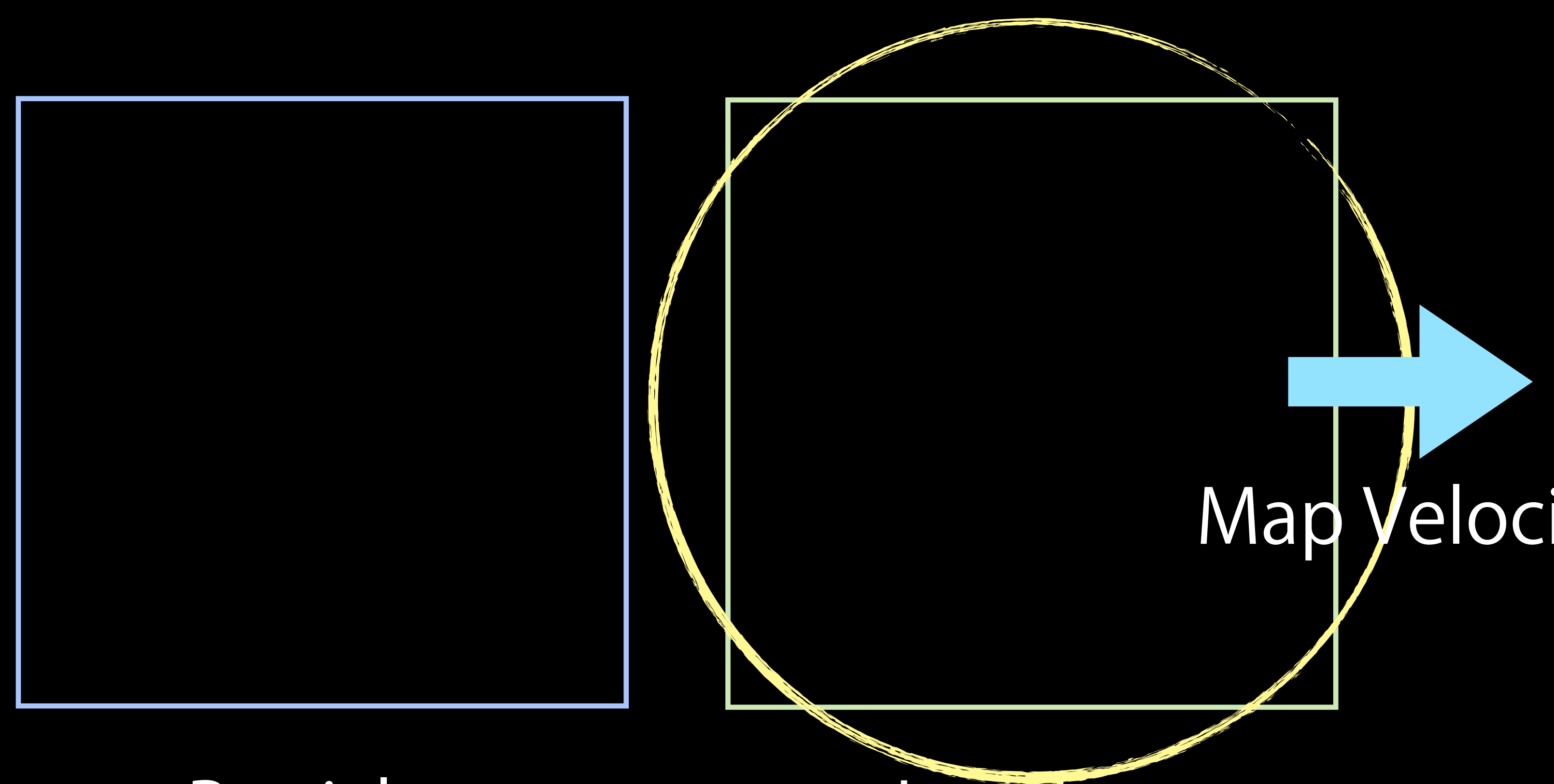
Particles

Levelset

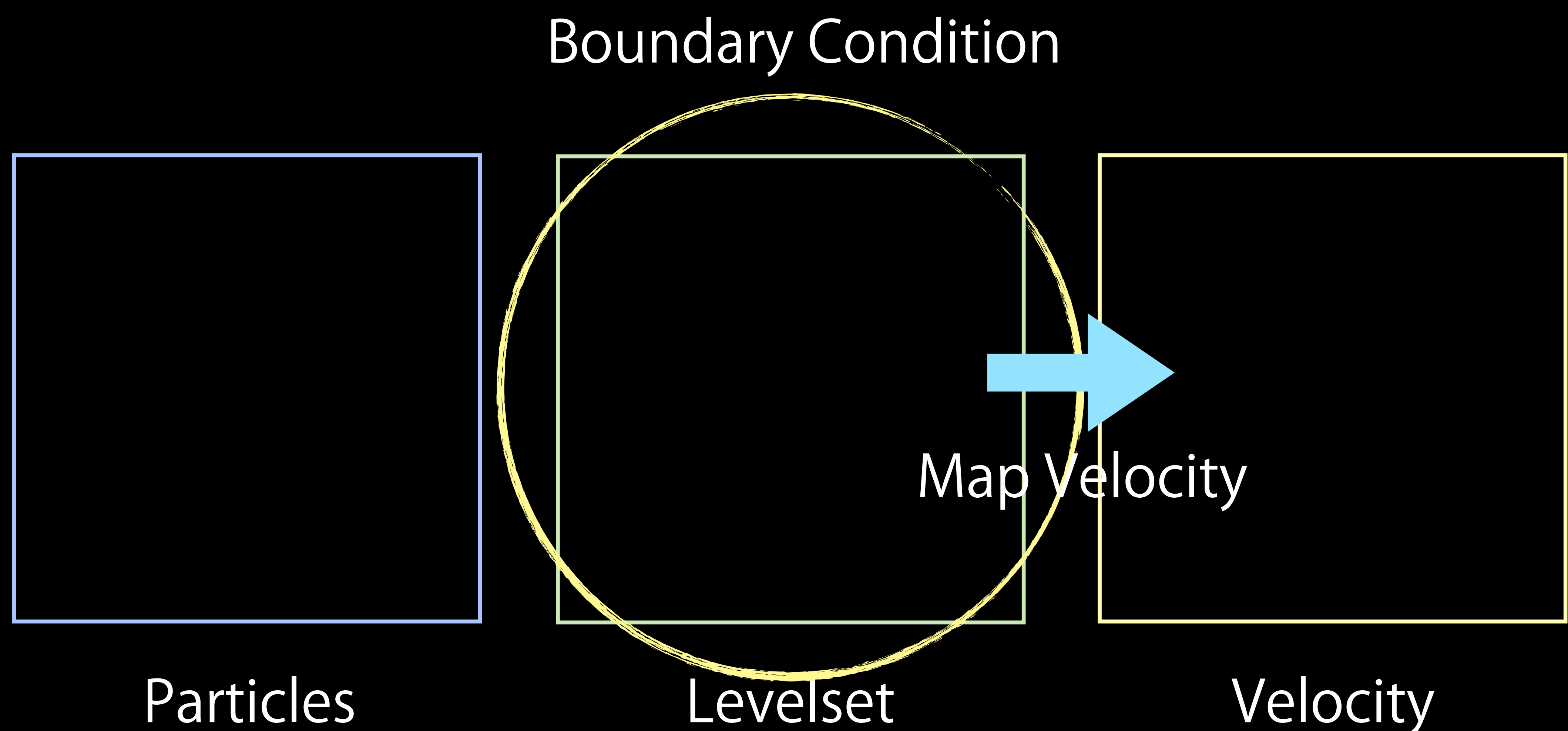
FLIP Solver



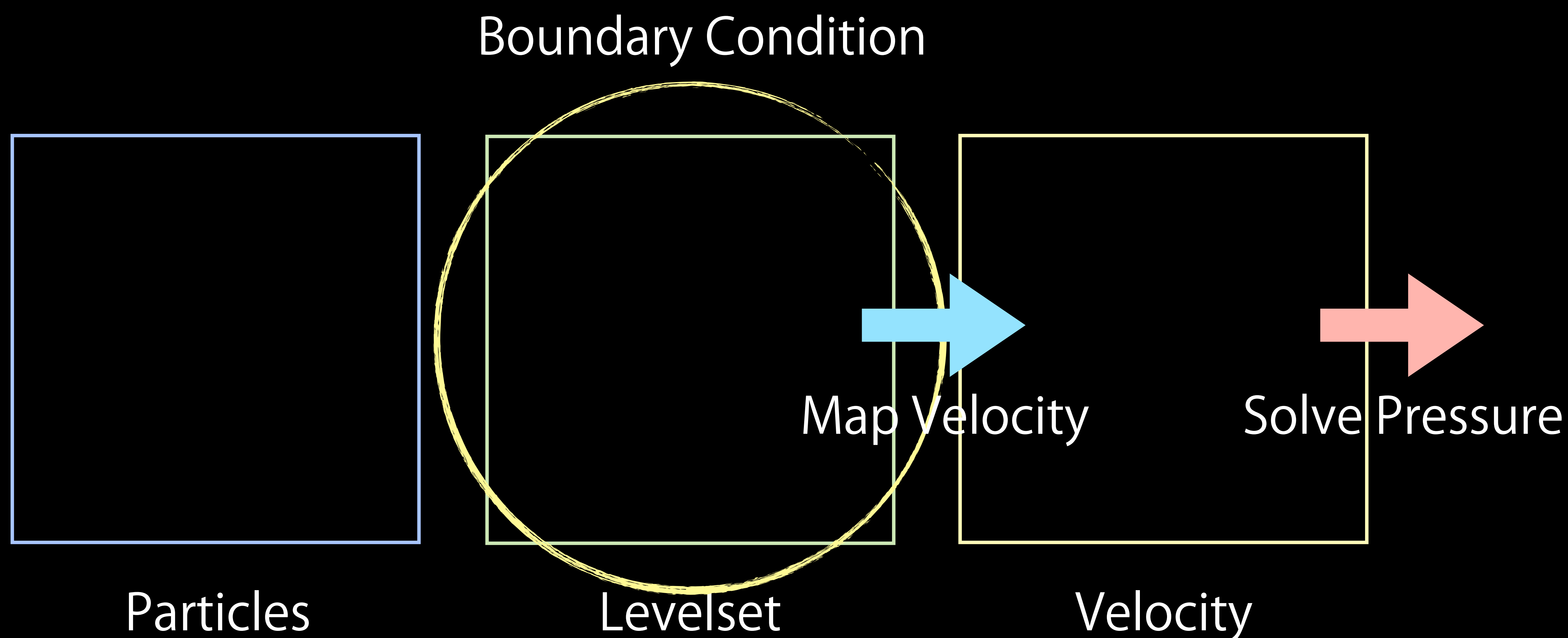
FLIP Solver



FLIP Solver

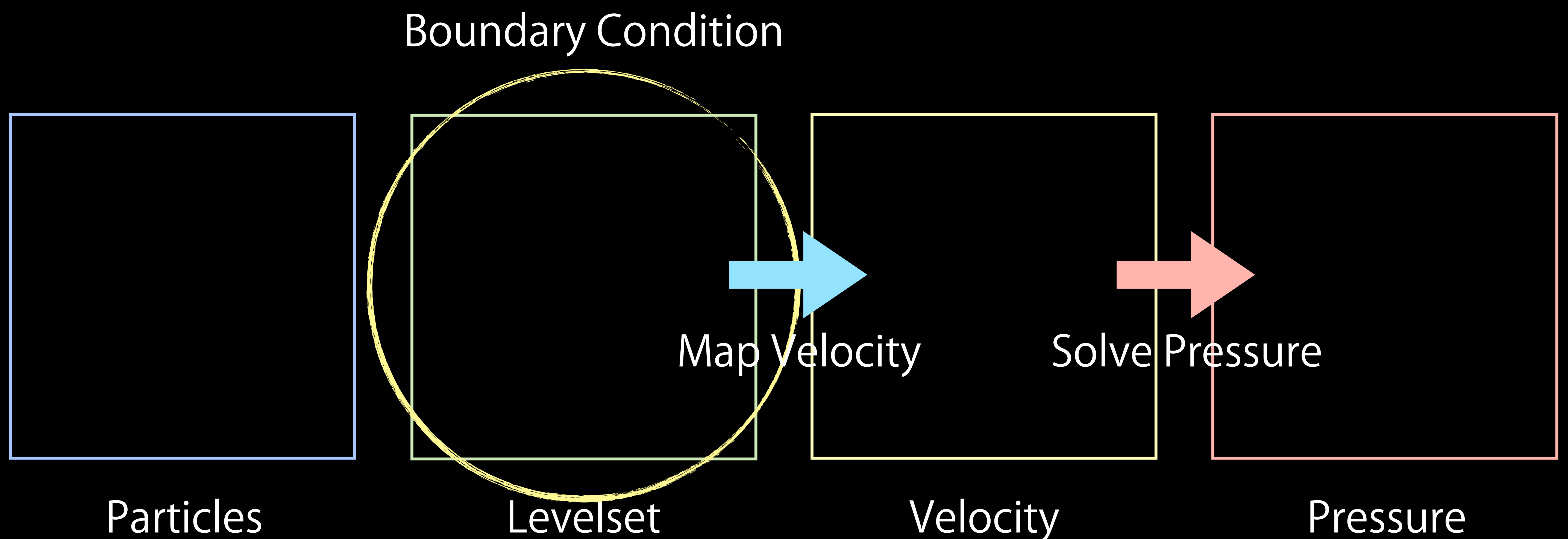

Boundary Condition
Map Velocity
Particles
Levelset
Velocity

FLIP Solver



Boundary Condition
Map Velocity
Solve Pressure
Particles
Levelset
Velocity

FLIP Solver


Boundary Condition
Map Velocity
Solve Pressure
Particles
Levelset
Velocity
Pressure

FLIP Solver

FLIP Solver

- Easy advection

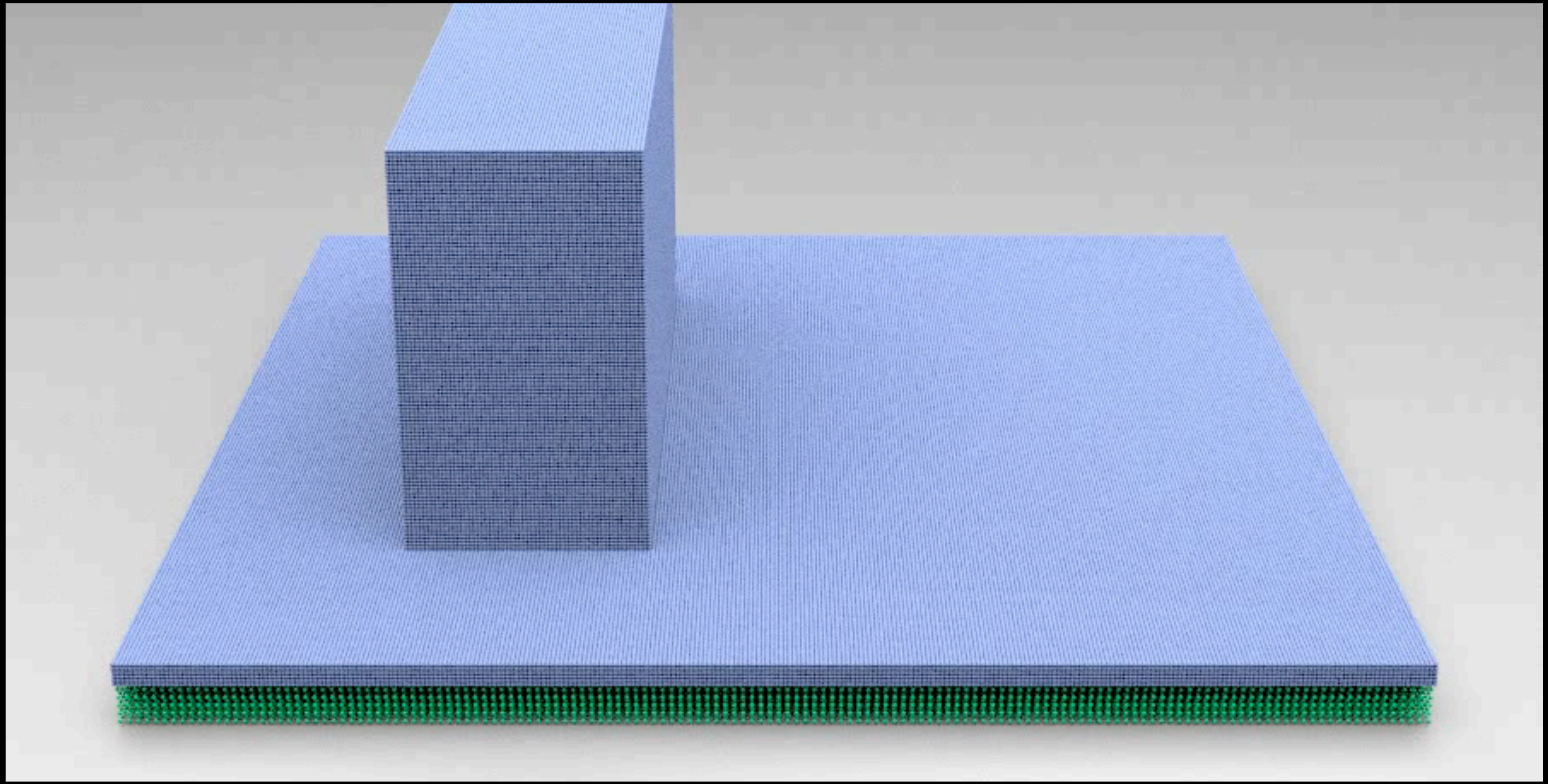
FLIP Solver

- Easy advection
- Fast pressure solver

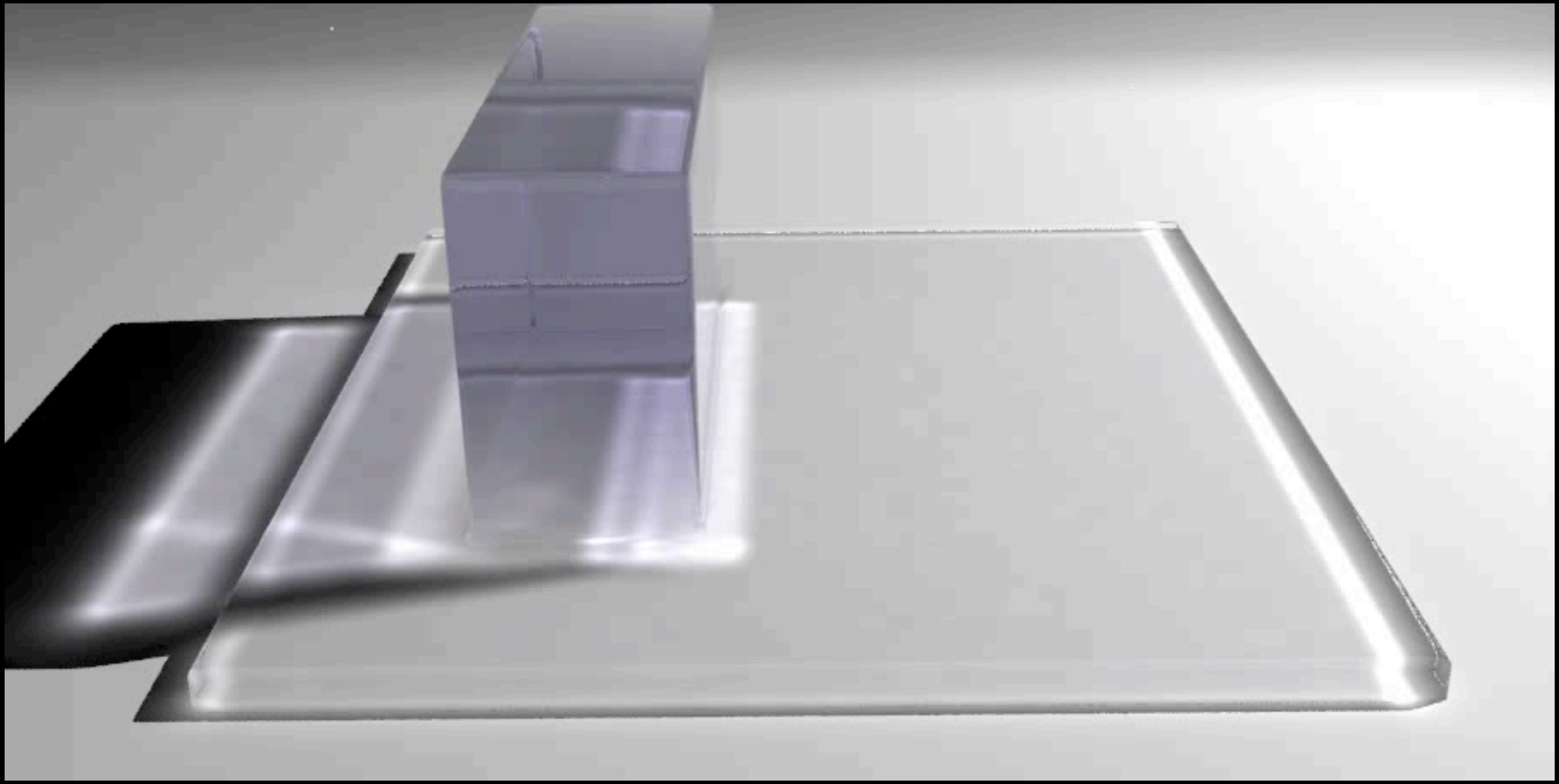
FLIP Solver

- Easy advection
- Fast pressure solver
- Splash !

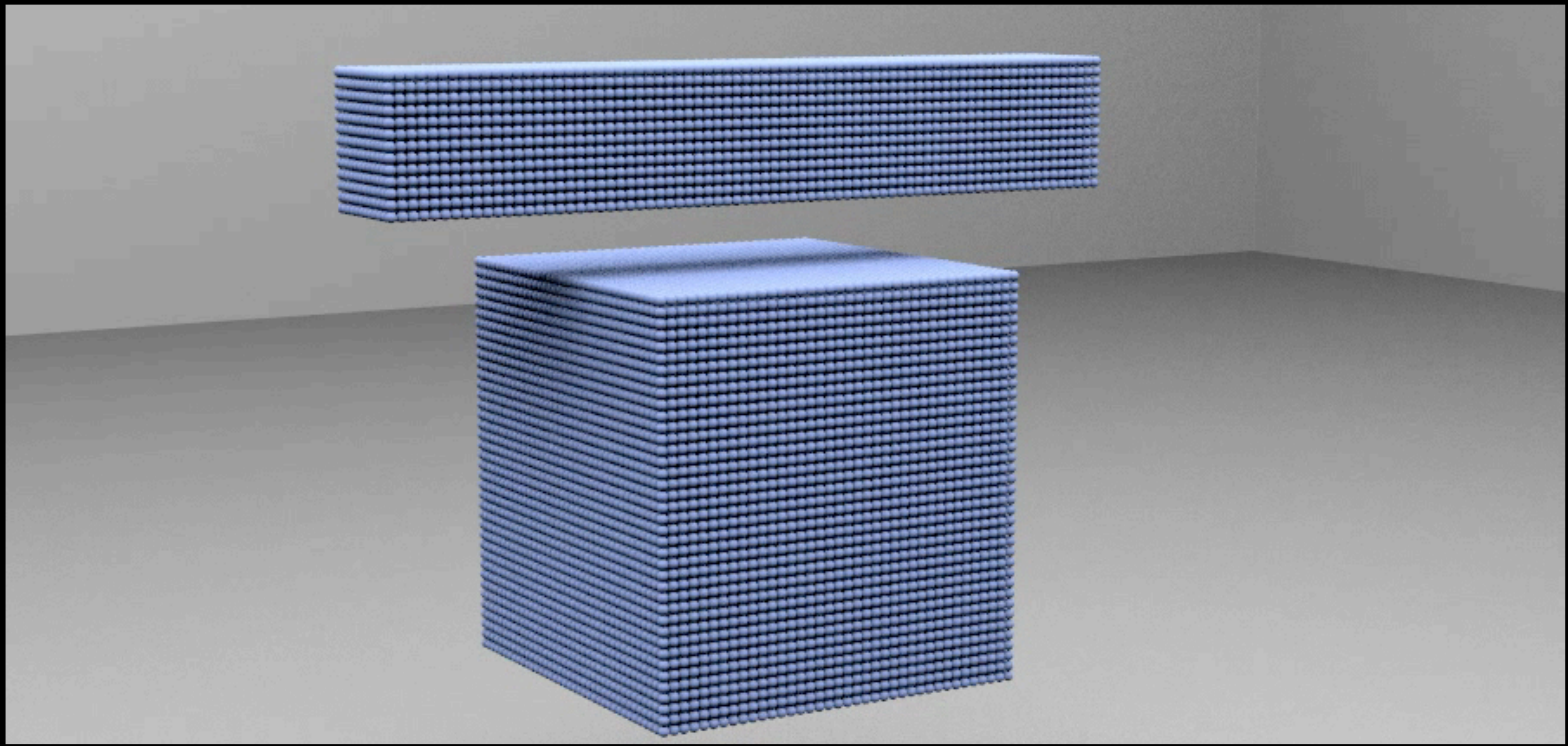
FLIP Solver



FLIP Solver

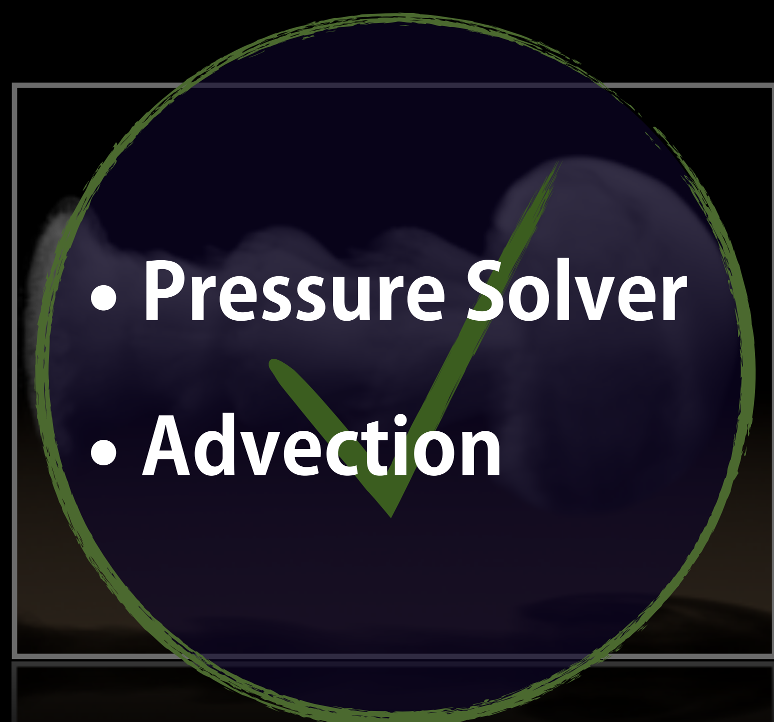


FLIP Solver



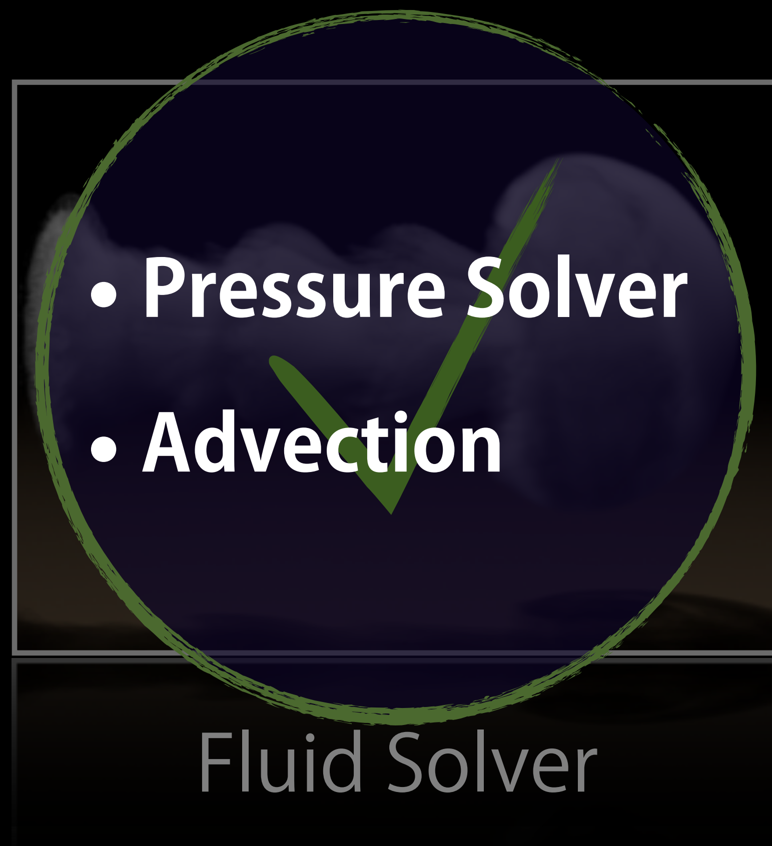
Summary

Summary

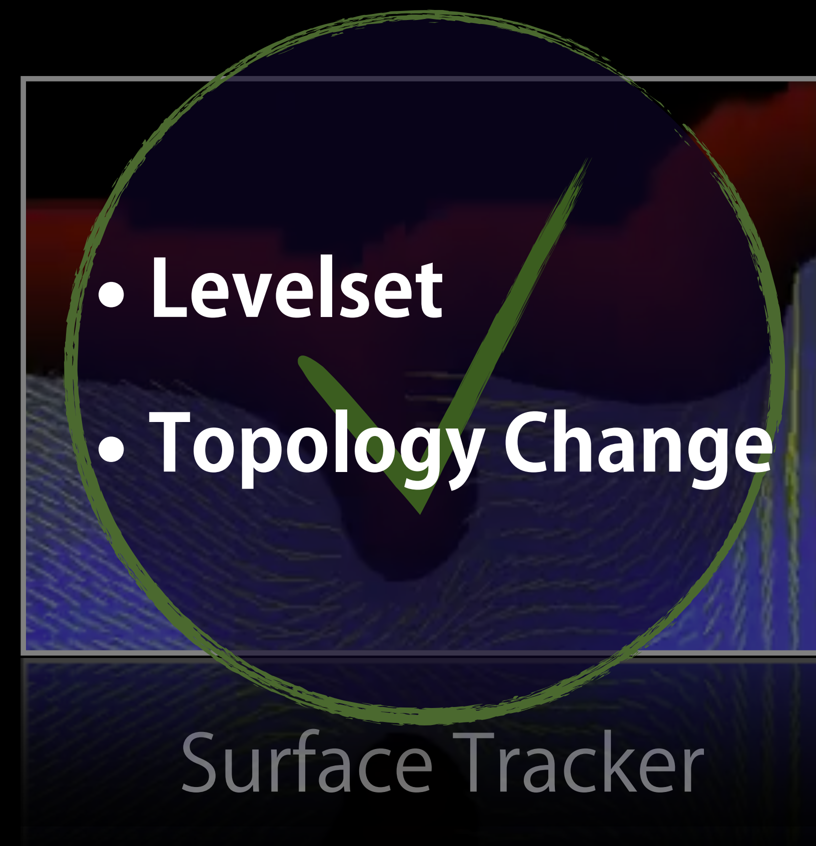


Fluid Solver

Summary

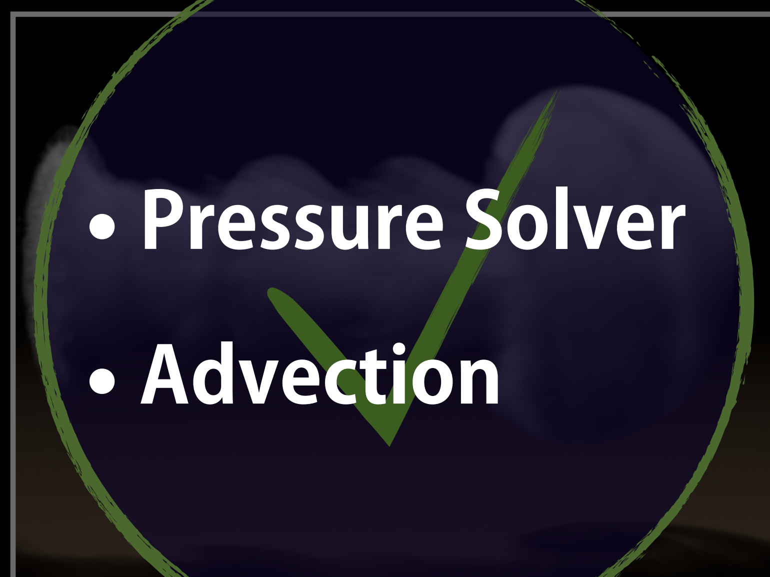


Fluid Solver



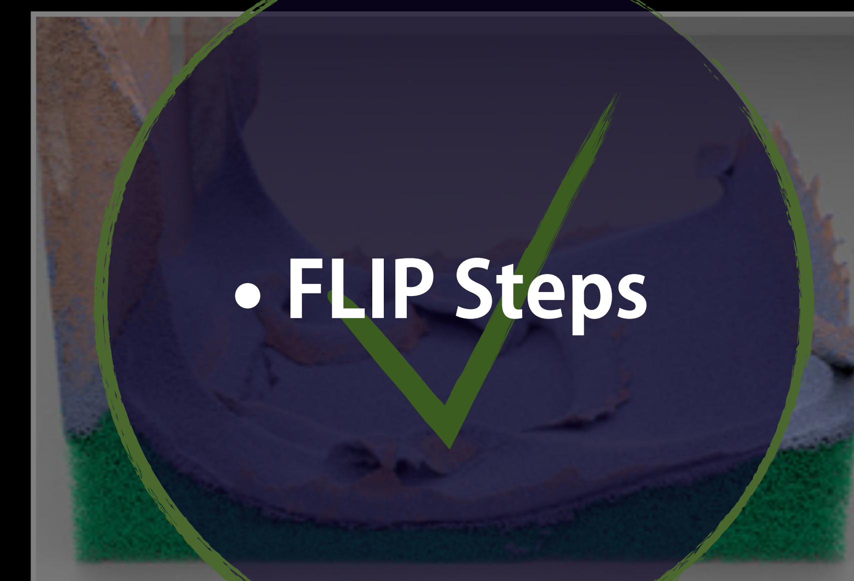
Surface Tracker

Summary



Fluid Solver

Surface Tracker



FLIP Solver