Introduction to Computer Graphics

Toshiya Hachisuka

Last Time

- Introduction to ray tracing
- Intersection algorithms with a ray
  - Sphere
  - Implicit surface
  - Triangle
- GLSL Sandbox

Last Time

```
for all pixels {
    ray = generate_camera_ray( pixel )
    for all objects {
        hit = intersect( ray, object )
        if “hit” is closer than “first_hit” {first_hit = hit}
    }
    pixel = shade( first_hit )
 }
```

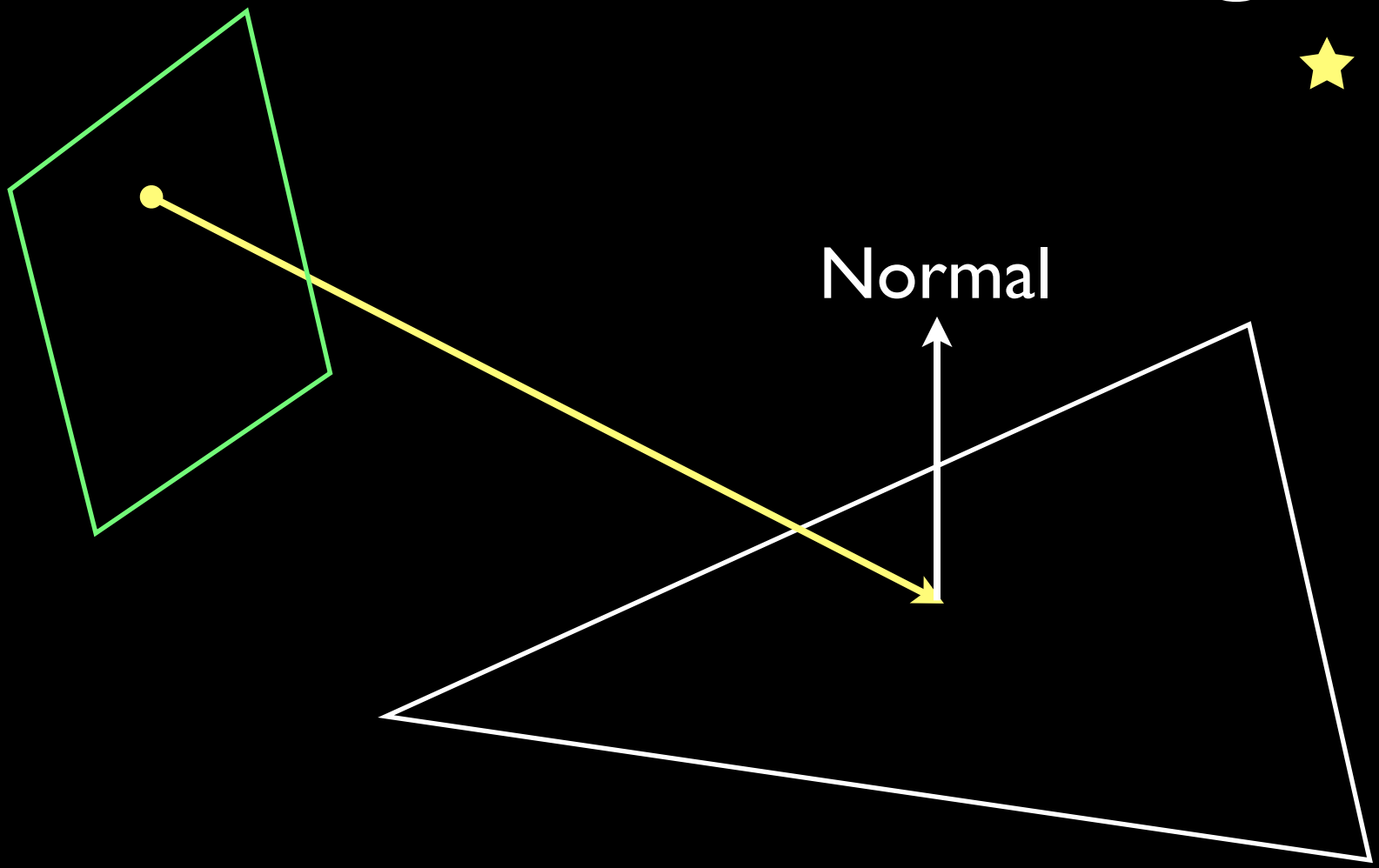
Today

```
for all pixels {
    ray = generate_camera_ray( pixel )
    for all objects {
        hit = intersect( ray, object )
        if “hit” is closer than “first_hit” {first_hit = hit}
    }
    pixel = shade( first_hit )
 }
```

Today

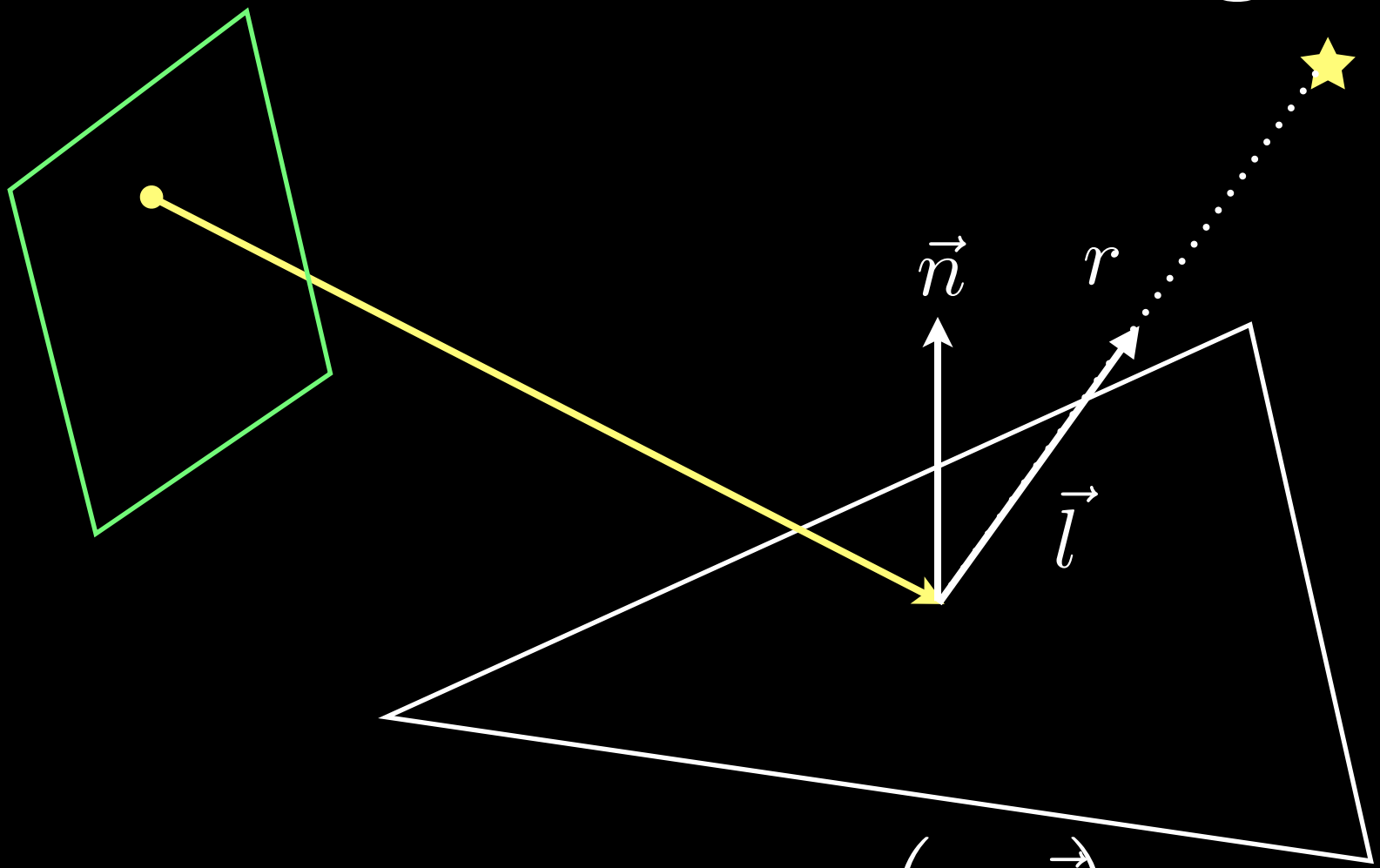
- Shading models
  - BRDF
  - Lambertian
  - Specular
- Simple lighting calculation
- Tone mapping

Basic Shading



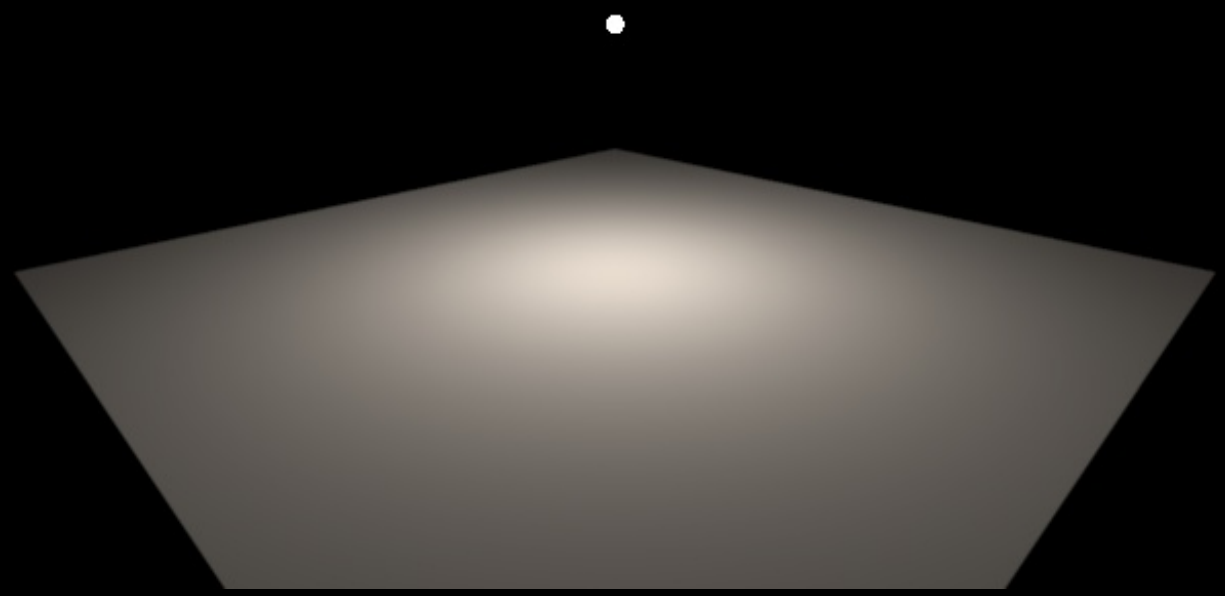
How much light illuminate this point?

Basic Shading



Irradiance $E = \dfrac{\Phi\left(\vec{n}\cdot\vec{l}\right)}{4\pi r^2}$

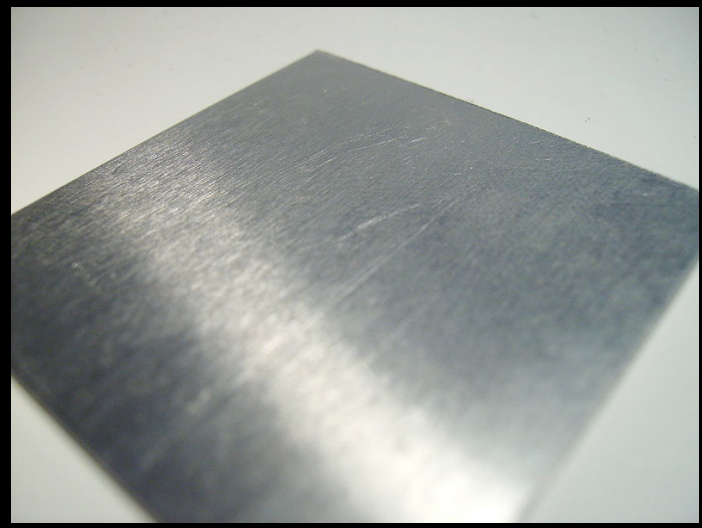
Basic Shading



Irradiance $E = \dfrac{\Phi \left(\vec{n} \cdot \vec{l} \right)}{4\pi r^2}$

Surface Appearance

- How is light reflected by
  - Mirror
  - Paper
  - Rough metallic surface

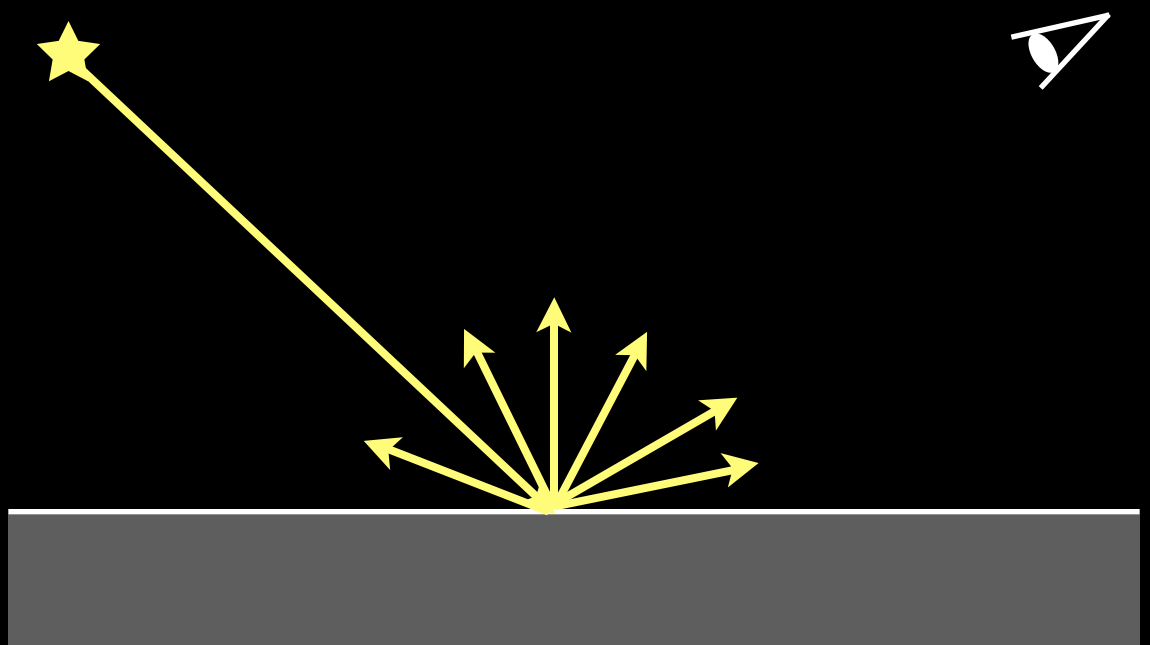


Types of Reflections

- Diffuse: matte paint, paper
- Glossy: plastic, rough metallic surface
- Specular: mirror
- Retro-reflective: the moon

BRDF

- Bidirectional Reflectance Distribution Function
  - How light is reflected off a surface
  - Capture appearance of surface



BRDF

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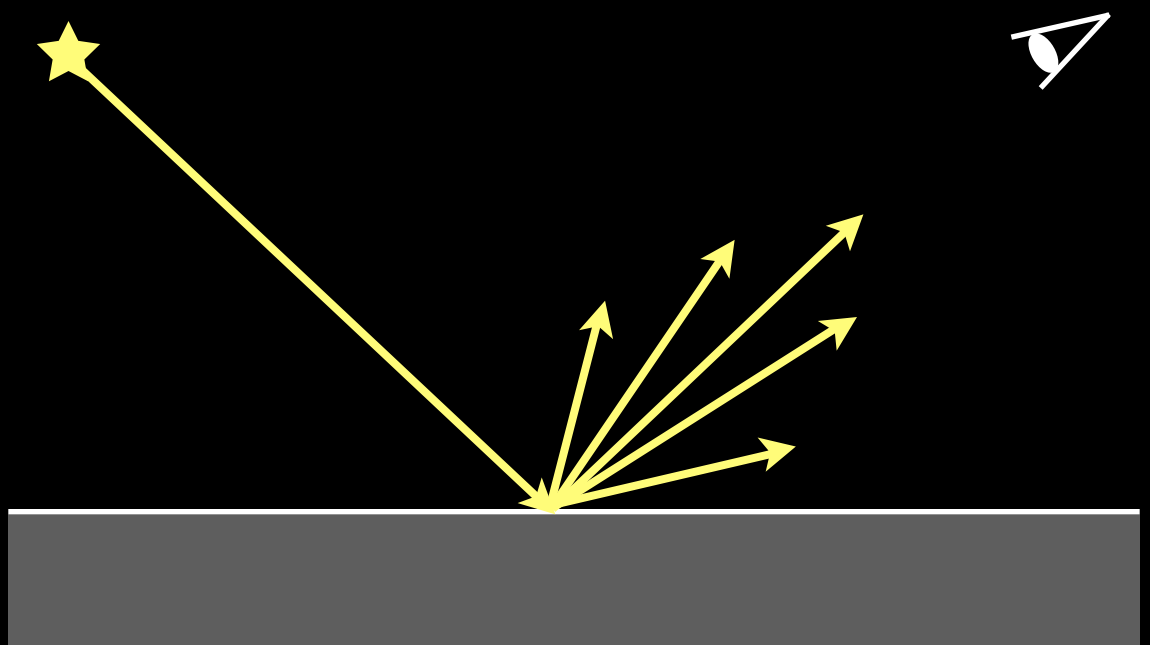
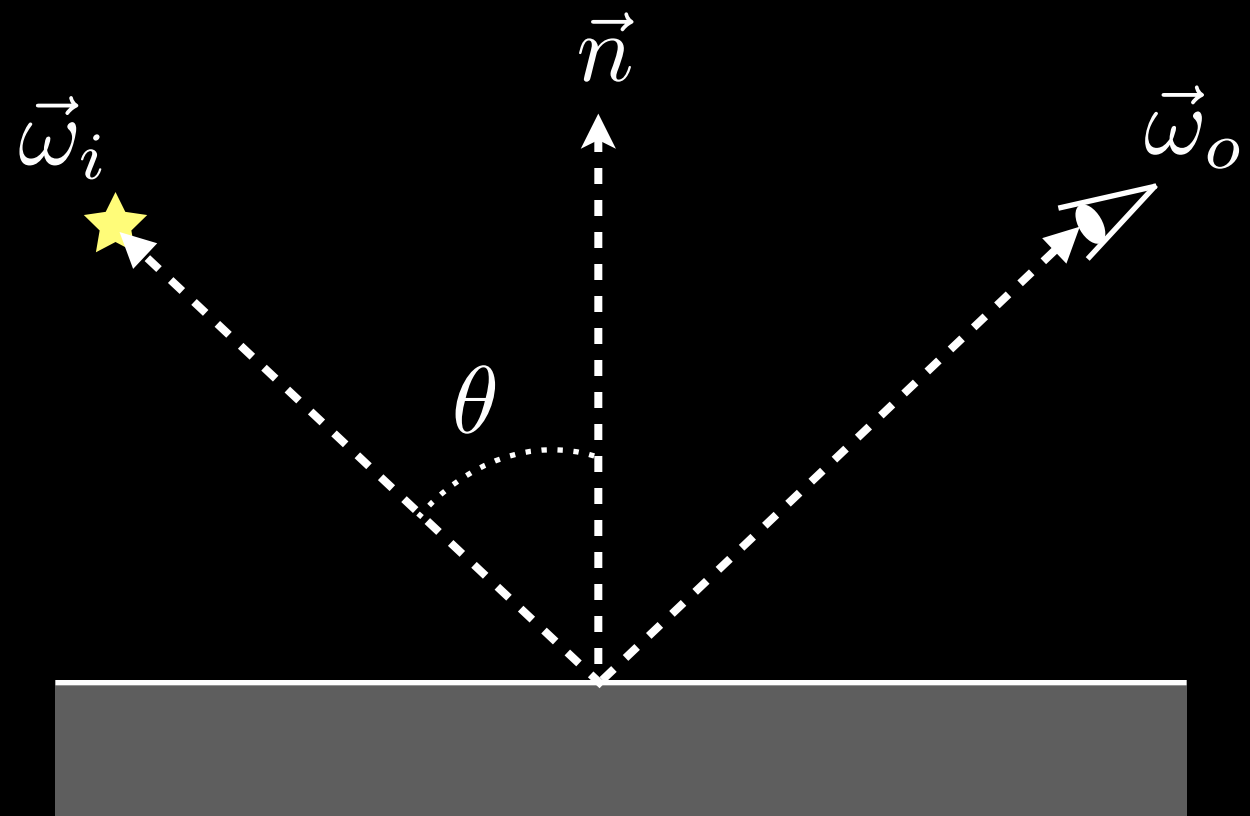


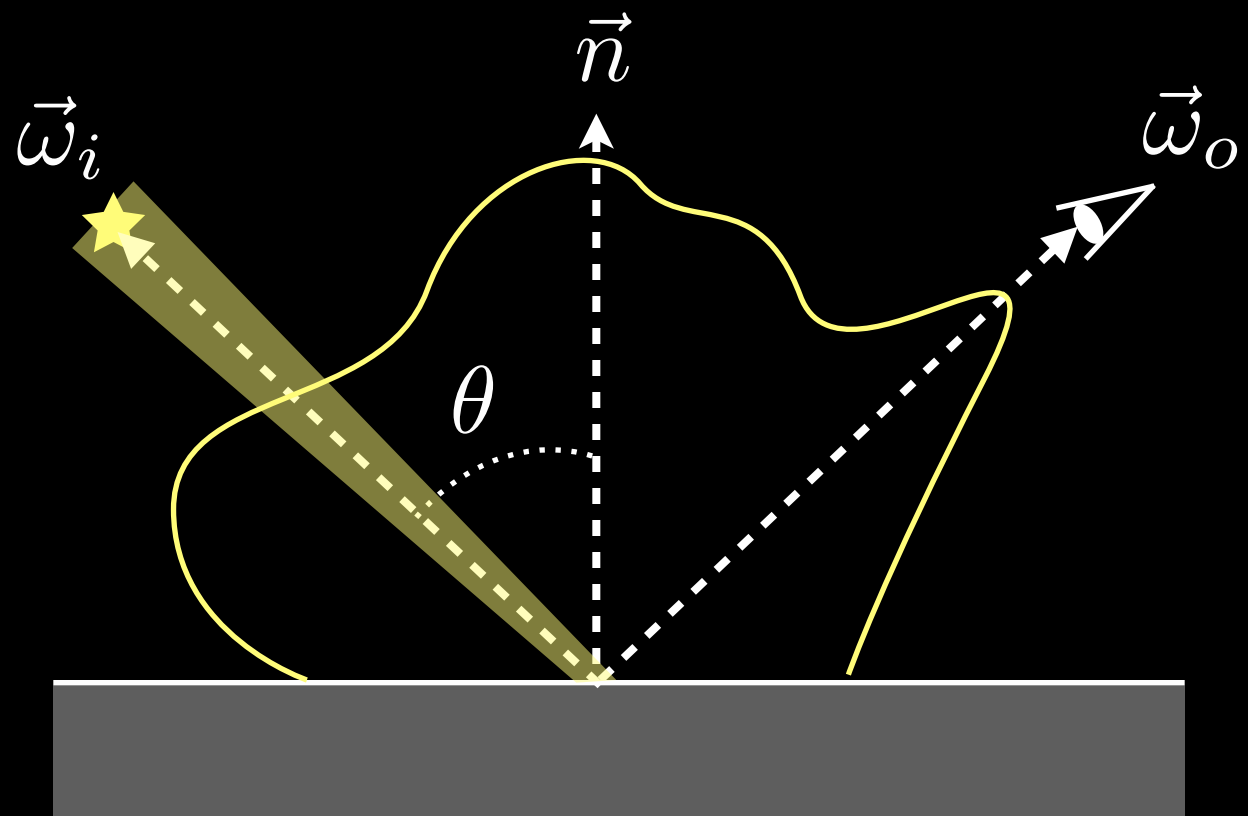
BRDF



BRDF

$$f(x, \vec{\omega}_o, \vec{\omega}_i) = \frac{dL(x, \vec{\omega}_o)}{dE(x, \vec{\omega}_i)}$$

$$dE(x, \vec{\omega}_i) = L(x, \vec{\omega}_i) \cos\theta d\omega_i$$



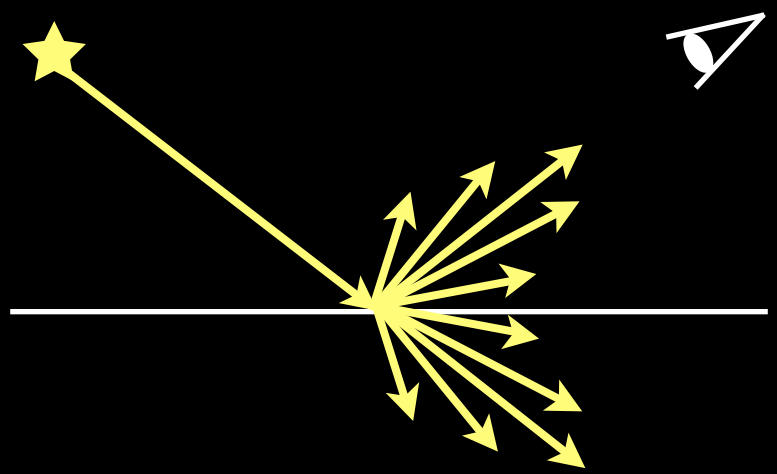
Reflected Radiance

- Sum of contributions from all directions

$$
\begin{aligned}
L_o(x, \vec{\omega}_o) &= \int_\Omega dL_o(x, \vec{\omega}_o) \\
&= \int_\Omega f(x, \vec{\omega}_o, \vec{\omega}_i) dE_i(x, \vec{\omega}_i) \\
&= \int_\Omega f(x, \vec{\omega}_o, \vec{\omega}_i) L_i(x, \vec{\omega}_i) \cos\theta d\omega_i
\end{aligned}
$$

BSDF

- Bidirectional Scattering Distribution Function
  - Generalization of BRDF to transparency
  - Defined over all the directions



Defining BRDF

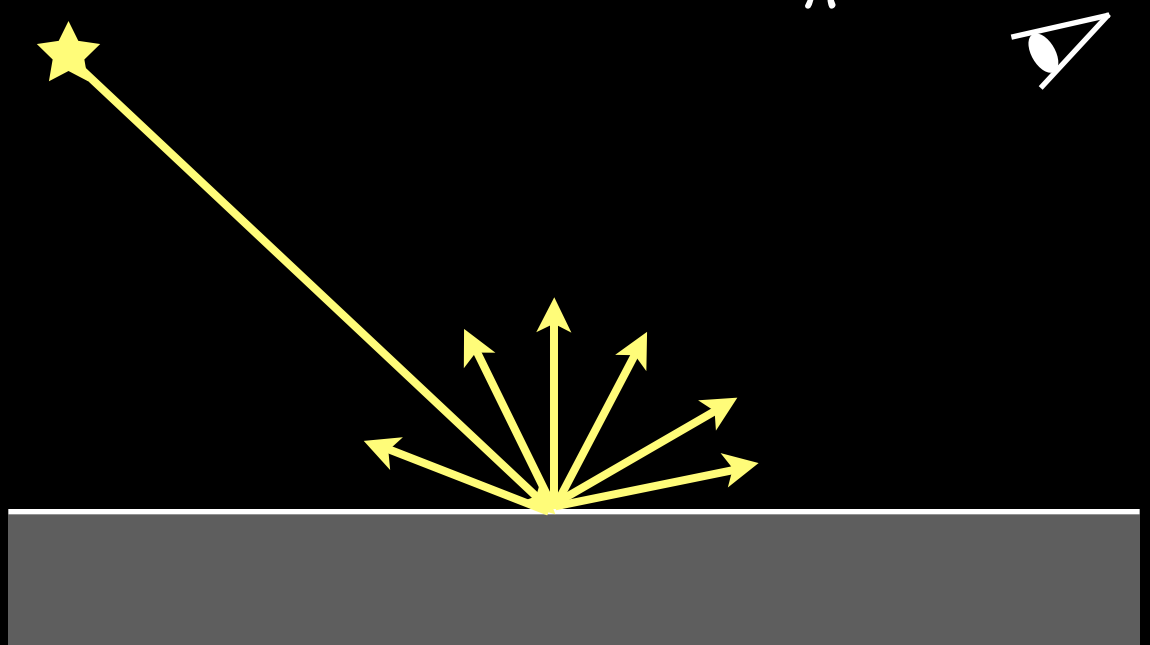
- Two approaches
  - Measurement
  - Analytical models

- We learn simple analytical models

Lambertian

- Uniformly reflects light over all directions
- “Matte” surfaces

$$f(\vec{\omega}_o, \vec{\omega}_i) = \frac{K_d}{\pi}$$



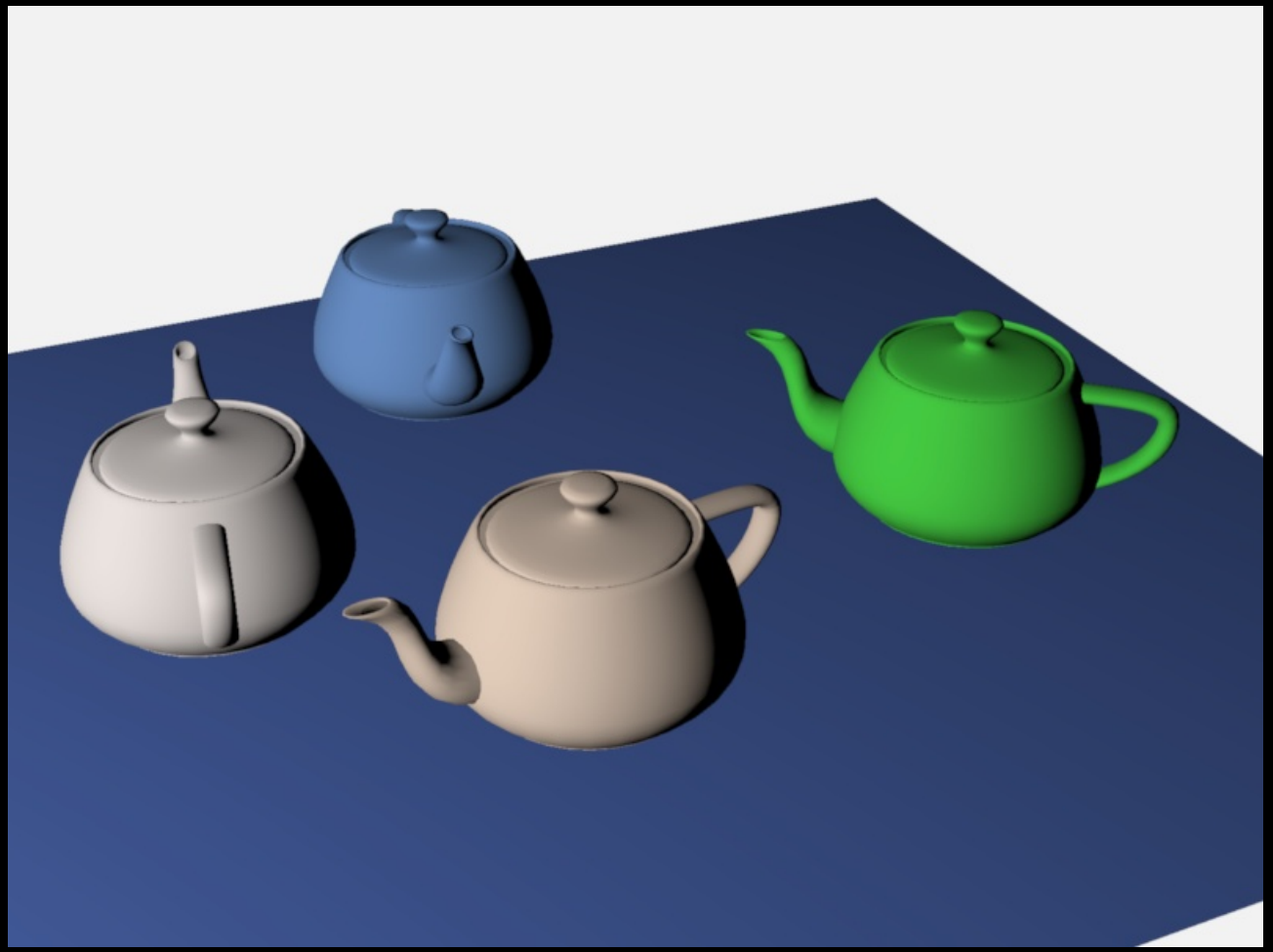
Lambertian

- Irradiance times BRDF = Lambertain shading

$$L_o(x, \vec{\omega}_o) = \int_\Omega dL_o(x, \vec{\omega}_o)$$
$$= \int_\Omega f(x, \vec{\omega}_o, \vec{\omega}_i) dE_i(x, \vec{\omega}_i)$$
$$= \frac{K_d}{\pi} \int_\Omega dE_i(x, \vec{\omega}_i) = \frac{K_d}{\pi} E_i$$

```
color shade (hit) {
  return (Kd / PI) * get_irradiance(hit)
}
```

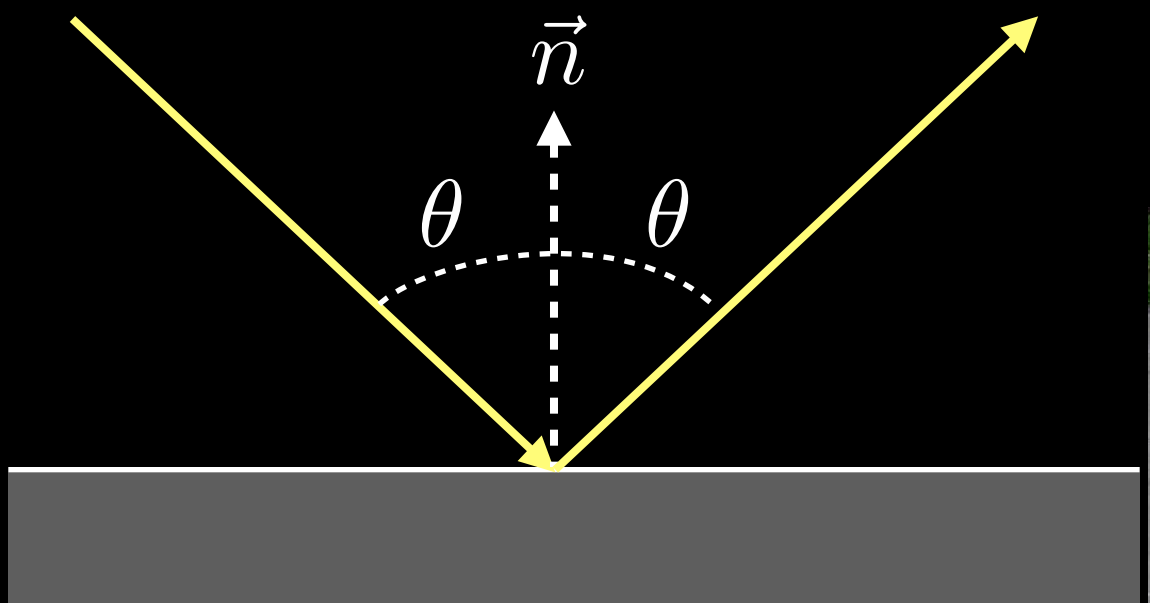
Lambertian - Example



Specular Reflection

- Mirror reflection

$$f(\vec{\omega}_o, \vec{\omega}_i) = \frac{K_s \delta(\vec{\omega}_o, \vec{\omega}_r)}{\cos\theta}$$



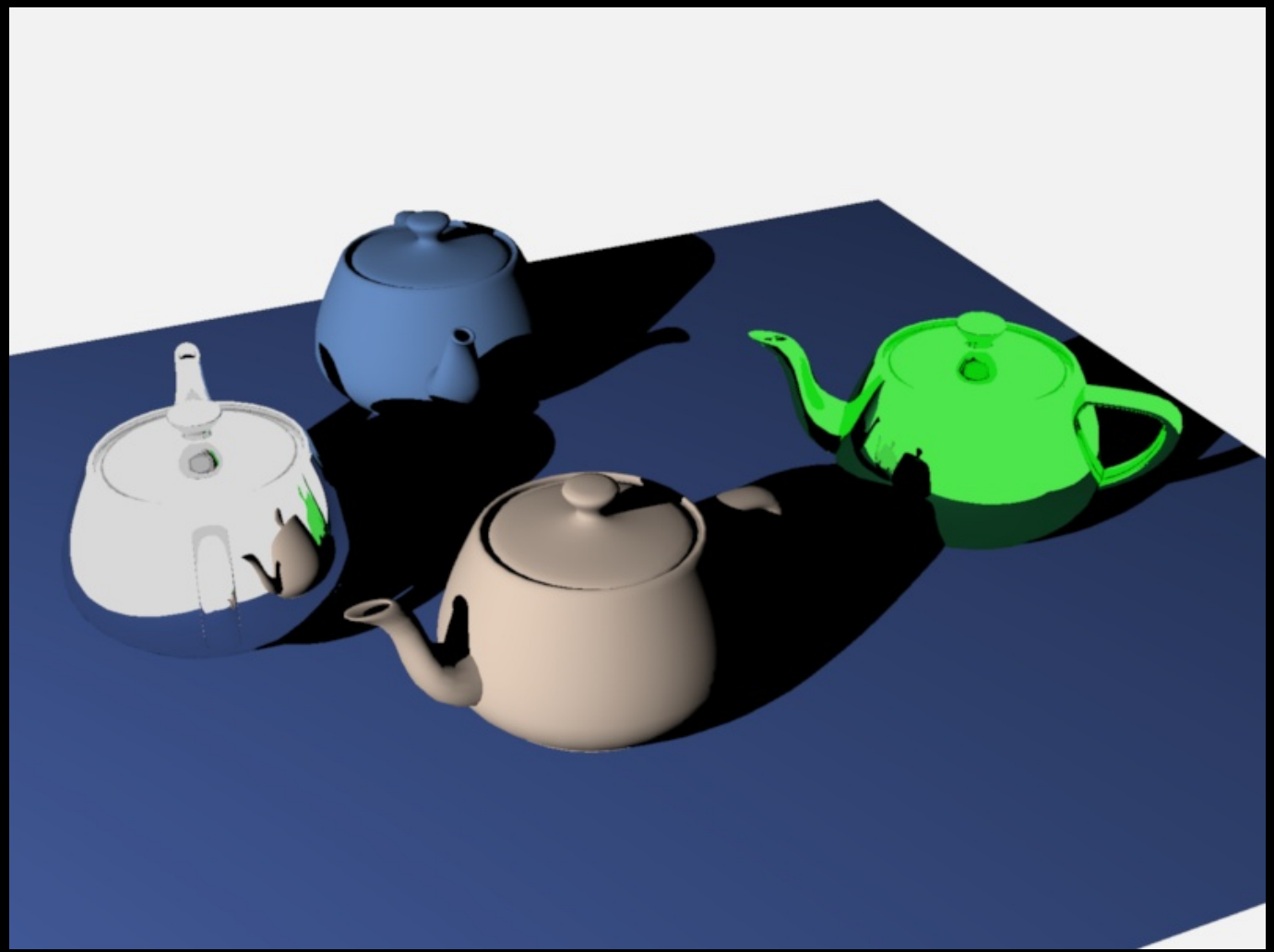
Specular Reflection

- Recursive tracing toward the reflected dir.

$$\vec{\omega}_r = -2(\vec{\omega}_i \cdot \vec{n})\vec{n} + \vec{\omega}_i$$

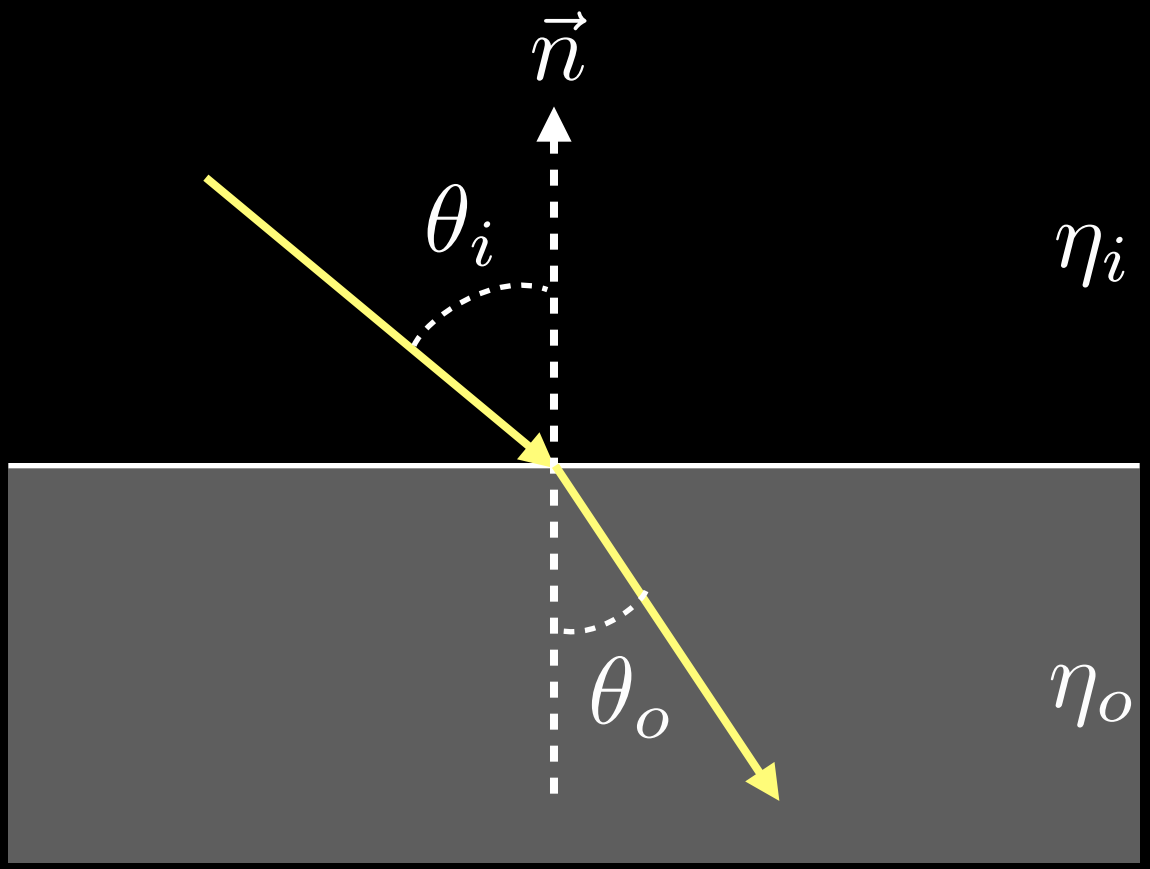
```
color shade (hit) {
   case hit.material
   diffuse:
      return (Kd / PI) * get_irradiance(hit)
   mirror:
      return Ks * shade(trace(reflected_ray))
}
```

Specular Reflection - Example



Specular Refraction

- Glass etc. $$f(\vec{\omega}_o, \vec{\omega}_i) = \frac{K_t \delta(\vec{\omega}_o, \vec{\omega}_t)}{\cos\theta_i}$$



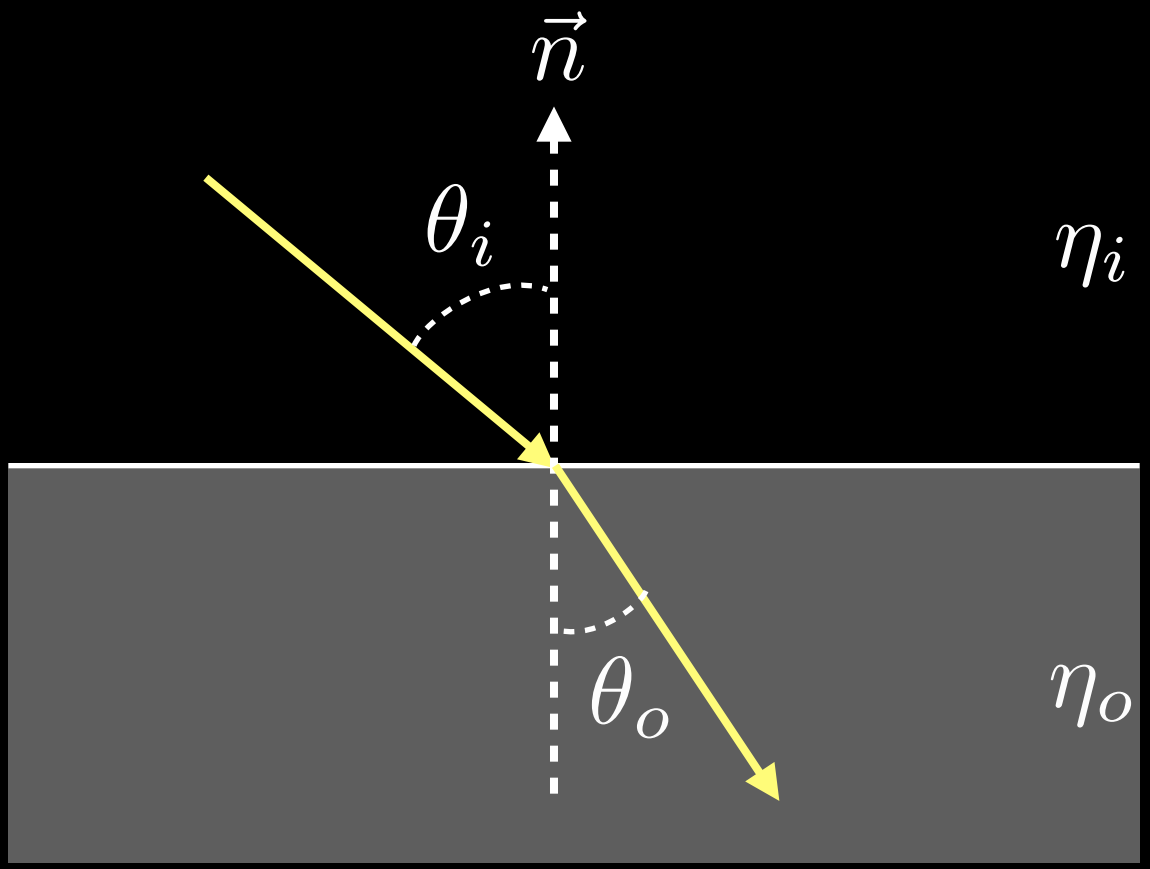
Specular Refraction

- Same as reflection: recursive tracing

```
color shade (hit) {
   case hit.material
   diffuse:
      return (Kd / PI) * get_irradiance(hit)
   mirror:
      return Ks * shade(trace(reflected_ray))
   glass:
      return Kt * shade(trace(refracted_ray))
}
```

Snell's Law

$$\eta_i \sin\theta_i = \eta_o \sin\theta_o$$

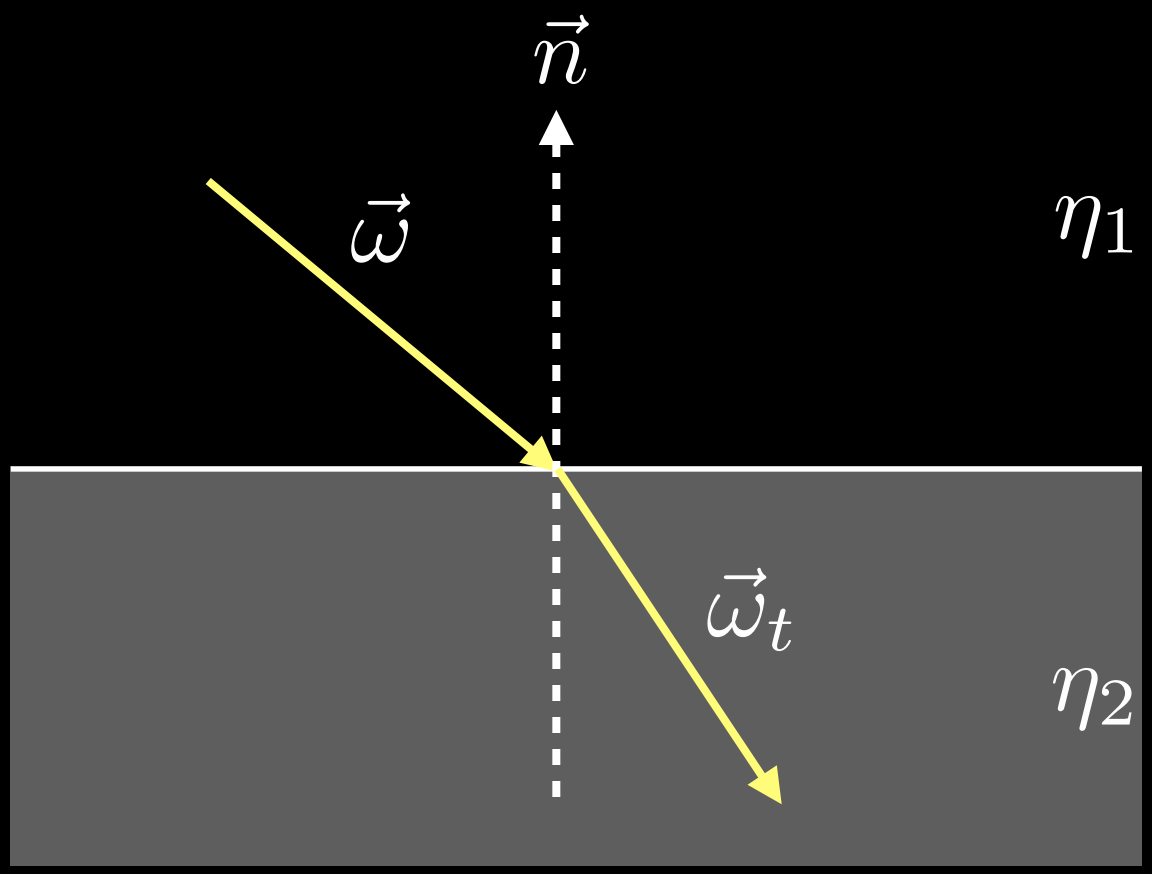


Index of Refraction

- Some values of η

Vacuum	1.0
Air	1.00029
Ice	1.31
Water	1.33
Crown glass	1.52 - 1.65
Diamond	2.417

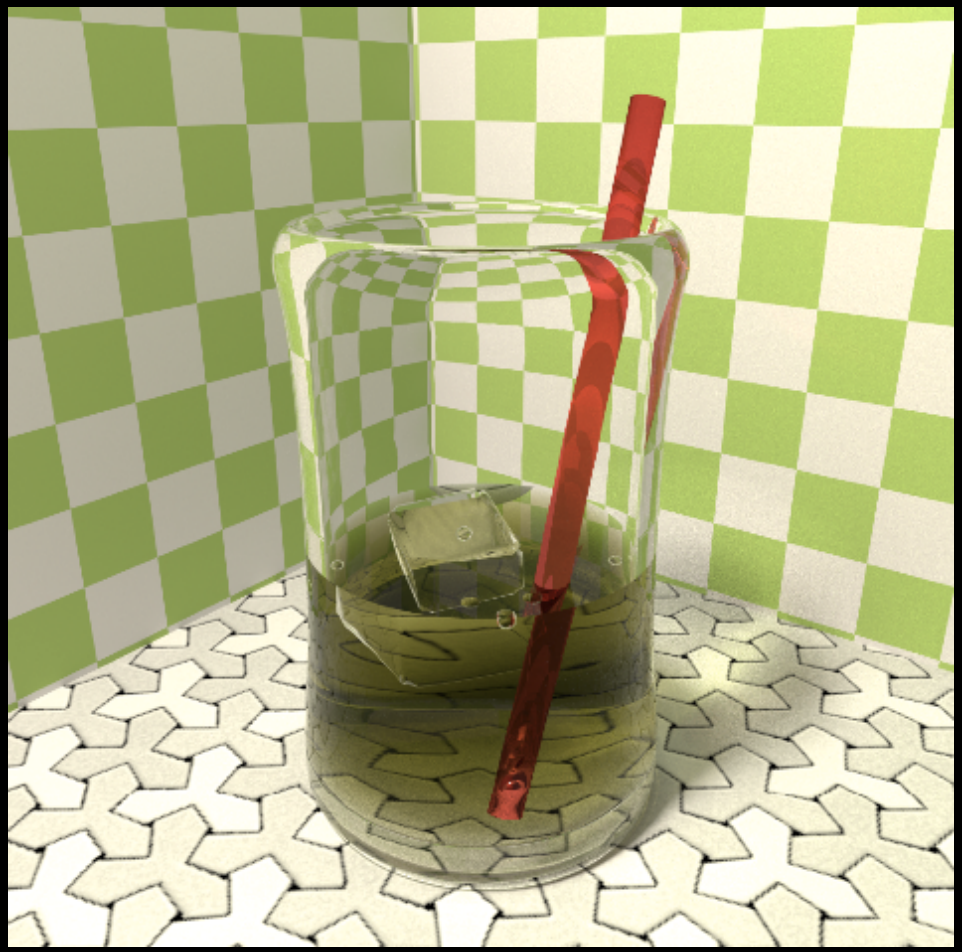
Refracted Direction



$$\vec{\omega}_t = \frac{\eta_1}{\eta_2}(\vec{\omega} - (\vec{\omega}\cdot\vec{n})\vec{n}) - \left(\sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2 (1 - (\vec{\omega}\cdot\vec{n})^2)}\right)\vec{n}$$

Refracted Direction

Be aware of what object ray is entering and existing!

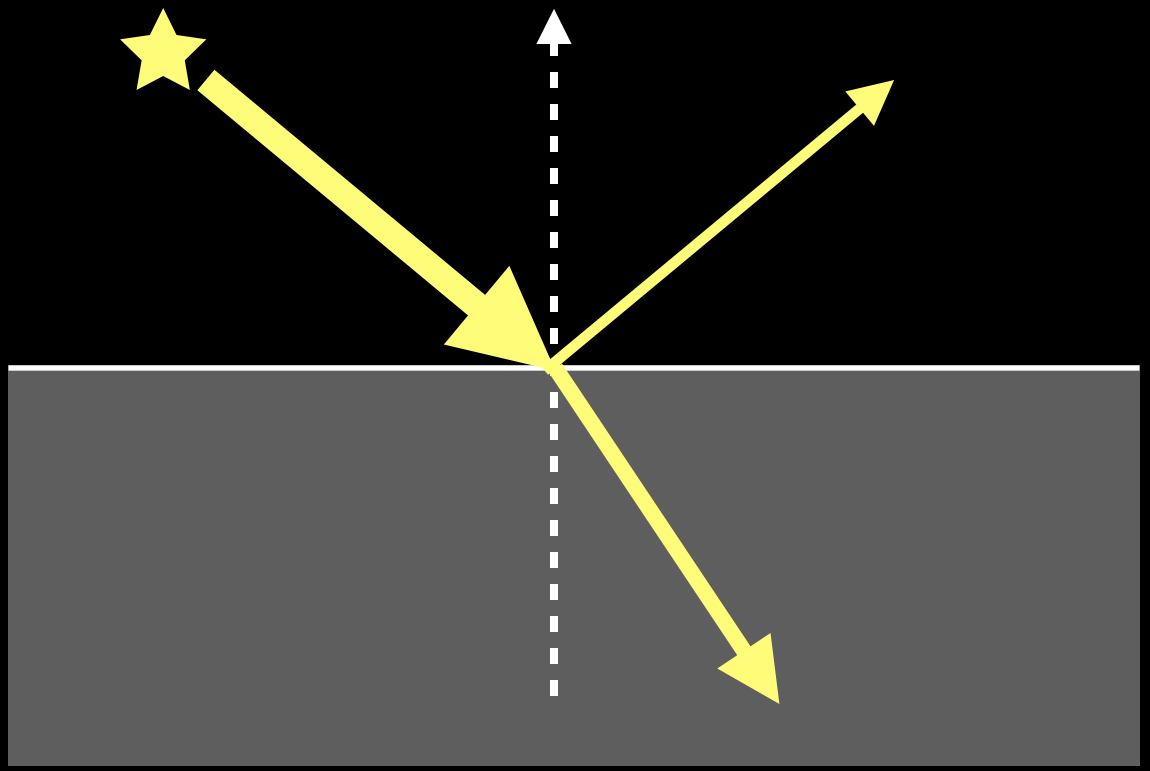


Stack-based solution

http://graphics.cs.ucdavis.edu/~bcbudge/deep/research/nested_dielectrics.pdf

Fresnel Reflection

- Both reflection and refraction occur at interface



Fresnel Reflection

- Based on Fresnel equations

$$\rho_s = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_o}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_o}$$

$$\rho_t = \frac{\eta_1 \cos\theta_o - \eta_2 \cos\theta_i}{\eta_1 \cos\theta_o + \eta_2 \cos\theta_i}$$

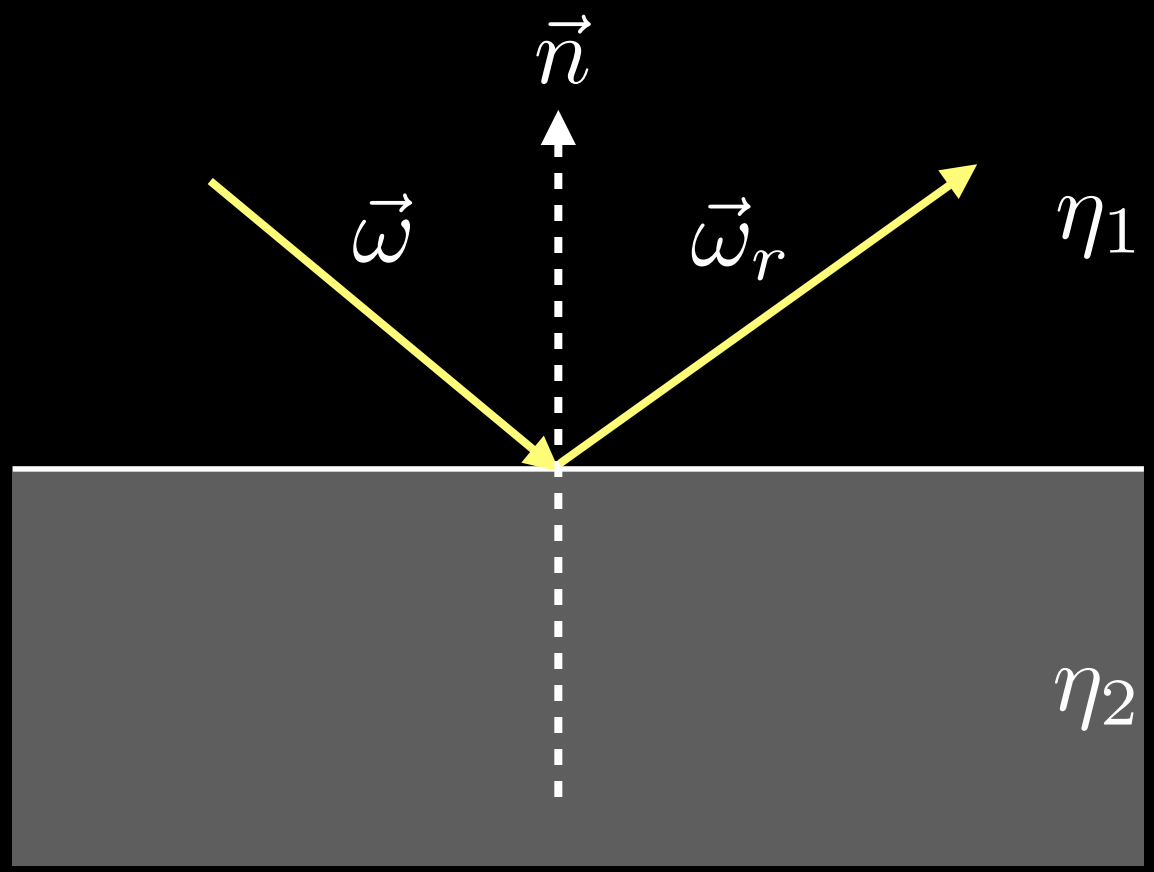
$$F(\theta_o, \theta_i) = \frac{1}{2}\left(\rho_s^2 + \rho_t^2\right)$$

Fresnel Reflection

- Recursive call with branch

```
glass:
   R = Fresnel(hit, ray);
   return R * shade(trace(reflected_ray))
   + (1 - R) * shade(trace(refracted_ray));
```

Total Internal Reflection



Trace only reflected ray with K_s = 1 if

$$1 - \left(\frac{\eta_1}{\eta_2}\right)^2 (1 - (\vec{\omega} \cdot \vec{n})^2) < 0$$ (inside of the sqrt)

Complete Glass

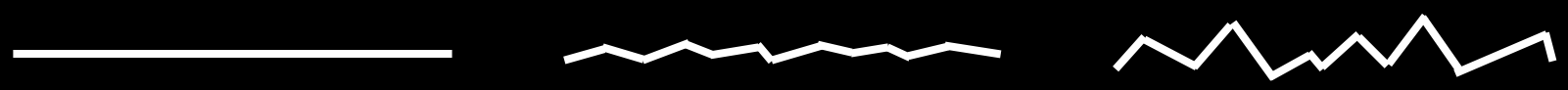
- Also known as dielectric materials
- Be careful about ray and normal directions

```
glass:
  if (TotalInternal) return shade(trace(reflected_ray))

  R = Fresnel(hit, ray);
  return R * shade(trace(reflected_ray))
  + (1 - R) * shade(trace(refracted_ray));
```

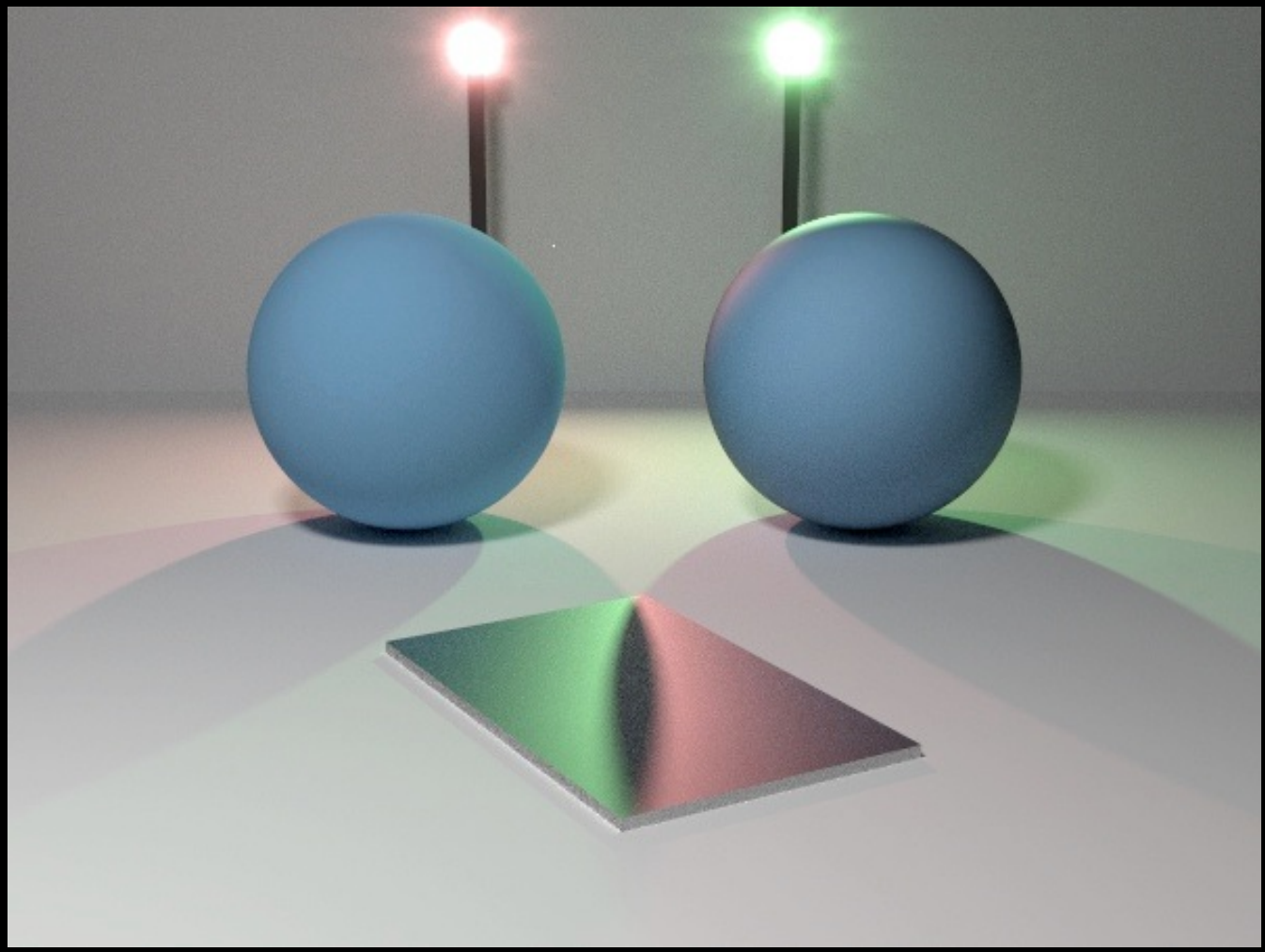
Other BRDFs

- Mircofacets model
  - Lots of tiny mirrors at random orientations
  - Distribution of orientations of microfacets decides the sharpness of reflection
  - e.g., Cook-Torrance model



Specular ⟶ Glossy

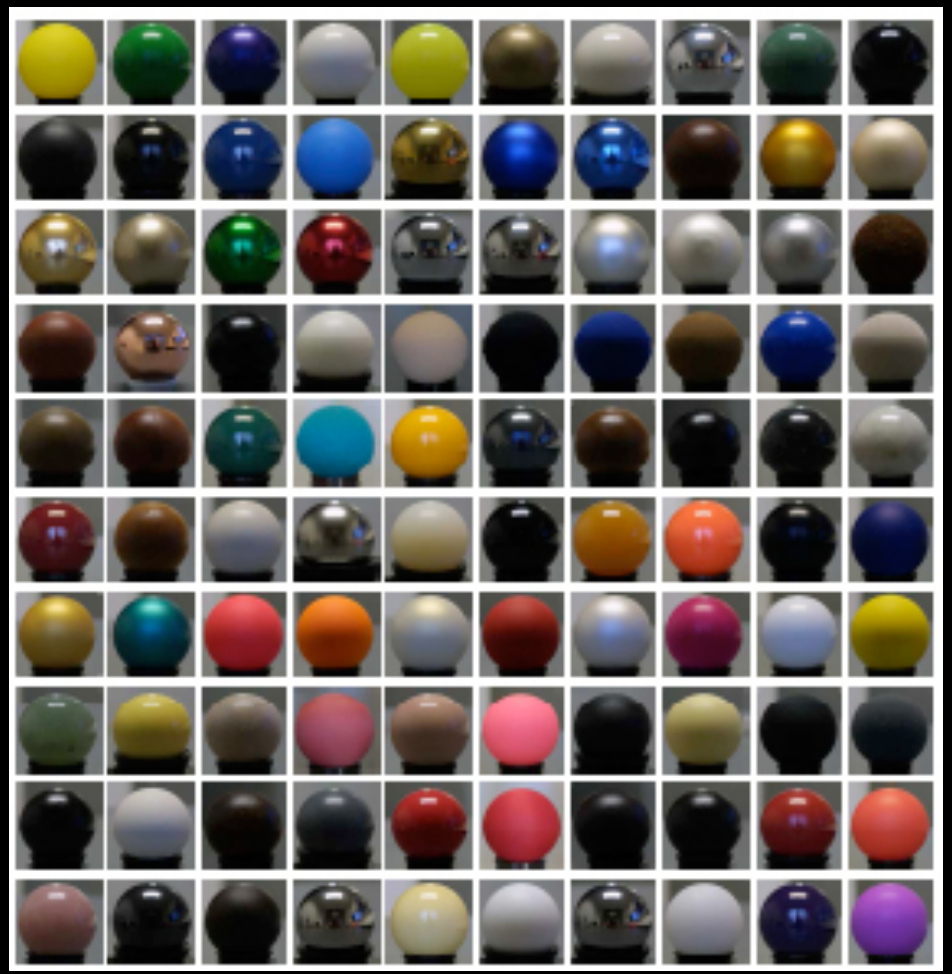
Other BRDFs



[Ashikmin & Shirley]

Other BRDFs

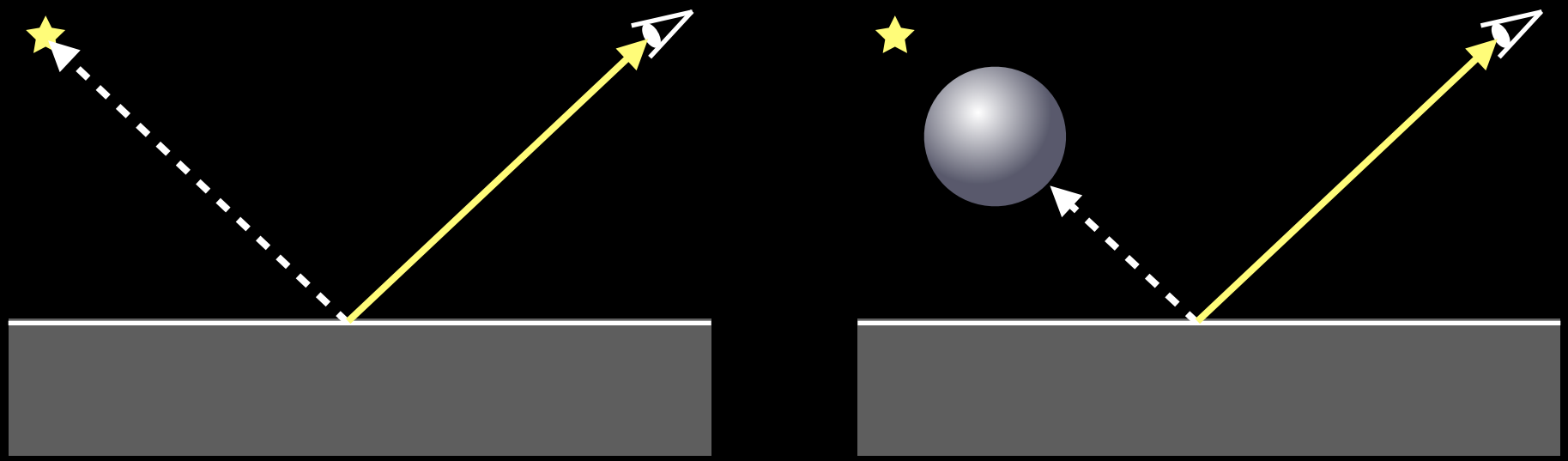
- Measured BRDFs



MERL BRDF Database

Shadows

- Return irradiance only if the light is visible

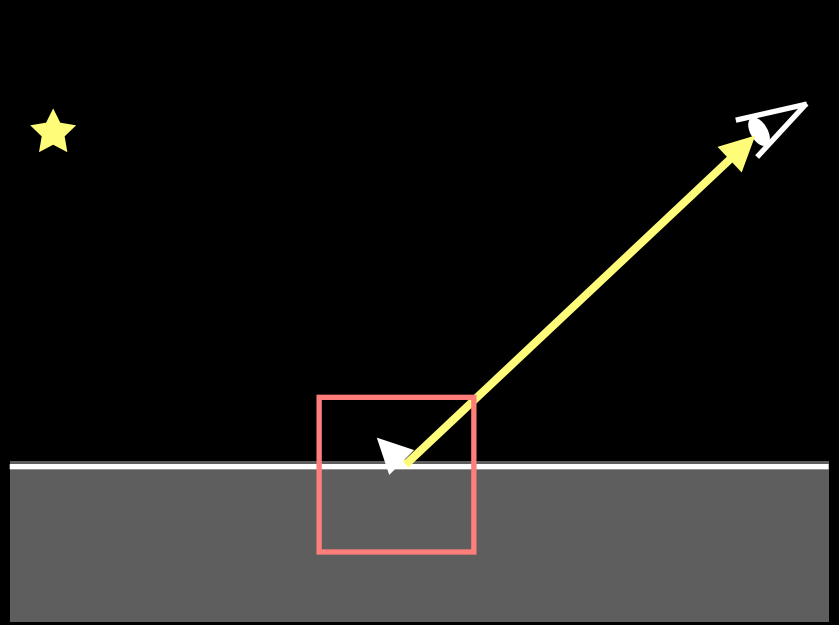


Illuminated

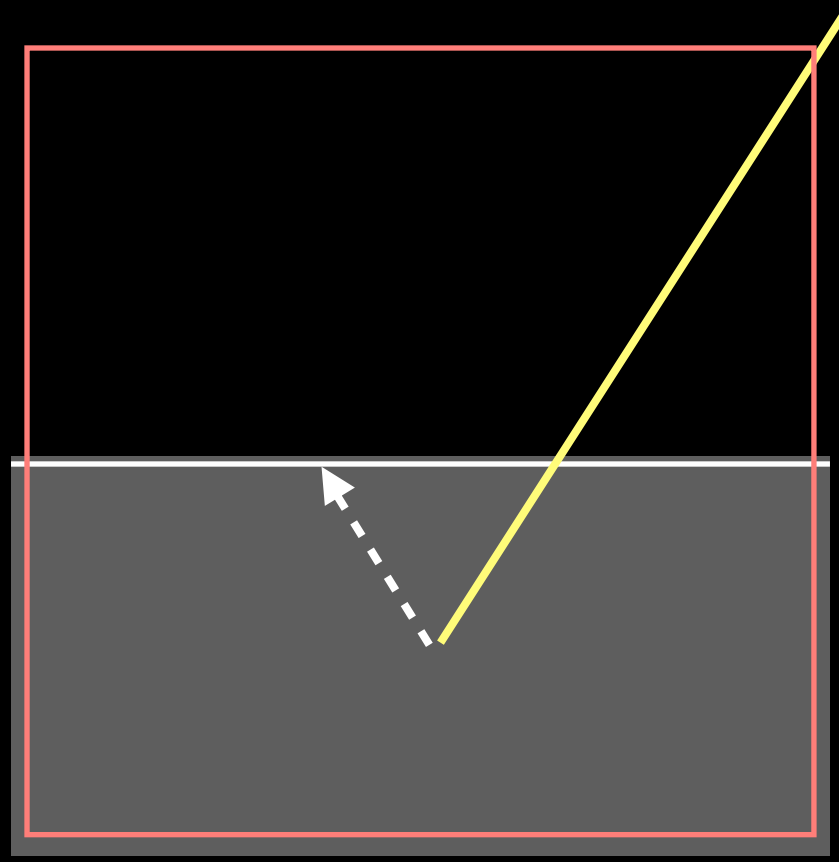
Occluded

Shadows

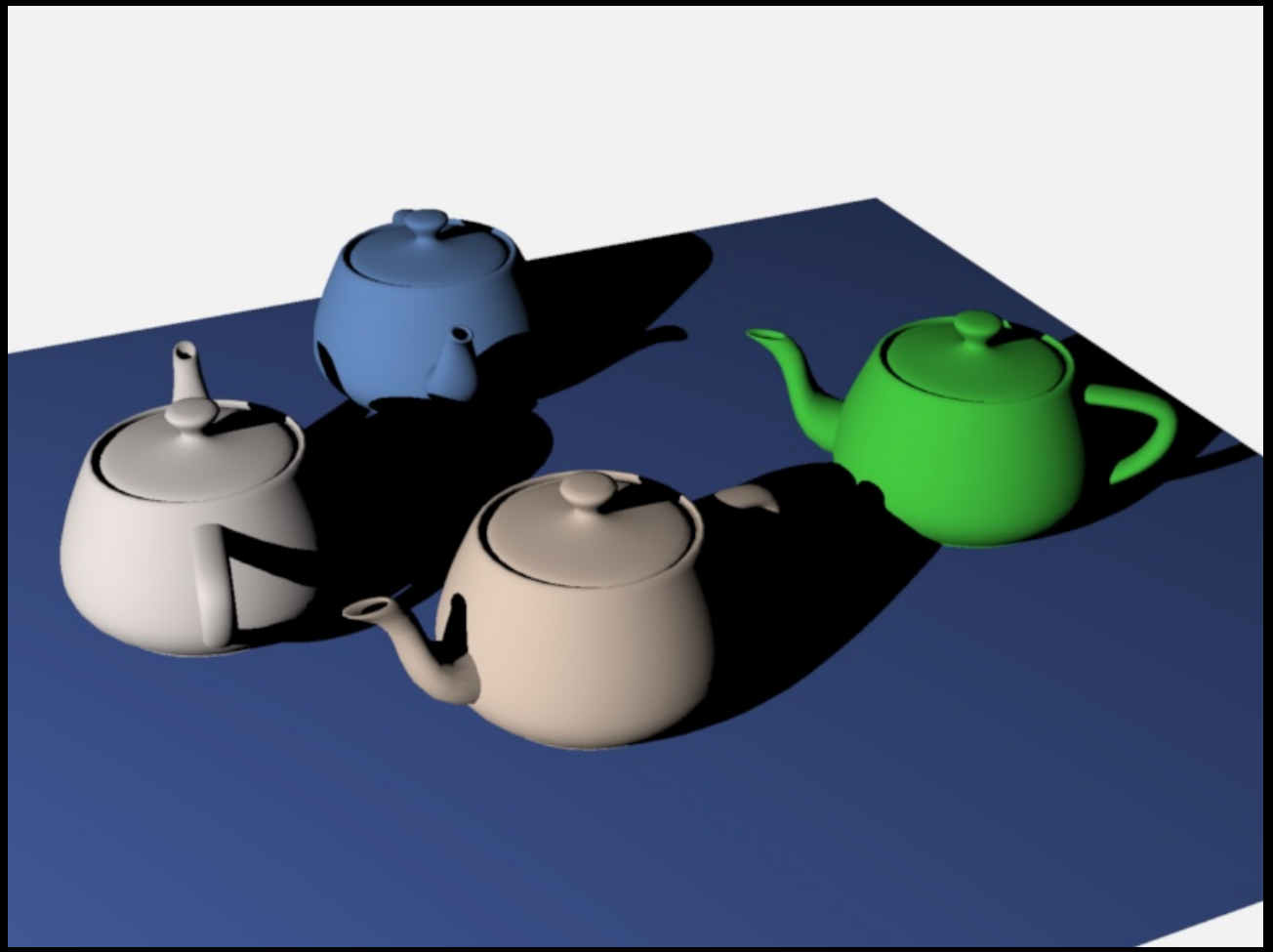
- Avoid self-intersection due to numerical error
  - Ignoring intersections that are too close etc.



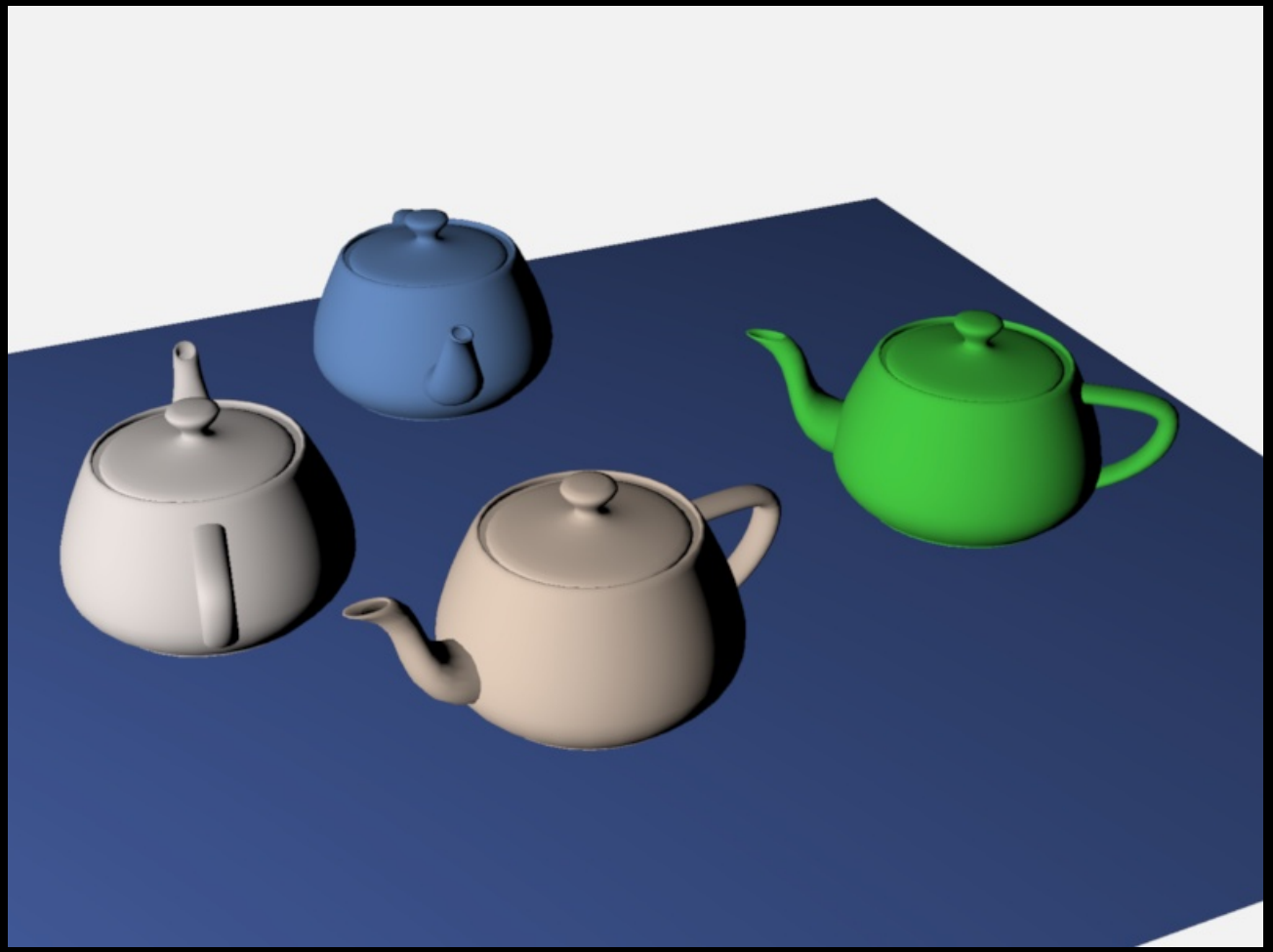
Occluded?



With Shadows

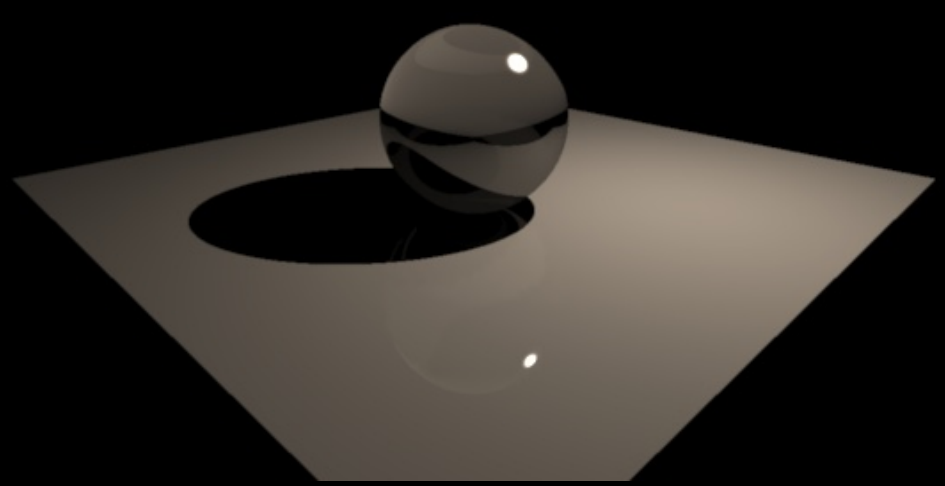


Without Shadows

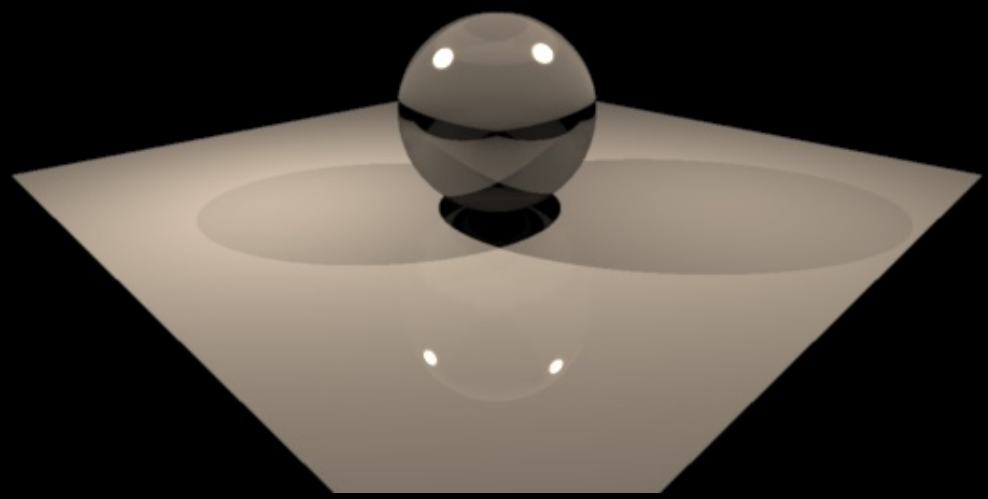


Multiple Light Sources

- Simply add up all the contributions
- Linearity of illumination



One light source



Two light source

Image Based Lighting

- Use of pixels in the image as light sources

Input illumination

Renderings

Image Based Lighting

- If ray hits nothing, return a value from image

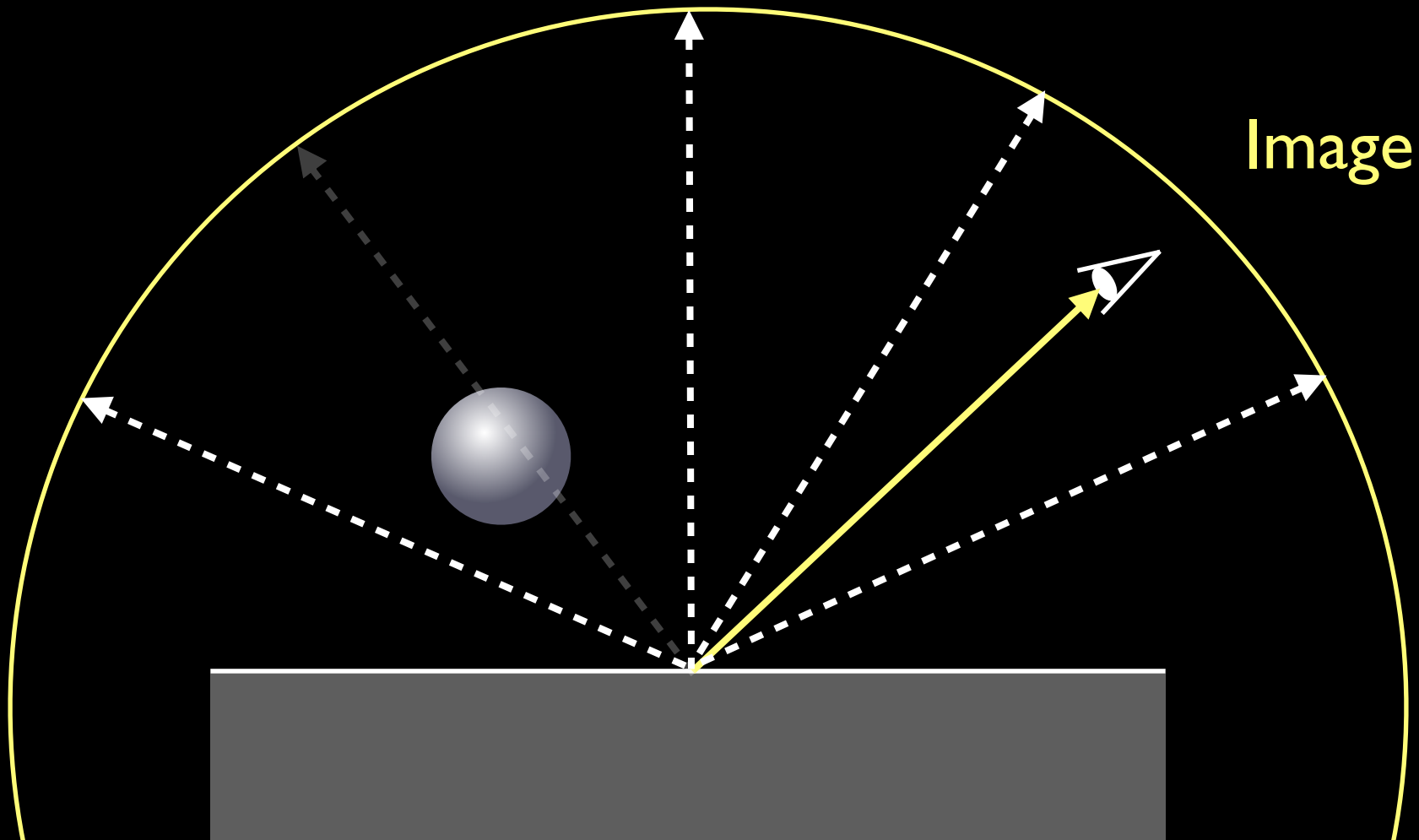


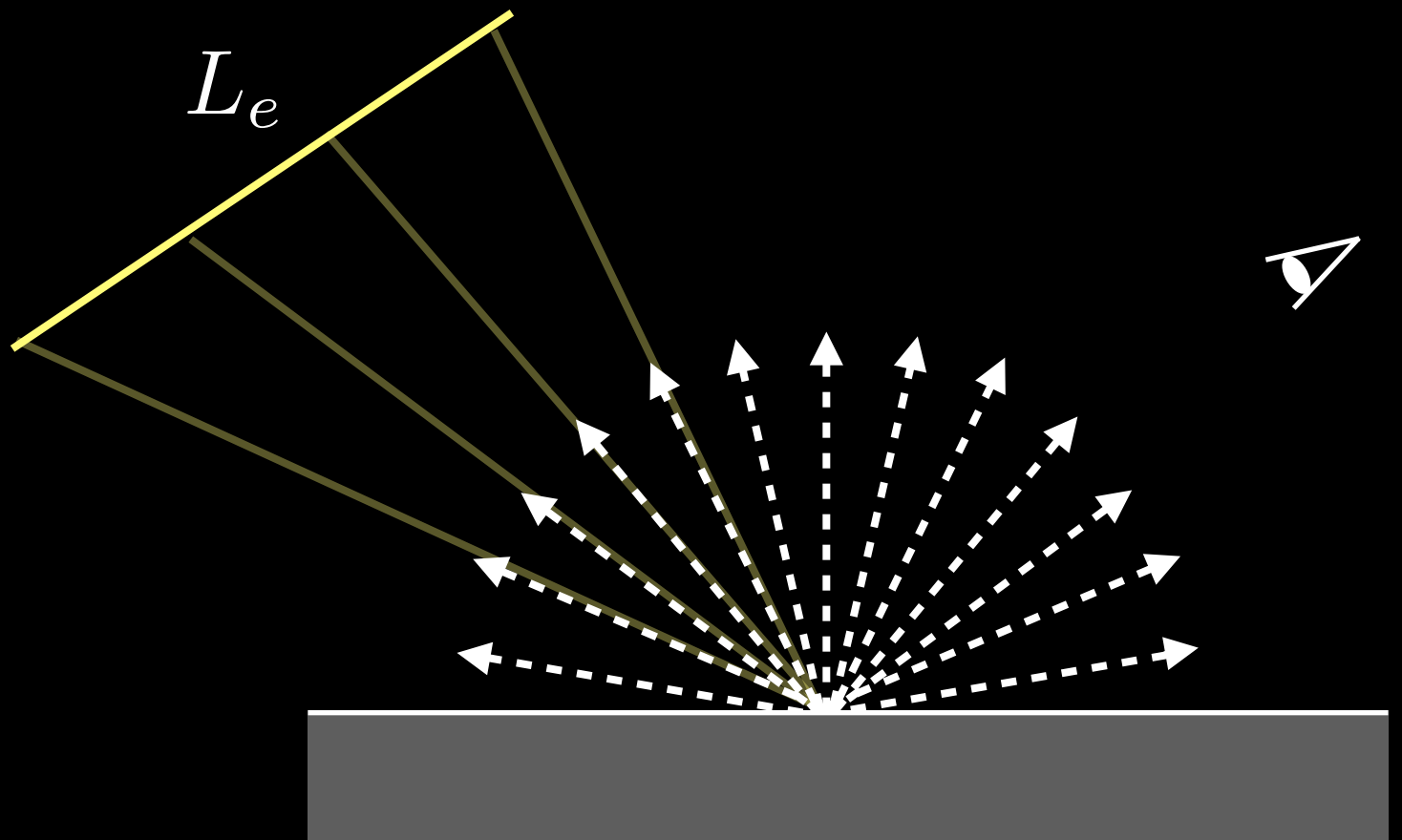
Image Based Lighting

- Trace random ray above the hemisphere

```
color get_irradiance (hit) {
  Ep = ... // irradiance from point light sources
  for (i = 1...N) {
    ray = gen_random_dir(hit.n)
    if (trace(ray) = no_hit) Ei = Ei + IBL(ray, image)
  }
  return Ep + (Ei / N)
}
```

Area Light Sources

$$L_o(x, \vec{\omega}_o) = \int_\Omega f(x, \vec{\omega}_o, \vec{\omega}_i) L_i(x, \vec{\omega}_i) \cos\theta_i d\omega_i$$

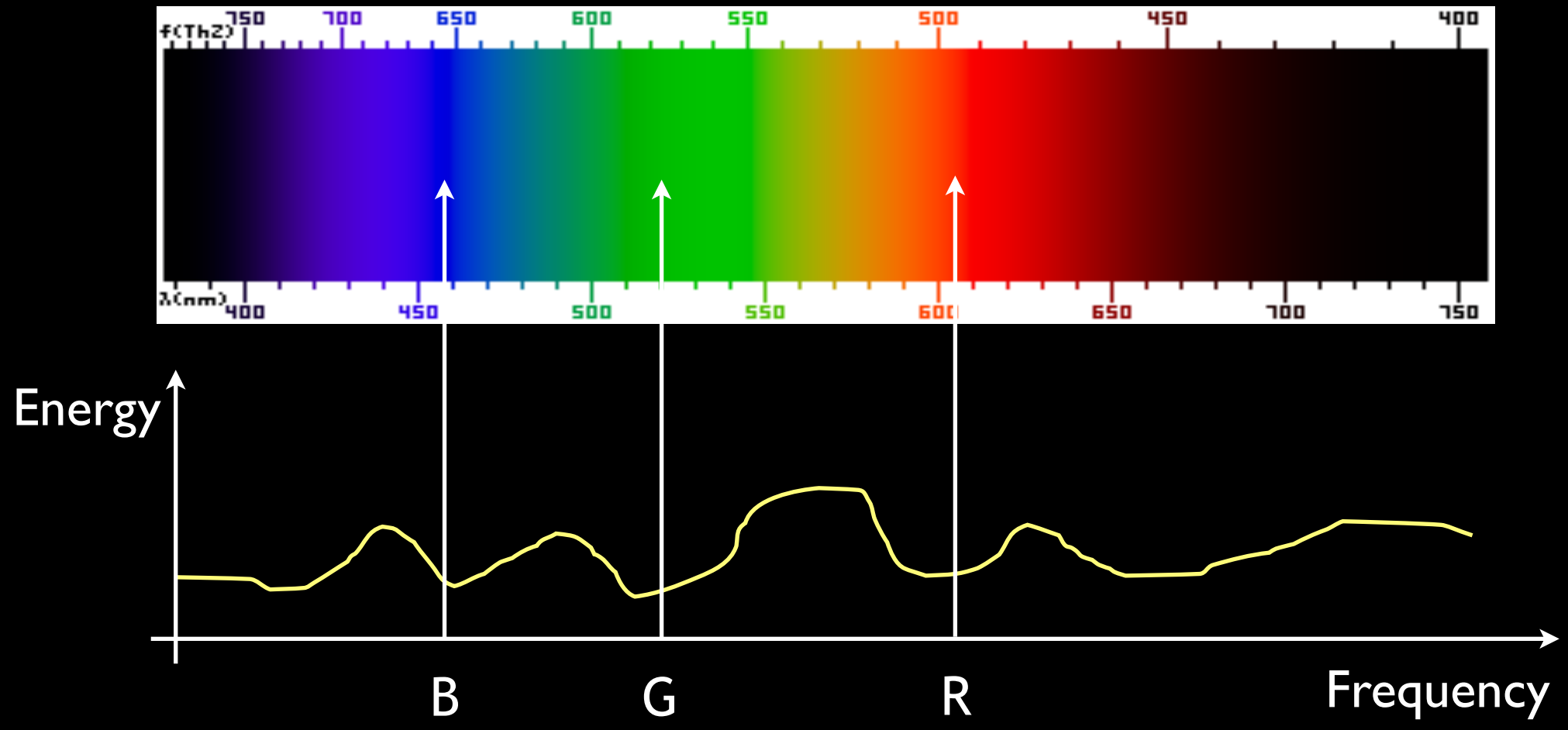


Some Details

- Generating random rays
  - http://people.cs.kuleuven.be/~philip.dutre/GI/
  - See Eqn. (34)
- Fetching pixels by rays
  - Depends on parameterization
  - http://www.pauldebevec.com/Probes/

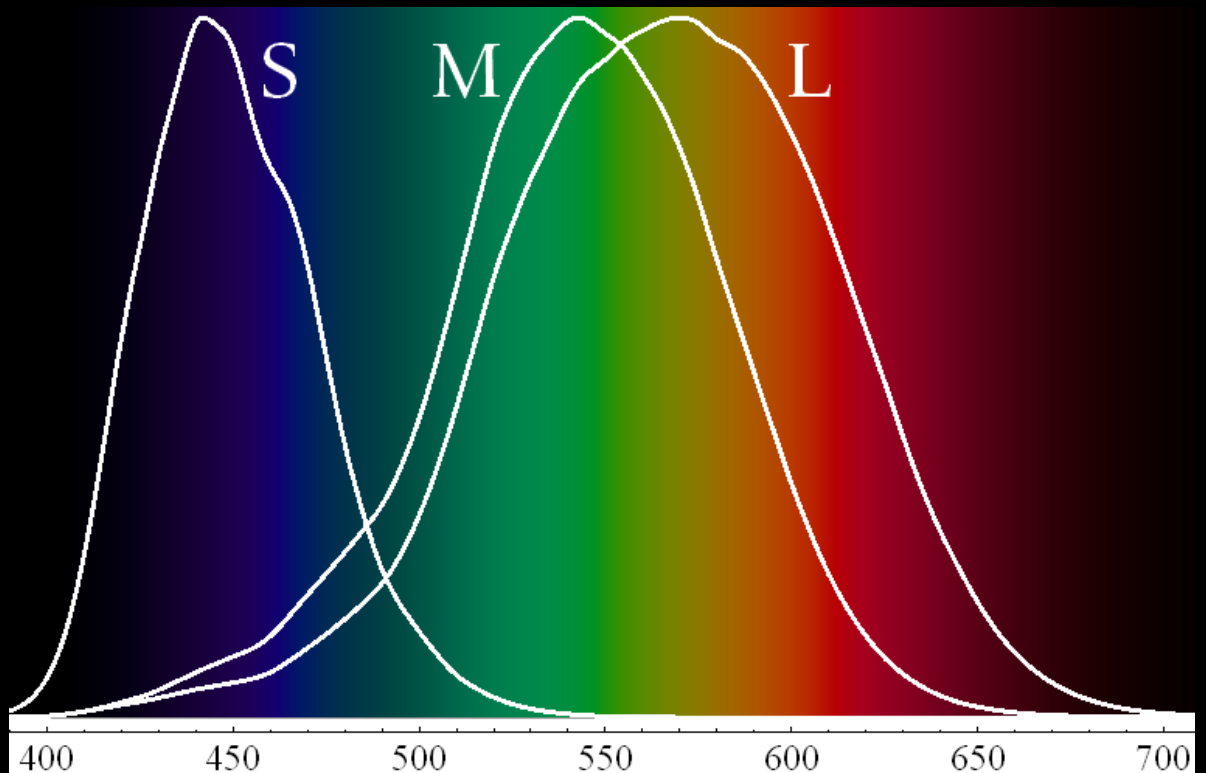
Color in Computer Graphics

- Store only three values (Red, Green, Blue)



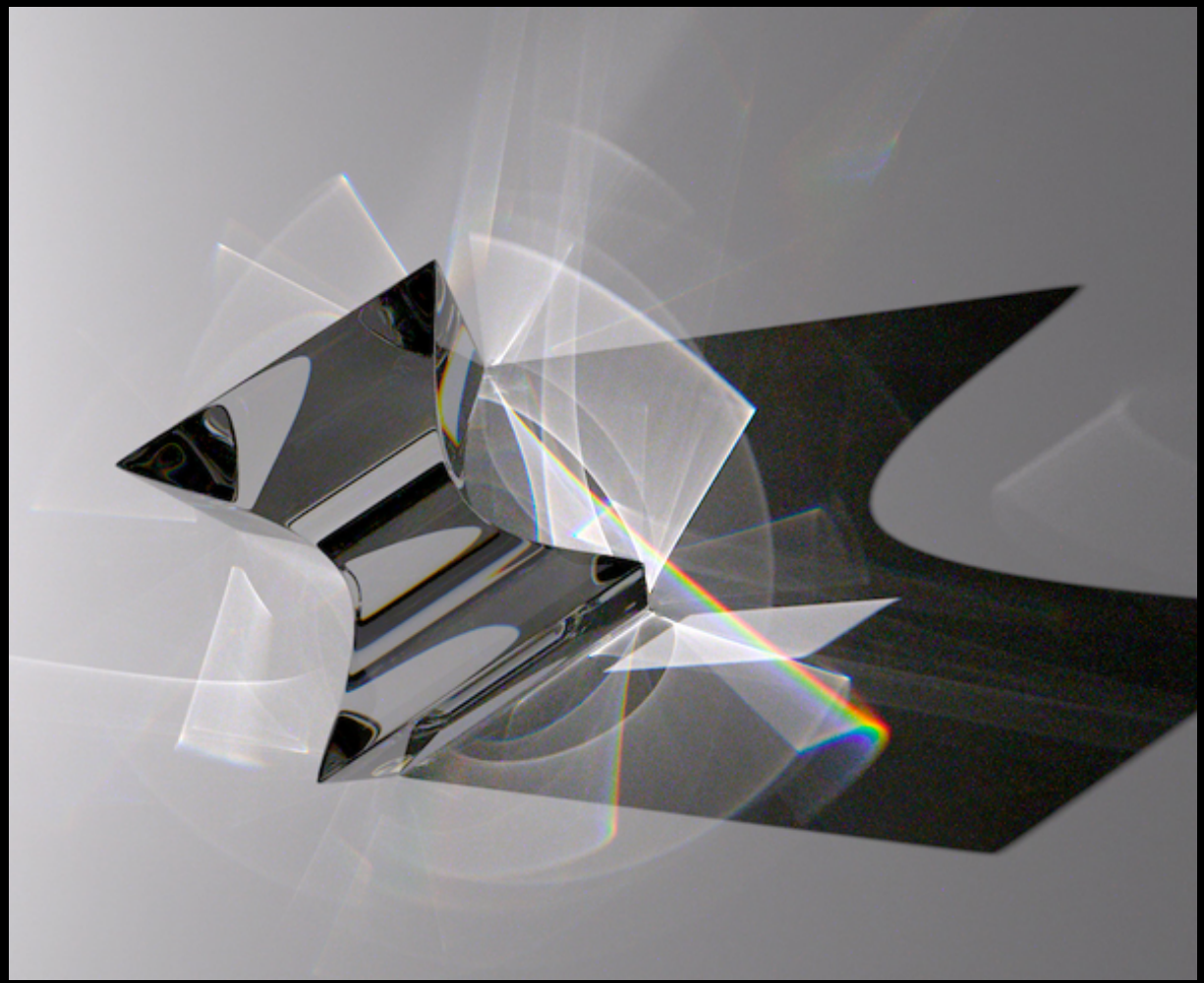
Why RGB?

- Because we see with RGB
  - Rods: Intensity sensors
  - Cones: Color sensors (typically three types)



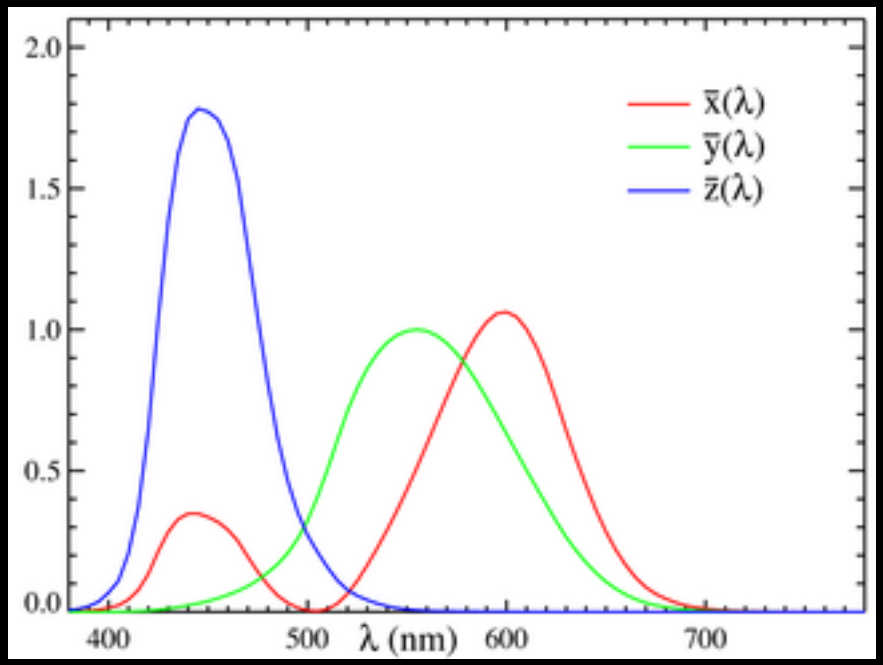
Full Spectrum Rendering

- Simulation at many wavelength ⇒ RGB



Spectrum to RGB

- Two steps
  - Compute responses to spectrum as XYZ
  - Convert XYZ to RGB



$$x = \int_{400}^{700} L(\lambda)\bar{x}(\lambda)d\lambda$$

Similar for y & z

$$r = c_0 x + c_1 y + c_2 z$$

RGB to Spectrum

- Ill-conditioned problem
  - Spectrum has much more info than RGB
  - “An RGB to Spectrum Conversion for Reflectances”

Human Eye vs Monitor

- Human eye
  - 2^{14}:1 brightness difference
  - 14 bits
- Monitor
  - 255:1 brightness difference
  - 8 bits

Tone Mapping

- Convert an HDR value into 0-1 (0-255)

Linear Scaling

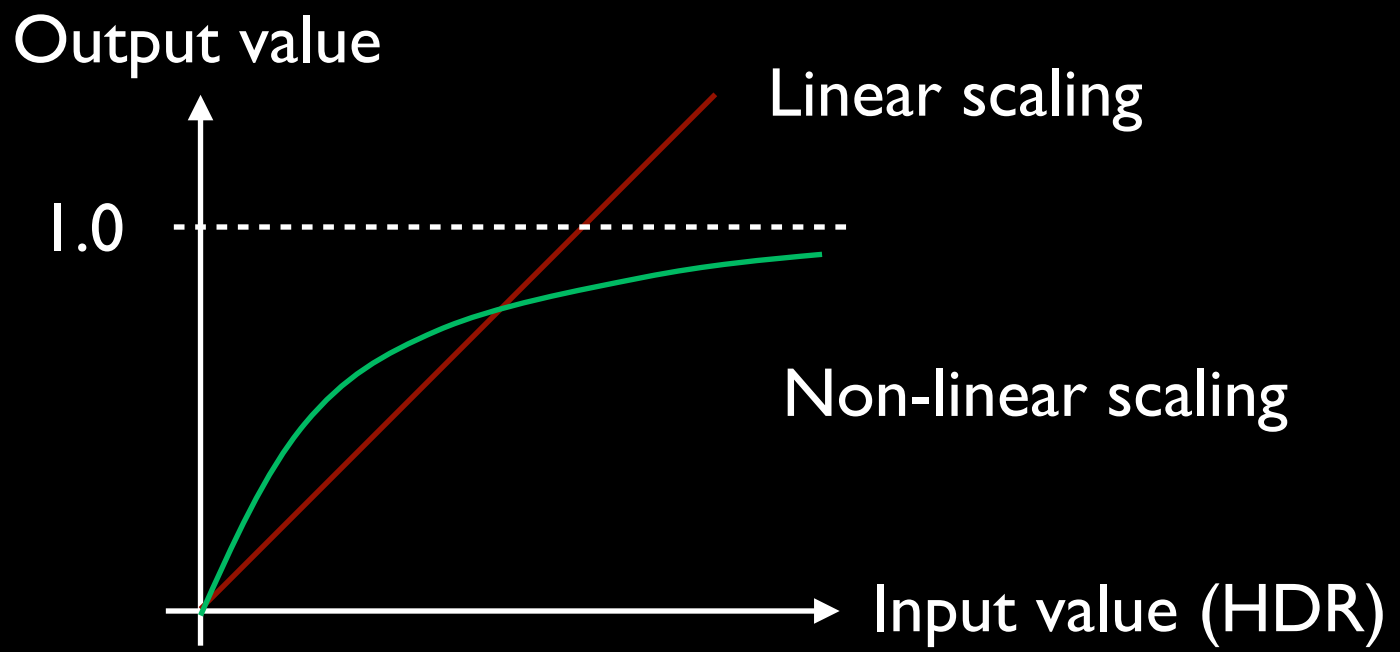
- Scale and clamp

$$l(x, y) = \min(cL(x, y), 1.0)$$

Non-linear Scaling

- Use a function to "compress" HDR into 0-1

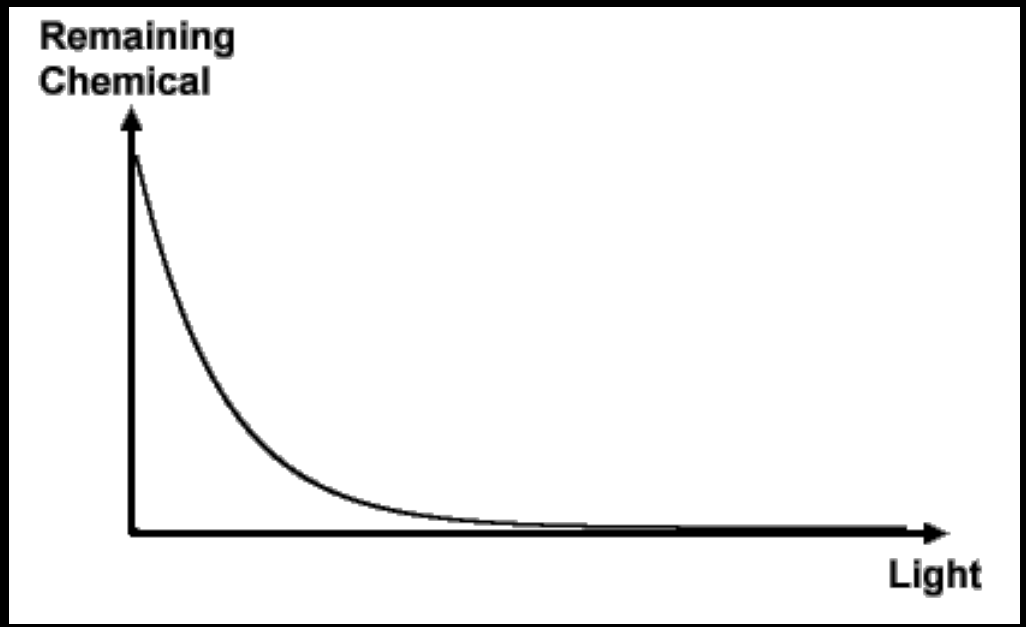
$$l(x, y) = f(L(x, y))$$



Film Response

- Inspired by the response curve of a film
  - Simple and works well often

$$l(x,y) = 1.0 - e^{-cL(x,y)}$$



http://freespace.virgin.net/hugo.elias/graphics/x_posure.htm

Photographic Tone Reproduction

- Inspired by photography [Reinhard]
  - Often works well
  - Desired brightness L_{white}

$$L_d(x,y) = \frac{L(x,y)\left(1 + \frac{L(x,y)}{L_{\text{white}}^2}\right)}{1 + L(x,y)}$$

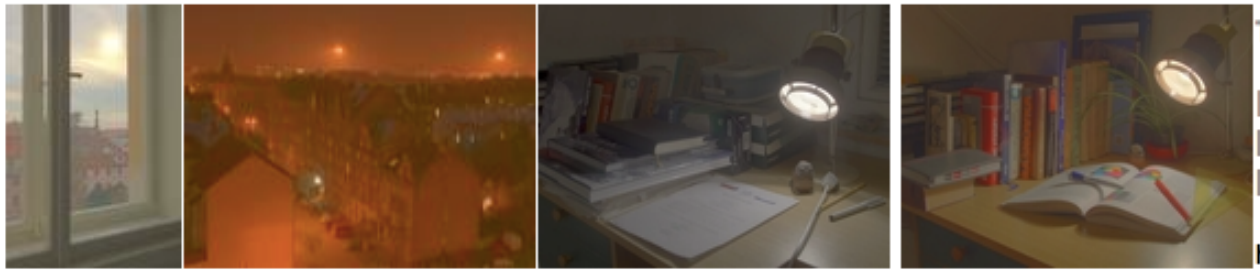
http://cadik.posvete.cz/tmo/

Comparisons

Drago, F., Myszkowski, K., Annen, T., Chiba, N.

Adaptive Logarithmic Mapping For Displaying High Contrast Scenes

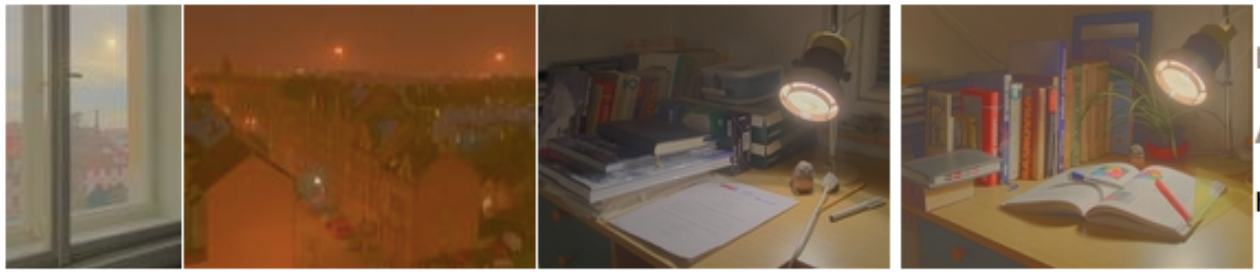
http://www-cg.cis.iwate-u.ac.jp/~frederic/logmap/
CODE AVAILABLE



Tumblin, J., Turk, G.

Low Curvature Image Simplifiers (LCIS): A Boundary Hierarchy for Detail-Preserving Contrast Reduction

http://www.cs.northwestern.edu/~jet/publications.html
CODE AVAILABLE



M. Ashikhmin

A tone mapping algorithm for high contrast images

http://www.cs.sunysb.edu/~ash/tm/

Fredo Durand, Julie Dorsey

Fast bilateral filtering for the display of high-dynamic-range images

http://graphics.csail.mit.edu/~fredo/PUBLI/Siggraph2002/
CODE AVAILABLE

Gamma Correction

- 0.5 is not necessary displayed as “0.5”
  - Nonlinear mapping on a monitor
  - Correction to input tone-mapped value
  - g is typically 1.7-2.2

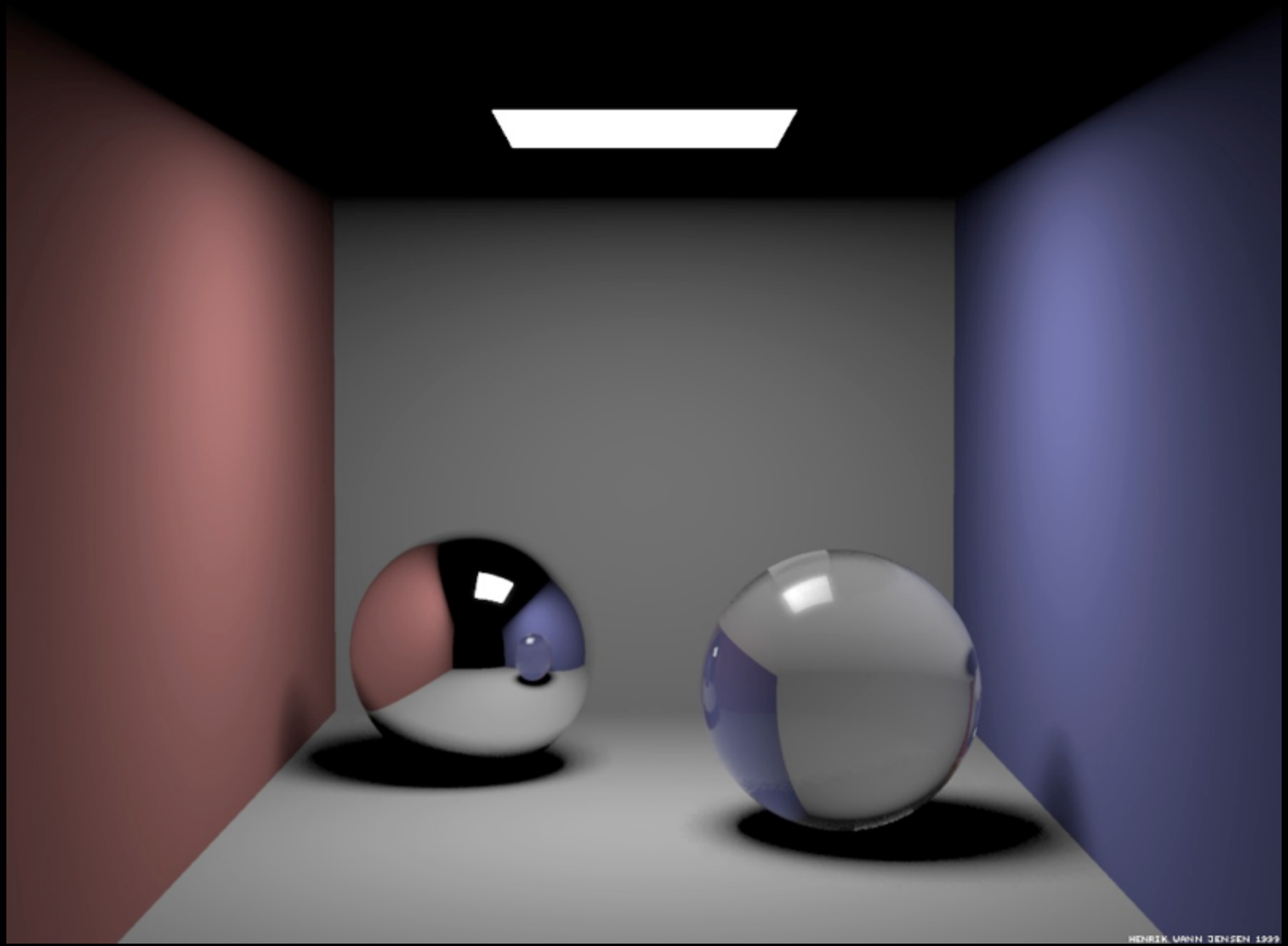
$$p(x, y) = l(x, y)^{\frac{1}{g}}$$

Gamma Correction

Corrected

No correction

Missing Piece



Missing Piece