Homework 1

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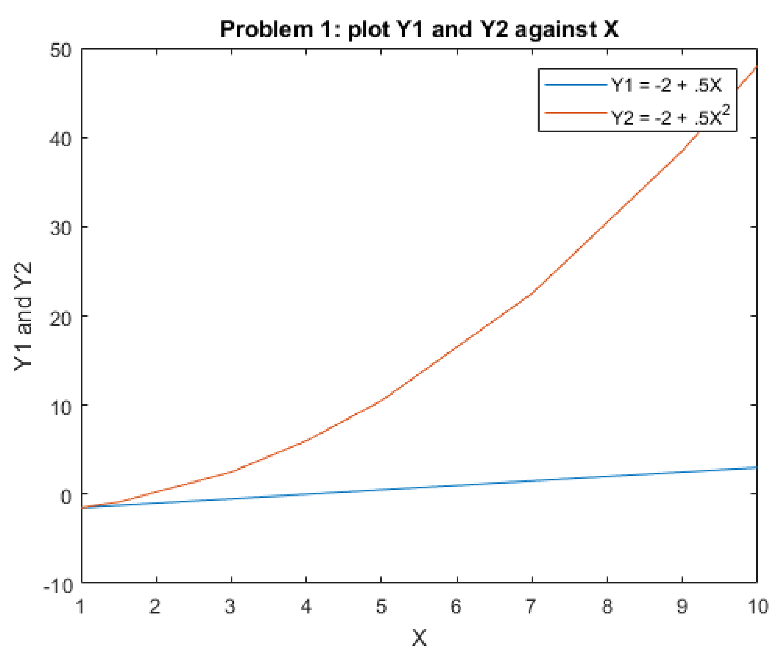
Problem 1

Define $\mathbf{X} = [1, 1.5, 3, 4, 5, 7, 9, 10]$ and construct the values of the function $Y1 = -2 * .5X$ and $Y2 = -2 + .5X^2$. I have

$$\mathbf{Y1} = \begin{bmatrix} -1.5 & -1.25 & -0.5 & 0 & 0.5 & 1.5 & 2.5 & 3.0 \end{bmatrix}$$
$$\mathbf{Y2} = \begin{bmatrix} -1.5 & -0.875 & 2.5 & 6.0 & 10.5 & 22.5 & 38.5 & 48.0 \end{bmatrix}$$

Plot $Y1$ and $Y2$ against X in a single graph



Problem 2

I use `linspace` command to create a vector containing evenly-spaced numbers between $[-10, 20]$. Fir summing elements of the vector, I use `sum( )` command. See the attached code.

Problem 3

For matrix algebra, I use "`*`" for matrix multiplication, use "'" for transpose, and use "`inv( )`" for getting inverse.

$$\mathbf{C} = \mathbf{A}'\mathbf{b} = \begin{bmatrix} 29.0 \\ 133.0 \\ 43.0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} -3.2505 \\ 0.3961 \\ 0.8037 \end{bmatrix}$$

To obtain $E = \sum_i \sum_j a_{ij} b_i$, I first calculate $\mathbf{E0} = \mathbf{A} \circ \left(\mathbf{b}\left[1, 1, 1\right]\right)$ where $\circ$ is element-wise multiplication. Then I sum all elements in **E0** by running `sum( )` twice, for summing over each column first and then for summing over raw.

$$\mathbf{E0} = \begin{bmatrix} -4.0 & -8.0 & -12.0 \\ 3.0 & 21.0 & 15.0 \\ 30.0 & 120.0 & 40.0 \end{bmatrix}$$
$$\Rightarrow E = 205$$

In creating $\mathbf{F}$, I firstly delete the 2nd row of $\mathbf{A}$ (defined as $\mathbf{F0}$) and then delete the 3rd column of $\mathbf{F0}$.

$$\mathbf{F} = \begin{bmatrix} 2.0 & 4.0 \\ 3.0 & 12.0 \end{bmatrix}$$

In solving the system of linear equation $\mathbf{Ax} = \mathbf{b}$ for $\mathbf{x}$, calculate $\mathbf{x} = A^{-1}b$ where I use `inv(A)` to get inverse.

$$\mathbf{x} = \begin{bmatrix} -0.1622 \\ 1.2432 \\ -1.1081 \end{bmatrix}$$

Problem 4

I use `blkdiag(A,A,A,A,A)` to create a 15×15 block diagonal matrix.

Problem 5

In creating a matrix of random draws from a normal distribution with mean 10 and standard deviation 5, we use `normrnd(10,5,[5,3])`. For example

$$\mathbf{A} - \begin{bmatrix} 6.96 & 9.6 & 12.2 \\ 6.31 & 14.5 & 10.5 \\ 1.25 & 10.9 & 23.9 \\ 14.6 & 11.5 & 4.17 \\ 14.3 & 10.6 & 0.729 \end{bmatrix}$$

Then using loop, we check if each element of $\mathbf{A}$ is smaller than 10 or not. If it is smaller than 10, replace it with 0 and replace it with 1 otherwise. This yields

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Problem 6

First we make $N \times 1$ vector of dependent variable ($\mathbf{Y}$), which is the 5th column of the dataset, where N is the number of observation. We then creat $N \times 4$ matrix of independent variables ($\mathbf{X}$), whose 1st column is ones, 2nd, 3rd and 4th columns are vectors of export dummy, R&D dummy, and capital stock, respectively.
By definition of OLS estimates, vector of coefficients $\hat{\boldsymbol{\beta}} = \left[\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\right]$ can be computed by

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1} \mathbf{X}'\mathbf{Y} = \begin{bmatrix} 0.082 \\ 0.120 \\ 0.140 \\ 0.030 \end{bmatrix}$$

To obtain the standard error, we first compute the vector of residual $\mathbf{e}$ as

$$\mathbf{e} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}$$

Then obtain the estimator for σ^2 (variance of the error)

$$\hat{\sigma} = \frac{\mathbf{e}'\mathbf{e}}{N-4}$$

Variance-covariance matrix for the estimates are

$$\mathbf{cov} = \hat{\sigma}(\mathbf{X}'\mathbf{X})^{-1}$$

Square-root of the diagonal of **cov** is the standard errors for the estimates. To obtain the diagonal elements, we use `diag(cov)`. Standard errors are

$$\mathbf{stderr} = \begin{bmatrix} 0.0167 \\ 0.0063 \\ 0.0085 \\ 0.0018 \end{bmatrix}$$

Matlab Code

```
% ECON512 Homework 1
% Kensuke Suzuki
clear all
%% Problem 1
X = [1,1.5,3,4,5,7,9,10];
Y1 = -2 + .5*X;
Y2 = -2 + 0.5 * X.^2;

plot(X, Y1, X, Y2)
title('Problem 1: plot Y1 and Y2 against X')
xlabel('X') % x-axis label
ylabel('Y1 and Y2') % y-axis label
legend('Y1 = -2 + .5X','Y2 = -2 + .5X^2')

%% Problem 2
% Create 200x1 vector X
clear X
X = linspace(-10,20,200)';
sumX = sum(X)

%% Problem 3
A = [2,4,6; 1,7,5; 3,12,4]
b = [-2;3;10]

% C
C = A'*b
% D
D = inv(A'*A) * b

% E
E0 = A .* (b*ones(1,3));
E = sum(sum(E0),2)

% F
F0 = [A(1,:);A(3,:)]
F = [F0(:,1), F0(:,2)]
```

```
% Solve linear equatuons
x = inv(A)*b

%% Problem 4
% block diagonal matrix
B =  blkdiag(A,A,A,A,A);

%% Problem 5
clear A

A = normrnd(10,5,[5,3])

for i = 1:size(A,1)
    for j = 1:size(A,2)
        if A(i,j) < 10
            A(i,j) = 0;
        else
            A(i,j) = 1;
        end
    end
end

disp(A)

%% Problem 6
clear X
clear Y

filename = 'datahw1.csv';
data = csvread(filename);
X = [ones(4392,1), data(:,3), data(:,4), data(:,6)];
Y = data(:,5);

% Pointe estimates
betahat = inv(X'*X)*X'*Y

% Standard error
% residual
e = Y - (X * betahat);
sigmahat = (e'* e)/(size(X,1)-size(X,2));
cov = sigmahat * inv(X'*X);
var = diag(cov);
stderr = var.^(1/2)
```