

---

# Homework 1

---

Kensuke Suzuki

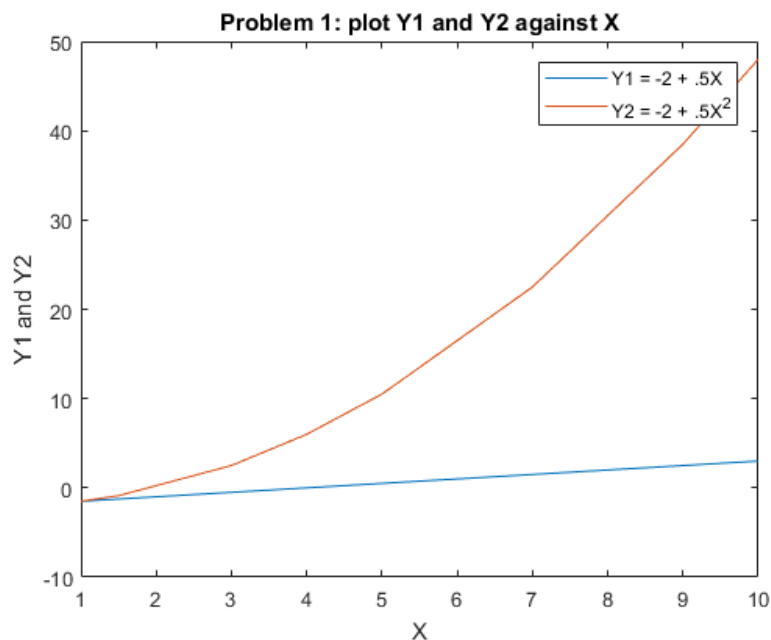
September 2, 2018

## Problem 1

Define  $\mathbf{X} = [1, 1.5, 3, 4, 5, 7, 9, 10]$  and construct the values of the function  $Y1 = -2 + .5X$  and  $Y2 = -2 + .5X^2$ . I have

$$\mathbf{Y1} = \begin{bmatrix} -1.5 & -1.25 & -0.5 & 0 & 0.5 & 1.5 & 2.5 & 3.0 \end{bmatrix}$$
$$\mathbf{Y2} = \begin{bmatrix} -1.5 & -0.875 & 2.5 & 6.0 & 10.5 & 22.5 & 38.5 & 48.0 \end{bmatrix}$$

Plot  $Y1$  and  $Y2$  against  $X$  in a single graph



## Problem 2

I use `linspace` command to create a vector containing evenly-spaced numbers between  $[-10, 20]$ . For summing elements of the vector, I use `sum( )` command. See the attached code.

## Problem 3

For matrix algebra, I use “\*” for matrix multiplication, use “'” for transpose, and use “`inv( )`” for getting inverse.

$$\mathbf{C} = \mathbf{A}'\mathbf{b} = \begin{bmatrix} 29.0 \\ 133.0 \\ 43.0 \end{bmatrix}$$
$$\mathbf{D} = \begin{bmatrix} -3.2505 \\ 0.3961 \\ 0.8037 \end{bmatrix}$$

To obtain  $E = \sum_i \sum_j a_{ij} b_i$ , I first calculate  $\mathbf{E0} = \mathbf{A} \circ (\mathbf{b} [1, 1, 1])$  where  $\circ$  is element-wise multiplication. Then I sum all elements in  $\mathbf{E0}$  by running `sum( )` twice, for summing over each column first and then for summing over row next.

$$\mathbf{E0} = \begin{bmatrix} -4.0 & -8.0 & -12.0 \\ 3.0 & 21.0 & 15.0 \\ 30.0 & 120.0 & 40.0 \end{bmatrix}$$
$$\Rightarrow E = 205$$

In creating  $\mathbf{F}$ , I firstly delete the 2nd row of  $\mathbf{A}$  (defined as  $\mathbf{F0}$ ) and then delete the 3rd column of  $\mathbf{F0}$ .

$$\mathbf{F} = \begin{bmatrix} 2.0 & 4.0 \\ 3.0 & 12.0 \end{bmatrix}$$

In solving the system of linear equation  $\mathbf{Ax} = \mathbf{b}$  for  $\mathbf{x}$ , calculate  $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$  where I use `inv(A)` to get inverse.

$$\mathbf{x} = \begin{bmatrix} -0.1622 \\ 1.2432 \\ -1.1081 \end{bmatrix}$$

## Problem 4

I use `blkdiag(A,A,A,A,A)` to create a  $15 \times 15$  block diagonal matrix. See the attached code.

## Problem 5

In creating a matrix of random draws from a normal distribution with mean 10 and standard deviation 5, we use `normrnd(10,5,[5,3])`. For example, this command returns

$$\mathbf{A} = \begin{bmatrix} 9.38 & 8.47 & 12.7 \\ -2.71 & 4.36 & 10.5 \\ 11.4 & 11.0 & 4.39 \\ 9.02 & 6.96 & 10.2 \\ 9.02 & 5.86 & 3.81 \end{bmatrix}$$

Then using loop, we check if each element of  $\mathbf{A}$  is smaller than 10 or not. If it is smaller than 10, replace it with 0 and replace it with 1 otherwise. This yields

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1.0 \\ 0 & 0 & 1.0 \\ 1.0 & 1.0 & 0 \\ 0 & 0 & 1.0 \\ 0 & 0 & 0 \end{bmatrix}$$

## Problem 6

First we make  $N \times 1$  vector of dependent variable ( $\mathbf{Y}$ ), which is the 5th column of the dataset, where  $N$  is the number of observation. We then creat  $N \times 4$  matrix of independent variables ( $\mathbf{X}$ ), whose 1st column is ones, 2nd, 3rd and 4th columns are vectors of export dummy, R&D dummy, and capital stock, respectively.

By definition of OLS estimates, vector of coefficient estimates  $\hat{\boldsymbol{\beta}} = [\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3]'$  can be computed by

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{Y} = \begin{bmatrix} 0.0817 \\ 0.1201 \\ 0.1399 \\ 0.0295 \end{bmatrix}$$

To obtain the standard error, we first compute the vector of residual  $\mathbf{e}$  as

$$\mathbf{e} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}$$

Then obtain the estimator for  $\sigma^2$  (variance of the error)

$$\hat{\sigma} = \frac{\mathbf{e}'\mathbf{e}}{N - 4}$$

Variance-covariance matrix for the estimates are

$$\mathbf{cov} = \hat{\sigma}(\mathbf{X}'\mathbf{X})^{-1}$$

Square-root of the diagonal of `cov` is the standard errors for the estimates. To obtain the diagonal elements, we use `diag(cov)`. Standard errors are

$$\mathbf{stderr} = \begin{bmatrix} 0.0167 \\ 0.0063 \\ 0.0085 \\ 0.0018 \end{bmatrix}$$

## Matlab Code

```
1 % ECON512 Homework 1
2 % Kensuke Suzuki
3 clear all
4 delete HW1log.txt
5 diary('HW1log.txt')
6 diary on
7
8 disp('ECON512 HOMEWORK1: Ken Suzuki')
9 disp(' ')
10
11 %% Problem 1
12 X = [1,1.5,3,4,5,7,9,10];
13 Y1 = -2 + .5*X;
14 Y2 = -2 + 0.5 * X.^2;
15
16 plot(X, Y1, X, Y2)
17 title('Problem 1: plot Y1 and Y2 against X')
18 xlabel('X') % x-axis label
19 ylabel('Y1 and Y2') % y-axis label
20 legend('Y1 = -2 + .5X', 'Y2 = -2 + .5X^2')
21
22 disp('————Problem 1————')
23 X
24 Y1
25 Y2
26 disp('Figure is saved in the directory')
27 disp(' ')
28
29 %% Problem 2
30 % Create 200x1 vector X
31 clear X
32 X = linspace(-10,20,200)';
33 sumX = sum(X);
```

```
34
35 disp('————Problem 2————')
36 disp('First define the vector X using linspace. (display of X is
    omitted)')
37 disp('Sum of all elements in X is')
38 sumX
39 disp(' ')
40
41
42 %% Problem 3
43 A = [2,4,6; 1,7,5; 3,12,4];
44 b = [-2;3;10];
45
46 % C
47 C = A'*b;
48 % D
49 D = inv(A'*A) * b;
50
51 % E
52 E0 = A .* (b*ones(1,3));
53 E = sum(sum(E0),2);
54
55 % F
56 F0 = [A(1,:);A(3,:)]';
57 F = [F0(:,1), F0(:,2)]';
58
59 % Solve linear equations
60 x = inv(A)*b;
61
62 disp('————Problem 3————')
63 A
64 b
65 C
66 D
67 E
68 F
69 x
70 disp(' ')
71
72 %% Problem 4
73 % block diagonal matrix
74 B = blkdiag(A,A,A,A,A);
75
76 disp('————Problem 4————')
77 B
78 disp(' ')
79
80 %% Problem 5
```

```
81 clear A
82
83 rng(2);
84 A = normrnd(10,5,[5,3]);
85
86 disp('———Problem 5———')
87 disp('Before converting with 1 and 0 matrices')
88 A
89
90 for i = 1:size(A,1)
91     for j = 1:size(A,2)
92         if A(i,j) < 10
93             A(i,j) = 0;
94         else
95             A(i,j) = 1;
96         end
97     end
98 end
99
100 disp('After converting with 1 and 0 matrices')
101 A
102 disp(' ')
103
104 %% Problem 6
105 clear X
106 clear Y
107
108 filename = 'datahw1.csv';
109 data = csvread(filename);
110 X = [ones(4392,1), data(:,3), data(:,4), data(:,6)];
111 Y = data(:,5);
112
113 % Pointe estimates
114 betahat = inv(X'*X)*X'*Y;
115
116 % Standard error
117 % residual
118 e = Y - (X * betahat);
119 sigmahat = (e'* e)/(size(X,1)-size(X,2));
120 cov = sigmahat * inv(X'*X);
121 var = diag(cov);
122 stderr = var.^(1/2);
123
124 betase = [betahat';stderr'];
125 rowNames = {'Coefficient','Std Error'};
126 colNames = {'beta0','beta1','beta2','beta3'};
127
128
```

```
129 disp('———Problem 6———')
130 disp('Estimated Coefficients and Standard Errors')
131 Table = array2table(betase, 'RowNames', rowNames, 'VariableNames', colNames
132 );
133 disp(Table)
134 disp('end of HW1')
135
136 %tab = [betaind; tab0]
137
138 diary off
```