Homework 2

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Problem 1

I define a function `bertrand.m` which returns vector of demand for each good, for given vector of price $\mathbf{p}$ and $\mathbf{v}$ (which are potentially $n \times 1$ vectors).

```
function fval = bertrand(p,v)

% for given vector of p and v, solve demand
fval = exp(v - p) ./ ( 1 + sum( exp(v - p) ) );

end
```

For $\mathbf{p} = [1, 1]^\top$ and $\mathbf{v} = [2, 2]^\top$, we obtain $[D_A, D_B]^\top = [0.422319, 0.422319]^\top$ and $D_0 = 1.55362$.

Problem 2

Each firm solves profit maximization problem:

$$\max_{p_i} p_i D_i$$

for $i = A, B$. The first order conditions yields:

$$D_i + \left[\frac{\partial D_i}{\partial p_i} p_i\right] = D_i - p_i D_i (1 - D_i) = D_i \left[1 - p_i (1 - D_i)\right] = 0$$

Provided $D_i \neq 0$, the FOC boils down to $[1 - p_i(1 - D_i)] = 0$. The Bertrand-Nash equilibrium is the set of prices which satisfy the system of equation:

$$1 - p_A(1 - D_A) = 0$$
$$1 - p_B(1 - D_B) = 0$$

In matrix notation, we have

$$\begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix} 1 - D_A & 0 \\ 0 & 1 - D_B \end{bmatrix} \begin{bmatrix} p_A \\ p_B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

We define the LHS of (1) as a function `bertrandfoc.m`

```
function fval = bertrandfoc(p,v)

% for given vector of p and v, solve demand
D = exp(v - p) ./ ( 1 + sum( exp(v - p) ) );

% First order condition boils down to
fval = ones(size(p,1),1) - diag(ones(size(p,1),1)-D)*p;

end
```

We then solve the system of nonlinear equation using Broyden's Method. The algorithm is completely analogue to what we did in the class. For the starting value of $\mathbf{p}$, we use $[1, 1]^\top$ and we use the identity matrix as an initial inverse of Jacovian. For convergence criterion, we use 1e-6. The iteration converges and we get the set of equilibrium prices $\mathbf{p} = [1.598942, 1.598942]^\top$

Problem 3

First we define the function `bertrandfocg.m` which return the FOC for gth good price.

```
function fval = bertrandfocg(p,v,g)

% for given vector of p and v, solve demand
D = exp(v - p) ./ ( 1 + sum( exp(v - p) ) );

% First order condition boils down to
foc = ones(size(p,1),1) - diag(ones(size(p,1),1)-D)*p;

fval = foc(g,1);

end
```

We then solve the system by using a Gauss-Seidel method. First we set the initial value for $\mathbf{p} = [1, 1]^\top$ and set $\mathbf{p}_{\text{old}} = [2, 2]$. Then, for given p_B we solve the FOC for good A price using the secant method. This sub-iteration solves p_A for given initial guess on p_B. Then next sub-iteration solves p_B using the FOC for good B price using the p_A obtained in the previous sub-iteration.

$$(1)\ p_A^{k+1} \Leftarrow 1 - p_A^k(1 - D_A(p_A^k, p_B^k)) = 0$$

$$(2)\ \ p_B^{k+1} \Leftarrow 1 - p_B^k(1 - D_B(p_A^{k+1}, p_B^k)) = 0$$

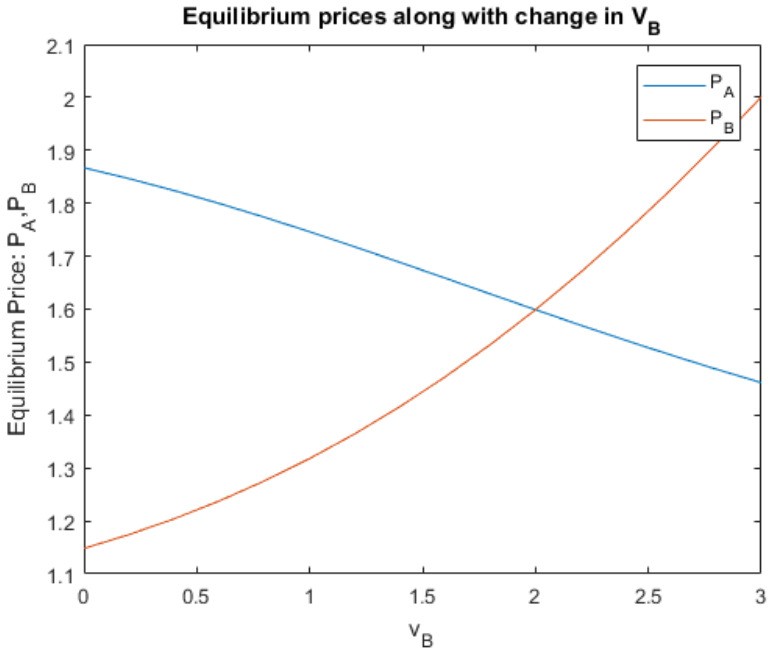
We iterate the set of two subiteration (indexed by k above) until the norm of the vector obtained using the function `bertrandfoc.m` (which returns the vector of the LHS of FOCs) is below the tolerance level. This algorithm also yields the same equilibrium price vector $\mathbf{p} = [1.598942, 1.598942]^\top$. The elapsed time for problem 2 (Broyden Method) is 0.006158 and the one for problem 3 (Gauss-Seidel Method) is 0.008184. Therefore, Gauss-Seidel Method is faster. **WHY?**

Problem 4

In this problem, we use the update rule specified in the problem set and solve the system. It again yields the same result: $\mathbf{p} = [1.598942, 1.598942]^\top$.

Problem 5

We define the vector of v_B values (0:.2:2) and for each v_B and $v_A = 2$, we solve the system of equation for $\mathbf{p}$. We store the equilibrium vector of prices in `result` matrix (where the first raw is v_B and the second and third raws contain the vector of equilibrium prices corresponding to each v_B value). 2-way plotted graph is demonstrated below.



Matlab Code

```
% ECON512 Homework 2
% Kensuke Suzuki
clear all
delete HW2log.txt
diary('HW2log.txt')
```

```
diary on

disp('ECON512 HOMEWORK2: Ken Suzuki')

%% Define bertrand and bertrandfoc function
% bertrand: return demand for each good
% bertrandfoc: return system of FOC (LHS)

%% Problem 1

p = [1;1];
v = [2;2];

Ans1 = bertrand(p,v)

D0 = 1 / (1+ sum(exp(v-p)) );

P1 = sprintf('Problem1: for vA=vB=2 and pA=pB=1, DA= %f, DA= %f, and D0
    = %f.', Ans1(1,1),Ans1(2,1),D0);
disp(P1);

%% Problem 2

clear all

v = [2;2];

p = [1;1];

fVal_foc = @(p) bertrandfoc(p,v);
i_fVal_foc = fVal_foc(p);

iJac = eye(size(p,1));

maxit = 100;
tol = 1e-6;

tic
for iter = 1:maxit
    fnorm = norm(i_fVal_foc);
    fprintf('iter %d: p(1)=%f, p(2)=%f, norm(f(x))=%.8f \n', iter, p(1)
        ,p(2),norm(i_fVal_foc));
    if norm(i_fVal_foc)<tol
        break
    end
    d = -(iJac*i_fVal_foc);
    p = p + d;
    fOld_foc = i_fVal_foc;
```

```
    i_fVal_foc = fVal_foc(p);
    u = iJac*(i_fVal_foc - fOld_foc);
    iJac = iJac + ( (d-u)* (d'*iJac) )/(d'*u);
end
elapsedTime_p2 = toc;

P2 = sprintf('Problem2: for vA=vB=2, equilibrium prices are: PA= %f, PB = %f; time elapsed is %f.', p(1,1),p(2,1), elapsedTime_p2);
disp(P2);



%% Problem 3

v = [2;2];
p = [1;1];


fVal_foc = @(p) bertrandfoc(p,v);
fVal_focg = @(p,g) bertrandfocg(p,v,g);

tic
for iter = 1:maxit

    fval = fVal_foc(p);
    if norm(fval) < tol
        break
    end
    fprintf('iter %d: p(1)=%f, p(2)=%f, norm(f(x))=%.8f \n', iter, p(1),p(2),norm(fval));

    % set pOld
    pOld = [2;2];
    pA_Old = pOld(1,1);

    % compute the LHS of FOC for good A price
    fOld_1 = fVal_focg(pOld,1);

    % for given pB, solve the first equation for pA
    % We use Secant Method
    for iter_1 =1:maxit
        fval_1 = fVal_focg(p(1,1),1);
        if abs(fval_1) < tol
            break
        else
            pA_New = p(1,1) - ( (p(1,1) - pA_Old) / (fval_1 - fOld_1) ) * fval_1;
            pA_Old = p(1,1);
```

```
97              p(1,1) = pA_New;
98              fOld_1 = fval_1;
99          end
100     end
101
102     % Use the solution for pA obtained above, solve for pB
103     pB_Old = pOld(2,1);
104     fOld_2 = fVal_focg(pOld,2);
105     for iter_2 = 1:maxit
106         fval_2 = fVal_focg(p,2);
107         if abs(fval_2) < tol
108             break
109         else
110             pB_New = p(2,1) - ( (p(2,1) - pB_Old) / (fval_2 - fOld_2) )
                    * fval_2;
111             pB_Old = p(2,1);
112             p(2,1) = pB_New;
113             fOld_2 = fval_2;
114         end
115     end
116
117 end
118 elapsedTime_p3 = toc;
119
120 P3 = sprintf('Problem3: for vA=vB=2, equilibrium prices are: PA= %f, PB
    = %f; time elapsed is %f.', p(1,1),p(2,1), elapsedTime_p3);
121 disp(P3);
122
123 %% Problem 4
124 clear all
125
126 clear all
127
128 v = [2;2];
129
130 p = [1;1];
131
132 fVal_bertrand = @(p) bertrand(p,v);
133 fVal_foc = @(p) bertrandfoc(p,v);
134 i_fVal_foc = fVal_foc(p);
135
136 maxit = 100;
137 tol = 1e-6;
138
139 tic
140 for iter = 1:maxit
141     fnorm = norm(i_fVal_foc);
142     fprintf('iter %d: p(1)=%f, p(2)=%f, norm(f(x))=%.8f \n', iter, p(1)
```

```
        ,p(2),norm(i_fVal_foc));
    if norm(i_fVal_foc)<tol
        break
    end
    p_next = 1./ ([1;1] - fVal_bertrand(p) );
    p = p_next;
    i_fVal_foc = fVal_foc(p);
end
elapsedTime_p4 = toc;

P4 = sprintf('Problem4: for vA=vB=2, equilibrium prices are: PA= %f, PB
    = %f; time elapsed is %f.', p(1,1),p(2,1), elapsedTime_p4);
disp(P4);

%% Problem 5

vB_5 = [0:.2:3];
v_5 = [2*ones(1,size(vB_5,2));vB_5 ];
result = [vB_5; ones(1,size(vB_5,2)); ones(1,size(vB_5,2)) ];

for vindex = 1:size(vB_5,2)

    p = [1;1];
    v = v_5(:,vindex);

    fVal_foc = @(p) bertrandfoc(p,v);
    i_fVal_foc = fVal_foc(p);
    iJac = eye(size(p,1));

    maxit = 100;
    tol = 1e-6;

    for iter = 1:maxit
        fnorm = norm(i_fVal_foc);
        %fprintf('iter %d: p(1)=%f, p(2)=%f, norm(f(x))=%.8f \n', iter,
            p(1),p(2),norm(i_fVal_foc));
        if norm(i_fVal_foc)<tol
            break
        end
        d = -(iJac*i_fVal_foc);
        p = p + d;
        fOld_foc = i_fVal_foc;
        i_fVal_foc = fVal_foc(p);
        u = iJac*(i_fVal_foc - fOld_foc);
        iJac = iJac + ( (d-u)* (d'*iJac) )/(d'*u);
    end
    result(2,vindex) = p(1);
    result(3,vindex) = p(2);
```

```
end

plot(vB_5,result(2,:),vB_5,result(3,:))
title('Equilibrium prices along with change in V_B')
xlabel('v_B')
ylabel('Equilibrium Price: P_A,P_B')
legend('P_A','P_B')

diary off
```