Stability for explicit schemes

Consider in 1D, with finite differences, an explicit scheme applied to

1) $u_t = C u_x$

and 2) $u_t = D u_{xx}$

> What are the stability requirement for Δt with respect to h ?

The schemes are

1) $\dfrac{u_j^n - u_j^{n-1}}{\Delta t} = C \dfrac{u_{j+1}^{n-1} - u_j^{n-1}}{\Delta h}$

and 2) $\dfrac{u_j^n - u_j^{n-1}}{\Delta t} = D \dfrac{u_{j+1}^{n-1} - 2u_j^{n-1} + u_{j-1}^{n-1}}{h^2}$

Requirement is

1) $\Delta t \leq Ch$

2) $\Delta t \leq Dh^2$

Can be shown with (e.g.)
von Neumann stability analysis (Mat 3360)

For finite elements the stencils
are different.

We still expect that

1) $\Delta t \sim Ch$ for transport

2) $\Delta t \sim Dh^2$ for diffusion