

MANDATORY EXERCISE

Exercise 1

Consider the following equation on the domain $\Omega = (0, 1)^2$:

- (1) $-\mu\Delta u + u_x = 0$ in Ω ,
- (2) $u = 0$ for $x = 0$,
- (3) $u = 1$ for $x = 1$,
- (4) $\frac{\partial u}{\partial n} = 0$ for $y = 0$ and $y = 1$

- a) Derive an expression for the analytical solution.
- b) Compute the numerical error for $\mu = 1, 0.3, 0.1$ at $h = 1/8, 1/16, 1/32, 1/64$.
- c) Compare against the expected error estimate, that is; assume:

$$\|u - u_h\|_1 \leq C_\alpha h^\alpha$$

and

$$\|u - u_h\|_0 \leq C_\beta h^\beta.$$

That is, the error estimates in the H^1 and $L^2 = H^0$ norms. Estimate C_α , C_β , α and β and check whether the expected error estimate is valid.

- d) Implement the Streamwise Upwinding Petrov-Galerkin (SUPG) method and compare against the results in b) and c).

Exercise 2

Consider the famous benchmark of " Schäfer, Michael, et al. "Benchmark computations of laminar flow around a cylinder." Flow simulation with high-performance computers II. Vieweg+ Teubner Verlag, 1996. 547-566".

- a) Implement a solver for the benchmark problem in FEniCS based on both a fully explicit time discretization and a semi-implicit discretization. Use piecewise linear for the pressure and both piecewise linear and quadratic elements for the velocity (command line option)

- b) Assess the stability requirement of both schemes, i.e., what is C, β in $\Delta t \leq Ch^\beta$ that make sure that the scheme is stable.

- c) Compute pressure difference and drag. Assess to what extent the numerical value approaches the true value.

Deadline: March 15. Include code. Typesetting in \LaTeX is preferred.
