1)

1) $$u^* = u^n + \Delta t\,(-u^n \cdot \nabla u^n) - \frac{1}{\rho}\nabla p^n + \nu \Delta u^n + f^n$$

2) $$-\nabla^2 \phi = -\frac{\rho}{\Delta t}\,\nabla \cdot u^*$$

3) $$u^{n+1} = u^* - \frac{\Delta t}{\rho}\,\nabla \phi$$

4) $$p^{n+1} = p^n + \phi$$

Variational ~~FEM~~ formulation

Find u^* in V such that

$$\int_\Omega u^* v \, dx = \int_\Omega \left(u^n + \Delta t\left(-u^n \cdot \nabla u^n - \frac{1}{\rho}\nabla p^n + \nu \Delta u^n + f^n\right)\right) \cdot v \, dx$$

$$\forall \; v \in V.$$

FEM formulation

Find $u^* = \sum_i u_i N_i$ such that

$v = N_j$

$$\int_\Omega u^* N_j \, dx = \int_\Omega \left(u^n + \Delta t \left(-u^n \cdot \nabla u^n - \frac{1}{\rho} \nabla p^n + \nu \Delta u^n + f^n \right) \right)$$

$$\int_\Omega u^* N_j \, dx = \int \sum_i u_i N_i N_j \, dx$$

$$= \sum u_j \int N_i N_j \, dx$$

$$= M u$$

where M is the mass matrix, u is the coefficient

Mass matrix entries

$$M_{ij} = \int_\Omega N_i N_j \, dx$$

$$b_j = \int_\Omega \left(u^n + \Delta t\left(-u^n \cdot \nabla u^n - \frac{1}{\rho} \nabla p^n + \nu \Delta u^n + f^n\right)\right) N_j$$

Solve

$$Mu = b.$$

We often use continuous finite elements like Lagrange elements of order 1.

N_i is then a linear polynomial ▯

What about

- $-u^n \cdot \nabla u^n$
- $-\Delta u^n$

?

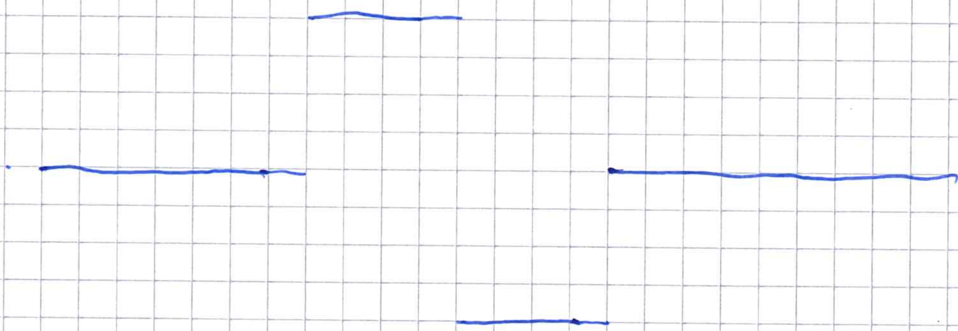
If u^h is continous and linear

Eg

$u^h =$

then

$\frac{\partial u^h}{\partial x} =$



$u^h \cdot \nabla u^h$ is linear and discontinous.

What about

$-\Delta u^h$?

It is zero inside the ~~elem~~ elements and Dirac delta at the element faces.

$\Rightarrow$ Piecewise - integration must be done on the

$$\int -\nu \Delta u^h \cdot N_j$$

term