MANDATORY EXERCISE

Exercise 1
Consider the following equation on the domain $\Omega = (0,1)^2$:

$$-\mu \Delta u + u_x = 0 \text{ in } \Omega, \tag{1}$$

$$u = 0 \text{ for } x = 0, \tag{2}$$

$$u = 1 \text{ for } x = 1, \tag{3}$$

$$\frac{\partial u}{\partial n} = 0 \text{ for } y = 0 \text{ and } y = 1 \tag{4}$$

a) Derive an expression for the analytical solution.
b) Compute the numerical error for $\mu = 1, 0.3, 0.1$ at $h = 8, 16, 32, 64$.
c) Compare against the expected error estimate, that is; assume:

$$\|u - u_h\|_1 \le C_\alpha h^\alpha$$

and

$$\|u - u_h\|_0 \le C_\beta h^\beta$$

Estimate C_α, C_β, α and β and check whether the expected error estimate is valid.
d) Implement the Streamwise Upwinding Petrov-Galerkin (SUPG) method and compare against the results in b) and c).

Exercise 2
Consider the famous benchmark of " Schäfer, Michael, et al. "Benchmark computations of laminar flow around a cylinder." Flow simulation with high-performance computers II. Vieweg+ Teubner Verlag, 1996. 547-566".
a) Implement a solver for the benchmark problem in FEniCS based on both a fully explicit time discretization and a semi-implicit discretization. Use piecewise linear for the pressure and both piecewise linear and quadratic elements for the velocity (command line option)

b) Assess the stability requirement of both schemes, i.e., what is C, β in $\Delta t \le C h^\beta$ that make sure that the scheme is stable.

c) Compute pressure difference and drag. Assess to what extent the numerical value approaches the true value.

Deadline: March 15. Include code. Typesetting in LaTeX is prefered.
