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ENERGY DEPENDENT PARTIAL-WAVE ANALYSIS OF $K^-p \to \Lambda\pi^0$ BETWEEN 1 540 AND 2 215 MeV*

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An energy-dependent partial-wave analysis of $K^-p \to \Lambda\pi^0$ has been performed over the energy range 1537 to 2215 MeV, using the world data sample for $d\sigma/d\Omega$ and $(d\sigma/d\Omega)P$ in the form of Legendre polynomial coefficients Improved parameters for the F15(1920) resonance have been obtained. E_R = 1920 MeV, Γ = 102 MeV, t = –0.09. In addition to the established D13 (1660), D15 (1765), and F17 (2040) states, resonance parameters are given for possible Σ resonances in S11(1700), D13(1950), P11(1670), S11(2000), F15(2250), P11(1985) and P13(1925). Details of the analysis are presented and tentative SU(3) assignments are discussed. A further paper will discuss the uniqueness of these solutions.

1. Introduction

Earlier experimental studies of the reaction $K^-p \to \Lambda\pi^0$ in the intermediate energy range below 2 GeV/c have shown evidence for s-channel resonant structures in the partial waves [1,2]. The dominant Σ resonances in D5(1765) and F7(2040) have been well established, but the resonant structure near 1900 MeV has been notably difficult to ascertain. The dominant structure in this energy region is the F5(1920), whose properties are of interest to several models of the strong interaction [3]. In addition, other isospin-one resonant states have been suggested near this energy [4] ***.

Using the new data of ref. [5] between 1865 and 2106 MeV, in conjunction with previous data, we have performed an energy-dependent partial-wave analysis of $K^-p \to \Lambda\pi^0$ over the broad energy interval 1537 to 2215 MeV. Legendre polynomial expansion coefficients have been used in order to facilitate the fitting procedure and reduce the total number of data points. Resonant structure was assumed for the D5(1765) and F7(2030). Consistent resonant structures appeared in

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*** The sign conventions for resonant couplings follow the convention of this compilation.

D3(1660) and F5(1920). Attempts to find alternative "non-resonant" behavior in the F5 partial wave were not successful.

A variety of resonant hypotheses were extensively tested. The number of acceptable solutions with reasonable χ^2 depended on the size of the energy interval over which the fits were performed. Differences among the fits obtained were primarily in the lower partial waves, which are not well constrained by the data. The statistical quality and energy resolution of the published data did not permit searching for resonant structures of less than 30 MeV total width. Resonance parameters are given for states which were consistent features of our best solutions over the entire energy range from 1537 to 2215 MeV. These include the following states: D5(1774), F7(2042), F5(1920), D3 (1659), S1(1697), D3(1949), P1(1668), S1(2004), F5(2251), P1(1985), and P3(1925).

Details of the analysis and tentative SU(3) assignments for the proposed Σ resonances are presented. The uniqueness of these resonant states is discussed in a companion paper [6].

2. Partial-wave formalism

In a reaction $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$, such as $K^-p \to \pi^0\Lambda$, the observables $d\sigma/d\Omega$ and $d\sigma/d\Omega\, P$ are related to the partial waves, $T_{l\pm}$, by the following formalism, which applies for an unpolarized target:

$$d\sigma/d\Omega = \frac{k_\pi}{k}\,[|f|^2 + |g|^2],$$

$$\frac{d\sigma}{d\Omega}P = \frac{2k_\pi}{k}\,\hat{n}\,\mathrm{Im}\,(fg^*),$$

where k and k_π are the center of mass momenta for the beam and the pion respectively; n is the normal to the production plane:

$$\hat{n} = \frac{k \times k_\pi}{|k \times k_\pi|}\,.$$

The amplitudes f and g are the familiar spin non-flip and spin flip amplitudes. They are related to the partial waves:

$$f(s, \cos\theta) = \frac{1}{\sqrt{k \cdot k_\pi}} \sum_l \,[(l+1)\,T_{l^+} + lT_{l^-}]\; P_l(\cos\theta),$$

$$g(s, \cos\theta) = \frac{1}{\sqrt{k \cdot k_\pi}} \sum_l \,[T_{l^+} - T_{l^-}]\; P_l^1(\cos\theta),$$

where s is the center-of-mass energy squared, $\cos\theta$ is the cosine of the c.m. scattering angle, $P_l^1 = (\sin\theta)\,(d/d\cos\theta)\,P_l\,(\cos\theta)$, P_l is the lth Legendre polynomial, and l is the orbital angular momentum with the superscript indicating a total angular momentum $J = l \pm \frac{1}{2}$.

The experimental angular distribution and polarization at each energy may be expanded in a Legendre polynomial series.

$$\frac{d\sigma}{d\Omega} = \lambda\!\!\!^{-2} \sum_n A_n P_n(\cos\theta),$$

$$P\, d\sigma/d\Omega = \hat{n}\, \lambda\!\!\!^{-2} \sum_n B_n P_n^1(\cos\theta),$$

with $\lambda\!\!\!^{-} = 1/k$. This expansion yields a well-known binomial relation between the experimental coefficients, A_n, B_n, and the partial waves [7].

To determine the partial waves a parametrization is chosen for each $T_{l\pm}$, and a χ^2 function is constructed from calculated quantities, C_n, the experimental Legendre coefficients ϕ_n, and their erros, $\Delta\phi_n$,:

$$\chi^2 = \sum_n (C_n - \phi_n)^2/(\Delta\phi_n)^2 .$$

The C_n are calculated from the relations given above. A variable metric method of minimization using the code VARMIT has been used to minimize the χ^2 with respect to the variable parameters. Details of the iterative procedure, which uses analytic derivatives of the χ^2 function, may be found in ref. [8].

3. Energy dependent parametrizations

The most general parametrization used for a partial wave was

$$T_{l\pm} = \frac{(\sqrt{\Gamma_r \Gamma_e}/2)\, e^{i\phi}}{E_R - E - \frac{1}{2} i\Gamma} + B_{l\pm},$$

where the first term is the non-relativistic Breit-Wigner form and the second term represents the nonresonant "background." The parametrizations had the following forms, where E is the c.m. energy and the q_i are fit parameters.

For backgrounds $B_{l\pm}$ was parametrized as either

$$\text{P: } (q_1 + q_2 E + q_3 E^2)\, e^{i(q_4 + q_5 E + q_6 E^2)}$$

or

$$\text{V: } q_1 (E - q_2)^2 e^{i(q_3 + q_4 E + q_5 E^2)}$$

For resonances the parametrization was

$$\text{B.W.} : \frac{q_1 q_3 d_1(E) e^{iq_4}}{(q_2 - E) - id_2(E) q_3} .$$

It should be noted that the above backgrounds are capable of exhibiting "resonant-like" behavior. The V background is constrained to vanish analytically for $E < q_2$. To represent more than one resonance in a partial wave, additional Breit-Wigners were simply added.

In the Breit-Wigner form the energy dependence of the partial width was given by the following relations.

$$v_i(E) = \frac{k_i}{E} \left(\frac{k_i^2}{k_i^2 + X^2} \right)^{l_i} ,$$

$$\Gamma_i = \gamma_i v_i(E) = \alpha_i \gamma \, v_i(E),$$

$$\Gamma = \sum_i \Gamma_i = \gamma \sum_i \alpha_i v_i(E).$$

The parameters were related by

$$q_1 = \sqrt{\alpha_e \alpha_r}, \quad q_2 = E_R, \quad q_3 = \tfrac{1}{2}\gamma ,$$

$$d_1(E) = \sqrt{v_e(E) \, v_r(E)},$$

$$d_2(E) = \sum_i \alpha_i v_i(E),$$

where k_i is the c.m. momentum for the ith outgoing channel, l_i is its orbital angular momentum, X is a mass related to the radius of interaction ($X = 0.35$ GeV was used), and γ_i is the reduced partial width. We have used the Glashow-Rosenfeld form factor for $v_i(E)$, where k_i/E is the two-body phase-space energy dependence and the remainder is the angular momentum barrier form factor [9]. We have estimated the energy independent branching fractions α_i into six channels in order to let $d_2(E)$ approximately represent the full width energy dependence. (Note that q_1 is independent of our choice of α_i, so long as the α_i have reasonable values.) The functions d_1 and d_2 depend only on the energy and appear as constants in the fitting. Other analyses using similar parametrizations have indicated that the results of resonant fits do not depend on either the precise form of the energy dependence $v_i(E)$ or the specific values chosen for X and α_i [2a, 10].

4. Fitting procedures and results

In order to determine the partial waves, we have compiled over 1 100 Legendre coefficients from refs. [1, 2, 5, 11]. These are shown in figs. 1 and 2. To save com-

Table 1

Summary of five energy dependent fits — 1540-2215 MeV *, †, **

	Partial waves									Average coefficients		Original coefficients	
	S1	P1	P3	D3	D5	F5	F7	G7	G9	χ^2/D. F.	$\langle\chi^2\rangle$/D.P.	χ^2/D. F.	$\langle\chi^2\rangle$/D. P.
Fit H	P(4) +R(4)+R(4)	P(4) +R(4)+R(4)	P(4) +R(4)	P(4) +R(3)+R(4)	V(5) +R(3)	V(5) +R(4)	R(4)	V(5)		444/419	0.91	1299/1011	1.20
Fit K	P(4) +R(4)+R(4)	P(4)+R(4)	P(6)	P(4) +R(4)+R(4)	V(5) +R(3)	V(5) +R(4)	R(4)	V(5)		457/424	0.93	1326/1016	1.23
Fit R	P(4) +R(4)+R(4)	P(6)	P(6)	P(4) +R(3)+R(4)	V(5) +R(3)	V(5) +R(4)	R(4)	V(5)		458/427	0.93	1317/1019	1.22
Fit HF	Identical to Fit H except resonance substituted for F5 background					R(4) +R(4)	R(4)	V(5)		481/420	0.99	1361/1012	1.26
Fit AH	P(4) +R(4)+R(4)	P(4) +R(4)+R(4)	P(4) +R(4)	P(4) +R(3)+R(4)	R(3) +V(5)	R(4) +R(4)	R(4)	V(5)	V(5)	444/415	0.91	1292/1007	1.20

*DF = Degrees of freedom = DP - No. variable parameters.
DP = Legendre coefficient data points: 74 A_0, 531 A's, 475 B's total.

For average DP: 33 A_0's, 231 A's, 224 B's were used.

†P(4) - polar background P with 4 variable parameters.
V(5) - vanishing background V with 5 variable parameters.
R(3) - resonant B. W. with 3 free parameters (phase fixed).

**Approximately 20 "best" solutions were found with χ^2/average D. P. < 1.0 and χ^2/original D. P. < 1.3. These five "best" fits represent the most consistent features of the partial waves. Fitting over a smaller energy range gave many more "good" fits with acceptable χ^2.

Table 2

Best values for energy dependent resonant parameters. *, †, **, ‡

Wave	$\sqrt{\alpha_e \alpha_r}$	$\gamma/2$ (GeV)	E_R (MeV)	Γ (MeV)	t	ϕ (radians)	Status
F17	0.16	0.63	2042± 11	178± 13	+0.20±0.01	$0.15^{+0.10}_{-0.16}$	Established
D15	0.22	0.70	1774± 10	146± 18	$-0.28^{+0.04}_{-0.05}$	3.14 fixed	Established
F15	0 08	0.44	1920^{+15}_{-20}	102± 18	-0.09±0.02	$3.6^{+0.5}_{-0.7}$	Established
D13	0.085	0.20	1659^{+12}_{-5}	32± 11	+0.09±0.02	0.0 ±0.1	Established
S11	0.085	0.20	1697^{+20}_{-10}	66^{+14}_{-12}	-0.13±0.04	$3.0^{+0.2}_{-0.3}$	Probable
D13	0.05	0.43	1949^{+40}_{-60}	160^{+70}_{-40}	$-0.05^{+0.03}_{-0.02}$	$3.7^{+0.5}_{-1.0}$	Probable
P11	0.08	1.0	1668± 25	230^{+165}_{-60}	$\mp 0.12^{+0.12}_{-0.04}$	1.8 ±0.3	Probable
S11	0.05	0.25	2004± 40	116± 40	$+0.07^{+0.02}_{-0.01}$	-0 4 ±0.3	Possible
F15	0.15	0.43	2251^{+30}_{-20}	192± 30	-0.16±0.03	2.8 ±0.2	Possible
P11	0 04	0.51	1985± 50	220±140	$+0.05^{+0.07}_{-0.02}$	-0.7 ±0.3	Possible
P13	0.05	0.17	1925±200	65^{+50}_{-20}	+0.06±0.04	0.1 ±0.2	Suggested

*Averaged from best fits with ± indicating the range of values in good fits.

†t = amplitude at resonance = $\pm\sqrt{\Gamma_e\Gamma_r}/\Gamma = \pm q_1 q_3 d_1(E_R)/q_3 d_2(E_R)$. Sign indicates SU(3) coupling with baryon first convention.

**Γ = full width = $2\, q_3 d_2(E_R)$.

‡$q_1 = \sqrt{\alpha_e \alpha_r}$ = energy independent branching fraction, $q_3 = \gamma/2$ = reduced width. These parameters are directly related to the SU(3) couplings.

puter time, "average" Legendre coefficients were fitted in 20 MeV intervals over the 675 MeV energy range of the data. Final χ^2's were calculated to both the average coefficients and to the original experimental coefficients and their errors.

In each fit a set of starting values was chosen for each partial wave up to G7 or G9. The established resonant states D5 (1765) and F7(2030) were assumed to behave as Breit-Wigner resonances with initial parameters set to values near those of ref. [4]. All other waves were initially parametrized as backgrounds with the G7 wave constrained to vanish below 1830 MeV. In subsequent fits the only

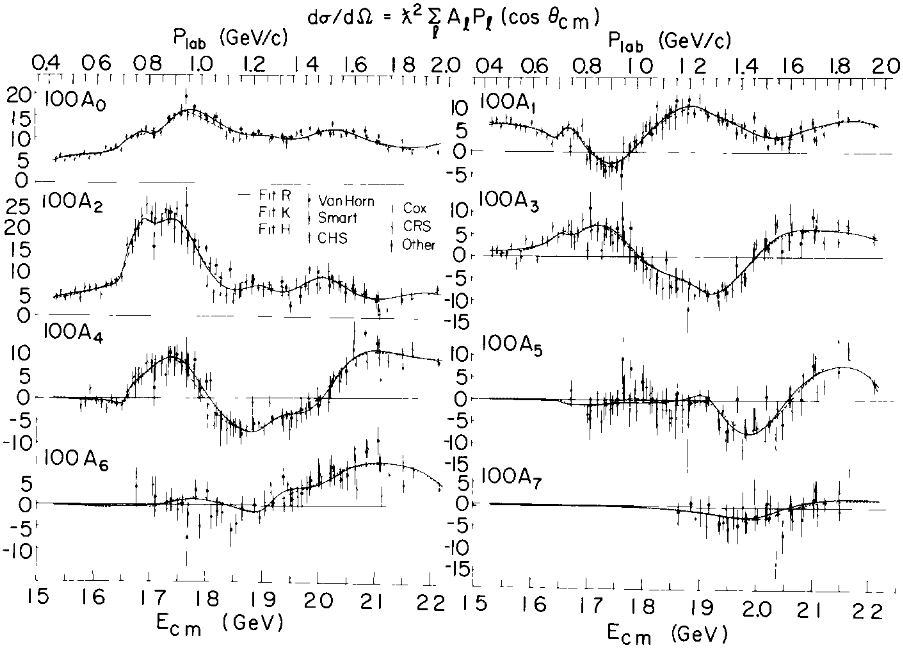

Fig. 1. Legendre expansion coefficients, A_n, for angular distributions as a function of energy from 1 537 to 2 215 MeV. Coefficients from refs. [1c, 2c, 2e, 5, 11] are plotted. The curves are from partial wave fits H, K, and R.

parameter not allowed to vary was the D5 phase parameter, q_4, which was fixed at $\phi = \pi$ by accepted convention [4].

A variety of resonance hypotheses were tried:

(i) To test the presence of the F5(1920) state under the assumption of resonant D5(1765) and F7(2030) states.

(ii) To test resonances suggested in previous $\overline{K}N$ analyses.

(iii) To test resonances predicted by SU(3) and quark models.

(iv) To test "resonant-like" structure appearing in the "background" parametrizations during the fitting procedure.

Several hundred fits were completed on the CDC 7600 computer, and about 20 "best" solutions with χ^2/data point ≈ 1.0 were found over the complete energy range. Acceptable fits were also found over more limited energy intervals, but many of these were incapable of being extended to the entire energy range when used as starting values for new fits. A summary of five representative fits is given in table 1. The Breit-Wigner resonant parameters for the 20 best fits are given in table 2. Argand diagrams of fits H, R, and K are presented in fig. 3, and the fits to the Legendre coefficients are illustrated by the curves in figs. 1 and 2. Additional details may be found in ref. [12]. It can be seen that these fits differ from

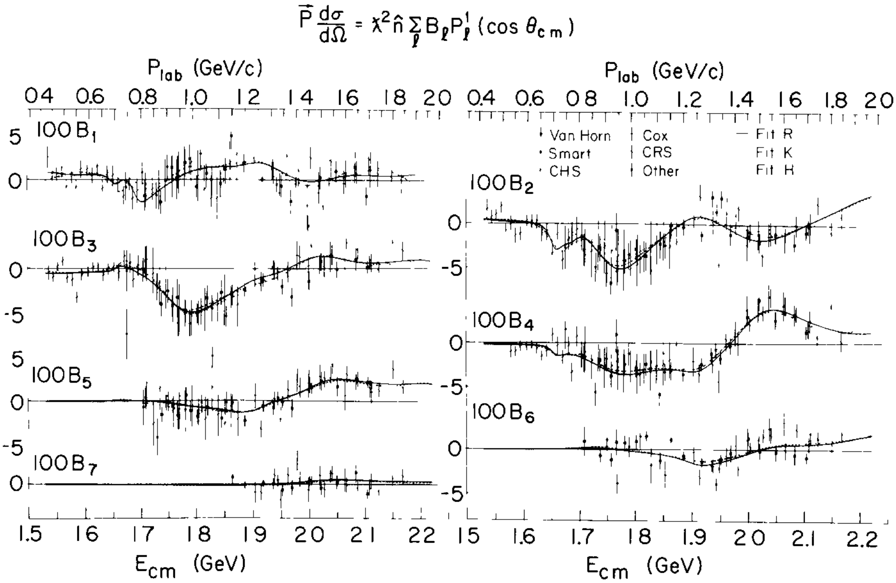

Fig. 2. Associated Legendre expansion coefficients, B_n, for the polarization distributions as a function of energy from 1 537 to 2 215 MeV. Coefficients from refs. [1c, 2c, 2e, 5, 11] are plotted. The curves are from partial wave fits H, K, and R.

one another primarily in the lower partial waves. It is understood that the lower waves are not well constrained by the data because of the peripheral nature of the interaction.

In our fits the phases of possible resonant waves were variable, except for the fixed D5(1765) phase parameter. In cases where there was only a small background in addition to a resonance, the fitted phase parameter was usually close to either 0 or π, the SU(3) prescribed value (table 2). This was also true when the background was predominantly real, as in the case of S1(1 700) (fits H and R, fig. 3a). However, when a "background" contained an appreciable imaginary part, a shift of the resonance phase occurred. In the case of P1(1670) a phase of 1.8 radians was required to account for the A_1 Legendre coefficient by $\boldsymbol{S} \cdot \boldsymbol{P}$ interference. Such a shifting of the resonance phase by background is expected (ref. [13]). However, in order to interpret resonance models and determine SU(3) couplings, it would be useful if a more quantitative understanding of the relation between "background" and resonant phases were available. There is, of course, always the possibility that such phase shifts are caused by heretofore undiscovered resonances.

We will briefly mention some features of our energy-dependent fits:

(a) The S1(1700) remained at this mass in almost all fits. In some analyses more than one S11 state has been suggested near this energy. Two S1 states below 1770 MeV were not compatible with our parametrizations. A possible resonance at higher energies is indicated near 2000 MeV.

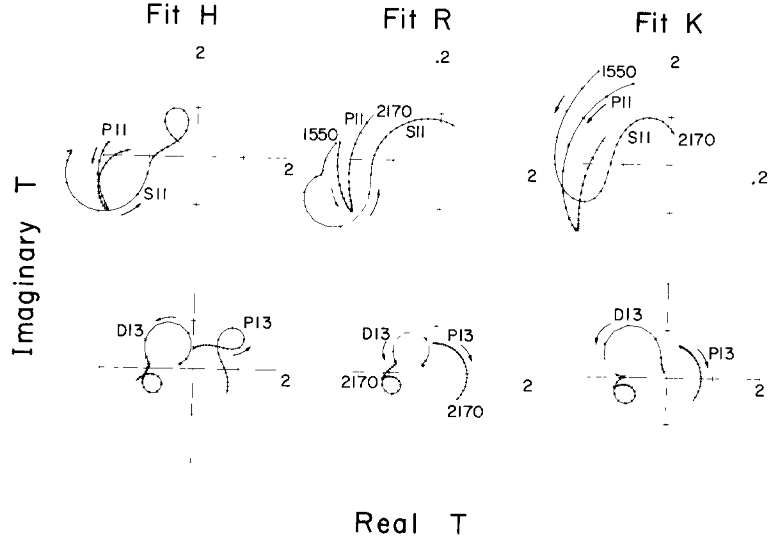

Fig. 3a.

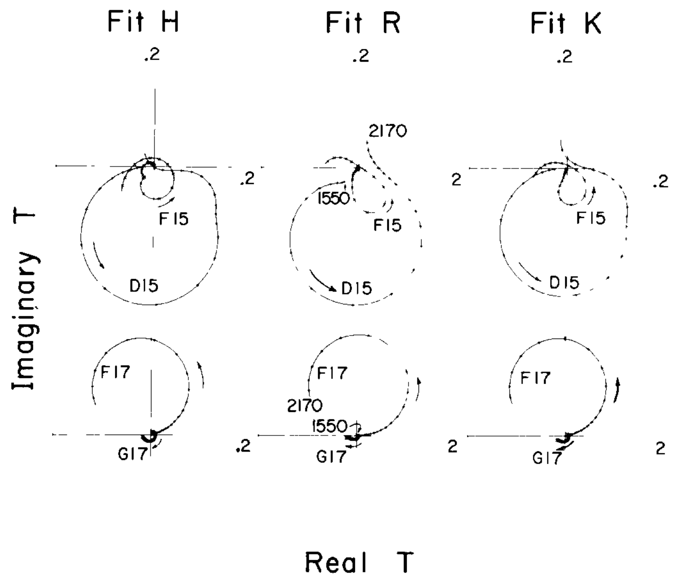

Fig. 3b.

Fig. 3. Argand diagrams for energy dependent fits H, R, and K Isospin-one partial waves are plotted every 5 MeV with symbols placed every 25 MeV from 1550 to 2170 MeV. (a) S1–D3 partial waves. (b) D5–G7 partial waves.

(b) Complicated structure at 1 670 MeV has been seen in both formation and production reactions. The P11(1 670) could account for some of this structure and might be the Σ companion to the N*(1470), the Roper resonance. The P11 state suggested at 1880 MeV (ref. [2a]) was not compatible with the P1 structure required over the complete energy range of our fits. A possible higher P1 resonance emerged at 1 990 MeV.

(c) The P3 wave was essentially featureless, with resonant parameters showing inconsistency among different fits. The best candidate for a P3 resonance is at 1 925 MeV.

(d) The established D3 (1 660) resonance has been a dominant feature of the energy interval below 1700 MeV. The structure of the D3 wave is complicated in the region 1 850 to 2 000 MeV. A resonance is indicated at 1 940–1 980 MeV with another possible resonance suggested between 1860–1900 MeV.

(e) "Non-resonant" behavior of the F5 wave near 1 900 MeV was found to be incompatible with our assumption of D5(1765) and F7(2 040) resonant states. No fits with reasonable χ^2 were obtained without a "resonant-like" F5 near 1 900 MeV. The resonance parameters we have determined for the F5(1 920) are E_R = 1 920 MeV, Γ = 102 MeV, and t = –0.09. This mass is about 5 MeV higher than previous determinations and the width is about 30 MeV greater than most earlier values (ref. [4]). Both the D5 and F5 waves required "backgrounds" in addition to the dominant resonances at 1 770 and 1 920 MeV. The background in F5 could be entirely accounted for by an additional resonance at 2 250 MeV. Production experiments have indicated structure at this energy and do not rule out a spin-parity assignment of $\frac{5}{2}^+$. However, since this state is just above our energy range, we can only suggest its presence.

(f) No background was required in the F7 wave, and little structure was revealed by either the G7 or G9 waves. Since the Legendre A_7 coefficient results from F7 · G7 interference, this lack of structure is not surprising.

5. SU(3) interpretation and conclusions

Although confirmation of the proposed resonant states in table 2 will await analyses of other channels, we make the following tentative assignments of these states to SU(3) multiplets, using the Gell-Mann-Okubo mass formula.

$\frac{1}{2}^-$ S11 (1 700) - possible octet partner of N(1535), Λ(1 670). S11(2 000) - possible decuplet companion of Δ(1 650) or octet partner of N(1 700) and Λ(1 870).

$\frac{1}{2}^+$ P11 (1 670) - possible octet partner of N(1 470), the Roper resonance. Could lie on an exchange degenerate Regge trajectory with D3 (1 950) and F5 (2 250) proposed in this analysis.

P11 (1 990) - possible octet partner of N(1 780) or decuplet Δ(1 910).

$\frac{3}{2}^+$ P13 (1 925) - possible octet partner of N(1 690)–1 850).

$\frac{3}{2}^-$ D13 (1 660) - already in established multiplet with N(1 520) and Λ(1 690).

D13 (1 950) - possible octet with N(1700). Could lie on trajectory with proposed P11 (1 670) and F5 (2 250).

$\frac{5}{2}^-$ D15 (1 765) - established octet with N(1 670), Λ(1 830).

$\frac{5}{2}^+$ F15 (1 920) - established octet with N(1 688), Λ(1 815). F15 (2 250) - possible octet with N(2 000).

$\frac{7}{2}^+$ F17 (2 040) - established decuplet with Δ(1 950).

From the results of our analysis, we conclude that an F5 resonance at 1 920 MeV is the simplest explanation for the observed structure in this energy interval. We have found no energy dependent fits with reasonable χ^2 without this F5 resonance. We have also confirmed the initial assumption of resonant D5 and F7 partial waves and give our parameters for these states in table 2. These fits have given evidence for the existence of several other resonances in the $\Lambda\pi$ channel, which are also presented in table 2. Because of the importance of these resonances to models of the strong interaction, we have proceeded to study the ambiguities of our partial wave solutions. The uniqueness of these resonant states will be discussed in ref. [6].

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* Only the events with seen spectators were used from these data.