Nuclear Physics B94 (1975) 39–66

ENERGY-INDEPENDENT PARTIAL-WAVE ANALYSIS OF $\bar{K}N \to \Lambda\pi$ BETWEEN 1540 AND 2150 MeV

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Received 17 March 1975

An exhaustive partial-wave analysis of the reaction $\bar{K}N \to \Lambda\pi$ has been carried out on a compilation of all available data using two separate semi-energy-independent methods. Particular attention has been given to finding all ambiguous solutions. Two possible solutions are found consistent with the physical assumptions imposed. The parameters of the resonances appearing in the two solutions are given.

1. Introduction

The reaction $\bar{K}N \to \Lambda\pi$ is the most studied of the strangeness -1 formation channels, yet there is no agreement on even the general form of the amplitudes. Many partial-wave analyses have been performed; refs. [1–4] give a selection of the latest efforts. Two well-known problems have bedevilled these analyses. They are:

(i) Large numbers of mathematical ambiguities would exist even with infinitely precise data because of the incomplete experimental data available.

(ii) The imposition of smoothess on the amplitudes introduces unknown biases.

A solution of the first problem has been attempted using the Barrelet zeros formalism to calculate all possible solutions and then to search amongst them for those are physically acceptable [5]. Because of the large experimental errors there are problems of connecting the zeros that have not yet been solved and no complete analysis has been published. The fact that several incompatible sets of amplitudes have been produced by partial-wave analyses indicates that in practice the ambiguities are a problem.

The majority of analyses of this channel have attempted to solve problem 2 by parametrising the amplitudes as a function of energy (energy-dependent p.w.a.). This has the obvious disadvantage that the form of the amplitudes is determined, *a priori*, by the analist. However, they produce rather simple amplitudes that give good fits to the experimental data.

Two previous attempts have been made at semi-energy-independent analyses of

this channel. One, by Langbein and Wagner [6], was a part of a multichannel analysis and used the shortest path method, the second, by the CHS collaboration [3], imposed continuity by smoothing the experimental data. The analysis of Langbein and Wagner produced very unsmooth amplitudes which have been shown to suffer from bad discontinuities in the Barrelet zero trajectories [7].

In this paper we present a new semi-energy-independent analysis of this channel including all published data [8–16] plus new data from the CHS collaboration [17]. Special attention has been paid to the search for ambiguous solutions by studying the Barrelet zeros and new techniques have been applied to ensure smoothness of the solutions. Two different methods of analysis have been used, producing similar results. Using a minimum structure criterion, i.e. rejecting solution variations that introduce extra structures in the amplitudes while producing little or no improvement of the fit to the data, two ambiguous solutions are found in the region below 1750 MeV. Between 1750 and 2050 MeV there appears to be only one single unique solution within the physical assumptions made. Finally fits have been made to the amplitudes found in the energy independent analyses using expressions of the type used in the energy dependent analyses, i.e. resonance plus background terms, and the parameters of the resonances required are given.

2. Semi-energy-independent partial-wave analysis

2.1. General methods

It is well-known that if only differential cross sections and polarisations are measured in our reaction there exists an infinite set of partial-wave solutions that fit the data equally well. These can be reduced to a finite set by cutting the partial-wave series at a maximum spin J and orbital angular momentum L. Examination of the data shows that over most of energy range studied in this analysis the cut-off may be made at the F7 amplitude but even this leaves a total of 64 (2^6) mathematically ambiguous solutions. The formalism of Barrelet [18] provides a convenient method of studying these ambiguities.

Clearly to enable further progress some assumptions are required on the physical content of the amplitudes. The usual assumption, and that made in this analysis, is that certain leading resonances may be parametrised in a Breit-Wigner resonance form. Below 1850 MeV the D15 $\Sigma(1765)$ has been so parametrised and in the high-energy region the F17 $\Sigma(2030)$. It will be shown below that this assumption, together with the condition that the amplitudes and the Barrelet zeros should have minimum structure, yields a unique solution between 1750 and 2050 MeV. Below 1750 MeV two solutions are found, one of which is preferred for reasons discussed below. Above 2050 MeV the solution depends weakly on the identity of the next leading resonance, the $\Sigma(2250)$.

The analysis is divided into two stages:

(a) Single energy fits at several points in the energy range.

(b) Using these results as starting values the solutions are continued in energy using the solution at one energy point as the starting values of the fit at a neighbouring energy.

2.2. Single-energy solutions

Single-energy fits were made at 3 points in the momentum range: at 0.453 Gev/c, 0.943 GeV/c, the peak of the $\Sigma(1765)$, and 1.519 GeV/c, the peak of the $\Sigma(2030)$.

At 0.453 GeV/c the absence of significant legendre A and B coefficients of order 3 and above implies that only S and P waves need be considered. With this restriction 4 distinct solutions are obtained. These are found to be the Barrelet ambiguities of each other and are the expected number of solutions obtainable by permuting the two zeros.

At 0.943 GeV/c coefficients up to the 4th order are significant, therefore D-waves are necessary but F-waves are not. 16 distinct solutions are expected from the 4 Barrelet zeros. However, fixing the D5 amplitude to the value expected from a Breit-Wigner with the accepted parameters reduced this to one unique low-chi-squared solution.

At 1.519 GeV/c coefficients up to 6th order are present giving a possible total of 64 ambiguous solutions. Imposing the $\Sigma(2030)$ with a resonant amplitude of 0.18 reduced these to 4 ambiguities.

2.3. Energy continuation

Two entirely separate programs using different methods of analysis were used. The first method (I) is a development of the CHS energy-independent analysis described in ref. [3] and involved preliminary smoothing of the experimental data. The second method (II) is presented here for the first time and uses the original data points.

In both methods it was found that just using the best values of the amplitudes at one energy point as starting values for the next minimisation was not sufficient constraint. The programs could slip into a different minimum instead of counting in the valley corresponding to the original minimum. In order to counteract this effect an extra χ^2 (χ^2_C) was added to the fit depending on the distance the new solution moved away from the old.

$$\chi^2_C = \sum_i \frac{T_i^{l\pm} - T_{i-1}^{l\pm}}{\sigma^2},$$

where $T_i^{l\pm}$ are the amplitudes at the ith energy and σ is a constant of the order of the expected change in the amplitudes from point to point.

The fits were insensitive to the value of σ provided that if too large a value of χ^2_C

was found the minimisation was repeated using the new values of $T^{l^{\pm}}$ as the constraining point and the process iterated until χ^2_C is small ($\leqslant 0.5$). Thus the procedures find the nearest possible acceptable solution and ensure a minimum path solution.

As already mentioned, a selection amongst the many mathematically ambiguous solutions is made by imposing two conditions:

(i) Imposition of a Breit-Wigner resonance amplitude on the two dominant amplitudes, D15 in the region of the $\Sigma(1765)$ and F17 in the region of the $\Sigma(2030)$. This was done by imposing in each single-energy fit the appropriate values of these amplitudes from the Breit-Wigner resonance form. The exact parametrisation of these amplitudes is given in sect. 3. Method II uses the normal energy-dependent Breit-Wigner form including the extra refinement of taking the energy dependence of the total width as the sum of the energy dependences of the partial widths as described in sect. 3.

Method I used the earlier method of approximating the energy dependence of the total width to be that of the elastic channel. Some differences were observed between the two types of parametrisation, principally Method II gave larger $\Lambda\pi$ amplitudes at low energies and continued the amplitude better when the Breit-Wigner constraint was removed from the D_{15} amplitude at around 1.1 GeV/c.

Method II also compensates for our uncertain knowledge of the parameters of the resonances by allowing the amplitudes to move away from the starting values given by the Breit-Wigner form within the constraints on the amplitudes described above. An iterative procedure was adopted; best known parameters were used in a fit, then the D_{15} or F_{17} resultant amplitudes were separately refitted as in sect. 3 to give better parameters for a new version of the partial-wave fit. In method I the values of the amplitudes D_{15} and F_{17} from the Breit-Wigner parametrisation were kept fixed throughout the fitting.

(ii) The smoothness of the Barrelet zero trajectories were checked as described in subsect 2.6.

2.4. Data

All data known to us [8–17] in the incident momentum region 0.450–1.800 GeV/c (centre-of-mass energies 1540–2140 MeV) was used in this analysis, 118 energy points in all.

The treatment of the data is the biggest difference between the two methods. Some screening of the data is necessary since inspection of the measured coefficients show points that are out of line by several standard deviations. In the rather critically balanced process of energy independent partial-wave analysis the sudden jumps in the amplitudes at these points can destroy the continuity of the amplitudes.

Method I solves this problem by an initial smoothing of the data as a function of energy using polynomials to fit each coefficient in turn. Fits are then done to data read off from the polynomials using the errors appropriate to the nearest measured data point. An advantage of the method is that data points at small energy intervals

(5 MeV) may be taken, making the imposition of continuity easier since the amplitudes are not expected to change very much in that range.

Method II uses the original data points with their errors. Because of the constraints on the energy variation of the amplitudes discussed in subsect 2.3, it was found that on going from one data point to the next in energy, one or two amplitudes may move significantly but generally most remain stable. However, at some energy points most of the amplitudes made sudden jumps. Examination of the data invariably showed that at these points one or more of the measured coefficients was badly out of line with the trend of the data. These points were omitted from the searches as being unreliable. A total of 13 out of the 118 data points were discarded in this way.

2.5. Energy-independent fits

Fits using techniques very similar to Method I have been described and published by the CHS collaboration [3]. We describe here in detail only the procedure followed in method II. The results of method I will be given at the end of this section.

Method II

Study of the experimental Legendre coefficients shows that below 1.15 GeV/c only D-waves are required and that G-waves are only significant above 1.6 GeV/c. Thus in our fits the G-waves were fixed to zero below 1.6 GeV/c and the F5 amplitude below 1.15 GeV/c. The F7 amplitude was fixed to the values given by the Breit-Wigner and was not allowed to vary below 1.15 GeV/c. The same was done for the D5 amplitude below 0.65 GeV/c. Finally at the lowest energies only solutions with small D3 amplitudes were accepted. From 1.15 GeV/c the Breit-Wigner starting values were no longer applied to the D5 amplitude as it does not necessarily continue to be described by a resonance form well above the resonance mass. It was then free to move like the other amplitudes.

As the analysis is heavily dependent on the assumption that the leading resonances can be described by Breit-Wigner forms, a careful study was made of how the fits varied as the resonance form was changed. The smoothest amplitudes were found when the momentum dependence of the total widths were parametrised as the sum of the dependences of the partial widths as described in sect. 3. This gives larger $\Lambda\pi$ amplitudes at low energies than the conventional form. Indeed, when the latter were used, the amplitudes were found to be pulled larger in these same regions.

Searches were started from each of the solutions found at 0.453 GeV/c. Two of the solutions could be connected to the one solution at 0.954 GeV/c, both following closely the D5 Breit-Wigner. The paths from the other two starting solutions did not reach the 0.953 GeV/c solution but the D5 amplitude was pulled strongly towards lower resonance amplitudes. All the amplitudes in these two fits were much more structured than the first two and could be rejected on a minimum structure hypothesis as well as for their lack of a Breit-Wigner resonance. This effect is expected since if the D5 amplitude is reduced then it has to be compensated by the addition of

complicated structure in the other amplitudes to reproduce the effects of the single large resonance.

Searches were also started from the 4 solutions found at 1.519 GeV/c moving downwards in energy. Only one of the solutions linked with the solution at 0.953 GeV/c and this reproduces rather well the upper half of the $\Sigma(1765)$ Breit-Wigner although this was not imposed above 1.15 GeV/c. Continuing the low-energy solutions up from 0.953 GeV/c also gave this same solution.

Above the peak of the $\Sigma(2030)$ the situation is complicated by our lack of knowledge of the next leading resonance, the $\Sigma(2250)$. This has been claimed to be either G7 or G9 [19]. G-waves begin to be significant at 1.6 GeV/c but only one of G7 or G9 is required to give good single-energy fits. Introduction of only the G7 gave an amplitude that was predominantly imaginary whereas the G9 amplitude was predominantly real. G9 was consistant with the tail of a resonance above our energy region while G7 was not. Therefore the G9 alternative was chosen. The other amplitudes are changed in detail if G7 is taken but not in general outline.

With the above choice of amplitudes the Legendre A and B coefficients are well fitted over the complete energy range. Between 0.9 and 1.6 GeV/c a unique solution is claimed, subject of course to the assumptions described above. Below 0.9 GeV/c there are two ambiguous solutions. The main distinguishing feature is that solution 1 has a large P3 amplitude, solution 2 a small one. In the recently performed analysis of the $\Sigma(1580)$ in this channel [20] the P3 amplitude was required to be large. The background amplitudes of that analysis (S1, P1 and P3) agree very well with the amplitudes found in solution 1. The amplitudes of solution 2 give a bad fit in this small region, a χ^2 of approximately 100 for solution 2 against 60 for solution 1 for 45 data points. The distinguishing feature is the interference with the D3 resonance amplitude in the same way that the interference with the D5 amplitude at higher energies has been used to discard several solutions. However, at present this is the only means of choosing between the two and cannot be considered absolutely overwhelming. Therefore we give both solutions in case future evidence should prefer solution 2.

A choice might in principle be made by continuing the solution down towards threshold. The situation is complicated by the presence of the $\Sigma(1385)$ below threshold which is expected to have a strong effect in the P3 amplitude. It is unfortunately so far below threshold that its contribution in the physical region is difficult to estimate. However, a Breit-Wigner form with reasonable angular momentum barrier factors and a $\bar{K}N$ partial width calculated from the $\Lambda\pi$ width by SU(3) can give a P3 amplitude near 450 MeV/c of the order of magnitude of that observed in solution 1 and in the correct part the Argand diagram. This is not a good test for solutions, however, since the distance from resonance is so far that clearly an extrapolation of solution 2 could also be devised which would be consistent with the $\Sigma(1385)$.

The measured amplitudes are given in table 1 for solution 1 and table 2 for that part of solution 2 that differs from solution 1. The energy continuation process

Table 1
The real and imaginary parts of the particle-wave amplitudes together with their errors for solution 1; the values are given at the momenta and energies of the data points used

MOMENTUM GEV/C	ENERGY GEV	AMPLITUDE S11	P11	P13	D13	D15	F15	F17	G19
.455	1.544	-.13 .11	-.04-.05	-.12 .18	.01 .03	-.02-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.00 .00	.00 .00	.00 .00	.00 .00
.475	1.552	-.10 .08	-.04-.04	-.11 .16	-.00-.01	-.02-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.02 .02	.00 .00	.00 .00	.00 .00	.00 .00
.514	1.569	-.11 .10	-.03-.01	-.12 .16	.02 .04	-.03-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.02 .02	.00 .00	.00 .00	.00 .00	.00 .00
.534	1.578	-.12 .08	-.03-.04	-.11 .18	.06 .00	-.03-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.00 .00	.00 .00	.00 .00	.00 .00
.554	1.586	-.12 .07	-.02-.00	-.11 .19	.09 .04	-.04-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.02 .02	.00 .00	.00 .00	.00 .00	.00 .00
.573	1.595	-.15 .05	-.01-.02	-.07 .18	-.00 .05	-.04-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .04	.03 .03	.03 .02	.02 .02	.00 .00	.00 .00	.00 .00	.00 .00
.594	1.604	-.14 .04	.01-.04	-.06 .19	.06 .02	-.05-.01	.00 .00	.00 .00	.00 .00
	+-	.03 .04	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00	.00 .00
.597	1.605	-.16 .06	-.03-.03	-.09 .19	.00 .03	-.05-.01	.00 .00	.00 .00	.00 .00
	+-	.03 .04	.03 .03	.03 .02	.02 .02	.00 .00	.00 .00	.00 .00	.00 .00
.617	1.614	-.14 .04	-.01-.04	-.07 .20	.02-.01	-.05-.01	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00	.00 .00
.637	1.624	-.16 .04	-.02-.05	-.07 .23	.00 .03	-.06-.01	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00	.00 .00
.658	1.633	-.14 .04	-.01-.05	-.07 .21	.01 .05	-.08-.03	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.02 .02	.02 .02	.00 .00	.00 .00	.00 .00
.668	1.638	-.14 .04	-.01-.08	-.06 .23	-.02 .01	-.03-.03	.00 .00	.01 .00	.00 .00
	+-	.03 .04	.04 .04	.04 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.677	1.642	-.14 .03	-.02-.07	-.06 .23	-.01 .08	-.07-.03	.00 .00	.01 .00	.00 .00
	+-	.03 .04	.04 .03	.03 .03	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00
.699	1.652	-.17 .07	.02-.03	-.06 .20	-.01 .07	-.06-.03	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.708	1.656	-.16 .07	.01-.06	-.06 .20	-.01 .06	-.04-.04	.00 .00	.01 .00	.00 .00
	+-	.03 .04	.04 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.719	1.661	-.19 .05	-.01-.04	-.05 .24	-.02 .06	-.12-.02	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.725	1.664	-.16 .06	.00-.06	-.05 .22	.02 .05	-.07-.06	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.04 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.740	1.671	-.15 .09	.02-.06	-.06 .21	-.05 .08	-.14-.02	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.04 .03	.03 .03	.03 .02	.02 .03	.00 .00	.00 .00	.00 .00
.741	1.672	-.13 .06	-.03-.08	-.05 .23	-.05 .05	-.06-.05	.00 .00	.01 .00	.00 .00
	+-	.03 .04	.03 .04	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.750	1.676	-.15 .05	-.03-.04	-.04 .22	-.06 .05	-.13-.04	.00 .00	.01 .00	.00 .00
	+-	.04 .04	.04 .04	.03 .04	.03 .04	.04 .03	.00 .00	.00 .00	.00 .00
.761	1.681	-.19 .06	-.02-.01	-.04 .23	-.06 .04	-.13-.03	.00 .00	.01 .00	.00 .00
	+-	.03 .04	.04 .04	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.768	1.684	-.18 .06	-.03-.02	-.05 .22	-.06 .05	-.11-.04	.00 .00	.01 .00	.00 .00
	+-	.04 .04	.04 .04	.03 .04	.04 .03	.03 .03	.00 .00	.00 .00	.00 .00
.773	1.687	-.19 .05	-.05 .01	-.06 .22	-.05 .06	-.15-.06	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.793	1.696	-.20 .04	-.07 .02	-.06 .23	-.07 .05	-.15-.04	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.04 .03	.03 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.806	1.702	-.21 .02	-.04 .01	-.03 .23	-.07 .08	-.10-.08	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.04 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.810	1.704	-.22 .01	-.05 .01	-.02 .22	-.09 .06	-.14-.07	.00 .00	.01 .00	.00 .00
	+-	.04 .03	.03 .03	.03 .04	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.820	1.709	-.18 .00	-.06-.01	-.03 .21	-.07 .07	-.13-.07	.00 .00	.02 .00	.00 .00
	+-	.03 .04	.04 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.821	1.709	-.18 .01	-.07-.03	-.05 .21	-.06 .10	-.13-.08	.00 .00	.02 .00	.00 .00
	+-	.04 .04	.04 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.838	1.717	-.18 .02	-.05-.03	-.03 .18	-.11 .07	-.15-.09	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.853	1.724	-.19 .02	-.08-.05	-.06 .18	-.09 .06	-.12-.11	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.865	1.730	-.21 .01	-.08-.01	-.05 .16	-.10 .07	-.14-.17	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.874	1.734	-.17-.01	-.09-.06	-.04 .16	-.11 .05	-.15-.14	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.882	1.738	-.17-.02	-.15-.07	-.04 .13	-.07 .02	-.14-.21	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.02 .03	.03 .02	.00 .00	.00 .00	.00 .00
.894	1.744	-.16-.02	-.16-.09	-.05 .12	-.07 .02	-.12-.21	.00 .00	.02 .00	.00 .00
	+-	.03 .04	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.904	1.748	-.16-.01	-.18-.10	-.07 .10	-.06 .01	-.10-.23	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.907	1.750	-.21-.03	-.15-.09	-.01 .09	-.08 .00	-.09-.22	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .02	.03 .03	.00 .00	.00 .00	.00 .00
.916	1.754	-.21-.04	-.15-.09	.00 .10	-.06 .02	-.06-.24	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.924	1.758	-.21-.03	-.16-.13	.01 .07	-.08 .02	-.06-.26	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.02 .03	.03 .02	.03 .02	.02 .02	.00 .00	.00 .00	.00 .00
.935	1.763	-.20-.03	-.16-.10	.01 .09	-.06-.01	-.03-.23	.00 .00	.03 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.943	1.767	-.19-.06	-.18-.14	-.02 .10	-.02-.01	-.07-.29	.00 .00	.03 .00	.00 .00
	+-	.04 .04	.04 .03	.04 .03	.03 .04	.03 .02	.00 .00	.00 .00	.00 .00
.950	1.770	-.19-.06	-.18-.15	-.02 .08	-.03-.00	-.00-.25	.00 .00	.03 .00	.00 .00

Table 1(continued)

MOMENTUM GEV/C	ENERGY GEV		AMPLITUDE S11	P11	P13	D13	D15	F15	F17	G19
		+-	.03 .03	.03 .03	.03 .02	.03 .02	.03 .03	.00 .00	.00 .00	.00 .00
.954	1.772	-	-.16-.06	-.21-.18	.05 .06	-.09-.02	.01-.23	.00 .00	.03 .00	.00 .00
		+-	.03 .03	.03 .03	.03 .02	.02 .03	.03 .02	.00 .00	.00 .00	.00 .00
.970	1.779	-	-.16-.07	-.21-.17	.04 .07	-.09-.03	.04-.24	.00 .00	.03 .00	.00 .00
		+-	.03 .03	.03 .03	.03 .02	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00
.975	1.782	-	-.15-.06	-.21-.18	.03 .06	-.07-.02	.02-.26	.00 .00	.03 .00	.00 .00
		+-	.03 .02	.03 .03	.03 .02	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00
.990	1.789	-	-.13-.07	-.22-.14	.02 .07	-.08-.01	.06-.22	.00 .00	.03 .00	.00 .00
		+-	.03 .03	.03 .03	.03 .02	.03 .02	.03 .03	.00 .00	.00 .00	.00 .00
.991	1.789	-	-.12-.06	-.21-.14	.02 .09	-.10-.00	.08-.22	.00 .00	.03 .00	.00 .00
		+-	.04 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.020	1.803	-	-.11-.07	-.20-.15	.01 .06	-.09-.02	.07-.24	.00 .00	.04 .00	.00 .00
		+-	.03 .03	.03 .03	.03 .02	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00
1.022	1.804	-	-.12-.04	-.20-.16	.03 .06	-.10 .02	.09-.19	.00 .00	.04 .00	.00 .00
		+-	.04 .03	.03 .03	.03 .03	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00
1.036	1.810	-	-.12-.06	-.20-.14	.02 .04	-.11 .03	.10-.18	.00 .00	.04 .00	.00 .00
		+-	.03 .03	.04 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.036	1.810	-	-.13-.03	-.20-.13	.04 .05	-.10-.04	.07-.20	.00 .00	.04 .00	.00 .00
		+-	.03 .03	.03 .03	.04 .02	.03 .02	.03 .03	.00 .00	.00 .00	.00 .00
1.044	1.814	-	-.12-.04	-.21-.12	.03 .06	-.12 .02	.10-.16	.00 .00	.04 .00	.00 .00
		+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.058	1.821	-	-.12-.04	-.21-.12	.03 .07	-.12 .02	.10-.22	.00 .00	.04 .01	.00 .00
		+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.061	1.822	-	-.11-.04	-.22-.11	.01 .03	-.11 .02	.11-.14	.00 .00	.04 .01	.00 .00
		+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.077	1.829	-	-.11-.00	-.21-.12	.03 .07	-.11-.00	.09-.14	.00 .00	.05 .01	.00 .00
		+-	.04 .03	.03 .04	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.078	1.830	-	-.12-.03	-.21-.11	.03 .03	-.11-.01	.09-.16	.00 .00	.05 .01	.00 .00
		+-	.03 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.080	1.831	-	-.09-.01	-.22-.09	.01 .09	-.11 .00	.11-.15	.00 .00	.05 .01	.00 .00
		+-	.04 .04	.03 .03	.04 .03	.03 .04	.02 .03	.00 .00	.00 .00	.00 .00
1.102	1.841	-	-.10 .00	-.22-.11	.02 .08	-.10-.01	.10-.12	.00 .00	.05 .01	.00 .00
		+-	.03 .03	.03 .04	.04 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
1.117	1.848	-	-.10-.01	-.22-.10	.03 .05	-.12-.02	.10-.15	.00 .00	.05 .01	.00 .00
		+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
1.120	1.849	-	-.10 .00	-.22-.09	.02 .07	-.11-.00	.12-.12	.00 .00	.06 .01	.00 .00
		+-	.04 .04	.04 .04	.04 .03	.03 .04	.03 .03	.00 .00	.00 .00	.00 .00
1.130	1.854	-	-.10-.01	-.20-.10	.03 .03	-.10-.01	.10-.13	.00 .00	.06 .01	.00 .00
		+-	.03 .03	.03 .03	.04 .02	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
1.134	1.856	-	-.12-.02	-.21-.09	.04 .04	-.10 .02	.11-.12	-.00-.02	.06 .04	.00 .00
		+-	.04 .04	.04 .04	.03 .03	.03 .04	.03 .03	.03 .03	.03 .03	.00 .00
1.153	1.865	-	-.10-.00	-.21-.10	.05 .05	-.10-.00	.11-.12	-.02-.04	.07-.03	.00 .00
		+-	.04 .03	.04 .04	.03 .03	.04 .03	.03 .03	.03 .03	.03 .03	.00 .00
1.153	1.865	-	-.12 .01	-.22-.10	.07 .07	-.12-.01	.13-.13	-.01-.04	.06-.00	.00 .00
		+-	.04 .03	.04 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00
1.164	1.870	-	-.11 .02	-.21-.10	.06 .07	-.10-.01	.12-.12	-.02-.04	.05 .00	.00 .00
		+-	.05 .04	.04 .04	.05 .04	.04 .04	.04 .03	.03 .04	.03 .03	.00 .00
1.174	1.874	-	-.11 .02	-.20-.10	.06 .04	-.08-.01	.11-.10	-.02-.01	.05 .01	.00 .00
		+-	.04 .04	.04 .04	.05 .03	.03 .04	.03 .03	.03 .03	.02 .03	.00 .00
1.183	1.879	-	-.11 .00	-.21-.10	.05 .02	-.07 .01	.14-.12	-.01-.02	.11 .04	.00 .00
		+-	.04 .04	.04 .04	.04 .03	.03 .04	.03 .03	.03 .03	.03 .03	.00 .00
1.196	1.885	-	-.10-.00	-.21-.10	.05 .03	-.07 .00	.14-.12	-.01-.02	.07 .03	.00 .00
		+-	.04 .04	.04 .04	.04 .04	.04 .04	.04 .03	.04 .04	.04 .04	.00 .00
1.201	1.887	-	-.11 .01	-.18-.08	.07 .06	-.10-.03	.13-.12	-.02-.05	.10 .01	.00 .00
		+-	.04 .03	.03 .03	.04 .02	.03 .03	.03 .02	.02 .03	.03 .02	.00 .00
1.206	1.889	-	-.12 .03	-.19-.08	.06 .06	-.10-.01	.14-.08	-.02-.04	.06 .03	.00 .00
		+-	.04 .04	.04 .04	.04 .04	.04 .04	.04 .03	.03 .04	.03 .03	.00 .00
1.220	1.895	-	-.13 .02	-.19-.08	.06 .06	-.09-.04	.14-.13	-.02-.06	.07 .03	.00 .00
		+-	.04 .03	.03 .03	.03 .02	.03 .03	.03 .02	.02 .02	.02 .02	.00 .00
1.240	1.905	-	-.12 .03	-.19-.07	.07 .06	-.10-.02	.15-.10	.00-.05	.10 .02	.00 .00
		+-	.04 .04	.04 .04	.04 .04	.03 .03	.04 .03	.03 .03	.03 .04	.00 .00
1.271	1.919	-	-.13 .02	-.17-.07	.07 .04	-.09-.02	.13-.10	.02-.05	.09 .05	.00 .00
		+-	.03 .03	.03 .03	.03 .03	.02 .03	.03 .03	.02 .02	.02 .03	.00 .00
1.316	1.939	-	-.13 .03	-.17-.07	.05 .03	-.12-.03	.09-.08	.03-.04	.10 .07	.00 .00
		+-	.03 .03	.03 .03	.03 .03	.03 .03	.02 .03	.02 .03	.02 .02	.00 .00
1.338	1.949	-	-.11 .03	-.16-.09	.04-.02	-.11-.05	.08-.09	.01-.01	.09 .05	.00 .00
		+-	.04 .04	.03 .03	.03 .03	.03 .03	.03 .02	.02 .03	.03 .03	.00 .00
1.346	1.953	-	-.10 .08	-.19-.09	-.02 .03	-.09-.08	.10-.10	.02-.02	.10 .08	.00 .00
		+-	.04 .03	.03 .04	.03 .03	.03 .03	.03 .03	.02 .03	.03 .03	.00 .00
1.368	1.963	-	-.09 .08	-.20-.07	.04 .04	-.08-.06	.10-.06	.02-.03	.10 .12	.00 .00
		+-	.03 .03	.03 .03	.03 .03	.03 .03	.02 .02	.02 .03	.03 .02	.00 .00
1.402	1.978	-	-.08 .08	-.17-.06	.03 .03	-.09-.04	.07-.05	-.00 .00	.07 .13	.00 .00
		+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00
1.415	1.984	-	-.09 .08	-.16-.06	.03 .02	-.08-.05	.10-.05	.04-.01	.10 .12	.00 .00
		+-	.03 .03	.03 .03	.03 .03	.03 .03	.02 .02	.02 .02	.02 .02	.00 .00
1.420	1.986	-	-.08 .07	-.16-.07	.05 .02	-.09-.04	.08-.05	.01-.01	.10 .17	.00 .00
		+-	.03 .03	.03 .03	.03 .02	.02 .03	.02 .02	.02 .02	.02 .02	.00 .00
1.480	1.999	-	-.07 .08	-.17-.05	.06 .00	-.06-.02	.09-.03	.00 .02	.05 .17	.00 .00
		+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00
1.468	2.006	-	-.10 .11	-.14-.03	.03-.02	-.06-.02	.09-.02	-.02-.00	.06 .18	.00 .00
		+-	.03 .02	.03 .03	.03 .02	.02 .02	.02 .02	.02 .02	.03 .01	.00 .00

Table 1 (continued)

MOMENTUM GEV/C	ENERGY GEV	AMPLITUDE S11	P11	P13	D13	D15	F15	F17	G19
1.478	2.012	-.09 .08	-.18-.03	.06-.01	-.08-.05	.07-.02	-.03 .02	.04 .17	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .02	.02 .02	.02 .02	.02 .02	.00 .00
1.495	2.019	-.08 .08	-.18-.04	.05-.00	-.08-.04	.06-.02	-.04 .02	.03 .18	.00 .00
	+-	.04 .03	.04 .03	.04 .03	.04 .03	.03 .03	.03 .03	.03 .02	.00 .00
1.501	2.022	-.07 .09	-.21-.04	.07-.01	-.08-.00	.04 .01	-.01 .02	-.00 .19	.00 .00
	+-	.04 .03	.04 .03	.03 .02	.04 .03	.03 .02	.03 .02	.04 .02	.00 .00
1.508	2.025	-.10 .09	-.17-.04	.04-.01	-.07-.02	.07-.03	-.01 .01	.02 .22	.00 .00
	+-	.03 .02	.02 .02	.02 .02	.02 .02	.02 .01	.02 .01	.03 .02	.00 .00
1.519	2.030	-.12 .11	-.16-.02	.05-.04	-.07-.02	.04 .00	-.05 .00	-.01 .16	.00 .00
	+-	.05 .03	.03 .03	.03 .03	.03 .03	.03 .02	.03 .02	.03 .02	.00 .00
1.541	2.040	-.11 .11	-.16-.02	.04-.02	-.07-.04	.04-.02	-.06 .00	-.01 .18	.00 .00
	+-	.03 .04	.04 .04	.03 .03	.03 .03	.03 .03	.03 .03	.03 .02	.00 .00
1.567	2.051	-.08 .10	-.19-.02	.06-.01	-.09-.05	.05-.03	-.03 .01	-.04 .17	.00 .00
	+-	.04 .04	.04 .04	.04 .03	.04 .04	.03 .03	.04 .03	.03 .03	.00 .00
1.567	2.051	-.09 .08	-.20-.03	.06-.01	-.11-.02	.03-.01	-.04 .02	-.03 .18	.00 .00
	+-	.04 .03	.03 .03	.03 .02	.02 .02	.03 .02	.02 .02	.02 .02	.00 .00
1.598	2.064	-.08 .10	-.19 .01	.02-.03	-.07-.00	.02 .00	-.06-.02	-.11 .13	.00 .00
	+-	.04 .04	.04 .04	.03 .03	.03 .04	.03 .03	.03 .03	.03 .03	.00 .00
1.600	2.065	-.06 .09	-.20 .01	.02-.04	-.09-.01	.05-.02	-.06 .01	-.08 .17	-.02-.01
	+-	.04 .04	.04 .04	.03 .04	.04 .04	.03 .03	.03 .04	.03 .03	.03 .03
1.606	2.068	-.10 .08	-.22 .03	.03-.05	-.10-.01	.02 .02	-.01 .00	-.04 .17	.01-.02
	+-	.04 .03	.03 .04	.03 .03	.03 .03	.03 .03	.02 .03	.03 .03	.03 .02
1.609	2.069	-.10 .07	-.16 .02	.04-.04	-.10-.02	.00-.01	-.03 .00	-.04 .16	.00 .02
	+-	.03 .03	.03 .03	.03 .03	.03 .02	.03 .02	.02 .02	.03 .02	.02 .02
1.645	2.085	-.10 .05	-.20 .04	.05-.04	-.11-.03	.01-.02	-.01 .04	-.07 .18	.00 .01
	+-	.04 .03	.03 .04	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03
1.652	2.088	-.08 .08	-.17-.00	.03-.05	-.10-.02	.02-.00	-.03 .02	-.06 .14	-.01 .01
	+-	.03 .03	.03 .03	.03 .03	.02 .03	.03 .02	.02 .02	.02 .02	.02 .02
1.652	2.088	-.09 .08	-.18 .00	.05-.04	-.11-.01	.02-.01	-.06 .01	-.07 .16	-.03-.02
	+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .02	.03 .02	.02 .02
1.690	2.104	-.08 .07	-.18-.01	.04-.05	-.08-.02	.02-.02	-.06 .00	-.07 .14	-.04 .03
	+-	.03 .03	.03 .03	.02 .03	.02 .03	.02 .02	.02 .02	.02 .02	.02 .02
1.694	2.106	-.09 .08	-.18-.00	.03-.04	-.11-.02	.01-.01	-.01 .05	-.09 .12	-.05 .00
	+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.02 .02	.02 .02
1.705	2.111	-.10 .09	-.19 .00	.04-.05	-.10-.01	.04-.02	-.03-.00	-.07 .12	-.06-.01
	+-	.04 .03	.03 .03	.03 .03	.03 .03	.02 .04	.02 .02	.02 .02	.02 .03
1.710	2.113	-.12 .13	-.20-.01	.04-.08	-.09-.01	.02-.02	-.04-.02	-.07 .13	-.01-.00
	+-	.04 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.02 .03
1.726	2.119	-.11 .11	-.17-.01	.02-.06	-.08-.03	.02-.00	-.05-.00	-.05 .09	-.01 .01
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.03 .03	.02 .03
1.739	2.125	-.09 .11	-.19 .01	.03-.04	-.09-.04	.05-.02	-.05-.01	-.09 .07	-.03-.01
	+-	.03 .03	.03 .03	.02 .03	.02 .03	.02 .03	.02 .02	.02 .02	.02 .02
1.798	2.150	-.09 .12	-.20 .01	.04-.01	-.11-.03	.04 .02	-.05-.02	-.06 .08	-.05 .01
	+-	.03 .03	.02 .03	.02 .02	.02 .03	.02 .02	.01 .02	.02 .02	.02 .02
1.800	2.151	-.10 .12	-.19 .02	.03 .00	-.12-.02	.03 .00	-.01-.02	-.06 .09	-.04-.02
	+-	.04 .03	.04 .03	.03 .03	.03 .03	.02 .03	.03 .03	.03 .03	.02 .02
1.800	2.151	-.10 .11	-.20 .00	.05-.01	-.12-.01	.01 .02	-.02-.00	-.07 .13	-.04-.00
	+-	.04 .04	.04 .04	.03 .03	.03 .03	.03 .03	.03 .03	.03 .02	.03 .02

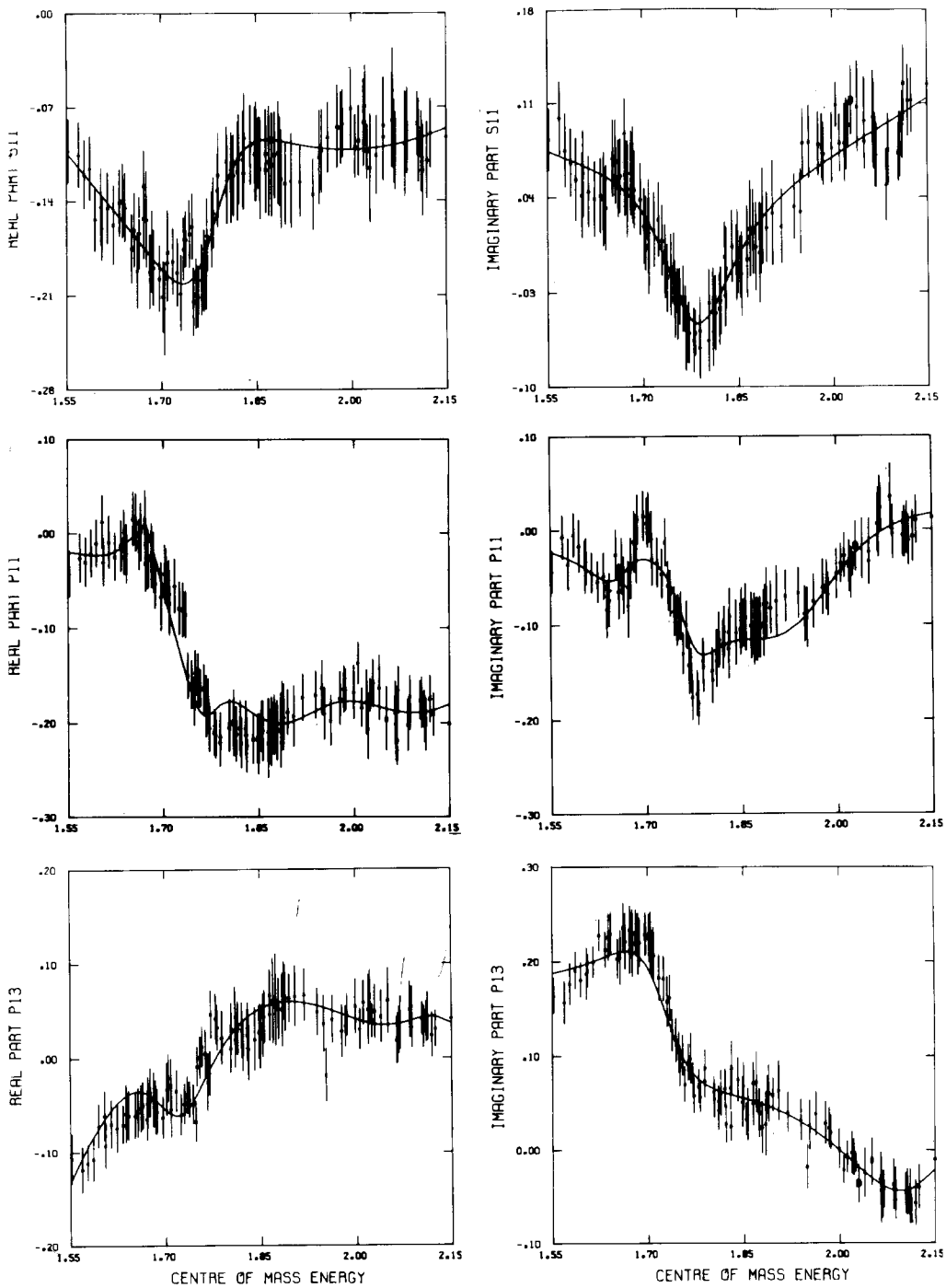
gives errors on the fitted amplitudes at each energy point but these will be distorted by the constraints on the amplitudes. The errors have thus been calibrated by multiplying a constant factor untill the fits of the Breit-Wigner to the D5 amplitude below 1.15 GeV/c and the F7 amplitude gave χ^2/NDF of approximately 1.0. The factor required was 2.0 and these errors are given in the tables. The values and errors of the real and imaginary parts of the amplitudes are plotted in fig. 1 for solution 1 and fig. 2 for the S and P waves of solution 2.

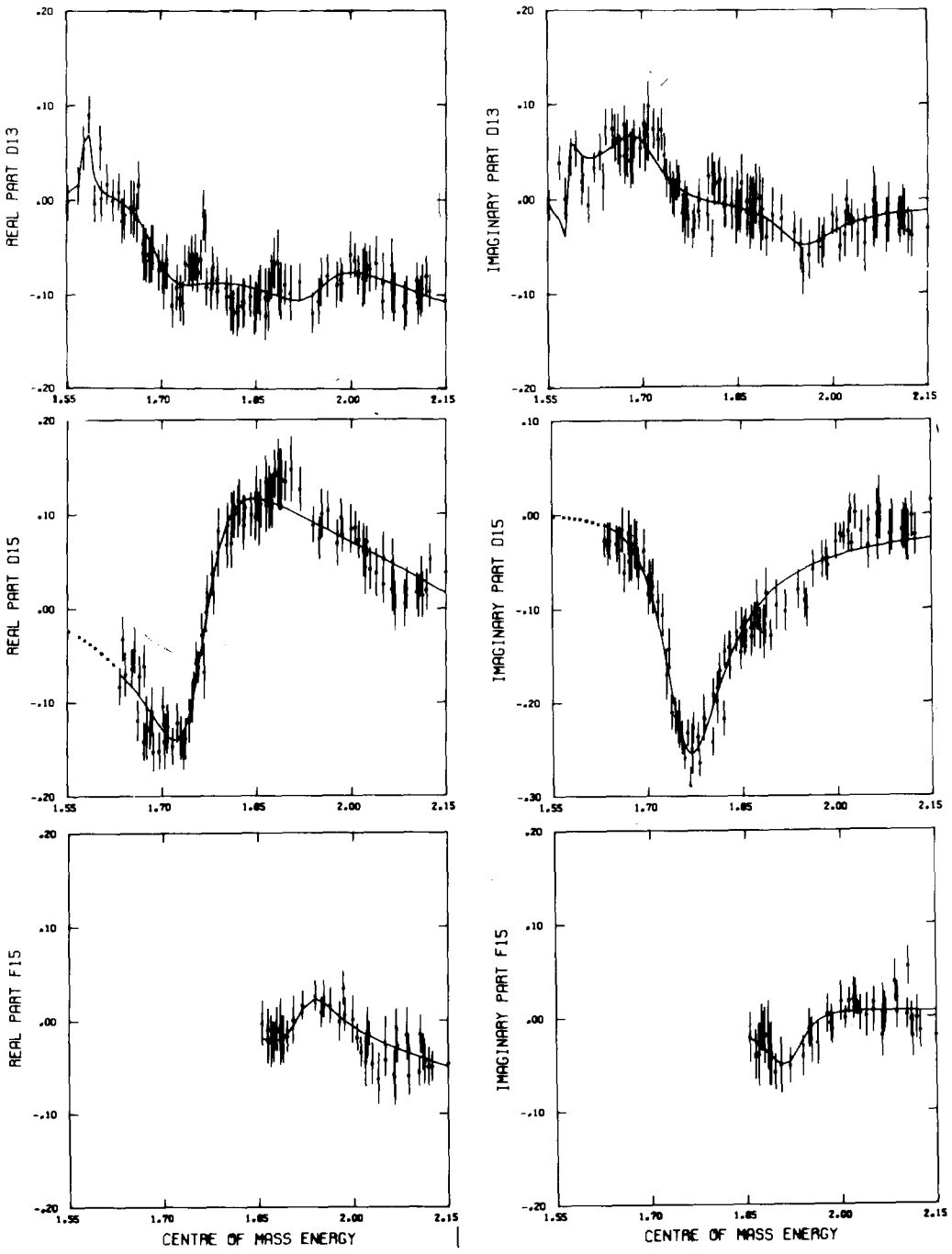
Method I

Method I finds similar results to the CHS results in the more limited energy range of their analysis [3] except that introduction of the χ^2 link removes the large structure below 1700 MeV which was probably due to the lack of a phase constraint on the amplitudes so far below the $\Sigma(1765)$. The two ambiguous solutions were again found.

Table 2
As table 1 for solution 2 at those energy points where it differs from solution 1

MOMENTUM GEV/C	ENERGY GEV	AMPLITUDE S11	P11	P13	D13	D15	F15	F17	G19
.455	1.544	-.31-.09	-.08-.03	-.06 .01	-.03-.01	-.02-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00	.00 .00
.475	1.552	-.28-.08	-.07-.03	-.05 .01	-.01-.02	-.02-.00	.00 .00	.00 .00	.00 .00
	+-	.02 .03	.02 .03	.02 .03	.02 .02	.00 .00	.00 .00	.00 .00	.00 .00
.514	1.569	-.26-.07	-.08-.03	-.08-.00	-.03 .05	-.03-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.02 .03	.00 .00	.00 .00	.00 .00	.00 .00
.534	1.578	-.29-.08	-.06-.05	-.070.00	.02 .06	-.03-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00	.00 .00
.554	1.586	-.29-.08	-.08-.03	-.06 .01	-.01 .10	-.04-.00	.00 .00	.00 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00	.00 .00
.573	1.595	-.27-.07	-.10-.05	-.04 .05	-.04 .06	-.04-.00	.00 .00	.00 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .03	.02 .03	.00 .00	.00 .00	.00 .00	.00 .00
.594	1.604	-.30-.10	-.13-.04	-.00 .02	.03 .04	-.05-.01	.00 .00	.00 .00	.00 .00
	+-	.03 .04	.03 .03	.03 .04	.03 .03	.00 .00	.00 .00	.00 .00	.00 .00
.597	1.605	-.29-.09	-.14-.06	-.03 .02	-.01 .01	-.05-.01	.00 .00	.00 .00	.00 .00
	+-	.02 .03	.02 .03	.02 .03	.02 .03	.00 .00	.00 .00	.00 .00	.00 .00
.617	1.614	-.29-.10	-.13-.06	-.00 .02	.01 .01	-.05-.01	.00 .00	.01 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .03	.02 .02	.00 .00	.00 .00	.00 .00	.00 .00
.637	1.624	-.30-.10	-.17-.08	-.01 .06	-.01 .04	-.06-.01	.00 .00	.01 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .03	.02 .03	.00 .00	.00 .00	.00 .00	.00 .00
.658	1.633	-.28-.11	-.17-.07	.00 .03	-.00 .06	-.08-.03	.00 .00	.01 .00	.00 .00
	+-	.02 .03	.02 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.668	1.638	-.28-.08	-.16-.12	.02 .05	.02 .04	-.07 .01	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.677	1.642	-.27-.09	-.18-.10	.00 .06	.01 .07	-.10-.03	.00 .00	.01 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .03	.02 .03	.02 .03	.00 .00	.00 .00	.00 .00
.699	1.652	-.28-.09	-.16-.09	-.00 .03	-.02 .06	-.07-.05	.00 .00	.01 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.708	1.656	-.29-.11	-.18-.09	.02 .07	-.01 .06	-.05-.02	.00 .00	.01 .00	.00 .00
	+-	.03 .04	.03 .03	.02 .04	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.719	1.661	-.30-.12	-.20-.08	.00 .06	-.04 .04	-.11-.05	.00 .00	.01 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.725	1.664	-.27-.13	-.18-.09	.05 .07	-.03 .05	-.05-.05	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.740	1.671	-.26-.12	-.20-.10	.01 .03	-.08 .10	-.11-.02	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.02 .03	.02 .03	.03 .02	.02 .02	.00 .00	.00 .00	.00 .00
.741	1.672	-.28-.12	-.17-.10	.04 .08	-.02 .03	-.09-.01	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.750	1.676	-.29-.13	-.17-.08	.03 .06	-.07 .03	-.12-.06	.00 .00	.01 .00	.00 .00
	+-	.04 .03	.04 .03	.03 .03	.03 .03	.03 .04	.00 .00	.00 .00	.00 .00
.761	1.681	-.30-.13	-.18-.09	.02 .05	-.08 .02	-.11-.05	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.768	1.684	-.27-.13	-.19-.08	.02 .06	-.06 .03	-.10-.04	.00 .00	.01 .00	.00 .00
	+-	.03 .04	.03 .03	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.773	1.687	-.28-.14	-.20-.07	.01 .05	-.07 .03	-.13-.08	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.793	1.696	-.28-.12	-.19-.10	-.00 .04	-.07 .01	-.16-.07	.00 .00	.01 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .04	.02 .03	.02 .03	.00 .00	.00 .00	.00 .00
.806	1.702	-.27-.10	-.18-.12	.04 .05	-.06 .04	-.13-.10	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.02 .02	.00 .00	.00 .00	.00 .00
.810	1.704	-.23-.10	-.18-.10	.04 .07	-.09 .00	-.13-.09	.00 .00	.01 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .02	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.820	1.709	-.19-.10	-.19-.13	.04 .07	-.07 .05	-.14-.09	.00 .00	.02 .00	.00 .00
	+-	.03 .04	.03 .04	.03 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.821	1.709	-.20-.10	-.19-.14	.02 .06	-.08 .06	-.13-.10	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .04	.03 .04	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.838	1.717	-.20-.10	-.17-.12	.04 .06	-.10 .03	-.15-.11	.00 .00	.02 .00	.00 .00
	+-	.02 .03	.03 .03	.02 .03	.03 .03	.02 .03	.00 .00	.00 .00	.00 .00
.853	1.724	-.19-.10	-.19-.11	.02 .06	-.07 .02	-.13-.13	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.03 .03	.02 .02	.00 .00	.00 .00	.00 .00
.865	1.730	-.17-.12	-.16-.08	.04 .06	-.12 .04	-.09-.17	.00 .00	.02 .00	.00 .00
	+-	.04 .03	.03 .03	.02 .03	.03 .03	.03 .03	.00 .00	.00 .00	.00 .00
.874	1.734	-.16-.10	-.18-.12	.04 .08	-.08 .00	-.14-.17	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .03	.02 .03	.02 .03	.00 .00	.00 .00	.00 .00
.882	1.738	-.15-.10	-.22-.11	.03 .08	-.06-.00	-.12-.21	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.02 .02	.03 .02	.03 .02	.00 .00	.00 .00	.00 .00
.894	1.744	-.16-.09	-.22-.14	.02 .07	-.05-.01	-.11-.21	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.904	1.748	-.16-.09	-.23-.14	.01 .06	-.05-.01	-.09-.22	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .03	.02 .02	.00 .00	.00 .00	.00 .00
.907	1.750	-.19-.09	-.18-.13	.05 .06	-.08-.00	-.08-.23	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .03	.03 .02	.03 .04	.00 .00	.00 .00	.00 .00
.916	1.754	-.17-.09	-.19-.14	.06 .09	-.07 .01	-.05-.23	.00 .00	.02 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.924	1.758	-.18-.09	-.19-.16	.05 .07	-.08 .01	-.04-.25	.00 .00	.02 .00	.00 .00
	+-	.03 .02	.03 .02	.03 .02	.03 .02	.02 .02	.00 .00	.00 .00	.00 .00
.935	1.763	-.16-.10	-.20-.14	.03 .10	-.05-.03	-.01-.22	.00 .00	.03 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .03	.03 .02	.00 .00	.00 .00	.00 .00
.943	1.767	-.15-.10	-.21-.18	.02 .09	-.03-.01	-.05-.28	.00 .00	.03 .00	.00 .00
	+-	.04 .03	.04 .04	.04 .03	.03 .04	.03 .02	.00 .00	.00 .00	.00 .00
.950	1.770	-.16-.07	-.21-.17	.05 .07	-.07-.01	-.00-.25	.00 .00	.03 .00	.00 .00
	+-	.03 .03	.03 .03	.03 .02	.03 .02	.03 .03	.00 .00	.00 .00	.00 .00


REAL PART S11
IMAGINARY PART S11
REAL PART P11
IMAGINARY PART P11
REAL PART P13
IMAGINARY PART P13
CENTRE OF MASS ENERGY
CENTRE OF MASS ENERGY


REAL PART D13
IMAGINARY PART D13
REAL PART D15
IMAGINARY PART D15
REAL PART F15
IMAGINARY PART F15
CENTRE OF MASS ENERGY
CENTRE OF MASS ENERGY

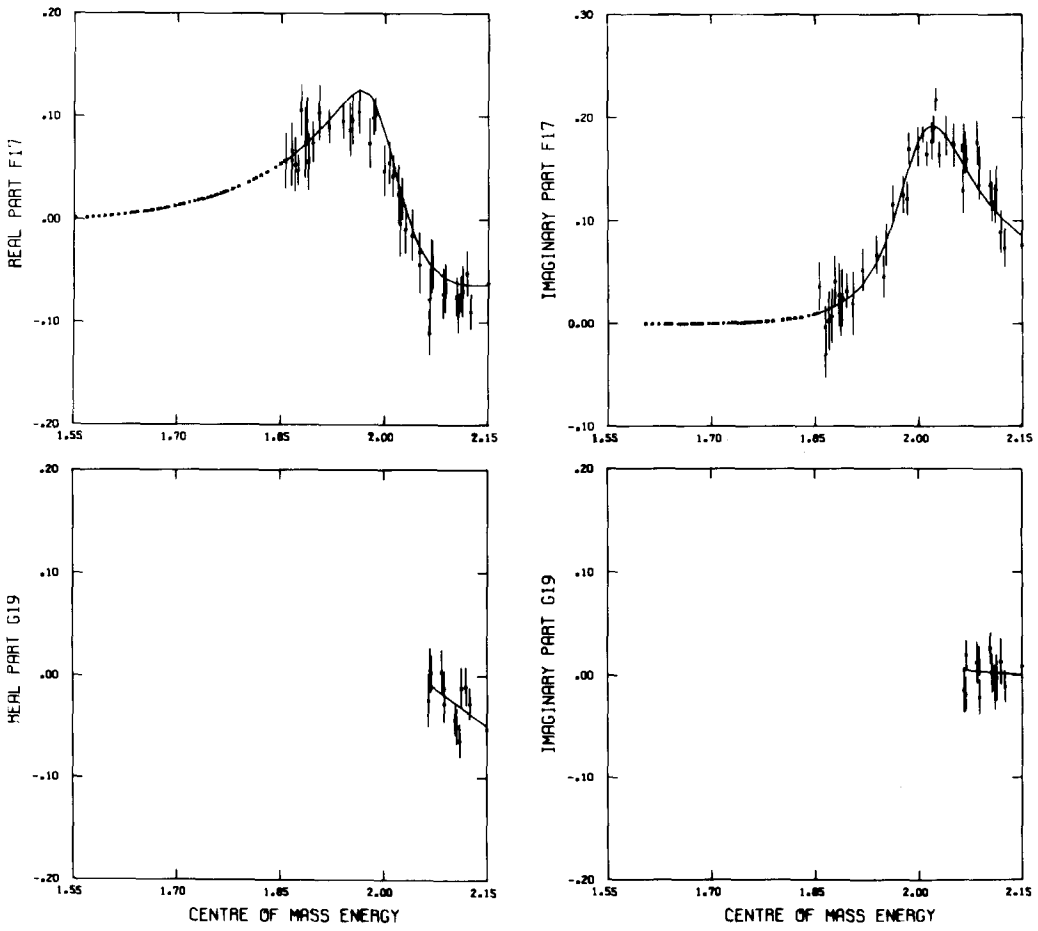


Fig. 1. The real and imaginary parts of the partial wave amplitudes from solution 1. The curves are the results of the energy dependent fits to the amplitudes described in the text.

Using this method studies were made of relaxing the χ_1^2 constraint since it was hoped that with the smooth continuous data smooth continuous solutions could be found. It was found however that when the χ_c^2 condition was relaxed excursions in the amplitudes were produced that tended to appear in several amplitudes together, i.e. the solutions were moving to different Barrelet zero trajectories. The solutions differed from the smoother solutions mostly in the predictions for the polarisation coefficients (B coefficients) which were different but remained within the error bars. We thus conclude that is mainly the poor measurements of the polarisation coefficients that allows this continuous ambiguity in the amplitudes and that the χ^2 link is necessary to obtain smooth solutions.

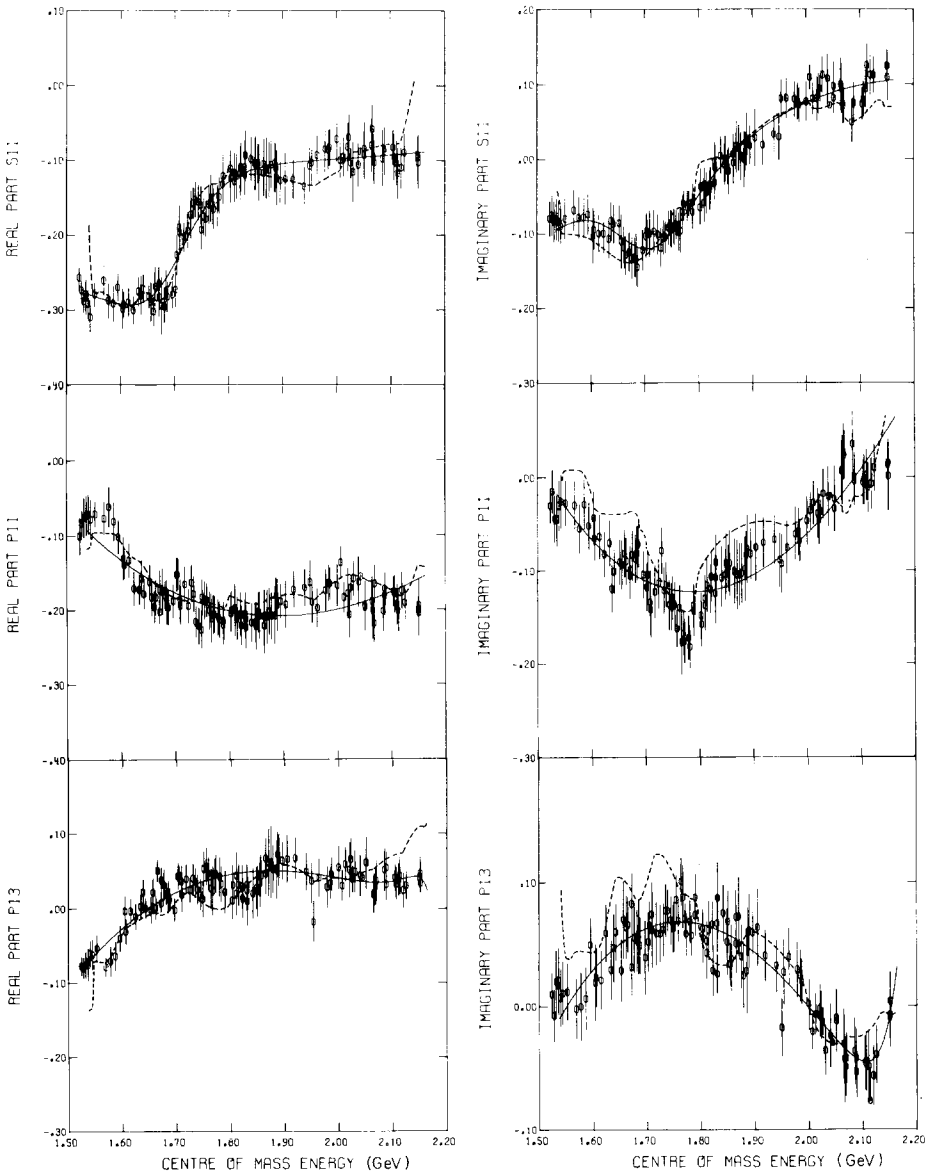

REAL PART S11
REAL PART P11
REAL PART P13
IMAGINARY PART S11
IMAGINARY PART P11
IMAGINARY PART P13
CENTRE OF MASS ENERGY (GeV)
CENTRE OF MASS ENERGY (GeV)

Fig. 2a

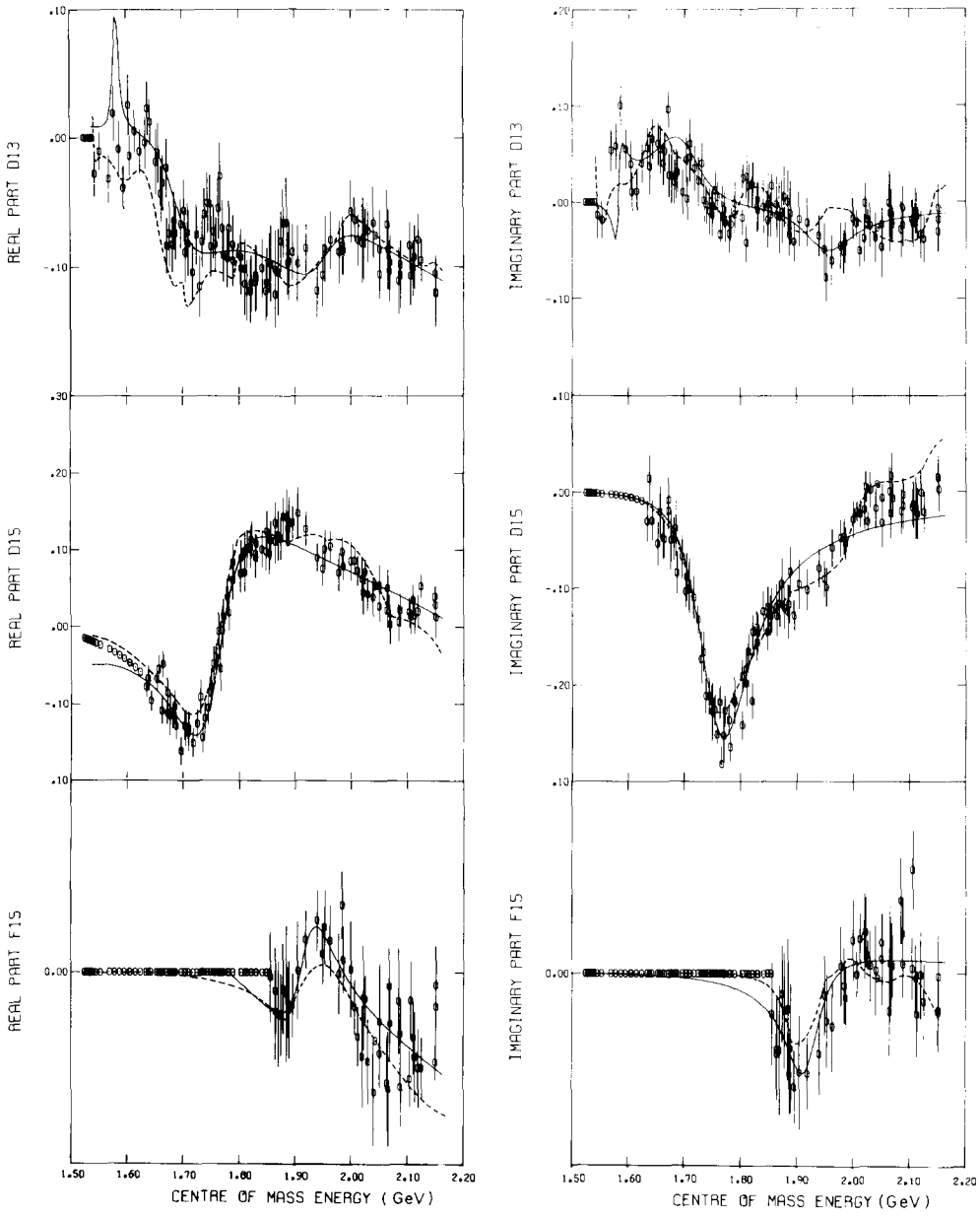

REAL PART D13
REAL PART D15
REAL PART F15
IMAGINARY PART D13
IMAGINARY PART D15
IMAGINARY PART F15
CENTRE OF MASS ENERGY (GeV)
CENTRE OF MASS ENERGY (GeV)

Fig. 2b.

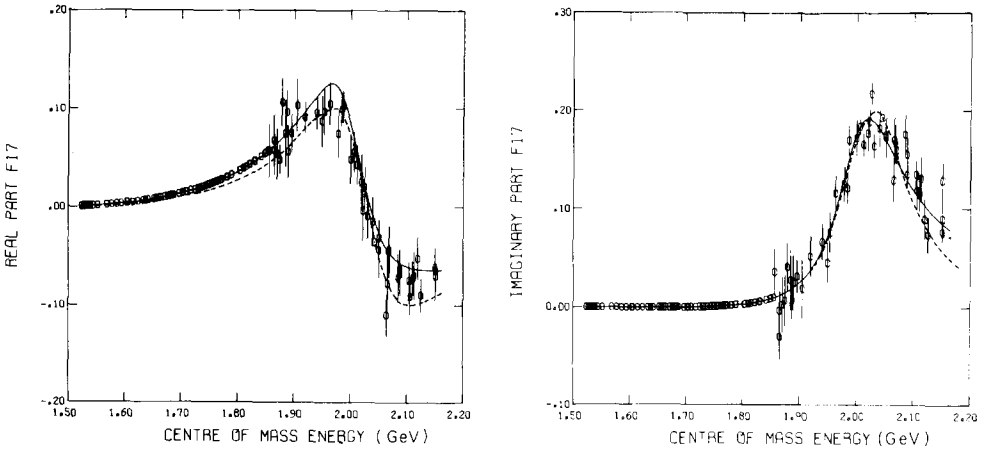


Fig. 2. The real and imaginary parts of the amplitudes from solution 2. The solid curves are the results of the energy dependent fits to the amplitudes described in the text. The dashed curves are the resultant amplitude of method I.

To show the good agreement of the two methods the results of method I are plotted as the dashed lines in fig. 2. The following comments can be made on the two sets of amplitudes:

(i) In order to show the difference in amplitudes produced by including a G7 instead of G9 at the higher energies, Method I uses this configuration, and this accounts for the differences in the amplitudes above 2100 MeV. It can be seen that G7 produces greater structure in the amplitudes than G9 and gives another reason for prefering G9.

(ii) The small systematic shifts of the amplitudes are probably due to the effects of the smoothing procedure. The out-of-line data points that were rejected in method II were included in the smoothing procedure and systematically pull the smoothed data from the mean of the data points used in method II.

(iii) Apart from these small effects the amplitudes agree very well and give us some confidence that the partial-wave analysis is correct.

2.6. Barrelet zeros

The Barrelet zeros of our solutions have been examined throughout the fitting to check the continuity of the trajectories. The zero trajectories for solution 1 of method II are plotted in fig. 3. Continuity of the amplitude requires that the new trajectories corresponding to the introduction of new partial waves enter from the inside or outside of the circle according to whether $J + L - \frac{1}{2}$ is odd or even respectively [5]. Using this property we associate zero 3 with the onset of the D3 amplitude, zero 4 with D5, zero 5 with F5 and zero 6 with F7. The zeros entering

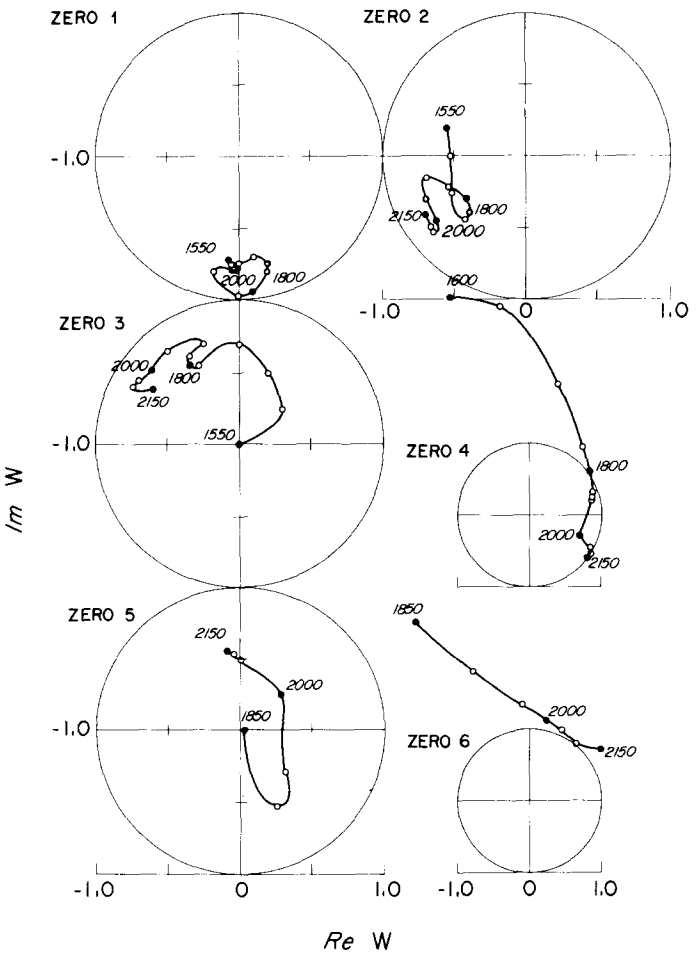


Fig. 3. The Barrelet zeros corresponding to solution 1.

with the G9 amplitude are not shown as they are still far from the physical region.

We agree with the conclusions of the Bellefon and Berthon [5] that a large D5 amplitude is found only if zeros 2 and 3 lie inside the circle and a large F7 if zeros 1, 2, 3 and 5 are inside. The difference between solution 1 and solution 2 is that 1 corresponds to zero 1 remaining inside the circle whilest in solution 2 it crosses outside the circle at around 1750 MeV. The two solutions are thus simple Barrelet ambiguities of each other.

Thus our assumption of large resonant D5 and F7 amplitudes together with continuity of the amplitudes only admit the two ambiguous solutions found up to 1760 MeV. The only other ambiguity comes about in the region of 1900–2000 MeV

for zero 4 which crosses the circle and moves downwards rather close to the circle. At 2030 MeV the zero has to lie inside the circle to correspond to the single-energy solution selected at the peak of the $\Sigma(2030)$. However, the exact point at which it crosses inside the circle may be varied, giving rise to slightly different amplitudes. The set chosen correspond to the smoothest zero trajectory and give the smoothest set of amplitudes. However, as the zero is rather close the circle, the amplitudes obtained by flipping it outside are not very different from those given, but change the S_1 and P_3 amplitudes outside the errors given.

3. Fits to the amplitudes

The amplitudes shown in figs. 1 and 2 for method II were fitted to a combination of resonances plus background in order to attempt the extraction of resonance parameters.

$$T = T_{\text{res}} + T_{\text{B}} ,$$

where T is the total amplitude; T_{res} is the resonant part and T_{B} the background part.

The resonances were parameterised as a function of the centre-of-mass energy (E) with the usual Breit-Wigner form

$$T_{\text{res}}(E) = \frac{\Gamma(E)\sqrt{xx'}(E)\, e^{i\phi}}{(M - E) - i\Gamma(E)} , \qquad \sqrt{xx'}(E) . = \frac{\sqrt{\Gamma e(E)\,\Gamma i(E)}}{\Gamma(E)} ,$$

where M is the resonance mass, ϕ is a phase to be fitted, $\Gamma(E)$ the total width, $\Gamma_e(E)$ the elastic partial width and $\Gamma_i(E)$ the partial width into the ith channel (in this case $\Lambda\pi$). The partial widths were given an energy dependence

$$\Gamma_i(E) = \frac{p_i B_l(p_i) M}{p_i^{\text{R}} B_l(p_i^{\text{R}})\, E}\, \Gamma_i ,$$

where Γ_i is the partial width at resonance, p_i is the centre-of-mass momentum in the ith channel corresponding to the energy E and $B_l(p_i)$ are the usual Blatt and Weisskopf angular momentum barrier functions with a radius of interaction of 1 fm. The quantities with superscript R are taken at the resonance energy. For the leading resonances, $\Sigma(1765)$ and $\Sigma(2030)$, where we have a reasonable knowledge of the branching fractions, the energy dependence of the total widths were taken as

$$\Gamma(E) = \sum \Gamma_i(E) = \sum x_i \frac{p_i B_l(p_i) M}{p_i^{\text{R}} B_l(p_i^{\text{R}}) E}\, \Gamma ,$$

where Γ and x_i are the total width and the branching fraction into the ith channel, taken at resonance. The values of x_i taken were [7]:

$\Sigma(1765)$	$\bar{K}N$	0.42	$\Sigma(2030)$	$\bar{K}N$	0.19
	$\Sigma\pi$	0.02		$\Sigma\pi$	0.05
	$\Lambda\pi$	0.17		$\Lambda\pi$	0.23
	$\Sigma(1385)\pi$	0.17		$\Sigma(1385)\pi$	0.07
	$\Lambda(1520)\pi$	0.22		$\Lambda(1520)\pi$	0.11
				$\Delta(1230)\bar{K}$	0.16
				$\Lambda(1815)\pi$	0.19

There are, of course, considerable errors on the above figures but they probably give a reasonable approximation to the make up of the branching fractions of these states. The shapes of the resonances are not sensitive to detailed changes.

For the other resonances, where their branching fractions are mostly unknown, the energy dependence of the elastic channel was taken for the total width as an approximation between the lighter $\Lambda\pi$ and $\Sigma\pi$ channels and the heavier resonance decays. The phases of the $\Sigma(1765)$ and $\Sigma(2030)$ were fixed to zero. The relative phases of the other resonances were allowed to vary.

Background amplitudes were parametrised as a polynomial in the centre-of-mass momentum up to second order

$$\text{Re } T_B = a + bp + cp^2 , \qquad \text{Im } T_B = d + ep + fp^2 .$$

Higher-order terms were not found to be necessary for any amplitude. T_B could be fixed to zero below a given (fitted) energy E_{min}.

Although the fitting program calculates errors for the resonance quantities more reliably than those usually obtained from energy-dependent partial-wave analyses, the parameters are still strongly dependent on the fit hypothesis e.g. how many terms in the background or whether the resonance phase is fixed. The errors quoted attempt to allow for these systematic uncertainties which are generally much larger than the statistical errors. Clearly, however, we cannot be sure that all possibilities have been taken into account.

The amplitudes are discussed in descending order of J^P i.e. increasing order of complexity and uncertainty. The resonance parameters obtained are given in table 3. The solid curves on figs. 1 and 2 are the results of the fits. The Argand diagrams of solution 1, taken from the fits, are plotted in fig. 4. The full lines are the total amplitudes, the dotted lines the background component. The Argand diagrams of the S and P waves of solution 2 are shown in fig. 5. Above 1750 MeV solution 1 and 2 are identical. Thus only for amplitudes P3 and below is there any significant difference between the two solutions.

We would like to emphasize that though the results of the energy-independent partial-wave analysis (i.e. the amplitudes of tables 1 and 2) are relatively model-independent, depending only cn the fairly weak assumptions described above, the results of the fits to the amplitudes described below are extremely model-dependent. We have chosen specific forms for the background parametrisation and specific numbers of resonances to be included in the fits. Clearly different choices could

Table 3
Resonance parameters for the two solutions; the parameters for the amplitudes D13 and above are the same in each solution

Partial wave	Mass (Mev)	Width (MeV)	Amplitude ($\sqrt{xx'}$)	ϕ (rad)
D13	1685 ± 20	85 ± 25	+0.06 ± 0.02	0.4 ± 0.3
D13	1950 ± 30	150 ± 75	−0.04 ± 0.02	0.0 ± 0.5
D15	1775 ± 10	125 ± 15	−0.25 ± 0.02	0.0 (fixed)
F15	1920 ± 30	70 ± 20	−0.06 ± 0.02	0.7 ± 0.3
F17	2035 ± 15	180 ± 20	+0.18 ± 0.02	0 0 (fixed)

Solution 1

Partial wave	Mass (MeV)	Width (MeV)	Amplitude ($\sqrt{xx'}$)	ϕ (rad)
S11	1780 ± 30	140 ± 30	−0.12 ± 0.02	−0.1 ± 0.3
P11	1660 ± 30	80 ± 40	−0.04 ± 0.02	1.1 ± 0.8
P11	1770 ± 20	80 ± 30	−0.08 ± 0.02	−0.5 ± 0.5
P11	1960 ± 30	260 ± 40	−0.12 ± 0.02	0.9 ± 0.4
P13	1720 ± 30	120 ± 30	+0.11 ± 0.02	1.2 ± 0.3
P13	2120 ± 40	240 ± 50	−0.13 ± 0.04	0.5 ± 0.3

Solution 2

Partial wave	Mass (MeV)	Width (MeV)	Amplitude $\sqrt{xx'}$)	ϕ(rad)
S11	1700 ± 30	160 ± 50	−0.13 ± 0.03	−0.4 ± 0.4
P13	2140 ± 40	200 ± 50	−0.13 ± 0.04	0.0 ± 0.4

produce different resonance parameters or even different sets of resonances. The results given below seem to us to comprise reasonable choices but other interpretations are certainly possible.

The results of the fits to the amplitudes of solution 1 are discussed first.

Solution 1

G19

The G19 amplitude is well fitted by a nearly real linear amplitude (χ^2/NDF = = 26.7/24). If the amplitude is assumed to be the tail of a resonance at 2250 MeV the amplitude at resonance would be −0.08 ± 0.03.

F17

F17 was constrained in the fits to approximate a Breit-Wigner resonance. There is no tendency to pull away from this form indicating that background is not im-

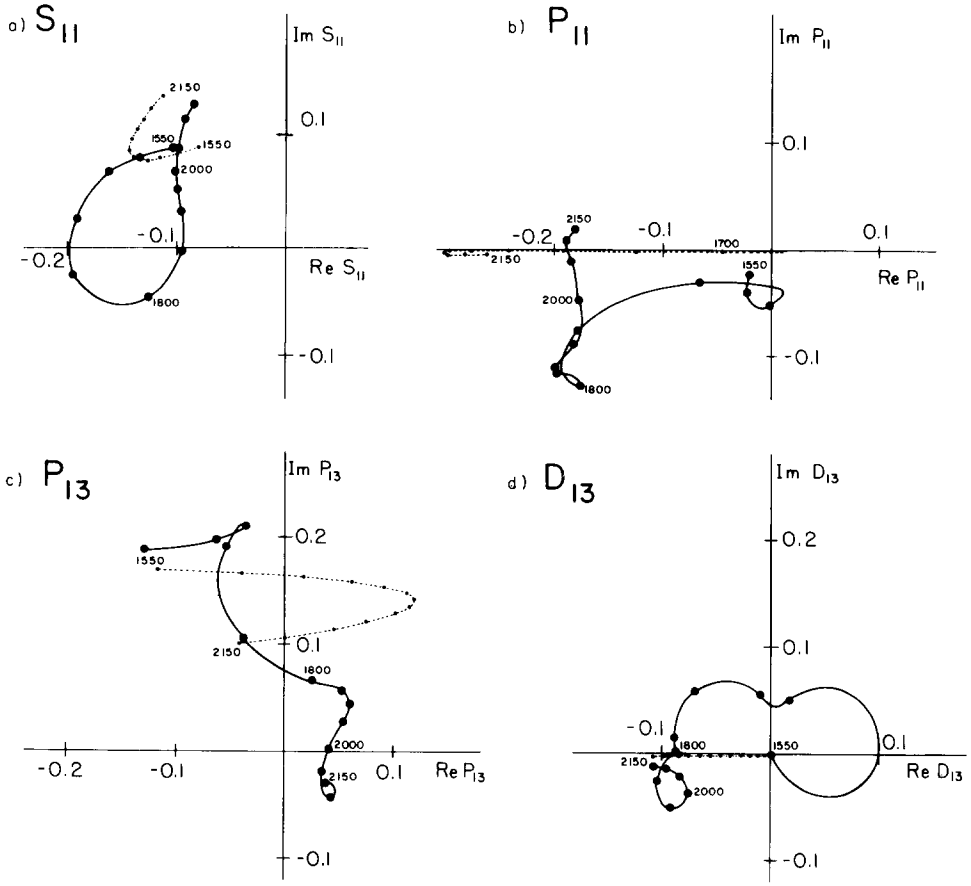


Fig. 4a.

portant. The fit gives a χ^2/NDF of 91.5/85. The parameters of the resonance are

$M = 2035 \pm 15$ MeV, $\sqrt{xx'} = 0.18 \pm 0.02$,
$\Gamma = 180 \pm 20$ MeV, $\phi = 0.0$ (fixed).

The slightly higher value of the mass then has been found in some analyses and it is probably due to the form used for the energy dependence of the total width.

F15

Fig. 1 shows a significant structure in F15 in the region of 1900 MeV which we associate with the $\Sigma(1915)$. At higher energies a predominantly real background is required. The fitted resonance mass showed a large spread depending on the assumptions made about the background and resonance phase, though the width and

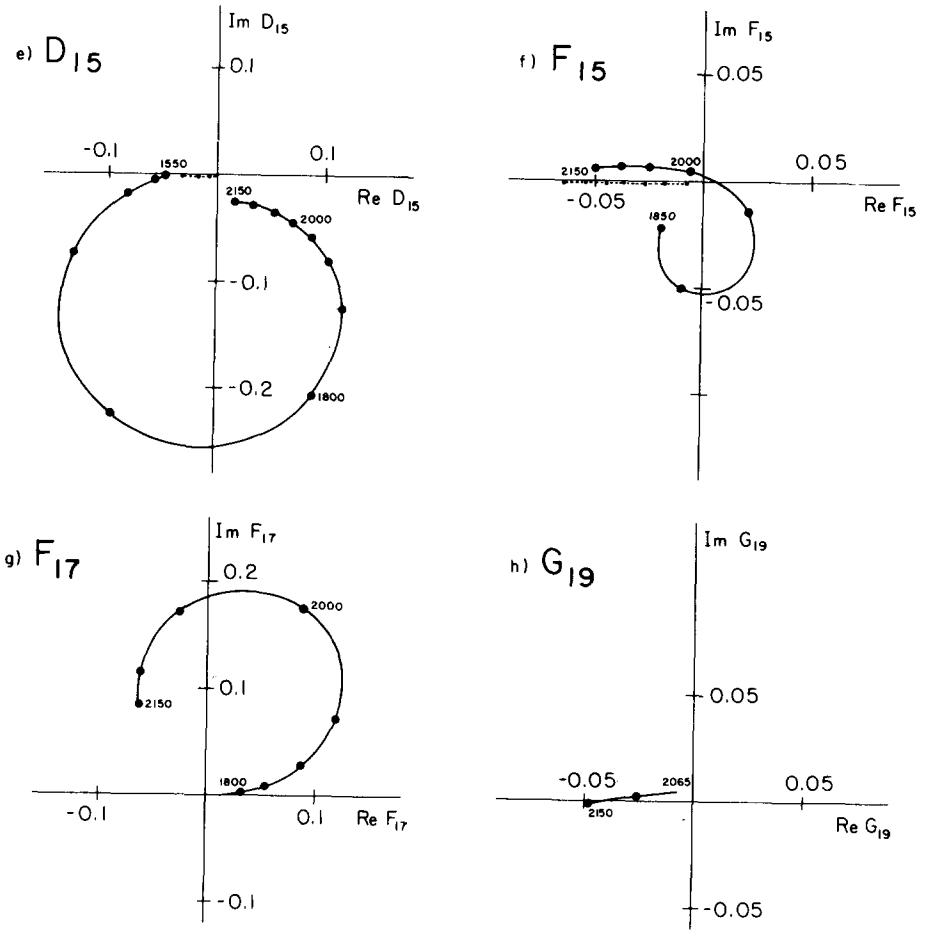


Fig. 4. Argand diagrams of the fitted amplitudes of solution 1. The circles are placed at 50 MeV centre-of-mass energy intervals. The solid lines are the total amplitudes, the dashed lines the background component of the amplitudes.

$\sqrt{xx'}$ remained more or less constant. Masses ranging from 1900–1965 could be obtained. Fits with the resonance phase fixed at zero, an assumption often made in earlier energy-dependent analyses, gave a mass of 1900 MeV in agreement with these analyses. However our preferred fit which has a purely real linear background has parameters:

$$M = 1920 \pm 30 \text{ MeV}, \quad \sqrt{xx'} = -0.06 \pm 0.02,$$
$$\Gamma = 70 \pm 20 \text{ MeV}, \quad \phi = 0.7 \pm 0.3 \text{ rad}.$$

The χ^2/NDF of this fit is 56/82. The best χ^2 obtainable with only a linear background and no resonance was 98.

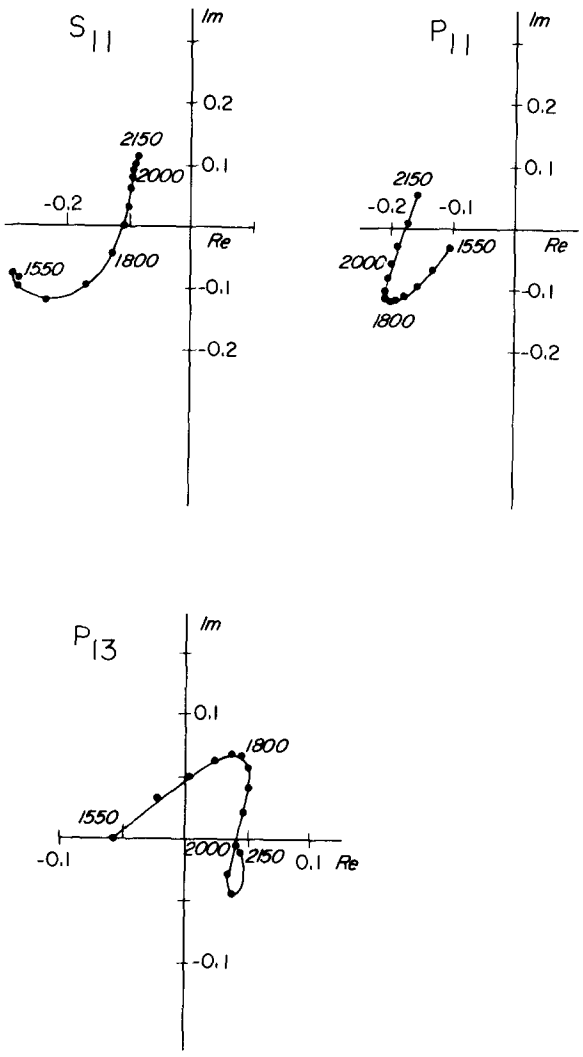


Fig. 5. Argand diagrams of the fitted S- and P-wave amplitudes of solution 2. The D-waves and above for the solution are the same as those shown in fig. 4. The solid circles are placed at 50 MeV centre-of-mass energy intervals.

D15

The D15 amplitude is, of course, dominated by the $\Sigma(1765)$. Although the resonance constraint was removed at 1850 MeV the amplitude continues to follow very closely the Breit-Wigner form with only a small purely real background entering at

higher energies. This is one of the strongest indications that our procedure extracts the true amplitudes. No further resonance structure is required up to 2150 MeV. The χ^2/NDF is 202/182 and the parameters of the $\Sigma(1765)$ are

$$M = 1775 \pm 10 \text{ MeV}, \quad \sqrt{xx'} = -0.25 \pm 0.02,$$
$$\Gamma = 125 \pm 15 \text{ MeV}, \quad \phi = 0.0 \text{ (fixed)}.$$

D13

The imaginary part of D13 shows clearly two structures, one around 1680 MeV and the other near 1950 MeV. In addition there is structure at low masses around 1580 MeV but the energy binning of this data is too coarse to clearly show the resonance as it is seen in ref. [17]. The real part while reflecting these resonances is dominated by a slowly varying background term. A fit including these resonances plus a purely real quadratic background gave a χ^2/NDF of 151/189. The mass width and resonant amplitude of the $\Sigma(1580)$ were fixed at the values found in ref. [20], but the phase relative to the other resonance was allowed to vary. A large value of -1.6 ± 0.4 was found in agreement with ref. [20]. The parameters of the $\Sigma(1670)$ and the $\Sigma(1915)$ are not well determined. The conventional fit to the $\Sigma(1670)$ with the phase fixed at zero gave a mass of 1670 MeV however, with a free phase the mass went to 1705 MeV. Because the $\sqrt{xx'}$ of the $\Sigma(1940)$ is very small the statistical errors on its parameters are large probably outweighing the systematic errors in this case. The best values for the parameters are

$$M = 1685 \pm 20 \text{ MeV}, \quad M = 1950 \pm 30 \text{ MeV},$$
$$\Gamma = 85 \pm 25 \text{ MeV}, \quad \Gamma = 150 \pm 75 \text{ MeV},$$
$$\sqrt{xx'} = +0.065 \pm 0.02, \quad \sqrt{xx'} = -0.040 \pm 0.015,$$
$$\phi = 0.4 \pm 0.3 \text{ rad}, \quad \phi = 0.0 \pm 0.5 \text{ rad}.$$

P13

The P13 amplitude shows a strong structure in the region around 1700 MeV and less significant structure at around 2100 MeV. The χ^2/NDF for a fit with two resonances plus a quadratic real and imaginary background is 129/194. The χ^2 rise on omitting each resonance is 158 for the $\Sigma(1730)$ and 47 for the $\Sigma(2120)$. The higher resonance is the same effect as that seen before at 2080 MeV in $\Lambda\pi$ [4, 14] but, as before, the χ^2 difference between a resonance solution and pure background is small, probably because of the large width of the resonance and its high mass near the end of the energy range. The parameters of the two resonances are

$$M = 1720 \pm 30 \text{ MeV}, \quad M = 2120 \pm 40 \text{ MeV},$$
$$\Gamma = 120 \pm 30 \text{ MeV}, \quad \Gamma = 240 \pm 50 \text{ MeV},$$
$$\sqrt{xx'} = 0.11 \pm 0.02, \quad \sqrt{xx'} = -0.13 \pm 0.04,$$
$$\phi = 1.2 \pm 0.3 \text{ rad}, \quad \phi = 0.5 \pm 0.3 \text{ rad}.$$

P11

The P11 amplitude shows a strong structure at around 1750 MeV together with a small imaginary dip at around 1650 MeV and a broad structure around 1950 MeV.

The amplitude could be fitted with a purely real quadratic background with E_{min} = 1667 MeV plus three resonances at 1660, 1770 and 1960 MeV, giving a χ^2/NDF of 140/192. The $\Sigma(1660)$ is not very significant in terms of χ^2, a rise of only 10 being produced by its omission, however its parameters are so close to those of the $\Sigma(1660)$ seen in the $\bar{K}N$ and $\Sigma\pi$ [1, 21] channel that is seems probable that the same object is being seen here. A χ^2 rise of 90 is produced by omission of the $\Sigma(1770)$ and also of 90 for replacing the broad $\Sigma(1960)$ by a quadratic background. The background thus produced, however, resembled a resonance circle. The resonance parameters are

$$\begin{array}{lll} M = 1660 \pm 30\,, & M = 1770 \pm 20\,, & M = 1960 \pm 30\,, \\ \Gamma = 80 \pm 40\,, & \Gamma = 80 \pm 30\,, & \Gamma = 260 \pm 40\,, \\ \sqrt{xx'} = -0.04 \pm 0.02\,, & \sqrt{xx'} = -0.08 \pm 0.02\,, & \sqrt{xx'} = -0.12 \pm 0.02\,, \\ \phi = 1.1 \pm 0.8\,, & \phi = -0.5 \pm 0.5\,, & \phi = 0.9 \pm 0.4\,. \end{array}$$

S11

This is also well fitted with a resonance plus quadratic background. A χ^2/NDF of 82/198 was obtained, without the resonance the best χ^2/NDF was 268/202. The resonance parameters are

$$\begin{array}{ll} M = 1780 \pm 30\,, & \sqrt{xx'} = -0.12 \pm 0.02\,, \\ \Gamma = 140 \pm 30\,, & \phi = -0.1 \pm 0.2\,. \end{array}$$

Whether this resonance is the same object that was seen predominantly in the elastic channel by LMMO [1] and Kim [22] with a mass of 1740 and width 60 MeV is not clear.

Solution 2

The conclusions from solution 2 for the D13 and higher amplitudes are identical to those from solution 1. The low-mass P13 and the P11 resonances are not required. Thus the solution could be considered a simpler and possibly a more acceptable solution. However, a preliminary analysis of data below 0.450 GeV/c [23] indicated that in this region solution 2 has strong structure which is not present in solution 1. As already discussed above the amplitudes found by ref. [20] in the region of the $\Sigma(1580)$ prefer solution 1.

P13

The χ^2/NDF for a quadratic background plus high mass resonance is 115/198.

P11

A quadratic background is sufficient to fit the data with a χ^2/NDF of 169/202.

S11

A resonance is required in the 1700 MeV region. The χ^2/NDF with the resonance is 90/198, the χ^2 rise for a simple quadratic background is 171. The resonance parameters are

$M = 1700 \pm 30$, $\sqrt{xx'} = -0.13 \pm 0.03$,
$\Gamma = 160 \pm 50$, $\phi = -0.4 \pm 0.4$.

Again the width is much broader than that claimed for the $\Sigma(1740)$ seen predominantly in the elastic channel.

4. Discussion of the results

We have compared our analysis with the following analyses of this channel, each using a different method.

(a) The energy-dependent single-channel analysis of Van Horn [2].

(b) The K-matrix analysis of Lea et al. (LMMO) [1]. This analysis only covers the momentum region up to 1.2 GeV/c.

(c) The energy-independent shortest-path method of Langbein and Wagner [6]. This analysis gives amplitudes which disagree strongly with those found here. It was shown in ref. [7] that there are serious problems with the continuity of the zeros of this analysis and it is not considered further.

In the region between 1750 and 2050 where we find a unique solution all the analyses agree well, within the errors, though our analysis probably has smoother amplitudes than the others. Below 1750 MeV there are strong differences. This is probably due to two problems: firstly the lower in mass the solution is taken the less effect the $\Sigma(1765)$ has and thus the phase of the solution becomes less well-defined. This allows the amplitudes to move and produce structure on the Argand diagrams which, since the relative phases of the amplitudes remain constant, is clearly not required by the data. Secondly, the presence of two solutions can cause extra structure if an analysis flips from one to another. The strong structure in the p waves of solution 1 could cause this to happen.

The three fits of Van Horn are all variants of solution 2, though fit K may suffer from the lack of definition of the overall phase at the lower energies. The fact that he only finds solution 2 is not surprising since the energy-dependent method would tend to favour its less structured P-waves. He did not include the $\Sigma(1580)$ which would point to solution 1 in its narrow energy region, which is poorly fitted in his analysis. He finds the resonances S11 $\Sigma(2004)$ and P13 $\Sigma(1925)$ which are not required by the analysis.

The K-matrix analysis of LMMO resembles our solution 2 below 1.0 GeV/c though above this it diverges from our solution, possibly a result of the parametrisation diverging at the end of the energy region studied. Although they see a strong loop in S11 they find a branching fraction into $\Lambda\pi$ for their $\Sigma(1740)$ of nearly zero which may be an indication that the resonance seen here at 1780 MeV is not the same object as that seen in $\bar{K}N$.

5. Conclusions

An energy-independent partial-wave analysis of the reaction $K^- p \to \Lambda\pi$ has been carried out between 1540 and 2150 MeV using two independent methods to provide energy continuity. Between 1750 and 2060 MeV the solution is believed to be unique given the assumption of the dominance of the two leading Breit-Wigner resonances, the $\Sigma(1765)$ and $\Sigma(2030)$, and smoothness of the Barrelet zeros. Above 2060 MeV the solutions depends slightly on the choice of the entering g-wave amplitude. Below 1750 there are two ambiguous solutions. We have chosen one (solution 1) as more probable because of its agreement with the solution found in the region of the narrow $\Sigma(1580)$ [20]. However the amplitudes of the second solution are also given in case further evidence, particularly from the very low energy region, should prefer solution 2.

The resonance parameters from energy-dependent fits to the two solutions are given in table 3. It must be stressed that whereas the partial-wave analysis is relatively model-independent the fits giving the resonance parameters are strongly dependent on the model used for the amplitudes. No sign is seen of the S11 resonance reported in the region 1600–1650 MeV [19] where our amplitude is very smooth. It seems probable that in the $\Lambda\pi$ channel this state has been generated by the processes described in sect. 4. Two possible new states are observed, a P11 at 1770 MeV and a P13 at 1720 MeV. If the analysis in the 1580 MeV region is accepted then structure in these amplitudes is inevitable since at 1580 the P3 amplitude has to be large and P1 small whereas by 1800 MeV the situation has reversed. The S-wave resonance has somewhat different parameters from those found for the $\Sigma(1740)$ in multichannel analyses and may be a different state. The P11 $\Sigma(1680)$ and P11 $\Sigma(1900)$ are seen here with compatible parameters to those previously observed. Structure is observed in the D13 amplitude in the region of the $\Sigma(1580)$ but the coarseness of the energy bins of the data used here makes it impossible to determine its parameters. The D13 $\Sigma(1660)$, D13 $\Sigma(1940)$ and F15 $\Sigma(1915)$ are observed with similar parameters to those determined previously in this channel.

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