POSSIBLE EXISTENCE OF $\bar{K}$NN BOUND STATES

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Evidence for a resonance called Y_0^* has been found with a mass of ~ 1405 MeV, although its spin and parity have not been determined [1]. The most promissing interpretation of Y_0^* appears to be that of Dalitz and Tuan [2]: Y_0^* is a $\bar{K}$-N bound state ($S_{\frac{1}{2}}$). On the basis of this interpretation, it is conceivable that $\bar{K}$NN bound states, or more generally. $\bar{K}$-nucleus bound states may exist. The purpose of this note is to examine this possibility.

There are three possible isospin configurations:

(A) $(\bar{K}(NN)_0)_{\frac{1}{2}}$, (B) $(\bar{K}(NN)_1)_{\frac{1}{2}}$, (C) $(\bar{K}(NN)_1)_{\frac{3}{2}}$,

For instance, in (A) two nucleons which are in an isosinglet state and $\bar{K}$ are combined so that the total isospin is $\frac{1}{2}$. We assume that all particles are in an S-state, then the system (A) has spin 1 and the other spin 0.

For simplicity we assume separable potentials [3] between the three particles, then as was shown by Mitra [4], the three body problem can be reduced to a soluble form. Of course, the separable potential would be rather unrealistic, but presumably we could depend on the shape independence theory.

For the two-nucleon problem [3], using a potential of the form

$$(\boldsymbol{p}|\,V_N\,|\boldsymbol{p}') = -(\lambda_N/m_N)\,f_N(p)\,f_N(p')\,, \qquad (1)$$

we find the S-wave scattering amplitude

$$F(p) = p^{-1}\exp[\mathrm{i}\delta(p)]\sin\delta(p) \qquad (2)$$
$$= 2\pi^2 f_N^2(p)[1+\lambda_N\int d^3\boldsymbol{g}\, f_N^2(g)(p^2-g^2+\mathrm{i}\epsilon)^{-1}]^{-1}.$$

Furthermore, we assume

$$f_N(p) = (p^2+\beta_N^2)^{-1}\,. \qquad (3)$$

Then the scattering length a_N is given by

$$a_N^{-1} = \tfrac{1}{2}\beta_N\{1-(\beta_N^3/\pi^2\lambda_N)\}\,. \qquad (4)$$

There is a bound state if $\lambda_N > \pi^{-2}\beta_N^3$. The deuteron binding energy, α_N^2/m_N, is obtained by the equation $F(\mathrm{i}\alpha_N)^{-1} = 0$ with real positive α_N, which gives the relation

$$\lambda_N = \pi^{-2}\beta_N(\alpha_N+\beta_N)^2\,. \qquad (5)$$

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Following Yamaguchi [3] we take ($c = \hbar = 1$): $\alpha_N = 0.232\ \mathrm{fm}^{-1}$, $\beta_N^{(0)} = \beta_N^{(1)} = \beta_N = 1.449\ \mathrm{fm}^{-1}$, $\lambda_N^{(0)} = 0.413\ \mathrm{fm}^{-3}$, $\lambda_N^{(1)} = 0.291\ \mathrm{fm}^{-3}$. Here the superscript denotes the isospin of the two-nucleon system. For instance, $\lambda_N^{(0)}$ and $\lambda_N^{(1)}$ are Yamaguchi's λ_t and λ_s, respectively.

For the $\bar{K}$-N interaction, let us take

$$(\boldsymbol{p}|\,V_K\,|\boldsymbol{p}') = -(\lambda_K/2m_{KN})\,f_K(p)\,f_K(p')\,, \qquad (6)$$

where

$$f_K(p) = (p^2+\beta_K^2)^{-1} \qquad (7)$$

and $m_{KN} = m_K m_N/(m_K+m_N)$ is the $\bar{K}$-N reduced mass. Because of the presence of the inelastic channel $\bar{K}+N \to \pi+Y$, the $\bar{K}$-N potential will be complex. For the present, however, let us ignore the imaginary part, partly because of the difficulty to solve the three-body problem with complex potentials, and partly because of our insufficient knowledge on the width of Y_0^* (ref. [6]). Considerable effort have been made to determine the low energy S-wave $\bar{K}$-N scattering amplitude, but there still seems to be fairly large ambiguity [2,5]. If the scattering lengths were known all the parameters wouldbe fixed. For β_K we assume the same value as β_N. From the binding energy of Y_0^*, we have $\alpha_K = 0.705\ \mathrm{fm}^{-1}$, then using eq. (5) we get $\lambda_K^{(0)} = 0.681\ \mathrm{fm}^{-3}$. For $\lambda_K^{(1)}$ we examine three cases: $\lambda_K^{(1)} = (\frac{1}{3}, \frac{1}{4}, \frac{1}{5})\,\lambda_K^{(0)}$. With these parameters, the $\bar{K}$-N scattering lengths are $a_K^{(0)} = 2.50$ fm, and $a_K^{(1)} = -(3.86,\ 1.70,\ 1.09)$ fm.

Now let us discuss the three-body system of $\bar{K}$NN. In the overall centre-of-mass system, let the momenta of the two nucleons and $\bar{K}$ be $\boldsymbol{P}_1$, $\boldsymbol{P}_2$, and $\boldsymbol{P}_3$, respectively, so that $\boldsymbol{P}_1+\boldsymbol{P}_2+\boldsymbol{P}_3 = 0$. The kinetic energy of the system is $(P_1^2+P_2^2)/(2m_{KN}) + \boldsymbol{P}_1\cdot\boldsymbol{P}_2/m_K$. The three-body Schrödinger equation is

$$\{P_1^2+P_2^2+(2m_{KN}/m_K)\,\boldsymbol{P}_1\cdot\boldsymbol{P}_2+\alpha^2\}\,\Psi(\boldsymbol{P}_1,\boldsymbol{P}_2) =$$
$$c_K\int d^3\boldsymbol{P}_1'\, f_K(\boldsymbol{P}_1+\tfrac{1}{2}\boldsymbol{P}_2)\, f_K(\boldsymbol{P}_1'+\tfrac{1}{2}\boldsymbol{P}_2)\,\Psi(\boldsymbol{P}_1',\boldsymbol{P}_2) \qquad (8)$$
$$+\,(1\rightleftharpoons 2) + c_N\int d^3\boldsymbol{p}'\, f_N(p)\, f_N(p')\,\Psi(\tfrac{1}{2}\boldsymbol{P}-\boldsymbol{p}',\ \tfrac{1}{2}\boldsymbol{P}+\boldsymbol{p}'),$$

Table 1
Coefficients in the Schrödinger eq. in units of f^{-3}.

	A	B	C
c_K	$\frac{1}{4}(\lambda_K^{(0)}+3\lambda_K^{(1)})$	$\frac{1}{4}(3\lambda_K^{(0)}+\lambda_K^{(1)})$	$\lambda_K^{(1)}$
$\lambda_K^{(1)}/\lambda_K^{(0)} = \frac{1}{3}$	0.341	0.568	0.227
$\frac{1}{4}$	0.298	0.553	0.170
$\frac{1}{5}$	0.272	0.545	0.136
c_N	$(2m_{KN}/m_N)\lambda_N^{(0)}$	$(2m_{KN}/m_N)\lambda_N^{(1)}$	$(2m_{KN}/m_N)\lambda_N^{(1)}$
	0.286	0.201	0.201

Table 2
The binding energy in the case (B) in MeV.

$\lambda_K^{(1)}/\lambda_K^{(0)}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\alpha^2/(2m_{KN})$	11.5	10.0	9.4

where $\boldsymbol{P} = \boldsymbol{P}_1+\boldsymbol{P}_2$, $2\boldsymbol{p} = \boldsymbol{P}_2-\boldsymbol{P}_1$, and $\alpha^2/(2m_{KN})$ is the binding energy. The coefficients c_K and c_N are given in table 1.

The eq. (8) can be reduced to a one-dimensional integral equation which can be solved numerically. For our qualitative argument, however, we content ourselves with a crude but analytic treatment as was illustrated by Mitra [4]. In addition to the approximations made by Mitra, we replace $(2m_{KN}/m_K)$ in the left hand side by unity. The effect of the K-N mass difference then appears only through the factor $(2m_{KN}/m_N)$ in c_N. The problem finally boils down to solve the following equation* for the unknown α:

$$\frac{\pi^2 c_K}{\beta(\alpha+\beta)^2-\pi^2 c_K}+\frac{2\pi^2 c_N}{\beta(\alpha+\beta)^2-\pi^2 c_N}=\frac{27}{8\alpha}\,\frac{(\alpha+\beta)(\alpha+2\beta)^3}{(\alpha+3\beta)^3+2\alpha\beta^2}\,. \tag{9}$$

As is seen from the structure of this equation, it is necessary for the existence of a bound state that at least one of the two coefficients, c_K and c_N, is larger than $\pi^{-2}\beta^3 = 0.308\ \mathrm{fm}^{-3}$. This condition is satisfied in the case (B) always, and in (A) if $\lambda_K^{(1)} = \frac{1}{3}\lambda_K^{(0)}$, but not in (C). The result in the case (B) is shown in table 2.

* Eq. (8) corresponds to Mitra's (4.16 ~ 17). There is a misprint in Mitra's (4.17).

The existence of these bound states will most directly be detected by observing resonances in the N-Λ or N-Σ scattering. Experimental data of the N-Λ scattering have been available up to the c.m. kinetic energy of ~ 500 MeV, but they are still meagre [7]. Our resonances would be expected around 300 MeV. Although our estimate is admittedly too crude to make any definite prediction, the behaviour of the N-Λ scattering cross section around this energy seems to deserve special attention of experimentalists. The $\overline{\mathrm{K}}$NN bound states may contribute to the high rate of the two-nucleon capture of $\overline{\mathrm{K}}$ at rest in heavy nuclei [8].

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