University of Waterloo
CS 341 — Algorithms
Spring 2014
Assignment 3

Written Portion Due: Wed, June 18, 2014, at 3:00pm.
Programming Portion Due: Mon, June 23, 2014, at 3:00pm.

Assignment Guidelines

- Use A3-coversheet.pdf for the first page of your assignment or format the first page of your assignment in *exactly* the same manner.
- Answers the questions in the same order that they are given in the assignment.
- Make it very clear to the marker where the answer to one question ends and the answer to the next one begins.
- You may lose up to 20% on a question because it is difficult to read or difficult to understand.
- You may only use the techniques discussed in class.
- Your whole assignment is considered late if either the programming question or the written portion is submitted late.

Assignment Questions

1. **[8 points]** Consider the following instance of the Stable Marriage Problem (SMP) with 3 men and 3 women, denoted M_1, M_2, M_3 and W_1, W_2, W_3, respectively. The preference lists are as follows (where ">" signifies "prefers over"):

$M_1 : W_1 > W_2 > W_3$	$W_1 : M_2 > M_1 > M_3$
$M_2 : W_1 > W_3 > W_2$	$W_2 : M_2 > M_3 > M_1$
$M_3 : W_3 > W_2 > W_1$	$W_3 : M_1 > M_2 > M_3$

   (a) List all possible matchings (with respect to the SMP). For each matching, find all instabilities (blocking pairs) by listing the 2-tuple of the couple that have caused the instability (i.e. the couple that would prefer to be together over their current arrangements).

   (b) Consider the Gale-Shapely algorithm with the alteration that the women propose to the men. Show each step of the execution of this algorithm on the problem instance given. Use statements of the form "x proposes to y, y accepts/rejects" to describe the execution and give the final matching.

2. **[10 points]** Yoshi is standing at position X on the left shore of a creek and wants to cross to the other side, to position Y. By using his "flutter jump", he is able to leap a distance of L feet. Unfortunately, the creek is much wider than L feet. However, fortunately, the creek has a path of n lily pads across it from X to Y where consecutive lily pads are less than L feet apart so Yoshi can cross the pond by jumping on them.

   Denote the lily pads by $lp_1, \ldots, lp_n$ and the distance $d[1] = distance(X, lp_1)$, $d[i] = distance(lp_{i-1}, lp_i)$ and $d[n+1] = distance(lp_n, Y)$.

   (a) Give an efficient greedy algorithm to determine the selection of lily pads that will minimize the number of jumps Yoshi must take from X to Y.

   (b) Prove that your algorithm gives an optimal solution.

3. **[10 points]** Suppose we need to deliver n distinct items to our home from the SuperCheapStore (SCS) where each item in the store is priced at \$1. Unfortunately,

   - we can fit only one item in our truck
   - it takes one day to drive home and back.

   The more unfortunate news is that SCS charges us for the storage of undelivered items and the charge for the storage of item i grows exponentially as the original price times a constant factor $c_i > 1$ each day: if item i is picked up d days from now, the charge will be $1 \cdot (c_i)^d$. You may assume that $c_i \neq c_j$ for $i \neq j$.

   (a) Give an efficient greedy algorithm that determines the order we should pick up the items from SCS and minimize the charges.

   (b) Give and briefly justify the runtime of your algorithm.

   (c) Prove that your algorithm gives an optimal solution.

4. **[12 points]** An orphanage has recently received a donation of a pair of shoes for each of the n children at the orphange. However, the shoes are of various sizes and so are the children's feet - some children will find a pair that fits well, others will find the shoes to be too big and others will find them to be too small. More formally, a child i has foot size f_i and pair of shoes j has shoe size s_j. The goal is to distribute the shoes to minimize the average difference of each child's foot size with the shoe size they are given.

   (a) Give an efficient greedy algorithm that assigns shoes to children and minimizes the absolute difference between a child's foot size and the size of the shoes they are given.

   (b) Give an exchange proof to show that your algorithm produces an optimal solution.

5. **[15 points]** Programming Question

   The programming question will be added by Monday June 9 and will be due Monday June 23.