L	I

Write the **first 2 characters of your lastname** in large format in the boxes above.

CS 341 Assignment #3

1	
2	
3	
4	
5	Programming See LEARN
Total	

Lastname	LIU
Given Names	Yinuo
Student #	20529186
Return in Section # (Circle One)	**8:30am** or **10:00am**

1

(a)

① m_1 w_1 m_2 prefers w_1 over w

m_2 w_2 w_1 prefers m_2 over m_1

m_3 m_3 then (m_2, w_1) causes instability

② m_1 w_1 w_1 prefers m_2 over m_1

m_2 w_3 m_2 prefers w_1 over w_3

m_3 w_2 then (m_2, w_1) causes instability

③ m_1 w_2

m_2 w_1 Stable marrige.

m_3 w_3

④ m_1 w_2 w_1 prefers m_2 over m_3

m_2 prefers w_1 over w_3

m_2 w_3 then (m_2, w_1) causes instability

m_3 w_1 m_3 prefers w_2 over w_1 and

w_2 prefers m_3 over m_1

then (m_3, w_2) also causes instability

(5) M_1 W_3

M_2 W_2

M_3 W_1

M_2 prefers W_1 over W_2

W_1 prefers M_2 over M_3

then (M_2, W_1) causes instability

M_1 prefers W_1 over W_3

W_1 prefers M_1 over M_3

then (M_1, W_1) also causes instability

(6) M_1 W_3

M_2 W_1

M_3 W_2

Stable marriage

b) initilize ~~Math~~ match $\Leftarrow \phi$

W_1 proposes to M_2, M_2 accepts

match $\Leftarrow \phi \cup \{M_2, W_1\}$

W_2 proposes to M_2, M_2 rejects

W_2 proposes to M_3, M_3 accepts

Match $\Leftarrow \{M_2, W_1\} \cup \{M_3, W_2\}$

W_3 Proposes to M_1, M_1 accepts

match $\Leftarrow \{(M_2, W_1), (M_3, W_2)\} \cup \{M_1, W_3\}$

final Match $= \{(M_2, W_1), (M_3, W_2), (M_1, W_3)\}$

2. a) idea: Yoshi needs to leap to the farthest pad he can reach.

initialize $s \leftarrow 0$. $B \leftarrow \phi$

~~while (true)~~

for i from 1 to $n+1$:

do

if $s + d[i] \leq L$

$s \leftarrow s + d[i]$

else:

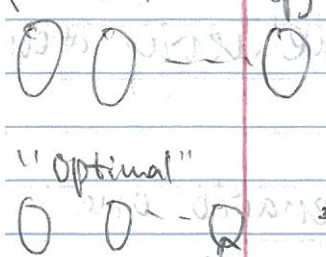
$B \leftarrow B \cup \{l_{p_{i-1}}\}$ $(i \geq 2)$

$s \leftarrow d[i]$

~~break for loop~~

~~break~~ return B

b) proof: ~~greedy selects~~ Suppose after j steps, greedy selects pad l_{p_j}, and "optimal" selects l_{p_i}. before that they select the same set of pad



Because at each step, greedy selects the farthest as possible, then distance(l_{p_j}, Y) > distance(l_{p_i}, Y).

Therefore, in order to reach Y within less steps compare to greedy, the "optimal" has to choose to leap farther than greedy could ~~within~~ in one later step, which is not possible because greedy already chooses the farthest pad Yoshi can reach, therefore "optimal" is not really optimal solution

3 a) Sort all the items by the constant factor so that $C_1 \geq C_2 \geq \cdots \geq C_n$

then deliver items by this order

b) sorting takes time $O(n \log n)$

So the run time of this algorithm is $O(n \log n)$

c) proof: Let $C_1 \geq C_2 \geq \cdots \geq C_n$

consider any ordering, there must be two deliveries that are consecutive, but differ from the greedy ordering. ~~Let $C_j >$~~

Suppose $C_i > C_j$. Consider swaping the delivery of item i and which means deliver item j on day k, and item i on day k+1, instead of delivering item i on day k and item j on day k+1

the total cost increase from scenario one to scenario two is ~~total cost scenario one is C~~

$$(C_i^k + C_j^{k+1}) - (C_i^k + C_j^{k+1}) = C_i^k(C_i - 1) - C_j^k(C_j - 1)$$

because $C_i > C_j > 1$ then $C_i - 1 > C_j - 1$, and $C_i^k > C_j^k > 0$

therefor $C_i^k(C_i - 1) \geq C_j^k(C_j - 1)$

$c_i^k(c_i - 1) - c_j^k(c_j - 1) > 0$

therefore the swapping causes the increase of storage cost, so we can keep swapping consecutive deliveries that are out of ~~ordering~~ order until we get the greedy ordering

4. a) sort children by their foot size such that

$f_1 < f_2 < \cdots < f_n$

sort shoes by shoe size such that

$s_1 < s_n < \cdots < f_n$.

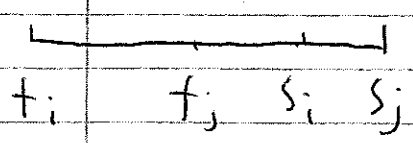
assign shoes to children according to this rule, the child with the largest foot size ~~with~~ will have the largest shoes, the child with the second largest foot size will have the second largest shoes, etc, so the final assignment will be $(f_1, s_1), (f_2, s_2), \ldots, (f_n, s_n)$

b) Suppose we have two children with foot size f_i and f_j, respectively. Let $f_i < f_j$ and $s_i < s_j$, under the greedy algorithm, shoes ~~j~~ i are assigned to child i and shoes j are assigned to child j. Now, let other assignments stay the same, swapping the shoes assigned to child i and child j.

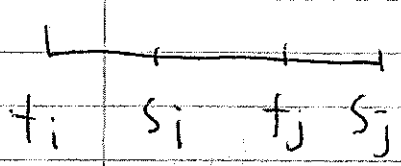
Now, the ~~differen~~ increase X in total shoe & foot absolute difference will be

$$X = (|f_i - s_j| + |f_j - s_i|) - (|f_i - s_i| + |f_j - s_j|)$$

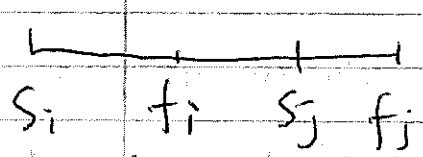
we need to consider differen scenarios

① 

$X = 0$

② 

$X = 2(s_j - s_i) > 0 \quad (\because s_j > s_i)$

③ 

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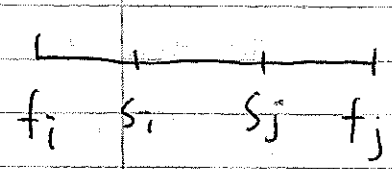
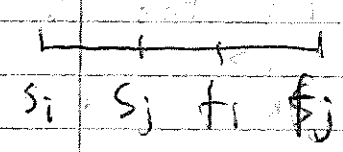
④ 

$X = 2(s_j - s_i) > 0 \quad (\because s_j > s_i)$

⑤ 

$X = 0$

therefore, the increase X will be non-negative, no matter what the relationship between s_i, s_j, t_i, f_j is. Therefore the greedy assignment will have the smallest total shoe & foot absolute difference. Any swap made to the greedy assignement will possibly cause the total difference to increase.