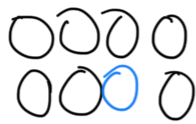52

Weigh 3v3

eq? → Weigh remaining unweighed 1v1. Heavier ball will be evident

→ Weigh 1v1 among the heavier 3

eq? → 3rd ball heavy

→ heavier side heavy
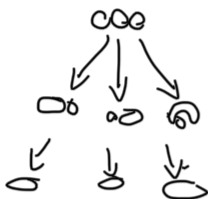
53

1

ways$_2$ = 1 $\binom{2}{2}$

$\binom{3}{2}$

2+1

ways$_3$ = 3 · ways$_2$

$\binom{4}{2}$

ways$_4$ = 6 · ways$_3$

3+2+1=6 ← 3 choices of merge for first, 2 rem. choices for 2nd, 1 rem choice for 3rd.

$$\text{ways}_n = \left(\sum_{i=1}^{n-1} i\right) \cdot \text{ways}_{n-1}$$

$$= \underbrace{\sum_{i=1}^{n-1} i \cdot \sum_{i=1}^{n-2} i \cdot \sum_{i=1}^{n-3} i \cdots \sum_{i=1}^{1} i}_{n-1 \text{ summations}}$$

$$= \bigcup_{i=1}^{n-1} \sum_{j=1}^{i} j$$

2   4   6   8   10   30   12 42
1+1   4+2   6   12   20   (5)|5
            9+3   (4+1)4

Does $n! \, n! = n!^2$?

EX.

$4! \, 4!$

$= 4 \cdot 3 \cdot 2 \cdot 4 \cdot 3 \cdot 2$

$= 24 \cdot 24$

$= 24^2$

$= 4!^2$

$$= \bigcup_{i=1}^{n-1} \frac{(i+1)i}{2} = \frac{(1+1)1}{2} \cdot \frac{(2+1)2}{2} \cdot \frac{(3+1)3}{2} \cdot \ldots \cdot \frac{(n(n-1))}{2}$$

$$= \frac{2 \cdot 6 \cdot 12 \cdot 20 \cdots (n^2-n)}{2^{n-1}}$$

$$= \frac{\bigcup_{i=1}^{n-1}(i+1)i}{2^{n-1}} = \frac{(n-1)! \cdot \bigcup_{i=1}^{n-1}(i+1)}{2^{n-1}} \Bigg\} \quad 2 \cdot 3 \cdot 4 \cdots n$$

$$\boxed{= \frac{(n-1)! \, n!}{2^{n-1}}}$$

$$= \frac{(n-1)!^2 \cdot n}{2^{n-1}}$$

Check

$n=2 \Rightarrow \dfrac{1! \, 2!}{2^1} = 1$ ✓

$n=3 \Rightarrow \dfrac{2! \, 3!}{2^2} = \dfrac{12}{4} = 3$ ✓

$n=4 \Rightarrow \dfrac{3! \, 4!}{2^3} = \dfrac{24 \cdot 6}{8} = 18$ ✓

54, 55

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ
1
✓ ✓ ✓ ✗ ✗ ✗

(blue) Actually 3rd to 6th may all be possible.

In scenario with 1 indivisible dollar, **3rd** senior gets it, while no one dies. B wouldn't vote against this because if A dies, he'll have to do a similar plan to stay alive, which would be contingent on C following through, which is less guaranteed than if he himself secured his own life by voting yes.

(blue, left) Assuming pirates wanna kill if it doesn't affect them either way, B makes no money. So the decision is just on what guarantees survival best.

(right) ○○○○○○○ } solution
2 9 8   1   1
✓ ✗ ✓ ✗ ✓ ✗

2 9 8
✓ ✗ ✓ ✗ ✓

2 9 9
✓ ✗ ✗ ✓ ✗

2 9 9
✗ ✓

2 9 9
✗ ✓

2 0 0
✗ ✗

Start. Work bottoms up from 2 pirates. Find the outcome of base case, then derive n+1 outcome given each pirate compares their situation to the n outcome.
curr

(red) ✱ A: if pirates want offers to live if it doesn't affect their outcome, then senior gets it.