THE UNIVERSITY OF CHICAGO
GRAHAM SCHOOL
CONTINUING LIBERAL AND PROFESSIONAL STUDIES

GRAHAM SCHOOL
UCHICAGO

Natural Language Processing
Session 6

Nick Kadochnikov

Session 6 Agenda

- Information retrieval and topic modeling
  - tf-idf weighting
  - Okapi BM25
  - The Vector Space Model (VSM)
- Ranked retrieval

Introduction to

Information Retrieval

Introducing Information Retrieval and Web Search

Information Retrieval

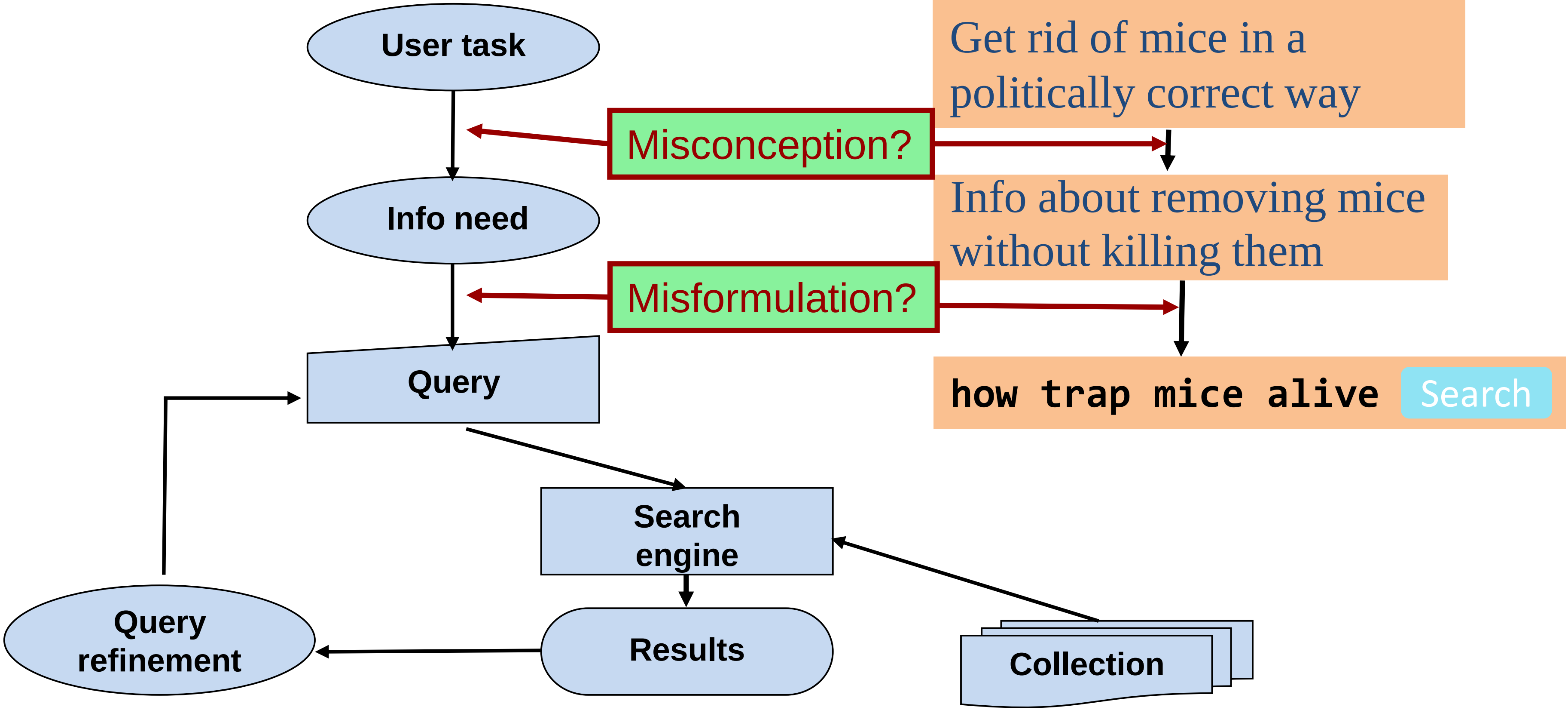
- Information Retrieval (IR) is finding material (usually documents) of an unstructured nature (usually text) that satisfies an information need from within large collections (usually stored on computers).

    - These days we frequently think first of web search, but there are many other cases:
        - E-mail search
        - Searching your laptop
        - Corporate knowledge bases
        - Legal information retrieval

Basic assumptions of Information Retrieval

- Collection: A set of documents
  - Assume it is a static collection for the moment
- Goal: Retrieve documents with information that is relevant to the user's information need and helps the user complete a task

The classic search model



How good are the retrieved docs?

- *Precision* : Fraction of retrieved docs that are relevant to the user's information need
- *Recall* : Fraction of relevant docs in collection that are retrieved

  - More precise definitions and measurements to follow later

Introduction to

Information Retrieval

Term-document incidence matrices

Unstructured data in 1620

- Which plays of Shakespeare contain the words ***Brutus*** *AND* ***Caesar*** but *NOT* ***Calpurnia***?
- One could `grep` all of Shakespeare's plays for ***Brutus*** and ***Caesar,*** then strip out lines containing ***Calpurnia***?
- Why is that not the answer?
  - Slow (for large corpora)
  - *NOT* ***Calpurnia*** is non-trivial
  - Other operations (e.g., find the word ***Romans*** near ***countrymen***) not feasible
  - Ranked retrieval (best documents to return)
    - Later lectures

Term-document incidence matrices

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Brutus *AND* ***Caesar*** *BUT NOT* ***Calpurnia***

1 if play contains word, 0 otherwise

Incidence vectors

- So we have a 0/1 vector for each term.
- To answer query: take the vectors for ***Brutus, Caesar*** and ***Calpurnia*** (complemented) ➔ bitwise *AND*.
  - 110100 *AND*
  - 110111 *AND*
  - 101111 =
  - **100100**

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Answers to query

- Antony and Cleopatra, Act III, Scene ii

Agrippa [Aside to DOMITIUS ENOBARBUS]: Why, Enobarbus,
When Antony found Julius ***Caesar*** dead,
He cried almost to roaring; and he wept
When at Philippi he found ***Brutus*** slain.

- Hamlet, Act III, Scene ii

Lord Polonius: I did enact Julius ***Caesar*** I was killed i' the Capitol; ***Brutus*** killed me.

Bigger collections

- Consider N = 1 million documents, each with about 1000 words.
- Avg 6 bytes/word including spaces/punctuation
  - 6GB of data in the documents.
- Say there are M = 500K *distinct* terms among these.

Can't build the matrix

- 500K x 1M matrix has half-a-trillion 0's and 1's.

- But it has no more than one billion 1's. ⇐ Why?
  - matrix is extremely sparse.

- What's a better representation?
  - We only record the 1 positions.

Introduction to

Information Retrieval

The Inverted Index

The key data structure underlying modern IR

Inverted index

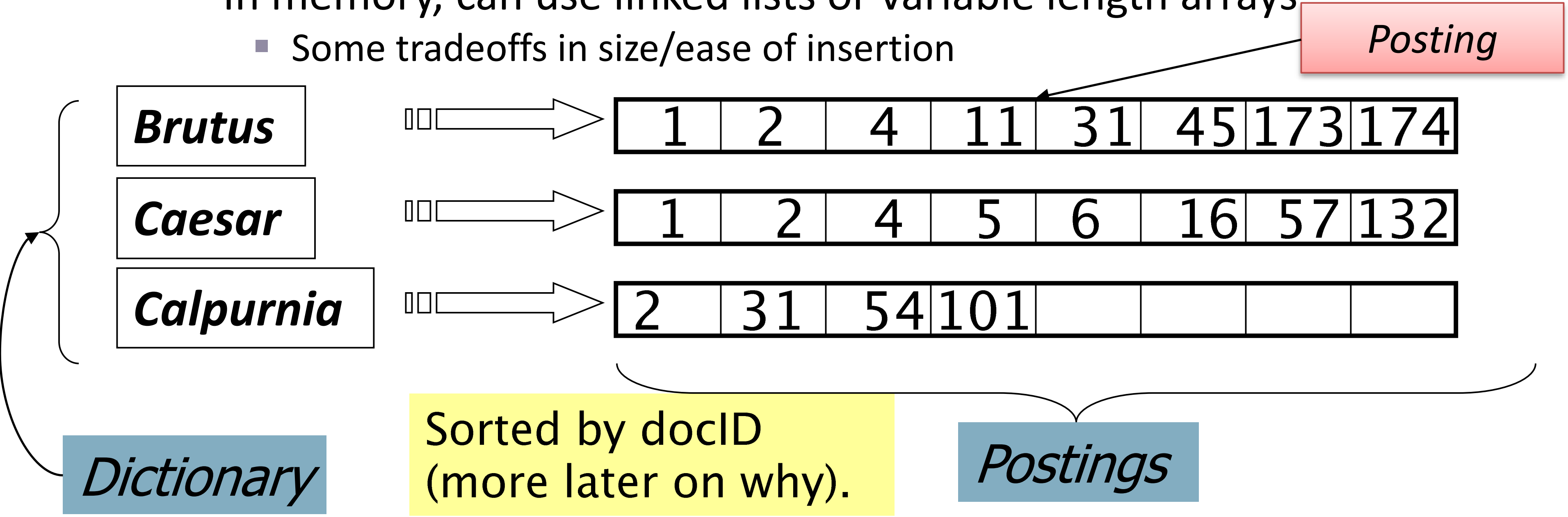
- For each term *t*, we must store a list of all documents that contain *t*.
  - Identify each doc by a **docID**, a document serial number
- Can we use fixed-size arrays for this?

Brutus	1	2	4	11	31	45	173	174
Caesar	1	2	4	5	6	16	57	132
Calpurnia	2	31	54	101				

What happens if the word ***Caesar*** is added to document 14?

Inverted index

- We need variable-size postings lists
  - On disk, a continuous run of postings is normal and best
  - In memory, can use linked lists or variable length arrays
    - Some tradeoffs in size/ease of insertion



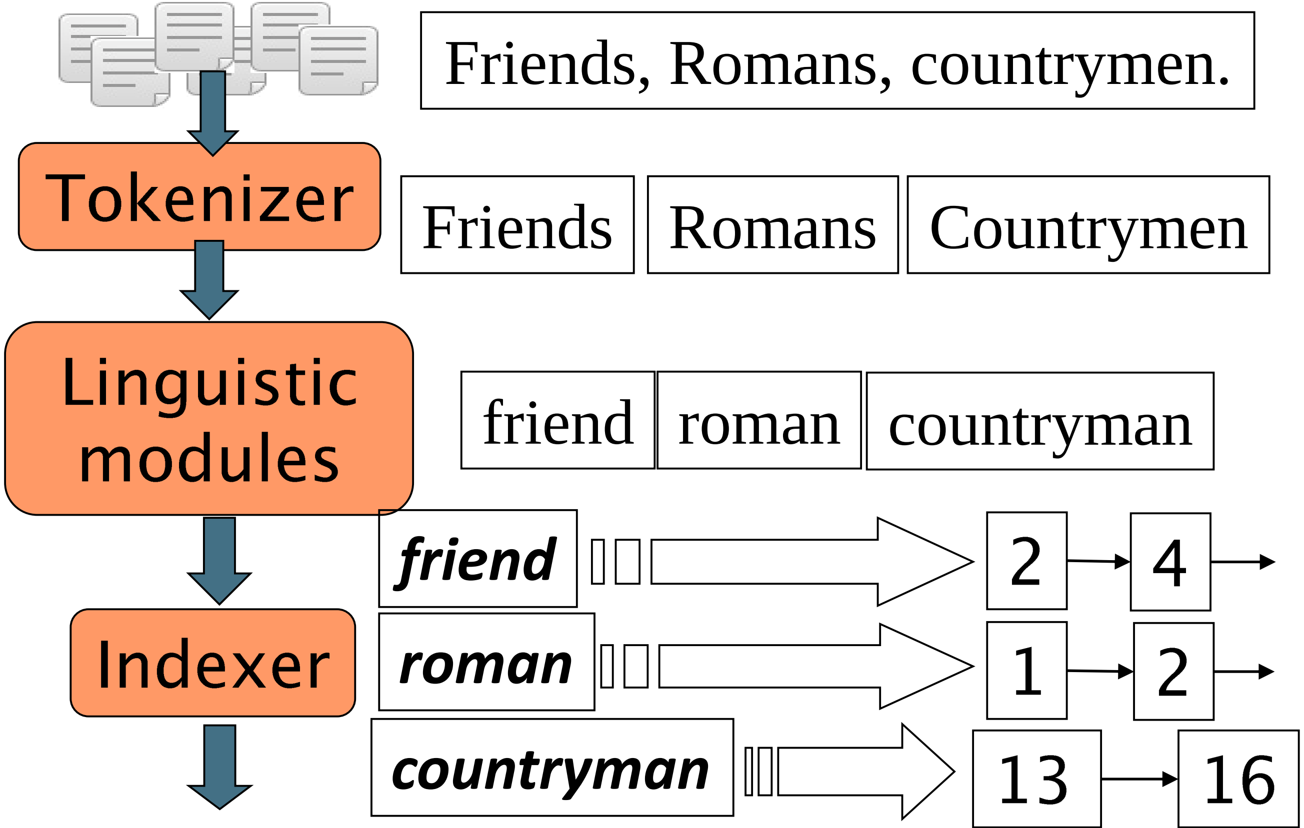
Inverted index construction

Documents to be indexed

Token stream

Modified tokens

Inverted index



Initial stages of text processing

- Tokenization
  - Cut character sequence into word tokens
    - Deal with ***"John's"***, ***a state-of-the-art solution***
- Normalization
  - Map text and query term to same form
    - You want ***U.S.A.*** and ***USA*** to match
- Stemming
  - We may wish different forms of a root to match
    - ***authorize***, ***authorization***
- Stop words
  - We may omit very common words (or not)
    - ***the, a, to, of***

Indexer steps: Token sequence

- Sequence of (Modified token, Document ID) pairs.

Doc 1

I did enact Julius Caesar I was killed i' the Capitol; Brutus killed me.

Doc 2

So let it be with Caesar. The noble Brutus hath told you Caesar was ambitious

Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2

Indexer steps: Sort

- Sort by terms
  - And then docID

Core indexing step

Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2

→

Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2

Indexer steps: Dictionary & Postings

- Multiple term entries in a single document are merged.
- Split into Dictionary and Postings
- Doc. frequency information is added.

Why frequency?
Will discuss later.

Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2

term	doc. freq.	→	postings lists
ambitious	1	→	2
be	1	→	2
brutus	2	→	1 → 2
capitol	1	→	1
caesar	2	→	1 → 2
did	1	→	1
enact	1	→	1
hath	1	→	2
i	1	→	1
i'	1	→	1
it	1	→	2
julius	1	→	1
killed	1	→	1
let	1	→	2
me	1	→	1
noble	1	→	2
so	1	→	2
the	2	→	1 → 2
told	1	→	2
you	1	→	2
was	2	→	1 → 2
with	1	→	2

Where do we pay in storage?

term	doc. freq.	→	postings lists
ambitious	1	→	2
be	1	→	2
brutus	2	→	1 → 2
capitol	1	→	1
caesar	2	→	1 → 2
did	1	→	1
enact	1	→	1
hath	1	→	2
i	1	→	1
i'	1	→	1
it	1	→	2
julius	1	→	1
killed	1	→	1
let	1	→	2
me	1	→	1
noble	1	→	2
so	1	→	2
the	2	→	1 → 2
told	1	→	2
you	1	→	2
was	2	→	1 → 2
with	1	→	2

Terms and counts

Lists of docIDs

Pointers

IR system implementation

- How do we index efficiently?
- How much storage do we need?

Introduction to

Information Retrieval

Query processing with an inverted index

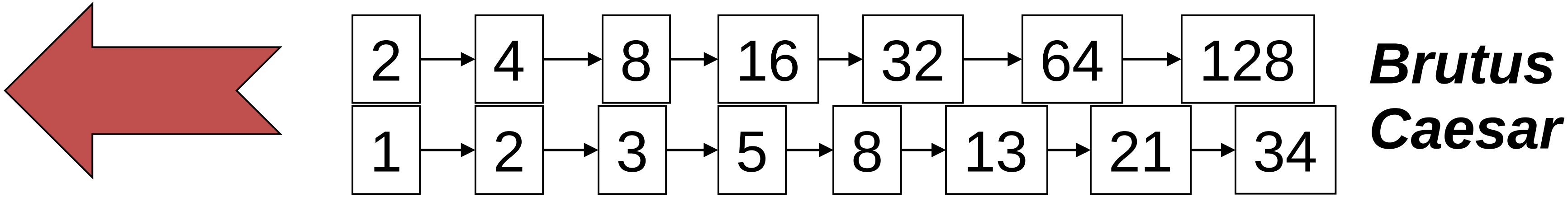
The index we just built

- How do we process a query?
  - Later - what kinds of queries can we process?

Our focus

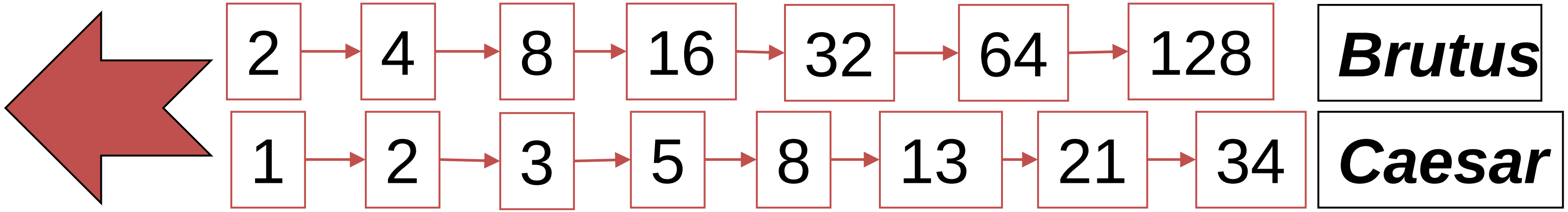
Query processing: AND

- Consider processing the query:
  Brutus *AND* ***Caesar***
  - Locate ***Brutus*** in the Dictionary;
    - Retrieve its postings.
  - Locate ***Caesar*** in the Dictionary;
    - Retrieve its postings.
  - “Merge” the two postings (intersect the document sets):



The merge

- Walk through the two postings simultaneously, in time linear in the total number of postings entries



If the list lengths are x and y, the merge takes $O(x+y)$ operations.

Crucial: postings sorted by docID.

Intersecting two postings lists (a "merge" algorithm)

```
INTERSECT(p1, p2)
answer ← ⟨ ⟩
while p1 ≠ NIL and p2 ≠ NIL
do if docID(p1) = docID(p2)
        then ADD(answer, docID(p1))
             p1 ← next(p1)
             p2 ← next(p2)
        else if docID(p1) < docID(p2)
                then p1 ← next(p1)
                else p2 ← next(p2)
return answer
```

Introduction to

Information Retrieval

Phrase queries and positional indexes

Phrase queries

- We want to be able to answer queries such as ***"stanford university"*** – as a phrase
- Thus the sentence *"I went to university at Stanford"* is not a match.
  - The concept of phrase queries has proven easily understood by users; one of the few "advanced search" ideas that works
  - Many more queries are *implicit phrase queries*
- For this, it no longer suffices to store only

<*term* : *docs*> entries

A first attempt: Biword indexes

- Index every consecutive pair of terms in the text as a phrase
- For example the text “Friends, Romans, Countrymen” would generate the biwords
  - ***friends romans***
  - ***romans countrymen***
- Each of these biwords is now a dictionary term
- Two-word phrase query-processing is now immediate.

Longer phrase queries

- Longer phrases can be processed by breaking them down
- ***stanford university palo alto*** can be broken into the Boolean query on biwords:

stanford university *AND* ***university palo*** *AND* ***palo alto***

Without examining the documents, we cannot verify that the documents matching the above Boolean query do actually contain the original 4 word phrase

Can have false positives!

Issues for biword indexes

- False positives, as noted before
- Index blowup due to bigger dictionary
  - Infeasible for more than biwords, big even for them

- Biword indexes are not the standard solution (for all biwords) but can be part of a compound strategy

Solution 2: Positional indexes

- In the postings, store, for each ***term*** the position(s) in which tokens of it appear:

<***term***, number of docs containing ***term***;
doc1: position1, position2 … ;
doc2: position1, position2 … ;
etc.>

Positional index example

<***be***: 993427;
1: 7, 18, 33, 72, 86, 231;
2: 3, 149;
4: 17, 191, 291, 430, 434;
5: 363, 367, …>

Which of docs 1,2,4,5 could contain "***to be or not to be***"?

- For phrase queries, we use a merge algorithm recursively at the document level
- But we now need to deal with more than just equality

Processing a phrase query

- Extract inverted index entries for each distinct term: ***to, be, or, not.***
- Merge their *doc:position* lists to enumerate all positions with "***to be or not to be***".
  - ***to****:*
    - *2*:1,17,74,222,551; *4*:8,16,190,429,433; *7*:13,23,191; ...
  - ***be****:*
    - *1*:17,19; *4*:17,191,291,430,434; *5*:14,19,101; ...
- Same general method for proximity searches

Proximity queries

- LIMIT! /3 STATUTE /3 FEDERAL /2 TORT
  - Again, here, /*k* means “within *k* words of”.
- Clearly, positional indexes can be used for such queries; biword indexes cannot.
- Exercise: Adapt the linear merge of postings to handle proximity queries. Can you make it work for any value of *k*?
  - This is a little tricky to do correctly and efficiently
  - See Figure 2.12 of *IIR*

Positional index size

- A positional index expands postings storage *substantially*
  - Even though indices can be compressed
- Nevertheless, a positional index is now standardly used because of the power and usefulness of phrase and proximity queries … whether used explicitly or implicitly in a ranking retrieval system.

Positional index size

- Need an entry for each occurrence, not just once per document
- Index size depends on average document size
  - Average web page has <1000 terms
  - SEC filings, books, even some epic poems … easily 100,000 terms
- Consider a term with frequency 0.1%

Why?

Document size	Postings	Positional postings
1000	1	1
100,000	1	100

Rules of thumb

- A positional index is 2–4 as large as a non-positional index
- Positional index size 35–50% of volume of original text
    - Caveat: all of this holds for “English-like” languages

Combination schemes

- These two approaches can be profitably combined
  - For particular phrases (***"Michael Jackson", "Britney Spears"***) it is inefficient to keep on merging positional postings lists
    - Even more so for phrases like ***"The Who"***
- Williams et al. (2004) evaluate a more sophisticated mixed indexing scheme
  - A typical web query mixture was executed in ¼ of the time of using just a positional index
  - It required 26% more space than having a positional index alone

Introduction to

Information Retrieval

Introducing ranked retrieval

Ranked retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don’t.
- Good for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
  - Most users incapable of writing Boolean queries (or they are, but they think it’s too much work).
  - Most users don’t want to wade through 1000s of results.
    - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (≈0) or too many (1000s) results.
  - Query 1: "*standard user dlink 650*" → 200,000 hits
  - Query 2: "*standard user dlink 650 no card found*" → 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval models, the system returns an ordering over the (top) documents in the collection with respect to a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval models have normally been associated with free text queries and vice versa

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
  - Indeed, the size of the result set is not an issue
  - We just show the top *k* (≈ 10) results
  - We don’t overwhelm the user

  - Premise: the ranking algorithm works

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this

Introduction to

Information Retrieval

Scoring with the Jaccard coefficient

Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets *A* and *B* is the Jaccard coefficient
- jaccard$(A,B) = |A \cap B| / |A \cup B|$
- jaccard$(A,A) = 1$
- jaccard$(A,B) = 0$ if $A \cap B = 0$
- *A* and *B* don't have to be the same size.
- Always assigns a number between 0 and 1.

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

Issues with Jaccard for scoring

- It doesn't consider *term frequency* (how many times a term occurs in a document)
  - Rare terms in a collection are more informative than frequent terms
  - Jaccard doesn't consider this information
- We need a more sophisticated way of normalizing for length
  - Later in this lecture, we'll use
    - . . . instead of $|A \cap B|/|A \cup B|$ (Jaccard) for length normalization.

$$|A \cap B| / \sqrt{|A \cup B|}$$

Introduction to

Information Retrieval

Term frequency weighting

Recall: Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a count vector in $\mathbb{N}^{|V|}$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary* and *Mary is quicker than John* have the same vectors

- This is called the **bag of words** model.
- In a sense, this is a step back: The positional index was able to distinguish these two documents
  - We will look at "recovering" positional information later on
  - For now: bag of words model

Term frequency tf

- The term frequency $\text{tf}_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*.
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
  - But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

NB: frequency = count in IR

Log-frequency weighting

- The log frequency weight of term *t* in *d* is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- Score for a document-query pair: sum over terms *t* in both *q* and *d*:
- score $= \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$

- The score is 0 if none of the query terms is present in the document.

Introduction to

Information Retrieval

(Inverse) Document frequency weighting

Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- → We want a high weight for rare terms like *arachnocentric*.

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance.
- → For frequent terms, we want positive weights for words like *high, increase, and line*
- But lower weights than for rare terms.
- We will use document frequency (df) to capture this.

idf weight

- df_t is the document frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - $\mathrm{df}_t \le N$
- We define the idf (inverse document frequency) of t by:

$$\mathrm{idf}_t = \log_{10}\left(N/\mathrm{df}_t\right)$$

  - We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

Will turn out the base of the log is immaterial.

idf example, suppose N = 1 million

term	df_t	idf_t
calpurnia	1	
animal	100	
sunday	1,000	
fly	10,000	
under	100,000	
the	1,000,000	

$$\mathrm{idf}_t = \log_{10}(N/\mathrm{df}_t)$$

There is one idf value for each term t in a collection.

Effect of idf on ranking

- Question: Does idf have an effect on ranking for one-term queries, like
  - iPhone

Effect of idf on ranking

- Question: Does idf have an effect on ranking for one-term queries, like
  - iPhone
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

Collection vs. Document frequency

- The collection frequency of *t* is the number of occurrences of *t* in the collection, counting multiple occurrences.
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is a better search term (and should get a higher weight)?

Introduction to

Information Retrieval

tf-idf weighting

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$\mathrm{w}_{t,d} = (1 + \log \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the “-” in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

Final ranking of documents for a query

$$\text{Score}(q,d) = \sum_{t \in q \cap d} \text{tf.idf}_{t,d}$$

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Introduction to

Information Retrieval

BM25, BM25F, and User Behavior

Okapi BM25 and BM25F

- BM25 “Best Match 25” (they had a bunch of tries!)
  - Developed in the context of the Okapi system
- The name of the actual ranking function is BM25
  - it is usually referred to as "Okapi BM25", since the Okapi information retrieval system, implemented at London's City University in the 1980s and 1990s, was the first system to implement this function.
- BM25 and its newer variants, e.g. BM25F (a version of BM25 that can take document structure and anchor text into account), are improving on the TF-IDF-like retrieval functions by incorporating normalization for *tf* using document length

Document length normalization

- Longer documents are likely to have larger tf_i values
- Why might documents be longer?
  - Verbosity: suggests observed tf_i too high
  - Larger scope: suggests observed tf_i may be right
- A real document collection probably has both effects
- … so should apply some kind of normalization

Okapi BM25

- Normalize *tf* using document length

$$tf_i' = \frac{tf_i}{B}$$

$$c_i^{BM25}(tf_i) = \log \frac{N}{df_i} \times \frac{(k_1+1)tf_i'}{k_1 + tf_i'}$$

$$= \log \frac{N}{df_i} \times \frac{(k_1+1)tf_i}{k_1((1-b)+b\frac{dl}{avdl})+tf_i}$$

- BM25 ranking function $RSV^{BM25} = \sum_{i \in q} c_i^{BM25}(tf_i);$

Okapi BM25

$$RSV^{BM25} = \sum_{i \in q} \log \frac{N}{df_i} \times \frac{(k_1+1)tf_i}{k_1((1-b)+b\frac{dl}{avdl})+tf_i}$$

- k_1 controls term frequency scaling
  - $k_1 = 0$ is binary model; k_1 = large is raw term frequency
- b controls document length normalization
  - $b = 0$ is no length normalization; $b = 1$ is relative frequency (fully scale by document length)
- Typically, k_1 is set around 1.2–2 and b around 0.75
- *IIR* sec. 11.4.3 discusses incorporating query term weighting and (pseudo) relevance feedback

Introduction to

Information Retrieval

The Vector Space Model (VSM)

Documents as vectors

- Now we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors – most entries are zero

Queries as vectors

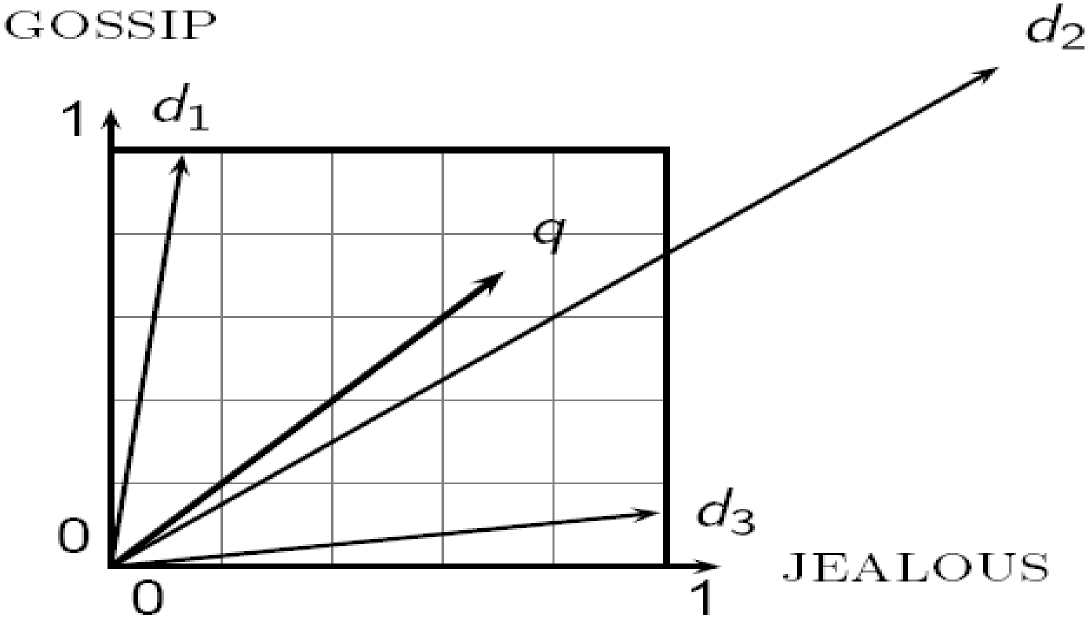
- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model
- Instead: rank more relevant documents higher than less relevant documents

Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

The Euclidean distance between $\vec{q}$ and $\vec{d_2}$ is large even though the distribution of terms in the query $\vec{q}$ and the distribution of terms in the document $\vec{d_2}$ are very similar.



Use angle instead of distance

- Thought experiment: take a document *d* and append it to itself. Call this document *d*′.
- “Semantically” *d* and *d*′ have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.

- Key idea: Rank documents according to angle with query.

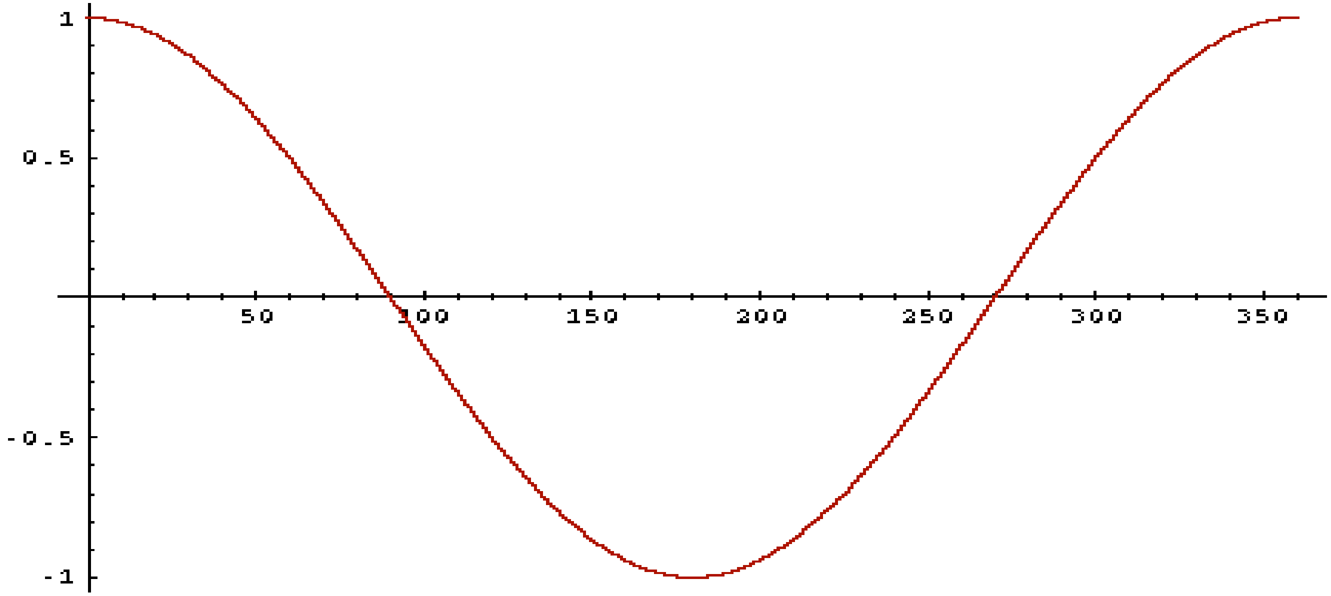
From angles to cosines

- The following two notions are equivalent.
  - Rank documents in <u>decreasing</u> order of the angle between query and document
  - Rank documents in <u>increasing</u> order of

    cosine(query,document)

- Cosine is a monotonically decreasing function for the interval [0^o, 180^o]

From angles to cosines



- But how – *and why* – should we be computing cosines?

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
  - Long and short documents now have comparable weights

cosine(query,document)

Dot product

Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term i in the query
d_i is the tf-idf weight of term i in the document

$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

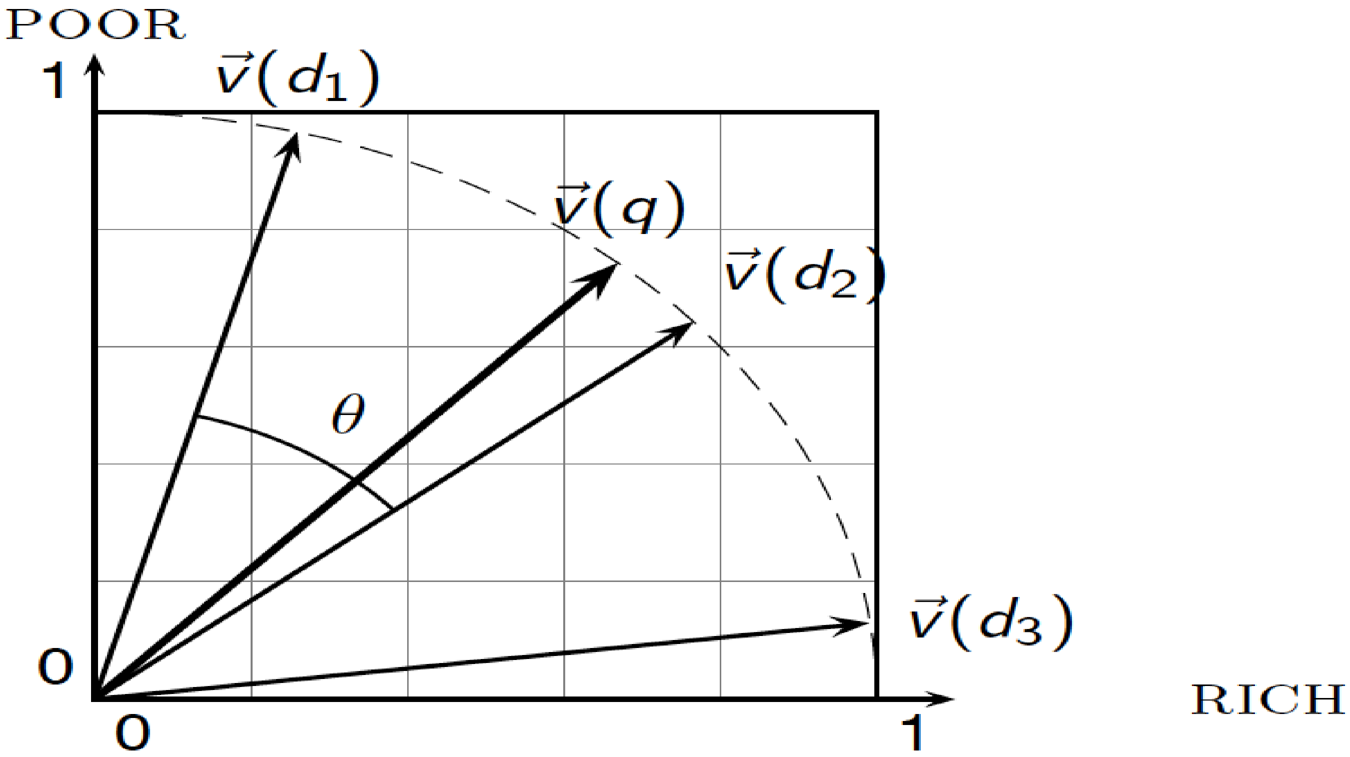
Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q}\cdot\vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for *q*, *d* length-normalized.

Cosine similarity illustrated



Cosine similarity amongst 3 documents

How similar are the novels
SaS: *Sense and Sensibility*
PaP: *Pride and Prejudice*, and
WH: *Wuthering Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈

$0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0 \approx 0.94$

cos(SaS,WH) ≈ 0.79

cos(PaP,WH) ≈ 0.69

Why do we have cos(SaS,PaP) > cos(SAS,WH)?

Introduction to

Information Retrieval

The Vector Space Model (VSM)

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1 + \log(\mathrm{tf}_{t,d})}{1 + \log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation *ddd.qqq,* using the acronyms from the previous table
- A very standard weighting scheme is: lnc.ltc
- Document: logarithmic tf (l as first character), no idf and cosine normalization

A bad idea?

- Query: logarithmic tf (l in leftmost column), idf (t in second column), cosine normalization …

lnc.ltc: popular vector-space term weight function

- "l": log (tf) + 1
- "n": No weight / normalization (i.e., 1.0)
- "t": log (N / df)
- "c": cosine normalization

$$\sqrt{\sum w_i^{\,2}}$$

- For example:

$$\frac{\sum d_i \cdot q_i}{\sqrt{\sum d_i^{\,2}} \cdot \sqrt{\sum q_i^{\,2}}} = \frac{\sum (\log(tf)+1) \cdot \left((\log(qtf)+1) \log \frac{N}{df} \right)}{\sqrt{\sum (\log(tf)+1)^2} \cdot \sqrt{\sum \left((\log(qtf)+1) \log \frac{N}{df} \right)^2}}$$

tf-idf example: lnc.ltc

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query						Document				Prod
	tf-raw	tf-wt	df	idf	wt	n'lize	tf-raw	tf-wt	wt	n'lize	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Exercise: what is *N*, the number of docs?

Doc length $= \sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

Computing cosine scores

CosineScore(q)

float $Scores[N] = 0$
float $Length[N]$
for each query term t
do calculate $\mathrm{w}_{t,q}$ and fetch postings list for t
    for each pair$(d, \mathrm{tf}_{t,d})$ in postings list
    do $Scores[d] += \mathrm{w}_{t,d} \times \mathrm{w}_{t,q}$
Read the array *Length*
for each d
do $Scores[d] = Scores[d]/Length[d]$
return Top K components of $Scores[]$

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., $K = 10$) to the user

Introduction to

Information Retrieval

Evaluating search engines

Measures for a search engine

- How fast does it index
  - Number of documents/hour
  - (Average document size)
- How fast does it search
  - Latency as a function of index size
- Expressiveness of query language
  - Ability to express complex information needs
  - Speed on complex queries
- Uncluttered UI
- Is it free?

Measures for a search engine

- All of the preceding criteria are *measurable*: we can quantify speed/size
  - we can make expressiveness precise
- The key measure: user happiness
  - What is this?
  - Speed of response/size of index are factors
  - But blindingly fast, useless answers won't make a user happy
- Need a way of quantifying user happiness with the results returned
  - Relevance of results to user's information need

Evaluating an IR system

- An **information need** is translated into a **query**
- Relevance is assessed relative to the **information need** *not* the **query**
- E.g., <u>Information need</u>: *I'm looking for information on whether drinking red wine is more effective at reducing your risk of heart attacks than white wine.*
- <u>Query</u>: ***wine red white heart attack effective***
- You evaluate whether the doc addresses the information need, not whether it has these words

Evaluating ranked results

- Evaluation of a result set:
  - If we have
    - a benchmark document collection
    - a benchmark set of queries
    - assessor judgments of whether documents are relevant to queries

  Then we can use Precision/Recall/F measure as before
- Evaluation of ranked results:
  - The system can return any number of results
  - By taking various numbers of the top returned documents (levels of recall), the evaluator can produce a *precision-recall curve*

Recall/Precision

		R	P
1	R		
2	N		
3	N		
4	R		
5	R		
6	N		
7	R		
8	N		
9	N		
10	N		

Assume 10 rel docs in collection

Two current evaluation measures…

- Mean average precision (MAP)
  - AP: Average of the precision value obtained for the top *k* documents, each time a relevant doc is retrieved
  - Avoids interpolation, use of fixed recall levels
  - Does weight most accuracy of top returned results
  - MAP for set of queries is arithmetic average of APs
    - Macro-averaging: each query counts equally

THE UNIVERSITY OF CHICAGO
GRAHAM SCHOOL
CONTINUING LIBERAL AND PROFESSIONAL STUDIES

Thank You!