**SMA 2409**

**Algebraic Number Theory Assignment**

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***Question.***

**Show that (Q√d) is a field.**

*Solution*

Q√d={a+b√d| a,b ∈ Q}

We consider two separate cases. If d has a square root in Q

then Q[√2] =Q, which is certainly a field.

Therefore, suppose that d does not have a square root in Q.

It is straight- forward to check that Q[√2] is commutative:

(a1+b1√d)(a2+b2√d)= (a1a2+db1b2) + (a1b2+a2b1)√d

= (a2a1+db2b1) + (a2b1+a1b2)√d

= (a2+b2√d)(a1+b1√d)

Also, 1 = 1 + 0√d ∈Q[√d], so Q[√d] is a commutative ring with unity.

We therefore need to check that every nonzero element of Q[√d] has a multiplicative inverse. So, let a+b√d be so that (a;b)≠ (0,0).

Then a2-b2d≠0,since then either b2= 0, in which case a and b are both

zero (which we're assuming is not the case), or else (a/b)2=d, so d has a

square root in Q, which we are also assuming is not the case.

Thus,a2-b2d≠ 0 and so a-b√d≠ 0 also (since a2-b2d= (a+b√d)(a-b√d).

Now, we have

(a+b√d). (1/( a2-b2d))(a-b√d) = 1;

so the multiplicative inverse of a+b√d is:

(a/(a2-b2d))-(b/ (a2-b2d))√d

Finally, note that a2-b2d∈ Q,

so a/(a2-b2d), -b/(a2-b2d) ∈Q, which means that every element of Q[√d] has a multiplicative inverse in Q[√d],

so it is a field, as required.