Trading Without Moving the Market

A Guide to the Almgren–Chriss Optimal Execution Model

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What Actually Happens When You Trade?

- When you decide to buy or sell a stock, you don't trade with "the market" directly — you trade with **other people's orders**.
- At any moment, there are two types of standing offers:
  - **Buy offers (Bids):** people waiting to buy at specific prices.
  - **Sell offers (Asks):** people waiting to sell at specific prices.
- These bids and asks form the **order book** — the list of who wants to buy or sell, and at what prices.
- When a new order arrives, it matches with existing ones in this book. This process — matching buyers and sellers — is what actually creates price movements.

The Price You See vs. The Price You Get

- The market always shows two key prices:
  - **Bid:** the highest price someone is willing to buy at.
  - **Ask:** the lowest price someone is willing to sell at.
- The midpoint between them — the **mid price** — is often used as a proxy for the fair value over short horizons.
- The difference between ask and bid is called the **spread** — this is the basic cost of trading quickly.
- If you place a very large order, you'll "use up" the best available offers and start trading at worse prices — this is called **market impact**.

Order Book

Order Book

Total	Amount	Price	Price	Amount	Total
10.68570	0.01000	1068.18	1069.99	1.61645	1,729.457
476.50640	0.53521	1068.21	1069.97	1.96304	2,100.315
53.41800	0.05000	1068.36	1069.96	9.24579	9,892.6254
6,410.520	6.0000	1068.42	1069.95	30.00000	32,098.50
10.99425	0.01029	1068.44	1069.93	1.96304	2,100.315
8,086.639	7.56850	1068.46	1068.89	6.25000	6,686.811
6,590.1713	6.16786	1068.47	1069.88	1.61645	1,729.4057
1,245.5040	1.40000	1068.48	1069.70	7.04620	7,537.331
12,822.360	12.0000	1068.53	1069.43	31.40287	33,583.13
496.50640	0.46465	1068.56	1069.37	0.01870	19.997222
10.68570	0.01000	1068.57	1069.30	2.50000	2,673.250

1,070.29

Last Market Price

Figure: Order book depth: large orders consume multiple price levels

Why Big Trades Move the Market

- The limit order book has **finite depth** at each price level — only a certain number of shares are available to buy or sell at any price.
- A **small order** interacts only with the best bid or best ask, so it executes with almost no price movement.
- A **large order** consumes the volume at the best price and must "walk the book," executing against deeper and worse-priced liquidity.
- Example:
  - Selling 100 shares might only hit the best bid $\rightarrow$ price impact of $Rs.0.01$.
  - Selling 1,000,000 shares exhausts multiple bid levels $\rightarrow$ price impact of about $Rs.0.50$.
- This **self-generated price movement** from consuming liquidity is called **market impact**. It occurs even when the trade conveys no new information.

Why Execution Matters

- Large investors — mutual funds, hedge funds, pension funds — often need to buy or sell in huge quantities.
- A poor trading plan can easily destroy the profit from a good investment idea.
- Two common mistakes:
  - Trading too fast: you push prices against yourself — high cost.
  - Trading too slowly: you wait for better prices — but risk the market moving against you.
- The trader's problem is to find the right balance between:

Impact Cost (speed) and **Risk (patience)**.

The Big Question

How can we trade a large position without moving the market too much?

This is the core question the Almgren–Chriss model set out to answer.

Historical Context: From Bertsimas–Lo to Almgren–Chriss

- **Bertsimas & Lo (1998):** One of the earliest formal treatments of optimal execution.
  - Framed execution purely as a cost-minimization problem.
  - Assumed execution is **risk-free** — price path uncertainty was ignored.
  - Optimal solution: trade at a **constant rate** over time.
- **Almgren & Chriss (2000):** Introduced the missing dimension — execution risk.
  - Incorporated price volatility explicitly into the objective.
  - Formulated execution as a **cost–risk tradeoff**.
  - Produced an efficient frontier of optimal trading schedules.
- **Key leap:** The problem shifted from deterministic optimization to a **mean–variance framework**, parallel to modern portfolio theory.

Why the Almgren–Chriss Model Was a Breakthrough

- Almgren and Chriss (1999, 2000) developed the first complete **quantitative model of optimal trade execution**.
- They unified two key ingredients:
  - A **market impact model** describing how trading affects prices,
  - A **mean–variance formulation** capturing execution risk.
- Their framework delivered:
  - A closed-form **optimal trading trajectory**,
  - A well-defined **efficient frontier** of cost–risk combinations.
- This transformed execution from a heuristic activity into a **rigorous optimization problem** grounded in market microstructure.

Why the Almgren–Chriss Model Still Matters

- Modern execution algorithms trace their foundations to the A–C framework.
- A–C introduced the vocabulary and structure to discuss:
  - **Temporary vs. Permanent** market impact,
  - **Execution risk** arising from price volatility,
  - The notion of an **optimal trading path**.
- Even in today's high-speed, data-rich markets, A–C remains the **baseline model** for research, benchmarking, and algorithm design.
- Studying it provides a unified understanding of how **trading, risk, and liquidity** interact—core to market microstructure.

The Almgren–Chriss Optimal Execution Framework

Trading Strategy in Discrete Time

- We start with an initial inventory of X shares that must be fully liquidated over a total horizon T.
- Split the horizon into N equal intervals of length $\tau = T/N$, giving discrete decision times

$$t_k = k\tau, \qquad k = 0, 1, \ldots, N.$$

- Let x_k denote the number of shares still held at time t_k:

$$x_0 = X, \qquad x_N = 0.$$

- The shares sold in interval k are simply the drop in inventory:

$$n_k = x_{k-1} - x_k.$$

  This is the key decision variable for the trader.
- The corresponding **trading rate** (how fast we trade) is

$$v_k = \frac{n_k}{\tau}.$$

  Higher v_k means more aggressive trading and greater market impact.

Definition: What is a Trading Strategy?

- A **trading strategy** is a rule that determines the sequence of trade sizes

$$n_1, n_2, \ldots, n_N,$$

or equivalently the inventory path x_k, based on information available up to the beginning of each interval.

- Intuitively, it answers the question: *"How much should I sell now, given what I know about the market so far?"*

- Strategies can be:
  - **Static (Pre-committed):** All n_k are fixed before trading starts. The entire schedule is predetermined and cannot respond to price moves or changes in liquidity.
  - **Dynamic (Adaptive):** Each n_k is chosen at time t_{k-1} using real-time information (price path, volatility, order book depth, volume patterns, signals, etc.). This reflects how real-world execution desks operate.

Market Price and Its Dynamics

- Let S_k denote the **market price** of the asset at time t_k.
- Over short execution horizons, the price is modeled as an Arithmetic Brownian Motion:

$$S_k = S_{k-1} + \sigma\sqrt{\tau}\,\xi_k,$$

  where
  - σ: volatility of price changes per unit time,
  - $\xi_k \sim \mathcal{N}(0,1)$ i.i.d. random shocks.
- Hence, price increments are normally distributed:

$$S_k - S_{k-1} \sim \mathcal{N}(0, \sigma^2\tau).$$

- The model assumes **additive** and **independent** price changes — appropriate for short horizons where relative (percentage) price moves are small.

Market Impact: Permanent and Temporary

Market impact refers to how trading activity itself influences prices. The Almgren–Chriss model decomposes this into two components:

1. **Permanent Impact**
   - A **lasting change** in the equilibrium price of the asset.
   - Arises because large trades reveal information to the market (e.g., a large sell program may signal negative information).
   - Buying pushes prices upward; selling pushes them downward.
2. **Temporary Impact**
   - A **short-lived price concession** due to limited liquidity while trading.
   - Reflects the cost of "crossing the spread" and consuming order-book depth.
   - Prices typically **revert** once the trade is completed and liquidity refills.

Permanent Impact: Mid-Price Dynamics

- The **market price** evolves as:

$$S_k = S_{k-1} + \sigma\sqrt{\tau}\,\xi_k - \tau\, g\!\left(\frac{n_k}{\tau}\right),$$

  where $g(v)$ is the **permanent impact function**.

- **Linear specification:**

$$g(v) = \gamma v, \quad \gamma > 0 \;\; \Rightarrow \;\; S_k = S_{k-1} + \sigma\sqrt{\tau}\,\xi_k - \gamma n_k.$$

- **Impact direction:**

$$n_k > 0 \Rightarrow \text{Sell} \Rightarrow S_k \downarrow, \qquad n_k < 0 \Rightarrow \text{Buy} \Rightarrow S_k \uparrow .$$

- Faster trades (v_k large) convey stronger information per unit time, causing a larger lasting shift in the market price.
- Permanent impact reflects the market's **long-term reaction** to your trades.

Intuition Behind Permanent Impact

- When you trade large volumes:
  - Other participants infer that you might have **information** (e.g., negative news, portfolio rebalancing).
  - They update their valuation — bids and offers shift permanently.
- Hence, your trade changes the **equilibrium price level** even after you stop trading.
- **Example:**
  - Selling 1M shares of a stock signals excess supply. Market makers widen spreads and lower quotes.
  - Once the order is filled, the mid-price stays lower on average.
- The constant γ measures how much price moves per unit traded:

$$\Delta S_{\text{permanent}} = \gamma\, n_k.$$

- Larger γ: less liquid asset $\rightarrow$ stronger lasting impact. Smaller γ: more liquid asset $\rightarrow$ impact decays faster.

Temporary Impact: Mechanics and Formula

- The **temporary impact function** measures the short-term price concession a trader faces due to limited liquidity.
- In the Almgren–Chriss model:

$$h(v_k) = \epsilon \operatorname{sgn}(n_k) + \eta\, v_k, \quad v_k = \frac{n_k}{\tau}.$$

  - ϵ: fixed cost per trade — captures half of the bid–ask spread or fees.
  - η: sensitivity of price to trade speed — higher η means illiquid market.
- The actual execution price obtained by the trader is:

$$\widetilde{S}_k = S_{k-1} - h(v_k),$$

meaning faster trades (large v_k) get worse prices.

Intuition Behind Temporary Impact

- Temporary impact represents the **immediate cost of liquidity**.
- Think of the market as a queue of resting orders:
  - The best prices have limited volume.
  - A large or fast order must "walk" down the order book to find enough volume.
- The faster you trade, the deeper into the book you go, paying progressively worse prices.
- Once the order finishes, new traders place fresh bids and offers — the book refills and prices return near their original level.
- This means:

$$\text{Temporary impact} = \text{price you pay for immediacy.}$$

Implementation Shortfall: The True Cost of Trading

- The moment you decide to trade, you have a **benchmark price** S_0: the price before your first order.
- The **Implementation Shortfall (IS)** measures how much worse your actual execution is compared to selling everything instantly at S_0:

$$\mathrm{IS} = XS_0 - \sum_{k=1}^{N} n_k \, \widetilde{S}_k .$$

- It combines three elements:
  - Random price moves (volatility risk),
  - Permanent market impact (lasting price change),
  - Temporary market impact (immediate liquidity cost).
- The goal: design a trading path that makes this shortfall **as small and stable as possible**.

Breaking Down the Implementation Shortfall

- Expanding the total proceeds from trading:

$$\sum_{k=1}^{N} n_k \widetilde{S}_k = XS_0 + \underbrace{\sum_{k=1}^{N} \sigma \sqrt{\tau}\, \xi_k x_k}_{\text{Volatility risk}} - \underbrace{\sum_{k=1}^{N} x_k g(n_k/\tau)}_{\text{Permanent impact}} - \underbrace{\sum_{k=1}^{N} n_k h(n_k/\tau)}_{\text{Temporary impact}}.$$

- **Linear impact assumptions:**
  - $g(v) = \gamma v$ (permanent impact per unit traded),
  - $h(v) = \epsilon \operatorname{sgn}(v) + \eta v$ (temporary impact with spread + slippage).

- Permanent impact pushes the whole price path downward. Temporary impact hurts only at the moment of execution.

Expected Cost and Risk of Execution

- Define the **adjusted temporary impact coefficient:**

$$\tilde{\eta} = \eta - \frac{\gamma\tau}{2}$$

  This accounts for the interaction between temporary and permanent impact.

- The expected (average) cost of a trading schedule:

$$\mathbb{E}[\mathrm{IS}] = \frac{\gamma}{2}X^2 + \frac{\tilde{\eta}}{\tau}\sum_{k=1}^{N} n_k^2 + \epsilon\sum_{k=1}^{N}|n_k|.$$

- The risk (variance) comes from price volatility:

$$\mathrm{Var}[\mathrm{IS}] = \sigma^2\tau\sum_{k=1}^{N} x_k^2.$$

Balancing Cost and Risk: The Optimization Problem

- The trader chooses a schedule $x_0, x_1, \ldots, x_N$ to minimize:

  $$U(x) = \mathbb{E}[\mathrm{IS}] + \lambda \, \mathrm{Var}[\mathrm{IS}],$$

  where λ reflects how much risk you're willing to tolerate.

- **Interpretation:**
  - High λ: risk-averse trader $\rightarrow$ sells quickly, accepts higher impact.
  - Low λ: risk-neutral trader $\rightarrow$ sells slowly, tolerates volatility.

- This is a **mean–variance optimization**, just like Markowitz portfolio theory — but for trading instead of investing.

Solving for the Optimal Path

- Taking the first-order condition for optimality:

$$\frac{\partial U}{\partial x_j} = 2\tau\left(\lambda\sigma^2 x_j - \frac{\tilde{\eta}}{\tau^2}(x_{j-1} - 2x_j + x_{j+1})\right) = 0, \quad j = 1, \ldots, N-1.$$

- Rearranging gives a **discrete difference equation:**

$$\frac{1}{\tau^2}(x_{j-1} - 2x_j + x_{j+1}) = \tilde{\kappa}^2 x_j,$$

where

$$\tilde{\kappa}^2 = \frac{\lambda\sigma^2\tau^2}{\tilde{\eta}}.$$

- The general solution has **hyperbolic form:**

$$x_j = Ae^{\kappa t_j} + Be^{-\kappa t_j},$$

where κ satisfies: $2\cosh(\kappa\tau) - 2 = \tau^2\tilde{\kappa}^2$.

The Optimal Trading Trajectory

- Applying boundary conditions $x_0 = X, x_N = 0$, we obtain:

$$x_j = X \frac{\sinh[\kappa(T - t_j)]}{\sinh(\kappa T)}, \quad \kappa = \sqrt{\frac{\lambda\sigma^2}{\tilde{\eta}}}.$$

- **Key Properties:**
  - Smooth exponential decay from X to 0
  - Front-loaded: trades more aggressively early
  - Slows down: tapers off as inventory decreases
- κ determines the **urgency** of liquidation:
  - Larger κ: high λ or $\sigma \to$ sell faster (cut risk)
  - Smaller κ: high $\tilde{\eta}$ (illiquid) $\to$ sell slower (reduce impact)

Intuition

The model finds the sweet spot: trade fast enough to avoid volatility risk, but slow enough to minimize market impact costs.

Half-Life: The Natural Time Scale of Liquidation

- The **half-life** is the time for inventory to fall to half its initial value:

$$x(t_{1/2}) = \frac{1}{2}x(0).$$

- From the exponential decay $x(t) \propto e^{-\kappa t}$, we get:

$$t_{1/2} = \frac{\ln(2)}{\kappa}, \quad \text{where} \quad \kappa = \sqrt{\frac{\lambda\sigma^2}{\tilde{\eta}}}.$$

- **Economic Interpretation:**
  - $t_{1/2}$ is the intrinsic time scale of the trade
  - If $T \gg t_{1/2}$: trader liquidates most position early (risk-dominant)
  - If $T \ll t_{1/2}$: forced to trade uniformly (liquidity-constrained)

- **Practical use:** Compare your execution horizon T to $t_{1/2}$ to assess whether you're trading "fast" or "slow" relative to market conditions.

Understanding the Parameters

The Role of Risk Aversion (λ)

- The parameter λ measures the trader's risk aversion — how much they care about reducing uncertainty (variance) relative to expected cost.
- In utility terms, it corresponds to:

$$\lambda = -\frac{u''(w)}{u'(w)},$$

  where $u(w)$ is the trader's utility function over wealth. Higher curvature $u''(w)$ means more risk-averse trader.
- **Economic interpretation:**
  - Large λ: high risk aversion $\rightarrow$ trade aggressively early to reduce price risk.
  - Small λ: risk-neutral or patient trader $\rightarrow$ trade slowly to minimize impact cost.
- Geometrically, λ determines where you sit on the **efficient frontier** — higher λ corresponds to lower risk but higher expected cost.

The Impact of Risk Aversion

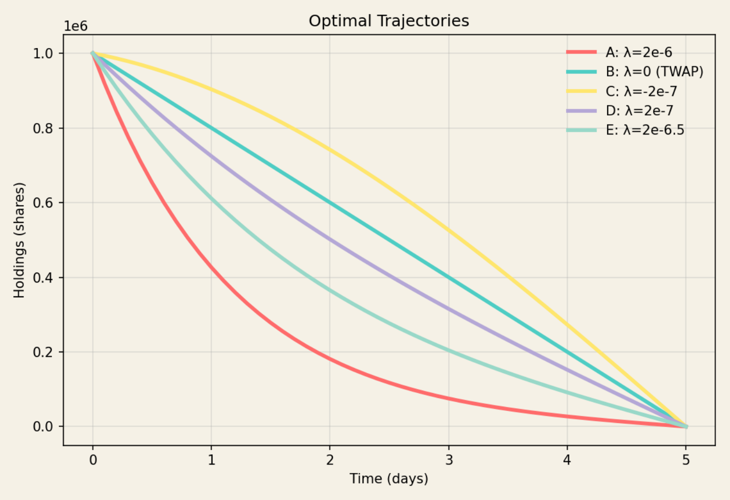


Figure: Optimal trajectories for different risk aversion levels

The Impact of Liquidity (η)

- η measures the **sensitivity of cost to trading velocity** — higher η means higher slippage per unit velocity.
- Larger η means smoother and slower trading paths to reduce temporary impact.

- **Market interpretation:**
  - High η: illiquid market (thin order book) $\rightarrow$ must trade very gradually
  - Low η: liquid market (deep order book) $\rightarrow$ can trade more aggressively

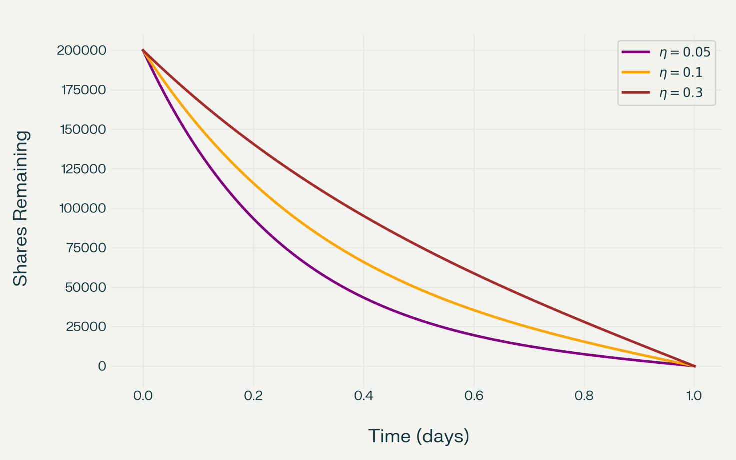
- Optimal decay rate:

$$\kappa = \sqrt{\frac{\lambda\sigma^2}{\tilde{\eta}}} \propto \frac{1}{\sqrt{\tilde{\eta}}}$$

So higher liquidity (lower η) allows faster execution.

How Liquidity Changes Your Strategy

The parameter η represents how sensitive the price is to your trading (a measure of illiquidity).



The Efficient Frontier: Cost–Risk Trade-off

- In the Almgren–Chriss framework, every trading schedule balances two opposing forces:

  $$\text{(i) Expected Cost} \quad \text{vs.} \quad \text{(ii) Execution Risk.}$$

  Varying the risk-aversion parameter λ generates different optimal schedules.
- Plotting the pair (Expected Cost, Risk) for each λ produces the **efficient frontier**—the set of all optimal cost–risk combinations.
- Every point on the curve corresponds to a fully optimized strategy. Points below the curve are unattainable; points above it are inefficient (dominated).
- **Key Insight:** The frontier is typically very steep at low risk levels—meaning a trader can dramatically reduce execution risk with only a modest increase in expected cost. This is why risk-aware execution matters.

The Efficient Frontier

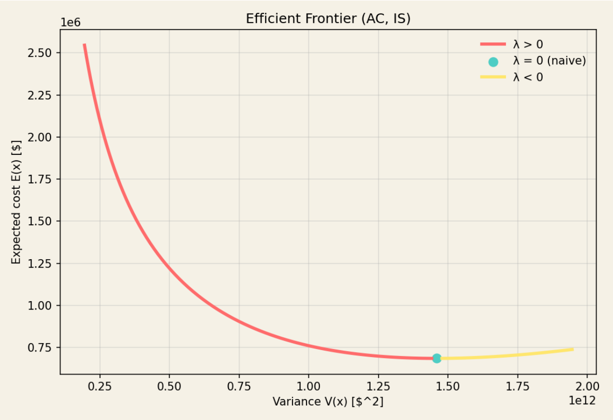


Figure: The efficient frontier generated by varying the risk-aversion parameter λ. Higher λ corresponds to lower risk but higher expected cost; lower λ corresponds to slower, cost-efficient but riskier schedules.

Scale Invariance of the Efficient Frontier

- A key property of the A–C model: the **shape** of the optimal trajectory depends only on market and preference parameters, not on the order size X.
- The optimal decay rate κ is:

$$\kappa = \sqrt{\frac{\lambda\sigma^2}{\tilde{\eta}}} \quad \text{(independent of } X\text{)}.$$

- **Implications:**
  - Liquidating 1M shares or 10M shares produces the *same trajectory shape*.
  - Only the **level** of costs scales with position size, not the strategy's geometry.
  - This property follows directly from the model's **linear** market impact functions.
- Therefore, the optimal execution path is **scale-invariant**: changing X rescales the solution but does not change its curvature or timing.

Serial Correlation in Returns: The Intuition

- So far, A–C assumes price changes are **i.i.d.** (independent shocks): no predictable pattern in short-term returns.
- In reality, prices often show **serial correlation:**

$$\text{Corr}(r_t, r_{t-1}) = \rho \neq 0.$$

- $\rho > 0$ (positive autocorrelation): short-term momentum — if price moves up, it likely continues up. $\rho < 0$ (negative autocorrelation): short-term mean reversion — a price rise is followed by a fall.
- Traders can exploit this pattern:
  - **Momentum** $\rightarrow$ delay selling to ride upward drift.
  - **Mean reversion** $\rightarrow$ accelerate selling before reversal.
- Serial correlation means execution cost isn't purely random — part of it becomes **predictable**.

Serial Correlation: Quantifying Information Value

- With serial correlation $\rho = \mathbb{E}[\xi_k \xi_{k-1}]$, price shocks are **predictable**.
- **Heuristic Analysis:** Consider shifting volume δn from period 1 to period 2.
  - Expected gain from timing: $G(\delta n) \approx \rho\sigma\sqrt{\tau} \cdot \delta n$
  - Additional impact cost: $\tilde{\eta}(\delta n)^2/\tau$
- Optimal shift:

$$\delta n^* = \frac{\rho\sigma\tau^{3/2}}{2\tilde{\eta}}.$$

- Maximum attainable gain:

$$G_{\max} = \frac{\rho^2\sigma^2\tau^2}{4\tilde{\eta}}.$$

- **Key insight:** Gain is *independent of position size X* — information value comes from microstructure, not scale.

Serial Correlation: Modified Optimal Trajectory

- Modify the price process to include serial correlation:

$$S_k = S_{k-1} + \sigma\sqrt{\tau}\,\xi_k - \tau g(v_k), \quad \mathbb{E}[\xi_k \xi_{k-1}] = \rho.$$

- Serial correlation introduces an **expected gain** in cost:

$$\Delta\mathbb{E}[\mathrm{IS}] \approx \frac{\rho^2\sigma^2\tau^2}{4\tilde{\eta}}.$$

- Larger $|\rho|$, σ, or smaller $\tilde{\eta} \to$ stronger impact of serial dependence on cost.
- **Practical implication:**
  - Positive ρ: slightly delay execution (ride momentum)
  - Negative ρ: slightly accelerate execution (exploit mean reversion)
  - Effect is typically **small** compared to impact costs

Serial Correlation: Trajectory Comparison

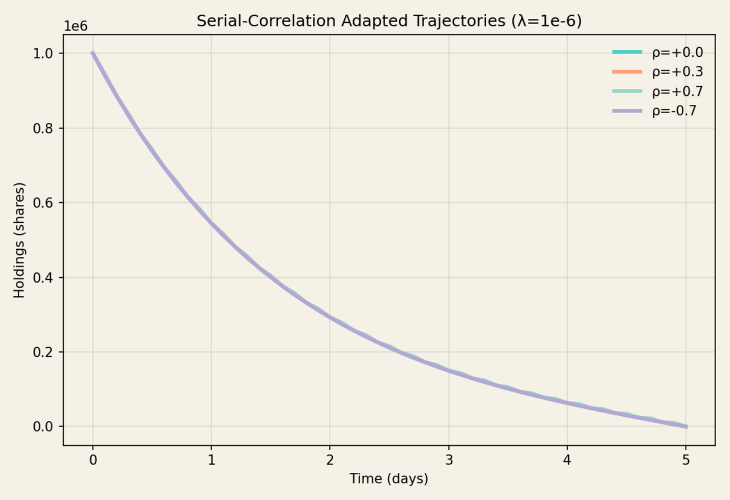


Figure: Optimal trajectories with and without serial correlation

Non-Zero Drift: Trading with a Price View

- So far, A–C assumed prices have **no directional bias**:

$$\mathbb{E}[S_{t+1} - S_t] = 0.$$

- Now introduce an expected trend (drift) α:

$$\mathbb{E}[S_{t+1} - S_t] = \alpha\tau.$$

- $\alpha > 0$: price expected to rise (bullish) $\rightarrow$ **trade slower**.
  $\alpha < 0$: price expected to fall (bearish) $\rightarrow$ **trade faster**.

- Drift introduces a **directional motive**: the trader no longer aims just to minimize cost and risk, but also to **exploit or protect** against expected price movements.

Drift Extension: Mathematical Formulation

- Modified price dynamics with drift α:

$$S_k = S_{k-1} + \alpha\tau + \sigma\sqrt{\tau}\,\xi_k - \tau g(v_k).$$

- Expected shortfall now includes a **benefit term:**

$$\mathbb{E}[\mathrm{IS}] = \frac{\gamma}{2}X^2 + \frac{\tilde{\eta}}{\tau}\sum n_k^2 + \epsilon\sum |n_k| - \alpha\tau\sum x_k.$$

- The last term represents expected gain from holding inventory while prices drift favorably.
- Optimal solution introduces a **target position:**

$$\bar{x} = \frac{\alpha\tau^2}{2\tilde{\eta}\kappa^2},$$

representing the "ideal" inventory to hold given the expected trend.

Including Drift in the Optimal Trajectory

- **Solution:**

$$x(t) = X\frac{\sinh[\kappa(T-t)]}{\sinh(\kappa T)} + \bar{x}\left(1 - \frac{\sinh[\kappa(T-t)] + \sinh(\kappa t)}{\sinh(\kappa T)}\right).$$

- **Components:**
  - First term: standard A-C trajectory (no drift)
  - Second term: adjustment for expected price movement
- Positive $\alpha \to$ trajectory shifted upward (hold more inventory longer)
- Negative $\alpha \to$ trajectory shifted downward (liquidate faster)
- **Special case:** If $\alpha = 0$, we recover the original A-C solution.

Drift: Trajectory Comparison

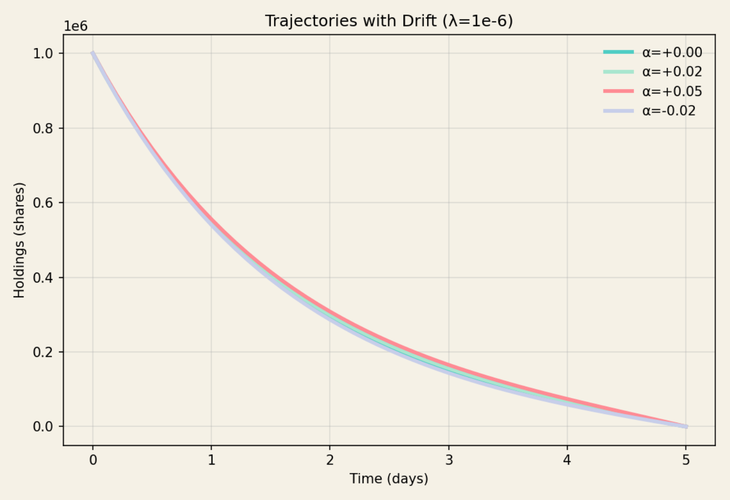


Figure: Optimal trajectories with different drift values

How Market Predictability Changes Optimal Execution

- **Serial correlation** introduces short-term predictability — traders can time execution within the day.
- **Drift** introduces long-term directional bias — traders change their overall urgency.
- Both add realism to the basic A–C model, linking execution with **market forecasts**.
- The structure of the model remains the same — cost, risk, and information now interact dynamically.

- **Key lesson:** Even with predictable price movements, the benefit is typically *small* compared to the cost of market impact — execution efficiency matters more than forecasting accuracy.

Limiting Cases: Fast vs Slow Execution

Two Extremes Illustrate the Tradeoff

1. **Immediate Execution** ($\lambda \to \infty$, $\kappa \to \infty$)

$$n_1 = X, \quad n_k = 0 \text{ for } k > 1.$$

- Cost: Maximum temporary impact $\sim \tilde{\eta} X / \tau$
- Risk: Zero variance (no exposure to price moves)

2. **Uniform Execution** ($\lambda \to 0$, $\kappa \to 0$)

$$n_k = \frac{X}{N} \quad \text{for all } k.$$

- Cost: Minimum impact $\sim \gamma X^2 + \tilde{\eta} X^2 / (N\tau)$
- Risk: Maximum variance $\sim \sigma^2 \tau X^2 T$

Key Assumptions and Their Implications

- **Linear market impact:** $g(v) = \gamma v,\ h(v) = \epsilon + \eta v$
  - Ensures unique convex optimization
  - Leads to scale invariance
  - Reality: often concave for large trades
- **Arithmetic random walk:** $S_{k+1} - S_k \sim \mathcal{N}(0, \sigma^2 \tau)$
  - Valid for short horizons.
  - Long-term: geometric Brownian motion more appropriate
- **Static strategy:** All trades predetermined at $t = 0$
  - Optimal under i.i.d. shocks and symmetric costs
  - Extensions allow dynamic adaptation to new information
- **No resilience:** AC assumes temporary impact decays instantly and permanent impact never decays.
  - Conservative assumption
  - Later models (Obizhaeva-Wang) add price recovery

Summary and Key Takeaways

- **The Core Trade-off:** Optimal execution is about balancing market impact costs and price risk.
- **Key Inputs:** The model needs your risk preference (λ), stock volatility (σ), and market impact parameters (η, γ).
- **The Solution:** It provides a smooth, curved trading trajectory that typically involves front-loading your trades to reduce risk.
- **Practical Use:** It's the theoretical basis for many real-world "smart order routers" and algorithmic trading strategies used by institutions.
- **It's a Model, Not Reality:** The model makes simplifying assumptions (e.g., linear impact, constant volatility). Real-world implementations are more complex, but the core principles remain the same.

Thank you!