

Q2

Convolution in matlab. Suppose you have a linear shift-invariant system with impulse response $r = [4 \ 2 \ 1 \ 0.5]$. Because it is LSI, the response of this system to any input vector \mathbf{x} can be computed as a convolution.

a)

Compute responses to the eight 8-dimensional impulse vectors, using MATLAB's `conv` function: $\mathbf{out} = \text{conv}(\mathbf{in}, \mathbf{r})$. How do these compare to what you'd expect from output 2 the convolution formula given in class,

$$y(n) = \sum_k r(n-k) x(k)$$

Specifically, compute the matrix that represents the linear system. What is the size, and organization of this matrix?

Response to impulse vector 1: $[4. \ 2. \ 1. \ 0.5 \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0.]$
 Response to impulse vector 2: $[0. \ 4. \ 2. \ 1. \ 0.5 \ 0. \ 0. \ 0. \ 0. \ 0. \ 0.]$
 Response to impulse vector 3: $[0. \ 0. \ 4. \ 2. \ 1. \ 0.5 \ 0. \ 0. \ 0. \ 0. \ 0.]$
 Response to impulse vector 4: $[0. \ 0. \ 0. \ 4. \ 2. \ 1. \ 0.5 \ 0. \ 0. \ 0. \ 0.]$
 Response to impulse vector 5: $[0. \ 0. \ 0. \ 0. \ 4. \ 2. \ 1. \ 0.5 \ 0. \ 0. \ 0.]$
 Response to impulse vector 6: $[0. \ 0. \ 0. \ 0. \ 0. \ 4. \ 2. \ 1. \ 0.5 \ 0. \ 0.]$
 Response to impulse vector 7: $[0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 4. \ 2. \ 1. \ 0.5 \ 0.]$
 Response to impulse vector 8: $[0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 4. \ 2. \ 1. \ 0.5]$

Matrix representation of the LSI system:

```
[[4. 2. 1. 0.5 0. 0. 0. 0. 0. 0. 0.]
 [0. 4. 2. 1. 0.5 0. 0. 0. 0. 0. 0.]
 [0. 0. 4. 2. 1. 0.5 0. 0. 0. 0. 0.]
 [0. 0. 0. 4. 2. 1. 0.5 0. 0. 0. 0.]
 [0. 0. 0. 0. 4. 2. 1. 0.5 0. 0. 0.]
 [0. 0. 0. 0. 0. 4. 2. 1. 0.5 0. 0.]
 [0. 0. 0. 0. 0. 0. 4. 2. 1. 0.5 0.]
 [0. 0. 0. 0. 0. 0. 0. 4. 2. 1. 0.5]]
```

b)

When MATLAB's `conv` function convolves two signals, it does extend their lengths by padding with zeros.

For example, given two signals:

$$\mathbf{a} = [1, 2, 3] \quad \mathbf{b} = [0.5, 1.5]$$

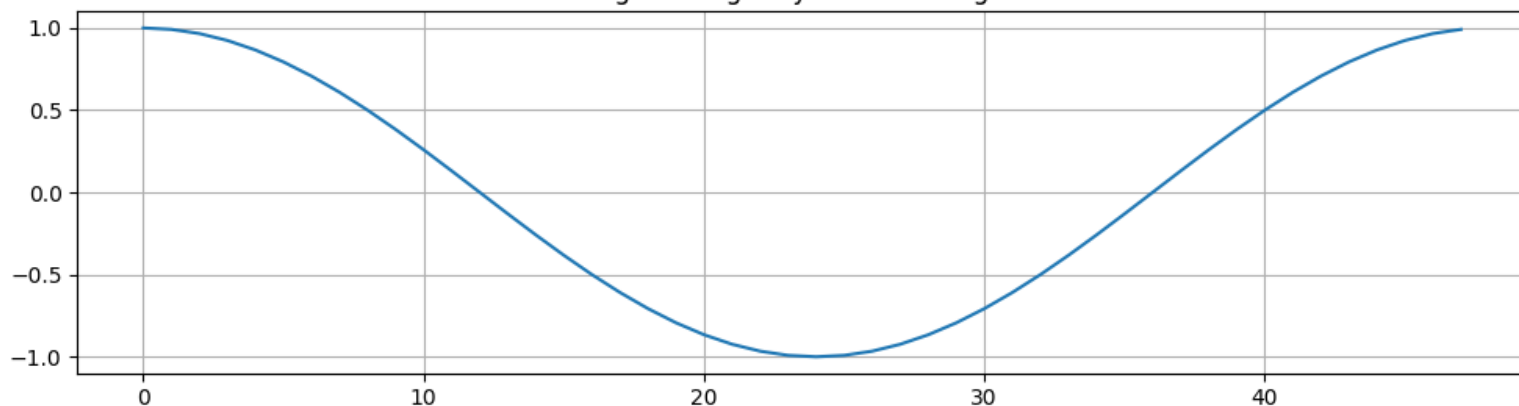
When we convolve them, it gonna extended like this:

$$\mathbf{a} = [0, 1, 2, 3, 0] \quad \mathbf{b} = [0, 0.5, 1.5, 0, 0]$$

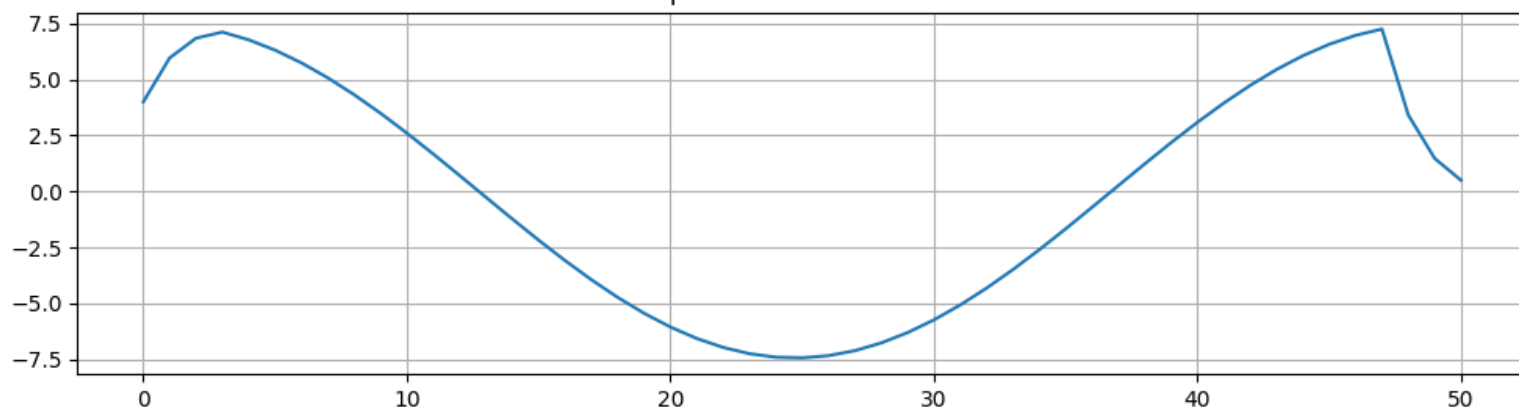
The output of the convolution will be $= \text{length}(\mathbf{a}) + \text{length}(\mathbf{b}) - 1 = 3 + 2 - 1 = 4$

$$\text{result} = [0.5 \ 2.5 \ 4.5 \ 4.5]$$

Original Single-Cycle Cosine Signal



Response after Convolution



Length of the response: 51

Is this a single-cycle sinusoid? Why or why not?

Based on the convolution with the provided impulse response r , the output is not a single-cycle sinusoid. The output is a sinusoid with a period of 48, which is 4 times the period of the original signal.

This is because the impulse response is a 4-point signal, and the convolution of the original signal with the impulse response is equivalent to the sum of 4 shifted copies of the original signal.

If not, what modification is necessary to the `conv` function to ensure that it will behave according to the “sinein, sine-out” behavior we expect of LSI systems?

To get "sine-in, sine-out" behavior, the impulse response should be a delta function. In other words, the impulse response should be a single 1 followed by zeros. This will result in a convolution that is equivalent to the original signal.