CSE 3302/5307 Programming Language Concepts

Homework6 - Fall 2023

Due Date: Oct.7, 2023, 8:00p.m. Central Time

Problem1 - 40%

We've seen how to define natural numbers using church encoding in untyped lambda calculus:

$$\begin{aligned}\mathbf{0} &= \lambda f.\lambda x.\ x\\ \mathbf{1} &= \lambda f.\lambda x.\ f\ x\\ &\dots\\ \mathbf{n} &= \lambda f.\lambda x.\ f^n\ x\\ &\dots\end{aligned}$$

Note that church encoding cannot represent negative integers, we try to encode all integers using **untyped** lambda calculus.

(a) Propose a method to extend church numerals to representation of integers.(Hint: you may try to use pairs). Give a concrete example for representation of integer **-5** with your proposed method.

(b) Define a function *nat2int* that converts a natural number to your representation of correspondent integer.

(c) Based on this definition of integers, define the following arithmetic operations in lambda calculus(you can directly use operations on natural numbers defined before like add, multi, etc.):

(1) negation: neg n

(2) addition: addint m n

(3) subtraction: subint m n

(4) multiplication: multint m n

(d) Bonus: Are there other ways to implement integers? Explain your idea briefly with some example for operations.

Problem2 - 30%

Given the definition of Fibonacci number

$$F_0 = 0, F_1 = 1, F_i = F_{i-1} + F_{i-2}$$

(a) Use *fix* to write a lambda function called *fib*: int → int to compute the n-th Fibonacci number.

(b) We want to extend simple *let* expression to recursive *let rec* expression:

$$letrec\ f = \lambda x.\ e_1\ in\ e_2$$

where f itself can appear in e_1.

Example usage of *letrec* for factorial:

$$fact = \lambda n.(letrec\ fact = (\lambda i.\ if\ i = 0\ then\ 1\ else\ i * (fact\ (i-1)))in\ fact\ n)$$

(1) Define semantic and typing rules for expression *letrec* ;

(2) Use *letrec* to redefine our Fibonacci function.

Problem3 - 30%

Given the following λ expression:

```
let x = 2 in
  let y = 4 in
    let f1 = \x.\y.x+2*y in
      let f2 = \x.\y.2*x-y in
      f2 (f1 y x) 3
```

Using the environment model for lambda calculus with let,
(a) Define closures. (Be careful and refer to lecture slides);
(b) Show detailed multi-step evaluation process of the λ expression above.