

# CSE 3302 Programming Language

Homework 3 - Spring 2026

Due Date: Feb. 9, 2026, 11:59 PM

## Problem 1 - 30%

Evaluate the following  $\lambda$  expressions using call-by-value and call-by-name operational semantics ( $e \rightarrow e'$ ) with **just one step**. (Hint: you may or may not be able to take a step on the given expression.) Show the complete derivation tree of this one step for all three expressions if you can take step.

- (a)  $(\lambda v. \lambda w. w) ((\lambda x. x) (y \lambda z. z))$
- (b)  $(\lambda x. x x) (\lambda y. y y)$
- (c)  $(\lambda x. \lambda y. x) ((\lambda z. z) (\lambda u. u))$

## Problem 2 - 40%

Prove by induction: If  $e_1$  is closed and  $e_1 \rightarrow^* e_2$  then  $e_2$  is closed.

(Hint: We need to first prove lemma: if  $e_1$  is closed and  $e_1 \rightarrow e_2$ , then  $e_2$  is closed. To prove this lemma, you may need another lemma on substitution.)

## Problem 3 - 30%

Recall the definition of natural numbers by  $n$  nat judgment taught in the lecture.

- (a) Give an inductive definition of the judgment form  $\text{fib } n_1 \ n_2$ , which indicates the  $n_1^{th}$  Fibonacci number is  $n_2$ .
- (b) Give an inductive definition of the judgment form  $\text{fibsum } n_1 \ n_2$ , which indicates the sum of the first  $n_1$  Fibonacci numbers is  $n_2$ .
- (c) Prove by induction: If  $\text{fibsum } n \ m$  then  $\text{fib succ}(\text{succ}(n)) \ \text{succ}(m)$ , that is

$$\sum_{i=1}^n F_i = F_{n+2} - 1.$$

**Submission Format:** Submit only the .pdf version of your homework (typed submissions are preferred; Scanned images must be readable). File must be named `lastname_studentID_hw3.pdf`.