CSE 3302/5307 Programming Language Concepts

Homework 3 - Fall 2025

Due Date: Sep. 8, 2025, 8:00PM Central Time

Name:_____ UTA ID:_____

Instructions

Please follow the proof structure introduced in the lecture slides, or points may be deducted even though your proof ideas are generally correct.

Problem1 - 40%

$$\frac{}{Z\ nat} \qquad \frac{n\ nat}{S(n)\ nat} \qquad \frac{n\ nat}{add\ Z\ n\ n} \qquad \frac{add\ n_1\ n_2\ n_3}{add\ S(n_1)\ n_2\ S(n_3)}$$

(a) Define $max\ a\ b\ c \iff c = \max(a, b)$

(b) Define $com\ n\ m\ k \iff k = \binom{n}{m}$

(c) (10 points) Prove that for all n such that $n\ nat$, $add\ n\ Z\ n$ holds

(d) (10 points) Prove that for all n_1, n_2, n_3, $add\ n_1\ n_2\ n_3 \rightarrow add\ n_1\ S(n_2)\ S(n_3)$ holds

(e) (10 points) Prove that for all n_1, n_2, n_3, $add\ n_1\ n_2\ n_3 \rightarrow add\ n_2\ n_1\ n_3$ (addition is commutative)

Problem2 - 30%

$$\frac{}{\epsilon\ tree} \qquad \frac{t_1\ tree \quad t_2\ tree}{t_1 \oplus t_2\ tree}$$

(a) Define $height\ a\ h \iff a\ tree$ and h is the height of a

(b) Define $size\ a\ n \iff a\ tree$ and n is the number of nodes contained in a

(c) Define $bst\ a \iff a$ is a binary search tree where each node contains a natural number defined in Problem 1. Hint: First define an order on natural numbers.

(d) (15 points) Prove: If $a\ tree$, $height\ a\ h$, and $size\ a\ n$, then $max\ n\ (2^h - 1)\ (2^h - 1)$.

Problem3 - 30%

$$\frac{}{\epsilon\ list} \qquad \frac{n\ nat \quad l\ list}{n :: l\ list}$$

(a) Define $append\ n\ l\ l' \iff l'$ is l appended with natural number n

(b) Define $reverse\ l\ l' \iff l'$ is the reverse of list l

(c) Define $sum\ l\ n \iff$ the sum of all elements in list l is n

(d) (15 points) Prove: If $sum\ l\ n$, and $reverse\ l\ l'$, then $sum\ l'\ n$.