Inductive Definition

Outline

- Judgements
- Inference Rules
- Inductive Definition
- Derivation
- Rule Induction

Language and Meta-language

- **Language** is the target programming language, e.g., Java, Python, ML.
  - Has its own identifiers, variables, etc.

- **Meta-language** is the language in which to describe the target language.
  - E.g. *Language L1 is more concise than language L2* (this is a statement in a meta-language, that's neither L1 or L2).

Meta-Variables

- A symbol in a meta-language that is used to describe some element in an object (target) language
  - E.g., Let **a** and **b** be two sentences of a language $\mathcal{L}$
  - E.g., Let **n** be a number, **d** be a digit and **s** be a sign in the language of *numerals*
    - 435, 535.23, -3847 are all numbers in the language of numerals
  - meta-variable doesn't appear in the language itself.
- **Meta-** is a prefix used to indicate a concept, which is an abstraction from another concept, used to complete or add to the latter.
- Similar use in "meta-data", "meta-theory", etc.
  - The syntax, semantics, etc. about a PL (e.g., Java) is the *meta-theory* about that language

Judgements

- A *judgement* is an *assertion* (in the meta-language) about one or more syntactic objects.

Judgement	Meaning
n nat	(n is a natural number)
$n = n_1 + n_2$	(n is the sum of n_1 and n_2)
τ type	(τ is a type)
e:τ	(expression e has type τ)
$e \Downarrow v$	(expression e has value v)

- "n nat" can also be written as "n isa nat", "n is a natural num", etc. as long as it's consistent and understandable.

Judgements (II)

- A judgement states one or more *syntactic objects* have a *property* or have a *relation* among one another.
- The property or the relation itself is called *predicate*.
  - E.g., n nat (this judgement involves one object n)
- The abstract structure (schema) of a judgement is called *judgement form*.
  - E.g., n nat.
- The judgement that a particular object or objects having that property is an *instance* of a judgement form.
  - E.g., 5 nat, succ(n) nat are all judgements
- W.L.O.G., we use "judgement" to mean the instance of judgement form usually.

INFERENCE RULES

- An inductive definition of a judgement form consists of a collection of rules of the form:

premises

$$\frac{J_1 \quad \dots \quad J_k}{J}$$

conclusion

- To show J, it is sufficient to show J_1, …, J_k.
- A rule without premises is called an *axiom*;
- Otherwise, it's called a *proper rule*.

Inductive Definition

- Definition of judgement form n nat:

$$\frac{}{zero \quad nat} \qquad \frac{n \quad nat}{succ(n) \quad nat}$$

- Definition of judgement form t tree:

$$\frac{}{empty\ tree} \qquad \frac{t_1\ tree \quad t_2\ tree}{node(t_1;\ t_2)\ \ tree}$$

Axioms!

Proper Rules!

QUIZ: JUDGEMENT AND JUDGEMENT FORM

- Please circle and indicate all the **judgement forms** and **judgements** you can find in the following inductive definition:

Definition of **t tree**:

$$\frac{}{\mathit{empty}\ \mathit{tree}} \qquad \frac{t_1\ \mathit{tree} \quad t_2\ \mathit{tree}}{\mathit{node}(t_1;\ t_2)\ \mathit{tree}}$$