

CSE 3302/5307 Programming Language Concepts

Homework 5 - Fall 2025

Due Date: Sep. 22, 2025, 9:00PM Central Time

Name: _____ UTA ID: _____

Problem1 - 30%

Prove the **exchange lemma**: If $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$, then $\Gamma, y : t_2, x : t_1, \Gamma' \vdash e : t$.
(proof by induction on derivation of $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$).

Problem2 - 30%

Prove the **weakening lemma**: If $\Gamma \vdash e : t$ then $\Gamma, x : t' \vdash e : t$ (provided x not in $\text{Dom}(\Gamma)$). Hint: you may need to use the Exchange lemma.

Problem3 - 40%

Prove the **substitution lemma**: If $\Gamma, x : t' \vdash e : t$ and $\Gamma \vdash v : t'$ then $\Gamma \vdash e[v/x] : t$.
Hint: you may need to use the Exchange and Weakening lemmas.