CSE 3302/5307 Programming Language Concepts

Homework 2 - Fall 2025

Due Date: Sep. 1, 2025, 8:00PM Central Time

Problem1 - 30%

(a) Consider looking at page 21 in slide "inductive-proof". In the proof of the second case $\frac{n\ nat}{S(n)\ nat}$, what is the assumption in this case and what is the difference between assumption and I.H.?

(b) We define a judgment form $IsNat\ x\ a$.

$$\frac{x\ nat}{IsNat\ x\ true}Nat \qquad \frac{x\ list}{IsNat\ x\ false}List \qquad \frac{x\ tree}{IsNat\ x\ false}Tree$$

For which rule we can use its inversion rule? If there exists such rule, point it out and give an explanation. If no rules can be inverted, give an explanation.

Problem2 - 35%

(a) Give an inductive definition of the judgment form max n_1 n_2 n_3, which indicates the max number between n_1 and n_2 is n_3.
Hint: think of how we defined *add* by knowledge of *nat*.

(b) Prove by induction: if max n_1 n_2 n_3, then max n_2 n_1 n_3.

Problem3 - 35%

Recall the definition of natural numbers by n nat judgement taught in the lecture.

(a) Give an inductive definition of the judgement form fib n_1 n_2, which indicates the n_1^{th} Fibonacci number is n_2.

(b) Give an inductive definition of the judgement form fibsum n_1 n_2, which indicates the sum of the first n_1 Fibonacci numbers is n_2.

(c) Prove by induction: If fibsum n m then fib succ(succ(n)) succ(m), that is

$$\sum_{i=1}^{n} F_i = F_{n+2} - 1.$$

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