CSE 3302/5307 Programming Language Concepts

Homework2 - Fall 2023

Due Date: Sep.4, 2023, 8:00p.m. Central Time

Problem1 - 30%

(a) Please look at page 21 in slide "inductive-proof". In the proof of the second case $\frac{n\ nat}{S(n)\ nat}$, what is the assumption in this case and what is the difference between assumption and I.H.?

(b) We define a judgment form $IsNat\ x\ a$.

$$\frac{x\ nat}{IsNat\ x\ true}NatRule \qquad \frac{x\ list}{IsNat\ x\ false}ListRule \qquad \frac{x\ tree}{IsNat\ x\ false}TreeRule$$

For which rule we can use its inversion rule? If there exists such rule, point it out and give an explanation. If no rules can be inverted, give an explanation.

(c) We define a judgment form $add'\ n_1\ n_2\ n_3$ (another definition for addition):

$$\frac{}{add'\ Z\ Z\ Z}add'Z \qquad \frac{add'\ n_1\ n_2\ n_3}{add'\ (Sn_1)\ n_2\ (Sn_3)}add'-l \qquad \frac{add'\ n_1\ n_2\ n_3}{add'\ n_1\ (Sn_2)\ (Sn_3)}add'-r$$

For which rule we can use its inversion rule? If there exists such rule, point it out and give an explanation. If no rules can be inverted, give an explanation.

Problem2 - 20%

(a) Give an inductive definition of the judgment form max $n_1\ n_2\ n_3$, which indicates the max number between n_1 and n_2 is n_3.

(b) Prove by induction: if max $n_1\ n_2\ n_3$, then max $n_2\ n_1\ n_3$.

Problem3 - 20%

(a) Recall the definition of addition by $add\ n_1\ n_2\ n_3$ judgment taught in the lecture.

(b) (15 points) Prove by induction: If $add\ n_1\ n_2\ n_3$, then $add\ n_2\ n_1\ n_3$ (Commutative law of add).

(**Hint**: You can begin with proof of this lemma: If $n\ nat$, then $add\ n\ Z\ n$.)

Problem4 - 30%

Recall the definition of natural numbers by n nat judgment taught in the lecture.

(a) Give an inductive definition of the judgment form fib n_1 n_2, which indicates the n_1^{th} Fibonacci number is n_2.

(b) Give an inductive definition of the judgment form fibsum n_1 n_2, which indicates the sum of the first n_1 Fibonacci numbers is n_2.

(c) Prove by induction: If fibsum n m then fib succ(succ(n)) succ(m), that is

$$\sum_{i=1}^{n} F_i = F_{n+2} - 1.$$