



INDUCTIVE DEFINITION

1

OUTLINE

- Judgements
- Inference Rules
- Inductive Definition
- Derivation
- Rule Induction

LANGUAGE AND META-LANGUAGE

- **Language** is the target programming language, e.g., Java, Python, ML.
 - Has its own identifiers, variables, etc.
- **Meta-language** is the language in which to describe the target language.
 - E.g. *Language L1 is more concise than language L2* (this is a statement in a meta-language, that's neither L1 or L2).

META-VARIABLES

- A symbol in a meta-language that is used to describe some element in an object (target) language
 - E.g., Let **a** and **b** be two sentences of a language \mathcal{L}
 - E.g., Let **n** be a number, **d** be a digit and **s** be a sign in the language of *numerals*
 - 435, 535.23, -3847 are all numbers in the language of numerals
 - meta-variable doesn't appear in the language itself.
- **Meta-** is a prefix used to indicate a concept, which is an abstraction from another concept, used to complete or add to the latter.
- Similar use in “meta-data”, “meta-theory”, etc.
 - The syntax, semantics, etc. about a PL (e.g., Java) is the *meta-theory* about that language

JUDGEMENTS

- A *judgement* is an *assertion* (in the meta-language) about one or more syntactic objects.

Judgement

n nat

$n = n_1 + n_2$

τ type

$e:\tau$

$e \Downarrow v$

Meaning

(n is a natural number)

(n is the sum of n_1 and n_2)

(τ is a type)

(expression e has type τ)

(expression e has value v)

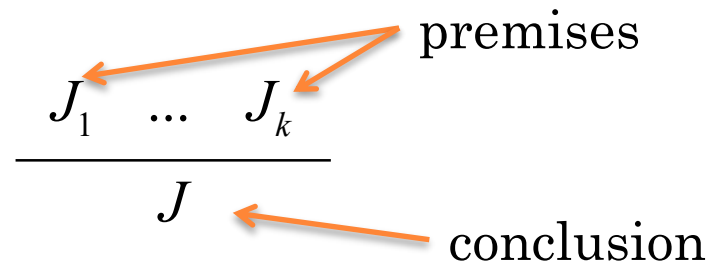
- “ n nat” can also be written as “ n isa nat”, “ n is a natural num”, etc. as long as it’s consistent and understandable.

JUDGEMENTS (II)

- A judgement states one or more *syntactic objects* have a *property* or have a *relation* among one another.
- The property or the relation itself is called *predicate*.
 - E.g., $n \text{ nat}$ (this judgement involves one object n)
- The abstract structure (schema) of a judgement is called *judgement form*.
 - E.g., $n \text{ nat}$.
- The judgement that a particular object or objects having that property is an *instance* of a judgement form.
 - E.g., 5 nat , $\text{succ}(n) \text{ nat}$ are all judgements
- W.L.O.G., we use “judgement” to mean the instance of judgement form usually.

INFERENCE RULES

- An inductive definition of a judgement form consists of a collection of rules of the form:



- To show J , it is sufficient to show J_1, \dots, J_k .
- A rule without premises is called an *axiom*;
- Otherwise, it's called a *proper rule*.

INDUCTIVE DEFINITION

- Definition of judgement form *n nat*:

$$\frac{}{\text{zero } \text{nat}} \qquad \frac{n \text{ nat}}{\text{succ}(n) \text{ nat}}$$

- Definition of judgement form *t tree*:

$$\frac{}{\text{empty tree}} \qquad \frac{t_1 \text{ tree} \quad t_2 \text{ tree}}{\text{node}(t_1; t_2) \text{ tree}}$$

Axioms!

Proper Rules!

QUIZ: JUDGEMENT AND JUDGEMENT FORM

- Please circle and indicate all the **judgement forms** and **judgements** you can find in the following inductive definition:

Definition of **t tree**:

$$\frac{}{\text{empty tree}} \qquad \frac{t_1 \text{ tree} \quad t_2 \text{ tree}}{\text{node}(t_1; t_2) \text{ tree}}$$