CSE 3302/5307 Programming Language Concepts

Homework4 - Fall 2023

Due Date: Sep.23, 2023, 8:00p.m. Central Time

Problem1 - 40%

Given the definition of *pred n* (predecessor of n):

$$pred = \lambda n.\lambda f.\lambda x.n\ (\lambda g.\lambda h.h\ (g\ f))\ (\lambda u.x)\ (\lambda u.u)$$

Please define following terms using lambda calculus:

1. sub m n (subtraction)
2. iszero n
3. leq m n (m is less or equal than n)
4. equal m n
5. factorial n (hint: try to define it using pair)

(You can directly use the definition in the slides and the last homework, like add, tru, etc.)

Problem2 - 20%

Prove the **exchange lemma**: If $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$, then $\Gamma, y : t_2, x : t_1, \Gamma' \vdash e : t$. (proof by induction on derivation of $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$).

Problem3 - 20%

Prove the **weakening lemma**: If $\Gamma \vdash e : t$ then $\Gamma, x : t' \vdash e : t$ (provided x not in Dom(Γ)).

Problem4 - 30%

Prove the **substitution lemma**: If $\Gamma, x : t' \vdash e : t$ and $\Gamma \vdash v : t'$ then $\Gamma \vdash e[v/x] : t$.

Remark:
Please email **.pdf** files to TA.
File name format: HW_X_FirstName_10digitID.pdf
Example: HW_3_Sinong_1001001000.pdf