CS-E4850 Computer Vision
Exercise Round 5

The problems should be solved before the exercise session and solutions returned via the MyCourses page. Handwritten solutions are fine if they are scanned to PDF format and clear to read. You do not need to solve all tasks as points will be rewarded also to partial solutions.

NOTE: In this round, there are also programming tasks for which the instructions are given separately. Return the solutions to pen and problem in a single PDF file, and another file for programming tasks.

Exercise 1. Total least squares line fitting. (Pen and paper problem)
An overview of least squares line fitting is presented on the slide 13 of Lecture 4. Study it in detail and present the derivation with the following stages:

1) Given a line $ax+by-d=0$, where the coefficients are normalized so that $a^2+b^2=1$, show that the distance between a point (x_i, y_i) and the line is $|ax_i + by_i - d|$.

2) Thus, given n points (x_i, y_i), $i = 1, \ldots, n$, the sum of squared distances between the points and the line is $E = \Sigma_{i=1}^{n}(ax_i + by_i - d)^2$. In order to find the minimum of E, compute the partial derivative $\partial E/\partial d$, set it to zero, and solve d in terms of a and b.

3) Substitute the expression obtained for d to the formula of E, and show that then $E = (a\ b)U^\top U(a\ b)^\top$, where matrix U depends on the point coordinates (x_i, y_i).

4) Thus, the task is to minimize $||U(a\ b)^\top||$ under the constraint $a^2 + b^2 = 1$. The solution for $(a\ b)^\top$ is the eigenvector of $U^\top U$ corresponding to the smallest eigenvalue, and d can be solved thereafter using the expression obtained above in the stage two.

Remember also the programming tasks...