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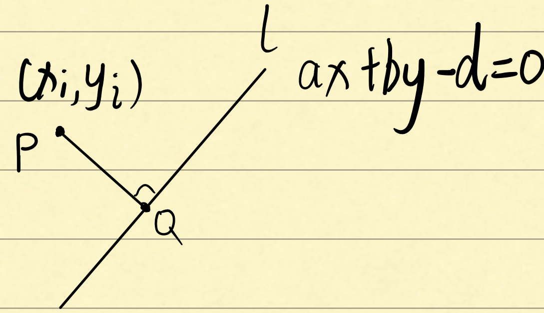
1:

1)



$\because PQ \perp l$

$\therefore k_{PQ} \cdot k_l = -1 \Rightarrow k_{PQ} \cdot (-\frac{a}{b}) = -1 \Rightarrow k_{PQ} = \frac{b}{a}$

$\therefore l_{PQ}: (y - y_1) = \frac{b}{a}(x - x_i)$

$\therefore$ The intersection of PQ and l is:

$$\begin{cases} (y - y_1) = \frac{b}{a}(x - x_i) \\ ax+by-d=0 \end{cases} \Rightarrow Q\left(\underset{Q_x}{\frac{b^2x_i - aby_0 + ad}{a^2+b^2}}, \underset{Q_y}{\frac{a^2y_i - abx_i + bd}{a^2+b^2}}\right)$$

$\therefore$ Distance of Point P and line l is:

$$|PQ| = \sqrt{(Q_x - x_i)^2 + (Q_y - y_i)^2} = \frac{|ax_i + by_i - d|}{\sqrt{a^2+b^2}}$$

since $a^2+b^2=1$, $|PQ| = |ax_i + by_i - d|$

2): $E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} -2(ax_i + by_i - d)$$

We set $\frac{\partial E}{\partial d} = 0$:

$$\sum_{i=1}^{n} -2(ax_i + by_i - d) = 0$$

$$\Rightarrow a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} = nd$$

$$\Rightarrow d = a\frac{\sum_{i=1}^{n} x_i}{n} + b\frac{\sum_{i=1}^{n} x_i}{n} = a\bar{x} + b\bar{y}$$

3) Using expression of d in 2), we get:

$$E = \sum_{i=1}^{n} (ax_i + by_i - a\bar{x} - b\bar{y})^2 = \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2$$

$$= \left\| \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = \left(U\begin{bmatrix} a \\ b \end{bmatrix}\right)^T \left(U\begin{bmatrix} a \\ b \end{bmatrix}\right)$$

$$= \begin{bmatrix} a \\ b \end{bmatrix}^T U^T U \begin{bmatrix} a \\ b \end{bmatrix} = N^T U^T U N$$

4) The derivation of E in (3):

$$\frac{\partial E}{\partial N} = 2(U^T U)N$$

$\because$ Solution to $2(U^T U)N = 0$, subject $\|N\|^2 = 1$, eigenvector of $U^T U$

$$\therefore \frac{\partial E}{\partial N} = 2(U^T U)N = 0$$

$$\Rightarrow (U^T U)N = 0$$