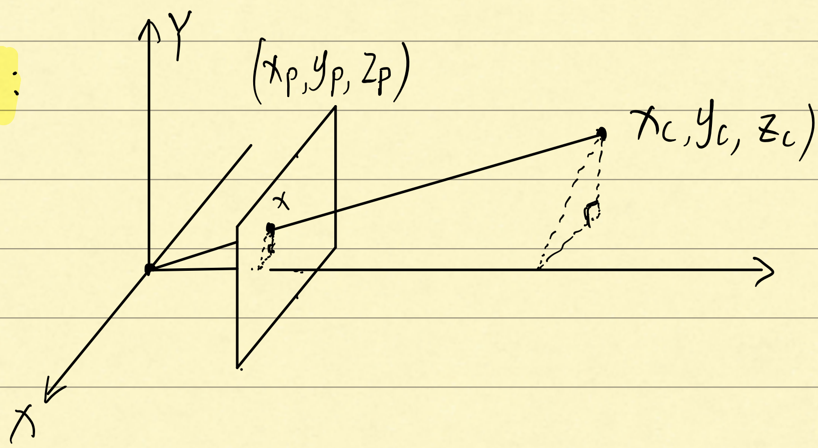
Yan Gengcong

E1:



According to Similar Triangle shown in the figure above, the image plane is perpendicular to the optical axis, and f is the focal length, So $f = Z_p$:

$$\frac{x_c}{z_c} = \frac{x_p}{z_p} = \frac{x_p}{f} \Rightarrow x_p = f\frac{x_c}{z_c}$$

$$\frac{y_c}{z_c} = \frac{y_p}{z_p} = \frac{y_p}{f} \Rightarrow y_p = f\frac{y_c}{z_c}$$

E2:

a):

Given u, v axis are parallel to x, y axis respectively, the equation (6.9) from Chapter 6:

$$K = \begin{bmatrix} a_x & & x_0 \\ & a_y & y_0 \\ & & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} fm_u & & u_0 & 0 \\ & fm_v & v_0 & 0 \\ & & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} m_u f \frac{x_c}{z_c} + u_0 \\ m_v f \frac{y_c}{z_c} + v_0 \\ 1 \end{bmatrix}$$

$\because$ $x_p = f\frac{x_c}{z_c}$, $y_p = f\frac{y_c}{z_c}$,

$\therefore$ $u = m_u x_p + u_0$, $v = m_v y_p + v_0$.

b):

Given u axis is parallel to x axis and angle between u and v axis is θ.

So $y' = \frac{y_p}{\sin\theta}$, $x' = x_p - \frac{y_p}{\tan\theta}$

$\therefore$ $u = m_u x' + u_0$, $v = m_v y' + v_0$

$\Rightarrow u = m_u x_p - \frac{m_u y_p}{\tan\theta}$, $v = m_v \frac{y_p}{\sin\theta} + v_0$

E3:

Using homogeneous coordinates to represent (2,a) with intrinsic calibration matrix,

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \cdot \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

According to result in (2,a), we can get:

$$\begin{bmatrix} fm_u & & u_0 \\ & fm_v & v_0 \\ & & 1 \end{bmatrix} \cdot \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

E4:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K x_c = K(Rx_w + t) = K[R\ t]\begin{bmatrix} x_w \\ 1 \end{bmatrix}$$
$$= P \begin{bmatrix} x_w \\ 1 \end{bmatrix}$$

$\therefore P = K[R\ t]$

E5:

R is a 3×3 matrix representing the orientation of camara coordinate frame.

$\bar{C}$ is the coordinate of the camera centre in world coordinate

$$X_{cam} = \begin{bmatrix} R & -R\bar{C} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} R & -R\bar{C} \\ 0 & -1 \end{bmatrix} X$$

$\because x = K[I|0]X_{cam}$

$\therefore x = KR[I|-\bar{C}]X$