

Yan Gengcong
1009903

2:

Equation in (10):

$$\Delta P = H^{-1} \sum_x \left[\nabla I \frac{\partial W}{\partial p} \right]^T [T(x) - I(W(x;p))]$$

And where $\Delta P = \begin{bmatrix} u \\ v \end{bmatrix}$, $H = \sum_x \left[\nabla I \frac{\partial W}{\partial p} \right]^T \left[\nabla I \frac{\partial W}{\partial p} \right]$

$$\Delta I = \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix}$$

We know that:

$$\frac{\partial W}{\partial p} = \begin{bmatrix} \frac{\partial W_x}{\partial p_1} & \frac{\partial W_x}{\partial p_2} \\ \frac{\partial W_y}{\partial p_1} & \frac{\partial W_y}{\partial p_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore H = \sum_x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix}$$

So equation in (10) can be written:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix}^{-1} \cdot \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix} [T(x) - I(W(x;p))]$$

$$\Rightarrow \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_x I_x I_t \\ \sum_x I_y I_t \end{bmatrix}$$

which is the same equation on slide 25 of lecture 7.