Exercise 1

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1

a): The equation of a line in the plane can be rewritten in homogeneous coordinates as:

$$ax+by+c = (x \ \ y \ \ 1)\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0,$$

where $l = (a \ b \ c)^T$.

We can present point $x = (x, y)^T$ as a 3-vector $(x, y, 1)^T$.

point $(x, y)^T$ lies on line l iff $x^T l = 0$

b): point x lies on both lines L and L'

$\therefore \ x^T L = x^T L' = 0$

From the triple scalar product, $L \cdot (L \times L') = L' \cdot (L \times L') = 0$

$\therefore \ x = L \times L'$

c): Both points x and x' lies on same line

$\therefore \ x^T L = (x')^T L = 0$

From the triple scalar product, $x \cdot (x \times x') = x' \cdot (x \times x') = 0$

$\therefore \ L = x \times x$

d): The line through points x, x':

$L = x \times x'$

If y lies on L:

$y^T L = 0$

$$(\lambda x+(1-\lambda)x')^T\cdot(x\times x')=0$$

$$\lambda x^T\cdot(x\times x')+(1-\lambda)(x')^T(x\times x')=0$$

From the triple scalar product, $x\cdot(x\times x')=x'\cdot(x\times x')=0$

$\therefore$ we can make $x^T\cdot(x\times x')$, $(x')^T\cdot(x\times x')$ equal to 0.

$\therefore$ y lies on the line through x, x'

2:

a), b):

Translation:

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

dof: 2

Euclidean:

$$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

dof: 3

Similarity:

$$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

dof: 4

Affine:

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \qquad dof = 6$$

Projective:

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \qquad dof = 8$$

c): We can scale the Matrix by scalar value like h_{33}, it won't change the representation of M because it's homogeneous coordinates, the ratio between element matters.

So, the dof is less than the number of element.

3:

a):

Given H for transforming points.

$$lx = 0$$

$$\Rightarrow lH^{-1}Hx = 0$$

$\therefore$ all points Hx lies on line lH^{-1}

$\therefore$ Line transformation is lH^{-1}

b):