$k_1y_1+k_2y_2=0$ on I only when $k_1=k_2=0$

* $y''+P(x)y'+Q(x)y=0$: Homogenous linear ODE ⇒ y_1, y_2 : linearly independent

↳ $y=c_1y_1+c_2y_2$: general sol.

WHEN are the solutions lineary independent ?

① homogenous linear 2nd-order ODE : has general solution

② $p(x)$. $q(x)$ are conti. on some open interval I.

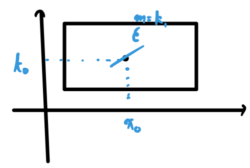
ex) $y''-y=0$, $p(x)=0$, $q(x)=-1$ ⇒ conti. on $\mathbb{R}'$

$y_1=e^x$, $y_2=e^{-x}$ ⇒ General sol. = $y=c_1e^x+c_2e^{-x}$

* IVP - ODE + initial condition.

★ Homo. linear. 2nd. & $p \cdot q$ are conti on I

→ unique solution



(신기해요)

ex) $y=c_1e^x+c_2e^{-x}$, $y(0)=6$, $y'(0)=-2$.

$C_1=2$, $C_2=4$

* How to solve IVP (H. L. 2 ODE)

⇒ $y''+p(x)y'+q(x)y=0$

1) y_1 를 알고 있다 ⇒ Reduction of order

ex) $(x^2-x)y''-xy'+y=0$

$(x\neq 0,\ x\neq 1)$

$y_1=x$ on $\mathbb{R}'$ (실수 전체)

(y_1 이랑 비슷할 거 같은데)

$y_2=u(x)y_1=u(x)x$

$y_2'=u(x)+u'(x)x$, $y_2''=u'(x)+u''(x)x+u'(x)$

→ $(x^2-x)(2u'+u''x)-x(u+u'x)+ux=0$.

⇒ $2u'x^2-2u'x+u''x^3-u''x^2-\cancel{ux}-u'x^2+\cancel{ux}=0$

⇒ $x((x^2-x)u''+(x-2)u')=0$: first-order ODE

$\ln|u'|=\ln|x-1|-2\ln|x|$

$u'=\frac{x-1}{x^2}=-\frac{1}{x^2}+\frac{1}{x}$ ∴ $u=\ln|x|+\frac{1}{x}$

Does it always work?)

이것만 알아도 충분

pf $y_2=uy_1$, $y_2'=u'y_1+uy_1'$, $y_2''=u''y_1+2u'y_1'+uy_1''$

ODE ($(u''y_1+2u'y_1'+uy_1'')+P(u'y_1+uy_1')+quy_1$

→ $u(y_1''+py_1'+qy_1)+u''y_1+2u'y_1'+pu'y_1$

생략

→ $u''y_1+2u'y_1'+pu'y_1=0$ $\ln|u'|=-2\ln|y_1|-\int p\,dx$

⇒ $u'=\frac{1}{y_1^2}\cdot e^{-\int p\,dx}$ ∴ $y_2=y_1\int\frac{1}{y_1^2}\cdot e^{-\int p\,dx}\,dx$ 기억하면 편해

continuous → $f(a)$ is defined, $f(a) = \lim_{x \to a} f(x)$,

2) case : $y'' + ay' + by = 0$ (a, b are constants)

+) 해를 예측한다 - 이 경우 exp func. 같다

→ $y_1 = e^{rx}$ → $r^2 e^{rx} + are^{rx} + be^{rx} = 0$, $(r^2 + ar + b) = 0$. $r = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$

>0, 안될수는 없다

characteristic eq.

① two real roots → $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ ex) $y'' - y = 0$

② double root → $y_2 = uy_1$, $u = x$

⇒ ③ case

③ complex root → if $a^2 - 4b < 0$, $w = \frac{\sqrt{4b - a^2}}{2}$,

$r = -\frac{a}{2} \pm wi$, $y_1 = e^{(-\frac{a}{2} + wi)x}$, $y_2 = e^{(-\frac{a}{2} - wi)x}$

근데 복소함수 취급 안함

* Euler formula

→ $e^{ix} = \cos x + i \sin x$ → $e^{i\pi} = -1$

↳ $y_1 = e^{-\frac{a}{2}x} e^{iwx} = e^{-\frac{a}{2}x}(\cos wx + i \sin wx)$

$y_2 = e^{-\frac{a}{2}x} e^{i(-wx)} = e^{-\frac{a}{2}x}(\cos wx - i \sin wx)$

⇒ $y_1 + y_2$, $y_1 - y_2$ → solutions (chain rule?)

⇒ $y_1 - y_2 = e^{-\frac{a}{2}x} \cdot \underline{2i} \sin wx$

$y_1 + y_2 = e^{-\frac{a}{2}x} \cdot \underline{2} \cos wx$

상수 안 중요 ($c_{1,2}$ 로 바뀜)

⇒ $\underline{y = e^{-\frac{a}{2}x}(c_1 \sin wx + c_2 \cos wx)}$

근동스럽다.

3) Euler - Cauchy Eq.

⇒ $x^2 y'' + axy' + by = 0$ → 다항함수가 아닐까? polynomial

→ $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

→ $m(m-1)x^m + amx^m + bx^m = 0$,

⇒ $(m^2 + (a-1)m + b) = 0$

① case : two real root ⇒ $y = c_1 x^{m_1} + c_2 x^{m_2}$

허수 화이팅!

② case : double root ⇒ $y_1 = x^m$, $y_2 = ux^m$, $u = \ln x$ $y = (c_1 + c_2 \ln x) x^m$

③ case : complex root

$m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$, $x^{\alpha + \beta i} = x^\alpha x^{\beta i} = x^\alpha e^{\beta i \ln x} = x^\alpha(\cos \beta \ln x + \sin \beta \ln x)$

∴ $\underline{y = c_1 x^\alpha \sin \beta \ln x + c_2 x^\alpha \cos \beta \ln x}$

요약.

* Existance

$\Rightarrow y'' + py' + qy = 0 \;,\; y(x_0) = k_0 \;,\; y'(x_0) = k_1$

<u>Thm 1.</u>

if func. p and q are continuos on I, $x_0 \in I$,

Then that ODE has an <u>unique</u> solution on I where is proof

↳ 하나 찾으면 끝

* Linear independence of solution

$\Rightarrow$ Wronskian,

$y'' + py' + q = 0$, p, q : conti. on I, y_1, y_2 : sol.

$$\rightarrow W(y_1, y_2) = y_1 y_2' - y_1' y_2 = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$ 이게 determinent 구나

<u>pf</u>

Linearly dependent $\Leftrightarrow$ $W(y_1, y_2)(x_0) = 0$ for some $x_0 \in I$

x_0 does not exist. $\rightarrow$ independent

Futhermore, $W(y_1, y_2)(x_1) \neq 0$, then y_1, y_2 are independent 신기하네요 극단적이다

$\Rightarrow$ suppose that y_1, y_2 are linearly <u>dependent</u>.

$y_1 = k_1 y_2$ or $y_2 = k_2 y_1$ $\rightarrow$ $W(y_1, y_2) = W(ky_2, y_2) \Rightarrow y_1 y_2' - y_1' y_2 = k y_2 y_2' - k y_2' y_2 = 0$

+) $x_0 \in I$, $W(y_1, y_2)(x_0) = 0$

$y_1(x_0)a + y_2(x_0)b = 0$, $y_1'(x_0)a + y_2'(x_0)b = 0$ (a,b are variables)

① $y_2(x_0) \neq 0$, $b = -\frac{1}{y_2(x_0)} y_1(x_0) \cdot a$ $\rightarrow$ $W(y_1, y_2)(x_0) = 0$ $\rightarrow$ (a, b) $(a \neq 0)$

$y = ay_1 + by_2$, $y(0) = 0$. $y'(0) = 0$

(unique) ⟶ 근데 $y=0$ 도 해임 $\rightarrow$ $y_1 = -\frac{b}{a} y_2$: dependent

$\Rightarrow$ $W(y_1, y_2)(x_0) = 0 \Leftrightarrow ay_1 + by_2 = 0$ in $\{(a,b) \mid a \neq 0, b \neq 0\}$ $\Rightarrow$ dependent