Square Root Decomposition

Arnar Bjarni Arnarson

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School of Computer Science
Reykjavík University

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- Sometimes we also want to update elements.

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- Also known as square root decomposition, and is a very powerful technique

Example problem: Supercomputer

- https://open.kattis.com/problems/supercomputer

Mo’s Algorithm