Segment Tree

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- Sometimes we also want to update elements.

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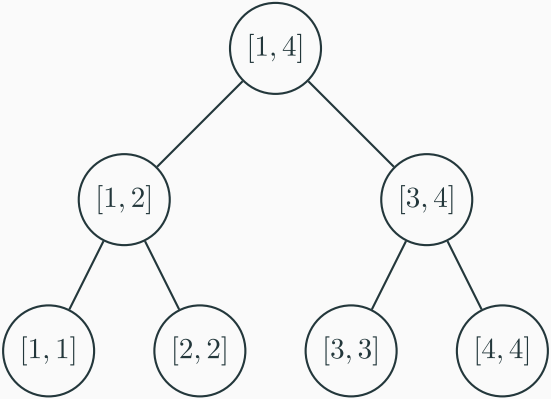
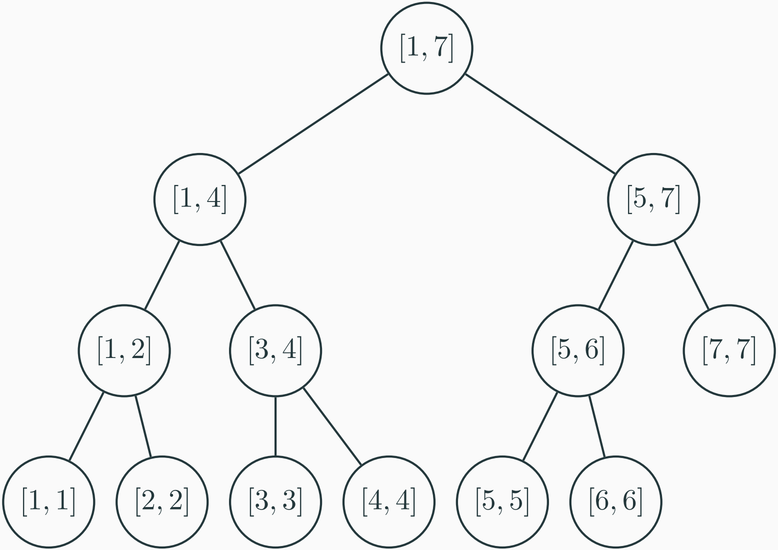
- We travel down the tree looking for the left and right end points, adding intervals that are completely inside our query range.
- When we update a value we only need to update the parents of that node up to the root, at most $\mathcal{O}(\log(n))$ nodes.

Drawn Segment Tree, $n = 4$



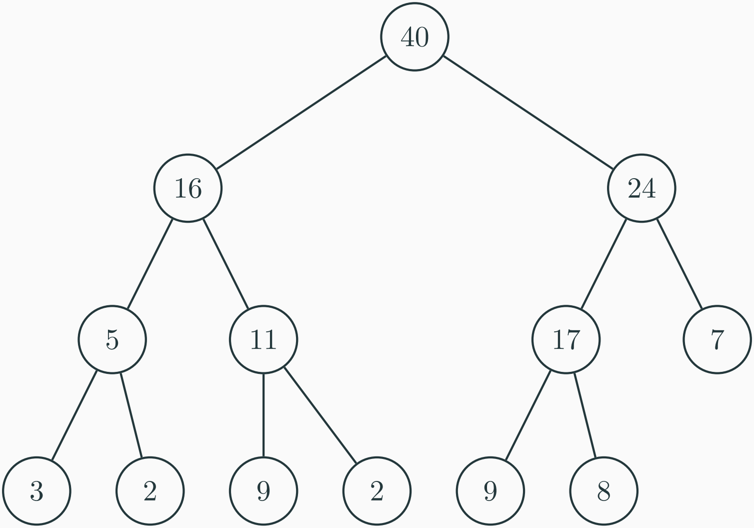
Drawn Segment Tree, $n = 7$



Segment Tree - Code

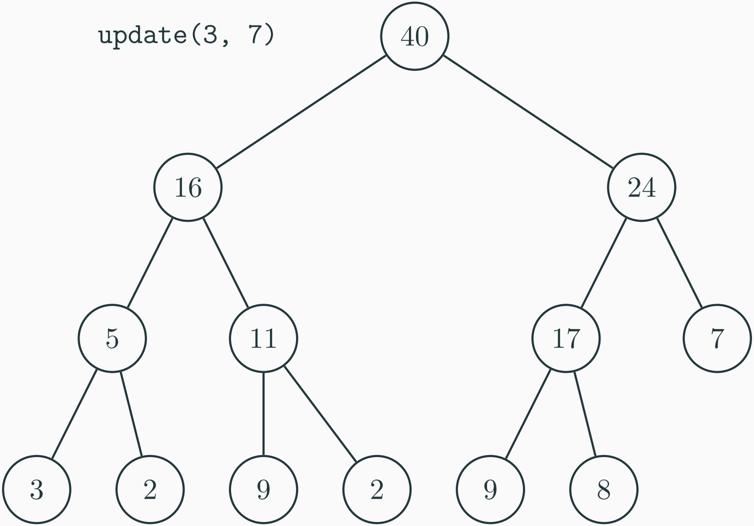
```cpp
struct segment_tree {
    segment_tree *left, *right;
    int from, to, value;
    segment_tree(int from, int to)
        : from(from), to(to), left(NULL), right(NULL), value(0) { }
};

segment_tree* build(const vector<int> &arr, int l, int r) {
    if (l > r) return NULL;
    segment_tree *res = new segment_tree(l, r);
    if (l == r) {
        res->value = arr[l];
    } else {
        int m = (l + r) / 2;
        res->left = build(arr, l, m);
        res->right = build(arr, m + 1, r);
        if (res->left != NULL) res->value += res->left->value;
        if (res->right != NULL) res->value += res->right->value;
    }
    return res;
}
```

Updates



Updates

`update(3, 7)`



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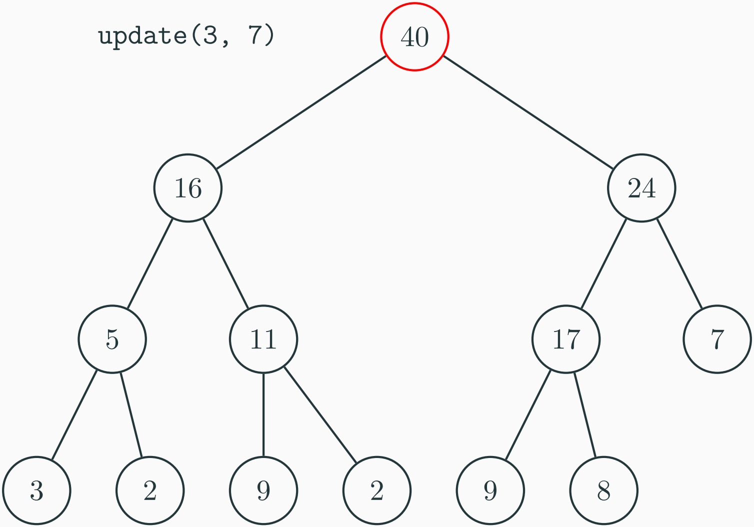

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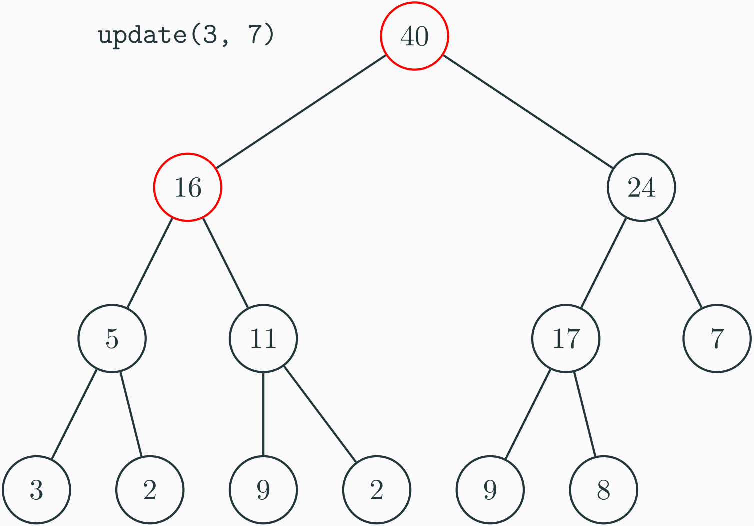

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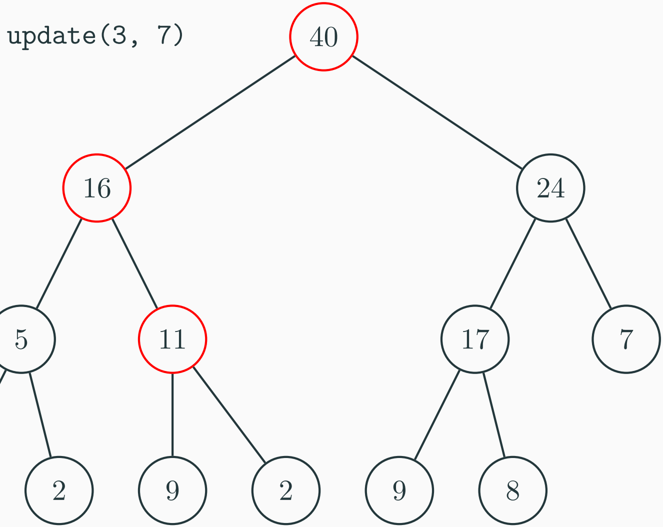

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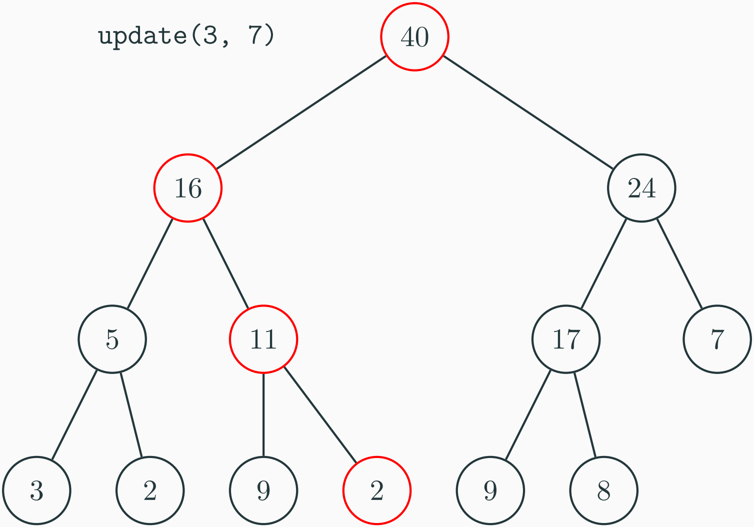

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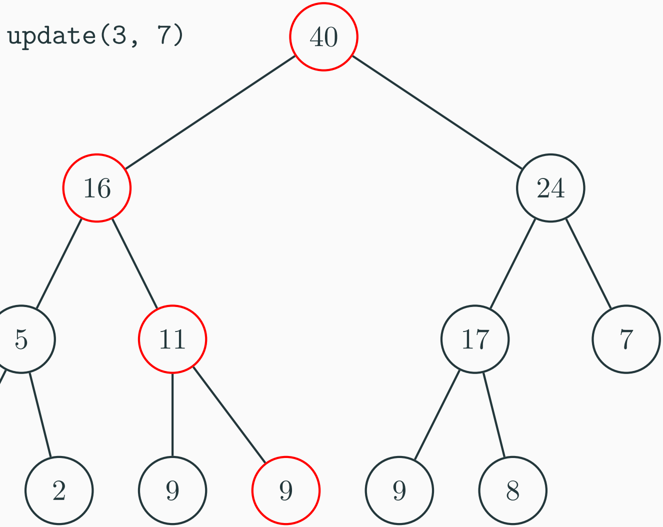

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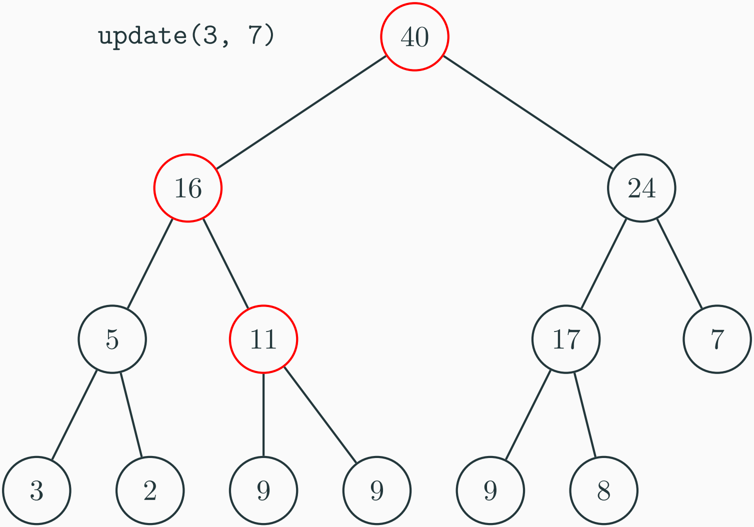

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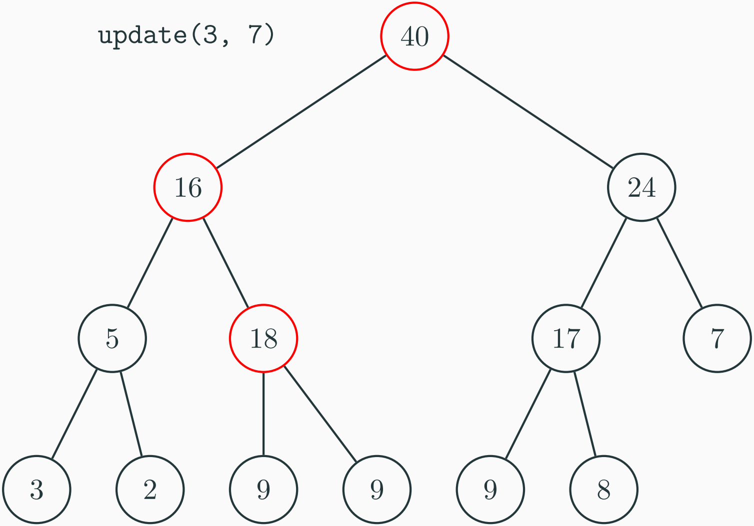

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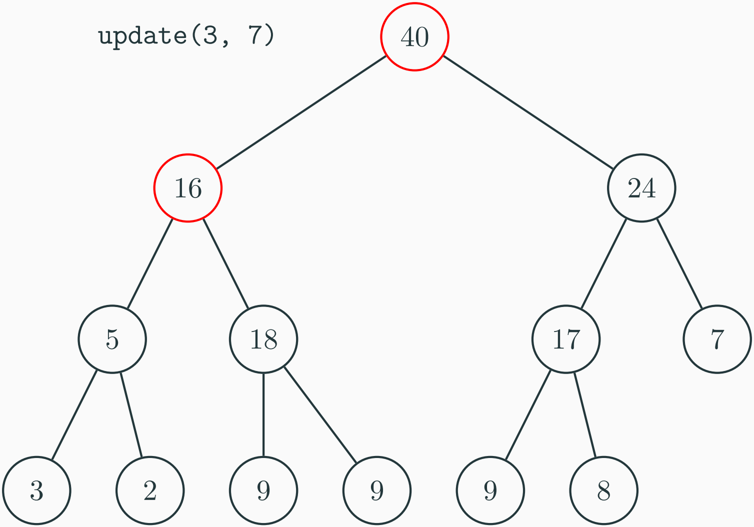

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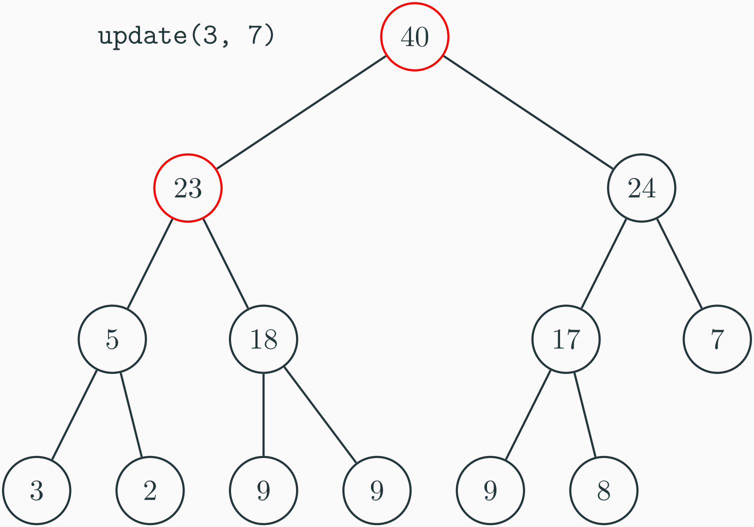

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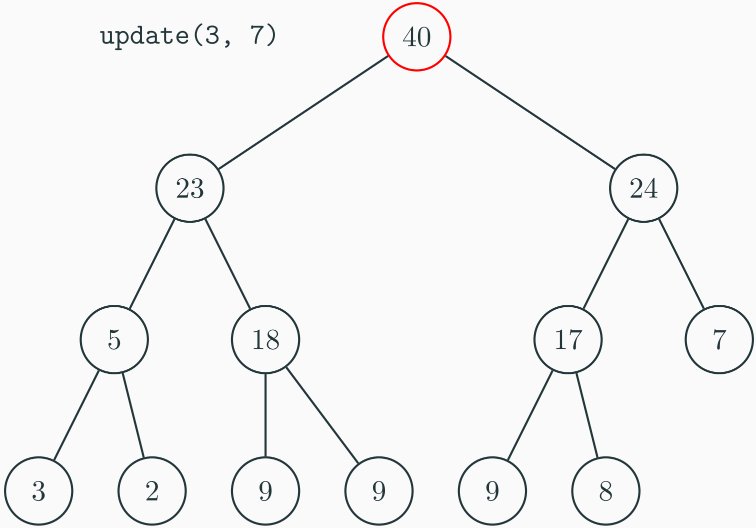

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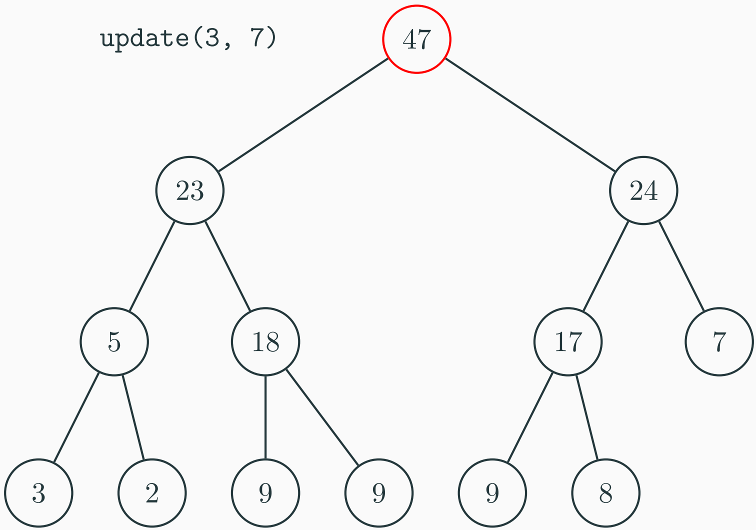

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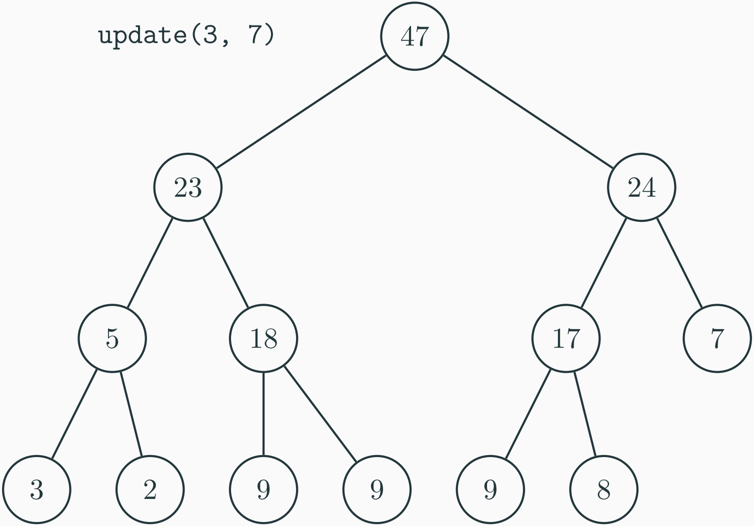

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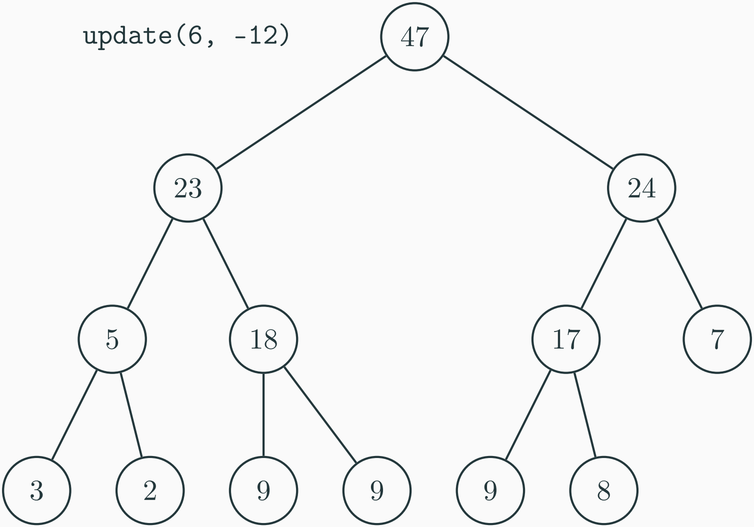

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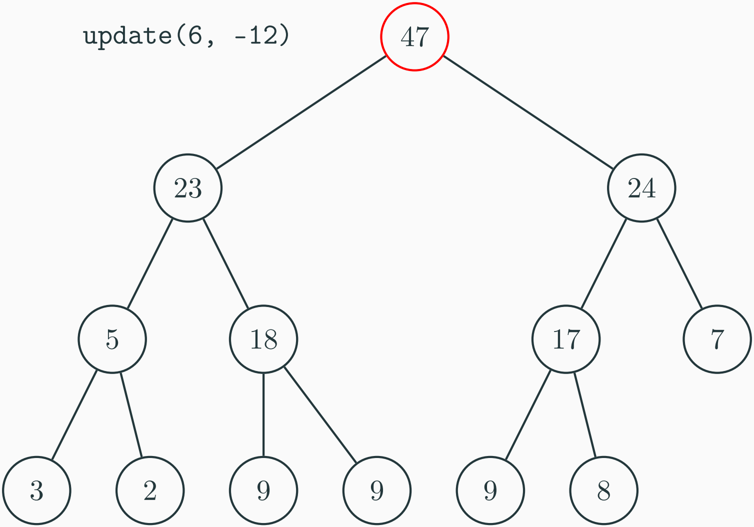

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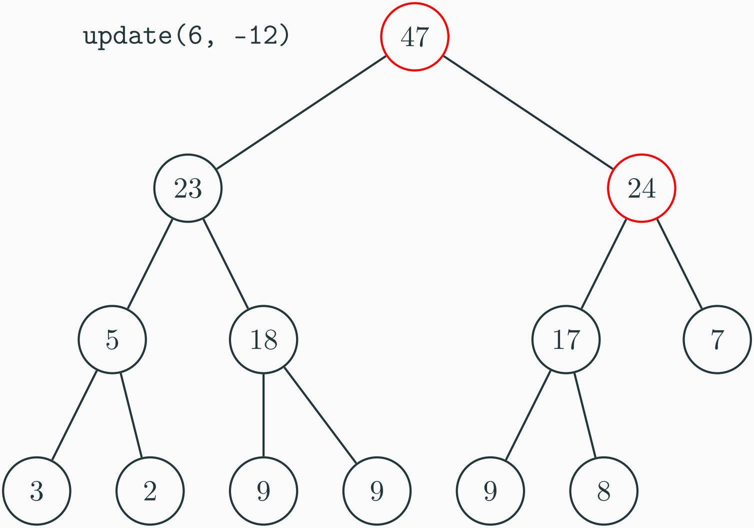

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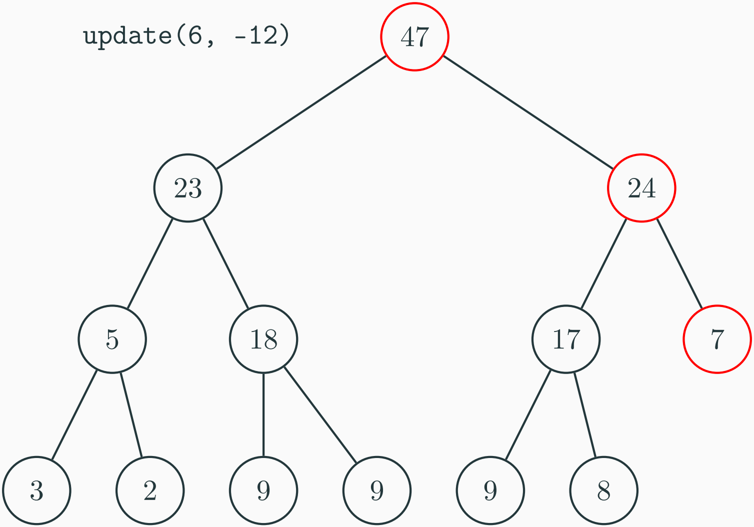

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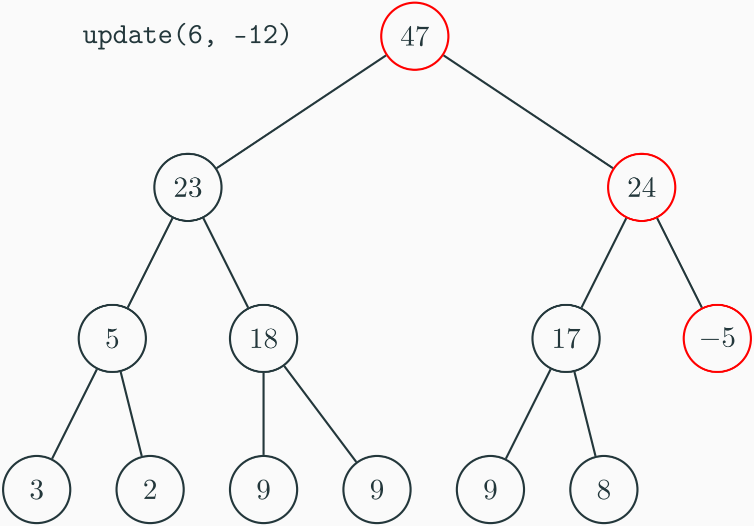

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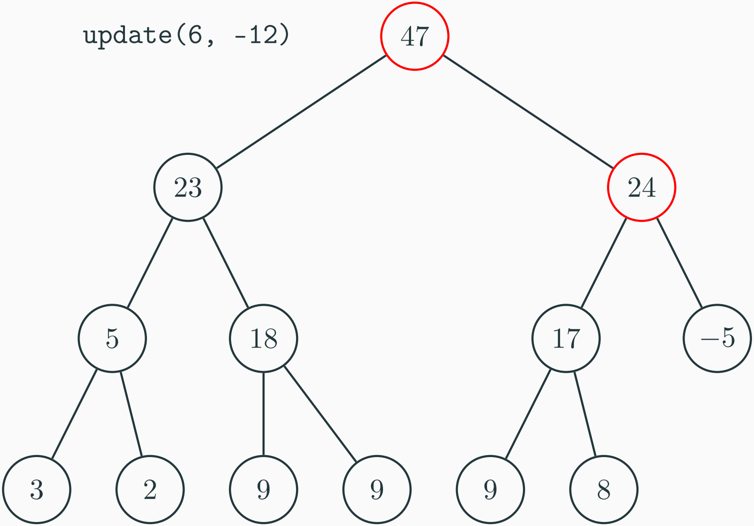

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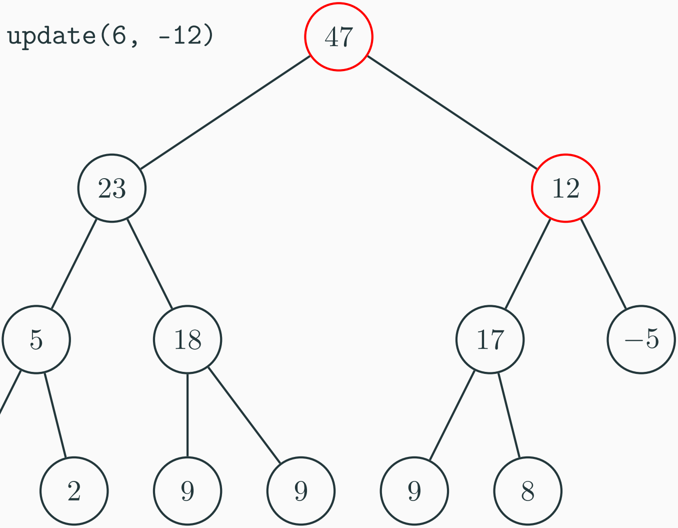

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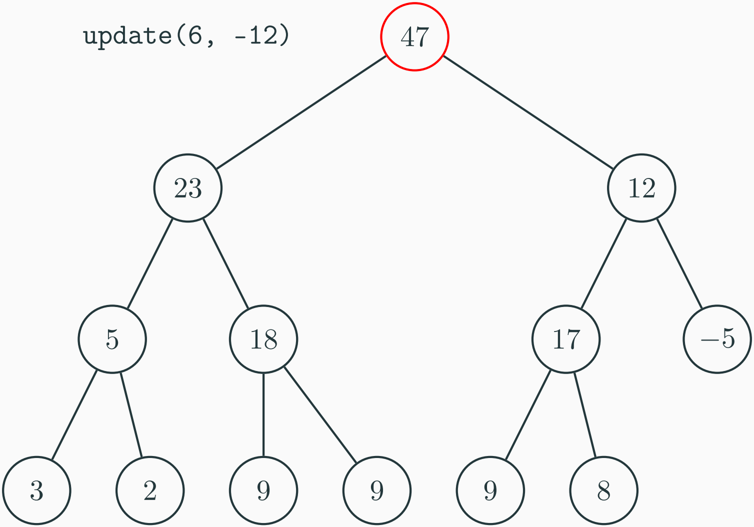

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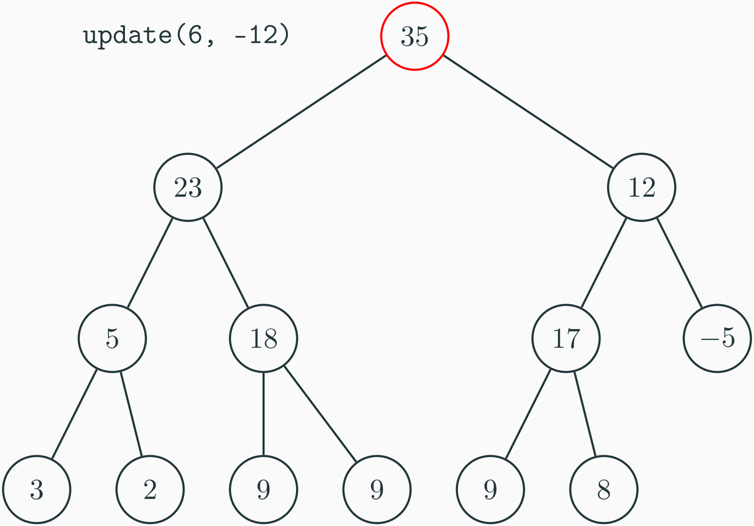

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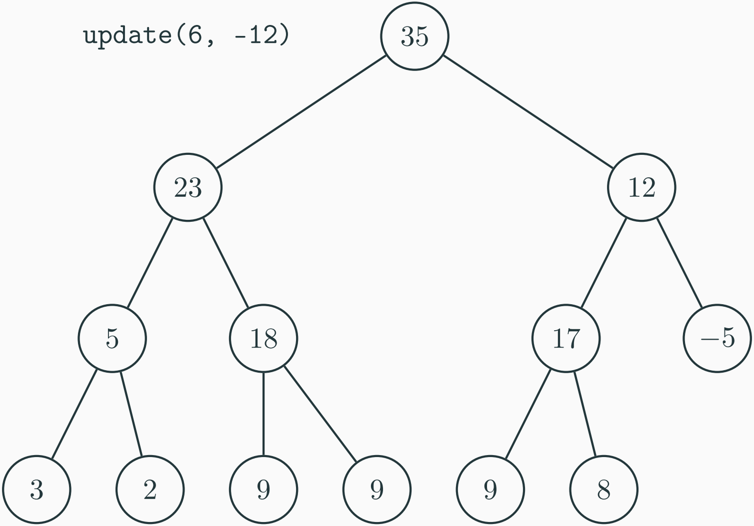

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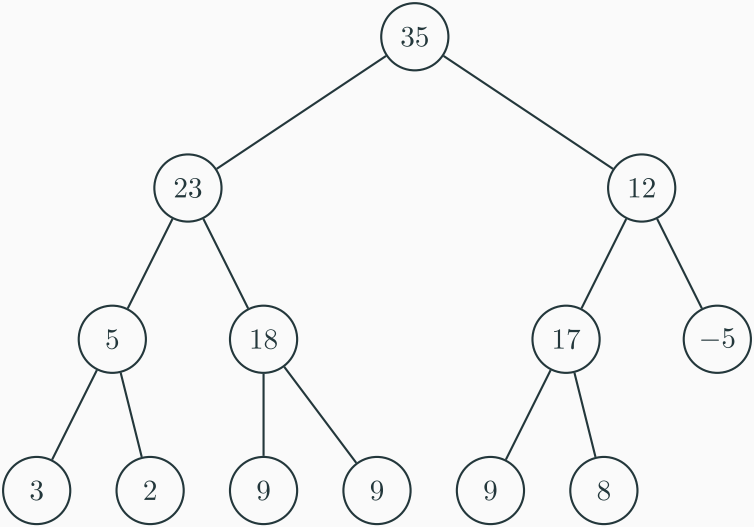
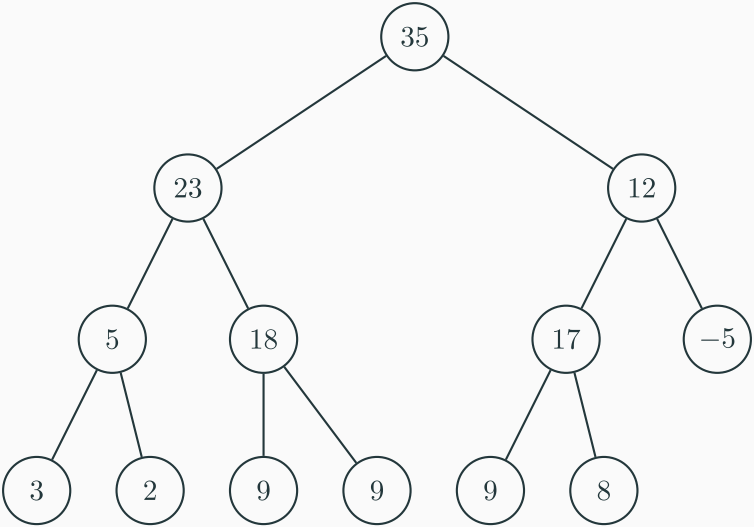


Updating a Segment Tree - Code

```
int update(segment_tree *tree, int i, int val) {
    if (tree == NULL) return 0;
    if (tree->to < i) return tree->value;
    if (i < tree->from) return tree->value;
    if (tree->from == tree->to && tree->from == i) {
        tree->value = val;
    } else {
        tree->value = update(tree->left, i, val) + update(tree->right, i, val);
    }
    return tree->value;
}
```

Querying



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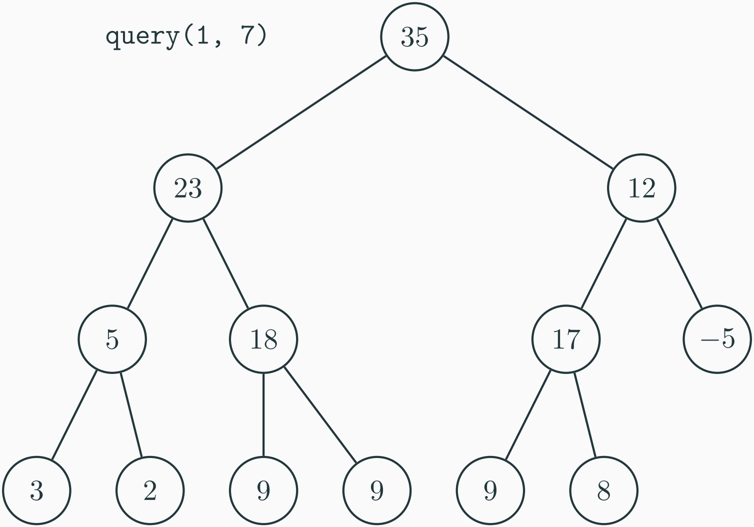

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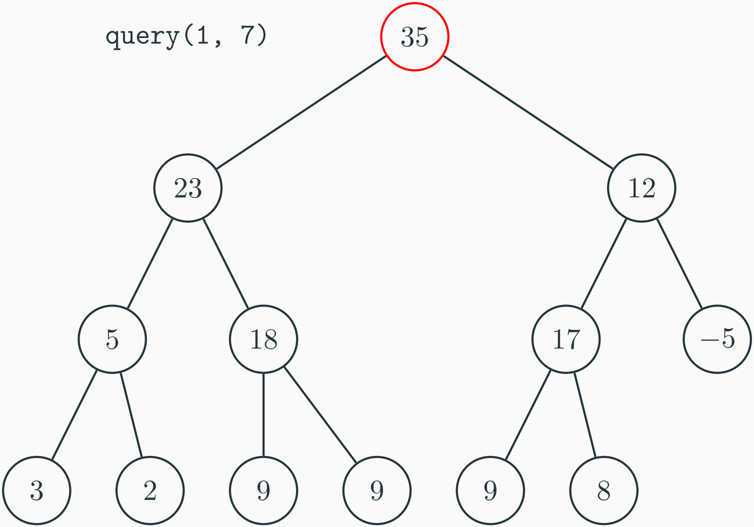
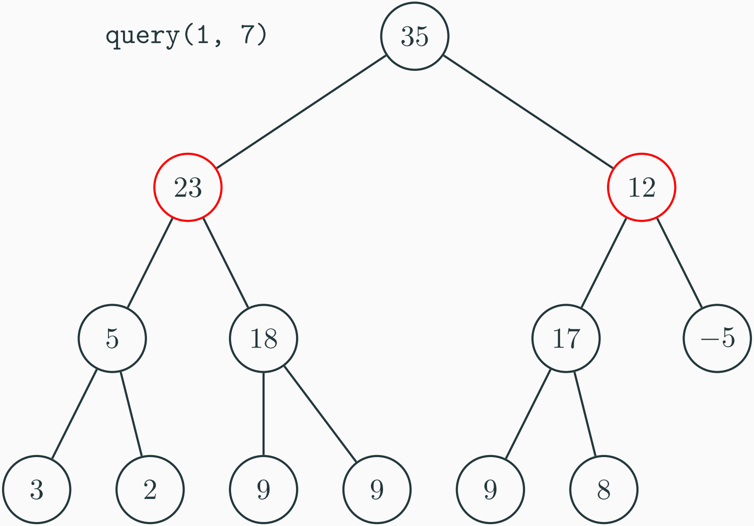


Querying

`query(1, 7)`



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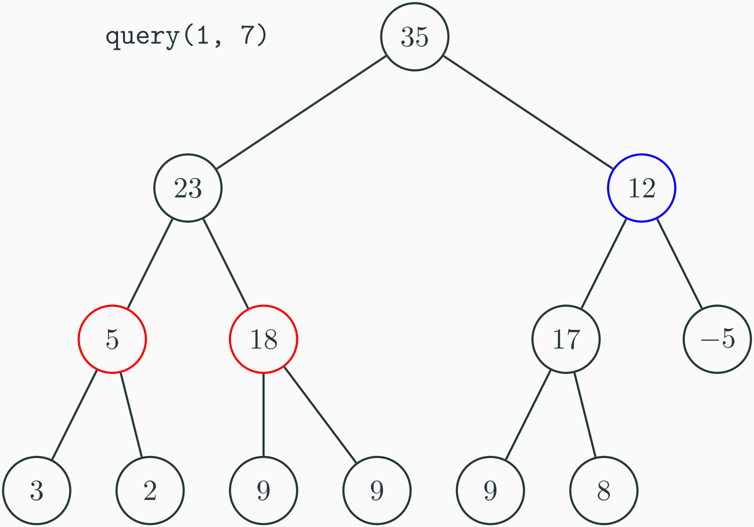

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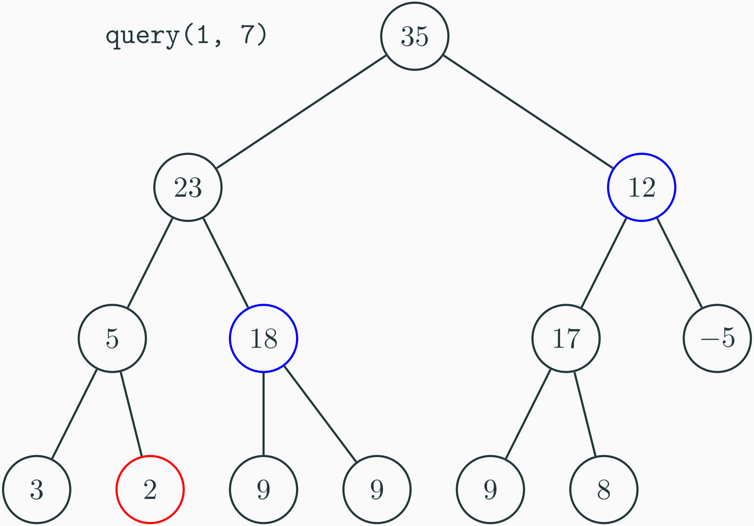

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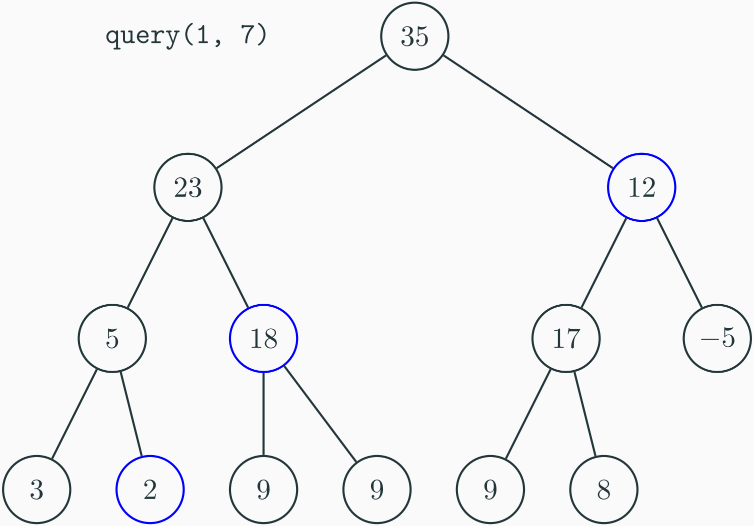
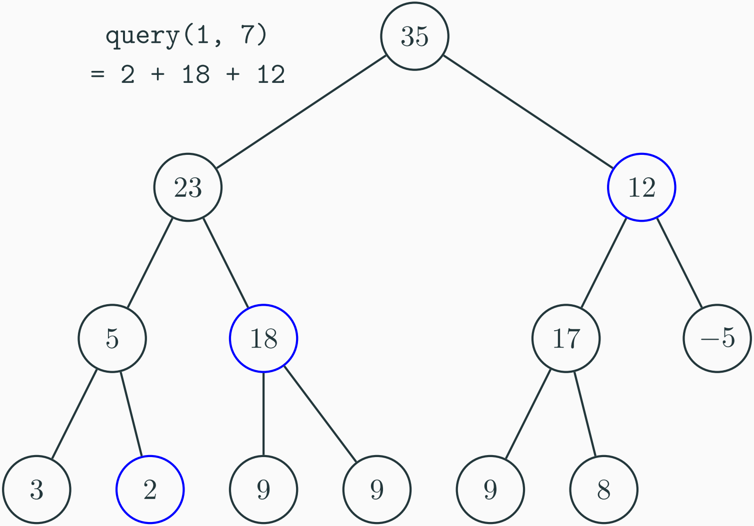


Querying

```
query(1, 7)
= 2 + 18 + 12
```



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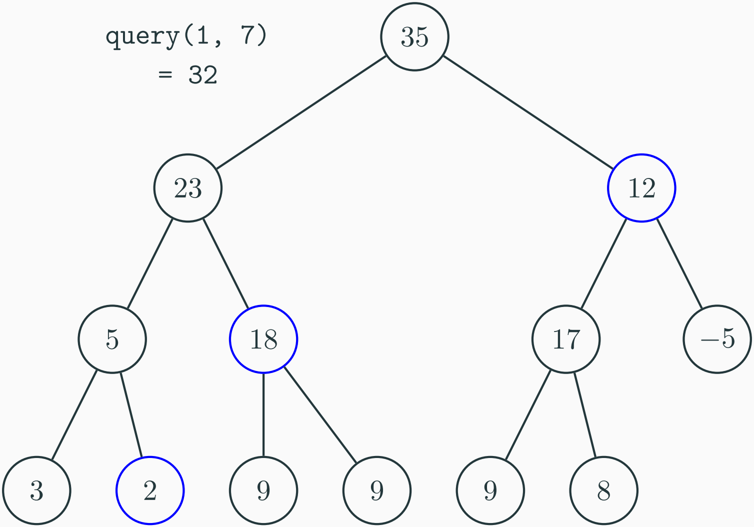

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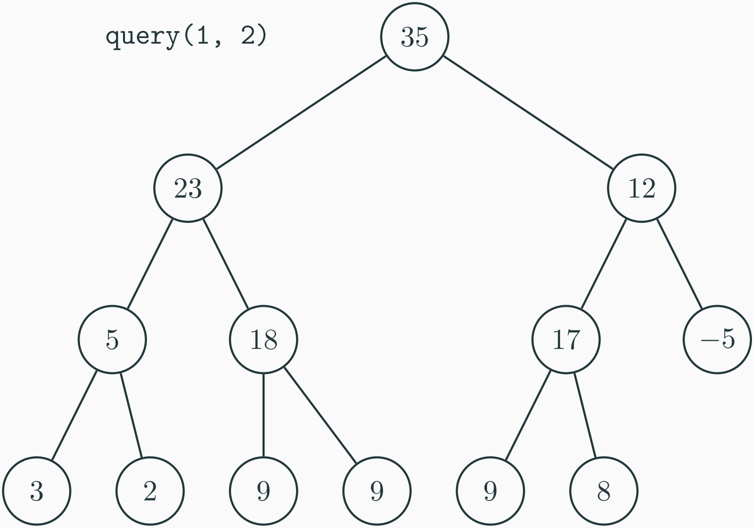

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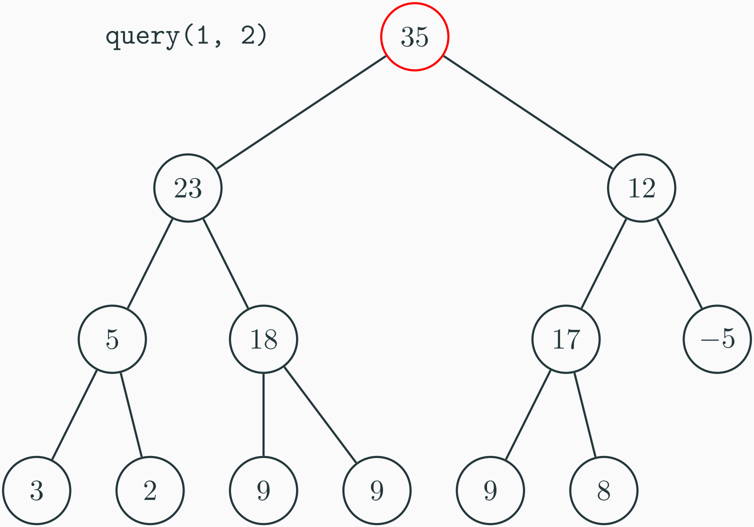
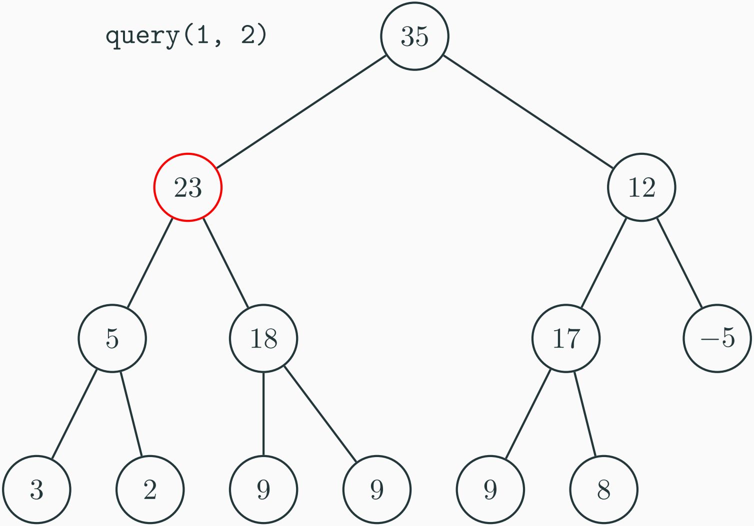


Querying

`query(1, 2)`



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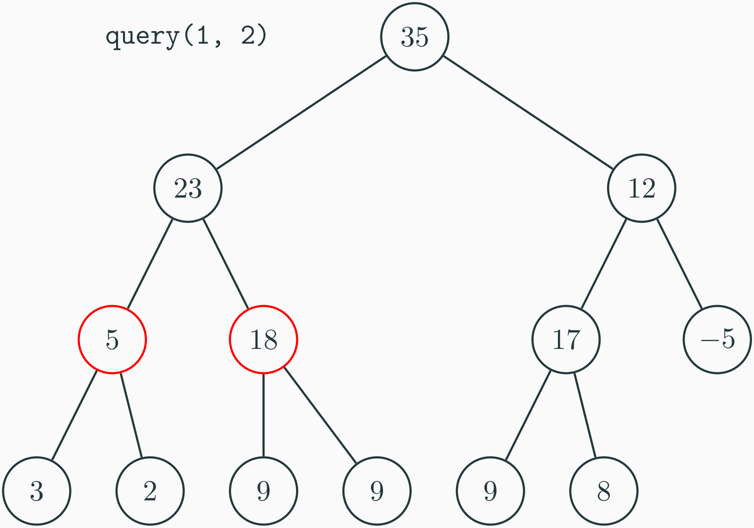

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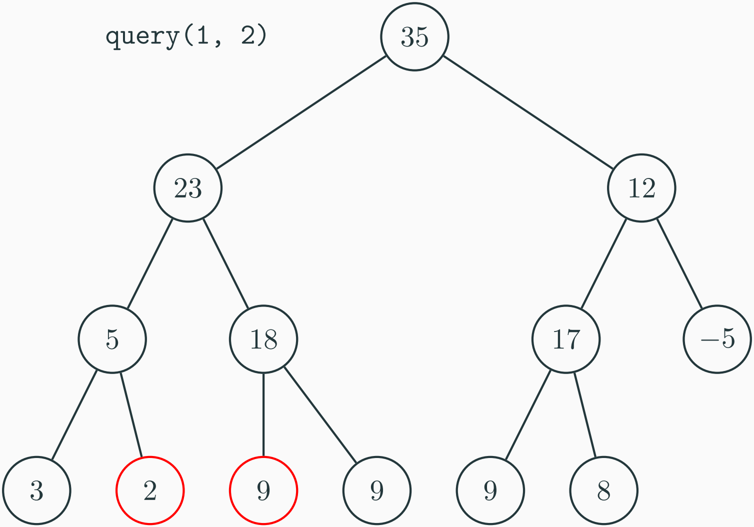

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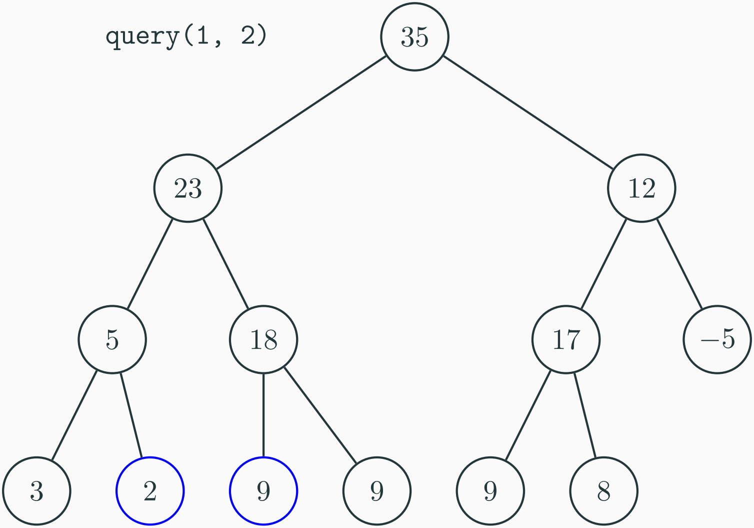
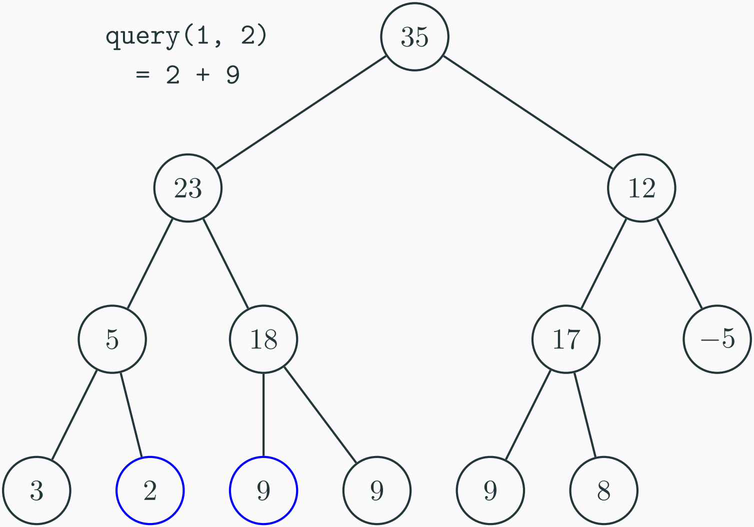


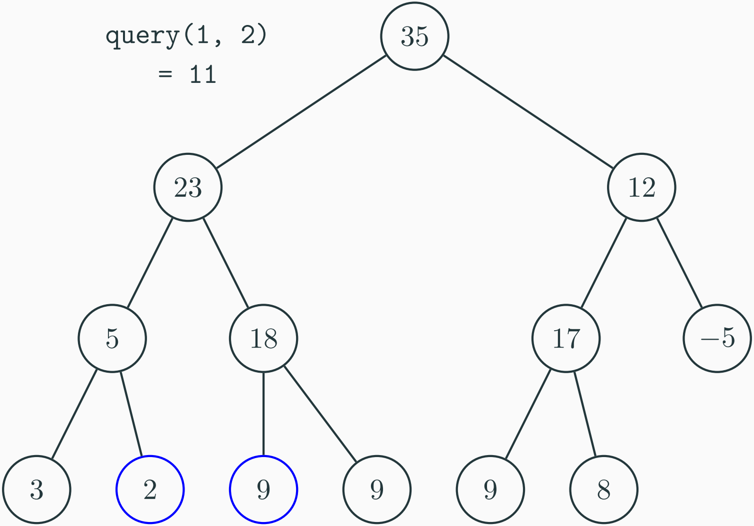
Querying

```
query(1, 2)
  = 2 + 9
```



Querying

`query(1, 2)`
`= 11`



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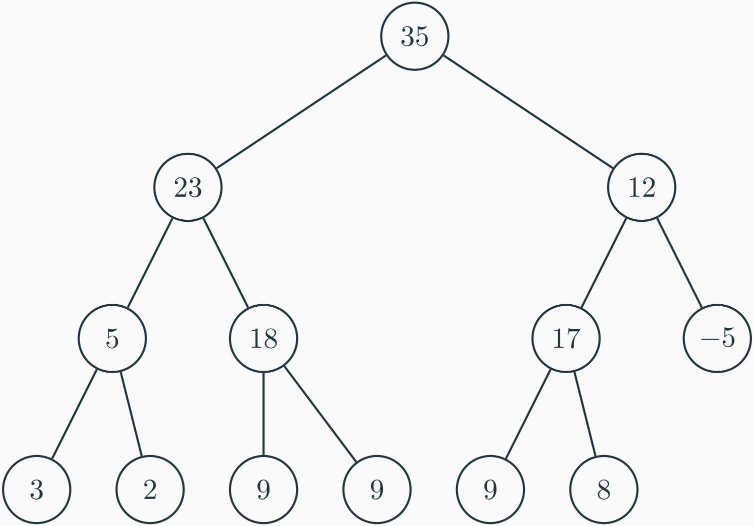

Querying a Segment Tree - Code

```c
int query(segment_tree *tree, int l, int r) {
    if (tree == NULL) return 0;
    if (l <= tree->from && tree->to <= r) return tree->value;
    if (tree->to < l) return 0;
    if (r < tree->from) return 0;
    return query(tree->left, l, r) + query(tree->right, l, r);
}
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- Simple to use Segment Trees for $\min$, $\max$, $\gcd$, and other similar operators, basically the same code.
- Any associative operator will work.
- So any operator f such that $f(a, f(b, c)) = f(f(a, b), c)$ for all a, b, c.

Example problem: Movie Collection

- https://open.kattis.com/problems/moviecollection

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- Lazy people tend to find efficient ways of doing all that **needs** to be done, but no more.
- After updating a value, there is no guarantee you will use the updated value afterwards.
- Idea: Be lazy and procrastinate changes until they are needed!

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- Reset the lazy value.

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- When looking at a node, apply the lazy value to the node
- After applying, push the lazy value to the two child nodes
- Reset the lazy value.
- Traverse to child nodes if needed.

Code example

See implementation example, for example here.