

Sliding Window

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A Sum Problem

Problem description

Write a program that, given an integer array of size N , finds the contiguous subarray of size K with the highest sum.

Input description

Input consist of two lines. The first line contains two space separated integers N , the size of the array, where $1 \leq N \leq 10^6$, and K , the size of the subarrays to consider, where $1 \leq K \leq N$. Then second line contains N space separated integers, the values of the array. Each value in the array is between -10^9 and 10^9 .

Output description

Output one line, the sum of the highest valued contiguous subarray of size K .

A Sum Problem

Sample input	Sample output
10 4 17 20 0 1 5 24 8 2 4 1	39

Straightforward Solution

```
n, k = map(int, input().split())
arr = list(map(int, input().split()))
highest = float('-inf')
for start in range(n-k+1):
    end = start + k
    total = 0
    for i in range(start, end):
        total += arr[i]
    highest = max((highest, total))
print(highest)
```

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- This solution constructs all size K contiguous subarrays.

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- This solution constructs all size K contiguous subarrays.
- What is the time complexity?

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- There are N starting points, each construction takes K steps, so $\mathcal{O}(NK)$.

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- There are N starting points, each construction takes K steps, so $\mathcal{O}(NK)$.
- Too slow!

Wasted Operations

- The subarray starting at index i has the sum
 $a_i + a_{i+1} + \cdots + a_{i+k-1}$.

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- We iterate over the indices $i + 1, i + 2, \dots, i + k - 1$ twice.

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- What changes between starting at i vs. starting at $i + 1$?

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- What changes between starting at i vs. starting at $i + 1$?
- We subtract a_i .

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- We iterate over the indices $i + 1, i + 2, \dots, i + k - 1$ twice.
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- What changes between starting at i vs. starting at $i + 1$?
- We subtract a_i .
- We add a_{i+k} .
- A shift from the subarray starting at i to the subarray starting at $i + 1$ takes $\mathcal{O}(1)$ time.

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- We iterate over the indices $i + 1, i + 2, \dots, i + k - 1$ twice.
- What changes between starting at i vs. starting at $i + 1$?
- We subtract a_i .
- We add a_{i+k} .
- A shift from the subarray starting at i to the subarray starting at $i + 1$ takes $\mathcal{O}(1)$ time.
- This is known as the sliding window technique, in this case with a fixed window size.

Sliding Window Solution

```
n, k = map(int, input().split())
arr = list(map(int, input().split()))
total = 0
for i in range(k):
    total += arr[i]
highest = total
for i in range(n - k):
    total -= arr[i]
    total += arr[i+k]
    highest = max((highest, total))
print(highest)
```

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- What is the time complexity?
- This solution constructs the first size K contiguous subarray.

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- Then, $N - K$ times, an element is removed and another added.

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- What is the time complexity?
- This solution constructs the first size K contiguous subarray.
- Then, $N - K$ times, an element is removed and another added.
- Subtracting and adding numbers is constant time so $\mathcal{O}(N)$.
- Fast enough!

A Substring Problem

Problem description

Write a program that, given a string of size N , finds the longest substring with K distinct elements.

Input description

Input consists of two lines. The first line contains two space-separated integers N , the size of the string, where $1 \leq N \leq 10^6$, and K , the number of distinct elements the substring must have, where $1 \leq K \leq 26$. Then the second line contains a string of length N consisting of English lowercase characters.

Output description

Output one line, the longest substring with K distinct elements. If no such string exists, output "DOES NOT EXIST", without quotations.

A Substring Problem

Sample input	Sample output
14 3 bacdcbcdbcabdb	cdcbcxcb

General Framework

```
from string import ascii_lowercase
n, k = map(int, input().split())
s = input()

best_ind, best_len = distinct_k(n, k, s)

if best_len == -1:
    print("DOES NOT EXIST")
else:
    print(s[best_ind:best_ind + best_len])
```

Straightforward Solution

```
def distinct_k(n, k, s):
    best_ind, best_len = -1, -1
    for start in range(n):
        for end in range(start, n+1):
            substring = s[start:end]
            distinct = 0
            for symbol in ascii_lowercase:
                if symbol in substring:
                    distinct += 1
            cur_len = len(substring)
            if distinct == k and cur_len > best_len:
                best_ind = start
                best_len = cur_len
    return best_ind, best_len
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- There are $\mathcal{O}(N^2)$ substrings of the string

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- What is the time complexity?
- There are $\mathcal{O}(N^2)$ substrings of the string
- Checking each one takes us $\mathcal{O}(N)$ time, so $\mathcal{O}(N^3)$ in total.
- Way too slow!

Constant optimization

```
def distinct_k(n, k, s):
    best_ind, best_len = -1, -1
    for start in range(n):
        for end in range(start, n+1):
            substring = s[start:end]
            present = [False for _ in range(26)]
            for symbol in substring:
                present[ord(symbol) - ord('a')] = True
            distinct = sum(present)
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- This is a little faster, by a factor of 26 approximately.

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- Time complexity is the same.

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- This is a little faster, by a factor of 26 approximately.
- Time complexity is the same.
- Note that present barely differs between adjacent values of end
- Build it as the substring grows.

Incremental

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            present[ord(s[end]) - ord('a')] = True
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            cur_len = end - start + 1
            if distinct == k and cur_len > best_len:
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- Now each substring is processed in constant time.

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- Now each substring is processed in constant time.
- Time complexity is $\mathcal{O}(N^2)$

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- Now each substring is processed in constant time.
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- For a given value of ind, adjacent start values have similar values of present.

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```

- Now each substring is processed in constant time.
- Time complexity is $\mathcal{O}(N^2)$
- For a given value of `ind`, adjacent start values have similar values of `present`.
- Note that adding characters will never decrease `distinct`.
- However, removing elements from the front may reduce `distinct`.

Sliding Window

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def distinct_k(n, k, s):
    best_ind, best_len = -1, -1
    start, end, distinct = 0, 0, 0
    count = [0 for _ in range(26)]
    while start < n:
        while end < n:
            c = ord(s[end]) - ord('a')
            if distinct == k and count[c] == 0:
                break
            count[c] += 1
            end += 1
            distinct = sum(x > 0 for x in count)
        cur_len = end - start
        if distinct == k and cur_len > best_len:
            best_ind = start
            best_len = cur_len
        count[ord(s[start]) - ord('a')] -= 1
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- What is the time complexity? It may seem quadratic at first

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- What is the time complexity? It may seem quadratic at first
- Each element gets added and removed once, so $\mathcal{O}(N)$.
- Lets introduce C , the number of different symbols possible.
- The time complexity is actually $\mathcal{O}(NC)$, but we can do better!

Sliding Window Improved

```
def distinct_k(n, k, s):
    best_ind, best_len = -1, -1
    start, end, distinct = 0, 0, 0
    count = [0 for _ in range(26)]
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            c = ord(s[end]) - ord('a')
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        cur_len = end - start
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        c = ord(s[start]) - ord('a')
        count[c] -= 1
        if count[c] == 0:
            distinct -= 1
        start += 1
        distinct = sum(x > 0 for x in count)
    return best_ind, best_len
```

- Now adding/removing an element is $\mathcal{O}(1)$.
- The time complexity is now $\mathcal{O}(N + C)$.

- This method is applicable when working with substrings or subarrays.
- The data has to be contiguous, or in other words, no gaps between selected elements.
- Usually you want the maximal or the minimal window fulfilling a certain condition.

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- Define an operation `add` which adds a_j to your subarray, finally increasing j by 1.
- Define an operation `remove` which removes a_i from your subarray, finally increasing i by 1.

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- Step 1: If performing `add` does not break your condition, perform it and repeat step 1. Otherwise go to step 2.
- Step 2: Check if the current window is a better answer and possibly update. Then go to step 3.
- Step 3: Perform `remove` and go to step 1.

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- Step 2: Check if the current window is a better answer and possibly update. Then go to step 3.
- Step 3: Perform `remove` and go to step 1.
- Time complexity is $\mathcal{O}(N \cdot (X + Y))$ where X and Y are the cost of `add` and `remove`, respectively.