Complexity and standard libraries

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Overview for today

- Comments on inputs
- Basic data types
- Complexities
- Standard libraries
- Why we need data structures
- Standard library data structures

Inputs

- As mentioned in the last lecture, when doing I/O **only print what is asked of in the output description**.
- Do not use `input("Please enter a number:")` or something equivalent, this prints to stdout.

Language	Stdin	Stdout
C	`scanf`	`printf`
C++	`cin`	`cout`
Python	`input()`	`print()`
Java	`Scanner(System.in)`	`System.out.print`

Speeding up I/O

- If fast I/O is needed, `sys.stdin` and `sys.stdout` are slightly faster in python.
- Similarly for fast I/O in Java, look up Kattio, it is much *much* faster. It can be found on github.
- To speed up C++ I/O somewhat one can use `ios_base::sync_with_stdio(false)`. Note that if this is done `scanf` or `printf` no longer sync up with `cin` and `cout`, so you have to pick one set and stick to it. Similarly `cin.tie(nullptr)` can speed things up as cout will no longer flush before cin is used, but this can cause problems when this behaviour is desired.

Basic data types

Basic data types

- Most languages contain some version of the following:
  - `bool`: a boolean (`true`/`false`)
  - `char`/`int8_t`: an 8-bit signed integer (often used to represent characters with ASCII)
  - `int32_t, int64_t, ...`: Signed fixed size integers
  - `uint32_t, uint64_t, ...`: Unsigned fixed size integers
  - `float`: an IEEE 32-bit floating-point number
  - `double`: an IEEE 64-bit floating-point number
  - `string`: a string of characters

Basic data types

Type	Bytes	Min value	Max value
bool	1		
int8_t	1	-128	127
int16_t	2	-32768	32767
int32_t	4	-2148364748	2147483647
int64_t	8	-9223372036854775808	9223372036854775807
	n	-2^{8n-1}	$2^{8n-1}-1$

Type	Bytes	Min value	Max value
uint8_t	1	0	255
uint16_t	2	0	65535
uint32_t	4	0	4294967295
uint64_t	8	0	18446744073709551615
	n	0	$2^{8n}-1$

Type	Bytes	Min value	Max value	Precision
float	4	$\approx -3.4\times 10^{38}$	$\approx 3.4\times 10^{38}$	≈ 7 digits
double	8	$\approx -1.7\times 10^{308}$	$\approx 1.7\times 10^{308}$	≈ 14 digits

Big integers

- What if we need to represent and do computations with very large integers, i.e. something that doesn't fit in a `__int128`
- Simple idea: Store the integer as a string
- But how do we perform arithmetic on a pair of strings?
- We can use the same algorithms as we learned in elementary school
  - Addition: Add digit-by-digit, and maintain the carry
  - Subtraction: Similar to addition
  - Multiplication: Long multiplication
  - Division: Long division
  - Modulo: Long division

Example problem: Simple Addition

- https://open.kattis.com/problems/simpleaddition
- As can be seen on the statistics for this problem, the fastest are C/C++. But big integer operations have to be implemented manually in those languages. For these kinds of problems it can be easier to use a language that has built-in support, like Java, Python or Haskell.

Time complexities

What is a time complexity?

- Saying a program runs in $\mathcal{O}(f(n))$ means that for some C, n_0 the program will take at most $Cf(n)$ steps to finish for $n \geq n_0$
- Ignoring constants is necessary, otherwise you could change the time complexity just by making the CPU faster or adding more cores
- Time complexities are very useful for napkin math on whether a solution will pass time constraints
- For example $\mathcal{O}(n^2)$ means that as n increases, the number of steps the program needs grows at most like n^2. So if we double n, the runtime might get multiplied by up to 4

Calculate time complexities

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- Say we want to sort $n \leq 10^6$ integers in 3 seconds.
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Calculate time complexities

- A good rule of thumb is that we have 10^8 operations per second
- Say we want to sort $n \leq 10^6$ integers in 3 seconds.
- Can we use a $\mathcal{O}(n^2)$ bubble sort or do we need to implement the more complex $\mathcal{O}(n\log(n))$ merge sort?
- Bubble sort would take $\sim 10^{12}$ operations or about 10^4 seconds, which is far too slow.
- The merge sort would be around 0.2 seconds, which suffices.

Time complexities cntd.

- Always use the simplest solution that suffices. If n had been 10^3 bubble sort would suffice.
- It can be good to be able to estimate these things quick in your head.
- Rules of thumb can be useful, things like $2^{10} \approx 10^3$.
- Logarithms are usually base 2, so like earlier if $n = 10^6$ for $n\log(n)$ we can estimate it as $10^6 \log_2(2^{20})$ or $2 \cdot 10^7$.

Complexity overview

n	Slowest Accepted Algorithm	Example
≤ 10	$O(n!)$	Enumerating a permutation
≤ 15	$O(2^n \times n^2)$	Traveling salesperson DP
≤ 20	$O(2^n), O(n^5)$	Bitmask DP
≤ 50	$O(n^4)$	Blossom algorithm
$\leq 10^2$	$O(n^3)$	Floyd Warshall algorithm
$\leq 10^3$	$O(n^2)$	Bubble/Selection/Insertion sort
$\leq 10^5$	$O(n \log_2 n)$	Merge sort, building a Segment tree
$\leq 10^6$	$O(n)$	Linear scans like prefix sums
$> 10^8$	$O(\log_2 n), O(1)$	Direct formulas or digit operations

Standard libraries

Language features

- Kattis allows the use of standard libraries, so get acquainted with what your language of choice has to offer.
- Kattis does not have other packages, like algs4 (Java) or boost (C++)
- C++ sorts with `sort(a.begin(), a.end())`, python has `a.sort()` and Java has `Arrays.sort(a)`.
- All three languages support common mathematical operations like square roots and complex numbers.
- C++ can do binary search with `lower_bound` and `upper_bound`, python can `import bisect`.
- There's plenty more! Regex, pseudo-randomness and plenty of data structures.

Language features

- The C++ standard library has many useful features like this
- `reverse` can reverse a vector or array in place
- `rotate` can rotate a vector or array in place
- `count` and `count_if` can count elements, or count elements satisfying a predicate function
- `find` and `find_if` can find an element, or find an element satisfying a predicate function
- And many more!

Data structures

Why do we need data structures?

- Sometimes our data needs to be organized in a way that allows one or more of
  - Efficient querying
  - Efficient inserting
  - Efficient deleting
  - Efficient updating
- Sometimes we need a better way to represent our data
  - How do we represent large integers?
  - How do we represent graphs?
  - How do we represent equivalence relations?
- Data structures help us achieve those things

Data structures you should (hopefully) be familiar with

- Static arrays
- Dynamic arrays
- Linked lists
- Stacks
- Queues
- Priority queues
- Ordered trees
- Hash maps

Data structures you should (hopefully) be familiar with

- Static arrays - T arr[n]
- Dynamic arrays - vector<T>
- Linked lists - list<T>
- Stacks - stack<T>
- Queues - queue<T>
- Priority queues - priority_queue<T>
- Ordered trees - set<T>
- Hash maps - unordered_map<K,V>

Data structures in standard libraries

- Usually it's best to use the standard library implementations
  - Almost surely bug-free and fast
  - We don't need to write any code
- Sometimes we need our own implementation
  - When we want more flexibility
  - When we want to customize the data structure
- And sometimes we need data structures not in the standard library, but that waits until later in the course

Applications of Arrays and Vectors

- Too many to list
- Most problems require storing data, usually in an array
- Vectors are similar but are dynamically allocated

Operation	Array complexity	Vector complexity
Access element	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Append element	$\mathcal{O}(n)$	$\mathcal{O}(1)$
Delete/insert element	$\mathcal{O}(n)$	$\mathcal{O}(n)$
Combine arrays/vectors	$\mathcal{O}(n)$	$\mathcal{O}(n)$

Applications of Linked lists

- Very rarely used
- Mostly used when concatenation of arrays or insertion outside of endpoints needs to be fast

Operation	Complexity
Access element	$\mathcal{O}(n)$
Append/prepend element	$\mathcal{O}(1)$
Delete/insert at pointer	$\mathcal{O}(1)$
Combine lists	$\mathcal{O}(1)$

Applications of Stacks

- Processing events in a last-in first-out order
- Simulating recursion
- Depth-first search in a graph
- Matching brackets
- And a lot more

Operation	Complexity
Push element	$\mathcal{O}(1)$
Access most recent element	$\mathcal{O}(1)$
Pop most recent element	$\mathcal{O}(1)$

Applications of Queues

- Processing events in a first-in first-out order
- Breadth-first search in a graph
- And a lot more

Operation	Complexity
Push element	$\mathcal{O}(1)$
Access oldest element	$\mathcal{O}(1)$
Pop oldest element	$\mathcal{O}(1)$

Applications of Heaps/Priority Queues

- Processing events in order of priority
- Finding a shortest path in a graph
- Some greedy algorithms
- And a lot more

Operation	Complexity
Push element	$\mathcal{O}(\log(n))$
Access highest priority element	$\mathcal{O}(1)$
Pop highest priority element	$\mathcal{O}(\log(n))$

Applications of Sets

- Keep track of distinct items
- If implemented as a binary search tree:
  - Find next greater element
  - Count how many elements are less than a given element
  - Find the kth largest element
- And a lot more

Operation	Complexity
Access/check for element	$\mathcal{O}(\log(n))$
Insert/delete element	$\mathcal{O}(\log(n))$

Applications of Hash maps

- Associating a value with a key
- As a frequency table
- And a lot more

Operation	Complexity
Access/check for element	$\mathcal{O}(1)$ average, $\mathcal{O}(n)$ worst case
Insert/delete element	$\mathcal{O}(1)$ average, $\mathcal{O}(n)$ worst case

Using data structures

How to use data structures

- In many cases you don't need fancy data structure, you just have to use simple ones in a smart way.
- Let us consider next the problem of NGE (Next Greater Element).
- We have an array of numbers. For each of the numbers we want to know where the next element to the right that is greater than it is located.
- Example: `[6, 2, 4, 7, 1] -> [3, 2, 3, NULL, NULL]`. Note that this is zero-indexed.

Naïve NGE

- We could always just walk to the right for every element.

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Naïve NGE

- We could always just walk to the right for every element.
- But what happens for a decreasing list then?
- The time complexity is $\mathcal{O}(n^2)$! Far too slow.

Smart NGE

- Let us consider a better algorithm. Let us start with a stack s.
- We then walk through the list from left to right and add values to the stack as we go.
- If what we want to put on the stack is bigger than the top element, the top element must have our current element as its NGE.
- Thus we set that in our output and pop the top element, repeating as necessary, before putting our new element on top.
- At the end our output vector will contain the answer.
- This will then give us a $\mathcal{O}(n)$ NGE solution. Let's see an example.

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
|
 0 1 2 3 4 5 6 7
[x x x x x x x x]
h: []
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
 |
 0 1 2 3 4 5 6 7
[x x x x x x x x]
h: []
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
 |
 0 1 2 3 4 5 6 7
[x x x x x x x x]
h: [0]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
   |
 0 1 2 3 4 5 6 7
[x x x x x x x x]
h: [0]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
 ^ |
 0 1 2 3 4 5 6 7
[x x x x x x x x]
h: [0]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
 ^ |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [0]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
   |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: []
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
   |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [1]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
     |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [1]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
   ^ |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [1]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
     |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [1]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
     |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [1 2]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
       |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [1 2]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
     ^ |
 0 1 2 3 4 5 6 7
[1 x x x x x x x]
h: [1 2]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
     ^ |
 0 1 2 3 4 5 6 7
[1 x 3 x x x x x]
h: [1 2]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
   ^   |
 0 1 2 3 4 5 6 7
[1 x 3 x x x x x]
h: [1]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
   ^   |
 0 1 2 3 4 5 6 7
[1 3 3 x x x x x]
h: [1]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
       |
 0 1 2 3 4 5 6 7
[1 3 3 x x x x x]
h: []
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
       |
 0 1 2 3 4 5 6 7
[1 3 3 x x x x x]
h: [3]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
        |
 0 1 2 3 4 5 6 7
[1 3 3 x x x x x]
h: [3]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
       ^ |
 0 1 2 3 4 5 6 7
[1 3 3 x x x x x]
h: [3]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
       ^ |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: [3]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
         |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: []
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
         |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: [4]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
           |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: [4]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
         ^ |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: [4]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
           |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: [4]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
          |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: [4 5]
```

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 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
             |
 0 1 2 3 4 5 6 7
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 0 1 2 3 4 5 6 7
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[2 3 1 5 7 6 4 8]
             |
 0 1 2 3 4 5 6 7
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[1 3 3 4 x x x x]
h: [4 5 6]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
             ^ |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x x x]
h: [4 5 6]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
             ^ |
 0 1 2 3 4 5 6 7
[1 3 3 4 x x 7 x]
h: [4 5 6]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
          ^   |
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h: [4 5]
```

```
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h: [4 5]
```

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 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
         ^     |
 0 1 2 3 4 5 6 7
[1 3 3 4 x 7 7 x]
h: [4]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
         ^     |
 0 1 2 3 4 5 6 7
[1 3 3 4 7 7 7 x]
h: [4]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
               |
 0 1 2 3 4 5 6 7
[1 3 3 4 7 7 7 x]
h: []
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
               |
 0 1 2 3 4 5 6 7
[1 3 3 4 7 7 7 x]
h: [8]
```

```
 0 1 2 3 4 5 6 7
[2 3 1 5 7 6 4 8]
 0 1 2 3 4 5 6 7
[1 3 3 4 7 7 7 x]
h: [8]
```

C++ NGE

```
template<typename T>
vector<size_t> nge(vector<T>& v) {
  vector<size_t> res(v.size(), -1);
  stack<pair<size_t,T>> s;
  s.push(make_pair(0,v[0]));
  for(size_t i = 0; i < v.size(); ++i) {
    while(!s.empty() && s.top().second <= v[i]) {
      res[s.top().first] = i;
      s.pop();
    }
    s.push(make_pair(i, v[i]));
  }
  return res;
}
```

Augmenting Data Structures

- Sometimes we can store extra information in our data structures to gain more functionality
- Usually we can't do this to data structures in the standard library (but there are exceptions, gnu_pbds)
- Need our own implementation that we can customize
- Sometimes this functionality is simply better time complexity

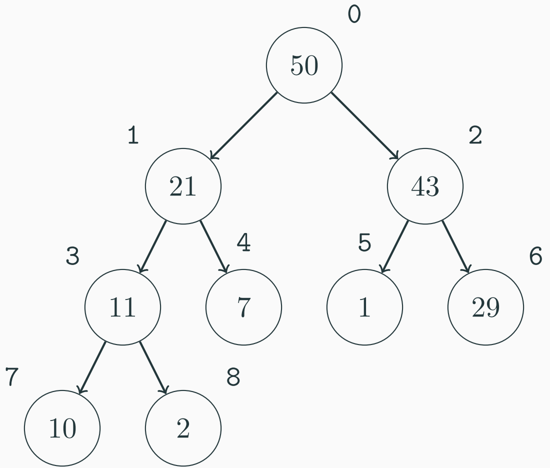
Heaps

- As an example of a data structure in the standard library but that sometimes requires a more powerful version, let us consider heaps.
- Heaps are implemented in most standard libraries in the forms of priority queues.
- A heap is nothing but a binary tree satisfying *the heap condition*.
- The heap condition (for a min heap) says that the value of any given node is not greater than that of its chilren.

Heaps

- Since arrays are linear, we want to smush this binary tree into an array for the implementation.
- We can do this by putting the root at index 1. Then the children of item at index i are simply at $2i$ and $2i + 1$. The parent of any item $i > 1$ is then $\lfloor i/2 \rfloor$.
- We could do this using raw arrays (then index 0 can be used to store its size), but the examples will be given in C++ using vectors.

Heaps




```
ARRAY: [SIZE, 50, 21, 43, 11, 7, 1, 29, 10, 2]
```

Heaps

- Items can be inserted by pushing them to the back and fixing the heap condition upwards from them.
- Items can be deleted by replacing the smallest value with a leaf and then fixing the heap condition downwards.
- Let us see how this would look in C++.

C++ implementation

```
template<typename T> struct Heap {
    vector<T> h; Heap() : h(1) { }
    size_t size() { return h.size() - 1; }
    T peek() { return h[1]; }
    void swim(size_t i) {
        while(i != 1 && h[i] < h[i / 2]) {
            swap(h[i], h[i / 2]);
            i /= 2; } }
    void sink(size_t i) {
        while(true) {
            size_t mn = i;
            if(2 * i + 1 < h.size() && h[mn] > h[2 * i + 1]) mn = 2 * i + 1;
            if(2 * i  < h.size() && h[mn] > h[2 * i]) mn = 2 * i;
            if(mn != i) swap(h[i], h[mn]), i = mn;
            else break; } }
    void pop() {
        h[1] = h.back();
        h.pop_back(); sink(1); }
    void push(T x) {
        h.push_back(x);
        swim(h.size() - 1); } };
```

Heaps

- We note that peek and size run in $\mathcal{O}(1)$ while all other operations run in $\mathcal{O}(log(n))$.
- This can be used to, for example, solve the Bastard's Peace problem listed earlier.
- This implementation isn't any better than the standard library one in C++.
- But let us consider a harder problem where the standard heap (and this one) aren't good enough.
- This particular implementation won't be needed for problems in the course, it is merely an example.

Hard heap problem (Trade Routes on Kattis)

- You are given a tree on $n \leq 300\,000$ vertices.
- Each of the nodes want to trade with Rome, located at node 1. A trade route from that node puts strain on all nodes on the way from that node to Rome.
- Each node has a trade value which is how much would be gained from it trading with Rome.
- Each node has a capacity which is the maximum strain it can tolerate from trade routes.
- How much trade value can be gained at most?

How to use heaps

- Imagine each node has a heap.
- At the start each heap just contains the trade value at that node.
- Then we move from the leaves inwards, merging together the heaps from children to parents as we go.
- This may make the heap larger than the capacity at some point, so we pop values from it until this is fixed.
- This would make the final heap at Rome contain the trade routes we want.

How to use heaps (?)

- There is just one problem.
- The STL (standard library) implementation has a merging operation that runs in $\mathcal{O}(n)$, so this algorithm would be $\mathcal{O}(n^2)$ which is far too slow (~ 15 minute runtime).
- Can we make our own heap that merges in $\mathcal{O}(\log(n))$ or better?

How to use heaps (?)

- There is just one problem.
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- Can we make our own heap that merges in $\mathcal{O}(\log(n))$ or better?
- The reason the problem is so hard is that this implementation is rather involved. I'll put it here for completeness's sake, but we will not delve much deeper here.

Pairing heap C++

```cpp
struct HeapNode {
    int32_t k, v; HeapNode *lch, *sib;
    HeapNode() : lch(NULL), sib(NULL) { }
    HeapNode(int32_t _k, int32_t _v, HeapNode *_lch,
        HeapNode *_sib) :
        k(_k), v(_v), lch(_lch), sib(_sib) { }
    void add_child(HeapNode *node) {
        if(lch == NULL) lch = node;
        else {
            node->sib = lch;
            lch = node;
        }
    }
};
struct PairingHeap {
    HeapNode* root; size_t sz;
    PairingHeap() : root(NULL), sz(0) { }
    HeapNode* merge(HeapNode* A, HeapNode* B) {
        if(A == NULL) return B;
        if(B == NULL) return A;
        if(A->k < B->k) {
            A->add_child(B);
            return A;
        }
        B->add_child(A);
        return B;
    }
    HeapNode* twopass(HeapNode *node) {
        if(node == NULL || node->sib == NULL)
            return node;
        HeapNode *A = node, *B = node->sib,
            *nw = node->sib->sib;
        A->sib = NULL; B->sib = NULL;
        return merge(merge(A, B), twopass(nw));
    }
    pair<int32_t,int32_t> top() {
        return make_pair(root->k, root->v);
    }
    void insert(int32_t k, int32_t v) {
        root = merge(root,
            new HeapNode(k, v, NULL, NULL));
        sz++;
    }
    void pop() {
        root = twopass(root->lch);
        sz--;
    }
    void join(PairingHeap oth) {
        root = merge(root, oth.root);
        sz += oth.sz;
    }
};
```

How to use (fancy) heaps

- This heap can also be used wherever you'd use the STL one as well.
- This one can peek, insert and merge in $\mathcal{O}(1)$ and pop in $\mathcal{O}(\log(n))$.
- It has more overhead though, so in practice it will be a fair bit slower than the STL one for non-merge operations.