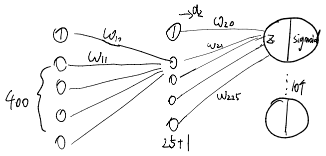
神经网络反向传播

$$J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[-y_k^{(i)}\log((h_\theta(x^{(i)}))_k)-(1-y_k^{(i)})\log(1-(h_\theta(x^{(i)}))_k)\right]+\frac{\lambda}{2m}\left[\sum_{j=1}^{25}\sum_{k=1}^{400}(\Theta_{j,k}^{(1)})^2+\sum_{j=1}^{10}\sum_{k=1}^{25}(\Theta_{j,k}^{(2)})^2\right].$$

反向传播的过程（以20x20像素数字识别为例）



$$J = -y\ln(h(x)) - (1-y)\ln(1-h(x))$$

以 w_{21} 的偏导为例

$$\frac{\partial J}{\partial w_{21}} = \frac{-y}{h(x)}\frac{\partial h(x)}{\partial w_{21}} + \frac{(1-y)}{1-h(x)}\frac{\partial h(x)}{\partial w_{21}}$$

$$h(z) = sigmoid(z) = \frac{1}{1+e^{-z}} \qquad h'(z) = h(z)(1-h(z))$$

$$\frac{\partial J}{\partial w_{21}} = \frac{-y}{h(x)}h(x)(1-h(x))\frac{\partial x}{\partial w_{21}} + \frac{(1-y)}{1-h(x)}h(x)(1-h(x))\frac{\partial x}{\partial w_{21}}$$

$$= -y(1-h(x))\frac{\partial x}{\partial w_{21}} + (1-y)h(x)\frac{\partial x}{\partial w_{21}}$$

$$= [-y + yh(x) + h(x) - yh(x)]\frac{\partial x}{\partial w_{21}} = [h(x)-y]\frac{\partial x}{\partial w_{21}}$$

$\frac{\partial x}{\partial w_{21}}$ 其实就是 theta 中的 w_{21} 对应的 x ⇐ 输入

这个就是隐藏层到输出层的梯度

然后基于上述结果继续求导，求出输入层到隐藏层的梯度

例 W_{11}，由于 W_{11} 输入隐藏层后，给输出层的多个输出了，

所以求 $\frac{\partial J}{\partial w_{11}}$ 除了需要 $\frac{\partial J}{\partial z_1}$ 的，还需要其它的 $\frac{\partial J}{\partial z_1}$

这里先以 $\frac{\partial J}{\partial w_{21}}$ 求导为例

也是一个 sigmoid 的函数，同理求导

$$\frac{\partial J}{\partial w_{11}} = [h(x)-y]\frac{\partial a_{21}}{\partial w_{11}} = [h(x)-y]\,theta2\,\underline{h(z)(1-h(z))}\,x$$