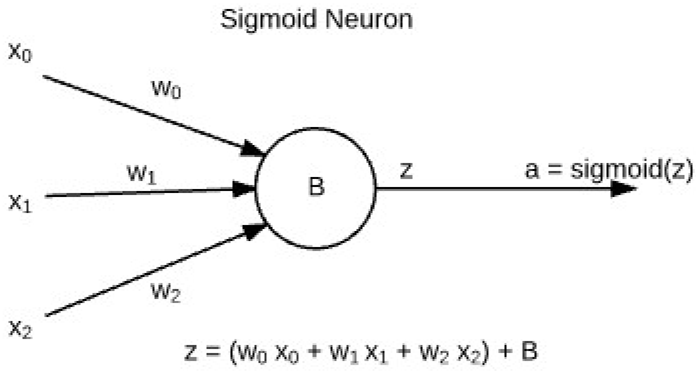
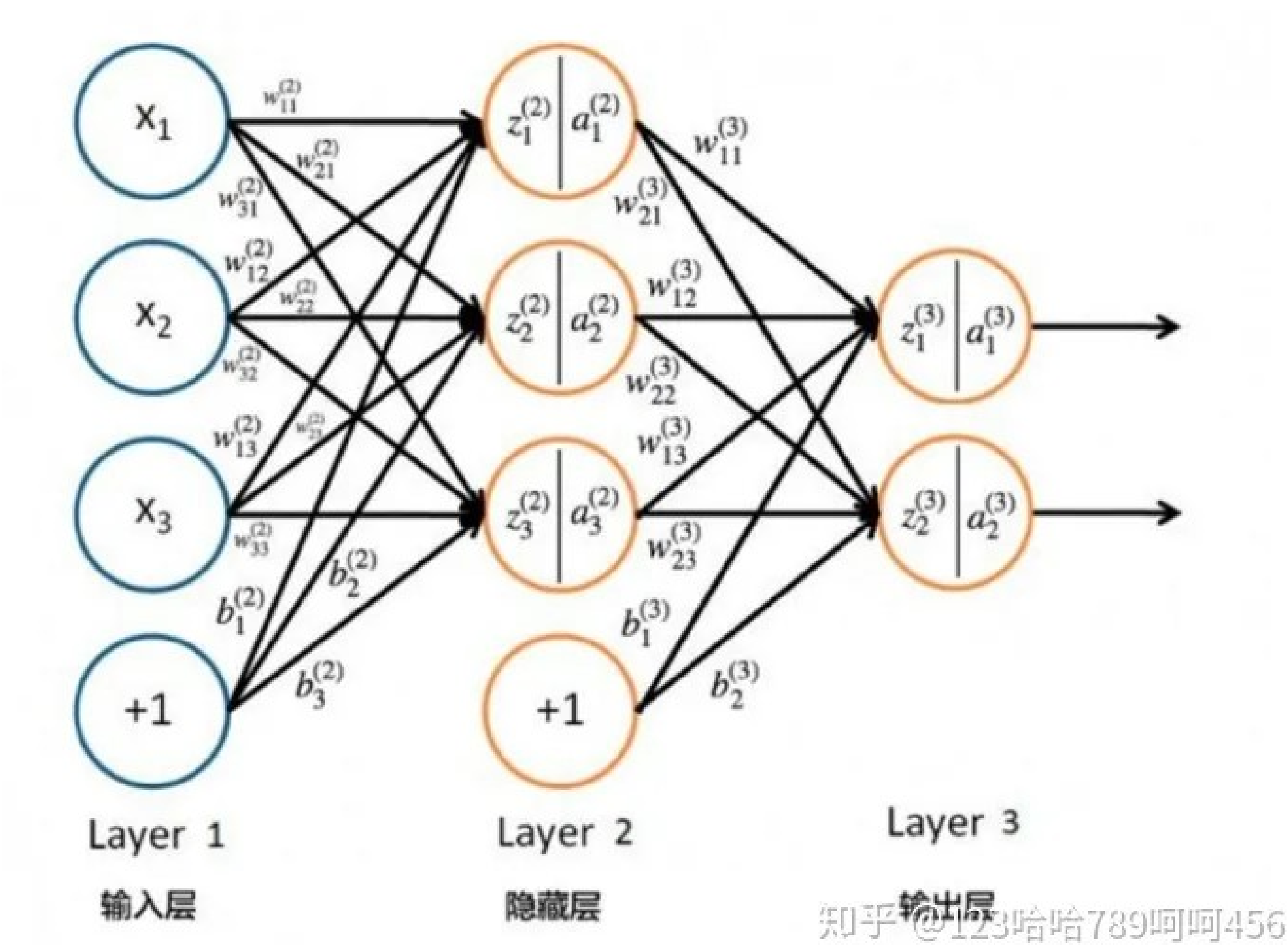
使用numpy实现简单的全连接神经网络 并在mnist数据集上进行测试



1.前向传播算法

将上一层的输出作为下一层的输入，并计算下一层的输出，一直到输出层为止



(图片源自网络) 使用矩阵表示

$z^{(l)} = w^{(l)} a^{(l-1)} + b^{(l)}$

$a^{(l)} = \sigma(z^{(l)})$

2.反向传播算法

记损失函数为 `L = C(W, b)`

我们的目标是求 $\frac{\partial L}{\partial w^{(l)}}$ 和 $\frac{\partial L}{\partial b^{(l)}}$

根据链式法则，可以求出输出层的梯度

$$\frac{\partial L}{\partial w^{(l)}} = \frac{\partial L}{\partial a^{(l)}}\frac{\partial a^{(l)}}{\partial z^{(l)}}\frac{\partial z^{(l)}}{\partial w^{(l)}} = \frac{\partial L}{\partial a^{(l)}}\frac{\partial a^{(l)}}{\partial z^{(l)}}a^{(l-1)}$$

$$\frac{\partial L}{\partial b^{(l)}} = \frac{\partial L}{\partial a^{(l)}}\frac{\partial a^{(l)}}{\partial z^{(l)}}\frac{\partial z^{(l)}}{\partial b^{(l)}} = \frac{\partial L}{\partial a^{(l)}}\frac{\partial a^{(l)}}{\partial z^{(l)}}$$

记$\delta^{(l)} = \frac{\partial L}{\partial a^{(l)}}\frac{\partial a^{(l)}}{\partial z^{(l)}}$

则隐藏层的梯度

$$\delta^{(l)} = \frac{\partial L}{\partial z^{(l+1)}}\frac{\partial z^{(l+1)}}{\partial z^{(l)}} = \delta^{(l+1)}\frac{\partial z^{(l+1)}}{\partial z^{(l)}}$$

又由 $z^{(l+1)} = w^{(l+1)}\sigma(z^{(l)}) + b^{(l+1)}$ 得到 $\frac{\partial z^{(l+1)}}{\partial z^{(l)}} = (w^{(l+1)})^T \odot \sigma'(z^{(l)})$

因此 $\delta^{(l)} = \delta^{(l+1)}(w^{(l+1)})^T \odot \sigma'(z^{(l)})$

由$\delta^{(l)}$的递推关系式，可以很容易的求解$w^{(l)}b^{(l)}$的梯度

$$\frac{\partial L}{\partial w^{(l)}} = \delta^{(l)}a^{(l-1)}$$

$$\frac{\partial L}{\partial b^{(l)}} = \delta^{(l)}$$

代码实现

引入需要的库

In [1]:
```
import numpy as np
import pandas as pd
```

定义损失函数和激活函数

In [2]:
```
def cross_entropy(y, y_hat):
    return -np.sum(y * np.log(y_hat))

def mse(y, y_hat):
    return np.sum((y - y_hat) ** 2) / (y.shape[1] * 2.0)

def sigmoid(x):
    return 1.0 / (1.0 + np.exp(-x))

def sigmoid_gradient(x):
    return sigmoid(x) * (1 - sigmoid(x))

def softmax(x):
    return np.exp(x) / np.sum(np.exp(x))

def relu(x):
    return np.maximum(x, 0)
```

```
def relu_gradient(x):
    return x > 0
```

加载数据集函数 注意维度

In [3]:

```
def load_mnist():
    train_data = pd.read_csv('./dataset/mnist/mnist_train.csv').values
    test_data = pd.read_csv('./dataset/mnist/mnist_test.csv').values

    train_images = train_data[:, 1:]
    train_images = train_images / 255.0
    train_labels_tmp = train_data[:, :1]
    train_images = [i.reshape(-1, 1) for i in train_images]
    train_labels = np.zeros((len(train_labels_tmp), 10))
    for i in range(len(train_labels_tmp)):
        train_labels[i][train_labels_tmp[i]] = 1
    train_labels = np.array([i.reshape(-1, 1) for i in train_labels])

    test_images = test_data[:, 1:]
    test_images = test_images / 255.0
    test_labels_tmp = test_data[:, :1]
    test_images = [i.reshape(-1, 1) for i in test_images]
    test_labels = np.zeros((len(test_labels_tmp), 10))
    for i in range(len(test_labels_tmp)):
        test_labels[i][test_labels_tmp[i]] = 1
    test_labels = np.array([i.reshape(-1, 1) for i in test_labels])

    return train_images, train_labels, test_images, test_labels
```

In [4]:

```
class NeuralNetwork:
    def __init__(self, layers, learning_rate, mini_batch_size, activation, loss):
        self.layers = layers
        self.learning_rate = learning_rate
        self.mini_batch_size = mini_batch_size
        self.num_layers = len(layers)
        self.activation_fn = {}
        self.activation_gradient = {}

        if loss == 'cross_entropy':
            self.loss = cross_entropy
        elif loss == 'mse':
            self.loss = mse

        for i, a in enumerate(activation):
            if a == 'sigmoid':
                self.activation_fn[i + 1] = sigmoid
                self.activation_gradient[i + 1] = sigmoid_gradient
            elif a == 'relu':
                self.activation_fn[i + 1] = relu
                self.activation_gradient[i + 1] = relu_gradient
            elif a == 'softmax':
                self.activation_fn[i + 1] = softmax

        self.weights = [np.array([0])] + [np.random.randn(y, x) / np.sqrt(x) for y, x in
        self.biases = [np.array([0])] + [np.random.randn(y, 1) for y in layers[1:]]

        self._zs = [np.zeros((bias.shape)) for bias in self.biases] # z = w * a + b
        self._activations = [np.zeros((bias.shape)) for bias in self.biases] # a = f(z)

    def forward(self, x):
        self._activations[0] = x
        for i in range(1, self.num_layers):
            self._zs[i] = self.weights[i] @ self._activations[i - 1] + self.biases[i]
```

```
            self._activations[i] = self.activation_fn[i](self._zs[i])

    def backward(self, y):
        nabla_b = [np.zeros((bias.shape)) for bias in self.biases]
        nabla_w = [np.zeros((weight.shape)) for weight in self.weights]

        # softmax + mse
        # delta = self.loss_gradient(y, self._activations[-1]) * self.activation_gradien

        # softmax + cross entropy
        # delta = self._activations[-1] - y

        # TODO: automatically calculate delta of the last layer
        delta = self._activations[-1] - y
        nabla_b[-1] = delta
        nabla_w[-1] = delta @ self._activations[-2].T

        for i in range(self.num_layers - 2, 0, -1):
            delta = self.weights[i + 1].T @ delta * self.activation_gradient[i](self._zs
            nabla_b[i] = delta
            nabla_w[i] = delta @ self._activations[i - 1].T

        return nabla_b, nabla_w

    def train(self, training_data, training_label, validation_data, validation_label, ep
        loss_history = []
        for epoch in range(epochs):
            mini_batches = [
                list(zip(training_data[k: k + self.mini_batch_size], training_label[k: k
            ]

            for mini_batch in mini_batches:
                nabla_b = [np.zeros((bias.shape)) for bias in self.biases]
                nabla_w = [np.zeros((weight.shape)) for weight in self.weights]

                for x, y in mini_batch:
                    self.forward(x)
                    delta_nabla_b, delta_nabla_w = self.backward(y)

                    nabla_b = [nb + dnb for nb, dnb in zip(nabla_b, delta_nabla_b)]
                    nabla_w = [nw + dnw for nw, dnw in zip(nabla_w, delta_nabla_w)]

                self.weights = [w - (self.learning_rate / self.mini_batch_size) * nw for
                self.biases = [b - (self.learning_rate / self.mini_batch_size) * nb for

            pre = self.predict_group(validation_data).reshape(validation_label.shape)
            loss = self.loss(validation_label, pre) / len(validation_data)
            loss_history.append(loss)
            accuracy = [np.argmax(pre) == np.argmax(y) for pre, y in zip(pre, validation
            print(f'Epoch {epoch}: accuracy: {accuracy} loss = {loss}')
        return loss_history

    def evaluate(self, test_data, test_label):
        results = [np.argmax(self.predict(x)) == np.argmax(y) for x, y in zip(test_data,
        return sum(results)

    def predict_group(self, data):
        res = np.array([])
        for x in data:
            res = np.append(res, self.predict(x))
        return res

    def predict(self, x):
        self.forward(x)
        return self._activations[-1]
```

```
    def save(self, path = 'model.npy'):
        np.save(path, [self.weights, self.biases])

    def load(self, path = 'model.npy'):
        self.weights, self.biases = np.load(path, allow_pickle=True)
```

导入数据集

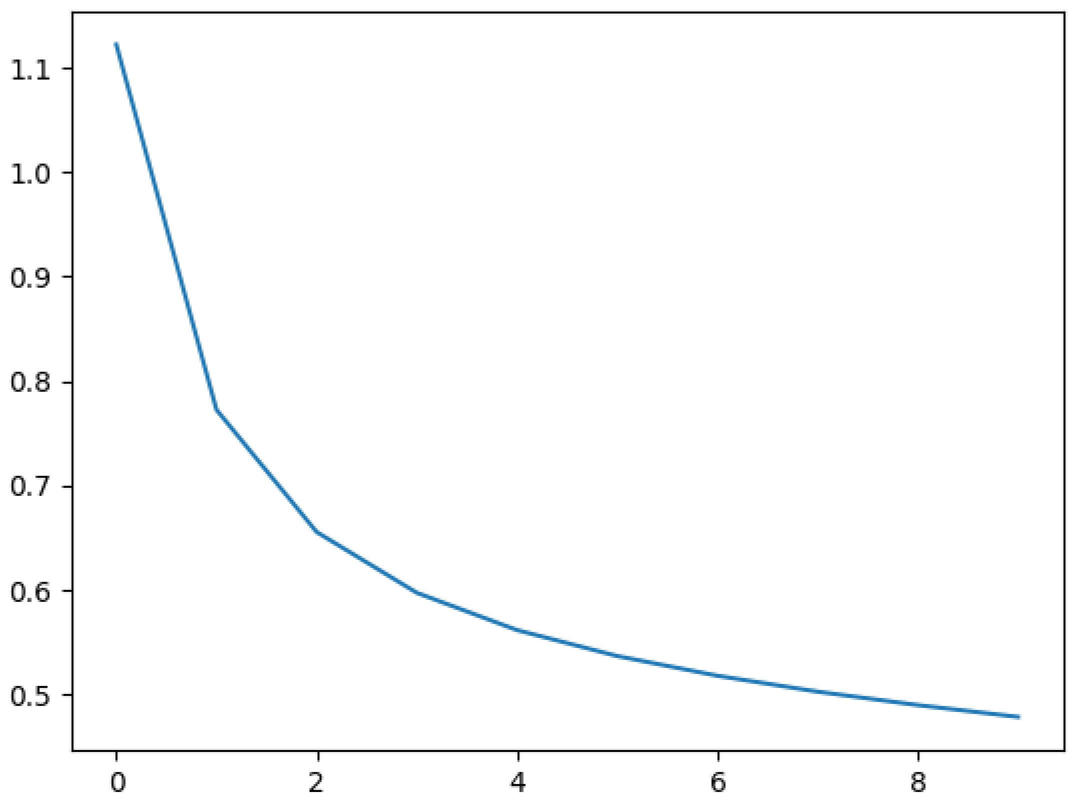
In [5]:
```
X_train, y_train, X_validation, y_validation = load_mnist()
```

In [6]:
```
layers = [784, 100, 10]
lr = 0.01
mini_batch_size = 100

net = NeuralNetwork(layers, lr, mini_batch_size, ['relu', 'softmax'], 'cross_entropy')
loss_history = net.train(X_train, y_train, X_validation, y_validation, 10)
```

```
Epoch 0: accuracy: 0.8534 loss = 1.1225781678620252
Epoch 1: accuracy: 0.8837 loss = 0.7722513923338015
Epoch 2: accuracy: 0.8963 loss = 0.6549254094006839
Epoch 3: accuracy: 0.9034 loss = 0.5965907708394834
Epoch 4: accuracy: 0.9078 loss = 0.5608345779337125
Epoch 5: accuracy: 0.9112 loss = 0.535947006383341
Epoch 6: accuracy: 0.914 loss = 0.5170416327746042
Epoch 7: accuracy: 0.9163 loss = 0.5017633139299483
Epoch 8: accuracy: 0.9187 loss = 0.48888829363873554
Epoch 9: accuracy: 0.9203 loss = 0.47757340656306424
```

In [7]:
```
import matplotlib.pyplot as plt
plt.plot(loss_history)
plt.show()
```

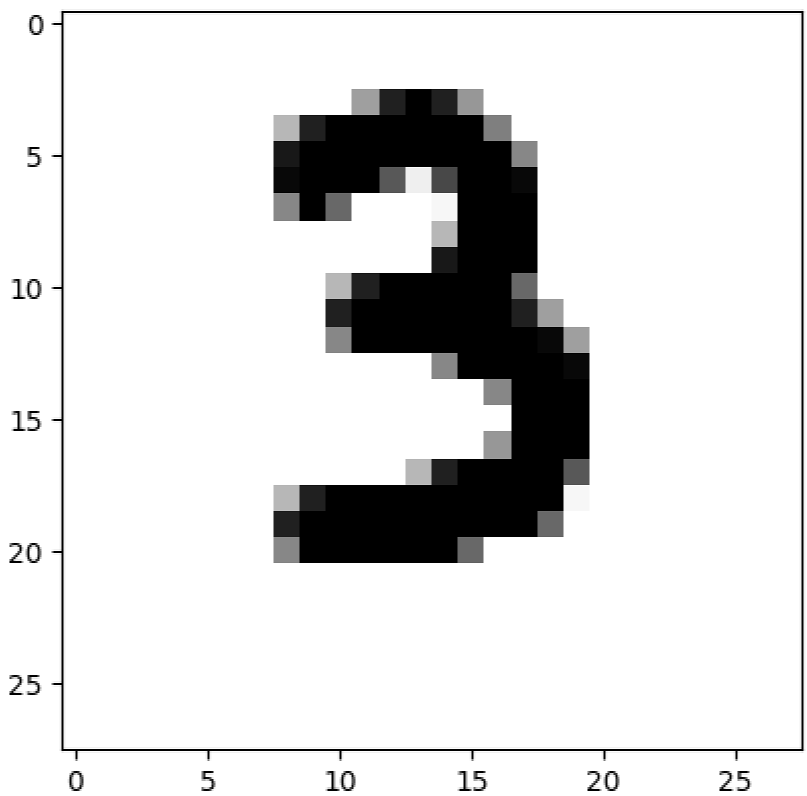


识别一个自己手写的数字

In [57]:
```
from PIL import Image
import matplotlib.pyplot as plt
```

```python
image = Image.open('digit.png')p
plt.imshow(image)
```

Out[57]: <matplotlib.image.AxesImage at 0x1171bd3adc8>



In [61]:
```python
img = np.dot(np.array(image)[...,:3], [0.299, 0.587, 0.114])
img = 255 - img

np.argmax(net.predict(img.reshape(-1, 1) / 255.0))
```

Out[61]: 3