ACPR 19 Tutorial

Digital Geometry in Pattern Recognition: Extracting Geometric Features with DGtal and Applications

– Part I –

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Overview of the presentation - Part I -

1. Motivation, Theory and Applications
2. Geometry with Digital Straight lines
    2.1 Main idea of DSS recognition algorithms
    2.2 Adaptation to noise
    2.3 Applications of DSS
3. DGtal Library Overview
    3.1 Short presentation of the library
    3.2 Extracting level sets contours with DGtal
    3.3 Example of geometric estimator
4. Practical session: Hands on DGtal

https://kerautret.github.io/ACPR19-DGPRTutorial

1. Motivation, Theory and Applications

Motivation

Digital Geometry

Study of shapes defined in a digital domain, generally images ($\mathbb{Z}^2$, $\mathbb{Z}^3$, ...) or sometimes regular lattices.

- 2D shapes = set of pixels = subsets of $\mathbb{Z}^2$

photo picture

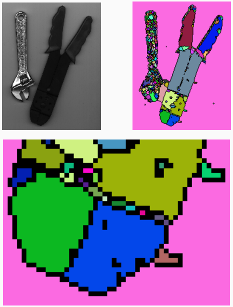
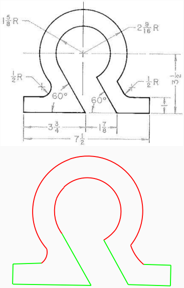


image segmentation



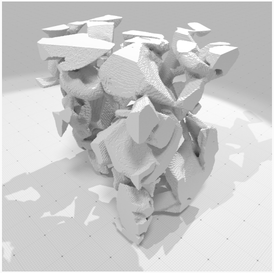
document analysis

Motivation

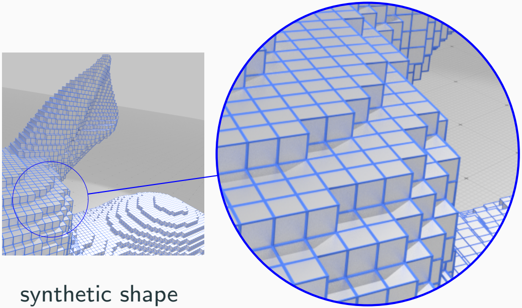
Digital Geometry

Study of shapes defined in a digital domain, generally images ($\mathbb{Z}^2$, $\mathbb{Z}^3$, ...) or sometimes regular lattices.

- 2D shapes = set of pixels = subsets of $\mathbb{Z}^2$
- 3D shapes = set of voxels = subsets of $\mathbb{Z}^3$



Micro-snow tomography

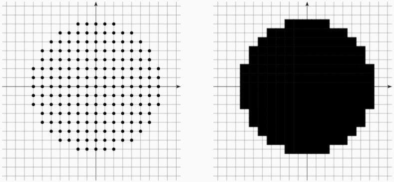


synthetic shape

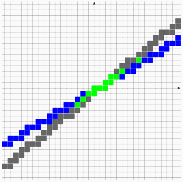
Motivation

Why a specific Digital Geometry ?

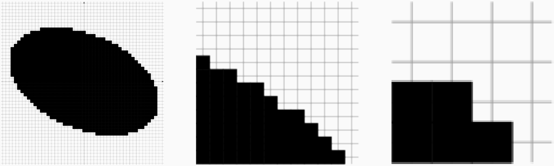
- geometry of pixels/voxels looks easy but is difficult for many reasons
- Euclidean definitions of connectedness, convexity, straight lines, differential geometric quantities fail



Convexity ?



Line Intersection ?



Infinitesimal differential geometry?

Applications require geometric tools

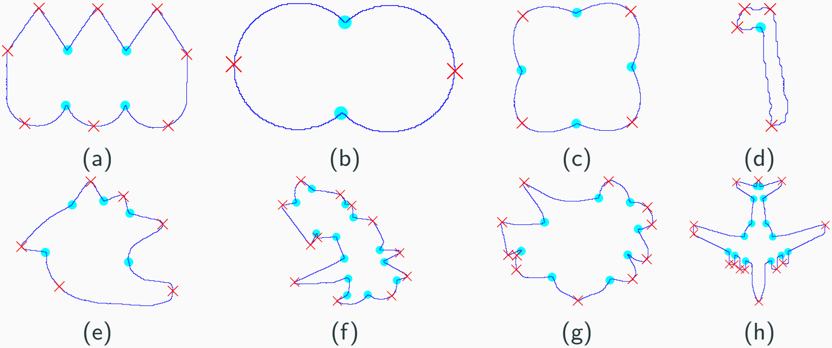
Classical image applications

- image restoration, noise identification/removal
- image segmentation with geometric priors
- shape matching, indexing
- precise shape measurements (biomedical and material imaging)

Desired geometric analysis

- identify linear or planar parts
- cut shape into convex / concave parts
- identify dominant points (high curvature) and inflexion points (perception)
- measure volume, perimeter, area, length, curvatures
- identify centerline of tubular objects
- compute skeleton, medial axis
- process shape geometry: remove noise, simplify, multi-scale decomposition

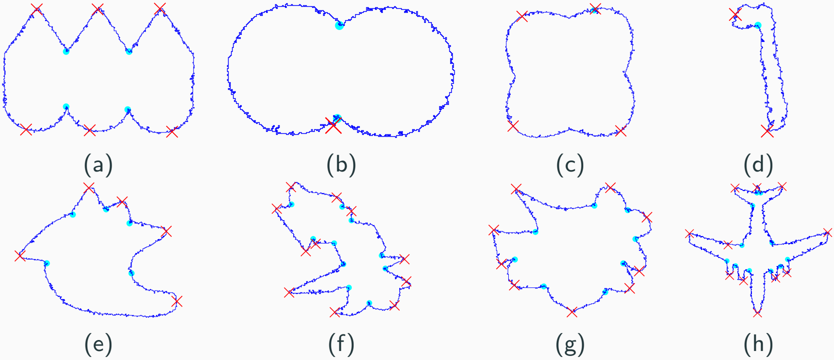
Applications where digital geometry is useful



Corner point detection

- digital contour tracking
- sound definition of digital straight segment
- stable and convergent digital curvature estimator

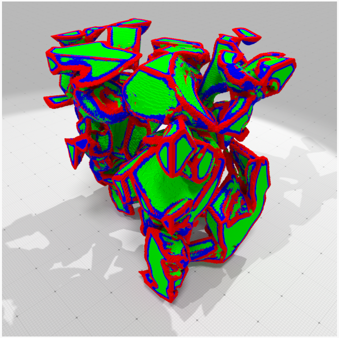
Applications where digital geometry is useful

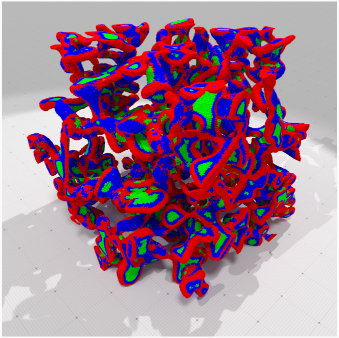


Corner point detection

- digital contour tracking
- sound definition of digital straight segment
- stable and convergent digital curvature estimator
- noise addressed with *thicker* digital straight segment

Applications where digital geometry is useful

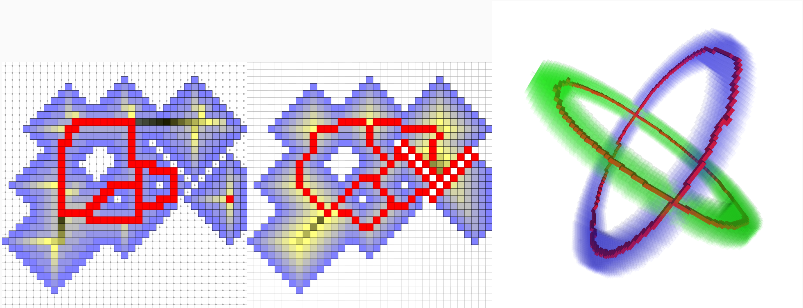




3D shape feature extraction on snow micro-structures

- 3D micro-tomography of snow ⇒ binary 3D images
- digital topology ⇒ digital surface tracking
- extracting linear parts along axes plane *xy*, *xz*, *yz*
- theoretical asymptotic analysis of length wrt gridstep *h*
- identify features according to length of linear parts

Applications where digital geometry is useful



Topology identification and control, skeleton extraction

- consistent definitions of connectedness
- topological invariant (here homotopy)
- simple points preserve topology: very efficient topological control

Applications where digital geometry is useful

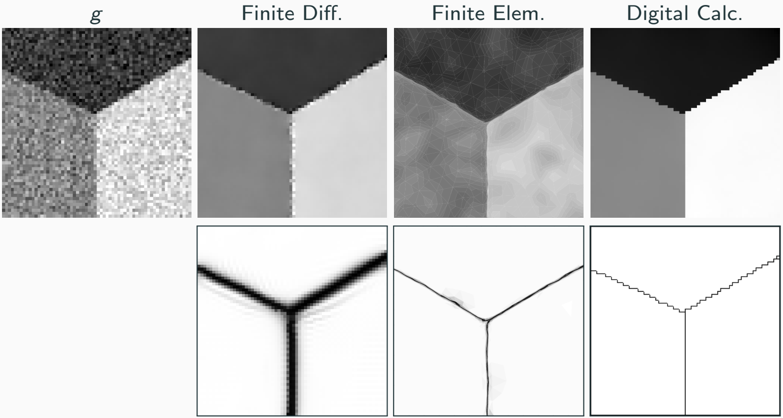
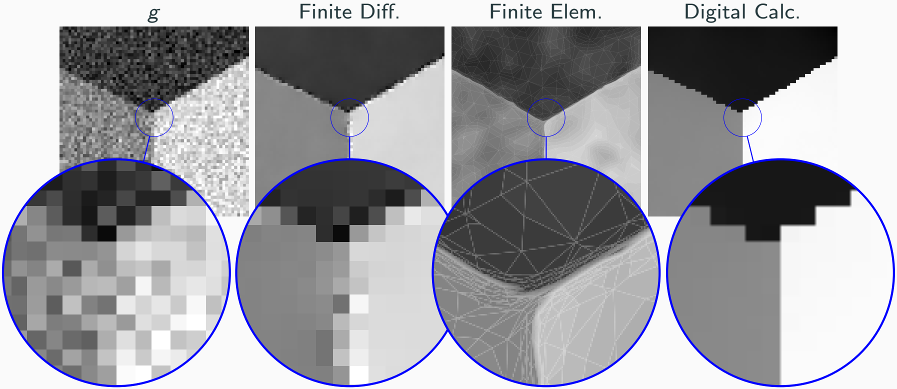


Image restoration, segmentation and inpainting

- most image processing task = variational formulation
- digital calculus = sound framework for variational problem in digital domain
- digital calculus formulation of Mumford-Shah model

Applications where digital geometry is useful



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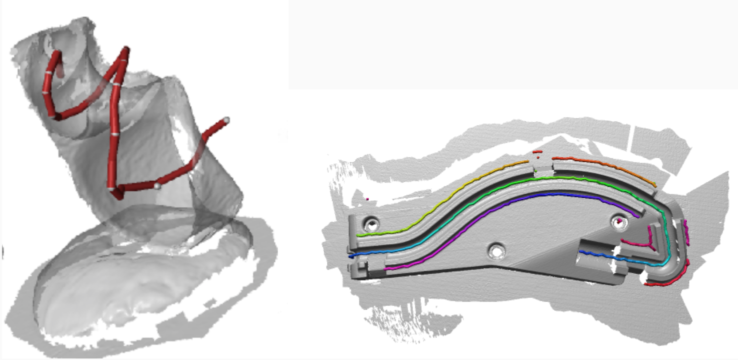
Applications where digital geometry is useful

Generate 3D surface model from 3D labelled images

- surface tracking in 3D labelled partitions
- convergent normal vector estimation on interfaces
- discrete variational model to align digital surface with estimated normals

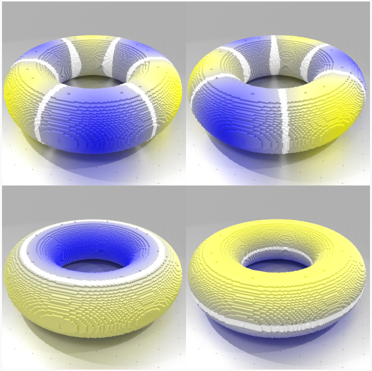
Applications where digital geometry is useful



Centerline extraction in arbitrary mesh / digital surfaces

- normal estimation on mesh / digital surfaces
- ray casting with 3D digital straight lines
- digital voting process

Applications where digital geometry is useful



Laplacian operator for shape analysis, simplification, matching

- convergent normal estimation on digital surfaces
- convergent surface integrals
- $\Rightarrow$ pointwise convergent Laplacian operator
- provide eigenvalues/eigenvector analysis

Summary

Applications require sound theoretical foundations

- digital topology
  - contour tracking
  - topological invariants and simple points
  - digital surfaces
- geometric primitives
  - digital straight segments
  - digital planes
- convergent geometric estimators
  - tangent and normal estimation
  - surface integrals
- digital calculus
  - variational image and geometry processing
  - multiscale analysis

Summary

Applications require sound theoretical foundations

- **digital topology**
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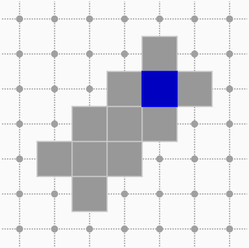
  - **digital surfaces**
- **geometric primitives**
  - **digital straightness**
  - digital planes
- **convergent geometric estimators**
  - **tangent** and normal **estimation**
  - curvatures estimation
  - surface integrals
- **digital calculus**
  - **variational image and geometry processing**
  - multiscale analysis

Main ingredients of digital geometry

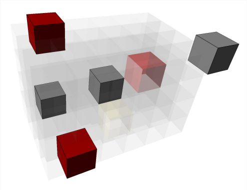
Topology: grid, adjacency, connectedness

- regular grid / lattice

2D discrete space



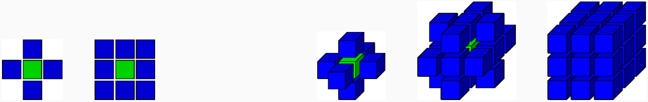
3D discrete space



Main ingredients of digital geometry

Topology: grid, adjacency, connectedness

- regular grid / lattice
- 4-/8-adjacency (2D), 6-/18-/26-adjacencies (3), play dual roles



4-adj 8-adj 6-adj 18-adj 26-adj

Main ingredients of digital geometry

Topology: grid, adjacency, connectedness

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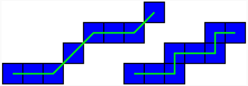
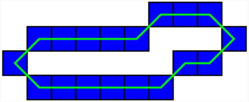
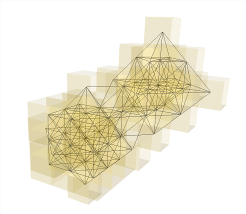
- curves, objects are related to adjacency pairs



8-Arc and 4-Arc



8-Curve

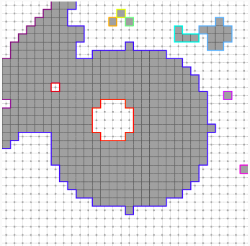


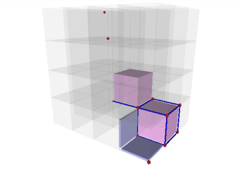
18-6-object

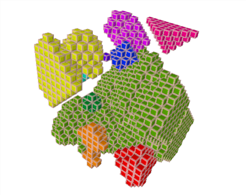
Main ingredients of digital geometry

Topology: grid, adjacency, connectedness

- regular grid / lattice
- 4-/8-adjacency (2D), 6-/18-/26-adjacencies (3), play dual roles
- curves, objects are related to adjacency pairs
- interpixel / cell topology, digital surfaces related to adjacency pairs
- sound definition of digital *d*-dimensional manifold



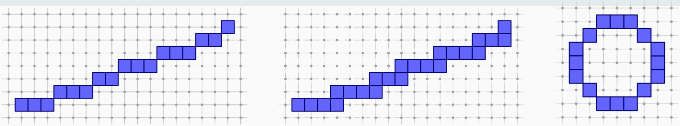




Main ingredients of digital geometry

Geometric primitives

- 2D/3D naive or standard digital lines, circles.



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Geometric primitives

- 2D/3D naive or standard digital lines, circles.
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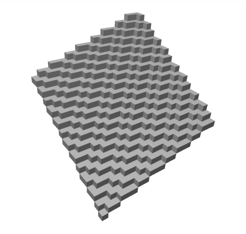

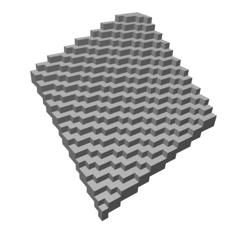

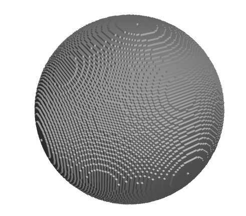

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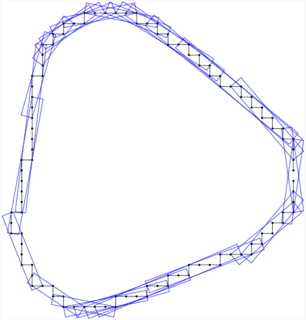

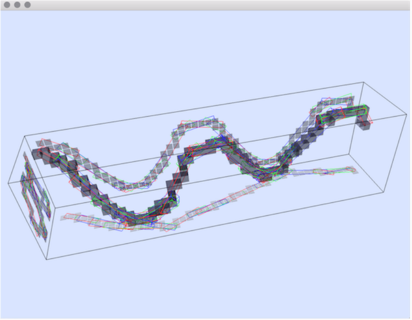

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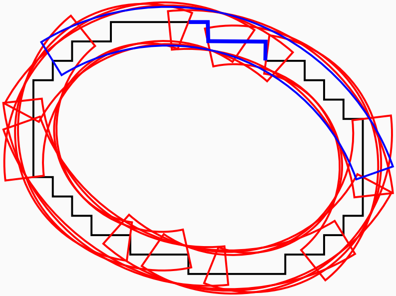
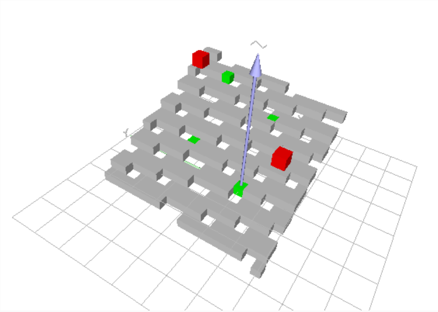
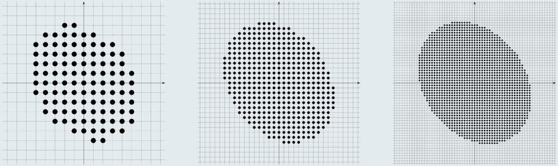




Main ingredients of digital geometry

Geometric estimators of area/volume/tangent/normals/curvatures

- multigrid convergence: the finer the sampling grid h, the better the geometric estimation



$\mathrm{Dig}_h(X)$ $\mathrm{Dig}_{h/2}(X)$ $\mathrm{Dig}_{h/4}(X)$ $\ldots$

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Geometric estimators of area/volume/tangent/normals/curvatures

- multigrid convergence: the finer the sampling grid h, the better the geometric estimation
- multigrid convergent estimators of (speed as a function of h)

area/volume pixel/voxel counting ($O(h)$ convex shapes, $O(h^{22/15})$ C^2-convex)

perimeter minimum length polygon ($O(h^{4/3})$ convex shapes, $O(h)$ otherwise)

tangent 2D max. digital straight segment ($O(h^{2/3})$ piecewise C^2 shapes), Voronoi Covariance Measure ($O(h^{2/3})$)

normal 3D integral invariant ($O(h^{2/3})$), Voronoi Covariance Measure ($O(h^{2/3})$),

curvatures 2D/3D integral invariant ($O(h^{1/3})$), corrected curvature measures ($O(h^{2/3})$)

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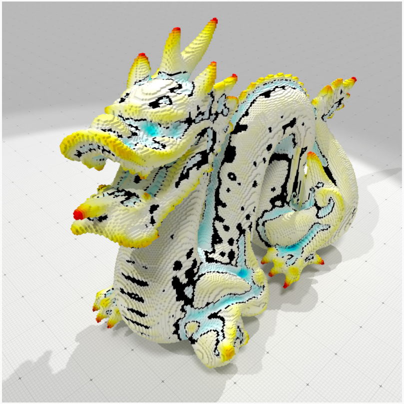
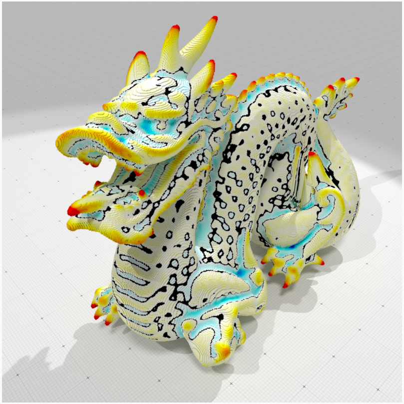
All results presented in the tutorial were obtained from the DGtal library!

Main ingredients of digital geometry

Example of convergent curvature estimator

- Mean curvature estimation with corrected curvature measures





2. Geometry with Digital Straight lines

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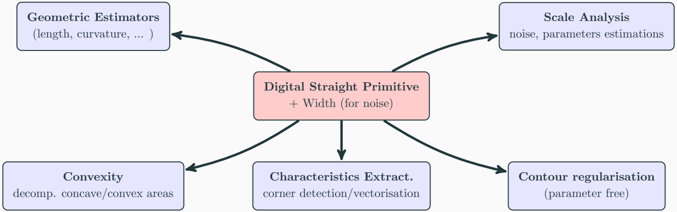
Main primitive for 2D analysis:

- **D**igital **S**traight **S**egment recognition algorithms (DSS).
- Take the noise into account (parameters).
- Adapted locally to the shape (scale adjustment).



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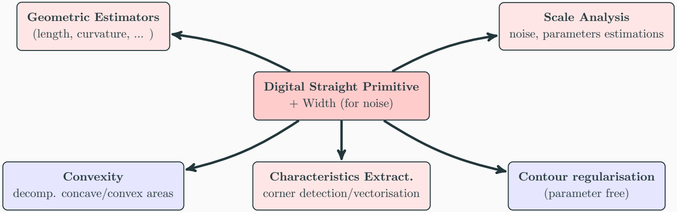

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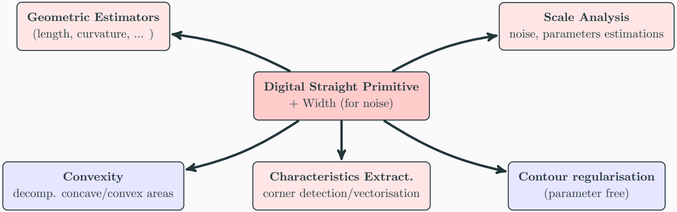

Overview of Geometry with DSS:

- 2.1 Main idea of DSS recognition algorithms.
- 2.2 Adaptation to noise.
- 2.3 Applications examples: curvature, scale detection and vectorisation.

2.1 Main idea of DSS recognition algorithms

Arithmetic definition of digital line [Réveilles 91]

A digital line with parameters (a, b, μ) and arithmetical thickness ω is defined as the set of integer points (x, y) verifying :

$$\mu \leq ax - by < \mu + \omega$$

- a, b, μ, ω in $\mathbb{Z}$
- $gcd(a, b) = 1$, (b, a) main vector of the line
- noted $\mathcal{D}(a, b, \mu, \omega)$

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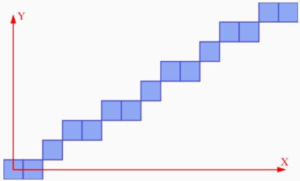

$\mathcal{D}(5, 8, -1, 8)$

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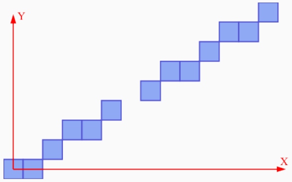
- if $\omega < max(|a|, |b|)$: $\mathcal{D}$ is deconnected.



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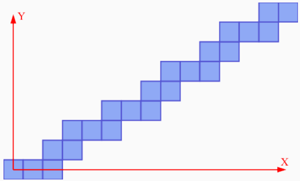

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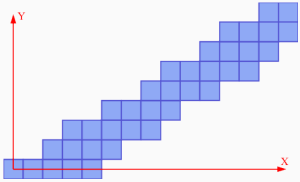
- if $\omega < max(|a|, |b|)$: $\mathcal{D}$ is deconnected.
- if $\omega = |a| + |b|$: $\mathcal{D}$ is 4-arc (standard line).
- if $\omega > |a| + |b|$: $\mathcal{D}$ is called a thick line.

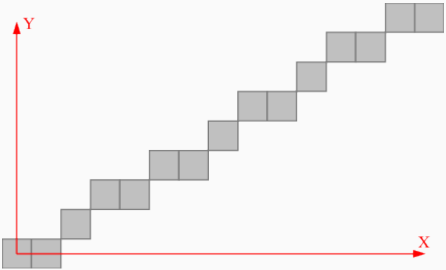


$\mathcal{D}(5, 8, -1, 8)$

2.1 Main idea of DSS recognition algorithms

Recognition Problem

- Goal: Recover the segment characteristics from a input sequence of points.
- Based on the remainder and periodicity detection.



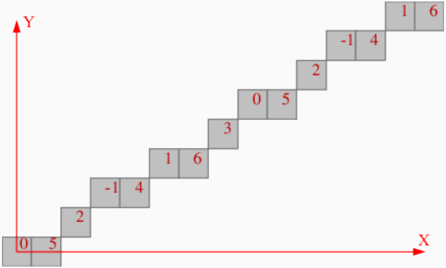
$\mathcal{D}(5, 8, -1, 13)$

2.1 Main idea of DSS recognition algorithms

Recognition Problem

- Goal: Recover the segment characteristics from a input sequence of points.
- Based on the remainder and periodicity detection.
- Remainder of a point M is defined as a function of $\mathcal{D}(a, b, \mu, \omega)$:

$$r_{\mathcal{D}}(M) = ax_M - by_M$$



$\mathcal{D}(5, 8, -1, 13)$

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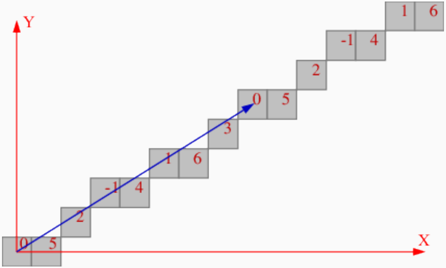

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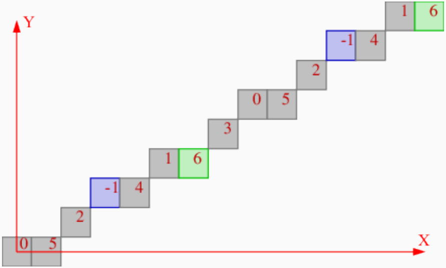
- Maintain the lower/upper leaning points.

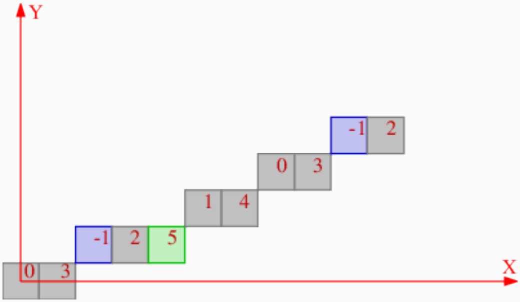


$\mathcal{D}(5, 8, -1, 13)$

2.1 Main idea of DSS recognition algorithms

Strategy of segment recognition ($\mathcal{S}$)

- Compute remainder of new point M.
- From $r(M)$ update characteristics.
- Update $\mathcal{S}$ parameters & leaning pts.



Recognized segment $\mathcal{S}$ of $\mathcal{D}_0(3, 7, -1, 7)$

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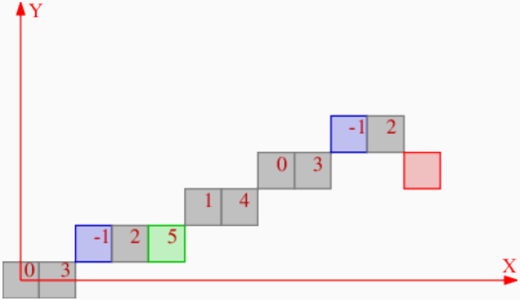

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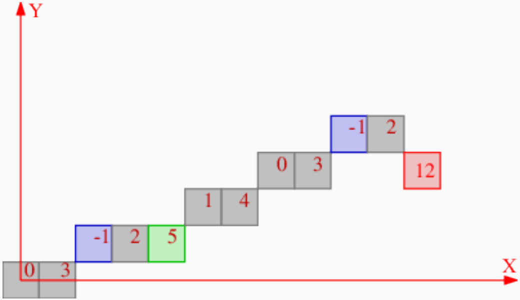

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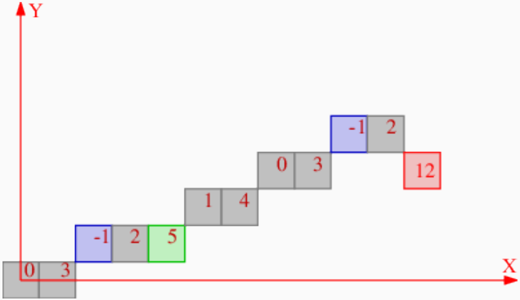
Rules to update the characteristics of $\mathcal{S}$:

(iv) $r_{\mathcal{D}}(M) > \mu + max(|a|, |b|)$: M is strongly exterior to $\mathcal{D}$ and M not added to $\mathcal{S}$.



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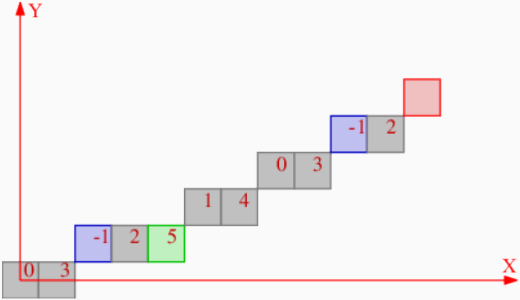

Recognized segment $\mathcal{S}$ of $\mathcal{D}_0(3, 7, -1, 7)$

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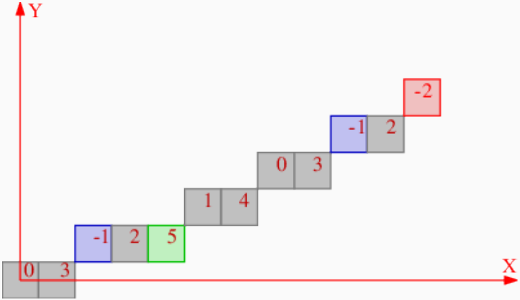
(*iii*) $r_{\mathcal{D}}(M) = \mu - 1$: M weakly exterior to $\mathcal{D}$,
M added to $\mathcal{S}$ and the slope is updated by the vector $U_F M$



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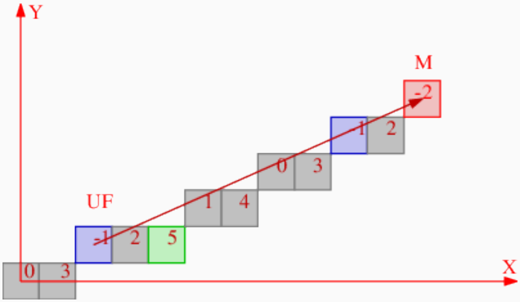

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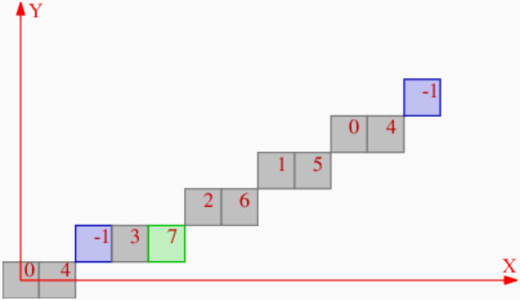
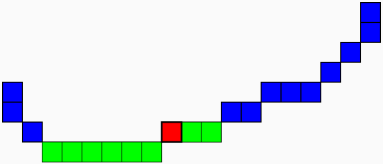


Recognized segment $\mathcal{S}$ of $\mathcal{D}_1(4, 9, -1, 9)$

2.1 Main idea of DSS recognition algorithms: maximal DSS

Primitive of Maximal Digital Straight Segment (MDSS)

Let $\mathcal{C}$ be a digital curve, a segment of a naïve digital line is said maximal if it cannot be extended at the right and left hand sides on $\mathcal{C}$.



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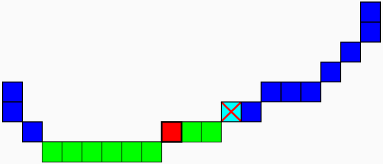

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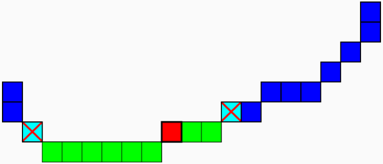

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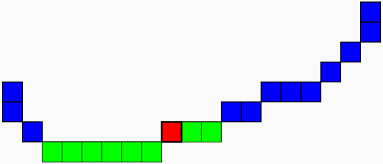

Sequence computation of maximal segments

Computable in linear type [Feschet and Tougne 99].

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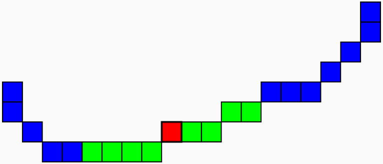

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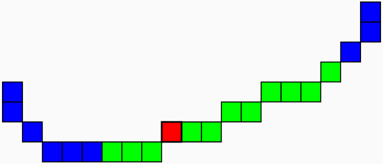

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2.1 Main idea of DSS recognition algorithms: maximal DSS (2)

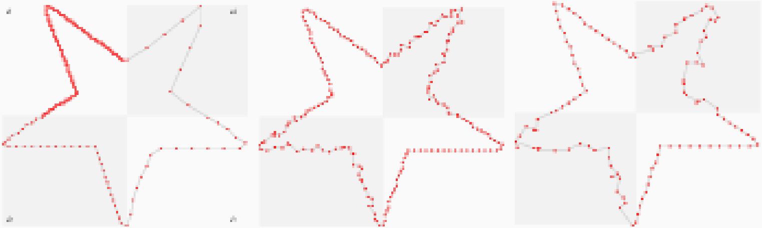
Advantage and limits of the MDSS

- + Gives a convergent technique to estimate geometric features like tangent, curvature.
- + Linear time algorithm.
- + Simple to implement and available in the DGtal Library.

2.1 Main idea of DSS recognition algorithms: maximal DSS (2)

Advantage and limits of the MDSS

- + Gives a convergent technique to estimate geometric features like tangent, curvature.
- + Linear time algorithm.
- + Simple to implement and available in the DGtal Library.
- - Limited to handle perfect digitized objects.
- - For real object it can be sensitive to noise.
- - Cannot process disconnected set of points.



2.2 Adaptation to noise

Primitive of Blurred Segment (or Alpha Thick Segments)

- Primitive able to handle noise.
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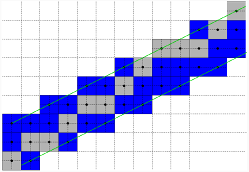

$\mathcal{D}(1, 2, -4, 6)$, bounding line of the sequence of grey points

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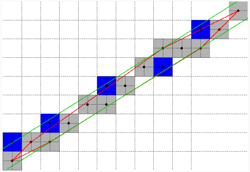
- Optimal Bounding line.



$\mathcal{D}(5, 8, -8, 11)$, optimal bounding line (width $\frac{10}{8} = 1.25$) of the sequence of grey points

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- Based on bounding line definition.
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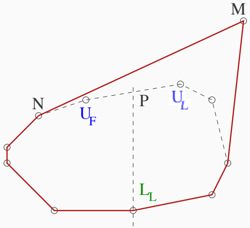

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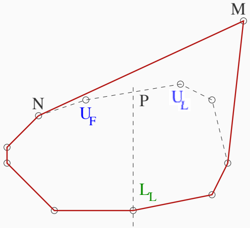
- Thick param. to take noise into account.



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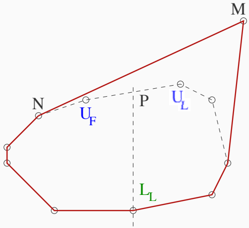

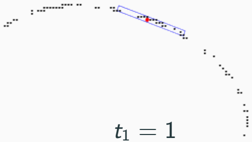

$t_1 = 1$

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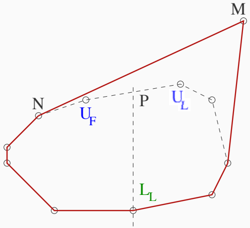

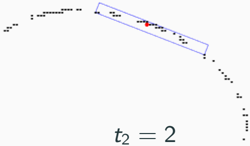

$t_2 = 2$

2.2 Adaptation to noise

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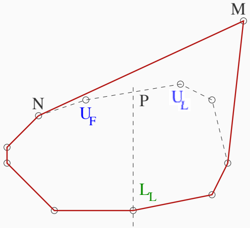

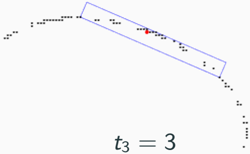

$t_3 = 3$

2.2 Adaptation to noise

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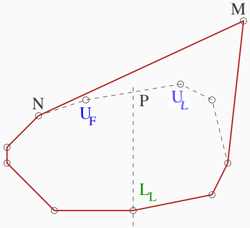

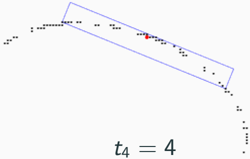

$t_4 = 4$

2.2 Adaptation to noise

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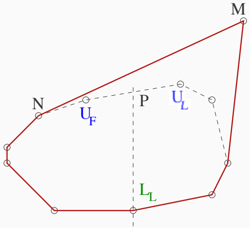
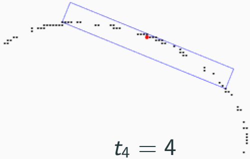
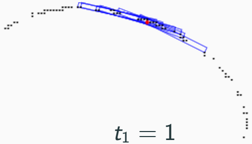




$t_4 = 4$



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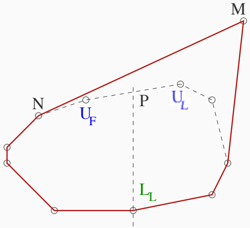
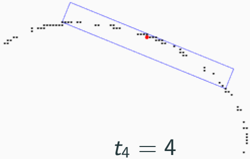
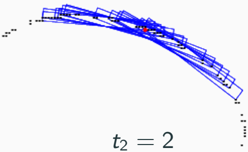




$t_4 = 4$



$t_2 = 2$

2.2 Adaptation to noise

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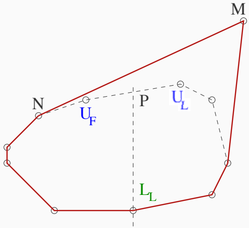
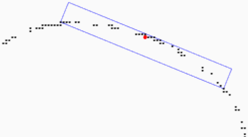
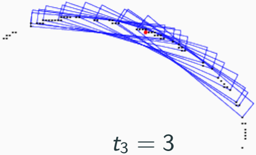




$t_4 = 4$



$t_3 = 3$

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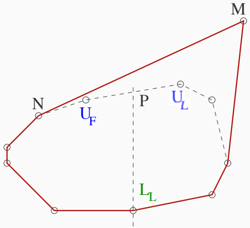

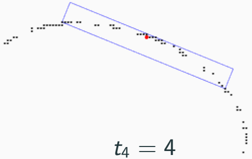

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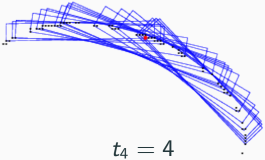
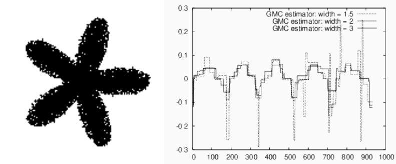


$t_4 = 4$

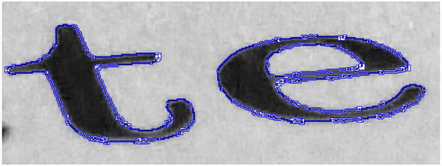
2.3 Applications of DSS

Overview of key applications:

- (1) Curvature estimator based on DSS.
- (2) Scale detection (noise).
- (3) Polygonalisation (arcs/segments).
- (4) Image vectorisation.



Curvature estimator (GMC)



noise estimator

Polygonalisation

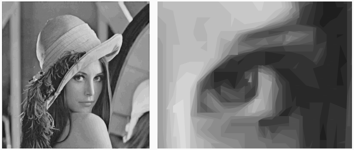


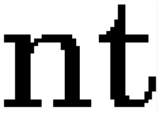
Image vectorisation

2.3 Applications of DSS: (1) Curvature estimator

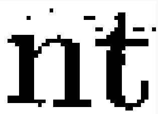
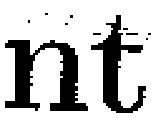
Objectives: [Kerautret and Lachaud 2009]

- Precise estimator even with low resolution.
- Adapted even on data with non perfect digitization or with noise.

Perfect Digitization

200 dpi	300 dpi	400 dpi
		

Result from printing and scan

200 dpi	300 dpi	400 dpi
		

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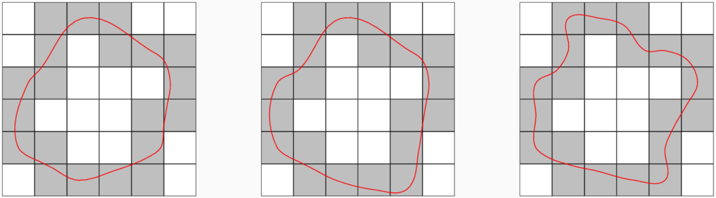
Main idea: (cf. deformable model)

- Take into account all the shapes having the same digitization.
- Retain the estimation corresponding to the shape having the highest probability (of lower energy).



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Realization:

- Best length estimator : minimize $\int ds$ [Sloboda *et al.* 98]

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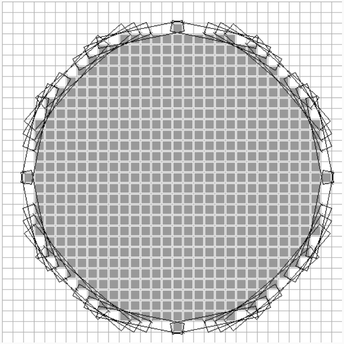
Realization:

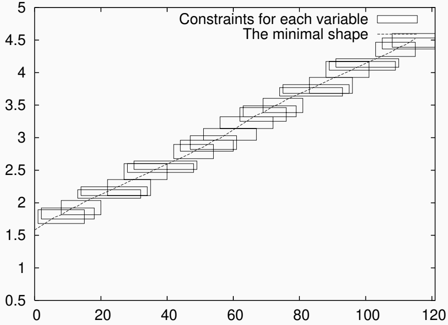
- Best length estimator : minimize $\int ds$ [Sloboda *et al.* 98]
- Best curvature estimator: minimize $\int \kappa^2 ds$
  $\Rightarrow$ Computed in the space of maximal segments (tangential cover).

2.3 Applications of DSS: (1) Curvature estimator

Examples of tangential cover with uncertainty on the slope

- Every maximal segment presents θ_{min}, θ_{max} values.
- For each surfel we can deduce the angle θ_{min} et θ_{max} of the tangent: $\min(\theta^i_{min})$ and $\max(\theta^i_{max})$.





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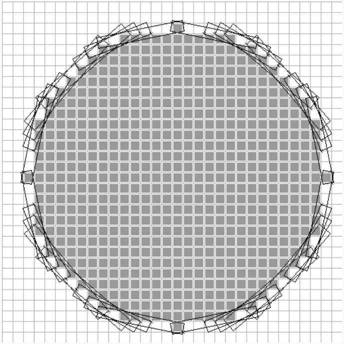
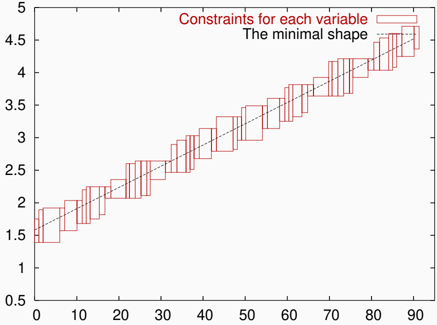
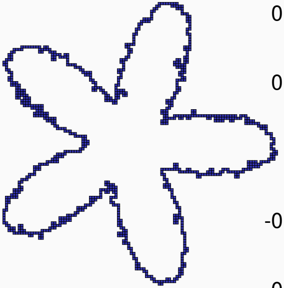
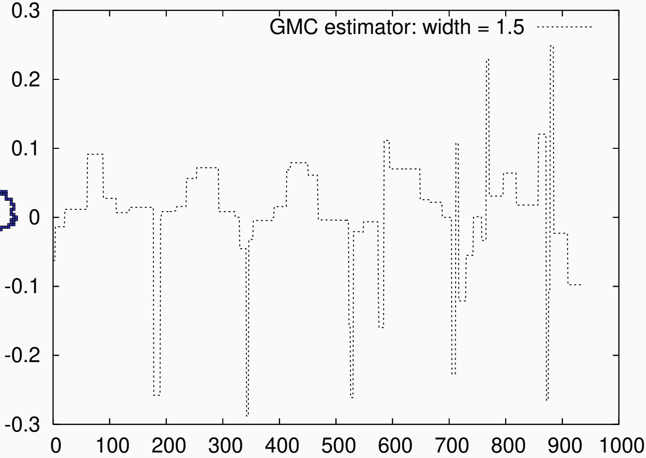




2.3 Applications of DSS: (1) Curvature estimator

Results of curvature : on noisy contours





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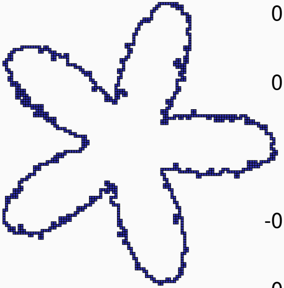

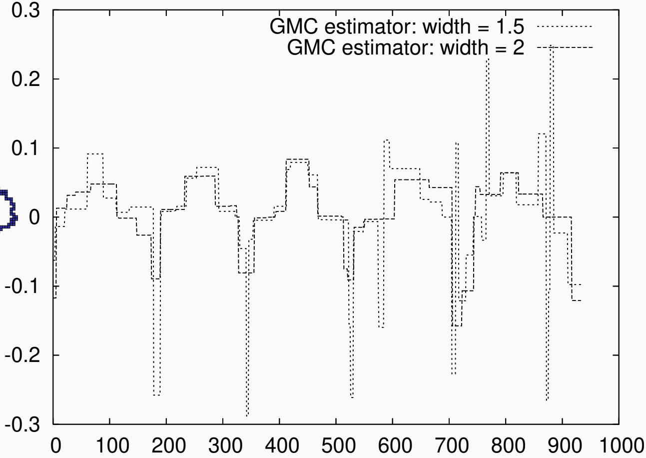

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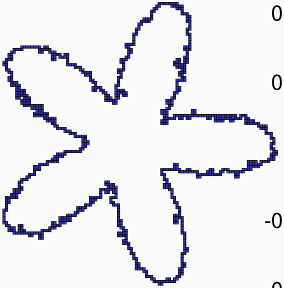

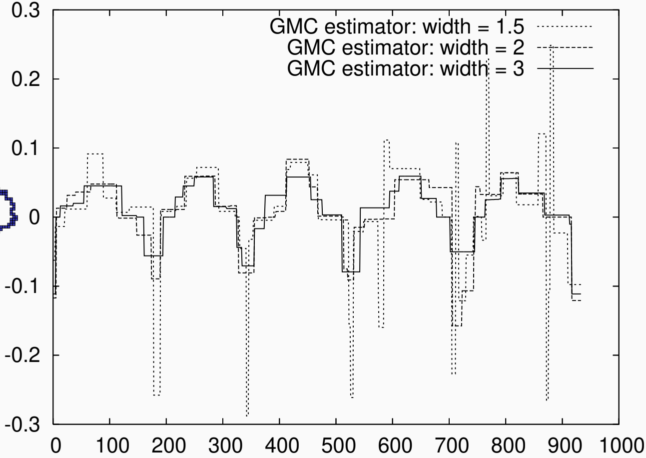

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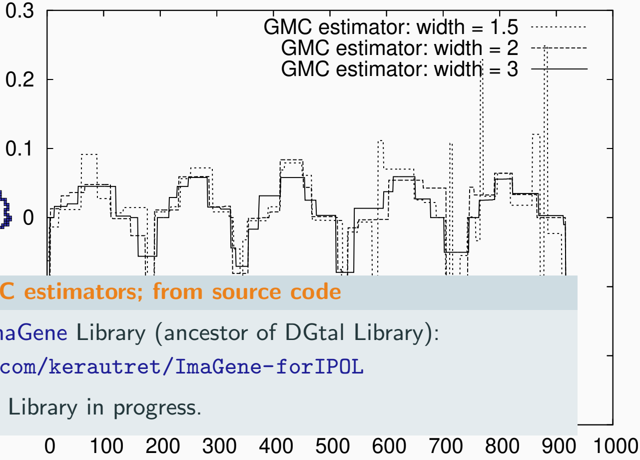


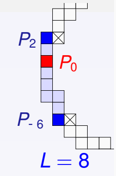
Reproducing the GMC estimators; from source code

- Available in the ImaGene Library (ancestor of DGtal Library): https://github.com/kerautret/ImaGene-forIPOL
- Backport in DGtal Library in progress.

2.3 Applications of DSS: (2) Scale detection

Main idea [Kerautret & Lachaud, 2012]

1. Exploit asymptotic properties of the Length (L) of maximal straight segments (valid on perfect shape digitizations).



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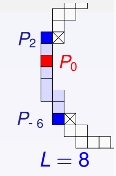

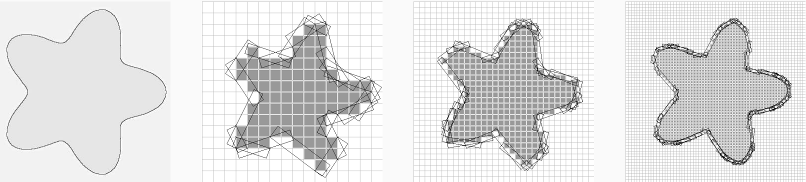

X $\mathrm{Dig}_2(X)$ $\mathrm{Dig}_1(X)$ $\mathrm{Dig}_{0,5}(X)$

- X some simply connected compact shape of $\mathbb{R}^2$.
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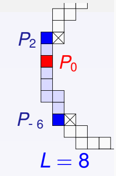

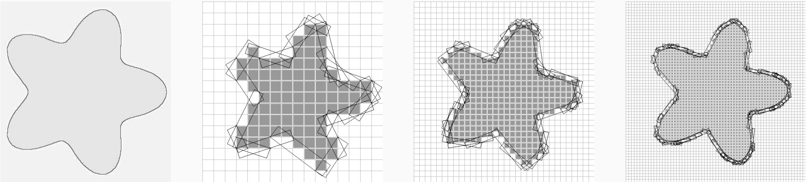

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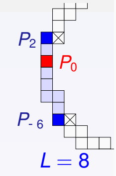

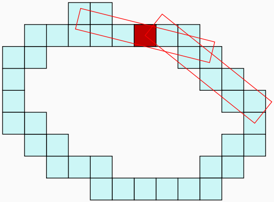

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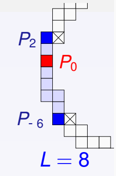

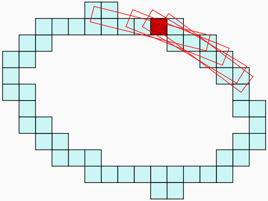

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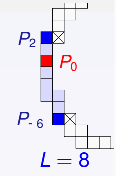

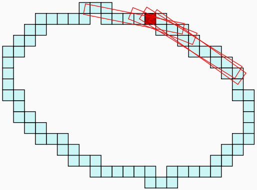

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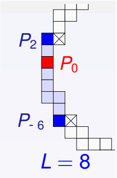

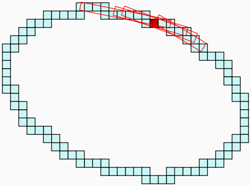

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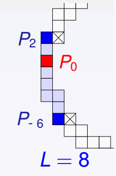

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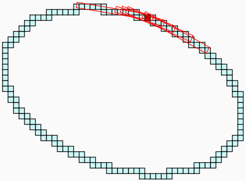

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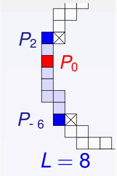

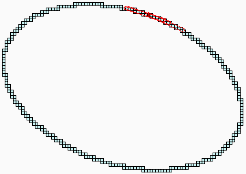

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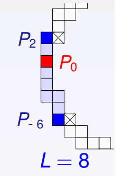

Theorem [Lachaud 06]: asymptotic behavior of the length of maximal segments

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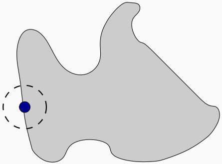

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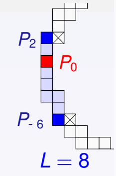

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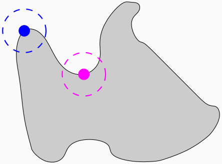

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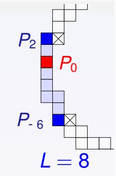

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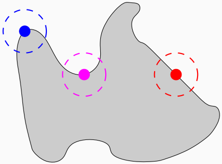
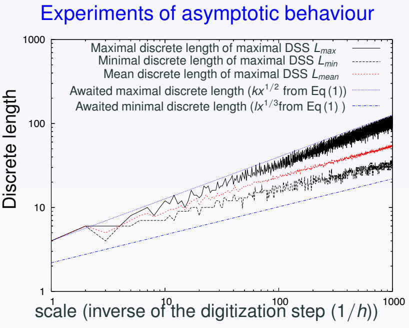


2.3 Applications of DSS: (2) Scale detection

Experiments about reverse asymptotic behavior:



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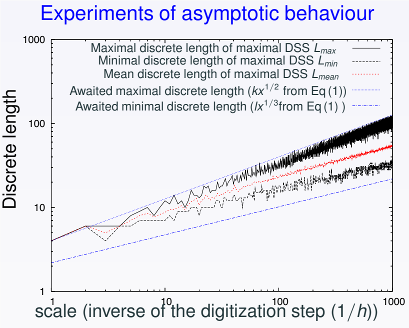

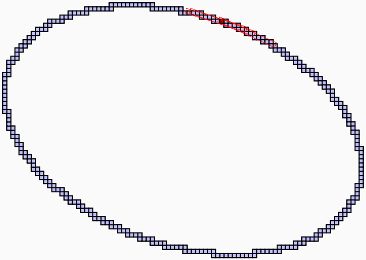

grid size 1

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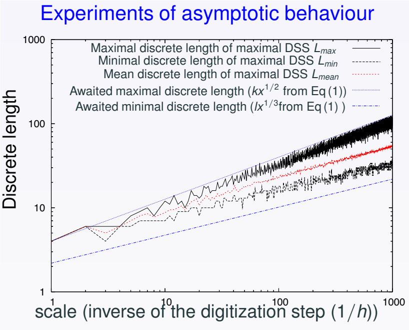

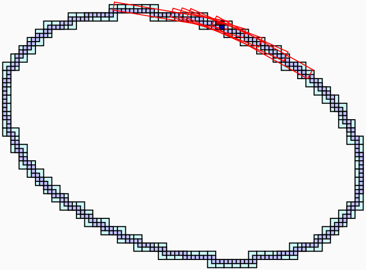

grid size 2

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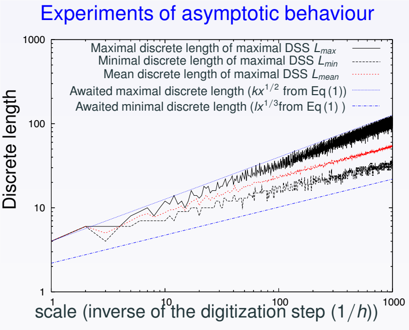

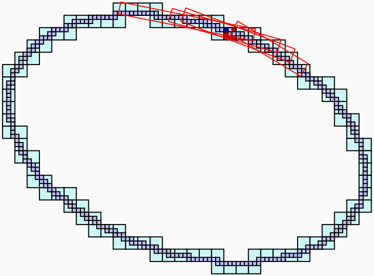

grid size 3

2.3 Applications of DSS: (2) Scale detection

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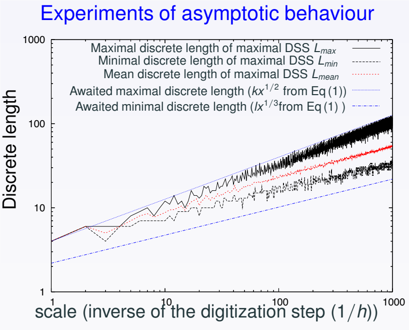

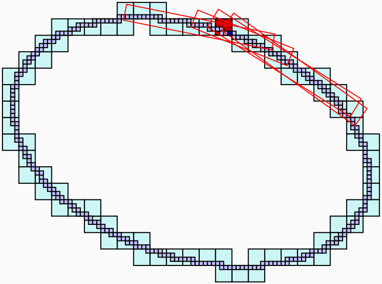

grid size 4

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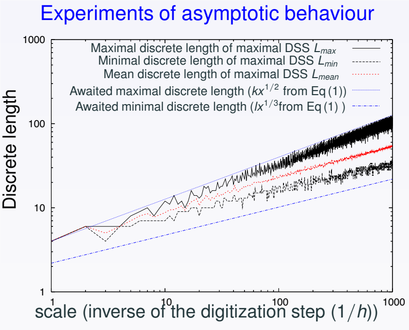

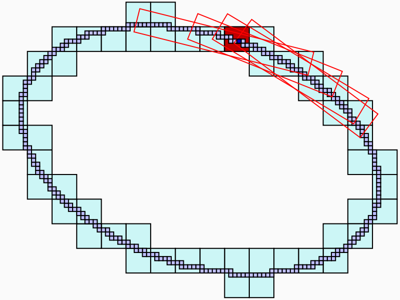

grid size 6

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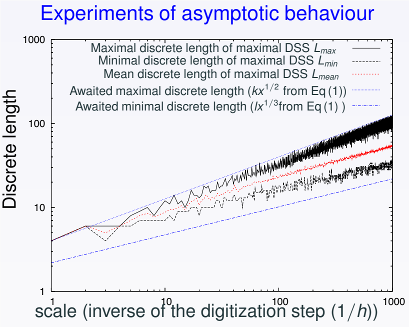
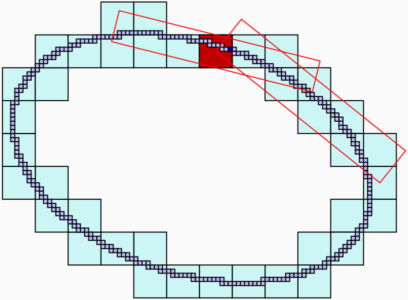
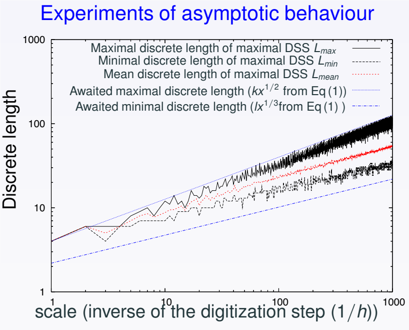


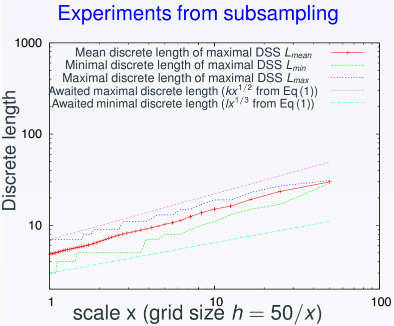


grid size 8

2.3 Applications of DSS: (2) Scale detection

Experiments about reverse asymptotic behavior:





Local meaningful scale and noise detection

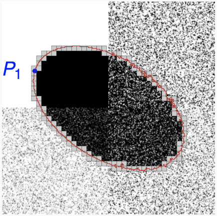
Meaningful scale:

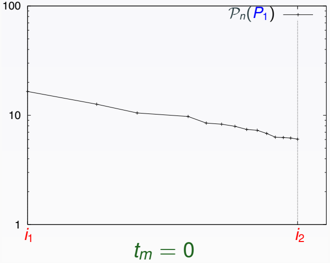
A meaningful scale of a multi-scale profile $(X_i, Y_i)_{1 \leq i \leq n}$ is the pair (i_1, i_2) $1 \leq i_1 \leq i_2 \leq n$ such that for all i, $i_1 \leq i < i_2$,

$$\frac{Y_{i+1} - Y_i}{X_{i+1} - X_i} \leq t_m,$$

while not true for $i_1 - 1$ and i_2.

Parameter t_m = noise threshold to discriminate curved from noisy areas.





Local meaningful scale and noise detection

Meaningful scale:

A meaningful scale of a multi-scale profile $(X_i, Y_i)_{1\leq i\leq n}$ is the pair (i_1, i_2) $1 \leq i_1 \leq i_2 \leq n$ such that for all i, $i_1 \leq i < i_2$,

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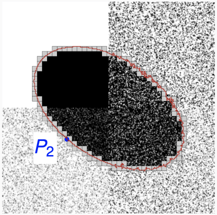

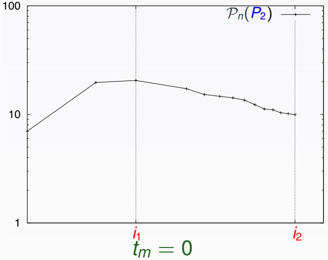

Local meaningful scale and noise detection

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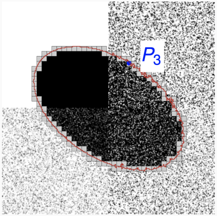
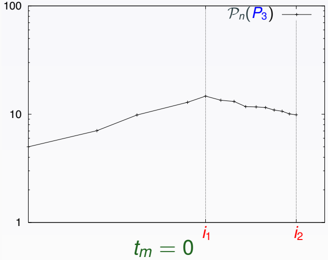
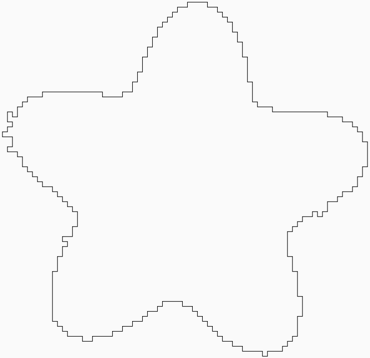
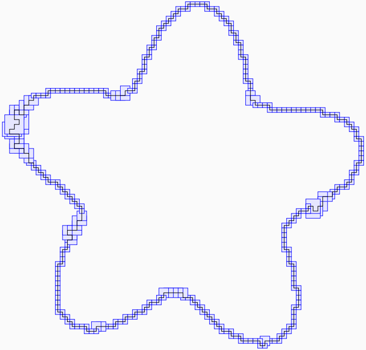




Experiments: local noise level detection

Flower with local noise

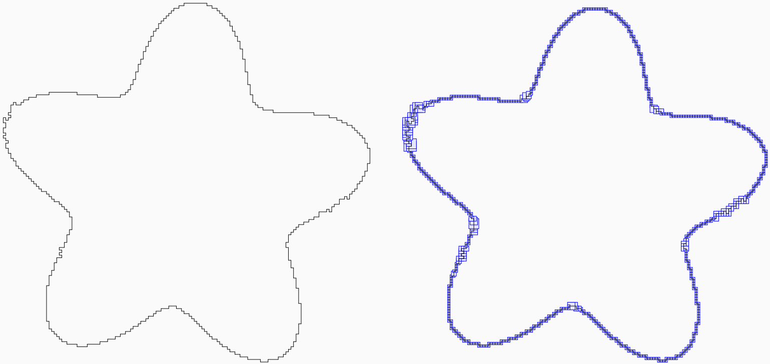




Local noise on resolution $R0$

Experiments: local noise level detection

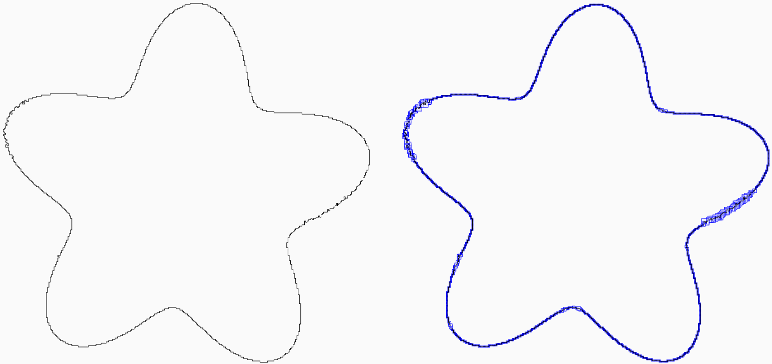
Flower with local noise



Local noise on resolution $R1$

Experiments: local noise level detection

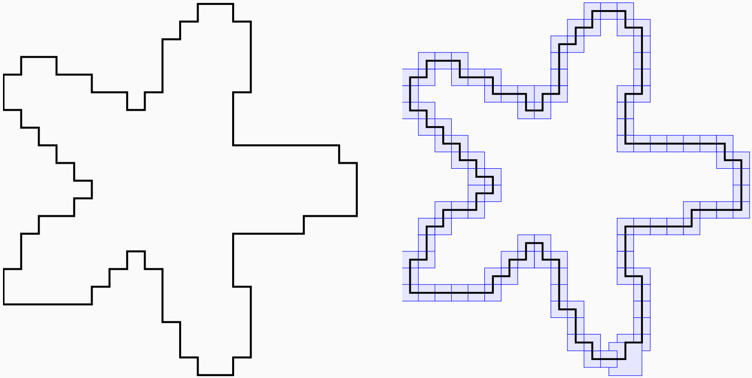
Flower with local noise



Local noise on resolution $R2$

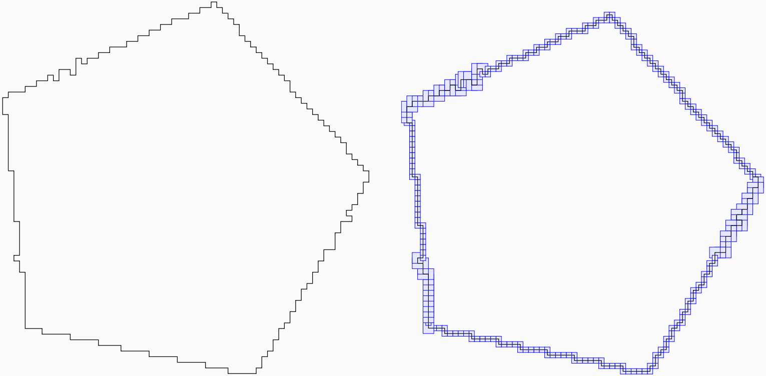
Experiments: local noise level detection

Tiny flower without noise



Local noise detection

Polygon with local noise



Local noise on resolution $R0$

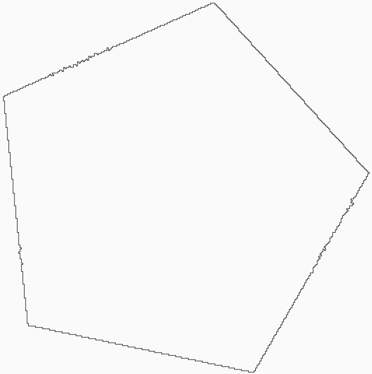
Local noise detection

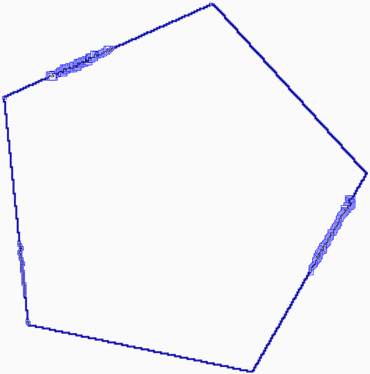
Polygon with local noise

Local noise on resolution $R1$

Local noise detection

Polygon with local noise





Local noise on resolution $R2$

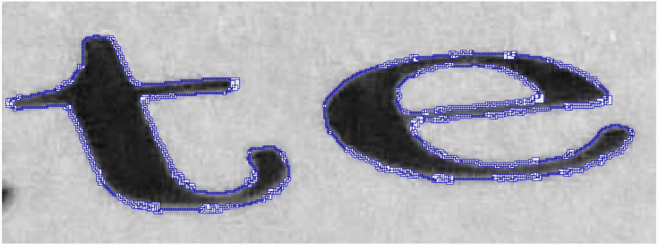
Local noise detection

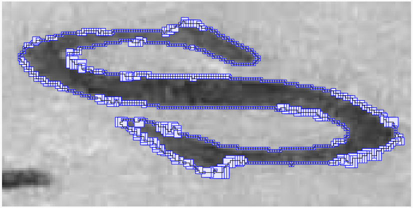
Polygon with local noise

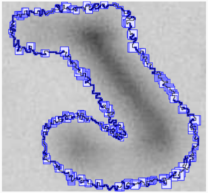
- Accuracy of noise detection independent of shape geometry, independent of shape resolution.
- Only one parameter : maximum level of subsampling (always 10 here).

Noise detection on real images

Noise detection on real images



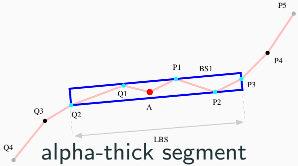




2.3 Applications of DSS: (2) Scale detection

Multi-thickness Profile

The **multi-thickness profile** $\mathcal{P}_n(P)$ of a point P is defined as the graph $(\log(t_i), \log(\overline{\mathcal{L}}^{t_i}/t_i))_{i=1,\dots,n}$.



alpha-thick segment

2.3 Applications of DSS: (2) Scale detection

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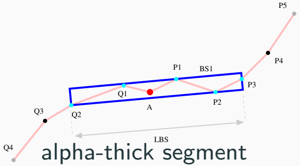
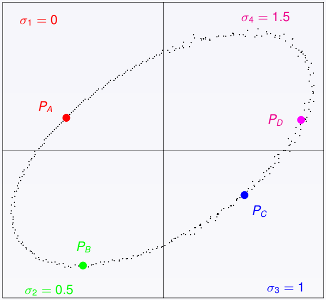
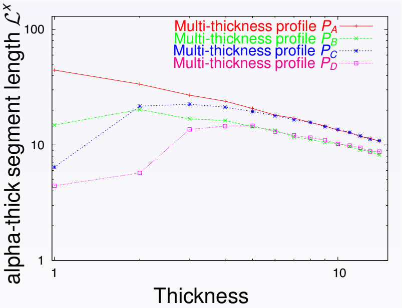
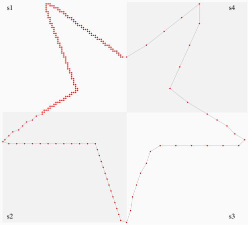
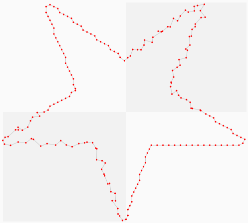
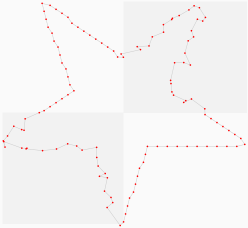




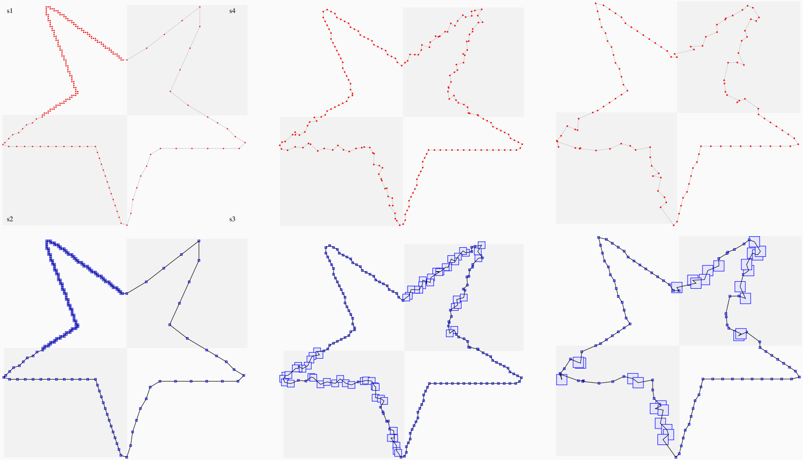


Experiments on polygonal shapes (1)

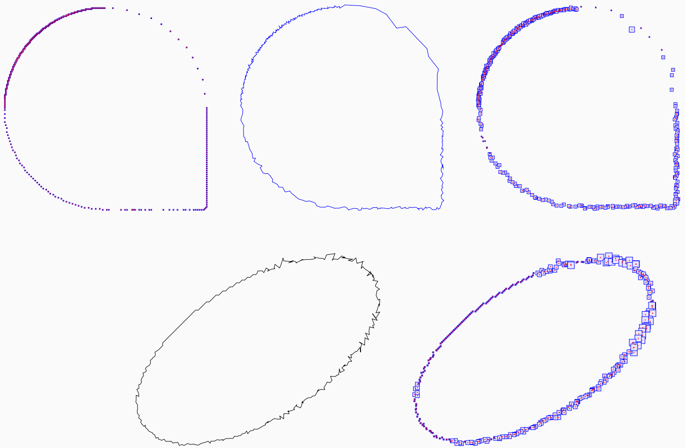


Experiments on polygonal shapes (1)



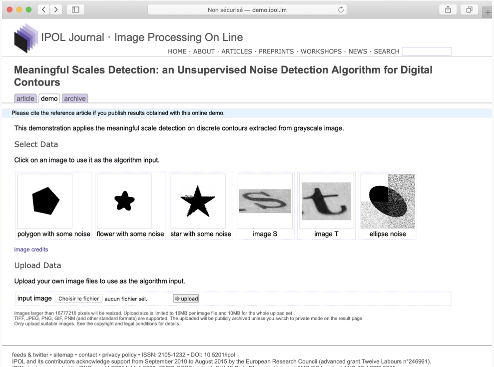
Experiments on polygonal shapes (2)



Reproduction of the results

Online demonstration available on IPOL: [Kerautret & Lachaud, 2014]

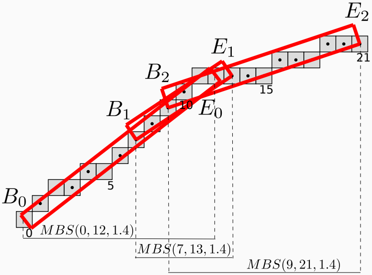
- The algorithm can be tested online: `http://www.ipol.im/pub/art/2014/75/`
- IPOL article with source code (based on the ImaGene Library).
- Reproducible in DGtal (with Alpha-Thick Segments), see examples of tutorial.



2.3 Applications of DSS: (3) Image vectorisation

Selection of polygonalisation algorithms [Kerautret *et al.* 17]

- Dominant points based polygonalization (DPP) [Nguyen 11].



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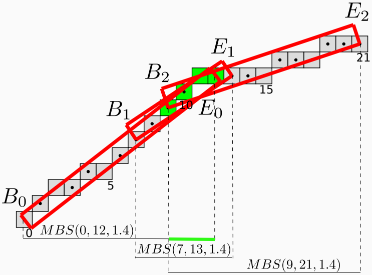

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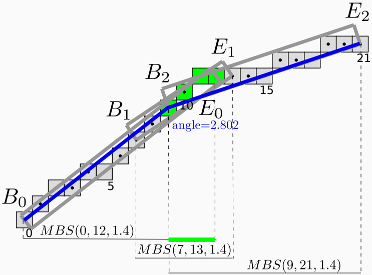

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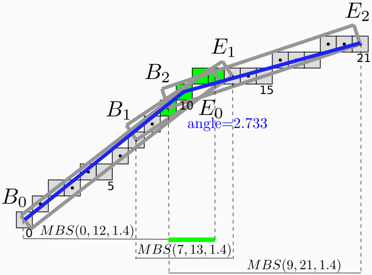

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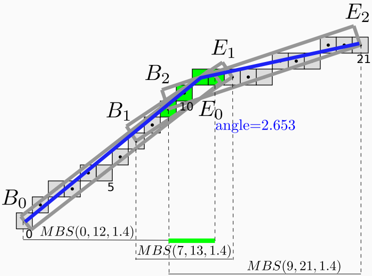

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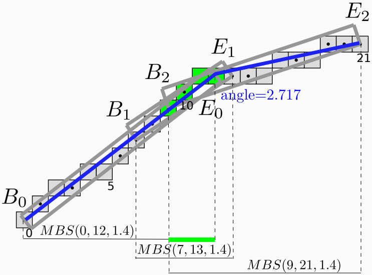

2.3 Applications of DSS: (3) Image vectorisation

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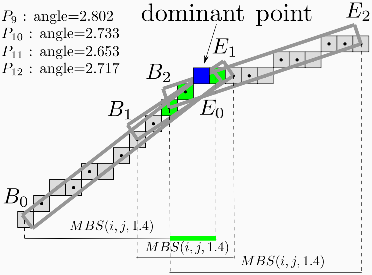
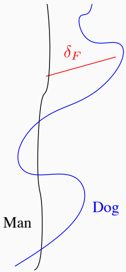


2.3 Applications of DSS: (3) Image vectorisation

Selection of polygonalisation algorithms [Kerautret *et al.* 17]

- Dominant points based polygonalization (DPP) [Nguyen 11].
- Polygonalization from Frechet dist. (minimal leash length) [Sivignon 11].



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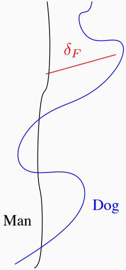

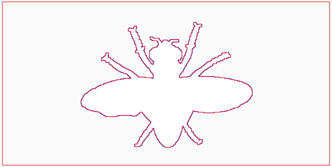

$\epsilon = 1$

2.3 Applications of DSS: (3) Image vectorisation

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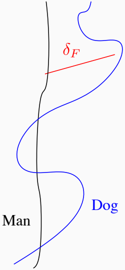

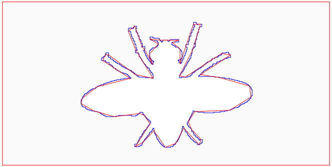

$\epsilon = 5$

2.3 Applications of DSS: (3) Image vectorisation

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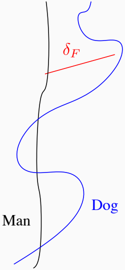
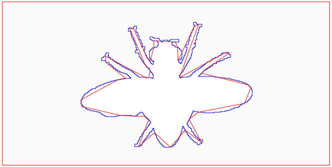
- Polygonalization from Frechet dist. (minimal leash length) [Sivignon 11].





$\epsilon = 10$

2.3 Applications of DSS: (3) Image vectorisation

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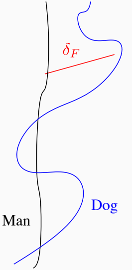

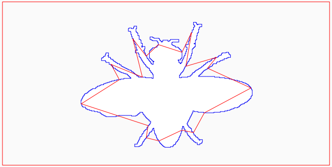

$\epsilon = 25$

2.3 Applications of DSS: (3) Image vectorisation

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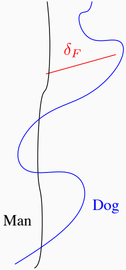
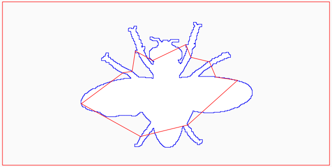
- Dominant points based polygonalization (DPP) [Nguyen 11].
- Polygonalization from Frechet dist. (minimal leash length) [Sivignon 11].



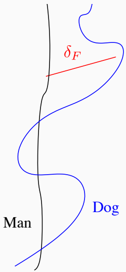


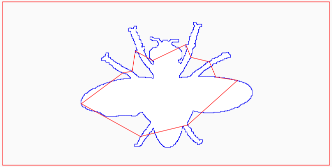
$\epsilon = 50$

2.3 Applications of DSS: (3) Image vectorisation

Selection of polygonalisation algorithms [Kerautret *et al.* 17]

- Dominant points based polygonalization (DPP) [Nguyen 11].
- Polygonalization from Frechet dist. (minimal leash length) [Sivignon 11].
- Extract from local maxima from curvature (GMC or other).



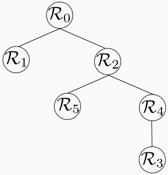


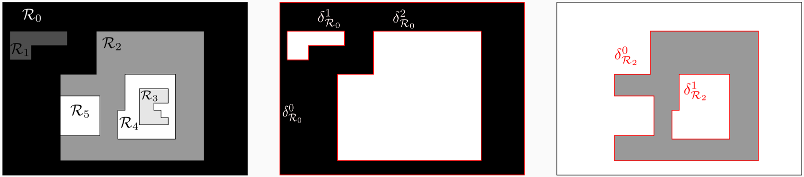
$\epsilon = 50$

2.3 Applications of DSS: (3) Image vectorisation

Component Tree representation [Najman & Couprie 06]

- Constructed from the intensity thresholds.
- Starting from the root (full image),
- to the node representing the included regions.





2.3 Applications of DSS: (3) Image vectorisation

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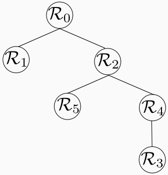
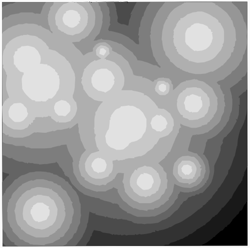


Representation from component tree



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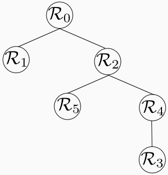
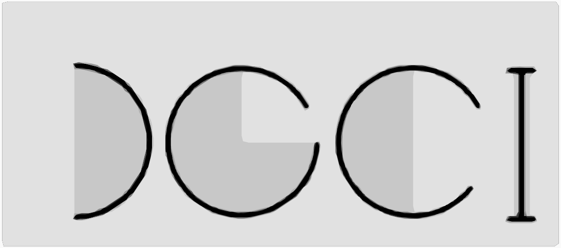
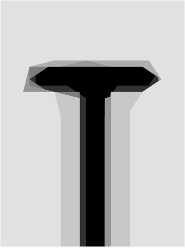


Representation from component tree





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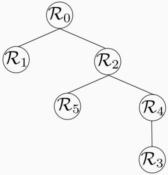
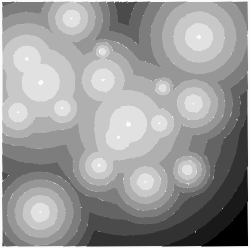


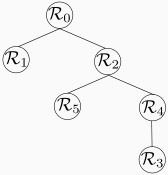
Representation using simple filling



2.3 Applications of DSS: (3) Image vectorisation

Component Tree representation [Najman & Couprie 06]

- Constructed from the intensity thresholds.
- Starting from the root (full image),
- to the node representing the included regions.



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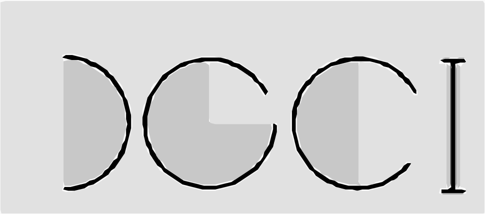
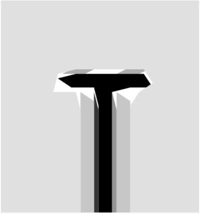


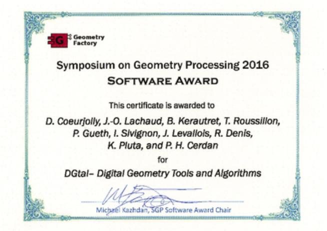


3. DGtal Library Overview

3.1 Short presentation of the library

Origin/evolution: (`www.dgtal.org`)

- DGtal: Digital Geometry tools and Algorithms
- Mainly a French initiative from the Discrete Geometry communauty.
- Born 10 year ago during the IWCIA workshop (end of november 2009)
- C++ based library: work (and tested) on *Linux*, *MacOS* and *Windows*.
- Current version 1.0 (from March 2019).
- SGP Software Award at the Symposium on Geometry Processing:


Geometry Factory

Symposium on Geometry Processing 2016

SOFTWARE AWARD

This certificate is awarded to

D. Coeurjolly, J.-O. Lachaud, B. Kerautret, T. Roussillon, P. Gueth, I. Sivignon, J. Levallois, R. Denis, K. Pluta, and P. H. Cerdan

for

DGtal– Digital Geometry Tools and Algorithms

Michael Kazhdan, SGP Software Award Chair

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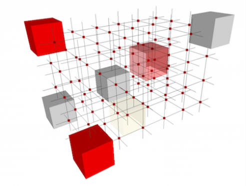
Main Objectives:

- Gathers in a unified setting many data structures and algorithms.
- For the discrete geometry communauty and related (digital topology, image processing, discrete geometry, arithmetic).
- It makes easier the appropriation of our tools for neophytes.
- Simplify comparisons of new methods with already existing approaches.
- Simplifies the construction of demonstration tools.

3.1 Short presentation of the library (2)

Main actual packages:

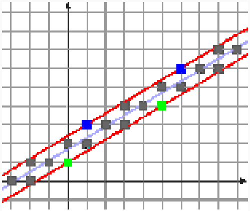
- Kernel package: number types, digital space, domain



3.1 Short presentation of the library (2)

Main actual packages:

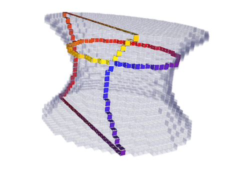
- Kernel package: number types, digital space, domain
- Arithmetic package: standard arithmetic computations
  $\Rightarrow$ greatest common divisor, Bézout vectors, continued fractions, convergent, intersection of integer half-spaces



3.1 Short presentation of the library (2)

Main actual packages:

- Kernel package: number types, digital space, domain
- Arithmetic package: standard arithmetic computations
- Topology package: classic topology tools
  $\Rightarrow$ Rosenfeld oriented tools, cartesian cellular topology, digital surface topology (Herman), tools to extract connected component, simple points,...



3.1 Short presentation of the library (2)

Main actual packages:

- Kernel package: number types, digital space, domain
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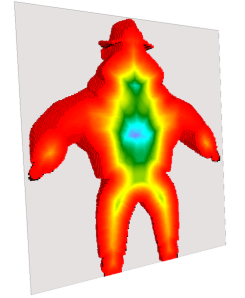
- Geometry package: geometric estimators 2D/3D: $\Rightarrow$ length, normal curvature estimators, 3D transform...



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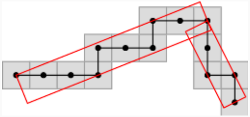

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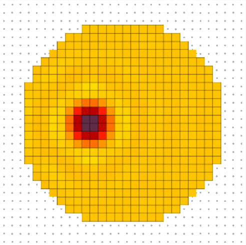
Main actual packages:

- Kernel package: number types, digital space, domain
- Arithmetic package: standard arithmetic computations
- Topology package: classic topology tools
- Geometry package: geometric estimators 2D/3D:
- DEC package: Discrete exterior calculus:
  $\Rightarrow$ provides an easy and efficient way to describe linear operator over various structure



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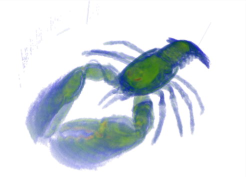
- DEC package: Discrete exterior calculus:
- Board & Viewer package: import/export image and visualization:
  $\Rightarrow$ interactive and non interactive viewer 2d/3d...



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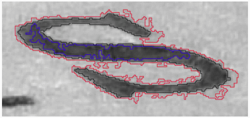
- Board & Viewer package: import/export image and visualization:
- Image package: implement image model and data-structures.



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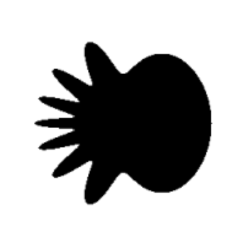
- Board & Viewer package: import/export image and visualization:
- Image package: implement image model and data-structures.
- Shape package: shape related concepts, models and algorithms.
  $\Rightarrow$ generic framework and tools to construct multigrid shapes in DGtal



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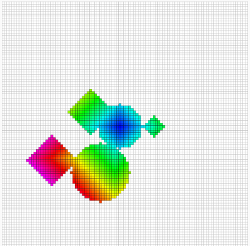

- Image package: implement image model and data-structures.
- Shape package: shape related concepts, models and algorithms.
- Graph package: gathers concepts and classes related to graphs.
  $\Rightarrow$ with wrappers to boost::graph

3.1 Short presentation of the library (2)

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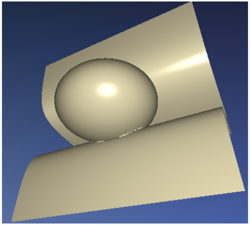
- DEC package: Discrete exterior calculus:
- Board & Viewer package: import/export image and visualization:
- Image package: implement image model and data-structures.
- Shape package: shape related concepts, models and algorithms.
- Graph package: gathers concepts and classes related to graphs.
- Math package: various mathematical subpackages.



3.1 Short presentation of the library (3)

Library organization and details:

- Three main projects:
  - Main DGtal library (`https://github.com/DGtal-team/DGtal`).
  - DGtal-Tools project: contains tools based on DGtal (`https://github.com/DGtal-team/DGtal-Tools`).
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Programming principle:

- Generic Programming.
- Concept, models of concepts and concept checking.

$\Rightarrow$ C++ with template programming

3.1 Short presentation of the library (4)

First example, see: `https://github.com/kerautret/ACPR19-DGPRTutorial`

- Example to read input contour.
- Display the digital contour.
- Export the visualization.

3.1 Short presentation of the library (4)

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(see file: tuto1_baseDGtal.cpp)

```cpp
#include "DGtal/base/Common.h"
#include "DGtal/helpers/StdDefs.h"
// To use the reading of input points:
#include "DGtal/io/readers/PointListReader.h"

// To display graphics elements
#include "DGtal/io/boards/Board2D.h"
...
typedef Z2i::Point Point;
std::vector<Point> contour = PointListReader<Point>::getPointsFromFile("
      contour.sdp");

//Displaying the input read contour:
Board2D aBoard;
for (auto&& p :contour) { aBoard << p; }
aBoard.saveEPS("res.eps");
```

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3.2 Extracting level sets contours with DGtal

Second tutorial exercise (see tuto2_LSC/README.md)

Three main steps in DGtal:

- Create a Khalimsky space:

(see file: tuto2_LSC.cpp)

```
Z2i::KSpace ks;
ks.init(image.domain().lowerBound(),
        image.domain().upperBound(), false);
```

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- Extract a set of pixel of the image:

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  Z2i::DigitalSet set (image.domain());
  SetFromImage<Z2i::DigitalSet>::append(set, image, 0, 108);
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```

- Track intergrid Cell and display them from Freman Chains objects:

```
SurfelAdjacency<2> sAdj(true);
std::vector<std::vector<Z2i::Point>> vCnt;
Surfaces<Z2i::KSpace>::extractAllPointContours4C(vCnt, ks, set, sAdj);
  ...
for (const auto &c: vCnt)
    FreemanChain<int> fc (c);
    ...
```

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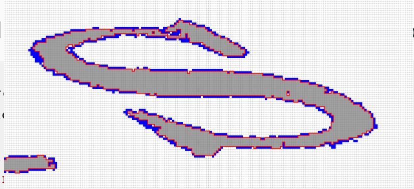

3.3 Example of geometric estimator

Third tutorial exercise (see `tuto3_curvatures/README.md`)

Computing curvature with DCA estimator [Roussillon & Lachaud 11].

⇒ Based on Digital Circular Arcs.

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- Defines types for Range and Iterator on input curve:

(see file: tuto3_curvatures.cpp)

```
typedef GridCurve<>::IncidentPointsRange Range;
typedef Range::ConstIterator ClassicIterator;
Range r = curve.getIncidentPointsRange();
std::vector<double> estimations;
```

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```
typedef GridCurve<>::IncidentPointsRange Range;
typedef Range::ConstIterator ClassicIterator;
Range r = curve.getIncidentPointsRange();
std::vector<double> estimations;
```

- Construct estimator and apply it:

```
SegmentComputer sc;
SCEstimator sce;
CurvatureEstimator estimator(sc, sce);
...

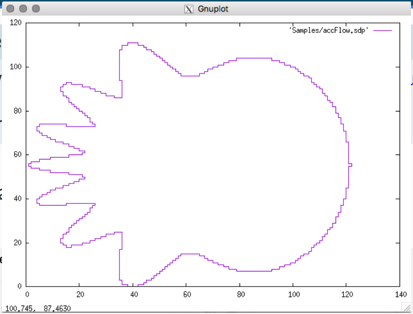
estimator.init( 1, r.begin(), r.end() );
estimator.eval( r.begin(), r.end(),
                std::back_inserter(estimations) );
```

3.3 Example of geometric estimator

Third tutorial exercise

Computing curvature w ...ud 11].

⇒ Based on Digital Cir



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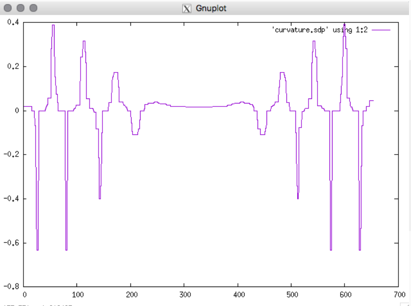

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4. Practical session: Hands on DGtal

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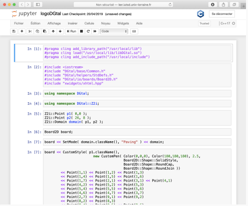
Pratical installation/exercices

Visit Github page: https://kerautret.github.io/ACPR19-DGPRTutorial

Test DGtal online with Jupyter notebook

- http://ker.iutsd.univ-lorraine.fr/notebook
- Login: follow our instructions



Thanks for your attention !

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