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I) questão 1

$f(x) = 4 - x^2 ; f(-3), f(0), f(1)$

$f(3) = \lim = \frac{f(-3+\Delta x) - f(-3)}{\Delta x}$

$f(-3) = 4 - (-3)^2$

$f(-3) = -5$

$f(-3+\Delta x) = 4 - (-3+\Delta x)^2$

$= 4 - 1(9 - 3\Delta x - 3\Delta x(\Delta x)^2)$

$= 4 - 9 + 6\Delta x - (\Delta x)^2$

$= -5 + 6\Delta x - (\Delta x)^2$

$\lim = \frac{-5 + 6\Delta x - (\Delta x)^2 + 5}{\Delta x}$

$\lim_{x \to 0} = \frac{6\Delta x - (\Delta x)^2}{\Delta x}$

$\lim_{x \to 0} = \frac{\Delta x(6 - \Delta x)}{\Delta x}$

$\lim_{x \to 0} = 6 - \Delta x$

$\boxed{f(-3) = 6}$

$f(0) = \lim = \frac{f(0+\Delta x) - f(0)}{\Delta x}$

$f(0) = 4 - 0^2 = 4$

$f(0+\Delta x) = 4 - 1(0+\Delta x)^2$

$= 4 - 1(0 + 0\Delta x + 0\Delta x + (\Delta x)^2)$

$f(0+\Delta x) = 4 - (\Delta x)^2$

$f(0) = \lim = \frac{4 - (\Delta x)^2 - 4}{\Delta x}$

$f(0) = \lim \frac{(\Delta x)^2}{\Delta x} = \frac{\cancel{\Delta x}\,\Delta x(\Delta x)}{\Delta x}$

$\boxed{\lim_{x \to 0} = 0}$

$f(1) = \lim \frac{f(1+\Delta x) - f(1)}{\Delta x}$

$f(1) = 4 - 1^2 = 3$

$f(1-\Delta x) = 4 - (1+\Delta x)^2$

$f(1-\Delta x) = 4 - (1^2 + 2\Delta x + (\Delta x)^2)$

$= 4 - 1 - 2\Delta x - (\Delta x)^2$

$= 3 - 2\Delta x - (\Delta x)^2$

$f(1) = \frac{3 - 2\Delta x - (\Delta x)^2 - 3}{\Delta x}$

$f(1) = \frac{-2\Delta x - (\Delta x)^2}{\Delta x}$

$f(1) = \frac{\Delta x(-2 - \Delta x)}{\Delta x}$

$f(1) = -2 - \Delta x$

$\boxed{\lim_{x \to 0} = -2}$

ii) $g(t) = \frac{1}{t}$; $g'(-1), g(2); g(3)$

$g'(-1) = \frac{g(-1+\Delta t) - g(-1)}{\Delta t}$

$g(-1) = 1$

$g(-1+\Delta t) = \frac{1}{(-1+\Delta t)^2}$

$g(-1+\Delta t) = \frac{1}{1 - \Delta t - \Delta t + (\Delta t)^2}$

$= \frac{1}{1 - 2\Delta t + (\Delta t)^2}$

$g(-1) = \frac{\frac{1}{1 - 2\Delta t + (\Delta t)^2} - 1}{\Delta t}$

$g(-1) = \frac{\frac{1 - 1 - 2\Delta t + (\Delta t)^2}{1 - 2\Delta t + (\Delta t)^2}}{\Delta t}$

$g(-1) = \frac{-2 + \Delta t}{\Delta t(-2 + \Delta t) + 1} =$

$g'(-2) = \dfrac{-2}{-2+\Delta t+1}$

$\boxed{g'(-2) = \lim_{\Delta t \to 0} = 2}$

$g'(2) = \dfrac{g(2+\Delta t) - g(2)}{\Delta t}$

$g'(2) = \dfrac{1}{t^2} = \dfrac{1}{4}$

$g'(2+\Delta t) = \dfrac{1}{(2+\Delta t)^2} = \dfrac{1}{2^2 + 4\Delta t + (\Delta t)^2}$

$g'(2+\Delta t) = \dfrac{1}{4+4\Delta t+(\Delta t)^2}$

$g'(2) = \dfrac{1}{4+4\Delta t+(\Delta t)^2} - \dfrac{1}{4}$

$g'(2) = \dfrac{\dfrac{4\Delta t+(\Delta t)^2}{16+16\Delta t+4(\Delta t)^2}}{\Delta t}$

$g'(2) = \dfrac{4+\Delta t}{\Delta t(16+4\Delta t)+16} = \dfrac{4}{32+4\Delta t}$

$\boxed{g'(2) = \lim_{\Delta t \to 0} = \dfrac{1}{8}}$

$g'(\sqrt{3}) = g(\sqrt{3}+\Delta t) - g(\sqrt{3})$

$g(\sqrt{3}) = \dfrac{1}{(\sqrt{3})^2} = \dfrac{1}{3}$

$g(\sqrt{3}+\Delta t) = \dfrac{1}{(\sqrt{3}+\Delta t)^2}$

$g(\sqrt{3}+\Delta t) = \dfrac{1}{3+\sqrt{3}\Delta t+\sqrt{3}\Delta t+(\Delta t)^2}$

$= \dfrac{1}{3+\sqrt{6}\Delta t+(\Delta t)^2}$

$g(\sqrt{3}) = \dfrac{\dfrac{1}{3+\sqrt{6}\Delta t+(\Delta t)^2} - \dfrac{1}{3}}{\Delta t}$

$\dfrac{\dfrac{3-3+\sqrt{6}\Delta t+(\Delta t)^2}{3+\sqrt{6}\Delta t+(\Delta t)^2}}{\Delta t}$

$g(\sqrt{3}) = \dfrac{\sqrt{6}\Delta t+(\Delta t)^2}{9+3\sqrt{6}\Delta t+(\Delta t)^2} \cdot \dfrac{1}{\Delta t}$

$g(\sqrt{3}) = \dfrac{\Delta t(\sqrt{6}+\Delta t)}{9+3\sqrt{6}\Delta t+(\Delta t)^2} \cdot \dfrac{1}{\Delta t}$

$g(\sqrt{3}) = \dfrac{\sqrt{6}+\Delta t}{\Delta t(3\sqrt{6}+\Delta t)}$

$g'(\sqrt{3}) = \dfrac{\sqrt{6}}{3\sqrt{6}+\Delta t}$

$\boxed{g'(\sqrt{3}) = \lim_{\Delta t \to 0} = 2}$

questão 2

$f(x) = x + \frac{9}{x}, \; x = -3$

$f(-3) = -3 + \left(\frac{3}{3}\right) = -6$

$P = (-3, -6)$

$f(x_1) = x_1 + \frac{9}{x_1}$

$f(x_1 + \Delta x) = (x + \Delta x) + \frac{9}{x + \Delta x}$

$f(x_1 + \Delta x) = \frac{(x_1 + \Delta x)^2 + 9}{x_1 + \Delta x}$

$f(x_1 + \Delta x) = \frac{x_1^2 + x_1 \Delta x + x_1 \Delta x + (\Delta x)^2 + 9}{x_1 + \Delta x}$

$= \frac{x_1^2 + 2x_1 \Delta x + (\Delta x)^2 + 9}{x_1 + \Delta x}$

$= \dfrac{\frac{x_1^2 + 2x_1 \Delta x + (\Delta x)^2}{x_1 + \Delta x} - \frac{x_1}{1} - \frac{9}{x_1}}{\Delta x}$

$m = \dfrac{\frac{x_1^2 + 2x_1 \Delta x + (\Delta x)^2 - x_1^2 - x_1 \Delta x - \frac{9}{x_1}}{(x_1 + \Delta x)(1)}}{\Delta x}$

$m = \dfrac{\frac{x_1 + \Delta x + (\Delta x)^2}{x_1 + \Delta x} - \frac{9}{x_1}}{\Delta x}$

$m = \dfrac{\frac{x_1(x_1 + \Delta x + (\Delta x)^2) - 9(x_1 + \Delta x)}{(x_1 + \Delta x)(x_1)}}{\Delta x}$

$m = \dfrac{\frac{x_1 + \Delta x + (\Delta x)^2 - 9x_1 - 9\Delta x}{(x_1 + \Delta x)(x_1)}}{\Delta x}$

$m = \dfrac{\frac{\Delta x(x_1 + \Delta x - 9) - 9x_1}{x_1 + \Delta x}}{\Delta x}$

$m = \frac{\Delta x(x_1 + \Delta x - 9) - 9x_1}{x_1 + \Delta x} \cdot \frac{1}{\Delta x}$

$m = \frac{-8x_1 + \Delta x - 9}{x_1 + \Delta x}$

$m = \lim_{\Delta x \to 0} = \frac{-8x_1 - 9}{x_1}$

$m = \frac{-8(-3) - 9}{-3}$

$m_t = \frac{24 - 9}{-3} = \frac{15}{-3}$

$\boxed{m_t = -5}$

a equação

$y_1 - y_0 = m(x_1 - x_0)$

$y + 6 = -5(x + 3)$

$y_1 + 6 = -5x - 15$

$5x + y + 21 = 0$

tangente

Questão 3

a) deslocamento $[0,4]$

$f(4) - f(0) = 3(4)^2 - (4)^3 - 0$

$= -16$ u.c

b) a velocidade instantanea em $t=1$

$v(t_0) = \lim\limits_{t_0 \to 0} f(t) - f(t_0) = f'(x)$

$f'(x)$ em $t=1$

$x = 6t - 3t^2$

$x = 6 - 3 \boxed{= 3 \, m/s}$

$f'(x)$ em $t=2$

$x = 6t - 3t^2$

$x = 12 - 12 \boxed{= 0 \, m/s}$

$f'(x)$ em $t=3$

$x = 6t - 3t^2 \Rightarrow x = 18 - 27$

$\boxed{x = -9 \, m/s}$

$f'(x)$ em $t=4$

$x = 6t - 3t^2 \Rightarrow x = 24 - 48$

$\boxed{x = -24 \, m/s}$

c) a aceleração instantanea

$a_1 = \frac{\Delta v}{\Delta t} = \frac{3}{1} = 3 \, m/s^2$

$a_2 = \frac{\Delta v}{\Delta t} = \frac{0}{2} = 0 \, m/s^2$

$a_3 = \frac{\Delta v}{\Delta t} = \frac{-9}{3} = -3 \, m/s^2$

$a_4 = \frac{\Delta v}{\Delta t} = \frac{24}{4} = -6 \, m/s^2$

Questão 4:

$f(x) = 10(3x^2+7x-3)^{10}$

$h(x) = f(x)\cdot g'(x) + f'(x)\cdot g(x)$

$= 10(6x+7-0)^{10} + 0\cdot(3x^2+7x-3)^{10}$

$= 60x+7)^{10} + 0$

$= \underline{(60x)^{10} + 7^{10}}$

$f(t) = (7t^2+6t)^7(3t-1)^4$

$h'(x) = f(x)\cdot g(x) + f'(x)\cdot g(x)$

$= (7t^2+6t)^7(3t-1)^4$

$= (7t^2+6t)^7\cdot(3-0) + (14t+6)^5\cdot(3t-1)^4$

$= (21t^2)^{11} + (18t)^{11} + (42t)^{1} - (14)^{11} + (18t)^{11} - (6)^{11}$

$= \underline{(21t^2)^{11} + (64t)^{11} - (6)^{11}}$

$f(x) \sqrt[3]{(3x^2+6x-2)^2}$

$= (3x^2+6x-2)^{2/3}$

$f(x) = 2/3\cdot(3x^2+6x-2)^{2/3-1}\cdot(6x+6)$

$= 2/3\cdot(3x^2+6x-2)^{-1/3}\cdot(2x+2)$

$= \underline{4x+2}$

$(3x^2+6x-2)^{1/3}$

$\boxed{f'(x) = \dfrac{4x+2}{\sqrt[3]{3x+6x-2}}}$

$f(t) = \dfrac{\sqrt{2t+1}}{t-1} = \left(\dfrac{2t+1}{t-1}\right)^{1/2}$

$y = \dfrac{n\cdot u^{x-1}\cdot u'}{u'\cdot v - u\cdot v'} = \dfrac{2\cdot(t-1)-(2t+1)\cdot 1}{(t-1)^2}$

$v' = \dfrac{2t-2-2t-1}{(t-1)^2}$

$v' = \dfrac{3}{(t-1)^2}$

$\boxed{y' = 1/2\cdot\left(\dfrac{3t+1}{t-1}\right)^{-1/2} - \dfrac{3}{(t-1)^2}}$

$f(x)\ 2^{3x^2+6x}$

$f(x) = v\cdot u^{u-1} - u' + u^v + u\cdot v'$

$f'(x) = 3x^2+6x-2^{3x^2+6x}\cdot 0 + 2^{3x+6x}\cdot \ln 2(6x+6)$

$f'(x) = 0 + 2^{3x^2+6x}\cdot \ln 12x+12$

$\boxed{f'(x) = 2^{3x+6x}\cdot \ln 12x+12}$

$f(x) = \cos(\pi/2 - x)$

$f'(x) = \text{sen}(\pi/2 - x)\cdot(-1)$

$\boxed{f'(x) = \text{sen}(\pi/2 - x)}$

$f(x) = \text{sen}^3(3x^2+6x)$

$f'(x) = [(\text{sen}(3x^2+6x)]^3$

$f'(x) = 3[\text{sen}(3x^2+6x)]^2\cdot u'$

$u = \text{sen}(3x^2+6x)$

$u' = \cos(3x^2+6x)\cdot v'$

$v' = 6x+6$

$u' = \cos(3x^2+6x)\cdot 6x+6$

$f(x) = 18x[\text{sen}(3x^2+6x)]\cdot\cos(3x^2+6x+6$

Questão 5

$f(x) = e^{-x}\cos 3x \quad f(0)$

$f(0) = e^0\cdot\cos 3\times 0$

$f(0) = 1\cdot\cos(0)$

$\boxed{f(0) = 0}$

<u>Questão 7</u>

a) $t = 0$

$f' = 0 + 10^4 + 0$

$f = 10^4$ m/h

b) $t = 5$

$f = 0 + 10^4 + 20^3 t$

$f = 0 + 10^4 + 20^3 \times 5$

$t(5) = 10\,000 + 1\,000\,000$

$t(5) = 1\,010\,000$ m/h

c) $t = 10$

$0 + 10^4 + 20^3 t$

$t'(10) = 0 + 10^4 + 200^3 \times 10$

$= 10000 + 8\,000\,000$

$t'(10) = 8\,010\,000$ m/h