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Questão 1

i) calcule os seguintes funções:

i) $\int (9t^2 + \frac{1}{\sqrt{t^3}})\, dt$

$\int 9t^2\, dt + \int \frac{1}{\sqrt{t^3}}$ $\quad \sqrt{t^3} = t^{3/2}$

$9 \cdot \frac{t^{2+1}}{3} + \frac{t^{-3/2+1}}{-1/2}$

$9 \cdot \frac{t^3}{3} + \frac{t^{-1/2}}{-1/2}$

$\boxed{3t^3 - 2t^{-1/2} + c}$

ii) $\int (ax^4 + 5x^3 + 3c)\, dx$

$a\int x^4\, dx + 5\int x^3\, dx + 3c$

$a\frac{x^{4+1}}{5} + 5\frac{x^{3+1}}{4} + 3c$

$\boxed{a\frac{x^5}{5} + 5\frac{x^4}{4} + 3c}$

iii) $\int (\sqrt{2y} - \frac{1}{\sqrt{2y}})\, dx$

$\int \sqrt{2y} - \int \frac{1}{\sqrt{2y}} + c$

$\int 2y^{1/2} - \int \frac{2y^{-1/2+1}}{1/2}$

$\int \frac{2y^{1/2+1}}{3/2} - \int \frac{2y^{1/2}}{1/2}$

$\frac{2y^{3/2}}{3/2} - \frac{2y^{1/2}}{1/2}$

$\boxed{\frac{4y^{3/2}}{3} - 4y^{1/2} + c}$

IV) $\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx$

$\int x + \int 2x^{-2} - \int x^{-4}$

$\frac{x^{1+1}}{2} + \frac{2x^{-2+1}}{-2+1} - \frac{x^{-4+1}}{-4+1}$

$\boxed{\frac{x^2}{2} + \frac{2}{x} + \frac{1}{3x^3} + c}$

VI) $\int \frac{8x^4 - 9x^3 + 6x^2 - 2x + 1}{x^2}\, dx$

$\int 8x^2 - \int 9x + \int 6 - \int 2x^{-1} + \int x^{-2}$

$\frac{8x^{2+1}}{3} - \frac{9x^{1+1}}{2} + 6x - \frac{2x^{-1+1}}{-1+1} + \frac{x^{-2+1}}{-2+1}$

$\boxed{\frac{8x^3}{3} - \frac{9x^2}{2} + 6x + \frac{1}{x} + c}$

Questão 2

encontra uma primitiva F da função $f(x) = x^{2/3} + x$ que satisfaz $F(1) = 1$

$f(x) = x^{2/3} + x$

$$F(x) = \int x^{2/3} + x \cdot dx = \frac{x^{2/3+1}}{2/3+1} + \frac{x^{1+1}}{1+1}$$

$$= \frac{x^{5/3}}{5/3} + \frac{x^2}{2} = \frac{3x^{\frac{5}{3}}}{5} + \frac{x^2}{2}$$

$$\boxed{F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} + c}$$

$F(1) = 1$

$$1 = \frac{3\sqrt[3]{1^5}}{5} + \frac{1^2}{2} + c = \frac{3}{5} + \frac{1}{2} + c$$

$$1 = \frac{11}{10} + c \Rightarrow 1 - \frac{11}{10} = c$$

$c = -\frac{1}{10}$, a função é $F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} - \frac{1}{10}$

questão 3

Determina a função f(x) tal que

$\int f(x)\,dx = x^2 + \frac{1}{2}\cos 2x + c$

Derivando a integral

$y = x^2 + \frac{1}{2}\cos 2x + c$

$y' = 2x + \frac{1}{2}\cdot(-\operatorname{sen} 2x \cdot 2)$

$y' = 2x - \operatorname{sen} 2x$

verificando

$\int 2x - \operatorname{sen} 2x \, dx$

$2\cdot\frac{x^2}{2} - \frac{1}{2}\cdot(-\cos 2x) + c$

questão 4

usando metodo de substituição

I) $\int (2x^2+2x-3)^{10}(2x+1)\,dx$

$u = 2x^2+2x-3$

$u = 4x+2-3$

$u = 4x+2$

$dx = \frac{du}{4x+2}$

$\int u^{10}\cdot 2x+1\cdot\frac{du}{4x+2}$

$\int u^{10}\,du$

$\frac{u^{11}}{11} + c$

$\boxed{\frac{(2x^2+2x-3)^{11}}{11} + c}$

tilibra

II) $\int 5x\sqrt{4-3x^2}\,dx$

$u=4-3x^2$

$u=-6x$

$\frac{du}{dx}=-6x$

$dx=\frac{du}{-6x}$

$\int 5x\cdot\sqrt{u}\cdot\frac{du}{-6x}$

$-5/6\int \sqrt{u}\,du$

$-5/6\int \frac{u^{1/2+1}}{3/2}+c$

$\frac{-5}{6}\cdot\frac{2^{3/2}}{3}\cdot u^{3/2}+c$

$\boxed{\frac{-5\,u^{3/2}}{9}+c}$

III) $\int (e^{2t}+2)^{1/3}e^{2t}\,dt$

$u=e^{2t}+2$

$\frac{du}{dt}=2e^{t}+0$

$du=2e^{2t}\,dt$

$\frac{du}{2}=e^{2t}\,dt$

$\int u^{1/3}\frac{du}{2}=1/2\int u^{1/3}\,du$

$1/2\,\frac{u^{4/3}}{4/3}=3/8\,u^{4/3}$

$\boxed{\frac{3}{8}(e^{2t}+2)^{4/3}+C}$

IV $\int sen^4 x\cdot \cos x\,dx$

$u=sen\,x$

$\frac{du}{dx}=\cos x$

$du=\cos u\,du$

$\int u^2\cdot du=\frac{u^3}{3}+c$

$\boxed{=\frac{sen^5 x}{5}+c}$

V $\int \frac{sen\,x}{\cos^5 x}\,dx$

$u=\cos x$

$du=-sen\,x$

$dx=\frac{du}{sen\,x}$

$\int \frac{sen\,x}{u^5}\left(-\frac{du}{sen\,x}\right.$

$\int -\frac{du}{u^5}$

$u^{5}\cdot du$

$\frac{-u^{-5+1}}{-5+1}+c$

$\frac{-u^{-4}}{-4}+c$

$\frac{1}{4u^4}+c$

voltando $u=\cos x$

$\frac{1}{4\cos^4}+c$

ou

$\boxed{\frac{1}{4}\sec^4+c}$

VI) $\int e^x \cos 2e^x \, dx$

$u = \cos 2e^x$

$du = \dfrac{\operatorname{sen} e^x}{2e^x}$

$dv = e^x \Rightarrow v = e^x$

$\dfrac{\operatorname{sen} 2e^x}{2e^x} + \int e^x \, dx$

$\dfrac{\operatorname{sen} 2e^x}{2e^x} + e^x + c$

$\boxed{\dfrac{1}{2} \operatorname{sen} 2e^x + c}$

questão 5

Calcule as integrais usando metodo por partes

I) $\int x \operatorname{sen}^5 x \, dx$

$u = x \quad dv = \operatorname{sen} 5x \, dx$

$u = \dfrac{-\cos 5x}{5}$

$-x \cos(5x) + \int \dfrac{\cos(5x) dx}{5}$

$\boxed{-\dfrac{x}{5} \cos 5x + \dfrac{\operatorname{sen} 5x}{5} + c}$

III) $\int x \ln 3x \, dx$

$u = \ln x \quad dv = x$

$du = \dfrac{dx}{3x} \quad v = 1/2 x^2$

$\ln x \cdot \frac{1}{2} x^2 - \int \frac{1}{2} x^2 \cdot \frac{dx}{3x}$

$\frac{1}{2} x^2 \cdot \ln x - \int \frac{1}{2} x \cdot \frac{dx}{3}$

$\Rightarrow \boxed{\frac{1}{2} \ln(3dx^2) x^2 - \frac{1}{2} x^2 + c}$

IX) $\int x^2 \cos x \, dx$

$u = x^2 \qquad dv = \cos x \, dx$

$du = \frac{dx}{2x} \qquad v = \frac{1}{a} \text{fecodo} (ax)$

$x^2 \cdot \cos x - \int \frac{1}{a} \text{fecodo}(ax) \frac{dx}{2x}$

$\boxed{(x^2 \cdot \cos ax - \frac{1}{a} \text{fecodo}(ax) 2x + C}$

X) $\int x^2 e^x \, dx$

$u = x^2 \quad dv = e^x$

$\frac{du}{dx} = 2x \quad v = e^x$

$x^2 \cdot e^x - \int e^x \cdot \frac{dx}{2x} \Rightarrow \boxed{x^2 \cdot e^x - 2xe^x + 2e^x + C}$

Questão 8

III) $y = \text{sen}\, x$ e $y = -\text{sen}\, x$, $y \in [0, 2\pi]$

Delimitada, ficaria assim

$\{(x, y) | 0 \leq x \leq 2\pi, \text{sen}\, x \leq y \leq \text{sen}\, x\}$

i) $\int_0^{2\pi} [y]_{-\text{sen}\, x}^{\text{sen}\, x} \, dx$

ii) $\int_0^{2\pi} 2 \text{sen}\, x \, dx = [-2\cos x]_0^{2} = -2\cos 2\pi + 2\cos 0$

$= 4 \, u.a$