

Kerby Lovince

data

5 1 0 0 5 0

Nome: Kerby Lovince, derivado em relação y

Questão 1

$$f_y(x,y) = \frac{1}{(x^2+y^2)^{3/2}} \cdot (x^2+y^2)^{1/2}$$

em $\sqrt{x^2+y^2}$

em $(x^2+y^2)^{1/2}$

$$f_y(x,y) = \frac{1}{(x^2+y^2)^{1/2} \cdot \frac{1}{2} \cdot 2y \cdot (x^2+y^2)^{1/2-1}}$$

derivado parcial em relação a x:

$$f_x(x,y) = \frac{1}{(x^2+y^2)^{3/2}} \cdot (x^2+y^2)^{1/2}$$

$$= \frac{1}{(x^2+y^2)^{1/2} \cdot \frac{1}{2} \cdot 2x \cdot (x^2+y^2)^{1/2-1}}$$

$$= \frac{1}{(x^2+y^2)^{1/2} \cdot x \cdot (x^2+y^2)^{-1/2}}$$

$$= \frac{x}{(x^2+y^2)^{1/2}} \times \frac{1}{(x^2+y^2)^{1/2}}$$

$$= \frac{x}{(x^2+y^2)}$$

ou seja

~~$$= \frac{x}{(x^2+y^2)}$$~~

$$= \frac{1}{(x^2+y^2)} \cdot \frac{2y}{2} \cdot (x^2+y^2)^{-1/2}$$

$$= \frac{1}{(x^2+y^2)^{1/2}} \cdot y \cdot (x^2+y^2)^{-1/2}$$

$$= \frac{y}{(x^2+y^2)^{1/2}} \cdot \frac{1}{(x^2+y^2)^{1/2}}$$

$$= \frac{y}{(x^2+y^2)}$$

Questão 4

$$x^3 - xy + 4xz - 5 = 0$$

F é diferenciável

$$\frac{\partial F}{\partial y}(x, y) = 3x^2 - y + 4z,$$

$$\frac{\partial F}{\partial z}(x, y) = -x$$

$$\text{e } \partial F(x, y) = 4x \neq 0$$

$$F(x, y) = 0 \Rightarrow$$

$$\frac{\partial F}{\partial x}(x, y, z) = \frac{\partial x}{\partial x} + \frac{\partial F}{\partial y}(x, y, z)$$

$$\frac{\partial y}{\partial x} + \frac{\partial F}{\partial z}(x, y, z) \cdot \frac{\partial z}{\partial x}(x, y) = 0$$

como x e y são

variáveis e não
funções $\frac{\partial x}{\partial x} = 1$ e

$$\frac{\partial y}{\partial x} = 0$$

Questão 5

$$f) f(x, y) = \frac{x+y}{x-y}$$

$$\frac{u}{v} = \frac{u'v - uv'}{v^2}$$

calculando a derivada em
relação a x

$$= \frac{(x+y)(x-y) - \frac{\partial}{\partial x}(x-y)(x+y)}{(x-y)^2}$$

$$= \frac{1 \cdot (x-y) - 1 \cdot (x+y)}{(x-y)^2} = \frac{x-y-x-y}{(x-y)^2}$$

$$\frac{\partial f}{\partial x} = \frac{-2y}{(x-y)^2}$$

derivando em relação a y

$$f_y(x, y) = \frac{\partial}{\partial y} \left(\frac{x+y}{x-y} \right) = \frac{\frac{\partial}{\partial y}(x+y)(x-y) - \frac{\partial}{\partial y}(x-y)(x+y)}{(x-y)^2}$$

$$= \frac{1 \cdot (x-y) - (-1) \cdot (x+y)}{(x-y)^2}$$

$$= \frac{x-y - (-x-y)}{(x-y)^2} = \frac{x-y+x+y}{(x-y)^2}$$

$$= \frac{2x}{(x-y)^2}$$

Questão 4

data

1 1 0 0 1 1 0

a) +101

$$\frac{\partial}{\partial x} (\sin(x, y, z) + x^2 \cdot \frac{y^3}{z})$$

$$= \frac{\partial}{\partial x} (\sin(x, y, z)) + \frac{\partial}{\partial x} \left(\frac{x^2 y^3}{z} \right)$$

Aplicando a soma:

$$\frac{\partial}{\partial x} (\sin(x, y, z)) = \cos(x, y, z) y z$$

$$\frac{\partial}{\partial x} \left(\frac{x^2 y^3}{z} \right) = \frac{2 y^3 x}{z}$$

$$\frac{\partial}{\partial x} = \cos(x, y, z) y z + \frac{2 y^3 x}{z}$$

$$\frac{\partial}{\partial y} (\cos(x, y, z) y z) + \frac{2 y^3 x}{z}$$

$$= \frac{\frac{\partial}{\partial y} (\cos(x, y, z) y z) z}{z} + \frac{2 y^3 x}{z}$$

$$\frac{\partial}{\partial y} = \frac{\frac{\partial}{\partial y} (\cos(x, y, z) y z) z + 2 y^3 x}{z}$$

$$\frac{\partial}{\partial x} = \frac{1}{z} \frac{\partial}{\partial x} [(x, y, z) \left(\frac{\partial}{\partial y} (\cos(x, y, z) y z) + 2 y^3 x \right)]$$

$$\frac{\partial}{\partial x} = (\cos(x, y, z) y z) = z^2 (-2 z x^2 \sin(z x^2) + \cos(z x^2) + 2 y^3 x)$$

$$\frac{\partial}{\partial y} (\cos(x, y, z) y z) = z^2 (-2 z x^2 \sin(z x^2) + \cos(z x^2))$$

$$\frac{\partial}{\partial x} = \frac{1}{z} (y, z, 1) (z^2 (-2 z x^2 \sin(z x^2) + \cos(z x^2) + 2 y^3 x) + (-z^2 (2 x \sin(z x^2) + 2 z x^3 \cos(z x^2) + 2 y^3)) (x, y, z))$$

b) f222

$$\frac{\partial}{\partial y} (\sin(xyz) + x^2 y^3 / z)$$

$$= \frac{\partial}{\partial y} (\sin(xyz) + \frac{\partial}{\partial y} (x^2 y^3 / z))$$

$$\frac{\partial}{\partial y} \sin(xyz) = \cos(xyz) xz$$

$$\frac{\partial}{\partial y} \left(\frac{x^2 y^3}{z} \right) = \frac{3x^2 y^2}{z} \quad \Rightarrow \quad \boxed{\cos(xyz) xz + \frac{3x^2 y^2}{z}}$$

$$= \frac{\frac{\partial}{\partial y} (\cos(xyz) xz) z}{z} + \frac{3x^2 y^2}{z}$$

$$\boxed{\frac{\partial}{\partial y} = \frac{\cos(xyz) xz + 3x^2 y^2}{z}}$$

Retirando constante

$$= \frac{1}{z} \left(\frac{\partial}{\partial x} (\cos(xyz) xz) + \frac{\partial}{\partial x} (3x^2 y^2) \right)$$

$$\frac{\partial}{\partial x} \cos(xyz) xz = z^2 (-yz \sin(yz) + \cos(yz))$$

$$\frac{\partial}{\partial x} (3x^2 y^2) = 6xy^2$$

$$\boxed{= z^2 (-xyz \sin(xyz) + \cos(xyz)) + 6xy^2}$$

Q/ 1332

$$\frac{\partial}{\partial z} \left(\sin(\pi y z) + \frac{\pi^2 y^3}{z} \right)$$

$$= \frac{\partial}{\partial z} \left(\sin(\pi y z) + \frac{\pi^2 y^3}{z} \right)$$

$$= \frac{\partial}{\partial z} (\sin(\pi y z)) = \cos(\pi y z) \pi y$$

$$= \cos(\pi y z) \pi y - \frac{\pi^2 y^3}{z^2}$$

$$\frac{\partial}{\partial z} (\cos(\pi y z) \pi y z^2 - \frac{\pi^2 y^3}{z^2})$$

$$\frac{\partial}{\partial z} (\cos(\pi y z) \pi y z^2 - \pi^2 y^3)$$

$$\frac{\partial}{\partial y} = \frac{1}{z^2} \left(\frac{\partial}{\partial y} (\cos(\pi y z) \pi y z^2) - \frac{\partial}{\partial y} (\pi^2 y^3) \right)$$

$$= \pi z^2 (-\pi z \sin(\pi z y) + \cos(\pi z y))$$

$$\frac{\partial}{\partial y} (\pi^2 y^3) = 3\pi^2 y^2$$

$$= \pi z^2 (-\pi y z \sin(\pi y z) + \cos(\pi y z)) - 3\pi^2 y^2$$

Questão 5

data . .

5 1 0 0 5 5 0

a) $f(x,y) = x^3 e^x + 10y$
derivando em relação
a x

$$f(x,y) = \frac{\partial}{\partial x} (x^3 e^x) + \frac{\partial}{\partial x} (10y)$$

$$= (x^3 e^x)' = 3x^2 e^x + e^x x^3$$

$$\Rightarrow (10y)' = 0$$

$$= \underline{\underline{3x^2 e^x + e^x x^3 + 0}}$$

Derivando em relação y

$$f(x,y) = \frac{\partial}{\partial y} (x^3 e^x) + \frac{\partial}{\partial y} (10y)$$

$$\frac{\partial}{\partial y} (10y) = 10$$

$$= 0 + 10$$

$$= \underline{\underline{10}}$$

b) $f(x,y) = 2y^2 \ln(x)$

derivando em relação a x

$$= 2y^2 \frac{\partial}{\partial x} \ln(x)$$

$$\ln x = 1/x$$

$$= 2y^2 \frac{1}{x}$$

$$= 2y^2 \frac{1}{x} \cdot \frac{2y^2}{x} = \underline{\underline{\frac{2y^2}{x}}}$$

derivando em relação

a y

$$\frac{\partial}{\partial y} 2y^2 \ln(x)$$

$$f(x,y) = 2 \ln(x) \frac{\partial}{\partial y} (y^2)$$

$$= 2 \ln(x) \cdot 2y^{2-1}$$

$$= \underline{\underline{4y \ln(x)}}$$

c) $f(x,y) = 3y^2 \cos(x)$

derivando em relação a x

$$= 3y^2 \frac{\partial}{\partial x} \cos(x)$$

$$\cos x = -\sin(x)$$

$$f(x,y) = 3y^2 (-\sin(x))$$

$$= \underline{\underline{-3y^2 \sin(x)}}$$

derivando em relação y

$$f_y(x,y) = 3 \cos(x) \frac{\partial}{\partial y} (y^2)$$

$$= 3 \cos(x) \cdot 2y^{2-1}$$

$$\underline{\underline{f_y(x,y) = 6y \cos(x)}}$$

d) $4y^2 e^y + 6x^2$

derivada em relação a x

$$f_x(x,y) = \frac{\partial}{\partial x} (4y^2 e^y) + \frac{\partial}{\partial x} (6x^2)$$

$$\frac{\partial}{\partial x} (4y^2 e^y) = 0$$

$$\frac{\partial}{\partial x} (6x^2) = 12x$$

$$f_x(x,y) = 0 + 12x$$

$$\underline{\underline{f_x(x,y) = 12x}}$$

Jandaia

derivando em relação

a y

$$\frac{\partial}{\partial y}(4y^2e^4 + 6x^2)$$

$$f_y(x,y) = \frac{\partial}{\partial y}(4y^2e^4) + \frac{\partial}{\partial y}(6x^2)$$

$$\frac{\partial}{\partial y}(4y^2e^4) = 0$$

$$4(2ye^4 + e^4y^2)$$

$$\frac{\partial}{\partial y}(6x^2) = 0$$

$$f_y(x,y) = 4(2ye^4 + e^4y^2) + 0$$

$$f_y(x,y) = 4(e^4y^2 + 2e^4y)$$

$$E) f(x,y) = 20x^2y^2 \sin(x)$$

derivando em relação a x

$$f(x,y) = 20y^2 \frac{\partial}{\partial x}(x^2 \sin(x))$$

$$(t \cdot g)' = t' \cdot g + t \cdot g'$$

$$t = x^2 \cdot g = \sin(x)$$

$$= 20y^2 \left(\frac{\partial}{\partial x} \right) (x^2) \sin(x)$$

$$\frac{\partial}{\partial x} \sin(x)(x^2)$$

$$\frac{\partial}{\partial x}(x^2) = 2x$$

$$\frac{\partial}{\partial x}(\sin(x)) = \cos$$

$$= 20y^2(2x \sin(x) + \cos(x) \cdot x^2)$$

derivando em relação a y

$$\frac{\partial}{\partial y} = 20x^2y^2 \sin(x)$$

$$f(x,y) = 20x^2 \sin(x) \frac{\partial}{\partial y}(y^2)$$

$$f_y(x,y) = 20x^2 \sin(x) + 2y^{2-1}$$

$$f_y(x,y) = 40x^2y \sin(x)$$

$$g) f(x,y) = \frac{e^x}{2x+3y}$$

derivando em relação a x

$$\frac{u'}{v} = \frac{u'v - v'u}{v^2}$$

$$f(x,y) = \frac{\frac{\partial}{\partial x}(e^x)(2x+3y) - \frac{\partial}{\partial x}(2x+3y)}{(2x+3y)^2}$$

$$\frac{\partial}{\partial x}(e^x) = e^x$$

$$\frac{\partial}{\partial x}(2x+3y) = 2$$

$$= \frac{e^x(2x+3y) - 2e^x}{(2x+3y)^2}$$

derivando em relação a y,

$$f_x(x, y) = e^x \frac{\partial}{\partial y} \left(\frac{1}{2x+3y} \right) \quad \text{ps: } \frac{1}{a} = a^{-1}$$

$$f_y(x, y) = e^x \frac{\partial}{\partial y} (2x+3y)^{-1}$$

Aplicar regra da cadeia

$$= \frac{1}{(2x+3y)^2} \frac{\partial}{\partial y} 2x+3y$$

$$\frac{\partial}{\partial y} (2x+3y) = 3$$

$$= e^x \left(- \frac{1}{(2x+3y)^2} \cdot 3 \right) = - \frac{3e^x}{(2x+3y)^2}$$

$$f_y(x, y) = \underline{\underline{- \frac{3e^x}{(2x+3y)^2}}}$$

Questão 2

$$a) f(x, y) = x^2 y^3 + x^3 y^2$$

$$= \left(\frac{\partial}{\partial x} \right) + \left(\frac{\partial}{\partial y} \right) + \left(\frac{\partial}{\partial t} \right) + \left(\frac{\partial}{\partial s} \right)$$

$$\begin{cases} x(t) = 1/t \\ y(t) = 1/t^2 \end{cases}$$

$$= \left(\frac{\partial}{\partial x} \right) + \left(\frac{\partial}{\partial y} \right) + \left(\frac{\partial}{\partial t} \right)$$

$$\frac{dy}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$a) \frac{d}{dt} = \left(\frac{\partial}{\partial x} \right) + \left(\frac{\partial}{\partial y} \right) + \left(\frac{\partial}{\partial t} \right)$$

$$\frac{\partial f}{\partial x} = 2x \cdot y^3 + 3x^2 \cdot y^2$$

$$b) f(x, y) = e^{x+y}$$

$$\frac{\partial f}{\partial y} = 3y^2 \cdot x^2 + 2y \cdot x^3$$

$$\begin{cases} x(t) = t^2 \\ y(t) = 2t^3 - 1 \end{cases}$$

$$\text{Sabe que: } 1/t = t^{-1}$$

$$e^{1/t} = t^{-1}$$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = -1 \cdot t^{-2} = t^{-2} = -1/t^2$$

$$= e^{x+y} \cdot 2t + e^{x+y} \cdot 6t^2$$

$$= e^{x+y} \cdot (6t^2 + 2t)$$

$$\frac{dy}{dt} = -2t^{-2-1} = -2t^{-3} = -\frac{2}{t^3}$$

$$\frac{dz}{dt} = e^{t^2+2t-1} \cdot (6t^2+2t)$$

$$\frac{dx}{dt} = (2x \cdot y^3 + 3x^2 \cdot y^2) \cdot (-1/t^2) +$$

$$(3y^2 \cdot x^2 + 2y \cdot x^3) \cdot (-2/t^2)$$

trocamos x e y

$$= \frac{2}{1} \cdot \left(\frac{1}{t} \right) \cdot \left(\frac{1}{t^2} \right)^3 + \frac{3}{1} \cdot \left(\frac{1}{t} \right)^2 \cdot \left(\frac{1}{t} \right)^2 + \frac{2}{1} \cdot \left(\frac{1}{t} \right)^2 \cdot \left(\frac{1}{t} \right)^3 + \frac{2}{1} \cdot \left(\frac{1}{t} \right)^3 \cdot \left(\frac{1}{t} \right)^3$$

$$\left(\frac{1}{t^2} \right)^2 + \frac{2}{1} \cdot \left(\frac{1}{t} \right) \cdot \left(\frac{1}{t} \right)^3 \cdot \left(\frac{1}{t} \right)^3$$

questão 3

a) calcule $f_x(10, 15)$

$$\frac{\partial f}{\partial x} = \frac{1}{e^{xy} - x^2 y^3} \cdot x^4 \cdot y - 2x \cdot y^3$$

$$= \frac{e^{xy} \cdot y - 2xy^3}{e^{xy} - x^2 y^3} \Rightarrow \frac{y - 2x}{-x^2}$$

$$f_x(10, 15) = \frac{15 - 2(10)}{(-10)^2} = \underline{\underline{-0,05}}$$

$$b) f(11, 15) = \ln(e^{11+15} - 11^2 \cdot 15^3)$$

$$= \ln(e^{165} - 121 \cdot 3 \cdot 375)$$

$$f(11, 15) = 165$$

$$f(10, 15) = \ln(e^{10+15} - 10^2 \cdot 15^3)$$

$$= \ln(e^{150} - 100 \cdot 3375)$$

$$f(10, 15) = \underline{\underline{150}}$$

c) $f_y(10, 15)$

$$\frac{\partial f}{\partial y} = \frac{1}{e^{xy} - x^2 y^3} \cdot e^{xy} \cdot x - 3x^2 \cdot y^2$$

$$= \frac{e^{xy} \cdot x - 3x^2 y^2}{e^{xy} - x^2 y^3} = \frac{x - 3}{-y}$$

Questão 7

data

S T Q Q S S O

$$3xy^2 + x^3 - 3x$$

derivando a função

$$f(x,y) = 3y^2 + x^3 - 3$$

$$f(x,y) = 6xy$$

total a igualdades
das derivas

$$3y^2 + 3x^2 - 3 = 0$$

$$3x^2 + 3y^2 = 3$$

$$x^2 + y^2 = 1 (*)$$

$$6xy = 0 (**)$$

$$x=0, y=0 \Rightarrow \text{se } x=0$$

substituindo

$$y=1, y=-1$$

se $y=0$ substituindo

$$x=1 \text{ e } y=-1$$

$$f''(x,y) = 6x$$

$$f''_y(x,y) = 6x$$

$$f''_{xy}(x,y) = 6y$$

$$H(x,y) = 6x \cdot 6x - (6y)^2$$

$$H(x,y) = 36x^2 - 36y^2$$

então neste caso

podso dizer o ponto

$$(0,1) \quad -36 \quad 6$$

$$(0,-1) \quad -36 \quad -6$$

O ponto ~~máximo~~ e mínimo
é $(1,0)$

$$g) f(x,y) = 2x^3 + 3y^2$$

$$(x,y) \text{ varia de } (1,2) \text{ a } (0,95, 2,05)$$

acha dz e dz

$$dz = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$

$$dz = 6x^2 \cdot dx + 6y \cdot dy$$