Package ‘GeoModels’

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Title Analysis of spatio (temporal/bivariate) Gaussian and non Gaussian Random Fields

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Depends R (>= 2.12.0)

Description This package provides a set of procedures for a) simulation and estimation of some spatial and spatio-temporal random fields using standard likelihood and a likelihood approximation method called composite likelihood and b) prediction using best linear unbiased prediction. Spatio (temporal) bivariate data estimation involves estimation of both regression and covariance parameters.
Gaussian and some non Gaussian Random fields can be analyzed using the GeoModels package. Among them gamma, Weibull, log-Gaussian, binomial, negative binomial, skewGaussian, StudendT and circular random fields can be analyzed.

Imports methods

Suggests spam, scatterplot3d, fields, mapproj, gpuR, gsl, pbivnorm, plot3D, shape, sphereplot,

License GPL (>= 2)

URL https://vmoprojs.github.io/GeoModels-page/

Repository GitHub

Encoding UTF-8

BugReports https://github.com/vmoprojs/GeoModels/issues

R topics documented:

`anomalies` *Annual precipitation anomalies in U.S.*

Description

A $(7252x3)$-matrix containing lon/lat and yearly total precipitation anomalies registered at 7.352 location sites in USA. For more details see `http://www.image.ucar.edu/Data/precip_tapering/`.

Usage

```
data(anomalies)
```

Format

A numerical matrix of dimension $7252x3$.

Source

Kaufman, C.G., Schervish, M.J., Nychka, D.W. (2008) Covariance tapering for likelihood-based estimation in large spatial data sets. *Journal of the American Statistical Association, Theory & Methods*, **103**, 1545–1555.

CheckBiv *Checking Bivariate covariance models*

Description

The procedure control if the correlation model is bivariate.

Usage

```
CheckBiv(numbermodel)
```

Arguments

numbermodel numeric; the number associated to a given correlation model.

Value

Returns TRUE or FALSE depending if the correlation model is bivariate or not.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

Examples

```
library(GeoModels)
CheckBiv(CkCorrModel("Bi_matern_sep"))
```

CheckDistance *Checking Distance*

Description

The procedure controls the type of distance.

Usage

```
CheckDistance(distance)
```

Arguments

distance String; the type of distance, for the description see GeoCovmatrix. Default is Eucl. Other possible values are Geod and Chor that is euclidean, geodesic and chordal distance.

Value

Returns 0,1,2 for euclidean,geodesic, chordal distances respectively. Otherwise returns NULL.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

GeoFit

CheckSph — *Checking if a covariance is valid only on sphere*

Description

Subroutine called by InitParam. The procedure controls if a covariance model is valid only on the sphere.

Usage

```
CheckSph(numbermodel)
```

Arguments

numbermodel	Numeric; the code number for the covariance model.

Value

Returns TRUE or FALSE

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

CheckST — *Checking SpaceTime covariance models*

Description

The procedure control if the correlation model is spacetime.

Usage

```
CheckST(numbermodel)
```

Arguments

numbermodel	numeric; the number associated to a given correlation model.

Value

Returns TRUE or FALSE depending if the correlation model is spatiotemporal or not.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

Examples

```
library(GeoModels)
CheckST(CkCorrModel("gneiting"))
```

CkCorrModel *Checking Correlation Model*

Description

The procedure controls if the correlation model inserted is correct.

Usage

```
CkCorrModel(corrmodel)
```

Arguments

corrmodel	String; the name of a correlation model, for the description see GeoCovmatrix.

Value

Return a number associated to a given correlation model if the model is considered in the package. Otherwise return NULL.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

CkInput *Checking Input*

Description

Subroutine called by the fitting procedures. The procedure controls the the validity of the input inserted by the users.

Usage

```
CkInput(coordx, coordy, coordt, coordx_dyn,corrmodel, data, distance,
           fcall, fixed, grid,likelihood, maxdist, maxtime,
            model, n,  optimizer, param, radius,
           start, taper, tapsep,  type, varest, vartype,
           weighted,X)
```

Arguments

`coordx`	A numeric ($d \times 2$)-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
`coordy`	A numeric vector assigning 1-dimension of coordinates; `coordy` is interpreted only if `coordx` is a numeric vector otherwise it will be ignored.
`coordt`	A numeric vector assigning 1-dimension of temporal coordinates.
`corrmodel`	String; the name of a correlation model, for the description see `GeoFit`.
`coordx_dyn`	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
`data`	A numeric vector or a ($n \times d$)-matrix or ($d \times d \times n$)-matrix of observations.
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details**.
`fcall`	String; `Fitting` to call the fitting procedure and `simulation` to call the simulation.
`fixed`	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if `list(nugget=0)` the nugget effect is ignored.
`grid`	Logical; if `FALSE` (the default) the data are interpreted as a vector or a ($n \times d$)-matrix, instead if `TRUE` then ($d \times d \times n$)-matrix is considered.
`likelihood`	String; the configuration of the composite likelihood. `Marginal` is the default.
`maxdist`	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
`maxtime`	Numeric; an optional positive value indicating the maximum temporal lag separation in the composite-likelihood.
`radius`	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6378.88, the radius of the earth.
`model`	String; the density associated to the likelihood objects. `Gaussian` is the default.
`n`	Numeric; the number of trials in a binomial random fields. Default is 1.
`optimizer`	String; the optimization algorithm (see `optim` for details). 'Nelder-Mead' is the default.
`param`	A numeric vector of parameters, needed only in simulation. See `GeoSim`.
`start`	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. `NULL` is the default.
`taper`	String; the name of the tapered correlation function.
`tapsep`	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see **Details**).
`type`	String; the type of the likelihood objects. If `Pairwise` (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
`varest`	Logical; if `TRUE` the estimate' variances and standard errors are returned. `FALSE` is the default.
`vartype`	String; the type of estimation method for computing the estimate variances, see `GeoFit`.
`weighted`	Logical; if `TRUE` the likelihood objects are weighted. If `FALSE` (the default) the composite likelihood is not weighted.
`X`	Numeric; Matrix of space-time covariates in the linear mean specification.

Author(s)

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See Also

GeoFit

CkLikelihood *Checking Composite-likelihood Type*

Description

Subroutine called by InitParam. The procedure controls the type of the composite-likelihood inserted by the users.

Usage

```
CkLikelihood(likelihood)
```

Arguments

likelihood	String; the configuration of the composite likelihood. `Marginal` is the default.

Author(s)

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See Also

GeoFit

CkModel *Checking Random Field type*

Description

Subroutine called by InitParam. The procedure controls the type of random field inserted by the users.

Usage

```
CkModel(model)
```

Arguments

model	String; the density associated to the likelihood objects. `Gaussian` is the default.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

GeoFit

CkType *Checking Likelihood Objects*

Description

Subroutine called by InitParam. The procedure controls the type of likelihood objects inserted by the users.

Usage

```
CkType(type)
```

Arguments

type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

GeoFit

CkVarType *Checking Variance Estimates Type*

Description

Subroutine called by InitParam. The procedure controls the method used to compute the estimates' variances.

Usage

```
CkVarType(type)
```

Arguments

type	String; the method used to compute the estimates' variances. If SubSamp the estimates' variances are computed by the sub-sampling method, see GeoFit.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

GeoFit

CompLik *Optimizes the Composite log-likelihood*

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

Usage

```
CompLik(bivariate, coordx, coordy ,coordt, coordx_dyn, corrmodel, data, distance,
               flagcorr, flagnuis, fixed, GPU, grid,likelihood,local,lower,
                 model, n, namescorr, namesnuis, namesparam,
               numparam, numparamcorr, optimizer, onlyvar, param,
               spacetime, type,upper, varest, vartype,
               weigthed, winconst, winstp,winconst_t, winstp_t, ns, X)
```

Arguments

bivariate	Logical; if TRUE then the data come froma a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric ($d \times 2$)-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a ($n \times d$)-matrix or ($d \times d \times n$)-matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
flagcorr	A numeric vector of binary values denoting which paramerters of the correlation function will be estimated.
flagnuis	A numeric vector of binary values denoting which nuisance paramerters will be estimated.
fixed	A numeric vector of parameters that will be considered as known values.
GPU	Numeric; if NULL (the default) no GPU computation is performed.

grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
likelihood	String; the configuration of the compositelikelihood, see GeoFit.
local	Numeric; number of local work-items of the GPU
lower	A numeric vector with the lower bounds of the parameters' ranges.
model	Numeric; the id value of the density associated to the likelihood objects.
n	Numeric; number of trials in a binomial random fields.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numparam	Numeric; the number of parameters to be maximised.
numparamcorr	Numeric; the number of correlation parameters.
optimizer	String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the default.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
param	A numeric vector of parameters' values.
spacetime	Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	A numeric vector with the upper bounds of the parameters' ranges.
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see GeoFit.
weigthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime)are used.
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
ns	Numeric; Number of (dynamical) temporal instants.
X	Numeric; Matrix of space-time covariates in the linear mean specification.

Author(s)

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See Also

GeoFit

CorrelationPar — *Lists the Parameters of a Correlation Model*

Description

Subroutine called by InitParam and other procedures. The procedure returns a list with the parameters of a given correlation model.

Usage

```
CorrelationPar(corrmodel)
```

Arguments

corrmodel	Integer; an integer associated to a given correlation model.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

GeoFit

CorrParam — *Lists the Parameters of a Correlation Model*

Description

Subroutine called by InitParam and other procedures. The procedure returns a list with the parameters of a given correlation model.

Usage

```
CorrParam(corrmodel)
```

Arguments

corrmodel	String; the name of a correlation model.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

GeoCovmatrix

Examples

```
require(GeoModels)
################################################################
###
### Example 1. Parameters of the  Matern  model
###
###############################################################

CorrParam("Matern")


################################################################
###
### Example 2. Parameters of the  Generalized Wendland  model
###
###############################################################

CorrParam("GenWend")


################################################################
###
### Example 3. Parameters of the  Generalized Wendland  model
###
###############################################################

CorrParam("GenCauchy")


################################################################
###
### Example 4. Parameters of the  space time Gneiting  model
###
###############################################################

CorrParam("Gneiting")


################################################################
###
### Example 5. Parameters of the bi-Matern separable  model
###
###############################################################

CorrParam("Bi_Matern_sep")
```

`DeviceInfo` *Prints Device Information*

Description

Prints the device details available in your computer. Device name, Max compute units, whether it supports double precision, among others.

Usage

```
DeviceInfo()
```

Details

The user can take this information into account so that the `local` parameter is set up in `GeoFit` when `GPU` computation is chosen.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

Examples

```
library(GeoModels)
DeviceInfo()
```

GeoCovariogram *Computes fitted covariance and/or variogram*

Description

The procedure computes and plots covariance or variogram estimated fitting a Gaussian, and non Gaussian spatio (temporal) bivariate random fields. Allows to add the empirical estimates in order to compare them with the fitted model.

Usage

```
GeoCovariogram(fitted, distance="Eucl",answer.cov=FALSE,
            answer.vario=FALSE, answer.range=FALSE, fix.lags=NULL,
             fix.lagt=NULL, show.cov=FALSE, show.vario=FALSE,
            show.range=FALSE, add.cov=FALSE, add.vario=FALSE,
            pract.range=95, vario, ...)
```

Arguments

`fitted`	A fitted object obtained from the `GeoFit` or `GeoWLS` procedures.
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See `GeoFit`.
`answer.cov`	Logical; if `TRUE` a vector with the estimated covariance function is returned; if `FALSE` (the default) the covariance is not returned.
`answer.vario`	Logical; if `TRUE` a vector with the estimated variogram is returned; if `FALSE` (the default) the variogram is not returned.
`answer.range`	Logical; if `TRUE` the estimated pratical range is returned; if `FALSE` (the default) the pratical range is not returned.
`fix.lags`	Integer; a positive value denoting the spatial lag to consider for the plot of the temporal profile.

fix.lagt	Integer; a positive value denoting the temporal lag to consider for the plot of the spatial profile.
show.cov	Logical; if TRUE the estimated covariance function is plotted; if FALSE (the default) the covariance function is not plotted.
show.vario	Logical; if TRUE the estimated variogram is plotted; if FALSE (the default) the variogram is not plotted.
show.range	Logical; if TRUE the estimated pratical range is added on the plot; if FALSE (the default) the pratical range is not added.
add.cov	Logical; if TRUE the vector of the estimated covariance function is added on the current plot; if FALSE (the default) the covariance is not added.
add.vario	Logical; if TRUE the vector with the estimated variogram is added on the current plot; if FALSE (the default) the correlation is not added.
pract.range	Numeric; the percent of the sill to be reached.
vario	A Variogram object obtained from the GeoVariogram procedure.
...	other optional parameters which are passed to plot functions.

Value

The returned object is eventually a list with:

covariance	The vector of the estimated covariance function;
variogram	The vector of the estimated variogram function;

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

References

Cressie, N. A. C. (1993) *Statistics for Spatial Data*. New York: Wiley.

Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. *Spring Verlang, New York*.

See Also

GeoFit.

Examples

```
library(GeoModels)
library(scatterplot3d)

################################################################
###
### Example 1. Plot of fitted covariance and fitted
### and empirical variograms from a  Gaussian RF
### with Matern correlation.
###
###############################################################
set.seed(21)
# Set the coordinates of the points:
x <- runif(300, 0, 1)
```

```
y <- runif(300, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel <- "Matern"
model <- "Gaussian"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5

param=list(mean=mean,sill=sill, nugget=nugget, scale=scale, smooth=smooth)
# Simulation of the Gaussian random field:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data

start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget,smooth=smooth)
# Maximum composite-likelihood fitting of the Gaussian random field:
fit <- GeoFit(data=data,coordx=coords, corrmodel=corrmodel,model=model,
                    likelihood="Marginal",type='Pairwise',start=start,
                    fixed=fixed,maxdist=0.05)

# Empirical estimation of the variogram:
vario <- GeoVariogram(data=data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE,show.vario=TRUE, vario=vario,pch=20)

################################################################
###
### Example 2. Plot of fitted covariance and fitted
### and empirical variograms from a  Binomial
###  RF with exponential correlation.
###
###############################################################
set.seed(2111)

model="Binomial";n=20
# Set the coordinates of the points:
x <- runif(500, 0, 1)
y <- runif(500, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel <- "exponential"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.2/3

param=list(mean=mean,sill=sill, nugget=nugget, scale=scale)
# Simulation of the Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param,n=n)$data

start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget)
```

```
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit <- GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
                    likelihood="Marginal",type='Pairwise',start=start,n=n,
                    fixed=fixed,maxdist=0.03)

# Empirical estimation of the variogram:
vario <- GeoVariogram(data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE,show.vario=TRUE, vario=vario,pch=20)


################################################################
###
### Example 3. Plot of fitted covariance and fitted
### and empirical variograms from a  RF
###  RF with Wend0 correlation.
###
###############################################################
set.seed(211)

model="Gamma";shape=4
# Set the coordinates of the points:
x <- runif(700, 0, 1)
y <- runif(700, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel <- "Wend0"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.3
power2=4


param=list(mean=mean,sill=sill, nugget=nugget, scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data

start=list(mean=0,scale=scale,shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit <- GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
                    likelihood="Marginal",type='Pairwise',start=start,
                    fixed=fixed,maxdist=0.03)

# Empirical estimation of the variogram:
vario <- GeoVariogram(data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE,show.vario=TRUE, vario=vario,pch=20)

################################################################
###
### Example 4. Plot of  fitted  and empirical variograms
### from a space time Gaussian random fields
```

```
### with double exponential correlation.
###
###############################################################
set.seed(92)
# Define the spatial-coordinates of the points:
x <- runif(50, 0, 1)
y <- runif(50, 0, 1)
coords=cbind(x,y)
# Define the temporal sequence:
time <- seq(0, 15, 1)


# Simulation of the spatio-temporal Gaussian random field:
data <- GeoSim(coordx=coords, coordt=time, corrmodel="Exp_Exp",param=list(mean=mean,
              nugget=nugget,scale_s=0.5/3,scale_t=2/2,sill=sill))$data

fixed=list(nugget=0, mean=0)
start=list(scale_s=0.2, scale_t=0.5, sill=1)
# Maximum composite-likelihood fitting of the space-time Gaussian random field:
fit <- GeoFit(data, coordx=coords, coordt=time, corrmodel="Exp_Exp", maxtime=2,
                    maxdist=0.1, likelihood="Marginal", type="Pairwise",
                    fixed=fixed, start=start)

# Empirical estimation of spatio-temporal covariance:
vario <- GeoVariogram(data,coordx=coords, coordt=time, maxtime=5,maxdist=0.5)

# Plot of the fitted space-time variogram
GeoCovariogram(fit,vario=vario,show.vario=TRUE)

# Plot of covariance, variogram and spatio and temporal profiles:
GeoCovariogram(fit,vario=vario,fix.lagt=1,fix.lags=1,show.vario=TRUE,pch=20)




################################################################
###
### Example 5. Plot of parametric and empirical variograms
### estimated from a Bivariate Gaussian random fields with
### Matern correlation.
###
###############################################################

# Simulation of a bivariate spatial Gaussian random field:
set.seed(892)
# Define the spatial-coordinates of the points:
x <- runif(200, -1, 1)
y <- runif(200, -1, 1)
coords=cbind(x,y)

# Simulation of a bivariate Gaussian Random field
# with matern (cross)  covariance function
scale_1 = 0.25/3
scale_2 = 0.2/3
scale_12 = 0.15/3
sill_1=1
sill_2=1
```

```
smooth=0.5
pcol=0.3
param=list(mean_1=0,mean_2=0,scale_1=scale_1,scale_2=scale_2,scale_12=scale_12,
           sill_1=sill_1,sill_2=sill_2,nugget_1=0,nugget_2=0,
           smooth_1=smooth,smooth_12=smooth,smooth_2=smooth,pcol=pcol)
data <- GeoSim(coordx=coords, corrmodel="Bi_Matern", param=param)$data

# Empirical bivariate variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=c(1,1,1))

# selecting fixed and estimating parameters
fixed=list(mean_1=0,mean_2=0,nugget_1=0,nugget_2=0,
          smooth_1=smooth,smooth_12=smooth,smooth_2=smooth)
start=list(sill_1=var(data[1,]),sill_2=var(data[2,]),
          scale_1=scale_1,scale_2=scale_2,scale_12=scale_12,
          pcol=cor(data[1,],data[2,]))

# Maximum likelihood fitting of the bivariate random field:
fit<- GeoFit(data, coordx=coords, corrmodel="Bi_Matern",likelihood="Marginal",
              type="Pairwise",start=start,fixed=fixed,maxdist=c(0.1,0.1,0.1))

GeoCovariogram(fit, vario=biv_vario,show.vario=TRUE,pch=20)
```

GeoCovmatrix *Spatial and Spatio-temporal (tapered) Covariance Matrix*

Description

The function computes the covariance matrix associated to a spatial or spatio-temporal or a bivariate spatial Gaussian or non Gaussian randomm field with given covariance model and a set of spatial location sites and temporal instants.

Usage

```
GeoCovmatrix(coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel,
           distance="Eucl", grid=FALSE, maxdist=NULL, maxtime=NULL,
           model="Gaussian", n=1, param, radius=6378.388, sparse=FALSE,
           taper=NULL, tapsep=NULL, type="Standard",X=NULL)
```

Arguments

`coordx` A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.

`coordy` A numeric vector giving 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.

`coordt`	A numeric vector giving 1-dimension of temporal coordinates. At the moment implemented only for the Gaussian case. Optional argument, the default is `NULL` then a spatial random field is expected.
`coordx_dyn`	A list of T numeric ($d_t \times 2$)-matrices containing dynamical (in time) coordinates. Optional argument, the default is `NULL`
`corrmodel`	String; the name of a correlation model, for the description see the Section **Details**.
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See `GeoFit`.
`grid`	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid). See `GeoFit`.
`maxdist`	Numeric; an optional positive value indicating the marginal spatial compact support in the case of tapered covariance matrix. See `GeoFit`.
`maxtime`	Numeric; an optional positive value indicating the marginal temporal compact support in the case of spacetime tapered covariance matrix. See `GeoFit`.
`n`	Numeric; the number of trials in a binomial random fields. Default is 1.
`model`	String; the type of RF. See `GeoFit`.
`param`	A list of parameter values required for the covariance model.
`radius`	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6378.88)
`sparse`	Logical; if `TRUE` the function return an object of class spam. This option should be used when a parametric compactly supporte covariance is used. Default is FALSE.
`taper`	String; the name of the taper correlation function if type is `Tapering`, see the Section **Details**.
`tapsep`	Numeric; an optional value indicating the separabe parameter in the space-time non separable taper or the colocated correlation parameter in a bivariate spatial taper (see **Details**).
`type`	String; the type of covariance matrix `Standard` (the default) or `Tapering` for tapered covariance matrix.
`X`	Numeric; Matrix of space-time covariates.

Details

In the spatial case, the covariance matrix of the random vector

$$[Z(s_1), \dots, Z(s_n,)]^T$$

with a specific spatial covariance model is computed. Here n is the number of the spatial location sites.

In the space-time case, the covariance matrix of the random vector

$$[Z(s_1,t_1), Z(s_2,t_1), \dots, Z(s_n,t_1), \dots, Z(s_n,t_m)]^T$$

with a specific space time covariance model is computed. Here m is the number of temporal instants.

In the bivariate case, the covariance matrix of the random vector

$$[Z_1(s_1), Z_2(s_1), \ldots, Z_1(s_n), Z_2(s_n)]^T$$

with a specific spatial bivariate covariance model is computed.

The location site s_i can be a point in the d-dimensional euclidean space with $d = 2$ or a point (given in lon/lat degree format) on a sphere of arbitrary radius.

Here there is the list of all the implemented space and space-time and bivariate correlation models. The argument `param` is a list including all the parameters of a given correlation model specified by the argument `corrmodel`. For each correlation model one can check the associated correlation parameters using `CorrParam`. In what follows $\kappa > 0$, $\beta > 0$, $\alpha, \alpha_s, \alpha_t \in (0, 2]$, and $\gamma \in [0, 1]$. The associated parameters in the argument `param` are `smooth`, `power2`, `power`, `power_s`, `power_t` and `sep` respectively. Moreover let $1(A) = 1$ when A is true and 0 otherwise.

- Spatial correlation models:
  1. $Cauchy$ defined as:
  $$R(h) = \left(1 + h^2\right)^{-\beta/2}$$
  It is a special case of the `Gencauchy` model.
  2. Exp defined as:
  $$R(h) = e^{-h}$$
  This model is a special case of the `Matern` and the `Stable` model.
  3. $Gauss$ defined as:
  $$R(h) = e^{-h^2}$$
  This model is a special case of the `stable` model.
  4. $GenCauchy$ (generalised $Cauchy$) defined as:
  $$R(h) = (1 + h^\alpha)^{-\beta/\alpha}$$
  If h is the geodesic distance then $\alpha \in (0, 1]$.
  5. $Matern$ defined as:
  $$R(h) = 2^{1-\kappa}\Gamma(\kappa)^{-1}h^\kappa K_\kappa(h)$$
  If h is the geodesic distance then $\kappa \in (0, 0.5]$
  6. $Stable$ defined as:
  $$R(h) = e^{-h^\alpha}$$
  If h is the geodesic distance then $\alpha \in (0, 1]$.
  7. $Wave$ defined as:
  $$R(h) = sin(h)/h$$
  This model is valid only for dimensions less than or equal to 3.
  8. $Wend0$ defined as:
  $$R(h) = (1 - h)^\mu 1(h \in [0, 1])$$
  where $\mu \geq 0.5(d + 1)$. If h is the geodesic distance then $\mu \geq 2$.
  9. $Wend1$ defined as:
  $$R(h) = (1 - h)^{\mu+1}(1 + (\mu + 1)h)1(h \in [0, 1])$$
  where $\mu \geq 0.5(d + 1) + 1$. If h is the geodesic distance then $\mu \geq 4$.

10. $Wend2$ defined as:

$$R(h) = (1-h)^{\mu+2}(1 + (\mu+2)h + (1/3)((\mu+1)^2 - 1)h^2)1(h \in [0,1])$$

where $\mu \geq 0.5(d+1)+2$. If h is the geodesic distance then $\mu \geq 6$.

11. $GenWend$ (Generalized Wendland) defined as:

$$R(h) = \int_h^1 [(1-x)^{\mu-1}(x^2-h^2)^{\kappa-1}1(h \in [0,1])]dx/B(2\kappa+1,\mu)$$

where $\mu \geq 0.5(d+1)+\kappa$. The cases $\kappa = 0, 1, 2$ correspond to the $Wend0$, $Wend1$ and $Wend2$ respectively.

12. $Multiquadric$ defined as:

$$R(h) = (1-\alpha 0.5)^{2\beta}/(1+(\alpha 0.5)^2 - \alpha cos(h))^{\beta}, \quad h \in [0,\pi]$$

This model is valid on the unit sphere and h is the geodesic distance.

13. $Sinpower$ defined as:

$$R(h) = 1 - (sin(h/2))^{\alpha}, \quad h \in [0,\pi]$$

This model is valid on the unit sphere and h is the geodesic distance.

14. $Smoke$ defined as:

$$R(h) = K * 1F2(1/\alpha, 1/\alpha + 0.5, 2/\alpha + 0.5 + \kappa), \quad h \in [0,\pi]$$

where $K = (\Gamma(a)\Gamma(i))/\Gamma(i)\Gamma(o))$. This model is valid on the unit sphere and h is the geodesic distance.

- Spatio-temporal correlation models.
  - Non-separable models:
    1. $Gneiting$ defined as:

    $$R(h,u) = e^{-h^{\alpha_s}/((1+u^{\alpha_t})^{0.5\gamma\alpha_s})}/(1+u^{\alpha_t})$$

    2. $Gneiting\_GC$

    $$R(h,u) = e^{-u^{\alpha_t}/((1+h^{\alpha_s})^{0.5\gamma\alpha_t})}/(1+h^{\alpha_s})$$

    where h can be both the euclidean and the geodesic distance

    3. $Iacocesare$

    $$R(h,u) = (1+h^{\alpha_s}+u_t^{\alpha})^{-\beta}$$

    4. $Porcu$

    $$R(h,u) = (0.5(1+h^{\alpha_s})^{\gamma} + 0.5(1+u^{\alpha_t})^{\gamma})^{-\gamma^{-1}}$$

    5. $Porcu1$

    $$R(h,u) = (e^{-h^{\alpha_s}(1+u^{\alpha_t})^{0.5\gamma\alpha_s}})/((1+u^{\alpha_t})^{1.5})$$

    6. $Stein$

    $$R(h,u) = (h^{\psi(u)}K_{\psi(u)}(h))/(2^{\psi(u)}\Gamma(\psi(u)+1))$$

    where $\psi(u) = \nu + u^{0.5\alpha_t}$

    7. $Wenx\_space$, $x = 0, 1, 2$ defined as:

    $$R(h,u) = \phi(u)^{3.5+2x} Wenx(h/\phi(u), \mu_s), \quad x = 0,1,2$$

    where $\phi(u) = (1+u^{0.5\alpha_t})^{-\gamma}$, $0 < \gamma \leq \alpha_t/2$, $\mu_s \geq 0.5(d+5)+x$.

8. $Wenx\_time$, $x = 0, 1, 2$ defined as:

$$R(h, u) = \phi(h)^{3.5+2x} Wenx(u/\phi(h); \mu_t), \quad x = 0, 1, 2$$

where $\phi(h) = (1 + h^{0.5\alpha_s})^{-\gamma}$, $0 < \gamma \leq \alpha_s/2$, $\mu_t \geq 0.5(d + 5) + x$.

9. $Multiquadric\_st$ defined as:

$$R(h, u) = ((1 - 0.5\alpha_s)^2/(1 + (0.5\alpha_s)^2 - \alpha_s\psi(u)cos(h)))^{a_s}, \quad h \in [0, \pi]$$

where $\psi(u) = (1 + (u/a_t)^{\alpha_t})^{-1}$. This model is valid on the unit sphere and h is the geodesic distance.

10. $Sinpower\_st$ defined as:

$$R(h, u) = (e^{\alpha_s cos(h)\psi(u)/a_s}(1 + \alpha_s cos(h)\psi(u)/a_s))/k$$

where $\psi(u) = (1+(u/a_t)^{\alpha_t})^{-1}$ and $k = (1+\alpha_s/a_s)exp(\alpha_s/a_s), \quad h \in [0, \pi]$ This model is valid on the unit sphere and h is the geodesic distance.

– Separable models.

Space-time separable correlation models are easly obtained as the product of a spatial and a temporal correlation model, that is

$$R(h, u) = R(h)R(u)$$

Several combinations are possible:

1. Exp_Exp defined as:

$$R(h, u) = Exp(h)Exp(u)$$

2. $Matern\_Matern$ defined as:

$$R(h, u) = Matern(h; \kappa_s)Matern(u; \kappa_t)$$

3. $Stable\_Stable$ defined as:

$$R(h, u) = Stable(h; \alpha_s)Stable(u; \alpha_t)$$

4. $Wendx\_Wendy$ defined as

$$R(h, u) = Wendx(h; \mu_s)Wendy(u; \mu_t), x, y = 0, 1, 2$$

.

Note that some models are nested. (The Exp_Exp with $Matern\_Matern$ for instance.)

- Spatial bivariate correlation models (see below):

1. Bi_Matern (Bivariate full Matern model)
2. Bi_Matern_contr (Bivariate Matern model with contrainsts)
3. Bi_Matern_sep (Bivariate separable Matern model)
4. Bi_LMC (Bivariate linear model of coregionalization)
5. Bi_LMC_contr (Bivariate linear model of coregionalization with constraints)
6. Bi_Wendx (Bivariate full Wendland model)
7. Bi_Wendx_contr (Bivariate Wendland model with contrainsts)
8. Bi_Wendx_sep (Bivariate separable Wendland model)

- Spatial taper.
For spatial covariance tapering the taper functions are:

1. $Bohman$ defined as:

$$T(h) = (1-h)(sin(2\pi h)/(2\pi h)) + (1-cos(2\pi h))/(2\pi^2 h)1_{[0,1]}(h)$$

2. $Wendlandx, \quad x = 0,1,2$ defined as:

$$T(h) = Wendx(h; x+2), x = 0,1,2$$

.

- Spatio-temporal tapers.
  For spacetime covariance tapering the taper functions are:
  1. $Wendlandx\_Wendlandy$ (Separable tapers) $x, y = 0,1,2$ defined as:

  $$T(h,u) = Wendx(h; x+2)Wendy(h; y+2), x, y = 0,1,2.$$

  2. $Wendlandx\_time$ (Non separable temporal taper) $x = 0,1,2$ defined as: $Wenx\_time$, $x = 0,1,2$ assuming $\alpha_t = 2$, $\mu_s = 3.5 + x$ and $\gamma \in [0,1]$ to be fixed using `tapsep`.
  3. $Wendlandx\_space$ (Non separable spatial taper) $x = 0,1,2$ defined as: $Wenx\_space$, $x = 0,1,2$ assuming $\alpha_s = 2$, $\mu_t = 3.5 + x$ and $\gamma \in [0,1]$ to be fixed using `tapsep`.
- Spatial bivariate taper (see below).
  1. $Bi\_Wendlandx, \quad x = 0,1,2$

Remarks:
In what follows we assume $\sigma^2, \sigma_1^2, \sigma_2^2, \tau^2, \tau_1^2, \tau_2^2, a, a_s, a_t, a_{11}, a_{22}, a_{12}, \kappa_{11}, \kappa_{22}, \kappa_{12}, f_{11}, f_{12}, f_{21}, f_{22}$ positive.

The associated parameters in `param` are `sill`, `sill_1`,`sill_2`, `nugget`, `nugget_1`,`nugget_2`, `scale`,`scale_s`,`scale_t`, `scale_1`,`scale_2`,`scale_12`, `smooth_1`,`smooth_2`,`smooth_12`, `a_1`,`a_12`,`a_21`,`a_2` respectively.

Let $R(h)$ be a spatial correlation model given in standard notation. Then the covariance model applied with arbitrary variance, nugget and scale equals to:

$$C(h) = (\sigma^2 + \tau^2 1(h=0))R(h/a, , ...), \quad h \geq 0$$

Similarly if $R(h,u)$ is a spatio-temporal correlation model given in standard notation, then the covariance model is:

$$C(h,u) = (\sigma^2 + \tau^2 1(h=0, u=0))R(h/a_s, u/a_t, ...) \quad h \geq 0, u \geq 0$$

Here '...' stands for additional parameters.

Let $R(h)$ be a spatial taper given in standard notation. Then the taper function applied with an arbitrary compact support (d_s) equals to:

$$T(h) = R(h/d_s)$$

Then the tapered covariance function is given by:

$$C^{tap}(h) = T(h)C(h)$$

Similarly if $R(h,u)$ is a spatio-temporal taper given in standard notation, then the taper function applied with arbitrary compact supports $(d_s, d_t)^T$ equals to:

$$T(h,u) = R(h/d_s, u/d_t)$$

Then the tapered covariance function is given by:

$$C^{tap}(h,u) = T(h,u)C(h,u)$$

Compact supports d_s and d_t can be set by the user with `maxdist` and `maxtime`.

The bivariate models implemented are the following :

1. Bi_Matern defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i\sigma_j + \tau_i^2 1(i=j, h=0)) Matern(h/a_{ij}, \kappa_{ij}) \quad i,j = 1,2. \quad h \geq 0$$

   where $\rho = \rho_{12} = \rho_{21}$ is the correlation colocated parameter and $\rho_{ii} = 1$. The model Bi_Matern_sep (separable matern) is a special case when $a = a_{11} = a_{12} = a_{22}$ and $\kappa = \kappa_{11} = \kappa_{12} = \kappa_{22}$. The model Bi_Matern_contr (constrained matern) is a special case when $a_{12} = 0.5(a_{11} + a_{22})$ and $\kappa_{12} = 0.5(\kappa_{11} + \kappa_{22})$

2. Bi_Wendx ($x = 0, 1, 2$) defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i\sigma_j + \tau_i^2 1(i=j, h=0)) Wendx(h/a_{ij}, \nu_{ij} + 1) \quad i,j = 1,2. \quad h \geq 0$$

   where $\rho = \rho_{12} = \rho_{21}$ is the correlation colocated parameter and $\rho_{ii} = 1$. The model Bi_Wendx_sep (separable wendland) is a special case when $a = a_{11} = a_{12} = a_{22}$ and $\mu = \mu_{11} = \mu_{12} = \mu_{22}$. The model Bi_Wendx_contr (constrained matern) is a special case when $a_{12} = 0.5(a_{11} + a_{22})$ and $\mu_{12} = 0.5(\mu_{11} + \mu_{22})$

3. Bi_LMC defined as:

$$C_{ij}(h) = \sum_{k=1}^{2} (f_{ik} f_{jk} + \tau_i^2 1(i=j, h=0)) R(h/a_k)$$

   where $R(h)$ is a correlation model. The model Bi_LMC_contr is a special case when $f = f_{12} = f_{21}$. Bivariate LMC models, in the current version of the package, is obtained with $R(h)$ equal to the exponential correlation model.

The bivariate spatial tapers implemented are the following :

1. $Bi\_Wendlandx$, $x = 0, 1, 2$ defined as:

$$T_{ij}(h) = r_{ij} Wendx(h/d_{ij}, x), \quad i,j = 1,2 \quad x = 0,1,2 \quad h \geq 0$$

   with $r_{ii} = 1$ and $r_{12} = r_{21}$ to be fixed using `tapsep`.

If $T_{ij}(h)$ is a bivariate taper, Then the tapered bivariate covariance function is given by:

$$C_{ij}^{tap}(h) = T_{ij}(h) C_{ij}(h)$$

Compact supports d_{11}, d_{12}, d_{22} can be set by the user with `maxdist`.

Value

Returns an object of class `CovMat`. An object of class `CovMat` is a list containing at most the following components:

`bivariate`	Logical:`TRUE` if the Gaussian random field is bivariaete otherwise `FALSE`;
`coordx`	A d-dimensional vector of spatial coordinates;
`coordy`	A d-dimensional vector of spatial coordinates;
`coordt`	A t-dimensional vector of temporal coordinates;
`coordx_dyn`	A list of t matrices of spatial coordinates;
`covmatrix`	The covariance matrix if `type` is`Standard`. An object of class spam if `type` is `Tapering` or `Standard` and `sparse` is TRUE.
`corrmodel`	String: the correlation model;
`distance`	String: the type of spatial distance;

grid	Logical:TRUE if the spatial data are in a regular grid, otherwise FALSE;
nozero	In the case of tapered matrix the percentage of non zero values in the covariance matrix. Otherwise is NULL.
maxdist	Numeric: the marginal spatial compact support if type is Tapering;
maxtime	Numeric: the marginal temporal compact support if type is Tapering;
n	The number of trial for Binomial RFs
namescorr	String: The names of the correlation parameters;
numcoord	Numeric: the number of spatial coordinates;
numtime	Numeric: the number the temporal coordinates;
model	The type of RF, see GeoFit.
param	Numeric: The covariance parameters;
tapmod	String: the taper model if type is Tapering. Otherwise is NULL.
spacetime	TRUE if spatio-temporal and FALSE if spatial covariance model;
sparse	Logical: is the returned object of class spam? ;

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See Also

GeoKrig, GeoSim, GeoFit

Examples

```
library(GeoModels)
library(spam)

################################################################
###
### Example 1. Spatial covariance matrix associated to
### a Matern correlation model
```

```
###
################################################################

# Define the spatial-coordinates of the points:
x <- runif(500, 0, 1)
y <- runif(500, 0, 1)
coords <- cbind(x,y)


# Correlation Parameters for Matern model
CorrParam("Matern")

# Matern Parameters
param=list(smooth=0.5,sill=1,scale=0.2,nugget=0)

matrix1 <- GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param)
dim(matrix1$covmatrix)

################################################################
###
### Example 2. Spatial tapered Covariance matrix associated to
### a Matern correlation model
###
################################################################


matrix2 <- GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param,
                     maxdist=0.2,taper="Wendland1",type="Tapering")
# Tapered covariance matrix
as.matrix(matrix2$covmatrix)[1:15,1:15]

# Percentage of no zero values in the tapered matrix
matrix2$nozero


################################################################
###
### Example 3. Spatial covariance matrix associated to
### a Generalized Wendland correlation model
###
################################################################

# Gen Wendland Parameters
param=list(sill=1,scale=0.2,nugget=0,smooth=0,power2=4)


matrix3 <- GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=param,
                     ,sparse=TRUE,maxdist=0.1)

# Percentage of no zero values in the tapered matrix
matrix3$nozero


################################################################
###
### Example 4. Spatial covariance matrix associated to
### a Generalized Cauchy correlation model
```

```
###
###############################################################

# Gen Cauchy Parameters
param=list(sill=1,scale=0.2,nugget=0,power1=1,power2=1)

# Correlation Parameters for Gen Cauchy model
CorrParam("GenCauchy")

matrix4 <- GeoCovmatrix(coordx=coords, corrmodel="GenCauchy", param=param)

matrix4$covmatrix[1:4,1:4]


################################################################
###
### Example 5. Covariance matrix associated to
### a space-time double exponential correlation model
###
###############################################################

# Define the temporal-coordinates:
times <- seq(1, 4, 1)

# Define covariance parameters
param=list(scale_s=0.3,scale_t=0.5,sill=1)

# Correlation Parameters for double exp model
CorrParam("Exp_Exp")

# Simulation of a spatial Gaussian random field:
matrix5 <- GeoCovmatrix(coordx=coords, coordt=times, corrmodel="Exp_Exp",
                     param=param)

dim(matrix5$covmatrix)

################################################################
###
### Example 6. Covariance matrix associated to
### a  skew gaussian RF with Exp correlation model
###
###############################################################


param=list(sill=1,scale=0.3/3,nugget=0,skew=4)
# Simulation of a spatial Gaussian random field:
matrix6 <- GeoCovmatrix(coordx=coords,  corrmodel="Exp", param=param,
                     model="SkewGaussian")

# covariance matrix
matrix6$covmatrix[1:10,1:10]


################################################################
###
### Example 7. Covariance matrix associated to
### a  Weibull RF with Genwend correlation model
```

```
###
###############################################################


param=list(sill=1,scale=0.3,nugget=0,shape=4,mean=0,smooth=1,power2=5)
# Simulation of a spatial Gaussian random field:
matrix7 <- GeoCovmatrix(coordx=coords,  corrmodel="GenWend", param=param,
                     sparse=TRUE,model="Weibull")

# covariance matrix
matrix7$nozero


################################################################
###
### Example 8. Covariance matrix associated to
### a  binomial gaussian RF with Wendland correlation model
###
###############################################################


param=list(sill=1,scale=0.2,nugget=0,power2=4)
# Simulation of a spatial Gaussian random field:
matrix8 <- GeoCovmatrix(coordx=coords,  corrmodel="Wend0", param=param,n=5,
                     model="Binomial")

# covariance matrix
matrix8$covmatrix[1:10,1:10]



################################################################
###
### Example 9.  Covariance matrix associated to
### a bivariate Matern exponential correlation model
###
###############################################################

set.seed(8)
# Define the spatial-coordinates of the points:
x <- runif(10, -1, 1)
y <- runif(10, -1, 1)
coords <- cbind(x,y)

# Parameters
param=list(scale=0.3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
             nugget_1=0,nugget_2=0,smooth=0.5,pcol=-0.25)

# Covariance matrix
matrix9 <- GeoCovmatrix(coordx=coords, corrmodel="Bi_matern_sep", param=param)$covmatrix

head(matrix8)
```

GeoFit *Max-Likelihood-Based Fitting of Gaussian and non Gaussian RFs.*

Description

Maximum weighted composite-likelihood fitting for Gaussian and some Non-Gaussian univariate spatial, spatio-temporal and bivariate spatial RFs The function returns the model parameters' estimates and the estimates' variances. Moreover the function allows to fix any of the parameters and setting upper/lower bound in the optimization.

Usage

```
GeoFit(data, coordx, coordy=NULL, coordt=NULL,coordx_dyn=NULL, corrmodel, distance="Eucl",
         fixed=NULL,GPU=NULL, grid=FALSE, likelihood='Marginal',  local=c(1,1),lower=NULL,
           maxdist=NULL, maxtime=NULL, method="cholesky", model='Gaussian',
           n=1, onlyvar=FALSE ,optimizer='Nelder-Mead',radius=6378.388,
           sparse=FALSE, start=NULL, taper=NULL, tapsep=NULL,type='Pairwise',
         upper=NULL, varest=FALSE, vartype='SubSamp', weighted=FALSE, winconst=NULL,
           winstp=NULL,winconst_t=NULL, winstp_t=NULL,X=NULL)
```

Arguments

- `data` — A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid). For the description see the Section **Details**.
- `coordx` — A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.
- `coordy` — A numeric vector assigning 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.
- `coordt` — A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is `NULL` then a spatial RF is expected.
- `coordx_dyn` — A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
- `corrmodel` — String; the name of a correlation model, for the description see the Section **Details**.
- `distance` — String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details**.
- `fixed` — An optional named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated.
- `GPU` — Numeric; if `NULL` (the default) no OpenCL computation is performed. The user can choose the device to be used. Use `DeviceInfo()` function to see available devices, only double precision devices are allowed

grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
likelihood	String; the configuration of the composite likelihood. Marginal is the default, see the Section **Details**.
local	Numeric; number of local work-items of the OpenCL setup
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or optimize. T he names of the list must be the same of the names in the start list.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite or tapered likelihood computation. See the Section **Details** for more information.
maxtime	Numeric; an optional positive value indicating the maximum temporal separation considered in the composite or tapered likelihood computation (see **Details**).
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section **Details**.
n	Numeric; number of trials in a binomial RF; number of successes in a negative binomial RF
optimizer	String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are nlm and L-BFGS-B. In this last case upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6378.88, the radius of the earth.
sparse	Logical; if TRUE then maximum likelihood is computed using sparse matrices algorithms (spam packake).It should be used with compactly supported covariance models.FALSE is the default.
start	An optional named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see **Details**).
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see **Details**).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods (see **Details**).
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.

`vartype`	String; (`SubSamp` the default) the type of method used for computing the estimates' variances, see the Section **Details**.
`weighted`	Logical; if `TRUE` the likelihood objects are weighted, see the Section **Details**. If `FALSE` (the default) the composite likelihood is not weighted.
`winconst`	Numeric; a positive value for computing the spatial sub-window in the subsampling procedure. See **Details** for more information.
`winstp`	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure. The case 1 correspond to no overlapping. See **Details** for more information.
`winconst_t`	Numeric; a positive value for computing the temporal sub-window in the subsampling procedure. See **Details** for more information.
`winstp_t`	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure. The case 1 correspond to no overlapping. See **Details** for more information.
`X`	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

Details

Note, that the standard likelihood may be seen as particular case of the composite likelihood. In this respect `GeoFit` provides maximum likelihood fitting. Only composite likelihood estimation based on pairs are considered. Specifically marginal pairwise likelihood is considered for each type of random field (Gaussian and not Gaussian). In the Gaussian case other types of estimation are available: conditional and difference pairwise likelihood, covariance tapering and cross-validation method.

The optimization method is specified using `optimizer`. The default method is `Nelder-mead` and other available methods are `nlm`, `BFGS` and `L-BFGS-B`. In this last case upper and lower bounds constraints in the optimization can be specified using `lower` and `upper`.

Depending on the dimension of `data` and on the name of the correlation model, the observations are assumed as a realization of a spatial, spatio-temporal or bivariate RF. Specifically, with `data`, `coordx`, `coordy`, `coordt` parameters:

- If `data` is a numeric d-dimensional vector, `coordx` and `coordy` are two numeric d-dimensional vectors (or `coordx` is $(d \times 2)$-matrix and `coordy=NULL`), then the data are interpreted as a single spatial realisation observed on d spatial sites;
- If `data` is a numeric $(t \times d)$-matrix, `coordx` and `coordy` are two numeric d-dimensional vectors (or `coordx` is $(d \times 2)$-matrix and `coordy=NULL`), `coordt` is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on `d` spatial sites and for `t` times.
- If `data` is a numeric $(2 \times d)$-matrix, `coordx` and `coordy` are two numeric d-dimensional vectors (or `coordx` is $(d \times 2)$-matrix and `coordy=NULL`), then the data are interpreted as a single spatial realisation of a bivariate RF observed on `d` spatial sites.
- If `data` is a list, `coordxdyn` is a list and `coordt` is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on dynamical spatial sites (different locations sites for each temporal instants) and for `t` times.

It is possible consider data on a regular grid using option `grid=TRUE`. Specifically:

- If `data` is a numeric $(d \times d)$-matrix, `coordx` and `coordy` are two numeric d-dimensional vectors, `grid=TRUE`, then the data are interpreted as a single spatial RF realisation observed on `d` equispaced spatial sites (named regular grid).

- If `data` is a numeric $(d \times d \times t)$-array, `coordx` and `coordy` are two numeric d-dimensional vectors, `coordt` is a numeric t-dimensional vector, `grid=TRUE`, then the data are interpreted as a single spatial-temporal realisation of a RF observed on `d` equispaced spatial sites and for `t` times.
- If `data` is a numeric $(d \times d \times 2)$-array, `coordx` and `coordy` are two numeric d-dimensional vectors, `grid=TRUE`, then the data are interpreted as a single realisation of a bivariate RF observed on `d` equispaced spatial sites.

When `grid=FALSE` is is possible to specify a matrix of covariates using `X`. Specifically:

- In the spatial case `X` must be a $(d \times k)$ covariates matrix associated to `data` a numeric d-dimensional vector;
- In the spatiotemporal case `X` must be a $(N \times k)$ covariates matrix associated to `data` a numeric $(t \times d)$-matrix, where $N = t \times d$;
- In the spatiotemporal case `X` must be a $(N \times k)$ covariates matrix associated to `data` a numeric $(t \times d)$-matrix, where $N = 2 \times d$;

The `corrmodel` parameter allows to select a specific correlation function for the RF. (See `GeoCovmatrix`).

The `distance` parameter allows to consider differents kinds of spatial distances. The settings alternatives are:

1. `Eucl`, the euclidean distance (default value);
2. `Chor`, the chordal distance;
3. `Geod`, the geodesic distance;

The `likelihood` parameter represents the composite-likelihood configurations. The settings alternatives are:

1. `Conditional`, the composite-likelihood is formed by conditionals likelihoods;
2. `Marginal`, the composite-likelihood is formed by marginals likelihoods;
3. `Full`, the composite-likelihood turns out to be the standard likelihood;

The `model` parameter indicates the type of RF considered. The available options are:

RF with marginal symmetric distribution:

- `Gaussian`, for a Gaussian RF
- `Logistic`, for a Logistic RF (see Bevilacqua, M. Caamano C., Gaetan , C. 2018)
- `StudentT`, for a StudentT RF (see Bevilacqua, M. Caamano C., 2018)

RF with positive right skewed marginal distribution:

- `Gamma` for a Gamma RF (see Bevilacqua, M. Caamano C., 2018)
- `Gamma` for a Weibull RF (see Bevilacqua, M. Caamano C., 2018)
- `LogGaussian` for a LogGaussian RF
- `LogLogistics` for a LogLogistic RF (see Bevilacqua, M. Caamano C., Gaetan , C. 2018)

RF with with possibly asymmetric marginal distribution:

- `SkewGaussian` for a skew Gaussian RF (see Hao Zhang, H. and El-Shaarawi A. (2010))
- `SinhAsinh` for Sinh-arcsinh RF (see Bevilacqua, M. Caamano C., 2018)

RF with for directional data

- `Wrapped` for a wrapped Gaussian RF (see Alegria A., Bevilacqua, M., Porcu, E. (2016))

Rf with marginal counts data

- `Binomial` for a Binomial RF (see Bevilacqua, M. Caamano C., Gaetan C. (2018))
- `BinomialNeg` for a negative Binomial RF(see Bevilacqua, M. Caamano C., Gaetan C. (2018))

For a given `model` the associated parameters are given by nuisance and covariance parameters. In order to obtain the nuisance parameter use `NuisParam`. In order to obtain the covariance parameter associated to a given covariance model use `CorrParam`.

All the nuisance and covariance parameters must be specified by the user using the `start` and the `fixed` parameter. Specifically:

The `start` parameter allows to specify (as starting values for the optimization) the parameters to be estimated. The `fixed` parameter allows to fix some of the parameters.

Regression parameters in the linear specfication must be specified as `mean,mean1,..meank` (see `NuisParam`). In this case a matrix of covariates with suitable dimension can be specified using the parameter `X`. In the case of a single mean then `X` should not be specified and it is interpreted as a vector of ones.

The `taper` parameter, optional in case that `type=Tapering`, indicates the type of taper correlation model. (See `GeoCovmatrix`)

The `type` parameter represents the type of likelihood used in the composite-likelihood definition. The possible alternatives are listed in the following scheme.

If a Gaussian or (any) non Gaussian RF is considered then the possible combination is marginal pairwise likelihoods (`likelihood=Marginal`) and `type="Pairwise"`).

If a Gaussian RF is considered (`model=Gaussian`) then:

- If the composite is formed by marginal likelihoods (`likelihood=Marginal`):
  - `Pairwise`, the composite-likelihood is defined by the pairwise likelihoods;
  - `Difference`, the composite-likelihood is defined by likelihoods which are obtained as difference of the pairwise likelihoods.
- If the composite is formed by conditional likelihoods (`likelihood=Conditional`)
  - `Pairwise`, the composite-likelihood is defined by the pairwise conditional likelihoods.
- If the composite is formed by a full likelihood (`likelihood=Full`):
  - `Standard`, the objective function is the classical multivariate likelihood;
  - `Restricted`, the objective function is the restricted version of the full likelihood (e.g. Harville 1977, see **References**);
  - `Tapering`, the objective function is the tapered 2 (unbiased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);
  - `Tapering1`, the objective function is the tapered 1 (biased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);
  - `CV`, the objective function is the cross validation estimation method ;

The `maxdist` parameter set the maximum spatial distance below which pairs of sites with inferior distances are considered in the composite-likelihood. This can be lower of the maximum spatial distance. **Note** that this corresponds to use a weighted composite-likelihood with binary weights. Pairs with distance less than `maxdist` have weight 1 and are included in the likelihood computation, instead those with greater distance have weight 0 and then excluded. The default is `NULL`, in this case the effective maximum spatial distance between sites is considered.

The same arguments of maxdist are valid for maxtime but here the weigthed composite-likelihood regards the case of spatial-temporal field. At the moment is implemented only for Gaussian RFs. The default is NULL, in this case the effective maximum temporal lag between pairs of observations is considered.

In the case of tapering likelihood maxdist and maxtime describes the spatial and temporal compact support of the taper (see GeoCovmatrix). If they are not specified then the maximum spatial and temporal distances are considered. In the case of space time adaptive taper the tapsep parameter allows to specify the spatio temporal compact support (see GeoCovmatrix).

The varest parameter specifies if the standard error estimation of the estimated parameters must be computed. For Gaussian RF and standard (restricted) likelihood estimation, standard errors are computed as square root of the diagonal elements of the Fisher Information matrix (asymptotic covariance matrix of the estimates under increasing domain). For Gaussian RF and tapered and composite likelihood estimation, standard errors estimate are computed as square root of the diagonal elements of the Godambe Information matrix. (asymptotic covariance matrix of the estimates under increasing domain (see Shaby, B. and D. Ruppert (2012) for tapering and Bevilacqua et. al. (2012) , Bevilacqua and Gaetan (2013) for weighted composite likelihood)). The vartype parameter specifies the method used to compute the estimates' variances in the composite likelihood case. In particular for estimating the variability matrix J in the Godambe expression matrix. This parameter is considered if varest=TRUE. The options are:

- SubSamp (the default), indicates the Sub-Sampling method;

The weighted parameter specifies if the likelihoods forming the composite-likelihood must be weighted. If TRUE the weights are selected by opportune procedures that improve the efficient of the maximum composite-likelihood estimator (not implemented yet). If FALSE the efficient improvement procedure is not used.

For computing the standard errors by the sub-sampling procedure, winconst and winstp parameters represent respectively a positive constant used to determine the sub-window size and the the step with which the sub-window moves.

In the spatial case (subset of R^2), the domain is seen as a rectangle $B \times H$, therefore the size of the sub-window side b is given by $b = winconst \times \sqrt(B)$ (similar is of h). For a complete description see Lee and Lahiri (2002). By default winconst is set $B/(4 \times \sqrt(B))$. This way we ensure to have at least eight spatial blocks. The winstp parameter is used to determine the sub-window step. The latter is given by the proportion of the sub-window size, so that when winstp=1 there is not overlapping between contiguous sub-windows. In the spatial case by default winstp=0.5. The sub-window is moved by successive steps in order to cover the entire spatial domain. Observations, that fall in disjoint or overlapping windows are considered indipendent samples.

In the spatio-temporal case the subsampling is meant only in time as described by Li et al. (2007). Thus, winconst_t represents the lenght of the temporal sub-window. By default the size of the sub-window is computed following the rule established in Li et al. (2007). By default winstp is the time step.

Value

Returns an object of class GeoFit. An object of class GeoFit is a list containing at most the following components:

bivariate	Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;
clic	The composite information criterion, if the full likelihood is considered then it coincides with the Akaike information criterion;
coordx	A d-dimensional vector of spatial coordinates;

coordy	A d-dimensional vector of spatial coordinates;
coordt	A t-dimensional vector of temporal coordinates;
coordx_dyn	A list of dynamical (in time) spatial coordinates;
convergence	A string that denotes if convergence is reached;
corrmodel	The correlation model;
data	The vector or matrix or array of data;
distance	The type of spatial distance;
fixed	The vector of fixed parameters;
iterations	The number of iteration used by the numerical routine;
likelihood	The configuration of the composite likelihood;
logCompLik	The value of the log composite-likelihood at the maximum;
maxdist	The maximum spatial distance used in the weigthed composite likelihood. If no spatial distance is specified then it is NULL;
maxtime	The maximum temporal distance used in the weigthed composite likelihood. If no spatial distance is specified then it is NULL;
message	Extra message passed from the numerical routines;
model	The density associated to the likelihood objects;
n	The number of trials in a binominal RF;the number of successes in a negative Binomial RFs;
nozero	In the case of tapered likelihood the percentage of non zero values in the covariance matrix. Otherwise is NULL.
numcoord	The number of spatial coordinates;
numtime	The number of the temporal realisations of the RF;
param	The vector of parameters' estimates;
param	The radius of the sphere in the case of great circle distance;
stderr	The vector of standard errors;
sensmat	The sensitivity matrix;
varcov	The matrix of the variance-covariance of the estimates;
varimat	The variability matrix;
vartype	The method used to compute the variance of the estimates;
type	The type of the likelihood objects.
winconst	The constant used to compute the window size in the spatial sub-sampling;
winstp	The step used for moving the window in the spatial sub-sampling;
winconst_t	The constant used to compute the window size in the spatio-temporal sub-sampling;
winstp_	The step used for moving the window in the spatio-temporal sub-sampling;
X	The matrix of covariates;

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Examples

```
library(GeoModels)
library(fields)

################################################################
############ Examples of spatial Gaussian RFs ################
```

```
###############################################################

################################################################
###
### Example 1, 2, 3:  Estimation of a spatial Gaussian RF with
### exponential correlation using and pairwise likelihood
### maximum likelihood and tapering likelihood
###############################################################

# Define the spatial-coordinates of the points:
set.seed(3)
N=400  # number of location sites
x <- runif(N, 0, 1)
set.seed(6)
y <- runif(N, 0, 1)
coords <- cbind(x,y)

# Define spatial matrix covariates
X=cbind(rep(1,N),runif(N))

# Set the covariance model's parameters:
corrmodel <- "Exp"
mean <- 0.2
mean1 <- -0.5
sill <- 1
nugget <- 0
scale <- 0.2/3
param<-list(mean=mean,mean1=mean1,sill=sill,nugget=nugget,scale=scale)

# Simulation of the spatial Gaussian RF:
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param,X=X)$data

fixed<-list(nugget=nugget)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)

################################################################
###
### Example 1. Maximum pairwise likelihood fitting of
### Gaussian RFs with exponential correlation.
###
###############################################################
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
                    maxdist=0.05,likelihood="Marginal",type="Pairwise",
                    start=start,fixed=fixed,X=X)
print(fit1)

################################################################
###
### Example 2. Standard Maximum likelihood fitting of
### Gaussian RFs with exponential correlation.
###
###############################################################
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
                    likelihood="Full",type="Standard",
                    start=start,fixed=fixed,X=X)
print(fit2)
```

```
################################################################
###
### Example 3. Tapered Maximum likelihood fitting of
### Gaussian RFs with exponential correlation.
###
###############################################################


fit3 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
                    likelihood="Full",type="Tapering",taper="Wendland1",
                    maxdist=0.1,start=start,fixed=fixed,X=X)
print(fit3)



###############################################################
############ Examples of spatial non-Gaussian RFs #############
###############################################################


################################################################
###
### Example 4. Maximum pairwise likelihood fitting of spatial
### Gamma and Weibull  RFs with Wendland correlation
###
###############################################################
set.seed(524)
# Define the spatial-coordinates of the points:
N=500
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
shape=2
scale=0.2

model="Weibull"
corrmodel="Wend0"
param=list(mean=mean,mean1=mean1,sill=1-nugget,scale=scale,shape=shape,nugget=nugget,power2=4)
# Simulation of a  non stationary weibull RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,
           param=param)$data

fixed<-list(nugget=nugget,power2=4,sill=1-nugget)
start<-list(mean=mean,mean1=mean1,scale=scale,shape=shape)

# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
                    start=start,fixed=fixed)
print(fit$param)

model="Gamma"
start<-list(mean=mean,mean1=mean1,scale=scale)
fixed<-list(nugget=nugget,power2=4,sill=1-nugget,shape=6)
```

```
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel="Wend0", model=model,
                    maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
                    start=start,fixed=fixed)

print(fit$param)


################################################################
###
### Example 5. Maximum pairwise likelihood fitting of
### StudendT spatial  RFs with Wendland correlation
###
###############################################################

set.seed(15274)
# Define the spatial-coordinates of the points:
N=300
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
sill=0.5
scale=0.2
df=4   ## degrees of freedom
model="StudentT"
corrmodel="Wend0"
param=list(mean=mean,mean1=mean1,sill=sill,scale=scale,df=1/df,nugget=nugget,power2=4)
# Simulation of a studentT RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,
           param=param)$data
##  estimation assuming df unknown
fixed<-list(nugget=nugget,power2=4)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill,df=1/df)
# Maximum pairwise composite-likelihood fitting of the RF:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
                    start=start,fixed=fixed)
print(fit1$param)

##  df must be rounded and fixed
df=round(1/(as.numeric(fit1$param['df'])))
 fixed<-list(nugget=nugget,power2=4,df=1/df)
 start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
                    start=start,fixed=fixed)
print(fit$param)
################################################################
###
### Example 6. Maximum pairwise likelihood fitting of
### SinhAsinh-Gaussian spatial  RFs with Wendland correlation
###
###############################################################
```

```
set.seed(261)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
x <- runif(500, 0, 1)
y <- runif(500, 0, 1)
coords <- cbind(x,y)

corrmodel="Wend0"
mean=0;nugget=0
sill=1
skew=-0.5
tail=1.5
power2=4
c_supp=0.2

# model parameters
param=list(power2=power2,skew=skew,tail=tail,
             mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param)$data

plot(density(data))
fixed=list(power2=power2,nugget=nugget)
start=list(scale=c_supp,skew=skew,tail=tail,mean=mean,sill=sill)
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    maxdist=0.05,likelihood="Marginal",type="Pairwise",
                    start=start,fixed=fixed)
print(fit1$param)
# Maximum likelihood:
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    likelihood="Full",type="Standard",
                    start=start,fixed=fixed)
print(fit2$param)


################################################################
###
### Example 7. Maximum pairwise likelihood fitting of
### Binomial and negative Binomial   RFs
### with exponential correlation.
###
###############################################################

set.seed(422)
N=350
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
mean=0.1; mean1=0.8; mean2=-0.5 # regression parameters
X=cbind(rep(1,N),runif(N),runif(N)) # marix covariates
corrmodel <- "Wend0"
param=list(mean=mean,mean1=mean1,mean2=mean2,sill=1,nugget=0,scale=0.2,power2=4)
# Simulation of the spatial Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="Binomial", n=10,X=X,
             param=param)$data

fixed <- list(nugget=nugget,power2=4,sill=1)
```

```
start <- list(scale=0.2,mean=mean,mean1=mean1,mean2=mean2)
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel,n=10, X=X,
                    maxdist=0.05,model="Binomial",  fixed=fixed,
                    start=start)
print(fit1)

set.seed(220)
# Simulation of the spatial Negative Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="BinomialNeg", n=5,X=X,
              param=param)$data
# Maximum pairwise likelihood:
fit2 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel, n=5,X=X,
                    maxdist=0.05,model="BinomialNeg", fixed=fixed,
                    start=start)

print(fit2)


###############################################################
######### Examples of spatio-temporal RFs ###########
###############################################################
set.seed(52)
# Define the temporal sequence:
time <- seq(1, 10, 1)

# Define the spatial-coordinates of the points:
x <- runif(20, 0, 1)
set.seed(42)
y <- runif(20, 0, 1)
coords=cbind(x,y)

# Set the covariance model's parameters:
corrmodel="Exp_Exp"
scale_s=0.2/3
scale_t=1
sill=1
nugget=0
mean=0

param<-list(mean=0,scale_s=scale_s,scale_t=scale_t,
            sill=sill,nugget=nugget)

# Simulation of the spatial-temporal Gaussian RF:
data <- GeoSim(coordx=coords,coordt=time,corrmodel=corrmodel,
              param=param)$data

################################################################
###
### Example 8. Maximum pairwise likelihood fitting of a
### space time Gaussian RF with double-exponential correlation.
###
###############################################################
# Fixed parameters
fixed<-list(nugget=nugget)
# Starting value for the estimated parameters
start<-list(mean=mean,scale_s=scale_s,scale_t=scale_t,sill=sill)
```

```
# Maximum composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,coordt=time,
                    corrmodel="Exp_Exp",maxtime=1,maxdist=0.3,
                    likelihood="Marginal",type="Pairwise",
                    start=start,fixed=fixed)
print(fit)
################################################################
###
### Example 9. Maximum standard likelihood fitting of a
### space time Gaussian RF observed on  dynamical spatial coordinates
### with double-exponential correlation.
###############################################################

maxN=50
coordx_dyn=list()
set.seed(31)
for(k in 1:length(time))
{
NN=sample(1:maxN,size=1)
x <- runif(NN, 0, 1)
y <- runif(NN, 0, 1)
coordx_dyn[[k]]=cbind(x,y)
}

data <- GeoSim(coordx_dyn=coordx_dyn, coordt=time, corrmodel="Exp_Exp",
                param=param)$data

fit <- GeoFit(data=data,coordx_dyn=coordx_dyn,coordt=time,
                    corrmodel="Exp_Exp",
                    likelihood="Full",type="Standard",
                    start=start,fixed=fixed)
print(fit)



###############################################################
######### Examples of  spatial bivariate RFs ###########
###############################################################


################################################################
###
### Example 10. Maximum, and pairwise  likelihood fitting of a
### bivariate Gaussian RF with separable Bivariate  matern
### (cross) correlation model.
###
###############################################################

# Define the spatial-coordinates of the points:
set.seed(5)
x <- runif(250, 0, 1)
y <- runif(250, 0, 1)
coords=cbind(x,y)
# parameters
param=list(mean_1=0,mean_2=0,scale=0.1,smooth=0.5,sill_1=1,sill_2=1,
```

```
          nugget_1=0,nugget_2=0,pcol=0.2)
# Simulation of a spatial Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel="Bi_Matern_sep",
             param=param)$data

# selecting fixed and estimated parameters
fixed=list(nugget_1=0,nugget_2=0,smooth=0.5)
start=list(mean_1=0,mean_2=0,sill_1=var(data[1,]),sill_2=var(data[2,]),
          scale=0.1,pcol=cor(data[1,],data[2,]))


# Maximum pairwise likelihood

fitcl<- GeoFit(data=data, coordx=coords, corrmodel="Bi_Matern_sep",
                     likelihood="Marginal",type="Pairwise",start=start,
                     fixed=fixed,maxdist=c(0.05,0.05,0.05))
print(fitcl)

# Maximum likelihood :
fitml<- GeoFit(data=data, coordx=coords, likelihood="Full",
                     corrmodel="Bi_Matern_sep", type="Standard",
                     start=start, fixed=fixed)
print(fitml)
```

GeoKrig *Spatial and spatio temporal standard, tapered and pairwise (simple and ordinary) kriging and bivariate cokriging for Gaussian and non Gaussian RFs.*

Description

The function computes simple or ordinary (tapered) kriging, in addition, for a set of unknown spatial location sites and temporal instants and a given space or space-time covariance model, it computes the Kriging variance.

Usage

```
GeoKrig(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel,distance="Eucl",
    grid=FALSE, loc, maxdist=NULL, maxtime=NULL, method="cholesky",
    model="Gaussian", n=1,nloc=NULL,mse=FALSE, lin_opt=TRUE,
    param, radius=6378.388, sparse=FALSE,taper=NULL,tapsep=NULL,
     time=NULL, type="Standard",type_mse=NULL,
     type_krig="Simple",weigthed=TRUE,which=1, X=NULL,Xloc=NULL)
```

Arguments

`data` A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.

`coordx` A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates used for prediction. qndd-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.

`coordy` A numeric vector giving 1-dimension of spatial coordinates used for prediction; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.

`coordt` A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is `NULL` then a spatial random field is expected.

`coordx_dyn` A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`

`corrmodel` String; the name of a correlation model, for the description see the Section **Details**.

`distance` String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details** of `GeoFit`.

`grid` Logical; if `FALSE` (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).

`lin_opt` Logical;If TRUE (default) then optimal (pairwise) linear kriging is computed. Otherwise optimal (pairwise) kriging is computed in the mean square sense.

`loc` A numeric $(n \times 2)$-matrix (where `n` is the number of spatial sites) giving 2-dimensions of spatial coordinates to be predicted.

`maxdist` Numeric; an optional positive value indicating the maximum spatial compact support in the case of covariance tapering kriging.

`maxtime` Numeric; an optional positive value indicating the maximum temporal compact support in the case of covasriance tapering kriging.

`method` String; the type of matrix decomposition used in the simulation. Default is `cholesky`. The other possible choices is `svd`.

`n` Numeric; the number of trials in a binomial random fields. Default is 1.

`nloc` Numeric; the number of trials of the locations sites to be predicted in a binomial random fields type II. Default is 1.

`mse` Logical; if `TRUE` (the default) MSE of the kriging predictor is computed

`model` String; the type of RF and therefore the densities associated to the likelihood objects. `Gaussian` is the default, see the Section **Details**.

`param` A list of parameter values required for the correlation model.See the Section **Details**.

`radius` Numeric: the radius of the sphere if coordinates are passed in lon/lat format;

`sparse` Logical; if `TRUE` kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.

`taper` String; the name of the taper correlation function, see the Section **Details**.

`tapsep` Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see **Details**).

`time`	A numeric $(m \times 1)$ vector (where `m` is the number of temporal instants) giving the temporal instants to be predicted; the default is `NULL` then only spatial prediction is performed.
`type`	String; if `Standard` then standard kriging is performed;if `Tapering` then kriging with covariance tapering is performed;if `Pairwise` then pairwise kriging is performed
`type_mse`	String; if `Theoretical` then theoretical MSE pairwise kriging is computed. If `SubSamp` then an estimation based on subsampling is computed.
`type_krig`	String; the type of kriging. If `Simple` (the default) then simple kriging is performed. If `ordinary` then ordinary kriging is performed. (See the Section **Details**).
`weigthed`	Logical; if `TRUE` then decreasing weigths coming from a compactly supported correlation function with compact support `maxdist` (`maxtime`)are used in the pairwise kriging.
`which`	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2
`X`	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
`Xloc`	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.

Details

Best linear unbiased predictor and associated mean square error is computed for Gaussian and some non Gaussian cases. Specifically, for a spatial or spatio-temporal or spatial bivariate dataset, given a set of spatial locations and temporal istants and a correlation model `corrmodel` with some fixed parameters and given the type of RF (`model`) the function computes simple or ordinary kriging, for the specified spatial locations `loc` and temporal instants `time`, providing also the respective mean square error. For the choice of the spatial or spatio temporal correlation model see details in `GeoCovmatrix` function. The list `param` specifies mean and covariance parameters, see `CorrParam` and `GeoCovmatrix` for details. The `type_krig` parameter indicates the type of kriging. In the case of simple kriging, the known mean can be specified by the parameter `mean` in the list `param` (See examples). In the Gaussian case, it is possible to perform kriging based on covariance tapering for simple kriging (Furrer et. al, 2008). In this case, space or space-time tapered function and spatial or spatio- temporal compact support must be specified. For the choice of a space or space-time tapered function see `GeoCovmatrix`. When performing kriging with covariance tapering, sparse matrix algorithms are exploited using the package `spam`.

Value

Returns an object of class `Kg`. An object of class `Kg` is a list containing at most the following components:

`bivariate`	`TRUE` if spatial bivariate cokriging is performed, otherwise `FALSE`;
`coordx`	A d-dimensional vector of spatial coordinates used for prediction;
`coordy`	A d-dimensional vector of spatial coordinates used for prediction;
`coordt`	A t-dimensional vector of temporal coordinates used for prediction;
`corrmodel`	String: the correlation model;
`covmatrix`	The covariance matrix if `type` is `Standard`. An object of class spam if `type` is `Tapering`

data	The vector or matrix or array of data used for prediction
distance	String: the type of spatial distance;
grid	TRUE if the spatial data used for prediction are observed in a regular grid, otherwise FALSE;
loc	A $(n \times 2)$-matrix of spatial locations to be predicted.
n	The number of trial for Binomial RFs
nozero	In the case of tapered simple kriging the percentage of non zero values in the covariance matrix. Otherwise is NULL.
numcoord	Numeric:he number d of spatial coordinates used for prediction;
numloc	Numeric: the number n of spatial coordinates to be predicted;
numtime	Numeric: the number d of the temporal instants used for prediction;
numt	Numeric: the number m of the temporal instants to be predicted;
model	The type of RF, see GeoFit.
param	Numeric: The covariance parameters;
pred	A $(m \times n)$-matrix of spatio or spatio temporal kriging prediction;
radius	Numeric: the radius of the sphere if coordinates are pssed in lon/lat format;
spacetime	TRUE if spatio-temporal kriging and FALSE if spatial kriging;
tapmod	String: the taper model if type is Tapering. Otherwise is NULL.
time	A m-dimensional vector of temporal coordinates to be predicted;
type	String: the type of kriging (Standard or Tapering).
type_krig	String: the type of kriging (Simple or Ordinary).
mse	A $(m \times n)$-matrix of spatio or spatio temporal mean square error kriging prediction;

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References

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York. Furrer R., Genton, M.G. and Nychka D. (2006). *Covariance Tapering for Interpolation of Large Spatial Datasets*. Journal of Computational and Graphical Statistics, **15-3**, 502–523.

See Also

GeoCovmatrix

Examples

```
library(GeoModels)
library(fields)

################################################################
############### Examples of Spatial kriging ###################
################################################################
```

```
# Define the spatial-coordinates of the points:
set.seed(79)
x <- runif(200, 0, 1)
y <- runif(200, 0, 1)
coords<-cbind(x,y)
# Set the exponential cov parameters:
corrmodel_1 <- "exponential"
mean<-0
sill<-1
nugget<-0
scale<-0.3/3
param<-list(mean=mean,sill=sill,nugget=nugget,scale=scale)

# Set the wendland parameters (two compatible correlations):
corrmodel_2 <- "Wend0"
mean<-0
sill<-1
nugget<-0
power2=3
c_supp<-0.3
param_wen<-list(mean=mean,sill=sill,nugget=nugget,scale=c_supp,power2=power2)



# Simulation of the spatial Gaussian random field:
data <- GeoSim(coordx=coords, corrmodel=corrmodel_1,
              param=param)$data

# locations to predict
xx<-seq(0,1,0.03)
loc_to_pred<-as.matrix(expand.grid(xx,xx))

################################################################
###
### Example 1. Spatial simple kriging of n sites of a
### Gaussian random fields with exponential correlation.
###
###############################################################
pr<-GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_1,
       param= param, data=data,mse=TRUE)

################################################################
###
### Example 2. Spatial tapered simple kriging of n sites of a
### Gaussian random fields with exponential correlation.
###
###############################################################
pr_tap=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_1,data=data,
       param= param,type="Tapering",maxdist=0.2,taper="Wendland1",mse=TRUE)


################################################################
###
### Example 3. Spatial  simple kriging of n sites of a
### Gaussian random fields using a compatible Wendland model
###
```

```
################################################################
param<-list(mean=mean,sill=sill,nugget=nugget,scale=scale,power2=power2)
pr_wen=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_2,data=data,
       param=param_wen,sparse=TRUE,mse=TRUE)

colour <- rainbow(100)
par(mfrow=c(3,2))
zlim=c(-2.6,2.6)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",
           main="Simple Kriging with  exponential model ")

# simple kriging map prediction variance
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")

# simple tapered kriging map prediction
image.plot(xx, xx, matrix(pr_tap$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",main="Simple Tapered Kriging")

# simple taperd kriging map prediction variance
image.plot(xx, xx, matrix(pr_tap$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")

# simple tapered kriging map prediction
image.plot(xx, xx, matrix(pr_wen$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",main="Simple Kriging with Wendland model")

# simple kriging map prediction variance
image.plot(xx, xx, matrix(pr_wen$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")



################################################################
###
### Example 4. Spatial  simple kriging of a binomial
###            random field
###
################################################################
set.seed(312)
model="Binomial";n=6

# Define the spatial-coordinates of the points:
x <- runif(1000)
y <- runif(1000)
coords=cbind(x,y)
#### mean and covariance parameters ###
mean<-0
sill<-1
nugget<-0
scale<-0.2
#################
param<-list(mean=mean,sill=sill,nugget=nugget,scale=scale,power2=4)
# Simulation of the Binomial Gaussian random field:
```

```
data <- GeoSim(coordx=coords, corrmodel="Wend0",model=model,n=n,
              sparse=TRUE,param=param)$data


par(mfrow=c(1,2))

#### map of simulated data
quilt.plot(x, y, data,nlevel=n+1,col=rainbow(n+1),zlim=c(0,n), main="Data")

## estimation with pairwise likelihood
fixed=list(nugget=nugget,power2=4,sill=1)
start=list(mean=0,scale=scale)
# Maximum pairwise likelihood fitting :
fit <- GeoFit(data, coordx=coords, corrmodel="Wend0",model=model,n=n,
                    likelihood='Marginal', type='Pairwise',maxdist=0.03,
                    start=start,fixed=fixed)

# locations to predict
xx<-seq(0,1,0.02)
loc_to_pred<-as.matrix(expand.grid(xx,xx))

## simple kriging
pr<-GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,n=n,
       sparse=TRUE,param= as.list(c(fit$param,fit$fixed)))

#standard   binomial kriging
map_binom=matrix(round(pr$pred),ncol=length(xx))
image.plot(xx, xx, map_binom,nlevel=n+1,col=rainbow(n+1),zlim=c(0,n),
           xlab="",ylab="",main="Simple Kriging ")




################################################################
###
### Example 5. Spatial  simple kriging of a Weibull
###            random field
###
###############################################################
set.seed(312)
model="Weibull";

# Define the spatial-coordinates of the points:
x <- runif(1000)
y <- runif(1000)
coords=cbind(x,y)
#### mean and covariance parameters ###
mean<-0
sill<-1
nugget<-0
scale<-0.2
shape=0.8

#################
param<-list(mean=mean,sill=sill,shape=shape,nugget=nugget,scale=scale,power2=4)
# Simulation of the Weibull random field:
data <- GeoSim(coordx=coords, corrmodel="Wend0",model=model,
```

```
              sparse=TRUE,param=param)$data


par(mfrow=c(1,2))

#### map of simulated data
quilt.plot(x, y, data, main="Data")

## estimation with pairwise likelihood
fixed=list(nugget=nugget,power2=4,sill=1)
start=list(mean=0,scale=scale,shape=shape)
# Maximum pairwise likelihood fitting :
fit <- GeoFit(data, coordx=coords, corrmodel="Wend0",model=model,
                    likelihood='Marginal', type='Pairwise',maxdist=0.03,
                    start=start,fixed=fixed)

# locations to predict
xx<-seq(0,1,0.02)
loc_to_pred<-as.matrix(expand.grid(xx,xx))

#optimal linear  kriging
pr<-GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,
       sparse=TRUE,param= as.list(c(fit$param,fit$fixed)))


map_weibull=matrix(pr$pred,ncol=length(xx))
image.plot(xx, xx, map_weibull,
           xlab="",ylab="",main="Simple Kriging ")





################################################################
###
### Example 5. Spatial  simple kriging of a t
###            random field
###
###############################################################
model="StudentT"
df=6
corrmodel <- "Wend0"
nsel=800
coords=cbind(runif(nsel),runif(nsel))

mean <- 0
sill <- 1.9
nugget <- 0
power2=4
scale <- 0.2

# Starting value for the estimated parameters
set.seed(3132)
param=list(nugget=nugget,mean=mean, scale=scale,sill=sill,df=1/df,power2=power2)
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
     model=model,sparse=TRUE)$data
```

```
fixed<-list(nugget=nugget,power2=4,df=1/df)
start<-list(mean=mean, scale=scale,sill=sill)

# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
                    maxdist=0.02,likelihood="Marginal",type="Pairwise",
                    start=start,fixed=fixed, model = model)

# locations to predict
xx<-seq(0,1,0.02)
loc_to_pred<-as.matrix(expand.grid(xx,xx))

#optimal linear  kriging
pr<-GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,
       sparse=TRUE,param= as.list(c(fit$param,fit$fixed)))

par(mfrow=c(1,2))
#### map of simulated data
quilt.plot(coords[,1], coords[,2], data, main="Data")

map_t=matrix(pr$pred,ncol=length(xx))
image.plot(xx, xx, map_t,
           xlab="",ylab="",main="Simple Kriging ")
################################################################
########### Examples of spatio temporal kriging ############
################################################################

# Define the spatial-coordinates of the points:
x <- runif(80, 0, 1)
y <- runif(80, 0, 1)
coords<-cbind(x,y)
times<-1:8

# Define model correlation and associated parameters
corrmodel<-"exp_exp"
param<-list(nugget=0,mean=0,scale_s=0.2/3,scale_t=1,sill=1)

# Simulation of the space time Gaussian random field:
set.seed(31)
data<-GeoSim(coordx=coords,coordt=times,corrmodel=corrmodel,
           param=param)$data

# Maximum pairwise likelihood fitting of the space time random field:
start <- list(scale_s=0.2/3,scale_t=1,sill=1,mean=0)
fixed <- list(nugget=0)
fit <- GeoFit(data, coordx=coords, coordt=times,
                    corrmodel=corrmodel, likelihood='Marginal',
                    type='Pairwise',start=start,fixed=fixed,
                    maxdist=0.1,maxtime=1)

################################################################
###
### Example 6. Spatio temporal simple kriging of n locations
### sites and m temporal instants for a Gaussian random fields
### with estimated double exponential correlation.
###
```

```
################################################################
set.seed(3)
param<-as.list(c(fit$param,fit$fixed))

# locations to predict
xx<-seq(0,1,0.03)
loc_to_pred<-as.matrix(expand.grid(xx,xx))
#  Define the times to predict
times_to_pred<-1:2

pr<-GeoKrig(loc=loc_to_pred,time=times_to_pred,coordx=coords,coordt=times,
       corrmodel=corrmodel, param=param,data=data,mse=TRUE)

par(mfrow=c(2,2))
zlim=c(-2.5,2.5)
colour <- rainbow(100)

for(i in 1:2) {
image.plot(xx, xx, matrix(pr$pred[i,],ncol=length(xx)),col=colour,
           zlim=zlim, main = paste(" Kriging Time=" , i),ylab="")
image.plot(xx, xx, matrix(pr$mse[i,],ncol=length(xx)),col=colour,
           main = paste("Std err Time=" , i),ylab="")
}

#################################################################
########### Examples of spatial bivariate cokriging ############
#################################################################

#################################################################
###
### Example 7. Bivariate simple cokriging  of n locations
### for a Gaussian random fields with separable Matern correlation
###
################################################################
# Define the spatial-coordinates of the points:
x <- runif(80, 0, 1)
y <- runif(80, 0, 1)
coords<-cbind(x,y)

# Simulation of a spatial bivariate Gaussian random field
# with Matern separable covariance model

set.seed(12)
param=list(scale=0.3/3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
          nugget_1=0,nugget_2=0,pcol=0.7,smooth=0.5)

data <- GeoSim(coordx=coords, corrmodel="Bi_matern_sep", param=param)$data

fixed=list(nugget_1=0,nugget_2=0,smooth=0.5,mean_1=0,mean_2=0)
start=list(sill_1=var(data[,1]),sill_2=var(data[,2]),scale=0.3/3,
      pcol=cor(data[1,],data[2,]))

# Maximum Composite likelihood fitting of the random field:
fitcl<- GeoFit(data, coordx=coords, corrmodel="Bi_matern_sep",
        likelihood="Marginal",type="Pairwise",maxdist=0.1,
        start=start,fixed=fixed)
```

```
# locations to predict
xx<-seq(0,1,0.03)
loc_to_pred<-as.matrix(expand.grid(xx,xx))


pr1<-GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
       param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=1,mse=TRUE)

pr2<-GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
       param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=2,mse=TRUE)

par(mfrow=c(2,2))


# simple kriging map prediction of the first variable
image.plot(xx, xx, matrix(pr1$pred,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="First Simple coKriging")

# simple kriging map prediction variance of the first variable
image.plot(xx, xx, matrix(pr1$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")


 # simple kriging map prediction of the second variable
image.plot(xx, xx, matrix(pr2$pred,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Second Simple coKriging")

# simple kriging map prediction variance of the second variable
image.plot(xx, xx, matrix(pr2$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")
```

GeoResiduals   *Computes fitted covariance and/or variogram*

Description

The procedure return a GeoFit object associated to the estimated residuals

Usage

```
GeoResiduals(fit)
```

Arguments

fit	A fitted object obtained from the GeoFit.

Value

A GeoFit object with the estimated residuals

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

GeoFit.

Examples

```
library(GeoModels)
set.seed(211)

model="Weibull";shape=4
# Set the coordinates of the points:
x <- runif(700, 0, 1)
y <- runif(700, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel <- "Wend0"
mean <- 5
sill <- 1
nugget <- 0
scale <- 0.3
power2=4


param=list(mean=mean,sill=sill, nugget=nugget, scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data

start=list(mean=mean,scale=scale,shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit <- GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
                    likelihood="Marginal",type='Pairwise',start=start,
                    fixed=fixed,maxdist=0.1)

res=GeoResiduals(fit)
# Empirical estimation of the variogram for the residuals:
vario <- GeoVariogram(res$data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(res, show.vario=TRUE, vario=vario,pch=20)
```

GeoSim *Simulation of Gaussian and non Gaussian Random Fields.*

Description

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields. The function return a realization of a Random Field for a given covariance model and covariance parameters. Simulation is based on Cholesky decomposition.

Usage

```
GeoSim(coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel, distance="Eucl",
     GPU=NULL, grid=FALSE, local=c(1,1),method="cholesky", model='Gaussian', n=1, param,
      radius=6378.388, sparse=FALSE, X=NULL)
```

Arguments

coordx	A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
GPU	Numeric; if NULL (the default) no GPU computation is performed.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
local	Numeric; number of local work-items of the GPU
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section **Details**.
n	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1.
param	A list of parameter values required in the simulation procedure of RFs, see **Examples**.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6378.88)
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake). It should be used with compactly supported covariance models.FALSE is the default.
X	Numeric; Matrix of space-time covariates.

Value

Returns an object of class GeoSim. An object of class GeoSim is a list containing at most the following components:

bivariate	Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;

coordx	A d-dimensional vector of spatial coordinates;
coordy	A d-dimensional vector of spatial coordinates;
coordt	A t-dimensional vector of temporal coordinates;
coordx_dyn	A list of dynamical (in time) spatial coordinates;
corrmodel	The correlation model; see GeoCovmatrix.
data	The vector or matrix or array of data, see GeoFit;
distance	The type of spatial distance;
model	The type of RF, see GeoFit.
n	The number of trial for Binomial RFs;the number of successes in a negative Binomial RFs;
numcoord	The number of spatial coordinates;
numtime	The number the temporal realisations of the RF;
param	The vector of parameters' estimates;
radius	The radius of the sphere if coordinates are passed in lon/lat format;
randseed	The seed used for the random simulation;
spacetime	TRUE if spatio-temporal and FALSE if spatial RF;

Author(s)

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Examples

```
library(GeoModels)
library(mapproj)
library(fields)

################################################################
###
### Example 1. Simulation of a spatial Gaussian RF on a regular grid
###
###############################################################

# Define the spatial-coordinates of the points:
x <- seq(0,1,0.045)
y <- seq(0,1,0.045)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Matern", param=list(smooth=0.5,
             mean=0,sill=1,scale=0.4/3,nugget=0))$data

# Simulation of a spatial Gaussian RF with Generalized Wendland correlation function
# using sparse alghorithm matrices
set.seed(261)
data2 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend", param=list(smooth=0,
              power2=4,mean=0,sill=1,scale=0.4,nugget=0))$data
par(mfrow=c(1,2))
image.plot(x,y,data1,col=terrain.colors(100),main="Matern",xlab="",ylab="")
image.plot(x,y,data2,col=terrain.colors(100),main="Wendland",xlab="",ylab="")
```

```
################################################################
###
### Example 2. Simulation of a spatial binomial RF based on
### the latent Gaussian RF with exponential correlation
### on a regular grid
###
################################################################

# Define the spatial-coordinates of the points:
x <- seq(0, 1, 0.022)
y <- seq(0, 1, 0.022)
coords <- cbind(x,y)
set.seed(251)
n=5
# Simulation of a spatial Binomial RF:
sim <- GeoSim(x,y,grid=TRUE, corrmodel="Wend0",
             model="Binomial",n=n,sparse=TRUE,
             param=list(nugget=0,mean=0,scale=.2,sill=1,power2=4))

image.plot(x,y,sim$data,nlevel=n+1,col=terrain.colors(n+1),zlim=c(0,n))


################################################################
###
### Example 3. Simulation of a spatial Weibull  RF
### with  exponential correlation
###
###############################################################
# Define the spatial-coordinates of the points:
x <- seq(0,1,0.032)
y <- seq(0,1,0.032)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Exponential",model="Weibull",
         param=list(shape=1.2,mean=0,sill=1,scale=0.3/3,nugget=0))$data
image.plot(x,y,data1,col=terrain.colors(200),main="Weibull RF",xlab="",ylab="")



################################################################
###
### Example 4. Simulation of a spatial t  RF
### with  exponential correlation
###
###############################################################
# Define the spatial-coordinates of the points:
x <- seq(0,1,0.03)
y <- seq(0,1,0.03)
set.seed(268)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend",model="StudentT", sparse=TRUE,
         param=list(df=1/4,mean=0,sill=1,scale=0.3,nugget=0,smooth=1,power2=5))$data
image.plot(x,y,data1,col=terrain.colors(100),main="Student-t RF",xlab="",ylab="")
```

```
################################################################
###
### Example 5. Simulation of a sinhasinh RF
### with  Wend0 correlation.
###
###############################################################

# Define the spatial-coordinates of the points:
x <- runif(800, 0, 2)
y <- runif(800, 0, 2)
coords <- cbind(x,y)
set.seed(261)
corrmodel="Wend0"
# Simulation of a spatial Gaussian RF:
param=list(power2=4,skew=0,tail=1,
             mean=0,sill=1,scale=0.2,nugget=0)  ## gaussian case
data0 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
plot(density(data0),xlim=c(-7,7))

param=list(power2=4,skew=0,tail=0.7,
             mean=0,sill=1,scale=0.2,nugget=0) ## heavy tails
data1 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data1),lty=2)

param=list(power2=4,skew=0.5,tail=1,
             mean=0,sill=1,scale=0.2,nugget=0)  ## asymmetry
data2 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data2),lty=3)



################################################################
###
### Example 6. Simulation of a bivariate Gaussian RF
### with separable bivariate exponential correlation model
### on a regular grid.
###
###############################################################

# Define the spatial-coordinates of the points:
x <- seq(-1,1,0.08)
y <- seq(-1,1,0.08)

# Simulation of a bivariate spatial Gaussian RF:
# with a separable Bivariate Matern
set.seed(12)
param=list(mean_1=0,mean_2=0,scale=0.12,smooth=0.5,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=0.5)
data <- GeoSim(x,y,grid=TRUE,corrmodel="Bi_matern_sep",
              param=param)$data
par(mfrow=c(1,2))
```

```
image.plot(x,y,data[,,1],col=terrain.colors(100),main="1",xlab="",ylab="")
image.plot(x,y,data[,,2],col=terrain.colors(100),main="2",xlab="",ylab="")




################################################################
###
### Example 7. Simulation of a  spatio temporal Gaussian RF.
### observed on  dynamical location sites with double exponential correlation
###
###############################################################

# Define the dynamical spatial-coordinates of the points:

coordt=1:5
coordx_dyn=list()
maxN=40
set.seed(8)
for(k in 1:length(coordt))
{
NN=sample(1:maxN,size=1)
x <- runif(NN, 0, 1)
y <- runif(NN, 0, 1)
coordx_dyn[[k]]=cbind(x,y)
}
coordx_dyn

param<-list(nugget=0,mean=0,scale_s=0.2/3,scale_t=2/3,sill=1)
data <- GeoSim(coordx_dyn=coordx_dyn, coordt=coordt, corrmodel="Exp_Exp",
                     param=param)$data
## spatial realization at first temporal instants
data[[1]]
## spatial realization at third temporal instants
data[[3]]




################################################################
###
### Example 8. Simulation of a Gaussian RF
###  with a Wend0 correlation on the planet earth
###
###############################################################

require(plot3D)
require(sphereplot)
distance="Geod";radius=6378.88

NN=4500 ## total point on the sphere on lon/lat format
set.seed(80)
coords=pointsphere(NN,c(-180,180),c(-90,90),c(radius,radius))
## Set the wendland parameters
corrmodel <- "Wend0"
param<-list(mean=0,sill=1,nugget=0,scale=radius*0.3,power2=3)
```

```
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(coordx=coords,corrmodel=corrmodel,sparse=TRUE,
        distance=distance,radius=radius,param=param)$data
#converting in 3d cartesian  coordinates
b=sph2car(coords[,1], coords[,2], radius = radius, deg = TRUE)
x0 = b[,1]; y0 = b[,2]; z0 = b[,3]
# plotting
scatter3D(x0,y0,z0,colvar=data,clim=c(min(data),max(data)),pch=20,cex=0.8,
col=rainbow(200))



################################################################
###
### Example 9. Simulation of a Gaussian RF
###  with Wend0 model  on USA
###
###############################################################

distance="Geod";radius=6378.88
NN=40
x=seq(-125,-64,length.out=NN)
y=seq(27,50, length.out =NN)
nrow(expand.grid(x,y))
## Set the wendland parameters
corrmodel <- "Wend0"
param<-list(mean=0,sill=1,nugget=0,scale=radius*0.3,power2=3)
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(x,y,grid=TRUE,corrmodel=corrmodel,sparse=TRUE,
        distance=distance,radius=radius,param=param)$data
image.plot(x,y,data,col=terrain.colors(100),xlab="",ylab="")
map("usa", add = TRUE)



################################################################
###
### Example 10. Simulation of a Wrapped  RF
### with exponential correlation
### on a regular grid
###
###############################################################

# Define the spatial-coordinates of the points:
x <- runif(200,0, 1)
y <- runif(200,0, 1)
coords <- cbind(x,y)
set.seed(251)

# Simulation of a spatial wrapped RF:
sim <- GeoSim(coordx=coords, corrmodel="Exp",
             model="Wrapped",
             param=list(nugget=0,mean=0,scale=.1,sill=1))$data
```

```
long <- 0.08;
x1 <- coords[,1] + long*cos(sim)
y1 <- coords[,2] + long*sin(sim)
eps <- 0.1
plot(0,xlim=c(0-eps,1+eps),ylim=c(0-eps,1+eps));
require(shape)
Arrows(coords[,1], coords[,2], x1, y1, arr.length = 0.2, code = 2,  arr.type = "triangle",col=1)
```

`GeoTests` *Statistical Hypothesis Tests for Nested Models*

Description

The function performs statistical hypothesis tests for nested models based on composite likelihood versions of: Wald-type, score-type and Wilks-type (likelihood ratio) statistics.

Usage

```
GeoTests(object1, object2, ..., statistic)
```

Arguments

`object1`	An object of class `GeoFit`.
`object2`	An object of class `GeoFit` that is a nested model within `object1`.
`...`	Further successively nested objects.
`statistic`	String; the name of the statistic used within the hypothesis test (see **Details**).

Details

The implemented hypothesis tests for nested models are based on the following statistics:

1. Wald-type (`Wald`);
2. Score-type, also known as Rao-type (`Rao`);
3. Wilks-type; also known as the composite likelihood ratio statistic. Available are variants of the basic version, in particular:
   - Rotnitzky and Jewell adjustment (`WilksRJ`);
   - Satterhwaite adjustment (`WilksS`);
   - Chandler and Bate adjustment (`WilksCB`);
   - Pace, Salvan and Sartori adjustment (`WilksPSS`);

More specifically, consider an p-dimensional random vector $\mathbf{Y}$ with probability density function $f(\mathbf{y};\theta)$, where $\theta \in \Theta$ is a q-dimensional vector of parameters. Suppose that $\theta = (\psi, \tau)$ can be partitioned in a q'-dimensional subvector ψ and q''-dimensional subvector τ. Assume also to be interested in testing the specific values of the vector ψ. Then, one can use some statistical hypothesis tests for testing the null hypothesis $H_0 : \psi = \psi_0$ against the alternative $H_1 : \psi \neq \psi_0$. Composite likelihood versions of 'Wald' and 'score' statistics have the usual asymptotic chi-square distribution with q' degree of freedom. The Wald-type statistic is

$$W = (\hat{\psi} - \psi_0)^T (G^{\psi\psi})^{-1}(\hat{\theta})(\hat{\psi} - \psi_0),$$

where $G_{\psi\psi}$ is the $q' \times q'$ submatrix of the Godambe information pertaining to ψ and $\hat\theta$ is the maximum likelihood estimator from the full model. The score-type statistic (Rao-type) is

$$W = s_\psi\{\psi_0, \hat\tau(\psi_0)\}^T H^{\psi\psi}(\hat\theta_\psi)\{G^{\psi\psi}(\hat\theta_\psi)\}^{-1} H^{\psi\psi}(\hat\theta_\psi) s_\psi\{\psi_0, \hat\tau(\psi_0)\},$$

where $H^{\psi\psi}$ is the $q' \times q'$ submatrix of the inverse of $H(\theta)$ pertaining to ψ (the same for G) and $\hat\theta_\psi$ is the constrained maximum likelihood estimate of θ for fixed ψ. These two statistics can be called from the routine `GeoTests` assigning at the argument `statistic` respectively the values: `Wald` and `Rao`.

Alternatively to the Wald-type and score-type statistics one can use the composite version of the Wilks-type or likelihood ratio statistic, given by

$$W = 2[C\ell(\hat\theta; \mathbf{y}) - C\ell\{\psi_0, \hat\tau(\psi_0); \mathbf{y}\}].$$

The asymptotic distribution of the composite likelihood ratio statistic is given by

$$W \dot\sim \sum_i \lambda_i \chi^2,$$

for $i = 1, \ldots, q'$, where χ_i^2 are q' iid copies of a chi-square one random variable and $\lambda_1, \ldots, \lambda_{q'}$ are the eigenvalues of the matrix $(H^{\psi\psi})^{-1} G^{\psi\psi}$. There exist several adjustments to the composite likelihood ratio statistic in order to get an approximated $\chi^2_{q'}$. For example, Rotnitzky and Jewell (1990) proposed the adjustment $W' = W/\bar\lambda$ where $\bar\lambda$ is the average of the eigenvalues λ_i. This statistic can be called within the routine by the value: `WilksRJ`. A better solution is proposed by Satterhwaite (1946) defining $W'' = \nu W/(q'\bar\lambda)$, where $\nu = (\sum_i \lambda)^2/\sum_i \lambda_i^2$ for $i = 1 \ldots, q'$, is the effective number of the degree of freedom. Note that in this case the distribution of the likelihood ratio statistic is a chi-square random variable with ν degree of freedom. This statistic can be called from the routine assigning the value: `WilksS`. For the adjustments suggested by Chandler and Bate (2007) and Pace, Salvan and Sartori (2011) we refere to the articles (see **References**), these versions can be called from the routine assigning respectively the values: `WilksCB` and `WilksPSS`.

Value

An object of class `c("data.frame")`. The object contain a table with the results of the tested models. The rows represent the responses for each model and the columns the following results:

`Num.Par`	The number of the model's parameters.
`Diff.Par`	The difference between the number of parameters of the model in the previous row and those in the actual row.
`Df`	The effective number of degree of freedom of the chi-square distribution.
`Chisq`	The observed value of the statistic.
`Pr(>chisq)`	The p-value of the quantile `Chisq` computed using a chi-squared distribution with `Df` degrees of freedom.

Author(s)

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See Also

GeoFit.

Examples

```
library(GeoModels)
set.seed(3451)
# Define the spatial-coordinates of the points:
x <- runif(700, 0, 1)
y <- runif(700, 0, 1)
coords=cbind(x,y)
################################################################
###
### Example 1. Composite likelihood-based hypothesis testing.
### Testing an exponential model vs Matern model
###
###############################################################

# Set the model's parameters:
corrmodel <- "matern"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth <- 0.5

# Simulation of the spatial Gaussian random field:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,
              sill=sill,nugget=nugget,scale=scale,smooth=smooth))$data

# Pairwise-likelihood fitting of the random field, full model:
start=list(mean=mean,sill=sill,scale=scale,smooth=smooth)
fixed=list(nugget=nugget)
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.1,
                    varest=TRUE,likelihood="Marginal",type="Pairwise",
                    fixed=fixed,start=start)

# Pairwise-likelihood fitting of the random field, with a nested model:
start=list(mean=mean,sill=sill,scale=scale)
fixed=list(nugget=nugget,smooth=0.5)
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.1,
```

```
                    varest=TRUE,likelihood="Marginal",type="Pairwise",
                    fixed=fixed,start=start)

# Hypothesis testing results:
# composite Wald-type statistic:
GeoTests(fit1, fit2 ,statistic='Wald')
# composite score-type statistic:
#GeoTests(fit1, fit2,statistic='Rao')
# composite likelihood ratio statistic with RJ adjustment:
#GeoTests(fit1, fit2, statistic='WilksRJ')
# composite likelihood ratio statistic with S adjustment:
#GeoTests(fit1, fit2, statistic='WilksS')
# composite likelihood ratio statistic with CB adjustment:
GeoTests(fit1, fit2, statistic='WilksCB')
# composite likelihood ratio statistic with PSS adjustment:
#GeoTests(fit1, fit2, statistic='WilksPSS')




################################################################
###
### Example 2. Composite likelihood-based hypothesis testing.
### Testing significance of a covariate parameter
###
###############################################################
set.seed(3451)
# Define the spatial-coordinates of the points:
N=2000
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
X=cbind(rep(1,N),runif(N))
coords=cbind(x,y)
################################################################
###
### Example 1. Composite likelihood-based hypothesis testing.
### Simulation of a Gaussian spatial random field with
### Matern correlation.
###
###############################################################

# Set the model's parameters:
corrmodel <- "Exp"
mean <- 1; mean1=0
sill <- 1
nugget <- 0
scale <- 0.15/3

# Simulation of the spatial Gaussian random field:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,mean1=mean1,
              sill=sill,nugget=nugget,scale=scale),X=X)$data

# Pairwise-likelihood fitting of the random field, full model:
start=list(mean=mean,mean1=mean1,sill=sill,scale=scale)
fixed=list(nugget=nugget)
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.05,
                    varest=TRUE,likelihood="Marginal",type="Pairwise",
```

```
                    fixed=fixed,start=start,X=X)

# Pairwise-likelihood fitting of the random field, with a nested model:
start=list(mean=mean,sill=sill,scale=scale)
fixed=list(nugget=nugget,mean1=0)
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.05,
                    varest=TRUE,likelihood="Marginal",type="Pairwise",
                    fixed=fixed,start=start,X=X)


# Hypothesis testing results:
# composite Wald-type statistic:
GeoTests(fit1, fit2 ,statistic='Wald')
# composite score-type statistic:
#GeoTests(fit1, fit2,statistic='Rao')
# composite likelihood ratio statistic with RJ adjustment:
#GeoTests(fit1, fit2, statistic='WilksRJ')
# composite likelihood ratio statistic with S adjustment:
#GeoTests(fit1, fit2, statistic='WilksS')
# composite likelihood ratio statistic with CB adjustment:
GeoTests(fit1, fit2, statistic='WilksCB')
# composite likelihood ratio statistic with PSS adjustment:
#GeoTests(fit1, fit2, statistic='WilksPSS')
```

GeoVariogram *Empirical Variogram(variants) estimation*

Description

The function returns an empirical estimate of the variogram for spatio (temporal) and bivariate random fields.

Usage

```
GeoVariogram(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, cloud=FALSE,
            distance='Eucl', grid=FALSE, maxdist=NULL,
            maxtime=NULL, numbins=NULL, radius=6378.388,
            type='variogram',bivariate=FALSE)
```

Arguments

data	A d-dimensional vector (a single spatial realisation) or a $(n \times d)$-matrix (n iid spatial realisations) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$-array (n iid spatial realisations on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$-array (n iid spatial-temporal realisations) or or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid) or an $(d \times d \times t \times n)$-array ($n$ iid spatial-temporal realisations on regular grid). See GeoFit for details.
coordx	A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.

coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
cloud	Logical; if TRUE the variogram cloud is computed, otherwise if FALSE (the default) the empirical (binned) variogram is returned.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.
maxdist	A numeric value denoting the spatial maximum distance, see the Section **Details**.
maxtime	A numeric value denoting the temporal maximum distance, see the Section **Details**.
numbins	A numeric value denoting the numbers of bins, see the Section **Details**.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6378.88)
type	A String denoting the type of variogram. The option available is : variogram.
bivariate	Logical; if FALSE (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are intrepreted as a a realization from a bivariate field.

Details

We briefly report the definitions of semi-variogram used in this function. In the case of a spatial Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h) = 0.5 \sum_{x_i, x_j \in N(h)} (Z(x_i) - Z(x_j))^2 / |N(h)|$$

where $N(h)$ is the set of all the sample pairs whose distances fall into a tolerance region with size h (equispaced intervalls are considered).

In the case of a spatio-temporal Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h, u) = 0.5 \sum_{(x_i, l),(x_j, k) \in N(h,u)} (Z(x_i, l) - Z(x_j, k))^2 / |N(h, u)|$$

where $N(h, u)$ is the set of all the sample pairs whose spatial distances fall into a tolerance region with size h and $|k - l| = u$. Note, that $Z(x_i, l)$ is the observation at site x_i and time l.

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum spatial distance below which the shorter distances will be considered in the calculation of the (weigthed) least squares.

The maxtime parameter indicates the maximum temporal distance below which the shorter distances will be considered in the calculation of the (weigthed) least squares.

Value

Returns an object of class Variogram. An object of class Variogram is a list containing at most the following components:

bins	Adjacent intervals of grouped spatial distances if cloud=FALSE. Otherwise if cloud=TRUE all the spatial pairwise distances;
bint	Adjacent intervals of grouped temporal distances if cloud=FALSE. Otherwise if cloud=TRUE all the temporal pairwise distances;
cloud	If the variogram cloud is returned (TRUE) or the empirical variogram (FALSE);
centers	The centers of the spatial bins;
distance	The type of spatial distance;
lenbins	The number of pairs in each spatial bin;
lenbinst	The number of pairs in each spatial-temporal bin;
lenbint	The number of pairs in each temporal bin;
maxdist	The maximum spatial distance used for the calculation of the variogram. If no spatial distance is specified then it is NULL;
maxtime	The maximum temporal distance used for the calculation of the variogram. If no temporal distance is specified then it is NULL;
variograms	The empirical spatial variogram;
variogramst	The empirical spatial-temporal variogram;
variogramt	The empirical temporal variogram;
type	The type of estimated variogram

Author(s)

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References

Cressie, N. A. C. (1993) *Statistics for Spatial Data*. New York: Wiley.

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York.

See Also

GeoFit

Examples

```
library(GeoModels)

################################################################
###
### Example 1. Empirical estimation of the semi-variogram from a
### spatial Gaussian random field with exponential correlation.
###
###############################################################
set.seed(514)
# Set the coordinates of the sites:
x <- runif(200, 0, 1)
```

```
y <- runif(200, 0, 1)
coords <- cbind(x,y)
# Set the model's parameters:
corrmodel <- "exponential"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.3/3

# Simulation of the spatial Gaussian random field:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,
              sill=sill, nugget=nugget, scale=scale))$data

# Empirical spatial semi-variogram estimation:
fit <- GeoVariogram(coordx=coords,data=data,maxdist=0.6)

# Results:
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
     ylim=c(0, max(fit$variograms)), pch=20,
     main="Semi-variogram")


################################################################
###
### Example 2. Empirical estimation of the variogram from a
### spatio-temporal Gaussian random fields with Gneiting
### correlation function.
###
###############################################################

set.seed(331)
# Define the temporal sequence:
# Set the coordinates of the sites:
x <- runif(400, 0, 1)
y <- runif(400, 0, 1)
coords <- cbind(x,y)
times <- seq(1,5,1)

# Simulation of a spatio-temporal Gaussian random field:
data <- GeoSim(coordx=coords, coordt=times, corrmodel="gneiting",
              param=list(mean=0,scale_s=0.1,scale_t=0.1,sill=1,
              nugget=0,power_s=1,power_t=1,sep=0.5))$data

# Empirical spatio-temporal semi-variogram estimation:
fit <- GeoVariogram(data=data, coordx=coords, coordt=times, maxtime=5,maxdist=0.5)

# Results: Marginal spatial empirical  semi-variogram
par(mfrow=c(2,2), mai=c(.5,.5,.3,.3), mgp=c(1.4,.5, 0))
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
     ylim=c(0, max(fit$variograms)), xlim=c(0, max(fit$centers)),
     pch=20,main="Marginal spatial semi-variogram",cex.axis=.8)

# Results: Marginal temporal empirical  semi-variogram
plot(fit$bint, fit$variogramt, xlab='t', ylab=expression(gamma(t)),
     ylim=c(0, max(fit$variogramt)),xlim=c(0,max(fit$bint)),
     pch=20,main="Marginal temporal semi-variogram",cex.axis=.8)
```

```
# Building space-time semi-variogram
st.vario <- matrix(fit$variogramst,length(fit$centers),length(fit$bint))
st.vario <- cbind(c(0,fit$variograms), rbind(fit$variogramt,st.vario))

# Results: 3d Spatio-temporal semi-variogram
require(scatterplot3d)
st.grid <- expand.grid(c(0,fit$centers),c(0,fit$bint))
scatterplot3d(st.grid[,1], st.grid[,2], c(st.vario),
              highlight.3d=TRUE, xlab="h",ylab="t",
              zlab=expression(gamma(h,t)), pch=20,
              main="Space-time semi-variogram",cex.axis=.7,
              mar=c(2,2,2,2), mgp=c(0,0,0),
              cex.lab=.7)

# A smoothed version
par(mai=c(.2,.2,.2,.2),mgp=c(1,.3, 0))
persp(c(0,fit$centers), c(0,fit$bint), st.vario,
      xlab="h", ylab="u", zlab=expression(gamma(h,u)),
      ltheta=90, shade=0.75, ticktype="detailed", phi=30,
      theta=30,main="Space-time semi-variogram",cex.axis=.8,
      cex.lab=.8)

################################################################
###
### Example 3. Empirical estimation of the (cross) semivariograms
### from a bivariate Gaussian random fields with Matern
### correlation function.
###
###############################################################
# Simulation of a bivariate spatial Gaussian random field:
set.seed(29)
# Define the spatial-coordinates of the points:
x <- runif(200, 0, 1)
set.seed(7)
y <- runif(200, 0, 1)
coords=cbind(x,y)

# Simulation of a bivariate Gaussian Random field
# with matern (cross)  covariance function
param=list(mean_1=0,mean_2=0,scale_1=0.15/3,scale_2=0.2/3,scale_12=0.15/3,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,
           smooth_1=0.5,smooth_12=0.5,smooth_2=0.5,pcol=-0.45)
data <- GeoSim(coordx=coords, corrmodel="Bi_matern", param=param)$data

# Empirical  semi-(cross)variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=c(0.5,0.5,0.5))

# Variograms plots
par(mfrow=c(2,2))
plot(biv_vario$centers,biv_vario$variograms[1,],pch=20,xlab="h",ylim=c(0,1.2),
                                      ylab="",main=expression(gamma[11](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",
                                      ylab="",main=expression(gamma[12](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",ylab="",
                                      main=expression(gamma[21](h)))
plot(biv_vario$centers,biv_vario$variograms[2,],pch=20,xlab="h",,ylim=c(0,1.2),
                                      ylab="",main=expression(gamma[22](h)))
```

GeoWLS *WLS of Random Fields*

Description

the function returns the parameters' estimates and the estimates' variances of a random field obtained by the weigthed least squares estimator.

Usage

```
GeoWLS(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel,
              distance="Eucl", fixed=NULL, grid=FALSE, maxdist=NULL,
              maxtime=NULL,  model='Gaussian', optimizer='Nelder-Mead',
              numbins=NULL, radius=6378.388, start=NULL, weighted=FALSE)
```

Arguments

data	A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$-array (n iid spatial realisations on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid). See GeoFit for details.
coordx	A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description (see GeoFit).
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
maxdist	A numeric value denoting the maximum distance, see **Details** and GeoFit.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation (see GeoFit.
model	String; the type of random field. Gaussian is the default, see GeoFit for the different types.

optimizer	String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the default.
numbins	A numeric value denoting the numbers of bins, see the Section **Details**
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6378.88)
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see GeoFit).
weighted	Logical; if TRUE then the weighted least square estimator is considered. If FALSE (the default) then the classic least square is used.

Details

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum distance below which the shorter distances will be considered in the calculation of the (weigthed) least squares.

Value

Returns an object of class WLS. An object of class WLS is a list containing at most the following components:

bins	Adjacent intervals of grouped distances;
bint	Adjacent intervals of grouped temporal separations
centers	The centers of the bins;
coordx	The vector or matrix of spatial coordinates;
coordy	The vector of spatial coordinates;
coordt	The vector of temporal coordinates;
convergence	A string that denotes if convergence is reached;
corrmodel	The correlation model;
data	The vector or matrix of data;
distance	The type of spatial distance;
fixed	The vector of fixed parameters;
iterations	The number of iteration used by the numerical routine;
maxdist	The maximum spatial distance used for the calculation of the variogram used in least square estimation. If no spatial distance is specified then it is NULL;
maxtime	The maximum temporal distance used for the calculation of the variogram used in least square estimation. If no temporal distance is specified then it is NULL;
message	Extra message passed from the numerical routines;
model	The type of random fields;
numcoord	The number of spatial coordinates;
numtime	The number the temporal realisations of the random field;
param	The vector of parameters' estimates;
variograms	The empirical spatial variogram;
variogramt	The empirical temporal variogram;
variogramst	The empirical spatial-temporal variogram;
weighted	A logical value indicating if its the weighted method;
wls	The value of the least squares at the minimum.

Author(s)

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References

Cressie, N. A. C. (1993) *Statistics for Spatial Data*. New York: Wiley.

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York.

See Also

GeoFit, optim

Examples

```
library(GeoModels)


# Set the coordinates of the sites:

set.seed(211)
x <- runif(200, 0, 1)
set.seed(98)
y <- runif(200, 0, 1)
coords <- cbind(x,y)

################################################################
###
### Example 1. Least square fitting of a Gaussian random field
### with exponential correlation.
###
###############################################################

# Set the model's parameters:
corrmodel <- "Exponential"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.15/3
param <- list(mean=0,sill=sill, nugget=nugget, scale=scale)
# Simulation of the Gaussian random field:
set.seed(2)
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=param)$data

fixed=list(nugget=0,mean=mean)
start=list(scale=scale,sill=sill)
# Least square fitting of the random field:
fit <- GeoWLS(data=data,coordx=coords, corrmodel=corrmodel,
         fixed=fixed,start=start,maxdist=0.5)

# Results:
print(fit)
```

```
################################################################
###
### Example 3. Least square fitting of a spatio-temporal
### Gaussian random field with double exponential correlation.
###
###############################################################

# Define the temporal sequence:
time <- seq(1, 10, 1)
mean <- 0
sill <- 1
scale_s <- 0.15/3
scale_t <- 2/3
param <- list(mean=0,scale_s=scale,scale_t=scale_t,sill=sill,nugget=nugget)
# Simulation of the Gaussian random field:
set.seed(35)
data <- GeoSim(coordx=coords,coordt=time, corrmodel="exp_exp",
             param=param)$data

fixed<-list(nugget=nugget,mean=0)
start<-list(scale_s=scale_s,scale_t=scale_t,sill=1)
# Weighted least square estimation:
fit <- GeoWLS(data=data, coordx=coords,coordt=time, corrmodel="exp_exp",
                 ,maxdist=0.5,maxtime=3,fixed=fixed,start=start)

# Results
print(fit)
```

`Lik` *Optimizes the Log Likelihood*

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximization of the log-likelihood.

Usage

```
Lik(bivariate,coordx,coordy,coordt,coordx_dyn,corrmodel,data,fixed,flagcor,flagnuis,
            grid,lower,method,model,namescorr,namesnuis,namesparam,numcoord,
            numpairs,numparamcor,numtime,optimizer,onlyvar,param,radius,setup,
            spacetime,sparse,varest,taper,type,upper,ns,X)
```

Arguments

`bivariate`	Logical; if `TRUE` then the data come froma a bivariate random field. Otherwise from a univariate random field.
`coordx`	A numeric ($d \times 2$)-matrix (where `d` is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates.
`coordy`	A numeric vector assigning 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a ($d \times 2$)-matrix.

coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$-matrix or $(d \times d \times n)$-matrix of observations.
flagcor	A numeric vector of flags denoting which correlation parameters have to be estimated.
flagnuis	A numeric verctor of flags denoting which nuisance parameters have to estimated.
fixed	A numeric vector of parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
lower	A numeric vector with the lower bounds of the parameters' ranges.
model	Numeric; the id value of the density associated to the likelihood objects.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numcoord	Numeric; the number of coordinates.
numpairs	Numeric; the number of pairs.
numparamcor	Numeric; the number of the correlation parameters.
numtime	Numeric; the number of temporal observations.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd (Singular values decomposition).
optimizer	String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the default.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
param	A numeric vector of parameters.
sparse	Logical; if TRUE then maximum likelihood is computed using sparse matrices algorithms.FALSE is the default.
radius	Numeric; the radius of the sphere when considering data on a sphere.
ns	Numeric: vector of number of location sites for each temporal instants
setup	A List of useful components for the estimation based on the maximum tapered likelihood.
spacetime	Logical; if the random field is spatial (FALSE) or spatio-temporal (TRUE).
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
taper	String; the name of the taper correlation function.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	A numeric vector with the upper bounds of the parameters' ranges.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

Author(s)

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See Also

GeoFit

MatDecomp *Matrix decomposition*

Description

Matrix decomposition.

Usage

```
MatDecomp(mtx, method)
```

Arguments

mtx	numeric; a square positive or semipositive definite matrix.
method	string; the type of matrix decomposition. Two possible choices: cholesky and svd.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

MatSqrt, MatInv, MatLogDet *Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.*

Description

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

Usage

```
MatSqrt(mat.decomp,method)
MatInv(mat.decomp,method)
MatLogDet(mat.decomp,method)
```

Arguments

mat.decomp numeric; a matrix decomposition.

method string; the type of matrix decomposition. Two possible choices: cholesky and svd.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>

See Also

MatDecomp

Examples

```
library(GeoModels)
################################################################
###
### Example 1. Inverse of Covariance matrix associated to
### a Matern correlation model
###
###############################################################
# Define the spatial-coordinates of the points:
x <- runif(15, 0, 1)
y <- runif(15, 0, 1)
coords <- cbind(x,y)
# Matern Parameters
param=list(smooth=0.5,sill=1,scale=0.2,nugget=0)
a=matrix <- GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param)

## decomposition with cholesky method
b=MatDecomp(a$covmat,method="cholesky")
## inverse of covariance matrix
inverse=MatInv(b,method="cholesky")
```

NuisParam *Lists the Nuisance Parameters of a Random Field*

Description

Subroutine called by InitParam and other procedures. The procedure returns a list with the nuisance parameters of a given random field model.

Usage

```
NuisParam(model, bivariate,num_betas)
```

Arguments

model String; the name of a random field.

bivariate Logical; if FALSE (the default) the correlation model is univariate spatial or spatial-temporal. Otherwise is bivariate.

num_betas Numerical; the nunber of mean parameters in the linear specification.

Author(s)

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See Also

GeoFit

Examples

```
library(GeoModels)

NuisParam("Gaussian", FALSE,1)
## note that in the bivariate case sill and nugget are considered as correlation parameteres
NuisParam("Gaussian", TRUE,1)

NuisParam("BinomialGaussian", FALSE,1)

NuisParam("Chisq", FALSE,2)

NuisParam("SkewGaussian", FALSE,3)

NuisParam("SinhAsinhGaussian",FALSE,1)
```

Prscores — *Computation of three predictive scores: RMSE, LSCORE, CRPS for spatial, spatiotemporal and bivariate Gaussian RF.*

Description

The function computes RMSE, LSCORE, CRPS predictive scores.

Usage

```
Prscores(data,  method="cholesky", matrix)
```

Arguments

data	A d-dimensional vector (a single spatial realisation) or a a$(t \times d)$-matrix (a single spatial-temporal realisation). or a a$(2 \times d)$-matrix (a single bivariate realisation).
method	String; the type of matrix decomposition used in the computation of the predictive scores. Default is cholesky. The other possible choices is svd.
matrix	An object of class matrix. See the Section **Details**.

Details

For a given covariance matrix object (GeoCovmatrix) and a given spatial, spatiotemporal or bivariare realization from a Gaussian random field, the function computes three predictive scores.

Value

Returns a list containing the following informations:

RMSE	Root-mean-square error predictive score
LSCORE	Logarithmic predictive score
CRPS	Continuous ranked probability predictive score

Author(s)

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References

Zhang H. and Wang Y. (2010). *Kriging and cross-validation for massive spatial data*. Environmetrics, **21**, 290–304.

Gneiting T. and Raftery A. *Strictly Proper Scoring Rules, Prediction, and Estimation*. Journal of the American Statistical Association, **102**

See Also

GeoCovmatrix

Examples

```
library(GeoModels)
library(fields)

################################################################
######### Examples of predictive score computation  ############
################################################################

  # Define the spatial-coordinates of the points:
x <- runif(500, 0, 2)
y <- runif(500, 0, 2)
coords=cbind(x,y)
matrix1 <- GeoCovmatrix(coordx=coords, corrmodel="Matern", param=list(smooth=0.5,
                      sill=1,scale=0.2,nugget=0))

data <- GeoSim(coordx=coords, corrmodel="Matern", param=list(mean=0,smooth=0.5,
                      sill=1,scale=0.2,nugget=0))$data

Pr_scores <- Prscores(data,matrix=matrix1)

Pr_scores
```

StartParam *Initializes the Parameters for Estimation Procedures*

Description

Subroutine called by the fitting procedures. The procedure initializes the parameters for the fitting procedure.

Usage

```
StartParam(coordx, coordy, coordt,coordx_dyn, corrmodel, data, distance, fcall,
          fixed, grid,likelihood,  maxdist, maxtime, model, n,  param,
          parscale, paramrange, radius,  start, taper, tapsep,
          type,typereal,varest, vartype, weighted, winconst,
          winstp, winconst_t, winstp_t,X)
```

Arguments

coordx	A numeric ($d \times 2$)-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model.
data	A numeric vector or a ($n \times d$)-matrix or ($d \times d \times n$)-matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
fcall	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
fixed	A named list giving the values of the parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a ($n \times d$)-matrix, instead if TRUE then ($d \times d \times n$)-matrix is considered.
likelihood	String; the configuration of the composite likelihood.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6378.88, the radius of the earth.
model	String; the density associated to the likelihood objects. Gaussian is the default.
n	Numeric; number of trials for binomial random fields.
param	A numeric vector of parameter values required in the simulation procedure of random fields.

parscale	A numeric vector of scaling factor to improve the maximizing procedure, see optim.
paramrange	A numeric vector of parameters ranges, see optim.
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure.
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see **Details**).
type	String; the type of likelihood objects. Temporary value set to be "WLeastSquare" (weigthed least-square) in order to compute the starting values.
typereal	String; the real type of likelihood objects. See GeoFit.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section **Details**.
weighted	Logical; if TRUE the likelihood objects are weighted, see GeoFit.
winconst	Numeric; a positive value for computing the spatial sub-window in the subsampling procedure.
winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the subsampling procedure.
winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
X	Numeric; Matrix of space-time covariates.

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See Also

GeoFit

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winds *Irish Daily Wind Speeds*

Description

A matrix containing daily wind speeds, in kilometers per hour, from 1961 to 1978 at 12 sites in Ireland.

Usage

```
data(irishwinds)
```

Format

A (6574×11)-matrix containing wind speed observations.

Source

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

`winds.coords` *Weather Stations of the Irish Daily Wind Speeds*

Description

A data frame containing information about the weather stations where the data are recorded in Ireland.

Usage

```
data(irishwinds)
```

Format

A data frame containing site - the name of the city (character), abbr - the abbrevation (character), elev - the elevation (numeric), lat - latitude (numeric) and lon - longitude.

Source

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

`WlsStart` *Computes Starting Values based on Weighted Least Squares*

Description

Subroutine called by GeoFit. The function returns opportune starting values for the composite-likelihood fitting procedure based on weigthed least squares.

Usage

```
WlsStart(coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance, fcall,
        fixed, grid,likelihood, maxdist, maxtime, model, n, param,
        parscale, paramrange, radius,start, taper, tapsep, type, varest,
        vartype, weighted, winconst,winconst_t, winstp_t, winstp,X)
```

Arguments

`coordx`	A numeric ($d \times 2$)-matrix (where `d` is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
`coordy`	A numeric vector assigning 1-dimension of coordinates; `coordy` is interpreted only if `coordx` is a numeric vector otherwise it will be ignored.
`coordt`	A numeric vector assigning 1-dimension of temporal coordinates.
`coordx_dyn`	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
`corrmodel`	String; the name of a correlation model, for the description.
`data`	A numeric vector or a ($n \times d$)-matrix or ($d \times d \times n$)-matrix of observations.
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details**.
`fcall`	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
`fixed`	A named list giving the values of the parameters that will be considered as known values.
`grid`	Logical; if `FALSE` (the default) the data are interpreted as a vector or a ($n \times d$)-matrix, instead if `TRUE` then ($d \times d \times n$)-matrix is considered.
`likelihood`	String; the configuration of the composite likelihood.
`maxdist`	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
`maxtime`	Numeric; an optional positive value indicating the maximum temporal separation considered in the composite-likelihood computation.
`model`	String; the name of the model. Here the default is `NULL`.
`n`	Numeric; number of trials in a binomial random field.
`param`	A numeric vector of parameter values required in the simulation procedure of random fields.
`parscale`	A numeric vector with scaling values for improving the maximisation routine.
`paramrange`	A numeric vector with the range of the parameter space.
`radius`	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6378.88)
`start`	A numeric vector with starting values.
`taper`	String; the name of the type of covariance matrix. It can be `Standard` (the default value) or `Tapering` for taperd covariance matrix.
`tapsep`	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see **Details**).
`type`	String; the type of estimation method.
`varest`	Logical; if `TRUE` the estimates' variances and standard errors are returned. `FALSE` is the default.
`vartype`	String; the type of estimation method for computing the estimate variances, see the Section **Details**.
`weighted`	Logical; if `TRUE` the likelihood objects are weighted, see `GeoFit`.
`winconst`	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.

winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

Author(s)

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See Also

GeoFit.

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