Solution of Electricity Exam

Exercise 1 (7pts)

1) Impedances of the reactive elements.

Recall that the impedance expression $Z = R + jX$, has a real part R referred to as resistance and an imaginary part X referred to as reactance. Reactive dipoles are dipoles for which the resistance is zero. They are reactive elements. Two are known: C and L.

$$\underline{Z}_{C_2} = \frac{-j}{C_2\omega} = \frac{-j}{0{,}125\cdot 4} = -2j \quad \Rightarrow \quad \underline{Z}_{C_2} = -2j \quad (\Omega) \qquad \textbf{(0.25pt)}$$

$$\underline{Z}_{C_1} = \frac{-j}{C_1\omega} = \frac{-j}{0{,}125\cdot 4} = -2j \quad \Rightarrow \quad \underline{Z}_{C_1} = -2j \quad (\Omega) \qquad \textbf{(0.25pt)}$$

$$\underline{Z}_L = jL\omega = j\cdot 0{,}05\cdot 4 = 0{,}2j \quad \Rightarrow \quad \underline{Z}_L = 0{,}2j = j/5 \ (\Omega) \qquad \textbf{(0.25pt)}$$

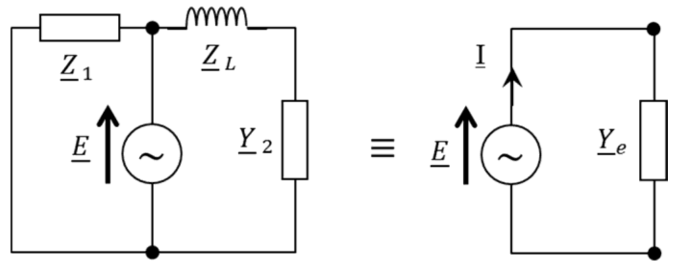
2) Complexe Admittance of (R_2, C_2).

(R_2, C_2) are in parallel: $\underline{Y}_2 = \underline{Y}_{R_2} + \underline{Y}_{C_2}$ $\quad \underline{Y}_2 = \frac{1}{R_2} + jC_2\omega$ $\quad \underline{Y}_2 = \frac{1}{2} + \frac{1}{-2j} = \frac{1}{2} + \frac{j}{2}$ $\quad \underline{Y}_2 = \frac{1}{2}(1+j)\ (S)$ **(0.5pt)**

Complexe impedance of (R_1, C_1).

(R_1, C_1) are in serie: $\underline{Z}_1 = R_1 + \underline{Z}_{C_1}$ $\quad \underline{Z}_1 = R_1 + \frac{-j}{C_1\omega}$ $\quad \underline{Z}_1 = 2 - 2j$ $\quad \underline{Z}_1 = 2(1-j)\ (\Omega)$ **(0.5pt)**

3) Equivalente complexe admittance of the circuit .



The source is connected in parallel with the left-hand branch containing admittance $\underline{Z_1}$ and with the right-hand branch containing impedance Z_L in series with impedance $\frac{1}{Y_2}$. Let, $\underline{Y_e}$ the equivalent admittance of the circuit:

$$\underline{Y_e} = \frac{1}{Z_1} + \frac{1}{Z_L + \frac{1}{Y_2}} \Rightarrow \underline{Y_e} = \frac{1}{2(1-j)} + \frac{1}{\frac{j}{5} + \frac{1}{\frac{1}{2}(1+j)}}$$

$$= \frac{1+j}{4} + \frac{5(5+4j)}{41} \Rightarrow \boxed{\underline{Y_e} = \frac{141}{164} + j\frac{121}{164} \ (Seimens)} \qquad \textbf{(1pt)}$$

The polar form of the complexe admittance is : $\underline{Y_e} = Y_e e^{j\varphi}$ where Y_e: module and φ : phase

$$Y_e = \left|\underline{Y_e}\right| = \sqrt{\left(\frac{141}{164}\right)^2 + \left(\frac{121}{164}\right)^2} = 1.133\ (\Omega^{-1}) or (Seimens) \quad \textbf{(0.75pt)}$$

Phase : $\varphi = tan^{-1}\left(\frac{121}{141}\right) = 40.64^{\circ}$ **(0.5pt)** $\quad \boxed{\underline{Y_e} = 1.133\angle 40.64^{\circ}}$ **(0.25pt)** or

$$\underline{Y}_e = 1,133\,\Omega^{-1}\, e^{\,j\,\frac{40,63\,\pi}{180}}$$

4- Complex expression of the current delivered by the source and its temporal expression

$e(t) = \sqrt{2}\cos 4t$ so the effective value $E = 1V$, $pulsation\ \omega = 4s^{-1}$ and the $phase\ \varphi_v = 0$

Using Ohm Law in the circuit loop $\underline{E} = \frac{1}{\underline{Y_e}}.\underline{I}$ so the complex value of the current is given by :

$$\underline{I} = \underline{Y_e}.\underline{E}$$

$$\underline{I} = 1.133\angle 40.64^{\circ}.1\angle 40.64^{\circ} \quad \mathbf{(0.75pt)}$$

$$Finally\ \underline{I} = 1.133\angle 40.64^{\circ} = \boxed{\mathbf{1.133e^{j40.63^{\circ}}\ (A)}} \quad \mathbf{(0.25pt)}\ \ or\ \ 1.133e^{j\frac{40.63.\pi}{180}}$$

The instantaneous expression : $i(t) = I\sqrt{2}\cos(\omega t + \varphi_i)$

$$\boxed{\boldsymbol{i(t) = 1.133\sqrt{2}\cos\left(4t + \frac{40.63.\pi}{180}\right)\ (A)}} \quad \mathbf{(0.75pt)}$$

5- Calculation of Power and Power coefficient

Active power $P = V_{rms}.I_{rms}.\cos(\varphi)$ *with* $\varphi: Shift\ phase\ (rad) = \varphi = \varphi_v - \varphi_i$

$\varphi = 0 - 40.63^{\circ} = -40.63^{\circ} \Rightarrow \boldsymbol{cos}(\varphi) = 0.758 \quad and\ \sin(\varphi) = -0.651$

So $P = 1x1.133x0.758 = \boxed{\mathbf{0.859\ Watt}}$ $\mathbf{(0.25pt)}$

Reactive power $Q = V_{rms}.I_{rms}.\sin(\varphi) = 1x1.133x(-0.651) = \boxed{\mathbf{-0.738\ VAR}}$ $\mathbf{(0.25pt)}$

Apparent power S= $V_{rms}.I_{rms} = 1x1.133 = \boxed{\mathbf{1.133VA}}$ $\mathbf{(0.25pt)}$

power factor cos$(\varphi) = \boxed{0.758}$ $\mathbf{(0.25pt)}$

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Ministry Of Higher Education
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وزارة التعليم العالي
و البحث العلمي
المدرسة الوطنية العليا في الأمن السيبراني

Exercise 2

$L_1\omega = 16\Omega\,,\ L_2\omega = 4\Omega,\ L_3\omega = 5\Omega\,,\ R_1 = R_2 = 10\Omega\,,\ \underline{E}_1 = 10V\angle\, 0°\ et\ \underline{E}_2 = 10V\angle\, 90$

1- Thevenin equivalent generator (E_{Th}, Z_{Th})

First step : calculate the equivalent impedance of the generator

- The equivalent circuit impedance Z_{AB} between A and B after switching off the voltage sources and disconnecting the current sources, and identify it as Z_{Th}.

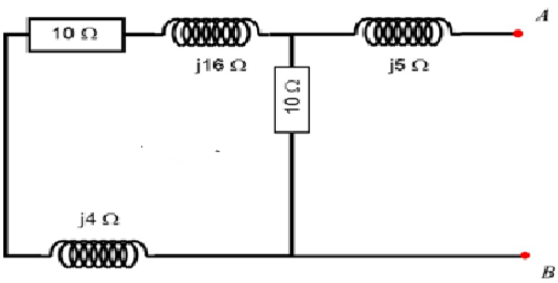
Second step: calculate Thevenin generator voltage

- Then determine the no-load voltage U_{AB} of the active circuit and identify it as E_{Th}.

First step : determine Z_{Th} (Z_{AB}) (1.5pt)

For inductor $_1$: $\underline{Z}_{L1}$ = j$L_1\omega$ = 16j Ω

In the circuit shown opposite, there are two shunt (parallel) branches: the first contains the dipoles j4Ω, 10Ω and j16 Ω connected in series, and the second contains the resistor 10Ω. The equivalent impedance of these two branches is in series with the inductor of j5 Ω.



$$\underline{Z}_{AB} = \underline{Z}_{th} = \frac{R_2.(R_1 + jL_1\omega + jL_2\omega)}{R_2 + (R_1 + jL_1\omega + jL_2\omega)} + jL_3\omega$$
$$= \frac{10.(10 + j20)}{10 + (10 + j20)} + j5 = \frac{5}{2}(1+2j)(1-j) + j5$$
$$= \frac{15}{2}(1+j)$$

Finaly : $\boxed{\underline{Z}_{th} = \frac{15}{2}(1+j)\ (\Omega)}$

Second Step : Détermine E_{Th} (1.5 pts)

We note that no current flows through inductor L_3 ($j5\Omega$), since the circuit is open between A and B. The voltage $E_{th} = U_{AB}$ is then the same as that across the branch containing the R_2 (10 Ω) resistor and the $10V\ \angle 90°$ generator. Among the various methods, we'll mention the Loop method, since we only have one here. Using KVL, we obtain :

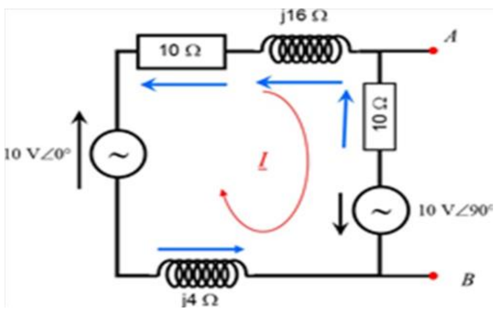
$$-\underline{E}_2 + R_2\underline{I} + jL_2\omega\underline{I} + R_1\underline{I} - E_1 + jL_1\omega\underline{I} = 0$$

$$-10\text{j} + 10 \cdot \underline{I} + \text{j}16 \cdot \underline{I} + 10 \cdot \underline{I} - 10 + \text{j}4 \cdot \underline{I} = 0$$

$$(10 + \text{j}16 + 10 + \text{j}4) \cdot \underline{I} = 10\text{j} + 10$$

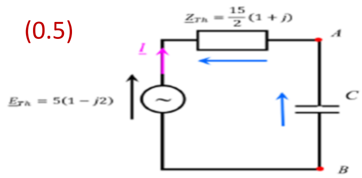
$$(20 + \text{j}20) \cdot \underline{I} = 10\text{j} + 10 \ \Rightarrow$$

$$\boxed{\underline{I} = \frac{1+j}{2(1+j)} = 0.5A}$$



Now let's calculate the voltage between A and B as the sum of the voltages across the 10 Ω resistor and the $10V\angle 90°$ voltage generator

(0.5)



$$\underline{E}_{th} = \underline{U}_{AB} = R_2\underline{I} - \underline{E}_2$$

$$\underline{E}_{th} = \underline{U}_{AB} = 10 \cdot I - 10j = 10 \cdot 0{,}5 - 10j = 5\,(1 - 2j)$$

Finally: $\boxed{\underline{E}_{Th} = 5\,(1 - 2j)V = 5\sqrt{5}V\ \angle -63{,}43° = 11.18V\angle -63{,}43°}$

2- Current through the capacitor (2 pts)

The circuit opposite shows the Thevenin generator equivalent in series with the capacitor impedance. The circuit opposite shows the equivalent of the Thevenin generator in series with the capacitor impedance. It is easy to establish that the current in the circuit is given by the expression:

$$\underline{I} = \frac{\underline{E}_{th}}{\underline{Z}_{th} + \underline{Z}_c} = \frac{5(1-2j)}{\frac{15}{2}(1+j) - 2j} = \frac{10(1-2j)}{15+11j} = \frac{5}{173}(-7-11j)A$$

- Effective value : Magitude : $I = |\underline{I}| = \frac{5}{173}\sqrt{7^2 + 41^2} = 1{,}20\ A$
- Argument: $\varphi_i = \arctan\left(\frac{-41}{-7}\right) = 80.3°$. However, we know that the tangent function admits two solutions to within π. The expression of the current shows a negative real part and an imaginary part that is also negative. So, the angle lies in quadrant 3 of the trigonometric circle. The solution is therefore:

$$\varphi_i = 80.3° - 180° = -99{,}7°.$$

In phaser notation $\boxed{\underline{I} = 1{,}20A\angle -99{,}7°}$

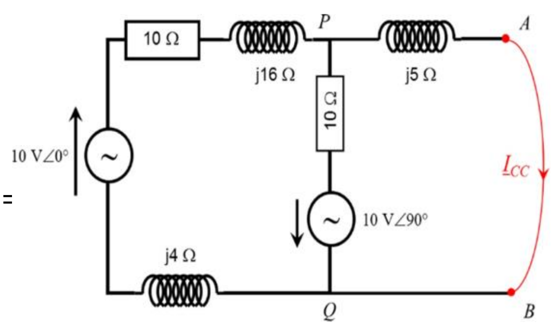
3- Norton equivalent generator ($\underline{I}_N, \underline{Z}_N$)

We need to determine the equivalent impedance to the circuit $\underline{Z}_{AB}$ between A and B after switching off the voltage sources and disconnecting the current sources, and identify it as $\underline{Z}_N$ .Then we need to determine the short-circuit current I_{AB}=I_{cc} by connecting A and B with a wire, and identify it as $\underline{I}_N$.

The equivalent Norton impedance (1 pt)

We apply the same method as Thévenin's, hence : $\boxed{\underline{Z}_N = \underline{Z}_{th} = \frac{15}{2}(1+j)\ (\Omega)}$

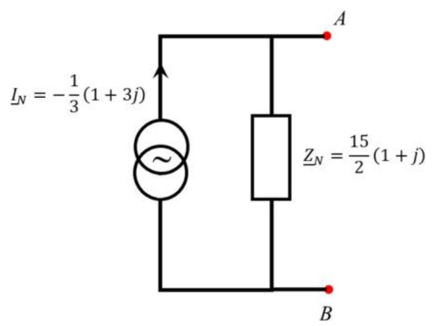
Current Norton generator (I_{AB} or I_{CC}) (1.pt)



Using conversion Thévenin- Norton transformation

$$\underline{I}_N = \frac{E_{th}}{\underline{Z}_N} = \frac{5\,(1-2j)}{\frac{15}{2}(1+j)} = \frac{5(1-2j)(1-j)}{15} = -\frac{1}{3}(1+3j) = \frac{\sqrt{10}}{3}A\angle -108{,}44° = 1{,}05A\angle -108{,}44°$$

$$\boxed{\underline{I}_N = -\frac{1}{3}(1+3j)A = 1{,}05A\angle -108{,}44°}$$

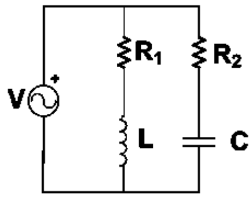


(0.5pt)

Exercise 3 (5 pts)

1- (2.5pt)

What is the value of C at resonance, given that $f_0 = 2500/\pi\ Hz$, $R_1 = 8\Omega$, $R_2 = 8.34\ \Omega$ and $Z_L = j\ 8\ [\Omega]$



$$\omega = 5000\ rad/s$$

$Z_1 = R_1 + jL\omega$; $\boldsymbol{Z_1 = 8 + j8}$

$Z_2 = R_2 - j\frac{1}{C\omega}$; $\boldsymbol{Z_2 = 8.34 - j\frac{0.0002}{c}}$

$Z_{eq} = Z_1 // Z_2$

$Y_{eq} = Y_1 + Y_2 \Rightarrow Y_{eq} = \frac{1}{Z_1} + \frac{1}{Z_1}$

$$\begin{cases} Y_1 = \frac{1}{Z_1} \\ Y_2 = \frac{1}{Z_2} \end{cases} \Rightarrow \begin{cases} Y_1 = \frac{1}{8+j8} \\ Y_2 = \frac{1}{8.34 - j\frac{0.0002}{C}} \end{cases} \Rightarrow \begin{cases} Y_1 = \frac{8-j8}{128} \\ Y_2 = \frac{8.34 + j\frac{0.0002}{C}}{(8.34)^2 + \left(\frac{0.0002}{C}\right)^2} \end{cases} \Rightarrow \begin{cases} Y_1 = 0.0625 - j\ 0.0625 \\ Y_2 = \frac{8.34}{(8.34)^2 + \left(\frac{0.0002}{C}\right)^2} + j\frac{\frac{0.0002}{C}}{(8.34)^2 + \left(\frac{0.0002}{C}\right)^2} \end{cases}$$

$$Y_{eq} = \left(0.0625 + \frac{8.34}{(8.34)^2 + \left(\frac{0.0002}{C}\right)^2}\right) + j\left(\frac{\frac{0.0002}{C}}{(8.34)^2 + \left(\frac{0.0002}{C}\right)^2} - 0.0625\right)$$

at resonance the imaginary part is null:

$$\frac{\frac{0.0002}{C}}{(8.34)^2 + \left(\frac{0.0002}{C}\right)^2} - 0.0625 = 0 \Rightarrow \frac{\frac{0.0002}{C}}{69.55 + \frac{4*10^{-8}}{C^2}} - 0.0625 = 0$$

$$\frac{0.0002}{69.55\,C + \frac{4*10^{-8}}{C}} - 0.0625 = 0$$

$$69.55\,C + \frac{4*10^{-8}}{C} = \frac{0.0002}{0.0625} \Rightarrow 69.55\,C + \frac{4*10^{-8}}{C} = 0.0032$$

$$69.55\,C^2 - 0.0032\,C + 4*10^{-8} = 0$$

It is a second-degree equation where $\Delta < 0$, so practically, there is no value of capacitance that leads to resonance at the mentioned frequency

2- **The resonant frequency of the circuit:** (1.5pt)

The equivalent impedance of the circuit is given by: $\underline{Z} = Z_L\ in\ serie\ with(\ R \parallel Z_c)$

$$\boxed{Z = jX_L + \frac{-jX_C R}{R - jX_C}} \qquad Z = jX_L + \frac{-jX_C R}{R - jX_C} \times \frac{R + jX_c}{R + jX_c} = jX_L + \frac{RX_c^2 - jR^2X_c}{R^2 + X_c^2}$$

we obtain $\boxed{Z = \frac{RX_c^2}{R^2 + X_C^2} + j\left(X_L - \frac{R^2X_c}{R^2 + X_C^2}\right)}$

At resonance, Im(Z)=0.

$$X_L - \frac{R^2X_C}{R^2 + X_C^2} = 0 \rightarrow X_L = \frac{R^2X_C}{R^2 + X_C^2}$$

$$X_LR^2 + X_LX_C^2 = R^2X_C \Longrightarrow \omega_o LR^2 + \frac{\omega_o L}{\omega_o^2C^2} = \frac{R^2}{\omega_o C} \Longrightarrow \omega_o^2CLR^2 + \frac{L}{C} = R^2$$

Finally,, we obtain

$$\omega_o^2 = \frac{1}{CL} - \frac{1}{C^2R^2} \rightarrow \omega_o = \sqrt{\frac{1}{LC} - \frac{1}{C^2R^2}}$$

So ω_o=435.9 rad/s and $\boxed{f_o = \frac{\omega_0}{2\pi} = 69.375Hz}$

The value of current at resonance is given by : (1pt)

$$I_m = \frac{V_m}{Re(Z)} = \frac{V_m}{\frac{RX_c^2}{R^2 + X_c^2}}\ (A)$$

$$Re(Z) = \frac{RX_c^2}{R^2 + X_c^2} = \frac{20\text{x}\left({}^1/_{435.9\text{x}0.510^{-3}}\right)^2}{20^2 + \left({}^1/_{435.9\text{x}0.510^{-3}}\right)^2} = 0.999A = \boxed{1A}$$