Ministry of Higher Education and Scientific Research

وزارة التعليــم العــالي والبحــث العــلمي

National School of Cyber Security

المدرسة الوطنيــة العليــا في الأمن السيبراني

Foundation Training Department

NSCS

قســـم التكويـــــن القـــــاعدي

MODULE'S TEACHER: Dr. B. CHAOUCHI

LEVEL : 1st Year Basic Training

SECTION / GROUP : A & B

MODULE : Analysis 1

DATE : 14 / 01 / 2025

DURATION : 2H00

Final Exam 1

NOTE :No documents are allowed.

Scale: 2 pts +7.5 pts +7.5 pts +3 pts

Exercise 1

Detremine, without the use of L'Hôpital's rule

1. $\displaystyle \lim_{x \to 0^+} \frac{\sin 2x}{\sqrt{1 - \cos 3x}}$

2. $\displaystyle \lim_{x \to -2} \frac{\arcsin(x+2)}{x^2 + 2x}$

Exercise 2

Let f be the real valued function given by

$$f(x) = x + 2\ln(\cosh(x))$$

1. Find D_f the domain of defintion of this function

2. Calculate $f(0), f(\ln 2)$ and $f(-\ln 3)$.

3. Find the first derivative f' of this function

4. Establish the variation table of f

5. For this function, determine the equation of the oblique asymptotes

6. Draw the graph of this function

Exercise 3

Let

$$T_n(x) = \cos(n \arccos(x)), \quad n \in \mathbb{N}$$

$\boxed{1}$ Find $D = D_{T_n}$ the domain of defintion of $x \mapsto T_n(x)$

$\boxed{2}$ Calculate $T_n(1)$, $T_n(0)$ and $T_n(-1)$.

$\boxed{3}$ Show that for suitable x: $\arccos(x) + \arccos(-x) = \pi$

$\boxed{4}$ Using question 3, study the parity of T_n (express it using n)

$\boxed{5}$ Detremine $T_0(x)$, $T_1(x)$ and $T_2(x)$

$\boxed{6}$ Calculate $T_{n+1}(x) + T_{n-1}(x)$ for all $n \in \mathbb{N}^*$ and all $x \in D$.

$\boxed{7}$ Deduce $T_3(x)$, $T_4(x)$ and $T_5(x)$.

Exercise 4

Let $f : \mathbb{R} \to \mathbb{R}$ be a real function such that for all $x \in \mathbb{R}$ with $f(x) \neq 3$, one has

$$f(x+1) = \frac{f(x) - 5}{f(x) - 3}$$

Show that f is a periodic function and find its period.

$$T_3 = \cos(3 \arccos x)$$

$$=$$

2