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| Ministry of Higher Education and Scientific Research<br>National School of Cyber Security | NSCS<br>المدرسة الوطنية العليا في الأمن السيبراني<br>NATIONAL SCHOOL OF CYBERSECURITY | وزارة التعليم العالي والبحث العلمي<br>المدرسة الوطنية العليا في الأمن السيبراني |
| **Foundation Training Department** | | قسم التكوين القاعدي |
| LEVEL : 1st Year Basic Training | | MODULE'S TEACHER : Dr. Hamza MOUFEK |
| SECTION / GROUP : A & B | | DATE : 24 / 11/ 2024 |
| MODULE : Algebra 1 | **Midterm 1 Test** | DURATION : 1h30 |
| FULL NAME : ................................ | | *NOTE : No documents are allowed.* |

## Exercise 1 : (4 points)

1. Find the inverse, converse, and contrapositive of the statement :

$$\forall x \in \mathbb{R}, x \notin \mathbb{Q} \Rightarrow 1 + x \notin \mathbb{Q}.$$

2. Determine whether each of these statements is true or false.

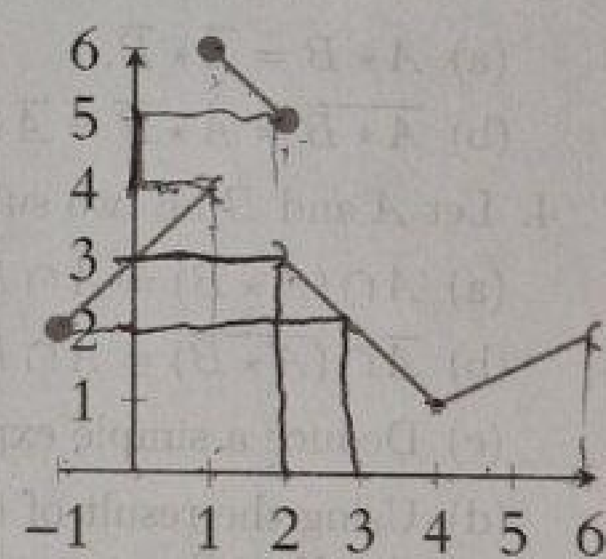
(a) $0 \in \emptyset$ F
(b) $\emptyset \in \{0\}$ F
(c) $\{0\} \subset \emptyset$ F
(d) $\emptyset \subset \{0\}$ T
(e) $\{0\} \in \{0\}$ F
(f) $\{\emptyset\} \subset \{\emptyset\}$ T

(g) $f$ is injective if and only if $f$ is not surjective. F
(h) $f$ is injective if and only if $\forall x_1, x_2 \in E, x_1 = x_2 \implies f(x_1) = f(x_2)$. F
(i) $f$ is injective if and only if $\forall x_1, x_2 \in E, x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$. T
(j) $f$ is injective if and only if $\forall x_1, x_2 \in E, f(x_1) = f(x_2) \implies x_1 = x_2$. T

## Exercise 2 : (1,5 points)

Let $f$ be the function defined on $[-1, 6[$ whose graph is shown opposite. Determine :

1. $f([-1, 6[)$
2. $f([1, 2[)$
3. $f^{-1}([0, 6])$
4. $f^{-1}([-2, 6[)$
5. $f^{-1}([4, 5])$
6. $f^{-1}(]2, 3])$




## Exercise 3 : (3 points)

Are the following functions injective ? Surjective ? Bijective ? (Use the definition)

1. $f : \begin{cases} \mathbb{R} \to \mathbb{Z} \\ x \mapsto \lfloor x \rfloor \end{cases}$

2. $g : \begin{cases} \mathbb{N} \to \mathbb{N} \\ n \mapsto \max(0, n-1) \end{cases}$

**Exercise 4 : (2,5 points)**

Using the Bernoulli inequality ( $(1+x)^n \geq 1+nx, \quad x > -1$), Prove the following inequality for $n \in \mathbb{N}$ :

$$n! < \left(\frac{n+1}{2}\right)^n, \quad n > 1.$$

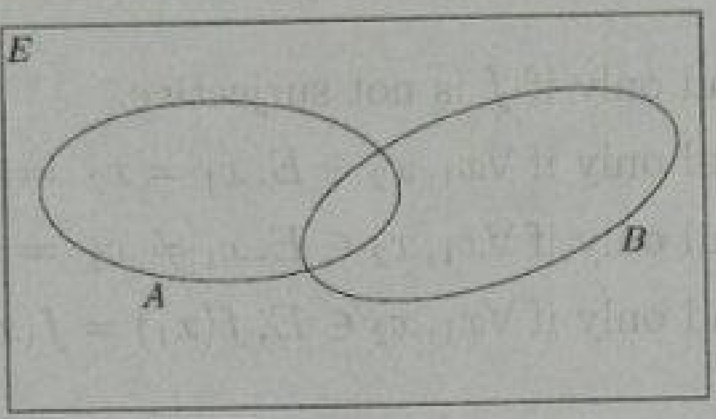
**Exercise 5 : (9 points)**

Let $E$ be a non-empty set. If $A$ and $B$ are two subsets of $E$, we define :

$$A \star B = [A \cap B] \cup [\overline{A} \cap \overline{B}]$$

where $\overline{A}$ denotes the complement of $A$ in $E$.

1. After copying the given diagram, shade the region corresponding to $A \star B$.




2. For any subset $A$ of $E$, determine $A \star A$, $A \star \emptyset$, and $A \star E$.
3. If $A$ and $B$ are two subsets of $E$, prove that :
   (a) $A \star B = \overline{A} \star \overline{B}$,
   (b) $\overline{A \star B} = A \star \overline{B} = \overline{A} \star B$.
4. Let $A$ and $B$ be two subsets of $E$. Prove that :
   (a) $A \cap (A \star B) = A \cap B$,
   (b) $\overline{A} \cap (\overline{A \star B}) = \overline{A} \cap B$,
   (c) Deduce a simple expression for $A \star (A \star B)$,
   (d) Using the result of the previous question, determine all subsets $X$ of $E$ such that $A \star X = B$.