

LEVEL : 1st Year Basic Training

SECTION / GROUP : A & B

MODULE : . Mathematical analysis 1 .

FULL NAME:.....

Midterm 1 Test

DURATION : 1h30

Exercice 1. [6] pts

Solve in \mathbb{R} the following equation $|x^2 - 1| + |x + 2| = |x - 3|$

Exercice 2. [5] nts

Let $A \subset \mathbb{R}$ the set defined by $A = \left\{ x_n = \frac{8}{n^2 + 2}, n \in \mathbb{N} \right\}$

1. Show that A is bounded set
 2. Determine $\sup A$ and $\max A$
 3. Determine $\inf A$ (with the complete proof)

Exercice 3. [3] pts

Determine

$$1. \lim_{x \rightarrow 0} \frac{(1+x)^b - 1}{x}, b \in \mathbb{R}$$

$$2. \lim_{x \rightarrow +\infty} 3x^2 \left(\cos\left(\frac{1}{x}\right) - \cos\left(\frac{3}{x}\right) \right)$$

$$3. \lim_{x \rightarrow 0} \frac{1 - \cos^b x}{x^2}, b \in \mathbb{R}$$

Exercice 4. [3] pts

Prove the following formula

$$\cos\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2^2}\right) \cdot \dots \cdot \cos\left(\frac{x}{2^n}\right) = \frac{\sin x}{2^n \cdot \sin\left(\frac{x}{2^n}\right)}$$

Exercice 5. [3] pts

Let $x, y \in \mathbb{R}$. Show that

$$x^2 + y^2 + 1 \geq x\sqrt{y^2 + 1} + y\sqrt{x^2 + 1}$$