

### Midterm Exam

Date: 24/11/2025

Duration: 1h30

Level: 1<sup>st</sup> Year

Material: Algorithms and Static Data Structures

1. Write a function **naturalLogarithm(x: real; n: integer): real** that calculates the approximation of  $\ln(x)$ .
  - Use only one loop for developing this function.
2. Write the main algorithm **CalculateNaturalLogarithm**.

**Hints:**

- Use the Function **powerXY(x, y: integer): integer**: integer that calculates  $x$  to the power of  $y$  ( $x^y$ ).
- Use the Function **abs(x: real): real**: real that returns the absolute value of  $x$  ( $|x|$ ).

**Exercise 3: Longest Collatz sequence under  $n$  (07 points)**

The Collatz sequence for a positive integer number  $n$  ( $1 \leq n \leq 1,000,000$ ) is given as follows:

- If  $n$  is even, the next number is  $\frac{n}{2}$ .
- If  $n$  is odd, the next number is  $3 * n + 1$ .
- Repeat until reaching 1.

Given a positive integer number  $n$ , find the number under  $n$  producing the longest sequence, and its length.

**Example:**

Number	Output	Length	Sequence
10	9	20	$9 \rightarrow 28 \rightarrow 14 \rightarrow 7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2$

1. Write a Procedure **longestSequence(n: integer; VAR bestNumber, bestLength: integer)** that returns the longest sequence.
2. Write the main algorithm **CollatzSequence**.

**Hints:**

- Develop the Function **lengthCollatz(n: integer): integer**: integer that returns the length of a sequence given an integer number  $n$ .

Good Luck

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### Exercise 1: Palindrome (07 points)

Consider a positive integer number  $n$  that has an even number of digits, and assume that  $n$  can be transformed into a palindrome, where each pair of identical digits is placed on opposite sides (one on the left side and the other on the right side).

1. Write a function `transfromPalindrome(n: integer): integer` that takes  $n$  as input and returns  $npal$ , which is the palindrome obtained from transforming  $n$ .

Example:

Number (n)	Palindrome (npal)
1212	2112
123123	123321
522522	225522

The standard deviation-like value is defined as the sum of the absolute differences ( $ad$ ) between the corresponding digits of two numbers at the same positions.

2. Write a function `standardDeviation(n, npal: integer): integer` that calculates the standard deviation-like value ( $sd$ ) between the original integer number  $n$  and its palindrome  $npal$ .

Example:

Number (n)	Palindrome (npal)	Absolute difference (ad)	Standard deviation (sd)
1212	2112	1100	2
123123	123321	000202	4
522522	225522	303000	6

3. Write the main algorithm `PalindromeStandardDeviation`.

Hints:

- Develop the Function `countDigits(n: integer): integer` that counts the number of digits of an integer number  $n$ .
- Develop the Function `checkEven(n: integer): boolean` that checks if an integer number  $n$  is even.
- Develop the Function `powerXY(x, y: integer): integer` that calculates  $x$  to the power of  $y$  ( $x^y$ ).

### Exercise 2: Natural logarithm (06 points)

The natural logarithm ( $\ln$ ) can be approximated using the following formula, where  $|x| \leq 1$  and  $n$  is a positive odd integer number:

$$\ln(x) = 2 * \left[ \left( \frac{x-1}{x+1} \right) + \frac{1}{3} \left( \frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left( \frac{x-1}{x+1} \right)^5 + \frac{1}{7} \left( \frac{x-1}{x+1} \right)^7 + \dots + \frac{1}{n} \left( \frac{x-1}{x+1} \right)^n \right]$$