Propositional Logic: Logical Equivalences

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Definition

A **statement** (or proposition) is a sentence that is true or false but not both.

Negation

If p is a statement variable, the **negation** of p is "not p" denoted $\sim p$.

p	$\sim p$
T	F
F	T

Conjunction and Disjunction

Conjunction

The **conjunction** of p and q is "p and q", denoted $p \wedge q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

The **disjunction** of p and q is "p or q", denoted $p \vee q$.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical Equivalence

Definition

Two statement forms are **logically equivalent** if they have identical truth values for each possible substitution. Denoted $P \equiv Q$.

Example

$p \wedge q \equiv q \wedge p$ (Commutative law)

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Logical Equivalences

Given statement variables p, q, r, a tautology t and contradiction c:

1. Commutative laws: $p \wedge q \equiv q \wedge p$, $p \vee q \equiv q \vee p$
2. Associative laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$, $(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$, $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws: $p \wedge t \equiv p$, $p \vee c \equiv p$
5. Negation laws: $p \vee \neg p \equiv t$, $p \wedge \neg p \equiv c$

Logical Equivalences (Continued)

6. Double negative law: $\neg(\neg p) \equiv p$
7. Idempotent laws: $p \wedge p \equiv p$, $p \vee p \equiv p$
8. Universal bound laws: $p \vee t \equiv t$, $p \wedge c \equiv c$
9. De Morgan's laws: $\neg(p \wedge q) \equiv \neg p \vee \neg q$, $\neg(p \vee q) \equiv \neg p \wedge \neg q$
10. Absorption laws: $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$
11. Negations of t and c: $\neg t \equiv c$

Simplifying Statement Forms: Example

Verify the logical equivalence:

$$\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$$

$$\begin{aligned}
\neg(\neg p \wedge q) \wedge (p \vee q) &\equiv (\neg(\neg p) \vee \neg q) \wedge (p \vee q) \quad \text{by De Morgan's laws} \\
&\equiv (p \vee \neg q) \wedge (p \vee q) \quad \text{by double negative law} \\
&\equiv p \vee (\neg q \wedge q) \quad \text{by distributive law} \\
&\equiv p \vee (q \wedge \neg q) \quad \text{by commutative law for } \wedge \\
&\equiv p \vee c \quad \text{by negation law} \\
&\equiv p \quad \text{by identity law}
\end{aligned}$$