Chapter 3. Fundamentals of electrical circuit analysis

I. Introduction

Linearity is the property of an element describing a linear relationship between cause and effect. For example, for a resistor, Ohm's law relates the input current to the output voltage v. It's linear: $V = Ri$. So, an electrical network is said to be "linear" if it consists solely of linear passive dipoles, ideal voltage sources (or ideal current sources) that are independent or linearly dependent. A linear circuit is one whose output is linearly related (or directly proportional) to its input.

An ideal voltage source (or ideal current source) is said to be "independent" if its voltage value (or current value) does not depend on the circuit to which it is connected. Linear passive dipoles have already been described. Essentially, they consist of resistors, inductors and capacitors.

The analysis of linear electrical circuits is based on the following general laws and theorems:

- Kirchhoff's laws
- Dividing bridges (Voltage divider; Current divider)
- Millman's theorem
- Superposition theorem
- Thevenin and Norton theorems
- Kennely's theorem

I.1 Definitions

Electrical circuit: An electrical circuit or network is made up of electrical dipoles connected together in any way. It usually includes at least one voltage or current source. In an electrical circuit, we distinguish: - **Node** (Junction), **Branch** (portion of the network between two consecutive nodes), **Mesh** (Loop): A mesh is any closed path in a circuit.

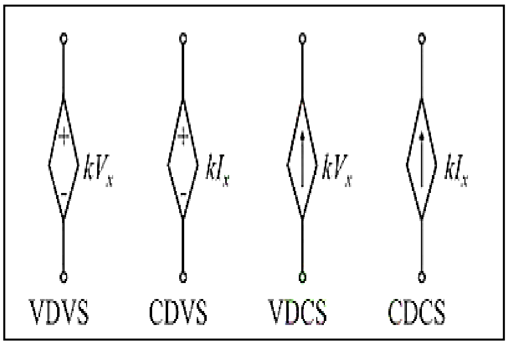
Source: In an electrical circuit, a **source** is a device that supplies electrical energy. Sources are the active components of a circuit that provide the power to drive the load. They are categorized as either voltage sources or current sources, which can be either independent (Stand-alone: Autonomous) or dependent (linked) where their characteristic is controlled by another voltage or current in the circuit. Linked sources are controlled sources.

- **Voltage Source:** Provides a specific voltage across its terminals, regardless of the current flowing through it.
  - **Independent Voltage Source:** The voltage is constant and does not depend on other circuit components.

- **Dependent Voltage Source:** The voltage is controlled by the current or voltage of another part of the circuit.

❑ **Current Source**: Provides a specific current, regardless of the voltage across its terminals.

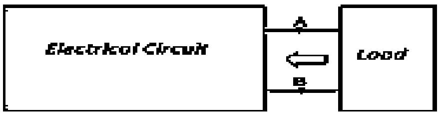
- **Independent Current Source:** The current is constant and does not depend on other circuit components.
- **Dependent Current Source:** The current is controlled by the current or voltage of another part of the circuit.



Voltage Dependent Voltage Source (VDVS)
Current Dependent Voltage Source (CDVS)
Voltage Dependent Current Source (VDCS)
Current Dependent Current Source (CDCS)

❑ **Load** : Source vs. Load

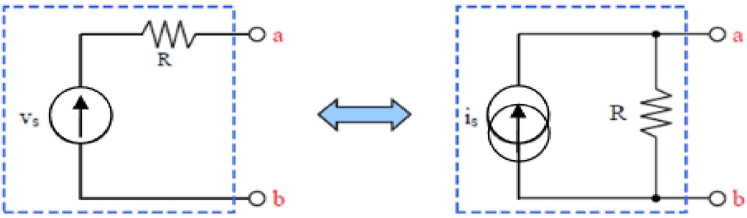
- A **source** supplies energy to the circuit.
- A **load** consumes energy from the circuit.



An electrical load is any component in a circuit that consumes electrical energy and converts it into another form, such as light, heat, or motion

I.2 Source Transformation

Source transformation is electrical circuit analysis tool for simplifying circuits. The source transformation is the process of replacing a voltage source in series with a resistor by a current source in parallel with a resistor, or vice versa.



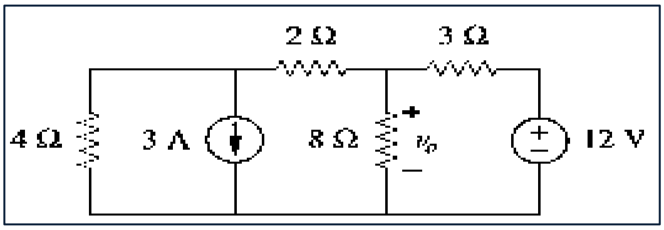
The above circuits have the same voltage-current relation at terminals a-b. It is easy to show that they are indeed equivalent. In both circuits R, i_s and v_s are the same. To ensure, that both circuits are equivalent. Hence, source transformation requires that.

$$\boldsymbol{v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}}$$

Keep the following points in mind when dealing with source transformation.

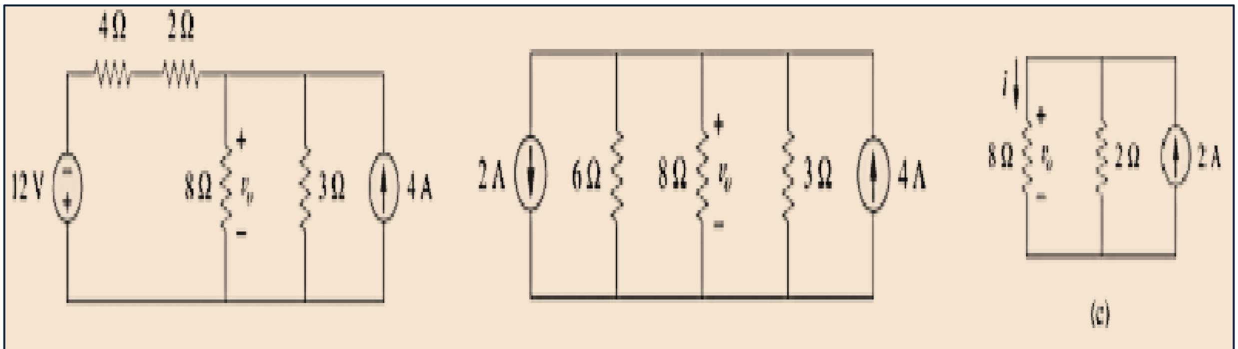
1- The arrow of the current source is directed toward the positive terminal of the voltage source.
2- Source transformation is not possible when R=0, which is the case with an ideal voltage source. Similarly, an ideal current source with R=∞ cannot be replaced by a finite voltage source.

Example 1: Use source transformation to find v_o in the circuit of shown figure.



Steps of the method using transform source:

- Transform the current and voltage sources to obtain the circuit in figure (a).
- Combining the 4Ω and 2Ω resistors in series and transforming the 12V voltage source gives us figure (b).
- Combine the 3Ω and 6Ω resistors in parallel to get 2Ω. Also combine the 2A and 4A current sources to get a 2A source to obtain figure (c).

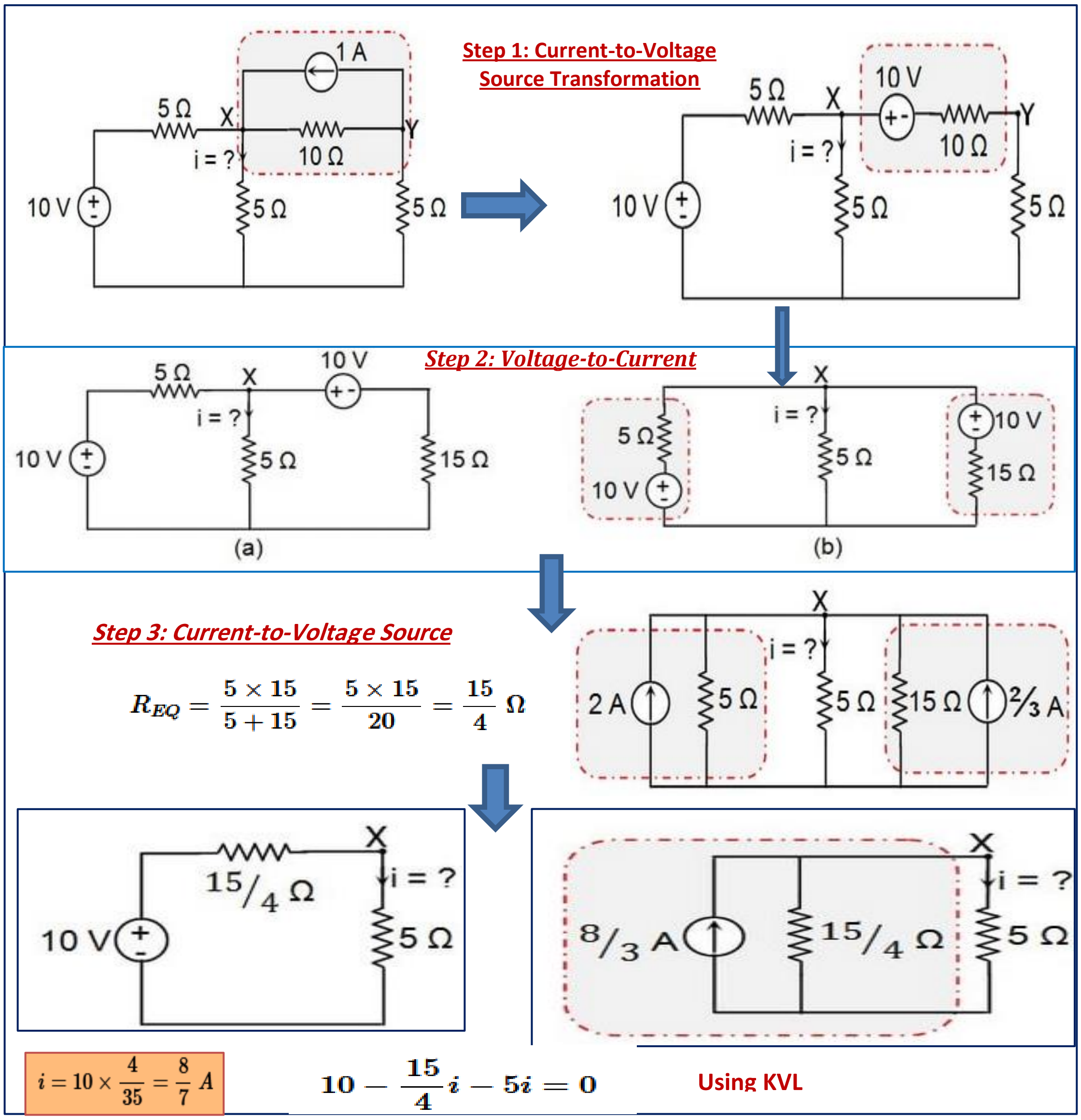


(a) (b) (c)

Then, we use current division in figure (c) to get:

$$i = \frac{2}{2+8} \times 2 = 0.4A \qquad \boxed{v_o = 8i = 8 \times 0.4 = 3.2V}$$

Example 2: Consider the circuit shown in Figure below; the goal is to find the current (denoted by i) through the central 5 Ω resistor.



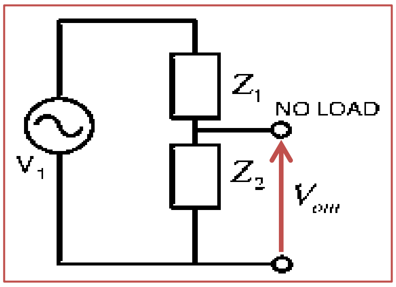
$$R_{EQ} = \frac{5 \times 15}{5+15} = \frac{5 \times 15}{20} = \frac{15}{4}\ \Omega$$

$$i = 10 \times \frac{4}{35} = \frac{8}{7}\ A$$

$$10 - \frac{15}{4} i - 5i = 0$$

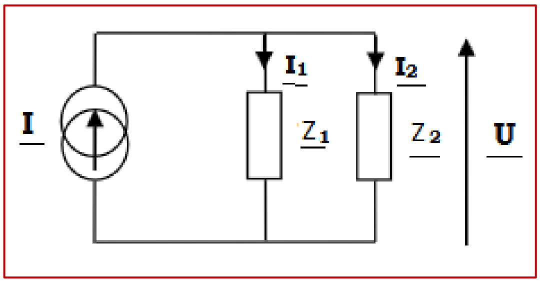
II.3. Bridges divider

➢ **AC Voltage Divider :** Any series circuit is a voltage divider, as shown in Figure. **An AC voltage divider is a circuit that divides an AC input voltage into a smaller AC output voltage.** **The voltage division equations for each of the impedances in the circuit are:**



$$V_{out} = \frac{Z_2}{Z_1 + Z_2} \cdot V_1$$

➢ **AC Current Divider**: Any parallel circuit is a current divider. A current divider in an AC circuit is a parallel circuit where the total current splits among branches with different impedances (resistance and reactance) in different currents. The value of each small current is given by ;



$$\underline{I}_1 = \underline{I}\,\frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2}$$

$$\underline{I}_2 = \underline{I}\,\frac{\underline{Y}_2}{\underline{Y}_1 + \underline{Y}_2}$$

II. AC Network analysis theorems

II.1. Kirchhoff Laws

In a circuit, Kirchhoff's laws consist of the Loop law (**KVL**), which deals with voltages, and the Nodes law (**KCL**), which deals with currents.

- ***Junction Law (First Kirchhoff's Law)*** : The law of knots expresses the conservation of charge in an electric circuit. The law of states that: “The algebraic sum of the intensities of the currents arriving at a node is zero”.

$$\sum_{k=1}^{N} \underline{I}_k = 0 \quad \text{or} \quad \sum_{e=1}^{N_1} \underline{I}_e = \sum_{s=1}^{N_2} \underline{I}_s$$

where $\underline{I}_e$ the incoming currents and $\underline{I}_s$ the outcoming current

- ***Loop law (Second Kirchhoff's law)*** : The loop law is used to study the behavior of voltages within an electric circuit. Kirchhoff's second law states: “The algebraic sum of the differences in potential (or voltage) along a mesh in a given direction is zero”.

$$\sum_{k=1}^{N} \underline{V}_k = 0$$

Steps of the Loop current method :

1- Find the number of independent meshes. We have the following relationship:

$$M = B - N + 1$$

With *M*: the number of independent meshes, *B:* the number of branches and *N:* the number of nodes in the network.

2- In each mesh is assigned a loop current and a path direction.

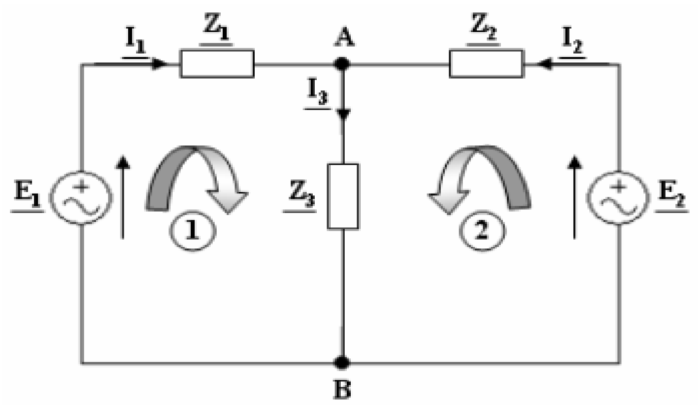
3- For each loop, we write the mesh equation whose unknowns are the currents, using the loop law.

4- Solve the system of equations.

5- Calculate the currents flowing in each branch from the mesh currents.

6- Deduct the potential difference between two nodes using dipole laws (ohm's law).

Example3 : Consider the following circuit:



1- Number of independent loops:

- **Loop (1)** : Composed of $\underline{E}_1$, $\underline{Z}_1$ and $\underline{Z}_3$
- **Loop (2)** : Composed of $\underline{Z}_3$, $\underline{E}_2$ and $\underline{Z}_2$

Number of nodes: (A, B) $N = 2$

Number of branches: ($E_1, Z_1; Z_3$ and E_2, Z_2) $B = 3$

Hence the number of **independent** meshes $M = 2$

2- The equation of node A: $\underline{I}_1 + \underline{I}_2 = \underline{I}_3$

3- Mesh equations :

$$(1): \underline{E}_1 = \underline{Z}_1 \times \underline{I}_1 + \underline{Z}_3 \times \underline{I}_3 = (\underline{Z}_1 + \underline{Z}_3) \times \underline{I}_1 + \underline{Z}_3 \times \underline{I}_2.$$

$$(2): \underline{E}_2 = \underline{Z}_2 \times \underline{I}_2 + \underline{Z}_3 \times \underline{I}_3 = \underline{Z}_3 \times \underline{I}_1 + (\underline{Z}_2 + \underline{Z}_3) \times \underline{I}_2.$$

4- The analysis of an electrical circuit is based on the determination of the currents flowing in all the branches of the circuit. The mesh (loop) equations are formulated in the following matrix form:

$$\boxed{[\underline{E}] = [\underline{Z}] \times [\underline{I}]}$$

With $[\underline{Z}]$: Square impedance matrix.

$$\boxed{\begin{bmatrix} \underline{E}_1 \\ \underline{E}_2 \end{bmatrix} = \begin{bmatrix} \underline{Z}_1 + \underline{Z}_3 & \underline{Z}_3 \\ \underline{Z}_3 & \underline{Z}_2 + \underline{Z}_3 \end{bmatrix} \times \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}}$$

To solve this matrix system, we can use **Cramer's rule**. The result is:

$$\underline{I_1} = \frac{\Delta I_1}{\Delta} = \frac{\begin{vmatrix} \underline{E_1} & \underline{Z_3} \\ \underline{E_2} & \underline{Z_2}+\underline{Z_3} \end{vmatrix}}{\begin{vmatrix} \underline{Z_1}+\underline{Z_3} & \underline{Z_3} \\ \underline{Z_3} & \underline{Z_2}+\underline{Z_3} \end{vmatrix}} = \frac{(\underline{Z_2}+\underline{Z_3})\times \underline{E_1} - \underline{Z_3}\times \underline{E_2}}{\underline{Z_3}\times(\underline{Z_1}+\underline{Z_2}) + \underline{Z_1}\times \underline{Z_2}}$$

$$\underline{I_2} = \frac{\Delta I_2}{\Delta} = \frac{\begin{vmatrix} \underline{Z_1}+\underline{Z_3} & \underline{E_1} \\ \underline{Z_3} & \underline{E_2} \end{vmatrix}}{\begin{vmatrix} \underline{Z_1}+\underline{Z_3} & \underline{Z_3} \\ \underline{Z_3} & \underline{Z_2}+\underline{Z_3} \end{vmatrix}} = \frac{(\underline{Z_1}+\underline{Z_3})\times \underline{E_2} - \underline{Z_3}\times \underline{E_1}}{\underline{Z_3}\times(\underline{Z_1}+\underline{Z_2}) + \underline{Z_1}\times \underline{Z_2}}$$

$$\underline{I_3} = \underline{I_1} + \underline{I_2} = \frac{\underline{Z_2}\times \underline{E_1} + \underline{Z_1}\times \underline{E_2}}{\underline{Z_3}\times(\underline{Z_1}+\underline{Z_2}) + \underline{Z_1}\times \underline{Z_2}}$$

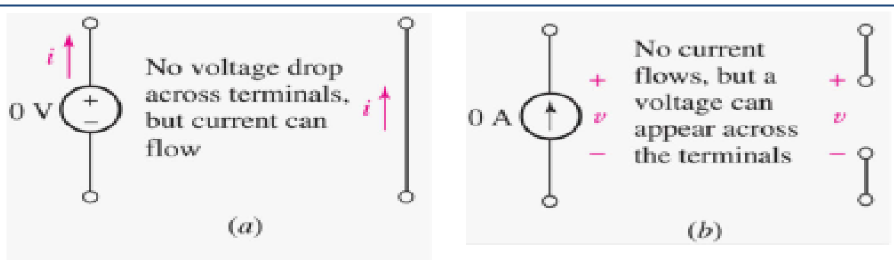
Where Δ: $the\ determinant\ of\ matrix$

II.2. Superposition Theorem

Superposition Theorem is another circuit analysis tool. We can use this theorem to find the voltages and currents around a linear electrical circuit. If a circuit contains one or more independent voltage and/or current sources, we can use ***superposition theorem*** to find the voltage and/or current contribution from each individual source and then algebraically added them together to find the actual voltage and/or current values at any point around the circuit.

So, we must "turn-off" all the sources around a circuit leaving us with just one ideal voltage source or one ideal current source for circuit analysis. This is easily done by open-circuiting all current sources and short-circuiting all voltage sources to find the effect of a particular voltage or current source on the circuit.

- That is, replacing **a voltage source** with **a short-circuit** effectively zero's it since the voltage drop across a short circuit is zero volts, **v = 0.**
- Whereas, replacing **a current source** with **an open-circuit**, effectively zero's it, **(i = 0)** since no current can flow through an open circuit (assuming ideal sources).



(a) A voltage source set to zero acts like a short circuit. (b) A current source set to zero acts like an open circuit.

Then the basic procedure for solving any circuit using **Superposition Theorem** is as follows:

Step 1: Identify all the independent sources in the circuit, such as voltage sources and current sources, and select just one source in the circuit.

Step 2: Turn off (short circuit or open circuit) all the other independent sources and analyse the circuit using only one active source.

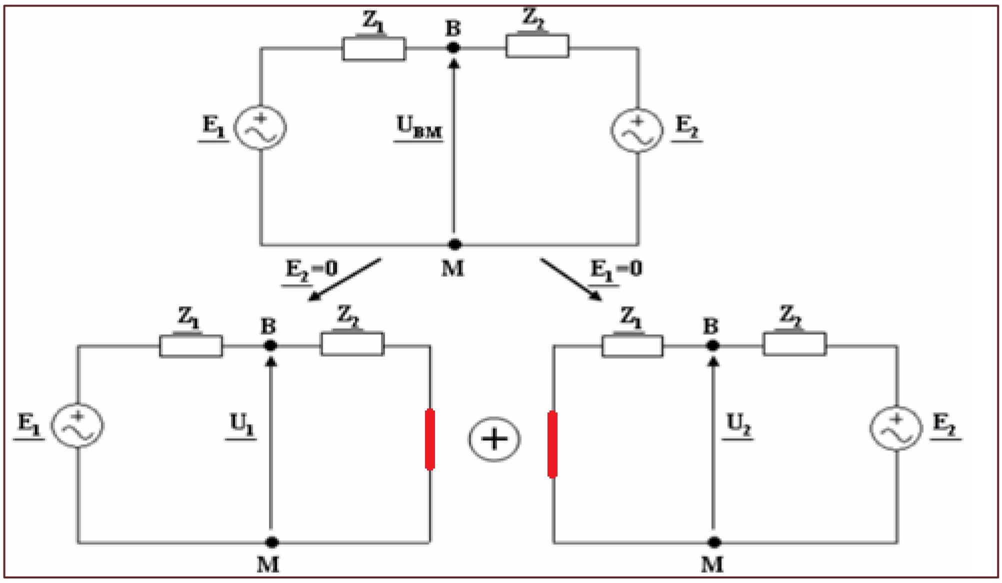
Step 3: Use standard circuit analysis techniques (Ohm's Law, Kirchhoff's laws) to determine the voltage across or current through the desired circuit element, branch or node due to the single selected source.

Step 3: Repeat for each independent source, one at a time, considering only the effects of that source while keeping all others turned off.

Step 4: Algebraically sum the individual responses obtained from each source to find the total response at the circuit element, branch or node of interest.

Note : Pay attention to polarities, sign conventions and direction of flow of combined responses when all the sources are acting simultaneously.

Example 3 : Take, for example, the circuit shown in the following figure, in which we calculate the voltage$\underline{U}_{BM}$



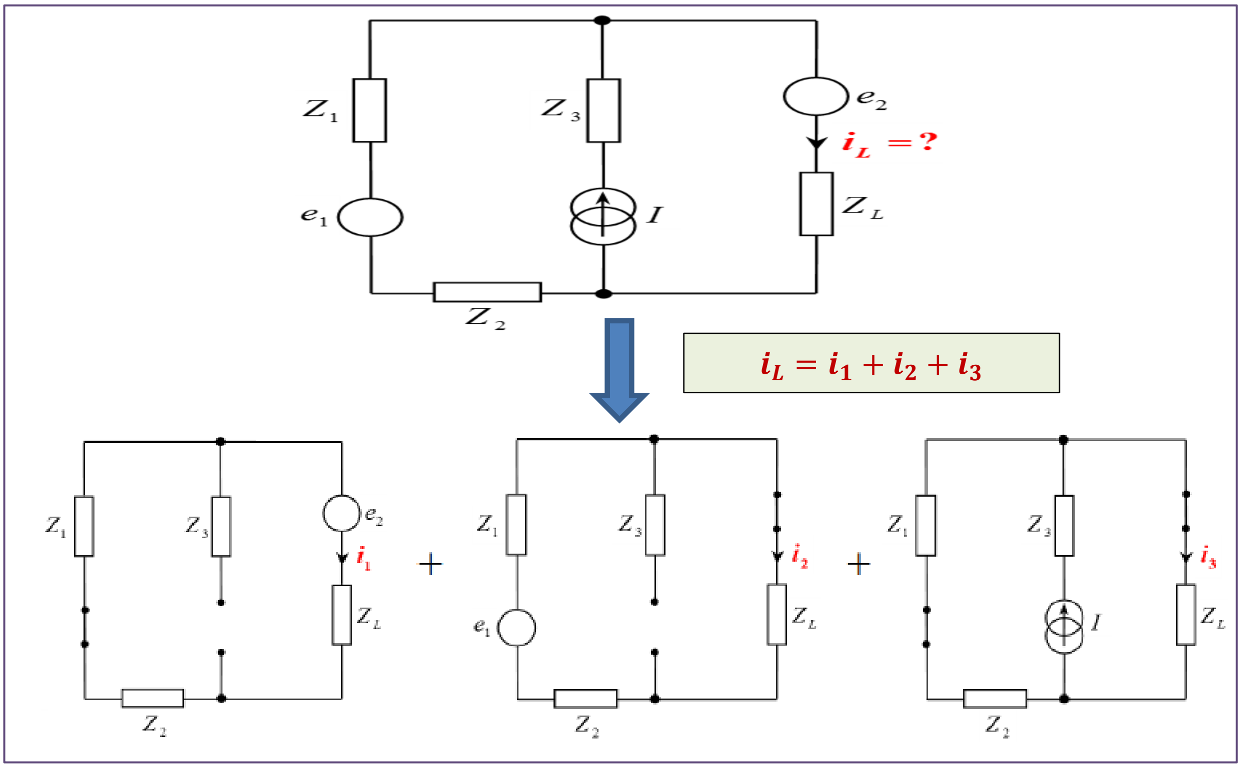
- **if** $\underline{E}_2 = 0: \ \underline{U}_1 = \underline{E}_1 \dfrac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$.

- **if** $\underline{E}_1 = 0: \ \underline{U}_2 = \underline{E}_2 \dfrac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2}$.

Taking into account both sources, we obtain :

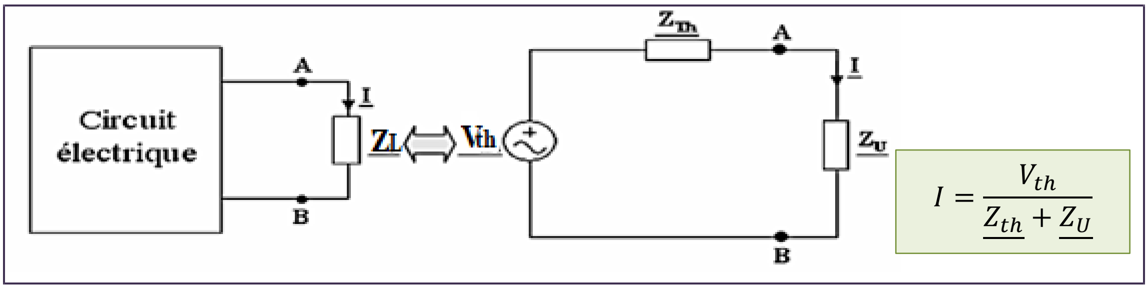
$$\underline{U}_{BM} = \underline{U}_1 + \underline{U}_2 = \underline{E}_1 \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} + \underline{E}_2 \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2}$$

Example 4:

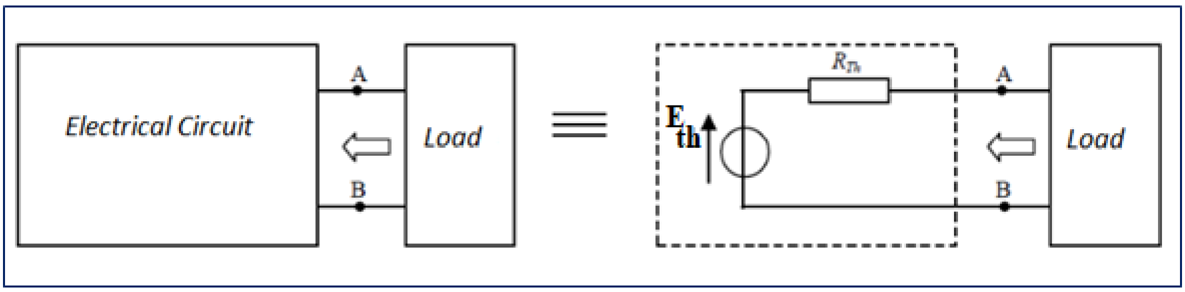


II.3. Thevenin's Theorem Léon Charles Thévenin: French Telegraph Engineer (1857–1926)

Thévenin's theorem is named after **Léon Charles Thévenin**. Thenvenin's theorem is valid for DC circuit and AC circuits. Thevenin equivalent of a circuit in DC and AC consists of a voltage source of value $\mathrm{E}_{\mathrm{thevenin}}$ and a series Resistance $R_{thevevin}$ (in DC) or impedance $\mathrm{Z}_{\mathrm{thevenin}}$ (in AC circuits). Thus, any linear AC sinusoidal network between two terminals A and B can be **replaced by an equivalent circuit consisting of** an equivalent generator of Thévenin $\underline{E_{th}}$ *or* $\underline{V_{AB}}$ **in series with Thevenin impedance $\underline{Z_{th}}$.**

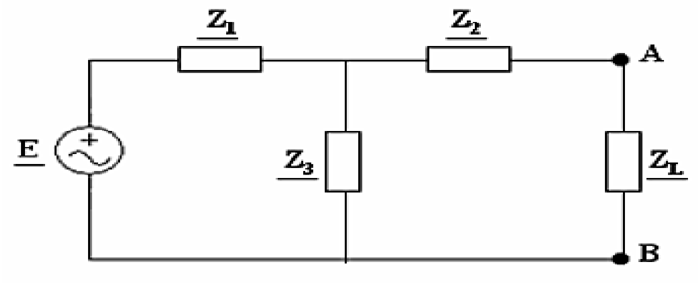


Or



It states that: **Any single port linear network can be reduced to a simple voltage source, E_{th}, in series with an internal impedance Z_{th}.**

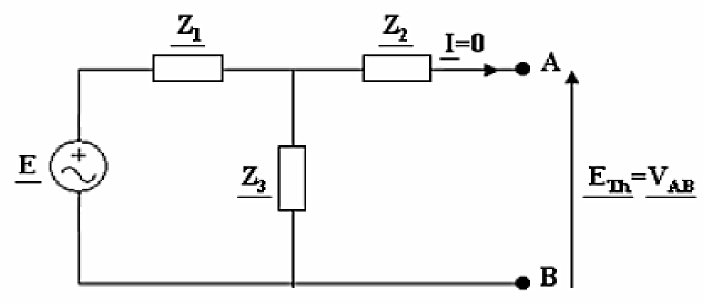
Example 5 : Application of Thévenin's theorem to the following circuit:



Step 1: Determination of E_{Th}

1- Disconnect Z_L between A and B.

2- Determine the voltage between A and B (no-load voltage).



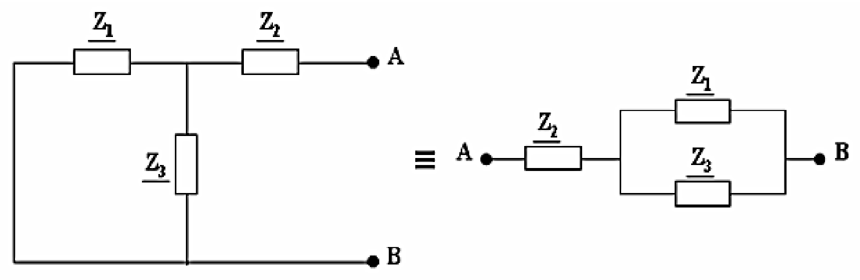
$$\underline{E_{Th}} = \underline{V_{AB}} = \underline{V_{Z_3}} = \underline{E}\frac{\underline{Z_3}}{\underline{Z_1} + \underline{Z_3}}$$

Step 2: Determination of $\underline{Z_{Th}}$

1- Disconnect Z_L between A and B.

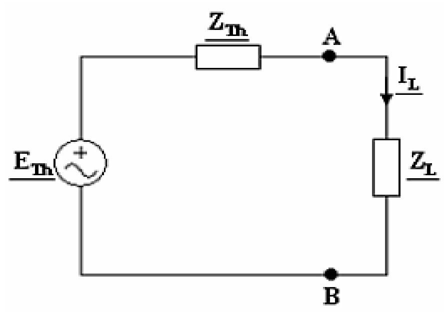
2- Switch off source

3- Determine the impedance between terminals A and B



$$\underline{Z_{Th}} = \underline{Z_2} + (\underline{Z_1} // \underline{Z_3}) = \underline{Z_2} + \frac{\underline{Z_1} \times \underline{Z_3}}{\underline{Z_1} + \underline{Z_3}} = \frac{\underline{Z_1} \times \underline{Z_2} + \underline{Z_1} \times \underline{Z_3} + \underline{Z_2} \times \underline{Z_3}}{\underline{Z_1} + \underline{Z_3}}$$

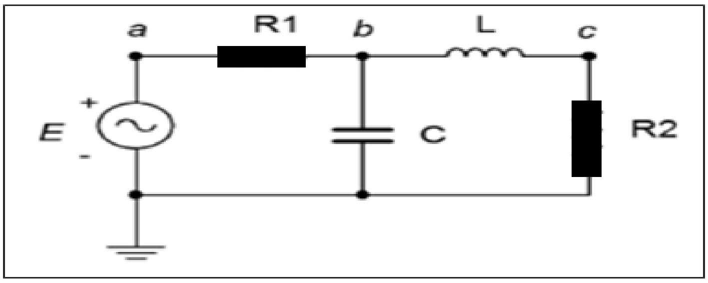
Step 3: Calculation of current $\underline{I_L}$ The Thévenin equivalent circuit appears as follows:



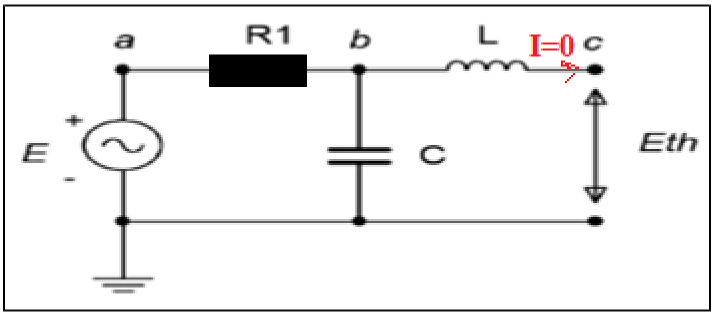
$$\underline{I_L} = \frac{\underline{E_{Th}}}{\underline{Z_{Th}} + \underline{Z_L}} = \frac{\underline{E}\dfrac{\underline{Z_3}}{\underline{Z_1}+\underline{Z_3}}}{\underline{Z_2} + \dfrac{\underline{Z_1}\times\underline{Z_3}}{\underline{Z_1}+\underline{Z_3}} + \underline{Z_L}} = \underline{E}\frac{\underline{Z_3}}{\underline{Z_1}\times\underline{Z_2}+\underline{Z_2}\times\underline{Z_3}+\underline{Z_1}\times\underline{Z_3}+\underline{Z_L}\times(\underline{Z_1}+\underline{Z_3})}$$

$$\underline{V_L} = \underline{Z_L}\times\underline{I_L} = \underline{E}\frac{\underline{Z_3}\times\underline{Z_L}}{\underline{Z_1}\times\underline{Z_2}+\underline{Z_2}\times\underline{Z_3}+\underline{Z_1}\times\underline{Z_3}+\underline{Z_L}\times(\underline{Z_1}+\underline{Z_3})}$$

Example 6: Consider the circuit shown in Figure. Suppose we want to find the Thévenin equivalent that drives R_2.



First Step : We cut the circuit immediately to the left of R2. We then determine the open circuit output voltage at the cut points (i.e., at the open port). This voltage is called the Thévenin voltage, $\mathbf{E_{th}}$.

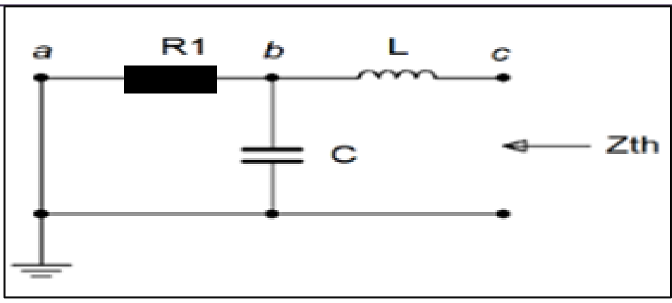


E_{th},the open circuit output voltage.

$$E_{th} = \frac{Z_C}{Z_C + Z_{R1}}.E$$

In a circuit such as this, basic series-parallel analysis techniques may be used to find $\mathbf{E_{th}}$. In this circuit, *due to the open, no current flows through the inductor*, **L**, and thus no voltage is developed across it. Therefore, E_{th} must equal the voltage developed across the capacitor, C.

Second Step: Finding the Thévenin impedance, $\boldsymbol{Z_{th}}$. Beginning with the "**cut**" circuit, replace all sources with their ideal internal impedance (**thus shorting voltage sources and opening current sources**). From the perspective of the cut point, look back into the circuit and simplify to determine its equivalent impedance. This is shown in figure. Looking in from where the cut was made (right side), we see that $\mathbf{R_1}$ **and** $\mathbf{X_C}$ are **in parallel**, and this combination **is then in series with** $\mathbf{X_L}$.



$Thus, Z_{th}$ is equal to

$Z_{th} = \boldsymbol{jX_L} + (\boldsymbol{R_1} \,||\, -\boldsymbol{jX_C}).$

The current flowing in R_2 is given by:

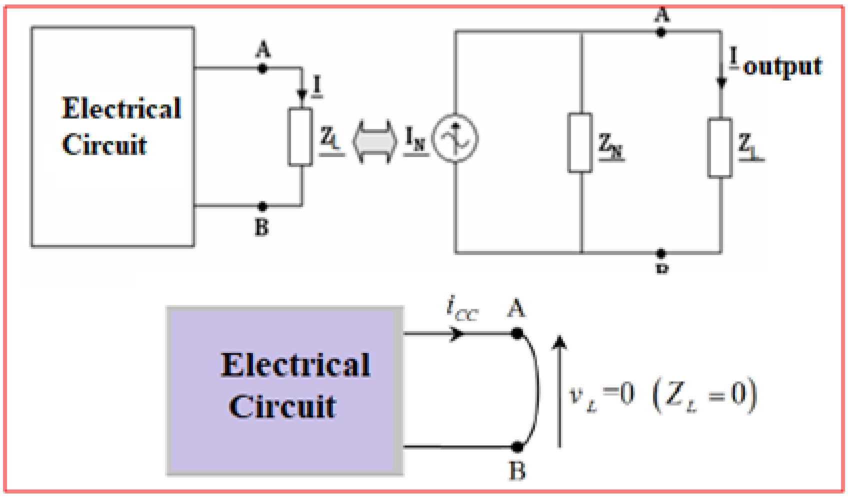
$$I_{R_2} = \frac{E_{th}}{Z_{th} + R_2}$$

II.4. Norton's Theorem : (Edward Lowry Norton: American electrical engineer (1898–1983))

Norton's theorem states that any linear circuit can be simplified to an equivalent circuit consisting of a single current source and parallel resistance that is connected to a load.

Thus, any linear AC sinusoidal electrical network placed between two terminals A and B can be replaced by an equivalent circuit consisting of an equivalent Norton generator current I_N in parallel with impedance Z_N .

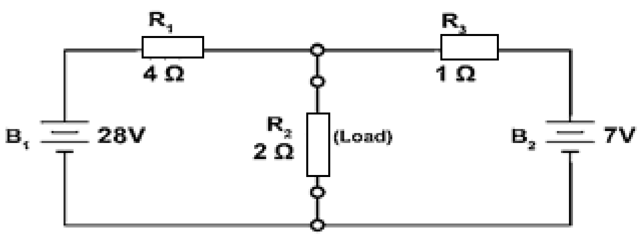
- The Norton current ($I_{CC} = I_N$) is obtained by calculation or measurement after short-circuiting terminals A and B.
- The internal impedance $Z_N$is obtained in the same way as Thévenin's theorem (Z_N=Z_{th}).



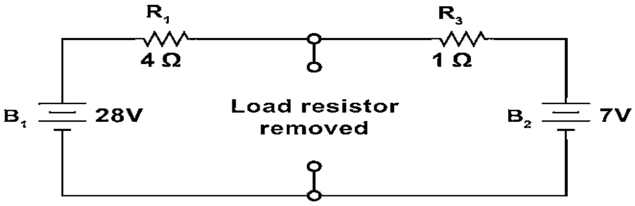
$$i_L = \frac{Z_N}{Z_N + Z_L} \cdot i_N$$

➢ **Example 7 : Applying Norton's Theorem to a Linear Circuit**

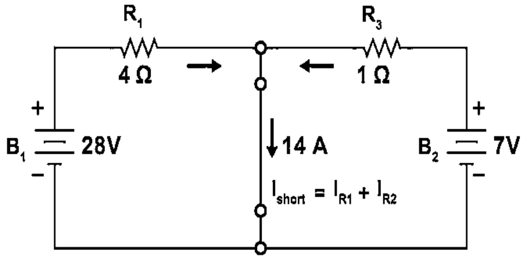
Let's explain Norton's theorem using the example circuit



Step 1: Remove the Load Resistor : The first step is to identify the load resistance and remove it from the original circuit, as shown in Figure.



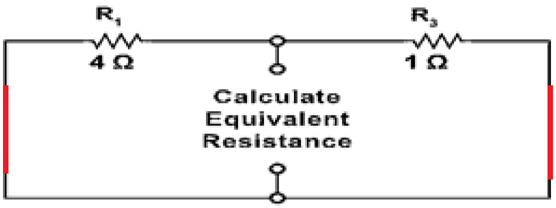
Step 2: Calculate the Norton Current : To find the Norton current (for the current source in the Norton equivalent circuit), place a direct wire (short circuit) connection between the load points and determine the resultant current



Using Kirchhoff's current law (KCL), we know that: $I_{short} = I_{R_1} + I_{R_2}$. Now, applying Ohm's law to each of the individual branch currents: $\boxed{I_{short} = I_{R_1} + I_{R_2} = \frac{V_1}{R_1} + \frac{V_2}{R_2}}$

We can solve for the short circuit current: $I_{Norton} = I_{short} = \frac{28}{4} + \frac{7}{1} = 14A$

Step 3: Replace the Power Sources : To find the Norton resistance for our equivalent circuit, we can now replace the power sources from our circuit, as shown in Figure.

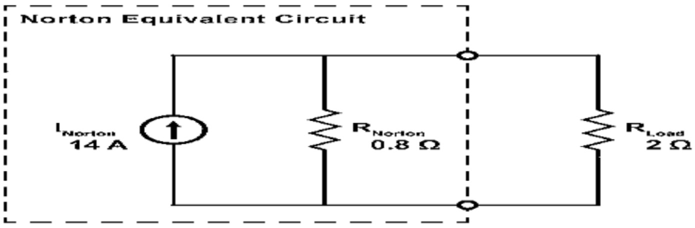


The voltage sources are replaced with short circuits, and the current sources are replaced with open circuits. This process of replacing the power supplies is identical to that used for the superposition theorem and Thevenin's theorem.

Step 4: Calculate the Nortan Resistance*:** After replacing the two voltage sources, the total resistance measured at the location of the removed load is equal to R_1 and R_3 in parallel. ***The Norton equivalent resistance is calculated as:

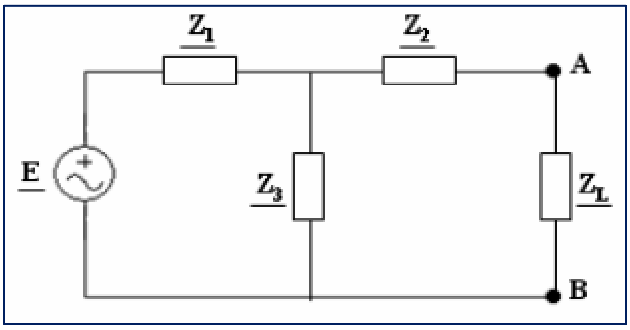
$$\boxed{R_{Norton} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = 0.8\Omega}$$

Step 5: Draw the Norton Equivalent Circuit: The simplified Norton equivalent circuit, shown in Figure.



➢ Example 8 : The Norton Equivalent Circuit

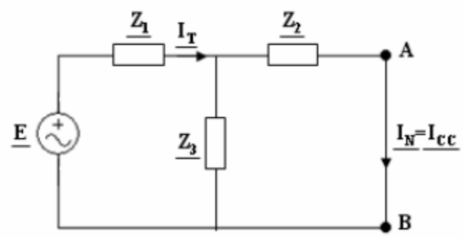
To give the Norton equivalent circuit of the following circuit:



Step 1: **Determination of Norton current $\underline{I}_N$:** To obtain this current, we proceed as follows:

1- Disconnect $\underline{Z}_L$ between A and B.

2- Short-circuit $\underline{Z}_L$.



$$\underline{I}_T = \frac{\underline{E}}{\underline{Z}_{eq}} = \frac{\underline{E}}{\underline{Z}_1 + (\underline{Z}_2 // \underline{Z}_3)} = \underline{E}\frac{\underline{Z}_2 + \underline{Z}_3}{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_2 \times \underline{Z}_3}$$

$$\underline{I}_N = \underline{I}_T \frac{\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \underline{E}\frac{\underline{Z}_2 + \underline{Z}_3}{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_2 \times \underline{Z}_3} \times \frac{\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \underline{E}\frac{\underline{Z}_3}{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_2 \times \underline{Z}_3}$$

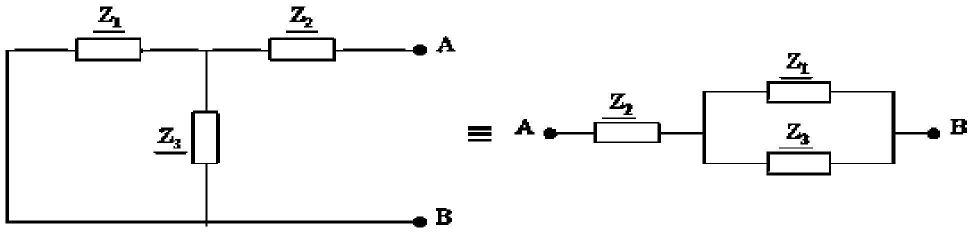
$$\boxed{\underline{I}_N = \underline{E}\frac{\underline{Z}_3}{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_2 \times \underline{Z}_3}}$$

Step 2: Determining $\underline{Z}_N$: To obtain the Norton impedance $\underline{Z}_N$, follow these steps:

1. The load impedance $\underline{Z}_L$ is always disconnected between A and B.

2. Short-circuit $\underline{E}$

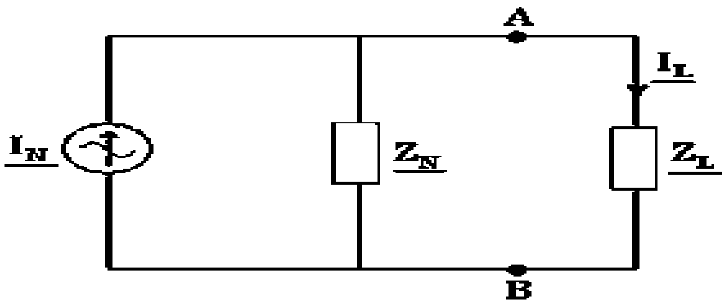
3. Determine the impedance between terminals A and B.



$$\underline{Z}_N = \underline{Z}_{Th} = \underline{Z}_2 + (\underline{Z}_1 // \underline{Z}_3) = \underline{Z}_2 + \frac{\underline{Z}_1 \times \underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3} = \frac{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_2 \times \underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3}$$

$$\boxed{\underline{Z}_N = \frac{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_2 \times \underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3}}$$

***Step 3*: Calculation of the current $\underline{I}_L$**, flowing through the load: The Norton equivalent circuit is given by :



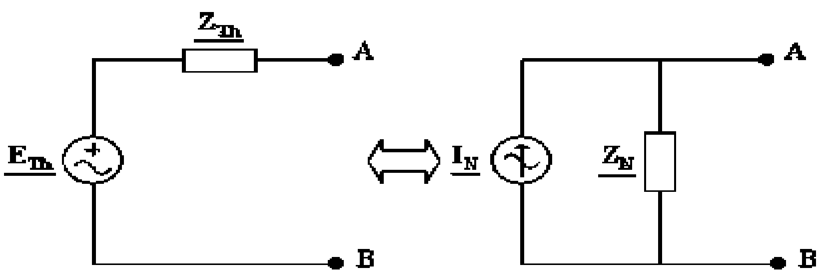
Using divider current and ohm law, we obtain:

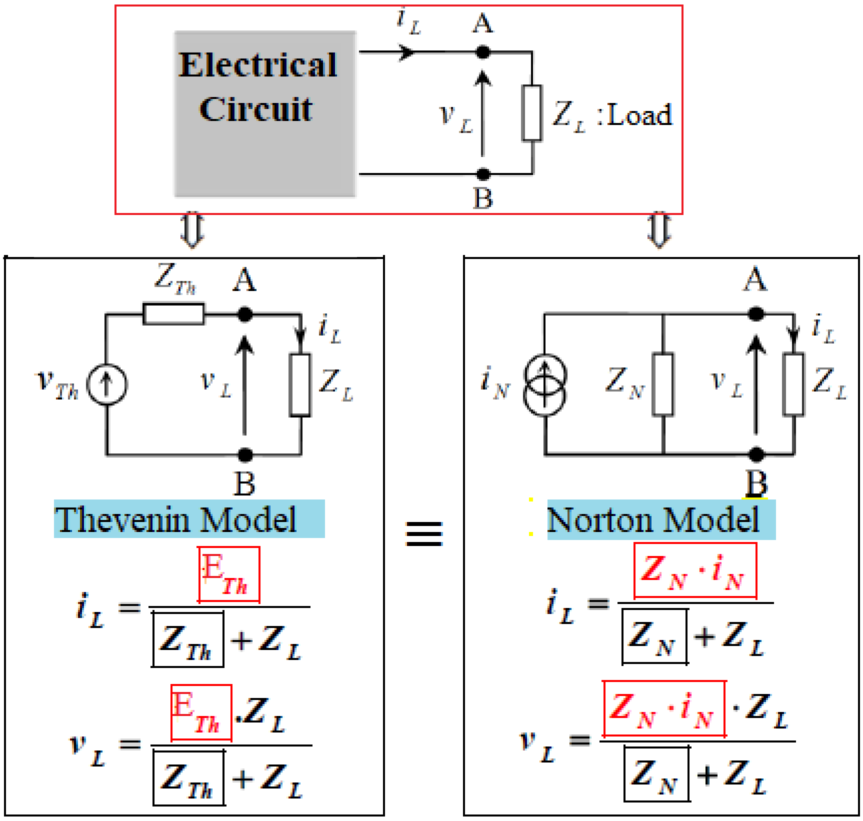
$$\underline{I}_L = \underline{I}_N \frac{\underline{Z}_N}{\underline{Z}_N + \underline{Z}_L} = \underline{E} \frac{\underline{Z}_3}{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_2 \times \underline{Z}_3 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_L \times (\underline{Z}_1 + \underline{Z}_3)}$$

$$\underline{V}_{Z_L} = \underline{Z}_L \times \underline{I}_L = \frac{\underline{Z}_N \times \underline{Z}_L}{\underline{Z}_N + \underline{Z}_L} \underline{I}_N = \underline{E} \frac{\underline{Z}_3 \times \underline{Z}_L}{\underline{Z}_1 \times \underline{Z}_2 + \underline{Z}_2 \times \underline{Z}_3 + \underline{Z}_1 \times \underline{Z}_3 + \underline{Z}_L \times (\underline{Z}_1 + \underline{Z}_3)}$$

❖ Conversion between a Thévenin and Norton circuit

Any Thévenin generator can be converted into a Norton generator (and vice versa).





We go directly from a Norton circuit to a Thévenin circuit and vice versa, using the following formulas:

- Norton to Thévenin transformation :

$$\begin{aligned} E_{Th} &= I_N \times Z_N \\ Z_{Th} &= Z_N \end{aligned}$$

- Thévenin to Norton transformation :

$$\begin{aligned} I_N &= \frac{E_{Th}}{Z_{Th}} \\ Z_N &= Z_{Th} \end{aligned}$$

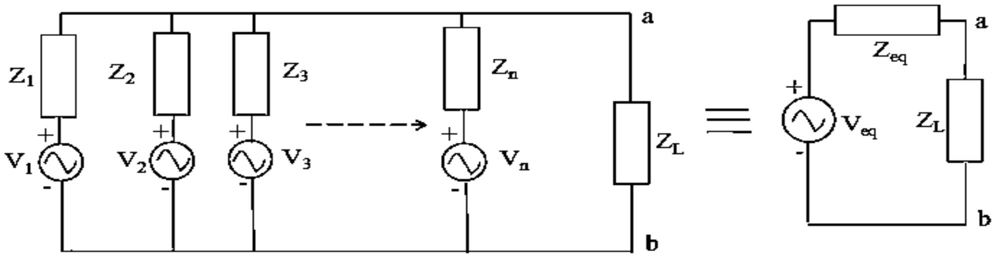
Example: For example 6, without performing the calculations, determine the equivalent Norton model using the Thévenin-Norton equivalence.

$$\begin{cases} E_{th} = \dfrac{Z_C}{Z_C + Z_{R1}} . E \\ Z_{th} = \boldsymbol{jX_L + (R_1 \,||\, -jX_C)} \end{cases} \Longleftrightarrow \begin{cases} \boldsymbol{I_N = \dfrac{E_{th}}{Z_{th}}} \\ \boldsymbol{Z_N = Z_{th}} \end{cases}$$

Thévenin Model | Norton Model

II.4 Millman's Theorem for networks

For AC networks Millman's theorem states that " if 'n' number of voltage sources $V_1, V_2, V_3, \ldots\ldots, V_n$ having internal impedances $Z_1, Z_2, Z_3, \ldots\ldots, Zn$ are connected in parallel across the load Z_L than this arrangement may be replaced by a single voltage source V_{eq} in series with equivalent impedance Z_{eq} . Millman's equivalent circuit is shown in figure. Millman's theorem is a very useful tool in pulse and digital circuit analysis.



$$V_{eq} = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3 - - - - - + V_nY_n}{Y_1 + Y_2 + Y_3 - - - - - - - - - + Y_n} = \frac{I_1 + I_2 + I_3 \text{ --------} + I_n}{Y_{eq}}$$

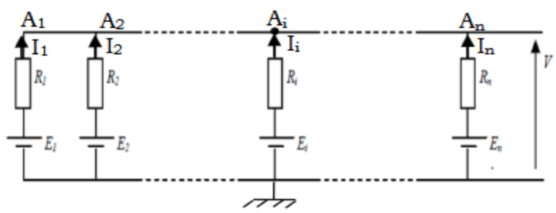
$$Z_{eq} = \frac{1}{Y_1 + Y_2 + Y_3 - - - - - - - - - + Y_n} = \frac{1}{Y_{eq}}$$

Where $Z_1, Z_2, Z_3, \ldots\ldots, Zn$ are the impedances and Y_1, Y_2 , Y_3,.......Yn are the admittances.

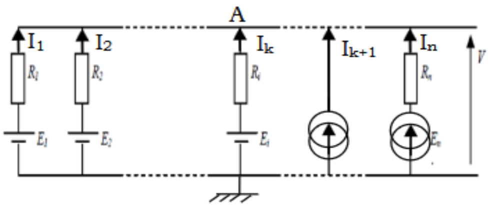
This theorem is simply the Junction law (node law) expressed in terms of potentials.

For DC circuit : Since the current intensity in branch i, of conductance G_i, arriving at node A has the expression $I_i = G_i(E_i - V_A)$, the law of neuds gives:

$$\sum_{i=1}^{n} I_i = G_1(E_1 - V_A) + G_2(E_2 - V_A) + \cdots \ldots + G_n(E_n - V_A) \Rightarrow V_A = \frac{G_1.E_1 + G_2.E_2 + \cdots \ldots + G_n.E_n}{G_1 + G_2 + \cdots \ldots + G_n}$$

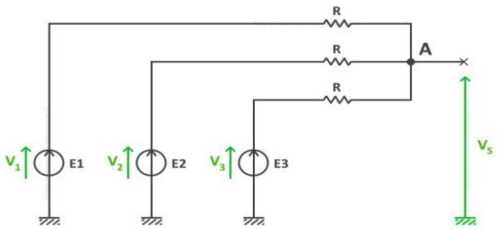


(a)



(b)

Example 9: Determine Vs as a function of V1, V2, and V3 using Millman's theorem at point A



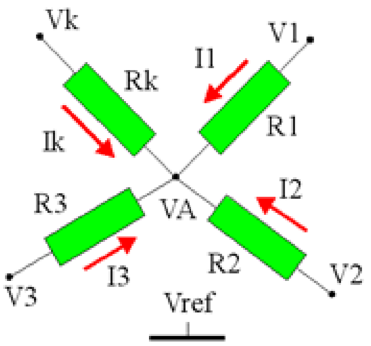
Using Millman theorem at point A, we obtain

$$V_S = \frac{\frac{V1}{R}+\frac{V2}{R}+\frac{V3}{R}}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} \quad \text{So:} \quad \boxed{V_S = \frac{V1+V2+V3}{3}}$$

So, as you can see, this circuit simply averages out all the voltages present at the input. It is therefore a '**voltage averaging'** circuit.

Example 10: Consider a node A to which branches k lead; the potentials V_i at the ends of the branches are all defined relative to the same reference potential (figure below).

R_i is the resistance of branch i and G_i is its conductance. The node law is written as:



$$\sum_{i=1}^{k} I_i = I_1 + I_2 + \cdots + I_k = 0$$

$$\frac{V_1 - V_A}{R_1} + \frac{V_2 - V_A}{R_2} + \cdots + \frac{V_k - V_A}{R_k} = O$$

$$(V_1 - V_A)\cdot G_1 + (V_2 - V_A)\cdot G_2 + \cdots + (V_k - V_A)\cdot G_k = 0$$

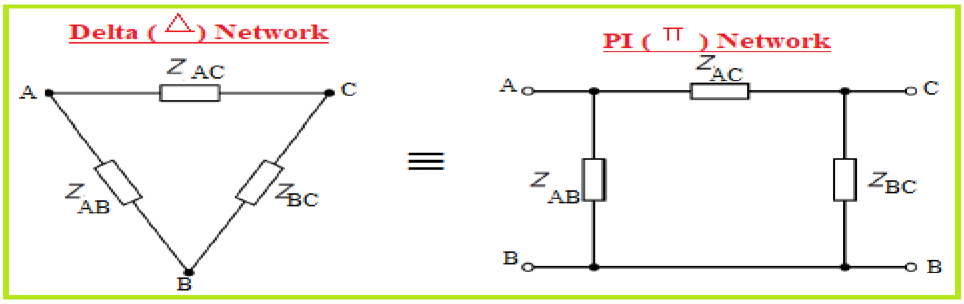
$$V_A \cdot \sum G_1 = \sum V_i \cdot G_i$$

The potential of point A relative to that of the common reference is therefore:

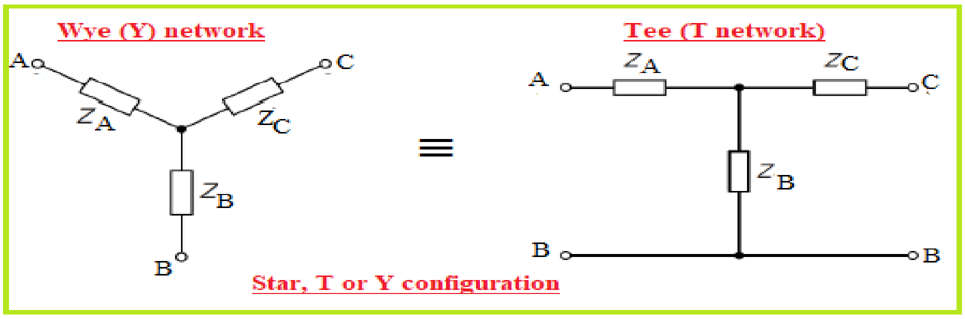
$$\boxed{V_A = \frac{\sum_i Vi \cdot G_i}{\sum_i Gi}}$$

II.5. Kennelly's theorem

Kennelly's theorem, or triangle-star transformation, or **Y-Δ** transformation, or **T-π** transformation, is a mathematical technique for simplifying the study of certain electrical networks. **Named after Arthur Edwin Kennelly**, this theorem allows you to switch from a "triangle" configuration (or Δ, or π, depending on how you draw the diagram) to a "star" configuration (or, similarly, Y or T). **It allows you to switch from a three-impedance star network to a three-impedance delta network, and vice versa**.

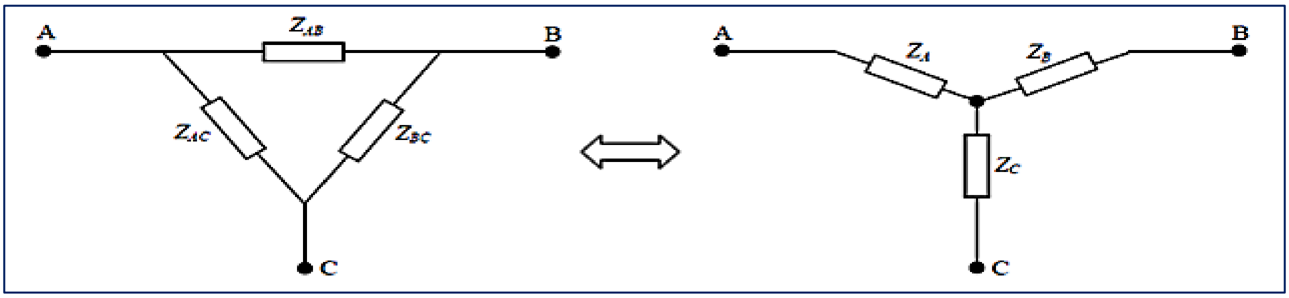


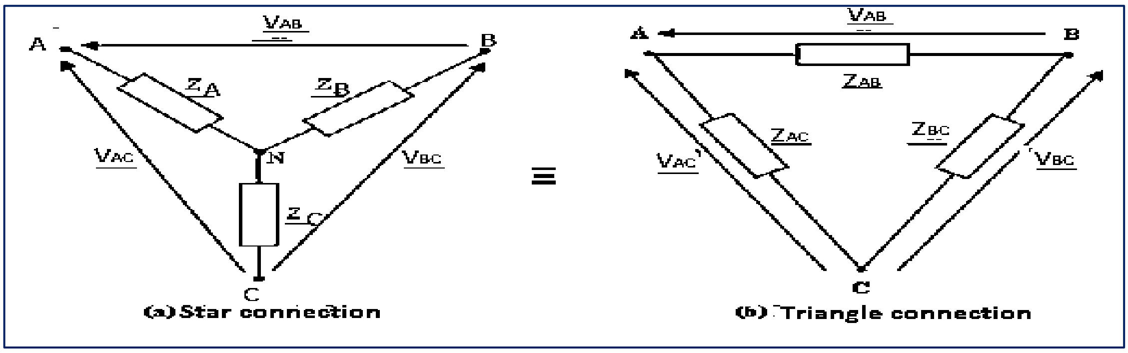
Triangle or delta or pi assembly



Δ-Y and Y-Δ Conversions (Star-delta equivalence)

The two circuits shown in figure are equivalent if their resistance values are linked by the relationships shown below.





From delta circuit (π) to star circuit (T) :

The impedance of the equivalent star branch is equal to the product of the adjacent impedances divided by the total sum of the impedances.

$$Z_A = \frac{Z_{AB} . Z_{AC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

$$Z_B = \frac{Z_{AB} . Z_{BC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

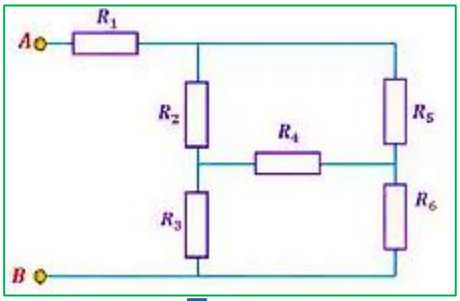
$$Z_C = \frac{Z_{AC} . Z_{BC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

From star circuit (T) to delta circuit (π) :

The impedance of one branch of the equivalent triangle is equal to the sum of the products of the impedances, divided by the impedance of the opposite branch.

$$Z_{AB} = \frac{Z_A.Z_B + Z_A.Z_C + Z_B.Z_C}{Z_C}$$
$$Z_{BC} = \frac{Z_A.Z_B + Z_A.Z_C + Z_B.Z_C}{Z_A}$$
$$Z_{AC} = \frac{Z_A.Z_B + Z_A.Z_C + Z_B.Z_C}{Z_B}$$

Example 11: Using the triangle-star transformation, determine the equivalent resistance of the dipole AB shown in the figure opposite:

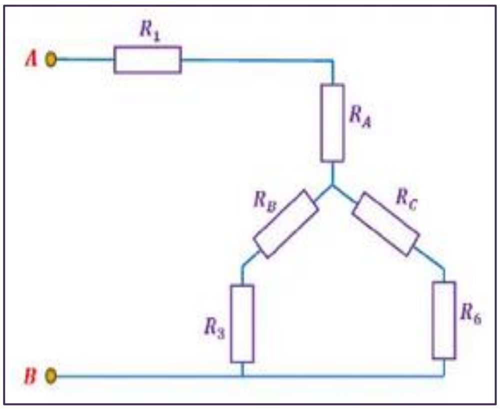


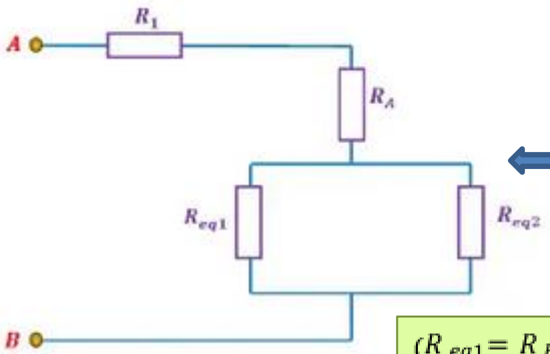
Tranformation

Delat-Tee

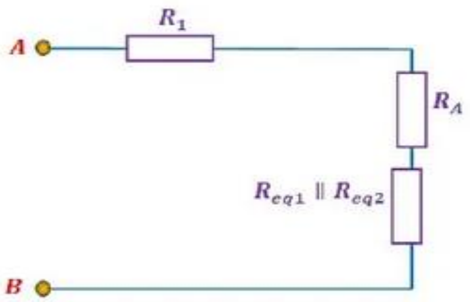
(Triangle-Star)

$$\begin{cases} R_A = \dfrac{R_2.R_5}{R_2 + R_4 + R_5} \\ R_B = \dfrac{R_2.R_4}{R_2 + R_4 + R_5} \\ R_C = \dfrac{R_4.R_5}{R_2 + R_4 + R_5} \end{cases}$$





$$\begin{cases} R_{eq1} = R_B + R_3 \\ R_{eq2} = R_C + R_6 \end{cases}$$



$$R_{eq3} = \frac{R_{eq1}.R_{eq2}}{R_{eq1} + R_{eq2}}$$

Finally the equivalent resistance between A and B is given by: $R_{eq} = R_1 + R_A + R_{eq3}$