

Level: 1st Year

Material: Algorithms and Static Data Structures

Date: 15/01/2025

Duration: 02h00

First semester final exam

Context: This exam consists of a single comprehensive problem divided into many interconnected parts. It is strongly recommended to use modularity and to call modules across the different parts. You can use the predefined functions `pow(x, p)` to calculate x^p and `log2(n)` to calculate $\log_2(n)$.

Exercise 01: (06 points)

1. Write a module that rearranges the digits of a positive integer n to form the largest possible number and deal with this number as a **base password** (03 points).
2. Write a module to evaluate the **entropy** of the generated number by analyzing the count of unique digits it contains using the following formula. The entropy value indicates the security level of the password as follows: **Weak** if $Entropy \leq 2$, and **Strong** otherwise (02 points).

$$\text{Entropy} = \log_2(\text{number of unique digits})$$

3. Write the main algorithm (01 point).

Exercise 2: (04 points)

1. Write a module to apply a transformation by reversing the digits of the **base password** (generated previously) and then subtracting the **smallest possible number** (obtained by rearranging the digits of n in the smallest number) from the **base password** (02 points).
2. Write the main algorithm (02 points).

Example

- Input: $n = 317$
- Smallest number: 137
- Base password: 731
- Subtract: $731 - 137 = 594$ (This is the final **obfuscated password**)

Exercise 3 (05 points)

1. Write a module that calculates the sum of the digits raised to the power of their index (index starts from 1). This value will serve as an **encryption key** for further transformations (02 points).
2. After that, shuffle the digits of n based on the encryption key (by rotating the digits left by a number of positions equal to the last digit of the encryption key) (02 points).
3. Write the main algorithm (01 points).

Example

$$n = 442775$$

$$\begin{aligned} \text{Encryption key} &= 4^1 + 4^2 + 2^3 + 7^4 + 7^5 + 5^6 \\ &= 4 + 16 + 8 + 2401 + 16807 + 15625 = 19901 \end{aligned}$$

Use the last digit (i.e., 1) of the **Encryption key** to rotate the digits of n by 1 position

$$\text{Shuffled number} = 427754$$

Exercise 04: (05 points)

1. Write a module that transforms a numeric password (positive Integer n) into a more secure representation by analyzing **consecutive repetitions** of its digits (04 points).
2. Write the main algorithm (01 points).

Example

- $n = 44455522$
- Output = 435322

Good Luck