الجمهـــوريـــــــــة الجزائريــــــــــة الديمـــقــراطيـــــــة الشعبيــــــــة

People`s Democratic Republic of Algeria

Ministry of Higher Education and Scientific Research
National School of Cyber Security
Foundation Training Department

وزارة التعـليــم العـــالــي والـبحـــث الـعـــلمــي
المدرســـة الوطنيـــة العليــا في الأمن السيبراني
قســـم التكويـــن القــــــاعدي

Surname : ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ **First Name :** ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░

N° Student Card : ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ **Date : 06 / 04 / 2025**

Section : A and B **Group :** ░ ░

Module : GRAPH THEORY ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ ░ **Academic Year : 2024/2025**

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Marker's Fullname :

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Exercise 1: (04 points)

Select the correct answer (+0.5 point for each correct answer, and -0.25 for each wrong answer)

1. **A graph is said to be connected if:**
   a) It contains no cycles
   b) There is a path between each pair of vertices
   c) It has at least one isolated vertex
   d) It contains a spanning tree

2. **In a bipartite graph, the vertices can be divided into:**
   a) Two disjoint sets with edges between vertices of the same set
   b) Three disjoint sets
   c) Two disjoint sets with edges only between these sets
   d) A single set containing all vertices

3. **What is the maximum number of edges in an undirected simple graph with n vertices?**
   a) n^2
   b) $n(n-1)$
   c) $n(n-1)/2$
   d) $n(n+1)/2$

4. **A directed graph is said to be symmetric if:**
   a) All its edges have the same weight
   b) It contains at least one cycle
   c) For every directed edge from u to v, there is an edge from v to u
   d) It is connected

5. **A graph is said to be asymmetric if:**
   a) It contains no loops
   b) It contains no cycles
   c) No edge has its inverse in the graph
   d) It has an isolated vertex

6. **A graph is said to be multigraph if:**
   a) It contains at least one loop
   b) It has multiple connected components
   c) It is both directed and undirected
   d) It has multiple edges between the same pair of vertices

7. **A graph is said to be simple if:**
   a) It contains only one vertex
   b) It is strongly connected
   c) It contains neither loops nor multiple edges
   d) It has a spanning tree

This area will be cut.

8. A cocycle in a graph is:

a) A path containing an odd number of edges

b) An Eulerian cycle that traverses all edges exactly once

c) A set of edges connecting two opposite vertices in a bipartite graph

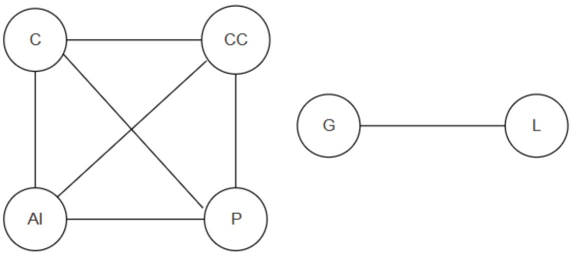
b) A subset of edges whose removal increases the number of connected components

Exercise 2: (04 points)

A company organizes training sessions for its employees. Each employee must attend a set of training sessions, and certain sessions cannot take place at the same time if they share common participants. The training sessions and their participants are grouped as follows (each group represents a set of sessions that share common participants and cannot be scheduled simultaneously):

- {Programming (P), Cybersecurity (C), Artificial Intelligence (AI)}
- {Project Management (G), Leadership (L)}
- {Cybersecurity (C), Cloud Computing (CC)}
- {Cloud Computing (CC), Artificial Intelligence (AI)}
- {Programming (P), Cloud Computing (CC)}

1. Represent the problem by a graph. (1.25 pts)



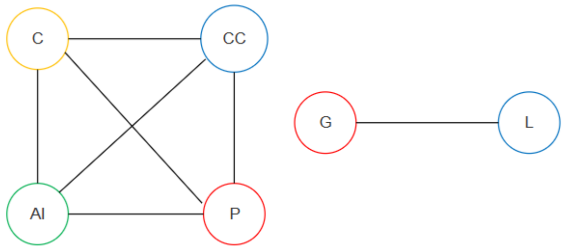
2. Find a minimal coloring of this graph and deduce the minimum number of sessions to be organized.

The graph is composed of two disjoint connected components, both of which are complete subgraphs. The first component consists of four vertices, necessitating a minimum of four colors for a proper coloring. The second component comprises two vertices, and thus requires two colors. Since the components are disjoint, the chromatic number of the entire graph is determined by the component with the highest chromatic requirement, which in this case is four. (1 pt)

Accordingly, it can be deduced that a minimum of four sessions is required. (0.5 pts)

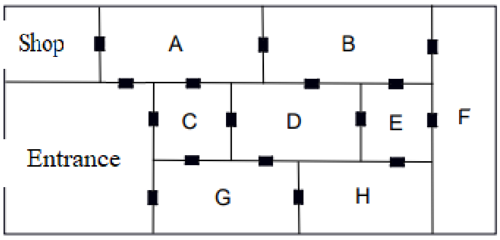
3. Use the graph to propose a schedule of training sessions (1.25 pts)

Session1	Session2	Session3	Session4
CC + L	P + G	C	AI



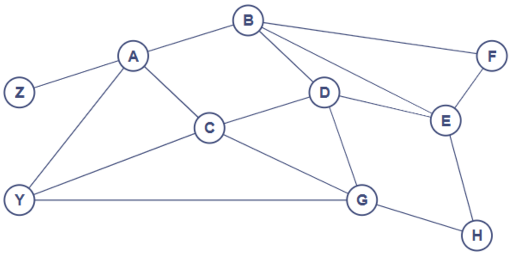
Exercise 3: (05 points)

Below is the plan of a museum:



The small black rectangles represent the doors. Visitors start from the entrance, visit the museum, and must finish the visit at the shop.

1. Represent the situation with a graph. We can represent Entrance by **Y** and Shop by **Z** (1 pts)

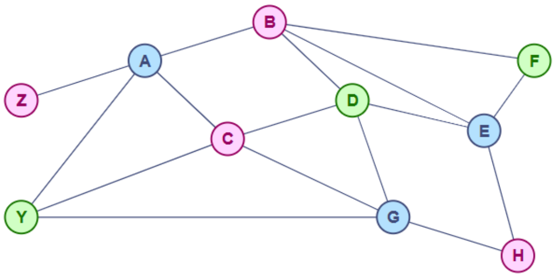


2. What is the degree of vertices of this graph (1 pt)

Vertex	A	B	C	D	E	F	G	H	Y	Z
Degree	4	4	4	4	4	2	4	2	3	1

3. Why is it possible to find a chain where visitors pass through each door exactly once?
The graph is connected and has two vertices of odd degree (Y and Z); therefore, it admits an Eulerian chain. (1 pt)
4. Give an example of such a chain.
Y-C-G-Y-A-B-F-E-B-D-G-H-E-D-C-A-Z (1 pt)

5. How can the rooms, including the entrance and the shop, be colored using a minimum number of colors so that two rooms connected by a door have different colors? (1 pt)

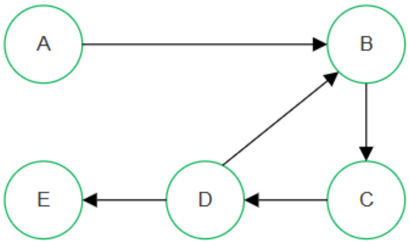


Exercise 4 : (07 points)
Let the graph G be as follows:

- Vertices: V={A,B,C,D,E}
- Edges: A→B, B→C, C→D, D→B, D→E

1. Study the properties of the graph (symmetry, connectivity and strong connectivity)

Symmetry	It is not symmetric because there is an arc from A to B and there is not an arc from B to A (0.25 pts)
Connectivity	It is a connected graph because there is a chain between each pair of vertices (0.25 pts)
Strong Connectivity	It is not strongly connected because by applying the Marking Algorithm we found 3 strongly connected component: {A}, {B, C, D} and {E} (0.5 pts)



2. Represent this graph as an adjacency matrix (M) and indicate the successors of each vertex
3. Calculate M^2. What does the resulting value represent?

	A	B	C	D	E
A	0	1	0	0	0
B	0	0	1	0	0
C	0	0	0	1	0
D	0	1	0	0	1
E	0	0	0	0	0

The Adjacency Matrix **M** (0.5 pts)

	A	B	C	D	E
A	0	0	1	0	0
B	0	0	0	1	0
C	0	1	0	0	1
D	0	0	1	0	0
E	0	0	0	0	0

The Matrix $\mathbf{M^2}$ (0.5 pts)

Vertex	A	B	C	D	E
Successors	B	C	D	B, E	/

(0.5 pts)

The Matrix M2 represents the set of all paths of length 2 in the graph (0.5 pts)

4. Find the longest simple path of G, and indicate its cardinality

The longest simple path of G is A→B →C →D →E. Its cardinality is 4 (1 pt)

5. Find the longest simple chain of G, and indicate its cardinality

The longest simple chain of G is A---B ---C ---D --- E. Its cardinality is 4 (1 pt)

6. Test for the existence of circuits and cycles in this graph G

The graph G contains a strongly connected component which is {B, C, D}. So, we can conclude the existence of circuits, and therefore a cycle (1 pt)

7. Is this graph Hamiltonian? If yes, provide the corresponding path.

This graph is not Hamiltonian, because it does not contain a hamiltonian circuit that visits each vertex exactly once. (1 pt)