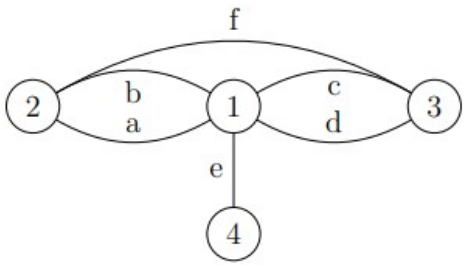
Exercise1:

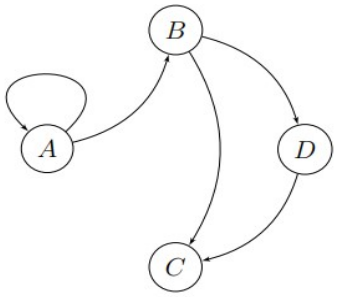
Consider the graph G = (X, U) shown below:



1) Provide the following:
   a) The sets **X** and **U**.
   b) The order of **G**.
   c) The degree of each vertex in **X**.
2) What changes would need to be made to **G** to obtain a simple graph?
3) Give the adjacency matrix of G.

Exercise 2:

Consider the directed graph G shown below:



1. State the out-degree (d^+) of each vertex in graph G.
2. Provide the adjacency matrix of G.
3. Find a cycle of length 3 in graph G.

Exercise 3:

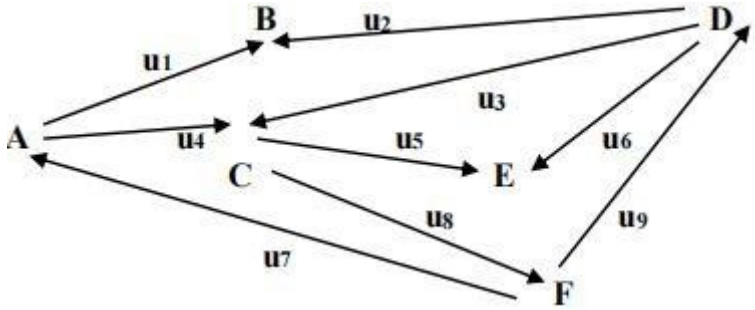
Consider an undirected graph G with the following matrix A as its adjacency matrix:

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

1. Give the order of G.
2. Provide the degree of each vertex in X.
3. Give the connectivity number of G (noted as p).
4. Describe the minimal modification to be made to A to obtain a new graph G1 that is not simple.
5. Describe the minimal modification to be made to A to obtain a new graph G2 that has a connectivity number greater than p.
6. Provide the adjacency matrix of G', the complement of graph G.

Exercise 4:

1) Let G = (X,U) be a directed graph with X = { 1,2,3,4,5,6 } and the arcs:u1=(1,2);u2=(2,3);u3=(4,3); u4=(6,4);u5=(6,5);u6=(5,1);u7=(2.5);u8=(4,2);u9=(3.5)andu10=(5.4).
   a) Plot the graph G.
   b) Find the degree of each vertex.
   c) Give an example of a circuit.
2) Let the graph G' as bellow:

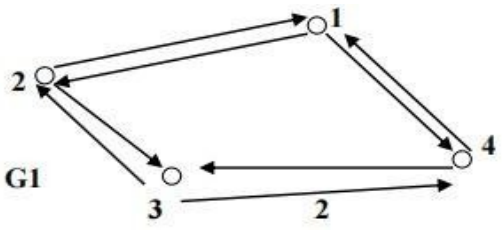


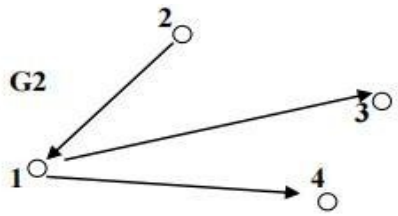
a) Is the path: {(AC),(CF),(FD),(DC),(CE)} an elementary path?
b) Is the path: {(AC),(CF),(FA),(AC),(CE)} a simple path?

3) From the graph G':
   a) Draw its equivalent undirected graph G"=(X,U) with X={A,B,C,D,E,F} and edges u1; u2; u3; u4; u5; u6; u7; u8 and u9.
   b) Find the degree of each vertex.
   c) Give an example of a cycle.
   d) Is the chain:{u4 u5 u6 u9 u8 u4}an elementary chain?
   e) Is the chain:{u1 u2 u3 u4 u7}a simple chain?

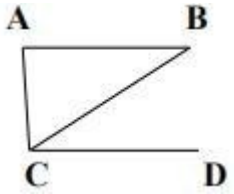
Exercise 5:

1) What is the nature of each graph?

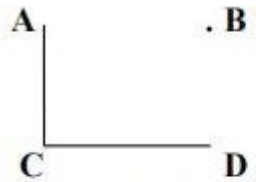




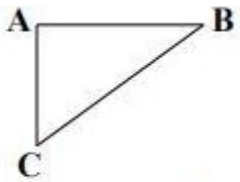
2) Let the undirected gr aph G be:



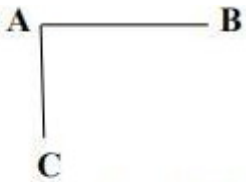
What is the nature of each of the following graphs extracted from G?



Graphe1



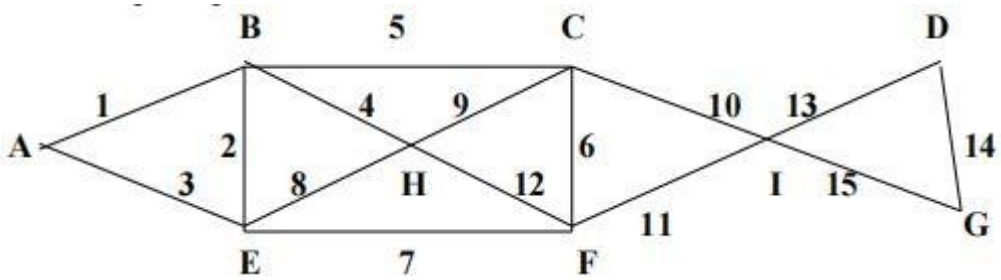
Graphe2



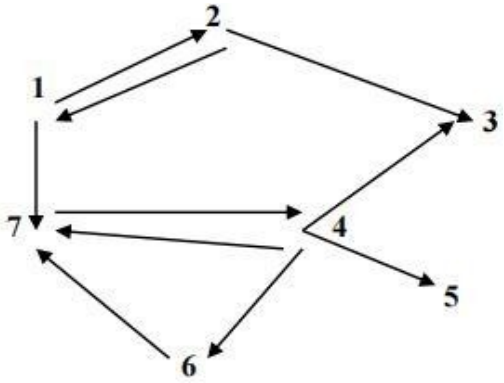
Graphe3

Exercise 6:

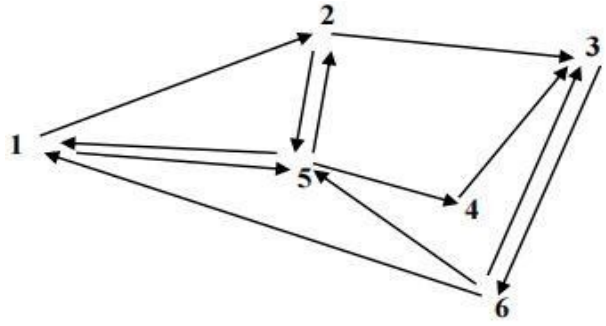
1. Does the following graph admit an Eulerian cycle? If yes, say why and find one?



2. Does the following graph have a Hamiltonian path?



3. Find at least one path and one Hamiltonian circuit in the following graph G':



Exercise 7:

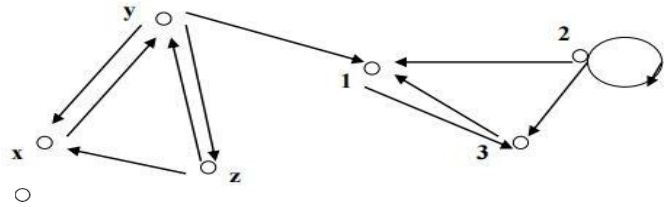
Let G be an undirected graph of order 10 having 22 edges. It has exactly 5 vertices of degree 3, the others are of degree 4 or 7. How many vertices of degree 4 are there?

Exercise 8:

What is the number of edges of a complete undirected graph G= (X, U) with |X|=n and |U|=m?

Exercise 9:

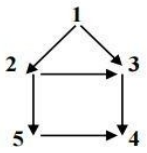
1. Is the following graph a strongly connected graph?



2. Find a reduced graph
3. Show that an undirected, loop-free graph with **n** number of vertices and having more than **(n-1)(n-2)/2 edges** is connected.

Exercise 10:

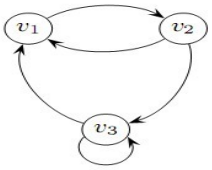
1. Propose three different data structures that can be used to represent a graph in a computer's memory.
2. Application to the following graph



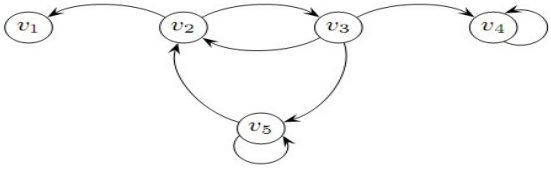
Exercise 11:(strong connectivity)

Determine the strongly connected components of the following directed graphs using a labeling algorithm:

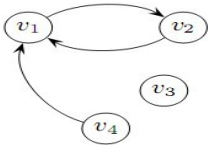
a)



b)



c)



d)