

LEVEL : 1st Year Basic Training

Tutorial Sheet
No. 1

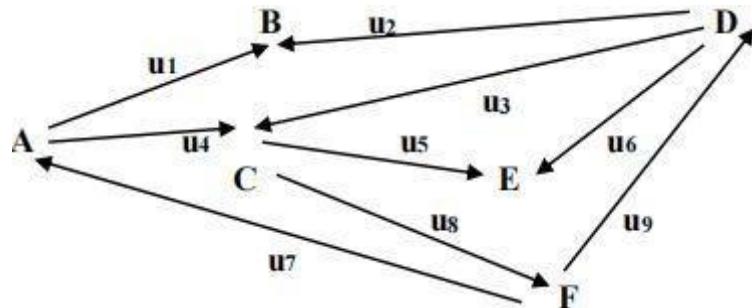
MODULE : Graph Theory

Exercise1:

- 1) Let $G = (X, U)$ be a directed graph with $X = \{1, 2, 3, 4, 5, 6\}$ and the arcs: $u_1 = (1, 2); u_2 = (2, 3); u_3 = (4, 3); u_4 = (6, 4); u_5 = (6, 5); u_6 = (5, 1); u_7 = (2, 5); u_8 = (4, 2); u_9 = (3, 5)$ and $u_{10} = (5, 4)$.

- a) Plot the graph G .
- b) Find the degree of each vertex.
- c) Give an example of a circuit.

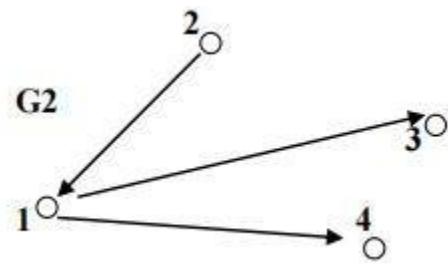
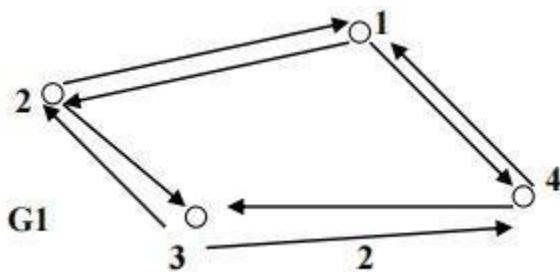
- 2) Let the graph G' as below:



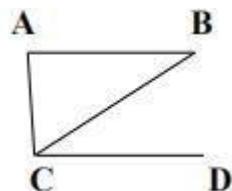
- a) Is the path: $\{(AC), (CF), (FD), (DC), (CE)\}$ an elementary path?
- b) Is the path: $\{(AC), (CF), (FA), (AC), (CE)\}$ a simple path?
- 3) From the graph G' :
 - a) Draw its equivalent undirected graph $G'' = (X, U)$ with $X = \{A, B, C, D, E, F\}$ and edges $u_1; u_2; u_3; u_4; u_5; u_6; u_7; u_8$ and u_9 .
 - b) Find the degree of each vertex.
 - c) Give an example of a cycle.
 - d) Is the chain: $\{u_4 \ u_5 \ u_6 \ u_9 \ u_8 \ u_4\}$ an elementary chain?
 - e) Is the chain: $\{u_1 \ u_2 \ u_3 \ u_4 \ u_7\}$ a simple chain?

Exercise2:

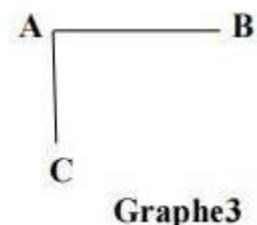
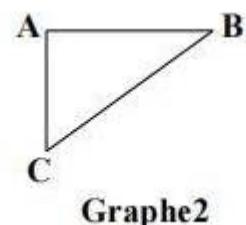
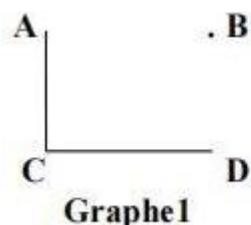
- 1) What is the nature of each graph?



2) Let the undirected graph G be:

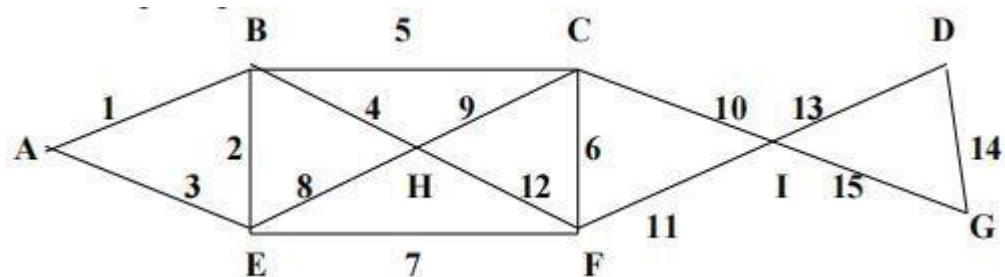


What is the nature of each of the following graphs extracted from G?

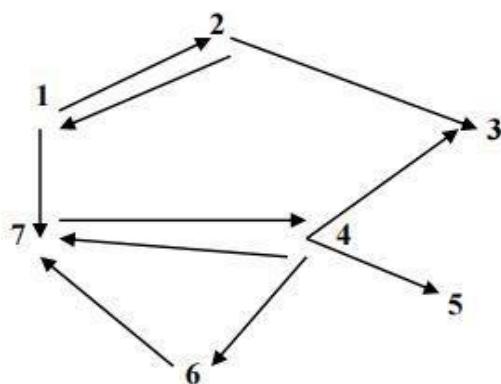


Exercise3:

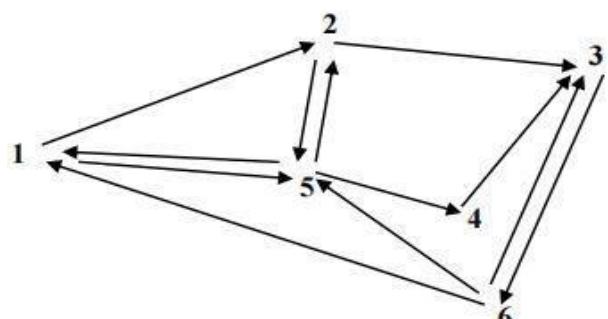
1. Does the following graph admit an Eulerian cycle? If yes, say why and find one?



2. Does the following graph have a Hamiltonian path?



3. Find at least one path and one Hamiltonian circuit in the following graph G':



Exercise4:

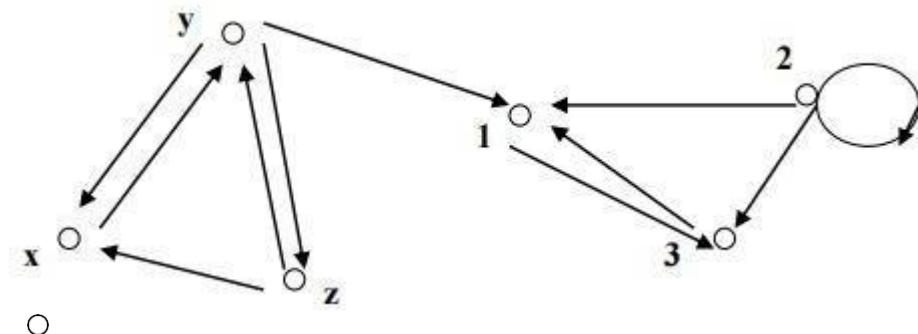
Let G be an undirected graph of order 10 having 22 edges. It has exactly 5 vertices of degree 3, the others are of degree 4 or 7. How many vertices of degree 4 are there?

Exercise5:

1. What is the number of edges of a complete undirected graph $G = (X, U)$ with $|X|=n$ and $|U|=m$?

Exercise6:

1. Is the following graph a strongly connected graph?



2. Find a reduced graph

3. Show that an undirected, loop-free graph with n number of vertices and having more than $(n-1)(n-2)/2$ edges is connected.

Exercise7:

1. Propose three different data structures that can be used to represent a graph in a computer's memory.
2. Application to the following graph

