

Algorithms and Static data structures

Series of exercises number 02

October 2024

Exercise 1: Sum of digits of N

Write an algorithm that computes the sum of the digits of a given number N.

Example: The sum of the digits of 4321 is $4+3+2+1=10$.

Exercise 2: Divisors of a Number

Write an algorithm that finds all divisors of a given number N.

Example: The divisors of 28 are 1, 2, 4, 7, 14, 28.

Exercise 3: Sum of the first N odd numbers

Write an algorithm to compute the sum of the first N odd numbers.

Example: The sum of the first 4 odd numbers is $1+3+5+7=16$

Exercise 4: Factorial

Write an algorithm to compute the factorial of a given number N, defined as $N! = N \times (N-1) \times \dots \times 1$

Example: The factorial of 5 is $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Exercise 5: Prime number

Knowing that a prime number is a number that has no divisors other than 1 and itself, construct a solution that allows us to determine if a number is prime or not.

Exercise 6: Perfect number

A perfect number is a number that is equal to the sum of all its divisors except itself. Find all the perfect numbers between 1 and N.

Example of a perfect number: 6

Exercise 7: Convert an integer to binary

How to convert an integer to binary. Example: 29 in base 10 gives 11101 in base 2.

Example: $29_{10} = 11101_2$

Exercise 8: Armstrong Number

An Armstrong number (or narcissistic number) is a number that is equal to the sum of its digits each raised to the power of the number of digits.

Write an algorithm to determine if a given number is an Armstrong number.

Example: 153 is an Armstrong number because $1^3 + 5^3 + 3^3 = 153$.

Exercise 9: Palindrome Number

Write an algorithm to check if a given integer N is a palindrome (it reads the same forwards and backwards).

Example: 121 is a palindrome, but 123 is not.

Exercise 10: Prime Numbers Up to N

Write an algorithm to find all prime numbers less than or equal to N.

Example: For $N=30$, the prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Exercise 11: Greatest Common Divisor (GCD)

Write an algorithm to find the GCD of two numbers A and B.

Example: $\text{GCD}(48, 18) = 6$.

Exercise 12: Fibonacci Number

Write an algorithm to check if a given number N is a Fibonacci number.

Hint: A number is a Fibonacci number if one or both of $5N^2 + 45N^2 + 45N^2 + 4$ or $5N^2 - 45N^2 - 45N^2 - 4$ is a perfect square.

Example: 21 is a Fibonacci number, but 22 is not.

Exercise 13: Co-prime Numbers

Write an algorithm to check if two numbers A and B are co-prime (i.e., their greatest common divisor (GCD) is 1).

Example: 8 and 15 are co-prime, but 12 and 15 are not.