2025/2026 | Basic Training Cycle

Lecturer: Pr. B, CHAOUCHI | **Analysis 1 - Tutorial 1-Part 1** | Real numbers

Only a few exercises will be covered during the tutorial session.

Exercise 1

Using the axiomatic properties of the real numbers, prove that

1. In the set $\mathbb{R}$ there exists a **unique** additive identity element;
2. In the set $\mathbb{R}$ every element has a **unique** additive inverse;
3. For given real numbers a and b the equation $a+x=b$ has a **unique** solution in $\mathbb{R}$

Exercise 2

Using the axiomatic properties of the real numbers and the previous exercise prove that for evry $x, y \in \mathbb{R}$ it holds

1. $x \cdot 0 = 0$
2. $-x = (-1).x$
3. $-(-x) = x$
4. $(-x).(-y) = xy$
5. $(-x).y = -xy$
6. $x \cdot (-y) = -(xy) = -(x)y$

Exercise 3

Solve in $\mathbb{R}$ the equation for the unknown x

1. $x^3+x^2+x = -\frac{1}{3}$
2. $\sqrt{6-x}+\sqrt{3-x} = \sqrt{5+x}+\sqrt{4-3x}$
3. $4\sqrt[3]{x}+5\sqrt[4]{x} = 9$

Exercise 4

Solve in $\mathbb{R}$ the equation for the unknown x

1. $(x-7)(x-5)(x+4)(x+6) = 608$
2. $\sqrt[4]{(19-x)(x-2)}+\sqrt{19-x}+\sqrt{x-2} = 7$
3. $2\sqrt[4]{x}+3\sqrt[3]{x} \geq \sqrt{x}$

Exercise 5

Solve in $\mathbb{R}$ the system of equations with unknowns x and y

$$\begin{cases} x^2+xy+y=3 \\ y^2+xy+x=-1 \end{cases}$$

Exercise 6

Let $x, y \in \mathbb{Q}^+$ such that $\sqrt{x}$ and $\sqrt{y}$ be irrational numbers. Prove that

$$\sqrt{x}+\sqrt{y}$$

is irrational

Exercise 7

Let

$$A = \frac{\sqrt[3]{54\sqrt{3}+41\sqrt{5}}}{\sqrt{3}} + \frac{\sqrt[3]{54\sqrt{3}-41\sqrt{5}}}{\sqrt{3}}$$

Show that $A \in \mathbb{Z}$ and find its value

Exercise 8

Let n be a nonzero natural number such that it is not the square of any integer. Show that

$$\sqrt{n} \notin \mathbb{Q}.$$

and deduce that

$$\sqrt{2}+\sqrt{3} \notin \mathbb{Q}.$$

Exercise 9

Let a, b be real positive numbers. Show that

$$a^2+b^2+c^2 \geq ab+bc+ca.$$

Then

$$(a+b+c)^2 \geq 3(ab+bc+ca).$$

Exercise 10

Let a, b be real nonnegative numbers. Show that

$$8\left(a^4+b^4\right) \geq (a+b)^4.$$

Exercise 11

Let M_a, M_g, M_h denote the arithmetic, geometric, harmonic means of n positive real numbers $x_1, x_2, \ldots, x_n$, respectively, where

$$M_a = \frac{\sum_{i=1}^{n} x_i}{n},\ M_g = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}},\ M_h = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

Show that

$$M_h \leq M_g \leq M_a.$$

Exercise 12

Using the mathematical induction principle, prove the following formulas for $n \in \mathbb{N}$

1. $\sum\limits_{k=1}^{n} k = \frac{n(n+1)}{2}$
2. $\sum\limits_{k=1}^{n} (2k-1) = n^2$
3. $\sum\limits_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum\limits_{k=1}^{n} (2k-1)^2 = \frac{n\left(4n^2-1\right)}{3}$
5. $\sum\limits_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$
6. $\sum\limits_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$

Exercise 13

Solve the following equations and inequations

$\boxed{1}$ $|x+2| \leq |2x+3|+1$

$\boxed{2}$ $|x-1|-|x+2| = 1$

$\boxed{3}$ $|3x+2| > 4|x-1|$

$\boxed{4}$ $\frac{|5x+2|}{|2x-3|} \geq 2, x \neq \frac{3}{2}$

Exercise 14

Show that for all x, y in $\mathbb{R}$

$\boxed{1}$ $|x+y| \leq |x|+|y|$

$\boxed{2}$ $|x-y| \geq ||x|-|y||$

$\boxed{3}$ $\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$