Analysis 1 - W.S 4

Exercise 1

1. Solve in $]0,+\infty[$:

$$\log_2(x)+\log_4(x)+\log_8(x)=\frac{11}{2}$$

2. Solve in $\mathbb{R}$

$$\cos^{11}(x)-\sin^{11}(x)=1$$

3. Solve in $\mathbb{R}$:

$$3^x+4^x=5^x$$

4. Solve in $\mathbb{R}^2$:

$$\begin{cases}\sin(x+y)=2x\\ \sin(x-y)=2y\end{cases}$$

5. Solve in $\mathbb{R}^2$:

$$\begin{cases}x+e^x=y+e^y\\ x^2+xy+y^2=27\end{cases}$$

Exercise 2

1. Show that for all $x\in\left[0,\frac{\pi}{2}\right]$:

$$\sin x\leq\frac{1}{2}\sqrt{\pi x}$$

2. Deduce that for all $x\in[0,\pi]$:

$$\sin x\leq\frac{\pi}{4}\sqrt{x(\pi-x)}$$

Exercise 3

Let $a\in\left[0,\frac{\pi}{2}\right]$, detremine

$$\lim_{n\to+\infty}\prod_{k=1}^{n}\cos\left(\frac{ka}{n}\right)$$

Exercise 4

Study and Sketch the graph of the function

$$\ln\left(x^2-\sqrt{x^2-1}\right)$$

Exercise 5

The aim of this exercise is only to show that the function $\arccos(x)$, this function is not even in an orthonormal cartesian coordinate system whose origin is $(0,0)$, this assertion is not true if we make a translation towards the point $\left(0, \dfrac{\pi}{2}\right)$

$$\forall x \in [-1,1] : \arccos(x) + \arccos(-x) = \pi$$

Exercise 6

$\boxed{1}$ Show that for all $x, y \in \mathbb{R}$ such that $0 < x < y$:

$$\frac{x-y}{\ln y - \ln x} < \frac{x+y}{2}$$

$\boxed{2}$ Deduce that for all $n \in \mathbb{N}^*$:

$$\sum_{k=1}^{n} \frac{k}{\ln\left(1+\dfrac{1}{k}\right)} < \frac{n(n+1)(4n+5)}{12}$$

Exercise 7

Show that for all $x > 0$:

$$\ln\left(1+\frac{1}{x}\right) \leq \frac{1}{\sqrt{x(x+1)}}$$

Exercise 8

$\boxed{1}$ Study and Sketch the graph of the function

$$\varphi(x) = \frac{x}{1+x^2}$$

$\boxed{2}$ The same qustions for the function

$$f(x) = \tan(\pi\varphi(x))$$

Exercise 9

$\boxed{1}$ Study and Sketch the graph of the function

$$f(x) = \arcsin\left(2x^2-1\right)$$

$\boxed{2}$ Study and Sketch the graph of the function

$$f(x) = \arctan\left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right)$$

$\boxed{3}$ Study and Sketch the graph of the function

$$f(x) = \arcsin\left(\frac{1-x^2}{1+x^2}\right)$$

Exercise 10

Show that

$\boxed{1}$ $\arctan x + \arctan 2x = \frac{\pi}{4}$

$\boxed{2}$ $2\arctan x = \arctan\left(\frac{2x}{1-x^2}\right) + \pi sgn(x)$

$\boxed{3}$ $\frac{\pi}{4} + \arctan x = \arctan\left(\frac{1+x}{1-x}\right)$

Exercise 11

$\boxed{1}$ Show that for $a, b \in [0,1] : \arctan a + \arctan b = \arctan\left(\frac{a+b}{1-ab}\right)$

$\boxed{2}$ Show that $1 + \cosh x + \cosh 2x + \cosh 3x + ... + \cosh nx = \frac{1}{2} + \frac{\cosh nx - \cosh(n+1)x}{2(1-\cosh x)}$

Exercise 12

Consider the function

$$f(x) = \frac{x}{2} - \arcsin\left(\sqrt{\frac{1+\sin x}{2}}\right)$$

$\boxed{1}$ Find the domain of definition of f denoted by D_f.

$\boxed{2}$ Show that

$$\forall x \in D_f : f(x+2\pi) = f(x) + \pi$$

$\boxed{3}$ Show that

$$\forall x \in D_f : f(x) + f(-x) = -\frac{\pi}{2}$$

$\boxed{4}$ Simpify the expression of f and draw its curve

Exercise 13

Let

$$f(x) = \arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

$\boxed{1}$ Find the domain of definition of this function and study its differentiability

$\boxed{2}$ Simplify the expression of this function

Exercise 14

Show that

$$\arctan\left(2\sqrt{2}\right) + 2\arctan\left(\sqrt{2}\right) = \pi.$$

Exercise 15

Compute

$$\sin\left(\frac{1}{2}\arcsin\left(\frac{3}{4}\right)\right)$$