

Exercise 1 : (7.5 Pts)

1) Perform the following conversions: 5.5 pts (0.25 pts for each correct answer)

Base =10	Base =2	Base =8	Base =16
39,875	100111.111	47.7	27.E
$16^2 + 2^5 + 2^3 + 16^{-1}$	100101000.0001	450.04	128.1
53.875	110101.111	65,7	35.E
61.25	00111101.0100	75.2	3D,4

$$E6A_{(16)} = 111001101010_{(2)} = 100101011111_{(Gray)}$$

$$1100011_{(Gray)} = 1000010_{(2)} = 66_{(10)}$$

Number	Base =2	BCD	Ecess-3
126 ₍₈₎	1010110	1000 0110	1011 1001
31 ₍₁₆₎	110001	0100 1001	0111 1100

2) Perform the following operation in BCD: 126₍₈₎ + 31₍₁₆₎

BCD

86 1000 0110

+ 49 + 0100 1001

 1100 1111 (1pts)

We have :

1111 > 1001

1100 > 1001

So, we must add 0110 to 1111 and to 1100. (0.5pts)

¹ 110¹0 1111

+ 011 0 0110

Final resut (0001 001 1 0101)_{BCD} (0.5pts)

(1 3 5)₁₀

Exercise 2 : (4.5 Pts)

a) Find the codes corresponding to the word "PNG" according to the above table: (0.25 pts for each correct answer)

The word	Code		
	Base =10	Base =16	Base =2
PNG	80 78 71	50 4E 47	1000000 01111000 01110001

b) Determine the Decimal, Sign and Magnitude, 1's complement, and 2's complement values for the following cases (using 9 bits): **(0.25 pts for each correct answer)**

Decimal	Sign and magnitude	1's complement	2's complement
+25	000011001	000011001	000011001
-40	100101000	111010111	111011000
-26	100011010	111100101	111100110

c) Perform the following operations using 7 bits in 2's complement, then provide the results in decimal:

- $-2D_{(16)} + 23_{(8)}$
- $+45_{(8)} + 2E_{(16)}$

$-2D_{(16)} = 1010011_{(C2)}$ (0.25 pts)	(sur 7 bits)	$+45_{(8)} = 0100101_{(C2)}$ (0.25 pts)
$+23_{(8)} = 0010011_{(C2)}$ (0.25 pts)		$+2E_{(16)} = 0101110_{(C2)}$ (0.25 pts)

$-2D_{(16)} \quad 1^1 010^1 0^1 11$ $+23_{(8)} \quad +0 \ 0 \ 10 \ 0 \ 11$ $\quad \quad 1 \ 1 \ 00 \ 110_{(C2)} = -011010_{(2)}$ (0.25 pts) $\quad \quad = -26_{(10)}$ (0.25 pts)	$+45_{(8)} \quad 0100101$ $+2E_{(16)} \quad +0101110$ $\quad \quad 1010011$ (0.25 pts) Incorrect result (Overflow) (0.25 pts)
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Exercise 3 : (4 pts)

1) Provide the ANSI/IEEE 754 representation in single precision (32 bits) for the following numbers:

$$-39.875 \times 2^{-107}_{(10)} = -100111.111_{(2)} \times 2^{-107} = -1.00111111_{(2)} \times 2^5 \times 2^{-107} = -1.00111111_{(2)} \times 2^{-102} \text{ (0.5 pts)}$$

The normalised number is $-1.00111111_{(2)} \times 2^{-102}$

- $S = 1$
- $M = 00111111$
- $\text{Exp} = -102 \Rightarrow \text{BE} = \text{Exp} + 127 = -102 + 127 = 25_{(10)} = 11001_{(2)}$

1	00011001	001111110000000000000000	(0.5 pts)
S	BE	M	

$$+53.25 \times 2^{-133}_{(10)} = +110101.01_{(2)} \times 2^{-133} = +1.1010101_{(2)} \times 2^5 \times 2^{-133} = +1.1010101_{(2)} \times 2^{-128} \text{ (0.5 pts)}$$

Impossible to represent $+53.25 \times 2^{-133}_{(10)}$ in single precision (32 bits) (0.5 pts)

OR:

The denormalised number is $+0.011010101 \times 2^{-126}$

- $S = 0$
- $M = 011010101$
- $\text{BE} = 0$

0	00000000	011010101000000000000000	(0.5 pts)
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1) Express the values of X and Y, corresponding to the following ANSI/IEEE 754 representations, in the form $\pm M \cdot 2^{E_r}$ (where M and E_r are decimals):

X = **10010011111000000000000000000000**₍₂₎

1	00100111	110000000000000000000000
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0 < BE < 255 => The number X is Normalised

- **S = 1 => X < 0**
- **BE = 00100111₍₂₎ = 39₍₁₀₎ => Exp = BE - 127 = 39 - 127 = -88₍₁₀₎**
- **M = 1.mantissa = 1.11₍₂₎ = 1.75₍₁₀₎**

Thus, X = -1.11₍₂₎ $\times 2^{-88}$ = -1.75₍₁₀₎ $\times 2^{-88}$ (1 pt)

Y = **10000000010000000000000000000000**₍₂₎

1	00000000	100000000000000000000000
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BE = 0 and mantissa $\neq 0$ => The number Y is Denormalised

- **S = 1 => Y < 0 (0.25 pts)**
- **BE = 0 (0.25 pts)**
- **M = 0.mantissa = 0.1₍₂₎ = 0.5₍₁₀₎ (0.25 pts)**

Thus, Y = -0.1₍₂₎ $\times 2^{-126}$ = -0.5₍₁₀₎ $\times 2^{-126}$ (0.25 pts)

Exercise 4 : (4 pts)

1) Consider the following Boolean function: $F(X,Y,Z) = X \cdot Z + X (\bar{Z} \cdot Y + Z \cdot \bar{Y})$

1. the truth table for F (1pt)

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

2. The two canonical form of F

➤ **The 1st** canonical form: Sum of Products (SOP)

$$F(X,Y,Z) = X\bar{Y}Z + XY\bar{Z} + XYZ = \Sigma(5, 6, 7) \text{ (0.5 pts)}$$

➤ **The 2nd** canonical form: Product of Sums (POS)

$$F(X,Y,Z) = (X + Y + Z)(X + Y + \bar{Z})(X + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + Y + Z) = \Pi(0,1,2,3,4) \text{ (0.5 pts)}$$

3. The simplified expression of F using Boolean algebra is:

$$\begin{aligned} F(X,Y,Z) &= XZ + X(ZY + \bar{Z}Y) \\ &= XY + X\bar{Y}Z + XYZ \\ &= XY(1 + \bar{Z}) + X\bar{Y}Z \\ &= XY + X\bar{Y}Z \end{aligned}$$

$$= X(Y + \bar{Y}Z)$$

$$= X(Y + Z)$$

$$= XY + XZ \text{ (1 pt)}$$

1. the Logic-Diagram of the simplified **F** using the **NANDs** logical gates is below **(1 pts)**

