

Exercise 2: (2 points)

1. How many positive integers can be represented in binary using one byte?

The total number of positive integers that can be represented in binary using one byte is $2^8 - 1 = 255$ (1pts)

2. How many bits are necessary to represent in binary the natural numbers less than or equal to n ?

Let k be the number of bits needed to represent a natural number $\leq n$.

We have $2^k - 1 \geq n \Rightarrow 2^k \geq n + 1 \Rightarrow k \geq \log_2(n + 1) \Rightarrow k = \lceil \log_2(n + 1) \rceil$ (1pts)

Exercise 3: (03 points)

Consider the following two numbers encoded according to the IEEE 754 standard (32 bits) and represented in hexadecimal: 3EE00000 and 3D800000.

1. Compute their sum and give the result in IEEE 754 format and in decimal form.

$$3EE00000 = 00111110111000000000000000000000 = (+1, 110 \cdot 2^{-2})_2$$

$$3D800000 = 00111110110000000000000000000000 = (+1, 0 \cdot 2^{-4})_2$$

$$3EE00000 + 3D800000 = (+1, 110 \cdot 2^{-2})_2 + (+1, 0 \cdot 2^{-4})_2 = (+111, 0 \cdot 2^{-4})_2 + (+1, 0 \cdot 2^{-4})_2 = (+1000, 0 \cdot 2^{-4})_2 = (+1, 0 \cdot 2^{-1})_2$$

$$(+1, 0 \cdot 2^{-1})_2 = 00111111000000000000000000000000 = \mathbf{3F000000} \text{ (1pts)}$$

$$(+1, 0 \cdot 2^{-1})_2 = (0.1)_2 = \mathbf{(0.5)_{10}} \text{ (0.5pts)}$$

2. Same question for the numbers: C8800000 and C8000000.

$$C8800000 = 11001000100000000000000000000000 = (-1.0 \cdot 2^{+18})_2 = (-10.0 \cdot 2^{+17})_2$$

$$C8000000 = 11001000000000000000000000000000 = (-1.0 \cdot 2^{+17})_2$$

$$C8800000 + C8000000 = (-10.0 \cdot 2^{+17})_2 + (-1.0 \cdot 2^{+17})_2 = (-11.0 \cdot 2^{+17})_2 = (-1, 1 \cdot 2^{+18})_2$$

$$(-1, 1 \cdot 2^{+18})_2 = 11001000110000000000000000000000 = \mathbf{C8C00000} \text{ (1pts)}$$

$$(-1, 1 \cdot 2^{+18})_2 = (-1, 5 \cdot 2^{+18})_{10} = \mathbf{(-393216)_{10}} \text{ (0.5pts)}$$

Exercise 4: (06 points)

In this exercise, numbers are stored in 1-byte words, i.e., 8 bits.

1. Give the two's complement representation of the following signed integers: -13 and -127.

$$(-13)_{10} = (10001101)_{SM} = (11110010)_{1c} = (1111\ 0011)_{2c} \text{ (0.5pts)}$$

$$(-127)_{10} = (11111111)_{SM} = (10000000)_{1c} = (10000001)_{2c} \text{ (0.5pts)}$$

2. Compute the opposite of the following signed integers represented in 1 byte: 10011101 and 00110011.

$$(10011101)_{2s} = (10011100)_{1s} = (11100011)_{SM} = (-99)_{10} \text{ its opposite is } (+99)_{10} = (01100011)_{SM} \text{ (0.5pts)}$$

$$(00110011)_{SM} = (+51)_{10} \text{ its opposite is } (-51)_{10} = (10110011)_{SM} = (11001100)_{1c} = (11001101)_{SM} \text{ (0.5pts)}$$

3. Give the decimal representation of the following signed integers (in two's complement): 11001101 and 00001101.

$$(11001101)_{2s} = (11001100)_{1s} = (10110011)_{SM} = (-51)_{10} \text{ (0.5pts)}$$

$$(00001101)_{2s} = (00001101)_{SM} = (+13)_{10} \text{ (0.5pts)}$$

4. Convert to binary, then compute on 8 bits the following operation: $(-13) + 13$, $23 - 46$, and $127 + 2$.

$$(+13)_{10} = (00001101)_{SM}, (-13)_{10} = (10001101)_{SM} = (11110010)_{1c} = (11110011)_{2c}$$

$$(+13)_{10} + (-13)_{10} = (00001101)_{SM} + (11110011)_{2c} = (11110000)_{2c} = (00000000)_{SM} \text{ (1pts)}$$

$$(+23)_{10} = (00010111)_{SM}, (-46)_{10} = (10101110)_{SM} = (11010001)_{1c} = (11010010)_{2c}$$

$$(+23)_{10} + (-46)_{10} = (00010111)_{SM} + (11010010)_{2c} = (11101001)_{2c} = (11101000)_{1c} = (10010111)_{SM} = (-23)_{SM} \text{ (1pts)}$$

$$(+127)_{10} = (01111111)_{SM}, (+2)_{10} = (00000010)_{SM}$$

$$(+127)_{10} + (+2)_{10} = (01111111)_{SM} + (00000010)_{SM} = (10000001)_{SM} \text{ with overflow. Invalid result (1pts)}$$

Exercise 5 : (05 points)

Let X, Y, Z be input variables, and let F be defined as:

$$F(A, B, C) = \begin{cases} 1 & \text{if the majority of variables is 1} \\ 0 & \text{sinon} \end{cases}$$

1. Construct the truth table for $F(A, B, C)$.(1pts)

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

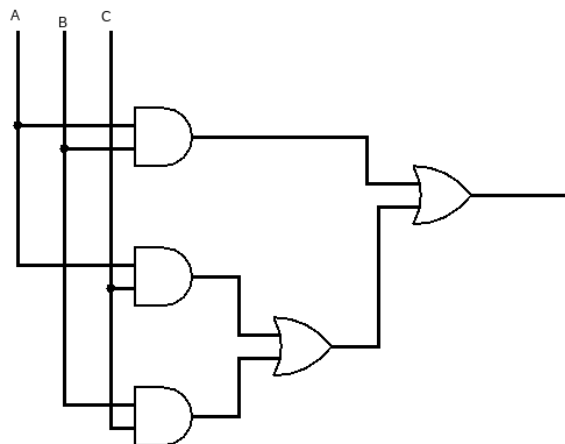
2. Simplify the function F using a Karnaugh map.

$$F = AB + AC + BC \dots (0.5pts)$$

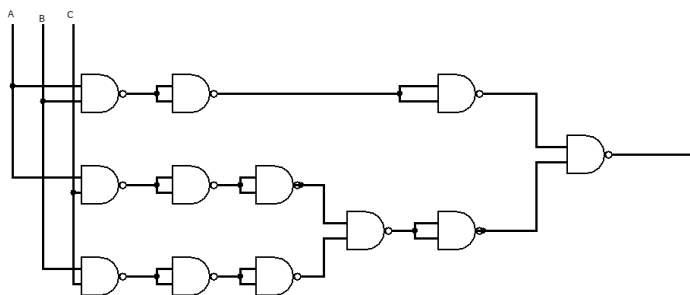
	C	0	1
AB			
00			
01			1
11	1		1
10			1

..(0.5pts)

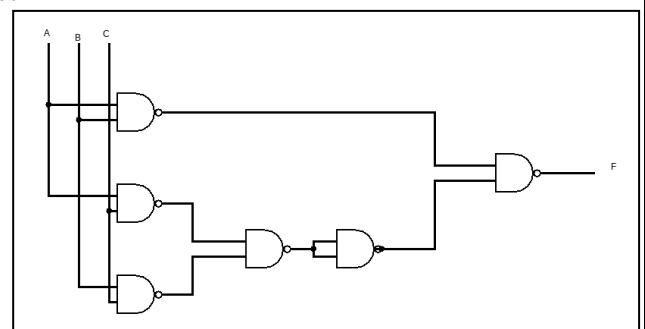
3. Draw the logical circuit implementing F (1pts)



4. Draw the logical circuit implementing F using NAND gates



(1pts)



(1pts)

