Chapter 1 Direct Current Circuits: Basic concepts

I. Introduction

What is electrokinetics?

Electrokinetics is a term applied to all the laws of physicochemical and mechanical phenomena involving the transport of charges, the action of charged particles, and the effects of applied electric potential in order to allow a desired movement (in one direction or slowly reversible) of electric charge carriers in conductors forming closed electrical circuits.

I.1. ELECTRIC CHARGES (symbol q, sometimes Q)

- Electric charge is a fundamental property of matter, noted **q or Q**, which allows objects to interact via electromagnetic fields, to attract or repel each other. It exists in two forms, **positive** (an excess of charges, such as protons) and **negative** (a deficit of charges, such as electrons). The **unit** of measurement for electric charge is **the coulomb (C).**
- In ordinary matter, there is a balance between positive and negative charges, known as electrical neutrality. One is called "**positive**" and is measured by a positive number, the other is called "negative" and is measured by a negative number.

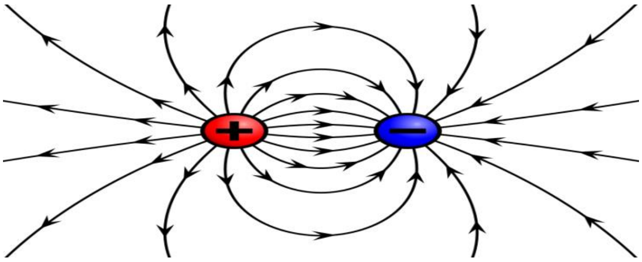


Fig 1: Electric field created by two charges of opposite signs.

Any charge is a multiple of the elementary charge (e) defined by:

$$e = 1{,}6 \cdot 10^{-19}\,(C)$$

Atoms are made up of charged particles, namely:

- ***Electrons***: (e^-) responsible for electrical conduction in metals:

$$charge: q_e = -e = -1{,}6 \cdot 10^{-19}\,(C)$$
$$mass: m_e = 9.1 \cdot 10^{-31} \quad (kg)$$

- ***Protons***: (H^+) :

$$charge: q_p = e = 1{,}6 \cdot 10^{-19}\,(C)$$
$$mass: m_p = 1.67 \cdot 10^{-24} \quad (kg)$$

II. ELECTROSTATICS

Electrostatics is the branch of physics that studies phenomena created by static electric charges, i.e. the study of charges in their resting state. ***Electrification*** is the phenomenon of the appearance of an electric charge or the appearance of quantities of electricity on a body.

Example: There is a simple experiment that anyone can do to feel an electrostatic force. All you have to do is rub a plastic ruler with a dry cloth and bring it close to small pieces of paper: this is electrification.

II.2. Electrical Field

To define the electric field at a point in space, we place a small positive test charge q there and look at the Coulomb force F exerted on it, due to the presence of the surrounding electric charges that create the electric field. The electric field at this point is defined as the force per unit charge:

$$\boxed{\vec{E} = \frac{\vec{F}}{q}\ , \qquad q > 0}$$

The electric field is therefore a vector quantity. The SI **unit of electric field** is the Newton per coulomb ($N.C^{-1}$). The test load must be small, so that it can be assumed that it does not itself disturb the surrounding electric field. At a distance r from a point charge Q, the electric field is given by **Coulomb's law**:

$$F = k\frac{qQ}{r^2} \qquad and \quad E = \frac{F}{q} = k\frac{Q}{r^2}$$

The principle of superposition that applies to Coulomb's law also applies to the electric field. To calculate the field created at a point by a set of n charges Q_i , we first determine separately the field E_1 due to Q_1 , the field E_2 due to Q_2 , etc. The resulting field E is equal to the vector sum of the individual fields E_i :

$$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n = \sum_{i=1}^{n} \vec{E}_i}$$

II.3 Electric Potential (*electric field potential*, potential drop, the **electrostatic potential**)

Electric potential, also known as electrostatic potential or voltage, is the amount of work required per unit of electric charge to move a charge from a reference point to a specific point within an electric field.

$$\boxed{\vec{E} = -\overrightarrow{grad}V = -\frac{dV}{dr} \ \Rightarrow \ V_{AB} = V_A - V_B = -\int_B^A \vec{E}.\vec{dl} = 0}$$

Electrical potential is in fact electrical potential energy per unit charge:

$$\boxed{V_E = U_E/q}$$

where : V_E = electrical potential (scalar) (J / C or V (volt)), U_E = electrical potential energy (scalar) (*J*: joule) and q = electrical charge (scalar) (C: coulomb).

The electrostatic potential energy, U_E, of one point charge q at position **r** in the presence of a point charge Q, taking an infinite separation between the charges as the reference position, is:

$$\boxed{U_E(r) = \frac{qQ}{4\pi\varepsilon_0 r}}$$

The potential at P due to the charge Q :

$$\boxed{V(r) = \frac{Q}{4\pi\varepsilon_0 r}}$$

with $k = \frac{1}{4\pi\varepsilon_0} = 9.10^9\ SI$ (The constant ε_0 is called the electrical vacuum permittivity (units: Farad/m).

III. Definition of a conductor in electrostatic equilibrium

So far, we've only been interested in electrical charges and their effects. What happens to a conducting body in which charges are free to move?

- In an insulator, charges remain where they were brought in (or removed).
- In a conductor, the charges are mobile (or free) and are therefore liable to move under the action of even a very weak electric field.

a- ***Definition of a conductor:*** A conductor is a material in which charges move when an electrostatic force is applied to them. In metals, only electrons are mobile. The network of positive charges has little mobility and can be considered fixed. In liquids and gases, ions are also mobile.

b- ***Definition of a conductor in electrostatic equilibrium***: the electrostatic equilibrium of a conductor is reached when no electric charge moves inside the conductor. This means that the distribution of charges remains constant over time.

III.1. Properties of an equilibrium conductor

1- The electrostatic field inside the conductor is zero $\overrightarrow{E} = 0$ because the charges q do not move (the charge q is at rest) and $\vec{F} = 0\ and\ E = \frac{F}{q} = 0$

2- The potential inside the conductor is constant; it's an equipotential volume because the field is zero.

The potential is constant, $\boxed{\overrightarrow{E} = -\overrightarrow{grad}V = -\frac{dV}{dr} = 0 \Longrightarrow V = Cste}$

III.2. Capacitance of a conductor in electrical equilibrium

The charge Q of an isolated conductor (away from any other conductor) is proportional to its potential V. If the potential becomes V_1 , then V_2 , then V_3 , the charge becomes q_1 , q_2 , q_3. Since charge-potential relationships are linear, we can write:

$$\boxed{\frac{q}{V} = \frac{q_1}{V_1} = \frac{q_2}{V_2} = \frac{q_3}{V_3} = C}$$

The proportionality coefficient C, independent of q and V, is called the conductor's capacity. It is measured in farads (F), if q is in coulomb and V in volt. This constant is called the intrinsic capacitance of the insulated conductor:

$$\boxed{Q = C \cdot V}$$

1 Farad corresponds to a charge of 1 Coulomb at a potential of 1 Volt. However, sub-multiples of the Farad, are used.

$$1\mu F\ (microfarad) = 10^{-6}F$$
$$1nF\ (nanofarad) = 10^{-9}F$$
$$1pF\ (picofarad) = 10^{-12}F$$

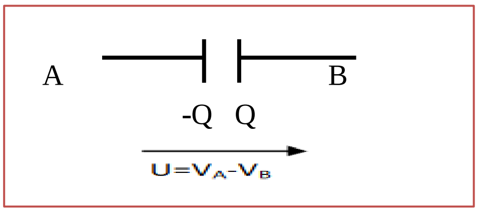
A conductor in electrostatic equilibrium carrying charge Q, let V be its potential and C its capacity. The Potential energy of a conductor in electrostatic equilibrium is written as:

$$\boxed{E_p = \frac{1}{2}Q.V = \frac{1}{2}.C.V^2 = \frac{1}{2}.\frac{Q^2}{C}\ \text{(Joule)}}$$

III.3. Capacitors

III.3.1 Definitions

A capacitor is a set of 2 conductors A and B in total influence. These two conductors are called the capacitor's armature. The capacitor's charge is that of its internal armature Q (Coulomb). V_A is the potential of the internal armature and V_B is the potential of the external armature, U= V_A- V_B is the potential difference of a capacitor (its unit is the volt). Its symbol in an electric circuit is:



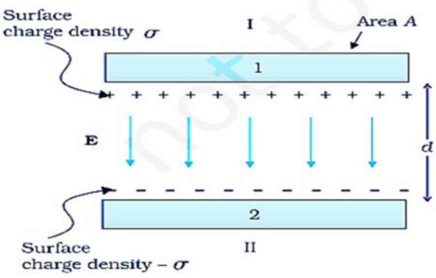
The charge of a capacitor is written as: Q= C.U

- The isolator (material placed between the armatures) increases the capacity of a capacitor.
- The capacitance of a capacitor depends on the geometry of the armatures.
- Capacity C is always positive.

Examples

- ***Planar capacitor***: Consider two uncharged flat conductors. The distance between these two planes is *d*. The surface charge density is σ.

$$\Delta V = V_+ - V_- = -\int \vec{E}d\vec{l} = +\int_0^d E\,dx = \frac{\sigma}{\epsilon_0}d \qquad \boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}}$$



This relationship remains approximately valid for two finite planes of surface area A and total charge Q, then :

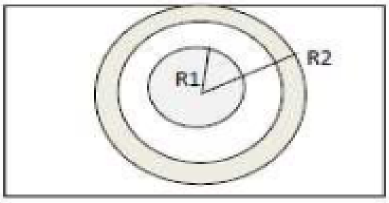
$$\boxed{\Delta V = \frac{Q.d}{\varepsilon_0.A} \quad and \quad C = \frac{\varepsilon_0.A}{d}}$$

- ***Spherical capacitor***

$$\Delta V = V_+ - V_- = -\int \vec{E}d\vec{l} = -\int_{R_2}^{R_1} E\,dr = +\int_{R_2}^{R_1} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dr$$

$$V_+ - V_- = -\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{Q}{4\pi\epsilon_0}\frac{(R_2 - R_1)}{R_1 R_2}$$

$$\boxed{C = 4\pi\epsilon_0 \frac{R_1 R_2}{(R_2 - R_1)}}$$



Remark: **The spherical conductor of radius R1, taken on its own, can be considered as an armature of a spherical capacitor whose second armature of radius R2 is rejected to infinity. By extending R2 to infinity in the previous expression, we find the capacitance of a spherical conductor spherical conductor $C = 4\pi\varepsilon_0 R1$**

- ***Cylindrical capacitor***

  Consider two coaxial conductive cylinders under total influence, one with charge +Q and the other with charge –Q

Step 1: Calculating the E field

The Gaussian surface is a cylinder of radius r and height h. Because of symmetry, the radial field is constant in the Gaussian surface. According to ***Gauss's Theorem*** and After calculation, we obtain: $\emptyset = E.2\pi rh = \frac{\sum Q_{ins}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$ $\Rightarrow$ $\boxed{E = \frac{Q}{2\pi rh\varepsilon_0}}$

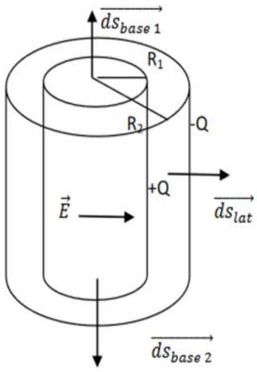
Step 2: Calculating the potential V

The potential: $\vec{E} = -\overrightarrow{grad}V \quad \Rightarrow \quad E = -\frac{dV}{dr} \quad so\ V = -\int Edr$

For $R_1 \leq r \leq R_2$ so $\Delta V = V_2 - V_1 = -\int_{R_1}^{R_2} \frac{Q}{2\pi rh\varepsilon_0} dr = -\frac{Q}{2\pi h\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r}$

$\Rightarrow\ V_2 - V_1 = -\frac{Q}{2\pi h\varepsilon_0}(lnR_2 - lnR_1)$, Finally :

$$\boxed{U = V_1 - V_2 = \frac{Q}{2\pi h\varepsilon_0}\left(ln\frac{R_2}{R_1}\right)}$$



Step 3: Capacity calculation

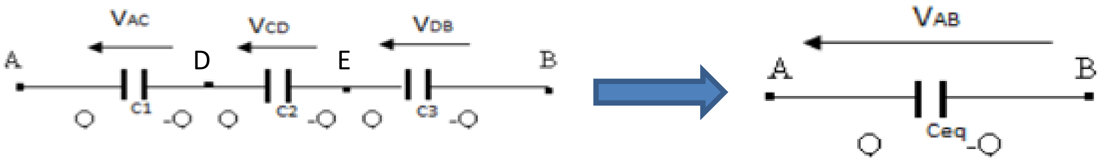
The charge is $Q = C.U \quad \Rightarrow \quad the\ capacity\ will\ be\ C = \frac{Q}{U}$ so,

$$\boxed{C = \frac{2\pi h\varepsilon_0}{\left(ln\frac{R_2}{R_1}\right)} = 2\pi h\varepsilon_0/\left(ln\frac{R_2}{R_1}\right)}$$

III.3.2. Association of capacitors

We can combine several capacitors of capacitance $C1, C2, \dots, Cn$ to obtain a system with some effective capacitance C. The effective capacitance depends on the way the individual capacitors are combined. Two simple possibilities are : **in series or parallel**, or a combination of both:

a- **Capacitors in series:** In a series connection of n capacitors, all capacitors store the same charge Q due to the total influence between the capacitor plates.



On the other hand, the voltage between all the capacitors is equal to the sum of the voltages of the individual capacitors (series connection).

$$V_A - V_B = (V_A - V_D) + (V_D - V_E) + (V_E - V_B) = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = \frac{Q}{C_{eq}}$$

Hence :

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For *n* capacitors in series:

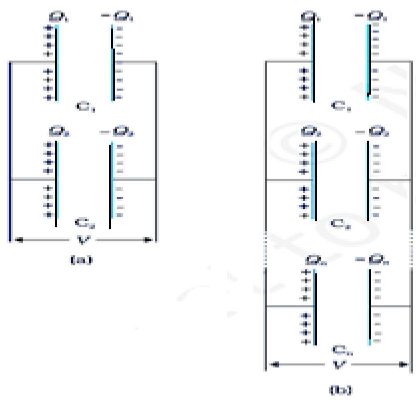
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \ldots \frac{1}{C_n} = \sum_{i=1}^{n} \frac{\mathbf{1}}{\boldsymbol{C_i}}}$$

b- ***Capacitors in parallel***

In a parallel connection of n capacitors, all the capacitors have the same voltage U and the total charge is the sum of the charges of the individual capacitors. For two capacitors in parallel, we have:

Figure (a) shows two capacitors arranged in parallel. In this case, the same potential difference is applied across both the capacitors. But the plate charges (±Q1) on capacitor 1 and the plate charges (±Q2) on the capacitor 2 are not necessarily the same: $Q_1 = C_1.V$, $Q_2 = C_2.V$, The equivalent capacitor is one with charge: $Q = Q_1 + Q_2$ and potential difference *V:* $\boxed{Q = C.V = C_1.V + C_2.V}$

The effective capacitance C is : $\boldsymbol{C = C_1 + C_2}$



Parallel combination of (a) two capacitors, (b) n capacitors

The general formula for effective capacitance C for parallel combination of n capacitors, Fig. (b), follows similarly,

$$\boxed{\begin{array}{c} Q = Q_1 + Q_2 + \cdots . + Q_n \\ i.e., \quad Q = C.V = C_1.V + C_2.V + \cdots . C_nV \\ which\ gives \quad : \boxed{C = C_1 + C_2 + \cdots . C_n} \end{array}}$$

IV. Energy stored in a capacitor

IV.1. Electrostatic energy

A capacitor can be charged by connecting a generator between its plates. This generator transfers charges from one armature to the other. The result is an increase in the capacitor's electrostatic potential energy. To calculate this energy, we assume that q is the charge of the capacitor at a certain instant during charging. At this instant, the potential difference between the two armatures is $\Delta V = q/C$. The work required to transfer an infinitesimal charge dq from the negative to the positive armature , when the charge of the armature (A) goes from the value q to the very close value $q + dq$ is given by:

$$dW = \Delta V dq = \frac{q}{C} dq$$

Total work W, stored in the capacitor, when the charge on the armature (A) changes from zero (capacitor discharged) to a value Q, is obtained by summing the elementary variations dW:

$$\boxed{W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}}$$

This work is stored in the form of electrical potential energy E_U . Since we have the relationship $Q = CV$, where V is the potential difference, this electrical potential energy can be expressed as a function of the potential difference $V_A - V_B$ between the armatures :

$$Q = C.(V_A - V_B) \quad we\ obtain \boxed{E_U = \frac{1}{2} C. \ (V_A - V_B)^2 = \frac{1}{2} Q \ \ (V_A - V_B)}$$

IV.2. Capacities with dielectric

In electromagnetism, a **dielectric** (or **dielectric medium**) is an electrical insulator that can be polarised by an applied electric field. A dielectric is a non-conductive material such as glass, rubber, etc.

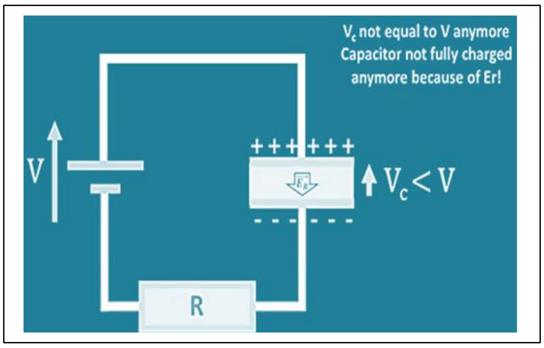
When a dielectric is inserted in all the space between the plates of a capacitor, the capacitance of the capacitor increases, multiplied by a factor κ called the dielectric constant (or relative permittivity) of the material:

$$C = \kappa . C_0 \qquad \kappa = \frac{\text{Capacitance with dielectric between the plates}}{\text{Capacitance with vacuum between the plates}}$$

Where C_0: is the capacity of the capacitor without the dielectric. κ varies from one material to another, for example **Parallel Plate Capacitor :** $\boldsymbol{C = \varepsilon_r \varepsilon_0 \frac{A}{d} = \varepsilon_r C_0 = \kappa C_0}$

Example:

Material	Dielectric constant	$E_{max}(10^6 V/m)$
Vacuum	1	
Air	1.00059	3
Bakelite	4.9	24
Mylar	3.2	7
Nylon	3.4	14
Porcelain	6	12
Pyrex glass	5.6	14
Paper	3.7	16



The table also gives the values of the maximum electric field E_{Max} that can be supported by the dielectric.

V. Electric current

V.1. Conductors and Insulator

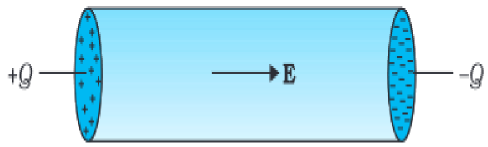
An electric current is an ordered movement of charged particles. This uniform motion of electrons is what we call electricity or electric current. A conductor is anybody, whether solid, liquid or, in some cases, a gas, that possesses mobile charged particles. These are electrons, but also positive or negative ions, i.e. atoms or molecules that have lost or captured one or more electrons.

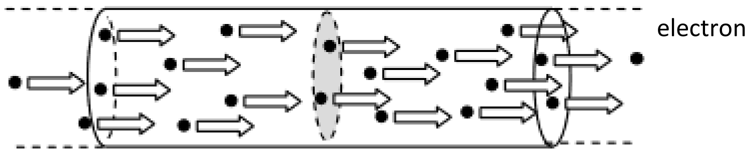
- ***Current in a metallic conductor***

Conductors are materials in which current flows easily, because free electrons are plentiful and can move easily from one place to another. Most metals are good conductors, with silver being the best conductor of all, followed by copper. Copper is generally used as a conductor in electrical wires.

In a metallic conductor, an electric current is a displacement of electrons.

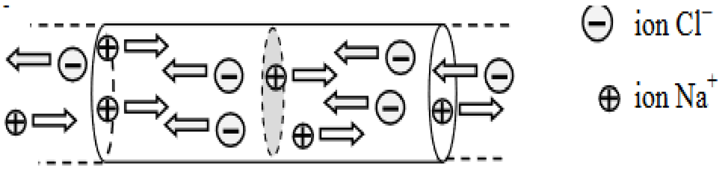
Charges +Q and –Q put at the ends of a metallic cylinder. The electrons will drift because of the electric field created to neutralise the charges. The current thus will stop after a while unless the charges +Q and –Q are continuously replenished.





- ***Current in a liquid***

In liquids, the electric current is made up of negative ions and positive ions always moving in opposite directions.



- **Insulating materials** are poor conductors, with few free electrons. They're most often used to prevent current flow (glass, plastic).

- ***Semiconductors*** tend to be insulators, but become conductors if the temperature is raised or if they contain impurities. Semiconductors are the basis of electronic circuits.

V.2. Intensity of Electric Current

Electric current is the ordered movement of electric charges under the effect of an electric force (obtained by the application of a potential difference, which gives rise to an electric field). This movement can be of different natures:

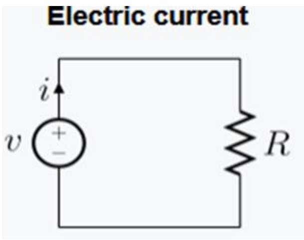
- Movement of electrons in a conductor (metals)
- Movement of ions in a liquid (electrolysis, human body, batteries)
- Movement of positive charges (holes) in semi-conductors

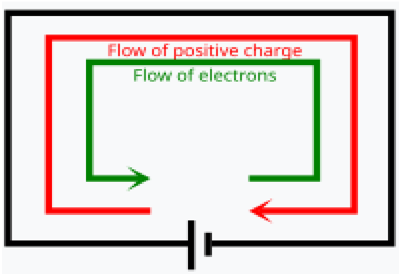
Electrons, ions and holes are charge carriers.

The intensity of an electric current, noted $i(t)$, represents the quantity of electric charge (expressed in coulomb) that has passed through a section of conductor over a period of $1s$. The international unit of electric current is the **Ampere,** noted [A]. **1 Ampere = 1 coulomb / second**

$$\boxed{i(t) = \frac{dq}{dt} \ (A) \ \ and \ q = \int_{t_0}^{t} i(t)dt}$$

The conventional direction of current, also known as *conventional current*, is arbitrarily defined as the direction in which *positive* charges flow.





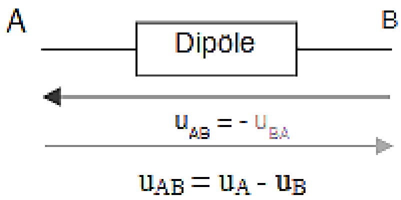
The electrons, the charge carriers in an electrical circuit, flow in the opposite direction of the conventional electric current.

There are essentially two types of current.

- ***Direct current***: Direct current (**DC**) is an electric current that is uni-directional, so the flow of charge is always in the same direction. As opposed to alternating current, ***the direction and amperage of direct currents do not change***. It is used in many household electronics and in all devices that use batteries. The intensity of DC is constant over time: $i(t) = I = constant$.
- ***Alternative Current:*** In alternating current (AC), the movement of electric charge periodically reverses direction. The variable intensity $i(t)$ of AC is identical to itself at regular intervals period T: $i(t) = i(t + T) = i(t + 2T) = \ldots\ldots = i(t + nT)$.

V-3. Elements of an electrical circuit : Electrical dipole

- An electrical circuit is a **closed loop** formed by conductors and electrical components, ***or dipoles***, through which electric current flows.
- A dipole is any component within this circuit with two terminals—***an input and an output***—that facilitates the passage of current and performs a specific function, either by supplying (**active**) or **consuming** (**passive**) energy.



There are 2 main categories of dipoles:

- ***Active dipoles*** that supply electrical energy: generators, batteries, photovoltaic cells, etc.
- ***Passive dipoles*** that consume electrical energy: resistors, inductors (coils), capacitors, etc,

VI. OHM's LAW

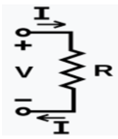
A basic law regarding flow of currents was discovered by Georg Simon Ohm in 1828(German physicist (1789-1854)). Ohm's Law describes the proportional relationship between three fundamental electrical quantities: voltage (V), current (I), and resistance (R). Imagine a conductor through which a current I is flowing and let V be the potential difference between the ends of the conductor. Then Ohm's law states that:

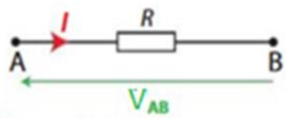
$V \propto I$ (*The electric current through a conductor between two points is directly proportional to the voltage across the two points*)

So $\boxed{\boldsymbol{V_{AB} = V_A - V_B = R.I}}$(*where the constant of proportionality* ***R is called the resistance*** *of the conductor*).

The SI unit of resistance is **Ohm**, and is denoted by the symbol $\boldsymbol{\Omega}$. The resistance R not only depends on the material of the conductor, but also on the dimensions of the conductor. At a given temperature, the resistance of a conductor with a uniform cross-section is written as (POUILLET law):

$$\boxed{R = \rho.\frac{l}{S}} \quad (\Omega)$$





Where, the constant of proportionality ρ depends on the material of the conductor but not on its dimensions. ρ is called ***resistivity (it's unit is* ohm.metre** $(\boldsymbol{\Omega.m})$) . l:the length (meter) and S: the cross sectional area. In an electrical conductor, the current density is proportional to the local electrostatic field. This relationship can be expressed as follows:

$$\boxed{\vec{j} = \gamma \vec{E}}$$

The proportionality coefficient γ is called the **conductivity of the medium**. Conductivity, γ, is the inverse of resistivity ρ: $\boxed{\gamma = \frac{1}{\rho}}$

Siemens per meter S/m or $\boldsymbol{\Omega^{-1}m^{-1}}$)

Similarly, we define the conductance G of an ohmic conductor, expressed in siemens (S):

$$G = \frac{1}{R} \quad or \quad i(t) = G.u(t)$$

Table of Resistivity and Conductivity at 20°C of some materials

Material	ρ (Ω·m) at 20 °C Resistivity	σ (S/m) at 20 °C Conductivity
Silver	1.59×10^{-8}	6.30×10^{7}
Copper	1.68×10^{-8}	5.96×10^{7}
Gold	2.44×10^{-8}	4.10×10^{7}
Aluminum	2.82×10^{-8}	3.5×10^{7}
Calcium	3.36×10^{-8}	2.98×10^{7}
Tungsten	5.60×10^{-8}	1.79×10^{7}
Zinc	5.90×10^{-8}	1.69×10^{7}
Nickel	6.99×10^{-8}	1.43×10^{7}
Lithium	9.28×10^{-8}	1.08×10^{7}
Iron	1.0×10^{-7}	1.00×10^{7}
Platinum	1.06×10^{-7}	9.43×10^{6}
Tin	1.09×10^{-7}	9.17×10^{6}
Carbon steel	(10^{10})	1.43×10^{-7}
Lead	2.2×10^{-7}	4.55×10^{6}
Titanium	4.20×10^{-7}	2.38×10^{6}
Stainless steel	6.9×10^{-7}	1.45×10^{6}

The resistance of a conductor varies with temperature (Mathiessen law). The resistivity of a material is found to be dependent on the temperature. The resistivity of a metallic conductor is approximately given by: $\boxed{\rho_T = \rho_0\big(1+\alpha(T-T_0)\big)}$

Where ρ_T is the resistivity at temperature T, ρ_0 is the same at a reference temperature T_0, α is called the temperature coefficient of resistivity at 20°C (e.g. copper 0.0038). For metals, α is positive

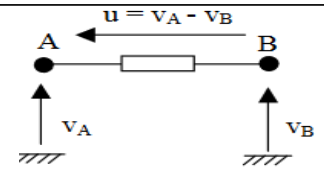
The following is a summary of the various rules of Ohm's law:

Ohms Law matrix Table

Ohm's law formulas		To Calculate: Voltage (V)	Current (I)	Resistance (R)	Power (P)
Given parameters	Power & Resistance	$V=\sqrt{P.R}$	$I=\sqrt{P/R}$	—	—
	Voltage & Power	—	$I=\frac{P}{V}$	$R=\frac{V^2}{P}$	—
	Voltage & Resistance	—	$I=\frac{V}{R}$		$P=\frac{V^2}{R}$
	Voltage & Current	—	—	$R=\frac{V}{I}$	$P=VI$
	Current & Power	$V=\frac{P}{I}$	—	$R=\frac{P}{I^2}$	—
	Current & Resistance	$V=I^*R$	—	—	$P=I^2R$

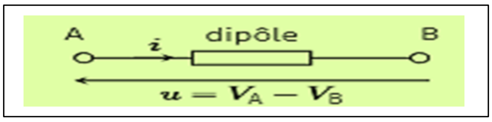
The potential difference

Potential difference (p.d.d.) or electric voltage is the difference in positive (+) and negative (-) charges between the 2 terminals of a generator. This value is expressed in VOLT (symbol V) and can be measured with a voltmeter.

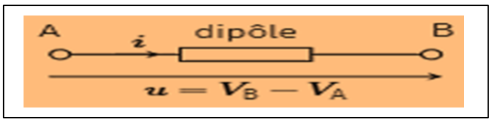


Current and voltage orientation convention:

Receiver convention:



Generator convention:



VII. Joule's Law (Joule effect formula)

In addition to current, voltage and resistance, a fourth parameter is very important in electricity: ***Electrical power***.

Electrical power: Power is a measure of the amount of work that can be delivered in a given time. Power is symbolized by the letter P and its unit of measurement is the watt (W). We can therefore conclude that electrical power is directly proportional to voltage and current: power = voltage x current. Hence :

$$\boxed{P = U.I\ (W)}$$

Power dissipation through a resistor takes the form of heat. If we replace I in the power formula by its equivalent in Ohm's law, we obtain :

$$\boxed{P = \frac{U^2}{R} = R.I^2 \quad (W)}$$

P: is the power (energy per unit time) converted from electrical energy to thermal energy,

The joule's first law (James Prescott Joule: British physicist (1818-1889)) shows the relationship between heat produced by flowing electric current through a conductor in direct current:

$$\boxed{W = P.t = R.I^2 t \quad (joule)}$$

W: energy dissipated in heat, t: denote time (Second s)

VIII. KIRCHHOFF'S RULES: Gustav Robert Kirchhoff: German physicist (1824-1887)

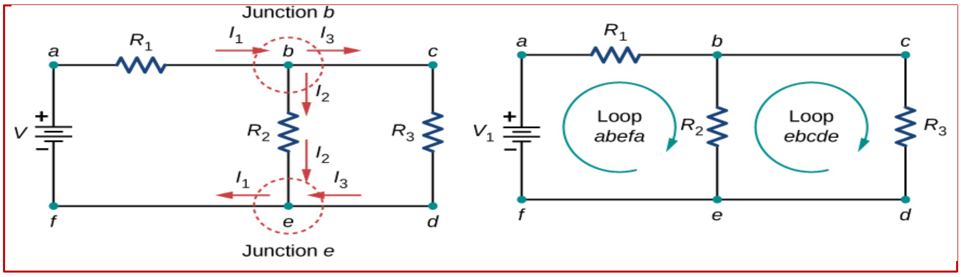
Electric circuits generally consist of a number of resistors and cells interconnected sometimes in a complicated way. Kirchhoff's laws are physical properties that apply to electrical circuits. They are named after the German physicist Gustav Kirchhoff, who established them in 1845.

Two rules, called Kirchhoff's rules are very useful for analysis of electric circuits:

- Junction rule
- Loop rule

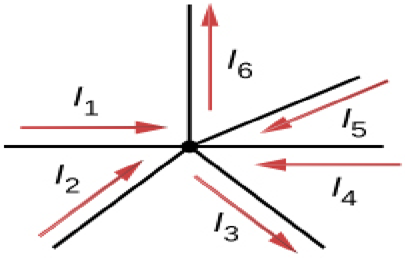
VIII.1. *Definitions*:

- A branch is a set of dipoles connected together and carrying the same current.
- A node is the junction point of at least three conductors.
- A loop is a closed path made up of successive network branches.
- An electrical dipole is an electrical component with two terminals (lamps, switches, generators, batteries, diodes, resistors, motors, etc.).). *A linear dipole is an electrical dipole whose current i(t) through it and voltage v(t) at its terminals are linked by a linear operator*.



VIII.2. Junction rule (Kirchhoff's first Rule)

It states that the sum of all currents directed towards a junction (point A) in an electrical network is equal to the sum of all the currents directed away from the junction (see figure below).



$$\sum I_{in} = \sum I_{out}$$
$$I_1 + I_2 + I_4 + I_5 = I_3 + I_6$$

In other words, the algebraic sum of all currents at a junction is zero.

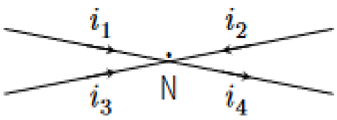
Kirchhoff's first rule tells us that there is no accumulation of charge at any point if steady current flows in it.

At each junction of a circuit, we have:

$$\boxed{\sum_{k=1}^{n} \alpha_k I_k = 0}$$

where α_k = +1 when the current is incoming and α_k = -1 when it is outgoing.

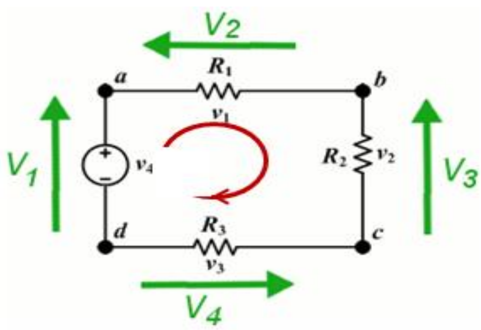
Example:



$i_1 + i_2 + i_3 - i_4 = 0$ or $i_4 = i_1 + i_2 + i_3$

VIII.3. Loop Rule (Kirchhoff's Second Rule)

This rule is an application of law of conservation of energy for electrical circuits. This law states that "in a loop of an electrical network, the sum of the voltages along this loop is always zero". In other words, if we go around a loop and add up all its voltages (paying attention to the direction), the sum will be zero.



Following the red direction, the voltages can be listed as follows:

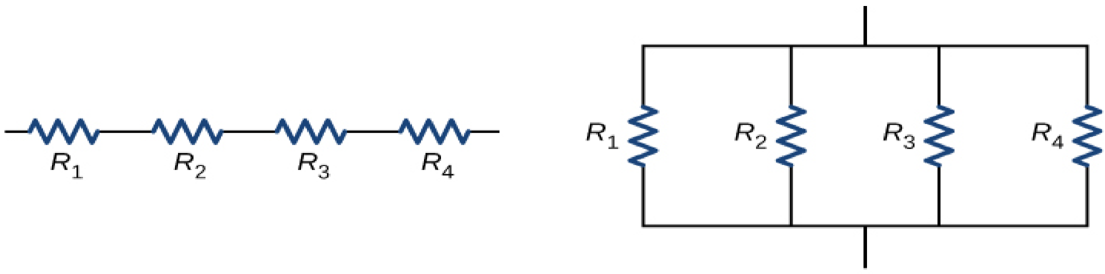
$+V_1 + -V_2 + -V_3 + -V_4 = 0$

The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero

IX- Resistance associations

The equivalent resistance of a combination of resistors depends on both their individual values and how they are connected. The simplest combinations of resistors are series and parallel connections (see figure below).

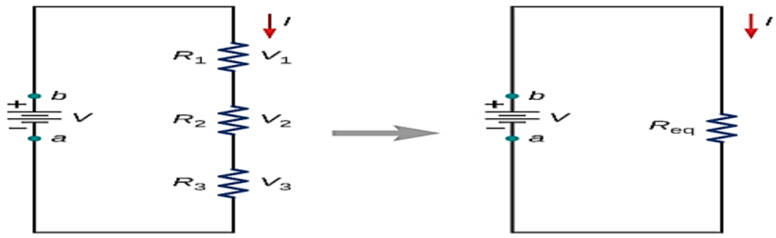
- ***In a series circuit***, the output current of the first resistor flows into the input of the second resistor; therefore, the current is the same in each resistor.
- ***In a parallel circuit***, all of the resistor leads on one side of the resistors are connected together and all the leads on the other side are connected together. In the case of a parallel configuration, each resistor has the same potential drop across it, and the currents through each resistor may be different, depending on the resistor. The sum of the individual currents equals the current that flows into the parallel connections.



(a) Resistors connected in series (b) Resistors connected in parallel

1- Resistors in Series (Series Association)

Consider Figure below, which shows three resistors in series with an applied voltage equal to V_{ab}. Since there is only one path for the charges to flow through, the current **is the same** through each resistor. **The equivalent resistance of a set of resistors in a series connection is equal to the algebraic sum of the individual resistances.**



(a) Original circuit (b) Equivalent circuit

Since energy is conserved, and the voltages equal to the potential energy per charge, the sum of the voltage applied to the circuit by the source and the potential drops across the individual resistors around a loop should be equal to zero:

$$\sum_{i=1}^{N} V_i = 0.$$

$$V - V1 - V2 - V3 = 0,$$

$$V = V1 + V2 + V3 \quad \rightarrow \quad V = R_1 I + R_2 I + R_3 I$$

Solving for :

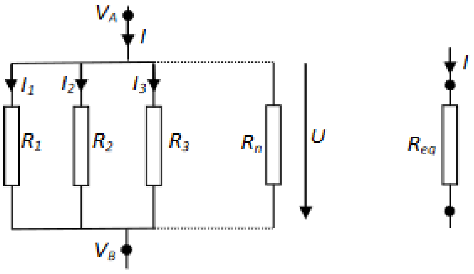
$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{V}{R_{eq}}$$

So the equivalent resistance is just the **sum of the resistances** of the individual resistors. Any number of resistors can be connected in series. If *N* resistors are connected in series, the **equivalent resistance** is:

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \cdots . R_N = \sum_{i=1}^{N} R_i}$$

2- Resistors in parallel : Parallel association

Each portion of the circuit shown in the figure below is supplied with an electric current which is distributed between the resistors (see figure), such that:



$$I = I_1 + I_2 + I_3 + \ldots\ldots\ldots\ldots + I_n$$

$$I = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3} + \ldots\ldots\ldots\ldots + \frac{U}{R_n} = \frac{U}{R_{eq}}$$

$$\Leftrightarrow \frac{U}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots\ldots + \frac{1}{R_n}\right).U$$

The equivalent resistance is given by :

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots\ldots + \frac{1}{R_n}$$

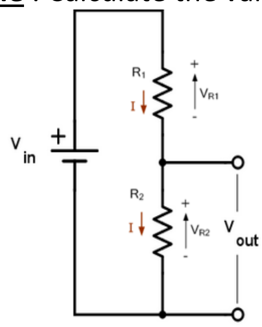
$$\Rightarrow \frac{1}{R} = \sum_{i=1}^{N} \frac{1}{R_i}$$

$$\boxed{\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}} \quad or \quad \boxed{G_{eq} = \sum_{i=1}^{N} G_i}$$

3. Voltage divider

Voltage Divider circuits are used to produce different voltage levels from a common voltage source but the current is the same for all components in a series circuit. The voltage divider is one of the most common and useful circuits used by engineers.

Example : Calculate the value the voltage across each resistor



Here the circuit consists of two resistors connected together in series: R_1, and R_2. Since the two resistors are connected in series, it must therefore follow that the same value of electric current must flow through each resistive element of the circuit

$$V_{out} = V_{in} . \frac{R_2}{R_2 + R_1}$$

Where:
V_{out}: Output voltage. This is the scaled down voltage; V_{in}: Input voltage.
R1 and R2 = Resistor values. The ratio $\frac{R_2}{R_2+R_1}$ determines the scale factor.

Applying Kirchhoff's voltage law around the closed loop

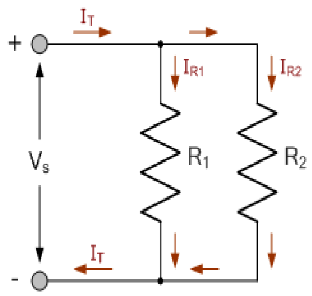
Conclusion: In the series combination of n ohmic conductors, the voltage u_k across the ohmic conductor with resistance R_k is:

$$\boxed{u_k = \frac{R_k}{R_{eq}} u = \frac{R_k}{\sum_k R_k} u}$$

4. Current divider

Current Divider circuits have two or more parallel branches for currents to flow through but the voltage is the same for all components in the parallel circuit. **Example**:

E



Here this basic current divider circuit consists of two resistors: R_1, and R_2 in parallel. This parallel combination splits the source current, I_S between them into two separate currents, I_{R1} and I_{R2} before the current joins together again and returns back to the source. Using KCL, we obtain:

$I_T = I_{R_1} + I_{R_2}$ and using Ohm's law, we have: $I_{R_1} = \frac{V_S}{R_1}$ and $I_{R_2} = \frac{V_S}{R_2}$

$$V_S = I_T \left[\frac{R_1 . R_2}{R_1 + R_2}\right]$$

$$I_{R_1} = I_T\left(\frac{R_2}{R_1 + R_2}\right) = I_T\left(\frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}}\right)$$

$$I_{R_2} = I_T\left(\frac{R_1}{R_1 + R_2}\right) = I_T\left(\frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}\right)$$

Conclusion:

In a parallel combination of n ohmic conductors, the current i_k through the ohmic conductor of conductance $G_k = \frac{1}{R_k}$ is:

$$\boxed{I_k = \frac{G_k}{G_{eq}} I = \frac{G_k}{\sum_k G_k} I}$$

3- Electric circuits

An electric circuit is a set of electrical components such as resistors, capacitors, diodes...etc. The electrical conductors are carried by an electric current. Electric Circuit is the closed loops or paths, in which the current flows.

Therefore, the electrokinetics of an electric circuit consists in finding the current intensity and voltage for each point in the circuit.

- **Direct Current Circuit** or DC Circuit is a closed electrical circuit in which the flow of electricity is in one direction. DC Circuit has a DC Power Supply which produces Direct Current in the circuit. As opposed to alternating current, Direct Current has a fixed magnitude and flows in one direction only. Direct Current Circuit forms a major backbone of the electronics industry.
- **Alternative Current Circuit** or AC circuits are powered by an alternating source such as alternating currents or voltages which are sinusoidal and change periodically in direction and magnitude. In other words, voltage or current oscillates in a sine wave pattern and varies with time.

3.1. Generators

An electric circuit needs a power source to supply it with energy. It is therefore necessary to connect these circuits with a device called a generator (electromotive forces) to transport electrical charges.

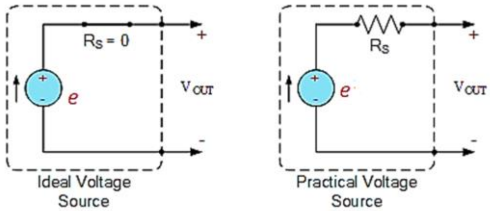
There are two categories of electric generators:

a- Voltage source (Voltage generator)

In the case of a voltage generator, the electromotive force (EMF) is equal to the potential difference between its terminals **when no current flows**. $e = EMF$.

The internal resistance R_s of an electric generator is responsible for the drop in voltage supplied by the generator as the current it delivers inc

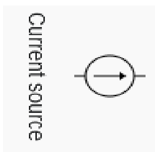
$$\boxed{V_{out} = e - R_s . I}$$

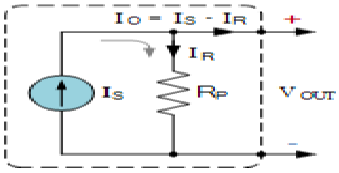


b- Current source (Current generator)

A current generator is a dipole characterized by a constant current, regardless of the potential difference between its terminals.

$$\boxed{I_{OUT} = Io = I_S - \frac{V_S}{R_P}}$$





3.2. Wheatstone Bridge

As an application of Kirchhoff's rules consider the circuit shown in figure below, which is called the **Wheatstone bridge**. The bridge has four resistors R1, R2, R3 and R4. Across one pair of diagonally opposite points (A and C in the figure) a source is connected.

The Kirchhoff's junction rule applied to junctions D and B (generally $I_G = 0$). Immediately gives us the relations $I_1 = I_3$ and $I_2 = I_4$. Next, we apply Kirchhoff's loop rule to closed loops ADBA and CBDC. The first loop gives:

$$-R_1 I_1 + 0 + I_2 R_2 = 0 \qquad (1)$$

and the second loop gives, upon using $I_1 = I_3$ and $I_2 = I_4$

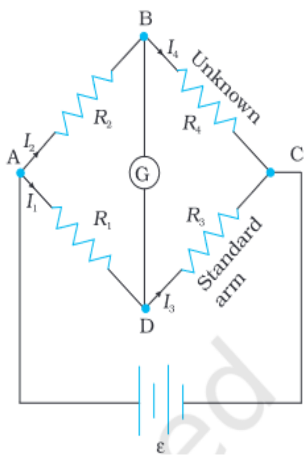
$$-R_3 I_1 + 0 + I_2 R_4 = 0 \qquad (2)$$

From Eq. 1, we obtain, $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

From Eq. 2, we obtain, $\frac{I_1}{I_2} = \frac{R_4}{R_3}$

Hence, we obtain the condition:

$$\boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$$



This last equation relating the four resistors is called the ***balance condition*** for the galvanometer to give zero or null deflection.

The bridge then is balanced, and from the balance condition the value of the unknown resistance $\boldsymbol{R_4}$ is given by,

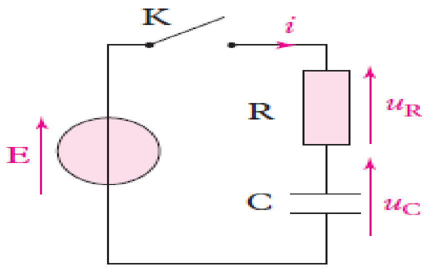
$$\boxed{R_4 = \frac{R_3 . R_2}{R_1}}$$

A practical device using this principle is called the ***meter bridge***.

IX. Capacitor charging and discharging

Consider an RC dipole consisting of a resistor R and a capacitor (condenser) of capacitance C connected to an e.m.f. generator e. Initially, the capacitor is not charged: q0 =0. We are interested in determining the charge q of the capacitor at any instant.

1- Expression of capacitor charge



- At time t, according to the Loop law, we have:

$$U_R + U_C = E \qquad (1)$$

- the voltage across the resistor is:

$$U_R = R.i(t) = R.\frac{dq}{dt} \qquad (2)$$

- the ddp (voltage) across a capacitor of capacitance C

We obtain: $U_C = \frac{q(t)}{C}$

$$R.\frac{dq}{dt} + \frac{q(t)}{C} = E \qquad (3)$$

Equation (3) is a 1st-order differential equation. It can be solved either:

- By the separation of variables method.
- By solving the general solution

The integration constants are found by using boundary conditions.

- A condition at t = 0.
- A condition when t tends to infinity.
  - ➢ The resolution of (3) by the separation of variables method

$$\frac{q}{C} - E = -R\frac{dq(t)}{dt} \Rightarrow -dt = \frac{Rdq}{\frac{q}{C} - E} = \frac{RC\,dq}{q - EC}$$

$$\Rightarrow \frac{dq}{q - EC} = -\frac{dt}{RC} = -\frac{dt}{\tau}$$

Where $\boldsymbol{\tau = RC}$ is a time-dimensional quantity, known as the ***circuit's characteristic time (s) (Time constant)***.

$$\Rightarrow \int \frac{dq}{q - EC} = -\int \frac{dt}{\tau} \Rightarrow \ln(q - EC) = -\frac{t}{\tau}$$

$$\Rightarrow q - EC = A\,e^{-\frac{t}{\tau}}$$

The constant A is determined from the initial conditions.

$$t = 0 \Rightarrow 0 - EC = A\,e^{-\frac{0}{\tau}} \Rightarrow A = -EC$$

$$\boxed{q(t) = C.E\left(1 - e^{\frac{-t}{RC}}\right)}$$

- ➢ $if\ t \to \infty, e^{\frac{-t}{RC}} \to 0\ \ so\ Q_f = C.E$
- ➢ $if\ t = \tau, then\ q(\tau) = C.E(1 - e^{-1})$

$$q(\tau) = CE(1 - 0.368) = 0.632.C.E = 63.2\%\ Q_f$$

We may define time constant of CR circuit as the time in which charge of capacitor grows to 63.2 of the maximum value of charge

Current and voltage across the capacitor.

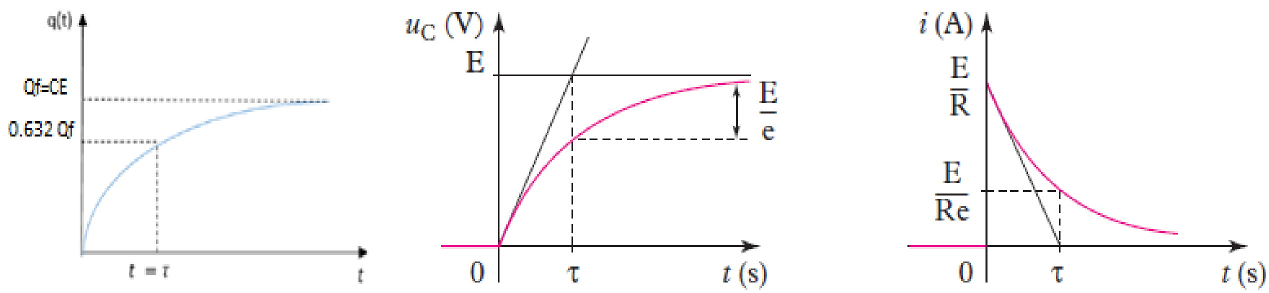
The current intensity during capacitor charging is given below:

$$\boxed{i(t) = \frac{dq}{dt} = C\frac{dU_c}{dt} = \frac{E}{R}e^{\frac{-t}{RC}}}$$

The voltage across the capacitor is :

$$\boxed{U_C(t) = \frac{q(t)}{C} = E\left(1 - e^{\frac{-t}{RC}}\right)}$$

Graphical representation



$$\boxed{u_R = Ri \text{ and } i = \frac{dq}{dt} = C\frac{du_C}{dt}, \text{ so } \quad u_R = RC\frac{du_C}{dt}}$$

- **Capacitor energy**

Initially, the capacitor's energy is zero, since its charge is zero. When it is charged, the voltage

between its terminals is E (generator e.m.f.) and its charge is $q = E.C$ and its energy is given by:

$$\boxed{W_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}C.E^2}$$

The energy supplied by the generator is :

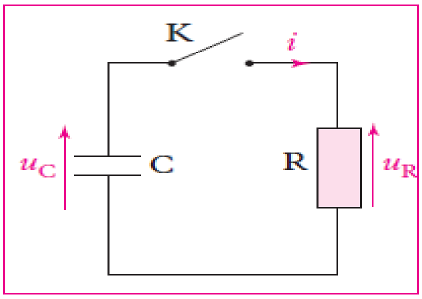
$$\boxed{W_G = qE = C.E^2}$$

In charging the capacitor of an RC circuit, half of the energy drawn from the battery is stored in the capacitor while the other half is dissipated as heat by the resistor (joule effect): $W_J == \frac{1}{2}C.E^2$

2- Capacitor discharge

Initially, the capacitor is fully charged. Its initial charge is given by $q_0 = E.C$. It discharges into the resistor.

In the closed discharge circuit, the sum of the voltages is zero. In this case, we obtain :



$$u_R = Ri \text{ and } i = \frac{dq}{dt} = -C\frac{du_C}{dt}, \text{ so } \quad u_R = -RC\frac{du_C}{dt}.$$

$$Ri(t) + \frac{q(t)}{C} = R.\frac{dq}{dt} + \frac{q(t)}{C} = 0$$

The equation is a 1st-order differential equation with no second member, with attention to the condition $q_0 = EC$. Applying the method of variable separation, we obtain the following differential equation:

$$Ri(t) + \frac{q(t)}{C} = 0 \Rightarrow R.\frac{dq(t)}{dt} + \frac{q(t)}{C} = 0 \Rightarrow \frac{dq}{q(t)} = -\frac{dt}{RC}$$

$$\Rightarrow \ln q(t) = -\frac{t}{RC} \Rightarrow q(t) = Ae^{-\frac{t}{RC}}$$

Considering the following initial condition: $q_0 = EC$. Consequently:

$$\boxed{q(t) = ECe^{-\frac{t}{RC}} = ECe^{-\frac{t}{\tau}}}$$

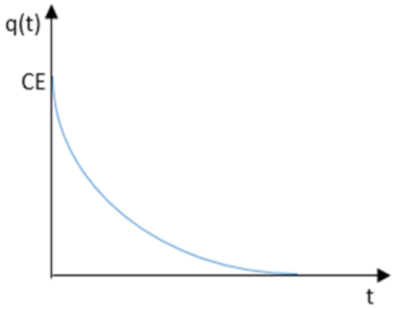
Current and voltage across the capacitor

The current during capacitor discharge is:

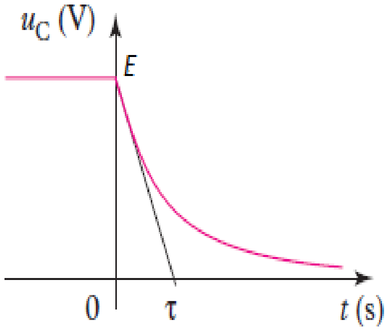
$$i(t) = -\frac{dq(t)}{dt} = \frac{E}{R}e^{-\frac{t}{\tau}}$$

The voltage across the capacitor is:

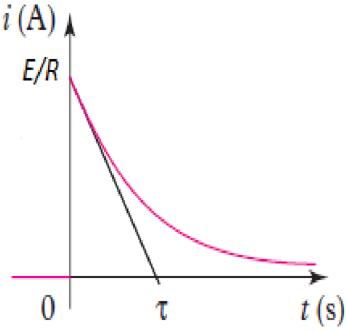
$$U_c(t) = \frac{q(t)}{C} = Ee^{-\frac{t}{RC}}$$



Charge



Voltage



Current

- **TYPES OF RESISTORS**

We use resistors in all electrical and electronic circuits to control the magnitude of current. Resistors usually are of two types :

- carbon resistors
- wire wound resistors

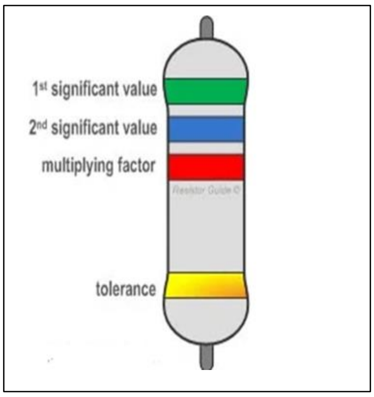
- ***In a wire wound resistor***, a resistance wire (of manganin, constantan or nichrome) of definite length, which depends on the required value of resistance, is wound two-fold over an insulating cylinder to make it non-inductive.
- ***In carbon resistors***, carbon with a suitable binding agent is molded into a cylinder. Wire leads are attached to the cylinder for making connections to electrical circuits. Resistors are colour coded to give their values :

$$\boxed{R = AB \times 10^C\ \Omega,\ \pm D}$$

where *A*, *B* and *C* are coloured stripes. The values of different colours are given in Table. As may be noted,

- ✓ first two colours (AB) indicate the first two digits of the resistance value;
- ✓ third colour gives the power of ten for the multiplier of the value of the resistance;
- ✓ fourth colour (the last one) gives the tolerance of the resistance, which is 5% for golden colour, 10% for silver colour and 20% for body colour.

Colour	Number	Multiplier
Black	0	1
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Grey	8	10^8
White	9	10^9



Suppose that four colours on a resistor are Blue, Grey, Green and Silver. Then

- The first digit will be 6 (blue)
- The second digit will be 8 (Grey)
- The third colour signifies multiplier 10^5 (Green)
- The fourth colour defines tolerance = 10% (Silver)

Hence value of the resistance is :

$$= 68 \times 10^5 \pm 10\%$$
$$= 68 \times 10^5 \pm (68 \times 10^5 \times 10/100)$$
$$= 68 \times 10^5 \pm 68 \times 10^4$$
$$= (6.8 \pm 0.68) M\Omega$$