Exercise 1

Evaluate the given integral

1 $\int\limits_{0}^{1}(x-1)\,e^x dx, \qquad \int\limits_{1}^{2}\ln(x)\,dx, \qquad \int\limits_{0}^{1}\arctan(x)\,dx.$

2 $\int\limits_{0}^{1}(x^2+1)\cos x dx, \qquad \int\limits_{1}^{2}\ln(x)\,dx, \qquad \int\limits_{0}^{\frac{\pi}{2}}\cos^2(x)\,dx.$

Exercise 2

Evaluate the given integral.

1 $\int\limits_{0}^{\frac{\pi}{2}}e^{-2x}\sin 3x dx$

2 $\int\limits_{-1}^{0}x^2\sqrt{1-x}dx$

Exercise 3

Evaluate the given integral.

1 $\int\limits_{1}^{2}\frac{\ln x}{x}dx, \qquad \int\limits_{0}^{1}e^x\cos(e^x)\,dx$

2 $\int\limits_{0}^{1}x\left(1+x^2\right)^4 dx, \qquad \int\limits_{e}^{e^2}\frac{1}{x(1+\ln x)}dx$

Exercise 4

1 Evaluate the given integral. $\int\limits_{0}^{\frac{\pi}{2}}\sin^4 x dx$

2 Evaluate the given integral. $I=\int\limits_{0}^{\frac{\pi}{2}}\cos^5 x dx$ and $J=\int\limits_{0}^{\frac{\pi}{4}}\frac{x}{\cos^2 x}dx$

3.1 Evaluate the given integral. $K=\int\limits_{0}^{\frac{\pi}{4}}\frac{1}{\cos^2 x}dx$

3.2 Let $f(x)=\dfrac{\sin^2 x}{\cos^4 x}$, find a and b such that

$$f(x)=\frac{a}{\cos^4 x}+\frac{b}{\cos^2 x}.$$

3.3 Let

$$L = \int_0^{\frac{\pi}{4}} \frac{1}{\cos^4 x} dx,$$

using the previous questions , determine L.

Exercise 5
Let $n \in \mathbb{N}^*$. Set

$$I_n = \int_1^e x(\ln x)^n dx$$

1 Evaluate I_0 and I_1

2 Show that for $n \in \mathbb{N}^* : 2I_n + + nI_{n-1} = e^2$ and deduce I_2

3 Show that $(I_n)_{n \in \mathbb{N}^*}$ is decreasing sequence

4 Deduce that for $n \in \mathbb{N}^* :$

$$\frac{e^2}{n+3} \le I_n \le \frac{e^2}{n+2}$$

5 Find $\lim_{n \to +\infty} I_n$ and $\lim_{n \to +\infty} (nI_n)$

Exercise 6
Evaluate the given integral.

1 $\int_0^1 |x-t| dt$

2 $\int_0^1 \max(x,t) dt$

3 $\int_{-1}^1 \frac{x^2 dx}{\sqrt{1+x}+\sqrt{1-x}}$

Exercise 7
Evaluate the given integral.

1 $\int_2^3 \frac{dx}{x(x-1)},$ $\int_2^3 \frac{(2x+1)dx}{x^2-1},$ $\int_0^{\frac{1}{2}} \frac{(x^2+1)dx}{x^2-3x+2},$

2 $\int_0^{\frac{1}{2}} \frac{(x+1)dx}{(x^2+1)(x-2)},$ $\int_0^1 \frac{dx}{(x^2+2x+5)(x+2)},$ $\int_0^2 \frac{dx}{(x^2+x+1)(x+1)}$

Exercise 8
Evaluate the given integral.

1 $\int_0^1 \frac{1}{(x^2+x+1)^2} dx,$ $\int_0^1 \frac{x}{(x^2-2x+2)^2} dx$

2

2 $\int\limits_{-1}^{0} \dfrac{1}{(x^2-3x+2)^2}dx, \quad \int\limits_{0}^{1} \dfrac{x-1}{(x^2+1)^2(x+2)}dx$

Exercise 9
Evaluate the given integral.

1 $\int\limits_{0}^{1} \dfrac{x-1}{(x^2+x+1)^3(x+2)}dx$

2 $\int\limits_{0}^{\frac{1}{2}} (3x^2-6x+1)^3 \ln(x-1)\,dx$

Exercise 10
Evaluate the given integral.

1 $\int\limits_{0}^{t} \sin^4 x\cos^5 x\,dx, \quad \int\limits_{0}^{t} \cos^4 x\,dx, \quad \int\limits_{0}^{t} \sin^3 x\cos^2 x\,dx$

2 $\int\limits_{0}^{\frac{\pi}{2}} \cos 2x\cos 3x\,dx, \quad \int\limits_{0}^{t} \dfrac{\sin x}{\cos^2 x}dx, \quad \int\limits_{0}^{t} \dfrac{1}{3+\tan x}dx$

3 $\int\limits_{0}^{t} \cosh x\,dx, \quad \int\limits_{0}^{t} \tanh x\,dx, \quad \int\limits_{\frac{1}{2}}^{t} \dfrac{1}{\sinh x}dx$

Exercise 11
Evaluate the given integral.

1 $\int \dfrac{(x-5)\,dx}{\sqrt{x^2-18x+106}}, \quad \int \dfrac{x^2 dx}{\sqrt{1-x^6}}, \quad \int \dfrac{x^2 dx}{\sqrt{2x-x^2}}$

2 $\int\limits_{-2}^{3} \sqrt{|x^2-1|}dx, \quad \int\limits_{0}^{1} (x-2)\sqrt{x^2+2x}\,dx$