Ministry of Higher Education and Scientific Research
National Higher School of Cybersecurity
1st Year Integrated Preparatory Cycle **Academic year : 2024 -2025**
Mathematical Analysis 1

Worksheet 3 : Differentiable functions
Part 1

Exercise 1. *Find by the definition the first derivatives of the following functions at the given points*

1. $f(x) = x(x-1)^2(x-2)^3, \quad x_0 = 0,$
2. $f(x) = \sqrt{x+1}, \quad x_0 = 1$
3. $f(x) = \sqrt[3]{x-1}, \quad x \in \mathbb{R}$
4. $f(x) = \ln(x+1), \quad x_0 > -1$

Exercise 2. *Find the first derivatives of the following functions*

1. $f(x) = \log_3 \log_5 \log_7 x \quad x > 7$
2. $f(x) = \ln\left(\ln^2\left(\ln^3 x^2\right)\right) \quad x \neq 0$
3. $f(x) = \sqrt[3]{x^4+1}, \quad x \in \mathbb{R}$
4. $f(x) = \sqrt{1+\sqrt[3]{1+\sqrt[4]{1+x^4}}}, \quad x \in \mathbb{R}$

Exercise 3. *Find the largest sets on which the first devatives of the following functions exist*

1. $f(x) = \begin{cases} 2-x & x < 2 \\ (2-x)(3-x) & 2 \leq x \leq 3 \\ -(3-x) & x > 3 \end{cases} \qquad f(x) = \begin{cases} x^3+1 & x \leq 0 \\ e^{-\frac{1}{x}}+1 & x > 0 \end{cases}$
2. $f(x) = \max\left(x^2-1, x+1\right) \qquad f(x) = \begin{cases} x^2 \sin(x) \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$
3. $f(x) = \begin{cases} e^x & x \leq 0 \\ ax2+bx+c & x > 0 \end{cases} \qquad f(x) = \sqrt{x - x^2[x]}$

Exercise 4. *Find the first derivative of the inverse function f^{-1} of the functions given below*

$$f(x) = 2x+1, x \in \mathbb{R} \quad f(x) = \sqrt{x}+2, x > 0$$
$$f(x) = x + \ln x, x > 0 \quad f(x) = \frac{x^2}{1+x^2}, x < 0$$

Exercise 5. *Find the $j-$derivative $(j \in \mathbb{N})$ of the functions given below*

1. $f(x) = 2^x \qquad f(x) = \ln(1+x), |x| < 1$

2. $f(x) = \dfrac{ax+b}{cx+d}, a, b, c, d \in \mathbb{R}^* \qquad f(x) = xe^{ax}, a \in \mathbb{R}^*$

Exercise 6. *Find the $n-$derivative ($n \in \mathbb{N}$) of the functions given below*

$$f(x) = xe^x \qquad f(x) = (x^3 - 2x + 7)e^x.$$

Exercise 7. *Using lagrange theorem prove that*

1. $|\sin ax - \sin bx| \leq |a| |x - y|$

2. $\dfrac{x-y}{x} < \ln \frac{x}{y} < \dfrac{x-y}{y}, 0 < y < x$

3. $\dfrac{x}{1+x} < \ln 1 + x < x, 0 < x$