NATIONAL SCHOOL OF CYBERSECURITY

المدرسة الوطنية العليا في الأمن السيبراني

2025/2026 | Basic Training Cycle

Lecturer: Pr. B, CHAOUCHI | **Analysis 1 - Tutorial 1-Part 2** | Real numbers

Only a few exercises will be covered during the tutorial session.

Exercise 1

Let x, y be two real numbers, show that

$$||x| - |y|| \leqslant |x+y| \leqslant |x| + |y|.$$

Exercise 2

Let x, y be two non-zero real numbers. Show that

$$\max(|x|, |y|)\left|\frac{x}{|x|} - \frac{y}{|y|}\right| \leqslant 2|x-y|.$$

Exercise 3

Let x and y be two real numbers. Show that

$\boxed{1}$ $E(x+1) = E(x) + 1$;

$\boxed{2}$ $E(x) + E(y) \leq E(x+y) \leq E(x) + E(y) + 1$.

Exercise 4

Solve in $\mathbb{R}$ the following equation

$$E(2x) + E\left(x + \frac{1}{3}\right) = E(x) + 5.$$

Exercise 5

Solve the following equations in $\mathbb{R}$

$$E(5x) = 4E(x) \quad \text{and} \quad E(2x+3) = E(3x+1).$$

Exercise 6

Let the subsets of $\mathbb{R}$ defined by

$$A = \left\{\frac{1}{2} - \frac{n}{2n+1}, n \in \mathbb{N} \cup \{0\}\right\}, B = \left\{1 + \frac{(-1)^n}{n}, n \in \mathbb{N}\right\}, C = \left\{\frac{1}{n} + (-1)^n, n \in \mathbb{N}\right\}.$$

Prove that they are bounded and determine theirs bounds, verify the results by using the definition (characterization of upper and lower bounds).

Exercise 7

Let A the subset given by

$$A = \left\{ \frac{1}{2} + \frac{n}{2n+1}, n \in \mathbb{N} \cup \{0\} \right\}.$$

Prove that A is bounded and determine its bounds.

Exercise 8

Let A, B be two nonempty subsets of $\mathbb{R}$. Define

$$A+B := \{x+y : x \in A \text{ and } y \in B\},$$

and

$$A-B := \{x-y : x \in A \text{ and } y \in B\}.$$

Show that

$$\sup(A+B) = \sup A + \sup B \quad \text{and} \quad \sup(A-B) = \sup A - \inf B.$$

Establish similar formulas for $\inf(A+B)$ and $\inf(A-B)$.

Exercise 9

Show that, if A and B are bounded subsets of real numbers, then

$$\sup(A \cdot B) = \max\{\sup A . \sup B, \sup A . \inf B, \inf A . \sup B, \inf A . \inf B\}.$$

Give an example of two nonempty bounded sets A and B for which

$$\sup(A \cdot B) \neq \sup A . \sup B.$$