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Neural Networks

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Sheet 1

→ Calculate the total net input for $h_1$

$$In_{h_1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$$
$$= 0,15 * 0,05 + 0,2 * 0,1 + 0,35 * 1 = 0,3775$$

→ use Sigmoid function

$$out_{h_1} = \frac{1}{1+e^{-In\,h_1}} = \frac{1}{1+e^{-0,3775}} = 0,593269992$$

→ Calculate the total net input for $h_2$

$$In_{h_2} = w_3 * i_1 + w_4 * i_2 + b_1 * 1$$
$$= 0,25 * 0,05 + 0,30 * 0,10 + 0,35 = 0,3925$$

→ use Sigmoid function

$$out_{h_2} = \frac{1}{1+e^{-0,3925}} = 0,596884378$$

→ $In_{o_1} = w_5 * out_{h_1} + w_6 * out_{h_2} + b_2 * 1$

$$= 0,4 * 0,593269992 + 0,45 * 0,596884378 + 0,6 * 1 = 1.105905967$$

→ $out_{o_1} = \frac{1}{1+e^{-In\,o_1}} = \frac{1}{1+e^{-1.105905967}} = 0,75136507$

→ $In\,O_2 = w_7 * out_{h_1} + w_8 * out_{h_2} + b_2 * 1$

$$= 0,50 * 0,593269992 + 0,55 * 0,596884378 + 0,60 * 1$$
$$= 1,2249214039$$

→ $out\,O_2 = \frac{1}{1+e^{-In\,O_2}} = \frac{1}{1+e^{-1,2249214039}} = 0,772928465$

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→ Calculate the total error

$$E_{total} = \sum \frac{1}{2} ( \text{actual} - \text{output} )^2$$

$$E_{total} = E_{o_1} + E_{o_2}$$
$$= \frac{1}{2} [0{,}01 - 0{,}75136507]^2 + \frac{1}{2} [0{,}99 - 0{,}772928465]^2$$
$$= 0{,}298371109$$

→ The Backpropagation [output layer]

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out\, o_1} * \frac{\partial out\, o_1}{\partial In\, o_1} * \frac{\partial In\, o_1}{\partial w_5}$$

→ $$\frac{\partial E_{total}}{\partial out\, o_1} = 2 * \frac{1}{2} (actual_{o1} - out_{o1})^{1} * -1 + 0 = -(0{,}01 - 0{,}75136507)$$
$$= 0{,}74136507$$

$$out_{o1} = \frac{1}{1+e^{-In\, o1}}$$

→ $$\frac{\partial out\, o_1}{\partial In\, o_1} = out_{o1} ( 1 - out_{o1} ) = 0{,}75136507 (1 - 0{,}75136507) = 0{,}186815602$$

$$In\, o_1 = w_5 * out_{h_1} + w_6 * out_{h_2} + b_2 * 1$$

→ $$\frac{\partial In\, o_1}{\partial w_5} = out_{h_1} + 0 + 0 = 0{,}593269992$$

$$\therefore \frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out\, o_1} * \frac{\partial out\, o_1}{\partial In\, o_1} * \frac{\partial In\, o_1}{\partial w_5}$$

$$= 0{,}74136507 * 0{,}186815602 * 0{,}593269992$$

$$= \boxed{0{,}082167041}$$

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Assumming that the learning rate is $\eta = 0,5$ then by updating weights $w_5, w_6, w_7, w_8$

$$w_5' = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5}$$

$$= 0,4 - [0,5 * 0,082167041] = \boxed{0,35891648}$$

with the Same Way We get new updated wights.

| |
|---|
| $w_6' = 0,408666186$ |
| $w_7' = 0,511301270$ |
| $w_8' = 0,561370121$ |

[For hidden layer]

- $$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out\, h_1} * \frac{\partial out\, h_1}{\partial In\, h_1} * \frac{\partial In\, h_1}{\partial w_1}$$

- $$\frac{\partial E_{total}}{\partial out\, h_1} = \frac{\partial E_{o1}}{\partial out\, h_1} + \frac{\partial E_{o2}}{\partial out\, h_2}$$

- $$\frac{\partial E_{o1}}{\partial out\, h_1} = \frac{\partial E_{o1}}{\partial In\, o_1} * \frac{\partial In\, o_1}{\partial out\, h1}$$

- $$\frac{\partial E_{o1}}{\partial In\, o_1} = \frac{\partial E_{o1}}{\partial out\, o_1} * \frac{\partial out\, o1}{\partial In\, o_1} = 0,74136507 * 0,186815602 = 0,138498562$$

- $\frac{\partial In\, o_1}{\partial out\, h_1}$ is equal to $w_5 = 0,40$

$$\therefore \frac{\partial E_{o1}}{\partial out\, h_1} = \frac{\partial E_{o1}}{\partial out\, h_1} * \frac{\partial In\, O_1}{\partial out\, h_1} = 0,138498562 * 0,40 = 0,055399425$$

$$\frac{\partial E_{o2}}{\partial out_{h_1}} = -0{,}019049119$$

$$\frac{\partial E_{total}}{\partial out_{h_1}} = \frac{\partial E_{o1}}{\partial out_{h_1}} + \frac{\partial E_{o2}}{\partial out_{h_1}} = 0{,}055399425 - 0{,}019049119 = 0{,}036350306$$

$$out_{h_1} = \frac{1}{1+e^{-In\, h_1}}$$

$$\frac{\partial out\, h_1}{\partial In\, h_1} = out_{h_1}(1 - out_{h_1}) = 0{,}59326999(1 - 0{,}59326999) = 0{,}241300709$$

$$In_{h_1} = w_1 * i_1 + w_3 * i_2 + b_1 * 1$$

$$\frac{\partial In\, h_1}{\partial w_1} = i_1 = 0{,}05$$

$$\therefore \frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h_1}} * \frac{\partial out\, h_1}{\partial In\, h_1} * \frac{\partial In\, h_1}{\partial w_1}$$

$$= 0{,}036350306 * 0{,}241300709 * 0{,}05 = 0{,}000438568$$

$$w_1' = w_1 - \eta \frac{\partial E_{total}}{\partial w_1} = 0{,}15 - [0{,}5 * 0{,}000438568] = \boxed{0{,}149780716}$$

| | |
|---|---|
| $w_2'$ = | 0,19956143 |
| $w_3'$ = | 0,24975114 |
| $w_4'$ = | 0,29950229 |

The error was 0,298371109 and now after the first round of propagation is down to 0,291027924