Please submit a pdf copy (at most 3 pages) of your own solutions to Problems 1 and 2, at SUCourse+ before March 10 (Monday), 23:30.

Problem 1 (25 points) The divide-and-conquer merge sort algorithm studied in class takes as input a list of numbers, and puts them into a nondecreasing numerical order.

(a) How can we modify this merge sort algorithm so that

- it takes as input a list of m-tuples of elements, such that the i'th elements of the tuples are comparable to each other for every i, and
- it puts these m-tuples into a nondecreasing lexicographic order?

Please explain the modifications.

(b) What is the best worst-case asymptotic time complexity of this modified merge sort, assuming that comparing the i'th elements of any two m-tuples takes constant amount of time in the worst-case.

Problem 2 (50 points) Consider a children's toy store where each toy is described by its id, its location in the store, and its features, such as character, color, size, and texture. For instance, the following set contains descriptions of five toys:

$$
\begin{aligned}
&\{\langle t7, f1r2v3h4, bird, blue, small, soft\rangle,\\
&\langle t3, f2r3s5v1h2, bear, blue, big, soft\rangle,\\
&\langle t6, f2r2v5h3, rabbit, red, small, hard\rangle,\\
&\langle t5, f3r3s2v6h8, fish, blue, medium, soft\rangle,\\
&\langle t2, f3r3s1v2h2, bird, blue, medium, hard\rangle\}.
\end{aligned}
$$

Consider a robot assistant that helps with the sale of toys in the store. In particular, when the sales person gives the robot the priorities and preferences of a child, and a positive number k, the robot sorts the toys with respect to the child's priorities and preferences, picks the k best toys, and then brings them to the sales person.

For instance, Alice gives more priority to the color of a toy, then to its character, then to its size, and then to its texture. She prefers red toys to blue toys, bird toys to fish toys to bear toys to rabbit toys, smaller toys, and softer toys. Then, Alice prefers toy $t6$ to toy $t2$, and toy $t2$ to toy $t3$:

$$t6 \prec_{Alice} t2 \prec_{Alice} t3.$$

Meanwhile, Bob gives more priority to the toy's texture, then to its character, then to its size, and then to its color. He prefers bear toys to fish toys to bird toys to rabbit toys, larger toys, blue toys to red toys, and harder toys. Then, Bob prefers toy $t2$ to toy $t6$, and toy $t6$ to toy $t3$:

$$t2 \prec_{Bob} t6 \prec_{Bob} t3.$$

(a) How can the robot use the modified merge sort algorithm (from Problem 1) to sort the five toys above with respect to Alice's priorities and preferences?

Please show every step of the algorithm: what is the input, which comparisons are done and in which order, and what is returned as the output?

(b) Note that the radix sort algorithm studied in class already has the desired properties of the modified merge sort algorithm:

- it can take as input a list of m-digit numbers ($m > 0$) (which can be considered as m-tuples of numbers $0..9$), and
- it can put these numbers into a nondecreasing numerical ordering (which can be considered as lexicographic ordering of the tuples).

Therefore, alternative to the modified merge sort algorithm, the robot can use the radix sort algorithm as it is, without any modifications, to sort toys with respect to the given priorities and preferences of a child.

Please illustrate how the radix sort algorithm can be used to sort the toy descriptions above with respect to Bob's priorities and preferences.

Note that in (a) and (b) above, you may need to rewrite the toy descriptions before you pass them as input to the modified merge sort algorithm and to the radix sort algorithm, respectively. In that case, please also describe briefly how the toy descriptions are rewritten.