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Should Central Bank respond to the Changes in the Loan to Collateral Value Ratio and in the House Prices?

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Should Central Bank respond to the Changes in the Loan to Collateral Value Ratio and in the House Prices?*

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Abstract

We study optimal policy in a New Keynesian model at zero bound interest rate where households use cash alongside with house equity borrowing to conduct transactions. The amount of borrowing is limited by a collateral constraint. When either the loan to value ratio declines or house prices fall we observe decrease in the money multiplier. We argue that the central bank should respond to the fall in the money multiplier and therefore to the reduction in house prices or in the loan to collateral value ratio. We also find that optimal monetary policy generates large and more persistent fall in the money multiplier in response to drop in the loan to collateral value ratio.

Keywords: optimal monetary policy, money supply, money multiplier, loan to value ratio, collateral constraint, house prices, zero bound interest rate

JEL classification: E44, E51, E52, E58

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1 Introduction

The recent economic crisis caused a significant decrease in credit availability (Dell Ariccia at al., 2008) which resulted in a sharp fall in house prices and output. Responding to worsening credit conditions, many developed countries significantly expanded their monetary bases. Indeed, many central banks went further and engaged in quantitative easing (QE) taking "unconventional" assets onto their balance sheets (Gambacorta at al. 2012). In this paper we provide a justification for QE and argue that monetary expansion is necessary for stabilizing price and output fluctuations when credit conditions tighten markedly.

The general idea of connecting financial markets and business cycles can be traced back to Fisher (1933), Bernanke (1983), Bernanke and Gertler (1989), who show that contraction in the financial sector can lead to economic slowdown. We wish to investigate whether, to what extent and how the monetary authorities should respond to worsening financial conditions in order to avoid economic recession. This question is not new to the academic literature[1]. For instance, Clarida (2012) in his review of recent monetary policy developments argues that there is still no consensus about how monetary policy makers should account for financial variables. On the one hand, he argues that financial variables are not *target* variables and should not be included in monetary policy rules. That opinion is also shared by Bernanke and Gertler (2001) and Iacoviello (2005), who argue that government should not react to changes in asset prices as this does not improve the economy in terms of inflation and output stabilization. In Iacoviello (2005) housing is modeled as the only asset in the economy. Similarly, Bernanke and Gertler (2001) find that the policy maker should not react to changes in stock prices because it also does not have any impact on either output or price stabilization.

On the other hand, Mishkin (2011) argues that after the 2007-2009 economic crisis monetary policy makers understood that the financial sector has a much greater impact on economic activity than was earlier realized. Further to this, Svensson (2009) recognises that credit capacity and asset prices may have a potentially negative impact on inflation and resources utilization, and therefore including them in the monetary policy rule is entirely consistent with stabilization of inflation and output gaps. That particular feature we will observe in our model.

We study optimal monetary policy in a New Keynesian economy with sticky prices where households use cash alongside equity borrowing to conduct transactions. The amount of borrowing is

[1] For a comprehensive survey on macroeconomics with financial frictions see Brunnermeier at al. (2012).

limited by a collateral constraint as in Kiyotaki and Moore (1997) or Iacoviello (2005). We simply assume that competitive financial intermediaries can costlessly create as much credit as it wishes. However, due to lack of contract enforcement, credit needs to be collateralized. The credit capacity can worsen because of a reduction in collateral value or when economy is hit by an exogenous shock causing a decline in the average recovery rate of collateral; we refer to this as a "credit shock". For example, the recovery rate may fall with an increase in the transaction costs associated with collateral repossession and sale. When the loan to collateral value (LTV) declines, credit capacity falls. Lower inside money reduces nominal expenditure and therefore nominal demand. In a flexible price economy, producers can adjust their prices accordingly and recession is avoided. However, when prices are sticky, only incomplete adjustment is possible, and credit tightening results in both deflation and recession unless an expansionary monetary policy is implemented.

We find that monetary policy can ensure perfect stabilization of output and prices when credit shock hits the economy. However, to achieve stability, government should increase monetary base in respond to a deterioration in credit availability and a corresponding fall in the money multiplier. The importance of the money multiplier has been discussed in Bernanke and Blinder (1988), Freeman and Kyndland (2000) and recently in Goodhart (2009) and Abrams (2011). Since the money multiplier affects the monetary transmission, optimal monetary policy should respond to the changes in the money multiplier. Our model shows how the money multiplier depends on the LTV and the relative price of collateral. Therefore if houses are used as collateral, monetary policy should react to the change in house prices.

The principle difference in our model compared to Iacoviello (2005) and Carlstrom and Fuerst (1995), is that we consider an economy which is already in a liquidity trap and the interest rate for loans is zero. As interest rate is at its zero bound, monetary authority stimulates the economy by providing direct monetary transfers to households. Unconventional monetary expansion at the lower bound interest rate has been advocated by Friedman (2000, 2006) and Bernanke at al. (2004). And since the interest rate is zero, direct monetary targeting cannot be criticized in the sense of McCallum (1985), because it does not cause any volatility in the short term interest rate.

To compare policy rules, we construct a second order approximation to social welfare as in Benigno and Woodford (2012) and obtain social loss function as in the conventional New Keynesian model (Benigno and Woodford, 2005). Therefore, the optimal policy generates the same dynamics for output and inflation gaps as in the standard New Keynesian model. However, in order to achieve the optimal dynamics, monetary authority should conduct monetary expansion when there is a fall in the loan to collateral value ratio or if the relative price of collateral declines. Interestingly, we find that optimal monetary expansion causes deep and very persistent fall in the money multiplier in response to a credit shock.

This paper is structured as follows. In Section 2 we present the model and define dynamic equations. In Section 3 we discuss optimal monetary policy and some other policy issues. We include a short discussion of what may happen if social planner ignores changes in the credit constraint or fluctuation in house prices. We also underline the importance of the money multiplier and its connection to credit constraints and the relative price of collateral. Finally, in Section 4 we provide a short discussion of the factors which can affect the LTV ratio. Section 5 concludes.

2 Model

In this section we present a stylized New Keynesian economy with binding collateral constraint and zero rate of borrowing[2].

2.1 Households

A representative household has a utility function that includes consumption of goods, Y_t, valuable collateral (housing), h_t, and labor, L_t:

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t u(Y_t, h_t, L_t) = E_t \sum_{t=0}^{\infty} \beta^t \left(\log Y_t + \eta \log h_t - \lambda \frac{L_t^{v+1}}{v+1} \right), \tag{1}$$

where v is the labor supply elasticity parameter, η captures the individual household preferences towards units of housing and λ is preferences towards labor supply.

For their transactions, households can use cash, i.e. outside money, M_t, and the money credited by the banking system, i.e. inside money, B_t. The broad money can be spent to buy consumption goods and to invest in collateral

$$P_t Y_t + Q_t(h_t - h_{t-1}) \leq M_t + B_t, \tag{2}$$

where P_t is the price of final goods, Q_t is the price of collateral, and $h_t - h_{t-1}$ is investment in

[2]It is a simplified model with assumptions similar to Midrigan and Philippon (2011).

collateral. The amount of private credit is subject to a collateral constraint

$$B_t \leq \theta_t Q_t h_t, \tag{3}$$

which implies that household cannot borrow more than the fraction θ_t of its collateral value $Q_t h_t$. Parameter θ_t denotes the tightness of the borrowing constraint. A smaller value of θ_t implies a smaller loan size, whereas a high value means that a household may obtain a relatively large loan. Government implements monetary policy by printing new bills and distributing them across households as a lump sum transfer

$$M^s_{t+1} = M_t + T_{t+1}. \tag{4}$$

The loan has to be repaid immediately after households obtain their wage and dividend income. Let W_t be the nominal wage and Π_t be the profit of firms owned by households and paid in the form of dividends. Then at the end of the period the liquidity position of the household is

$$M^d_{t+1} = W_t L_t + \Pi_t + T_{t+1} - (1 + r_t) B_t. \tag{5}$$

2.1.1 Household's optimization problem

We will consider the case where broad money constraint (2) and collateral constraint (3) are binding. Combining these with equilibrium in the money market $M^s_{t+1} = M^d_{t+1} = M_{t+1}$ and (4), (5) we get a set of household budget constraints

$$\begin{aligned} M_t &= P_t Y_t + Q_t(h_t - h_{t-1}) - \theta_t Q_t h_t; && (6) \\ M_{t+1} &= W_t L_t + \Pi_t + T_t - (1 + r_t)\, \theta_t Q_t h_t. && (7) \end{aligned}$$

As we show in the appendix, maximization of household utility (1) subject to constraints (6) and (7), results in the following Euler equation

$$U'_{ht} + \theta_t \frac{Q_t}{P_t} \left(U'_{ct} - \beta E_t U'_{ct+1} \frac{P_t}{P_{t+1}} (1 + r_t) \right) = U'_{ct} \frac{Q_t}{P_t} - \beta E_t U'_{ct+1} \frac{Q_{t+1}}{P_{t+1}}. \tag{8}$$

The left-hand side of the equation shows the marginal benefit from an extra unit of collateral: it consists of a direct boost to utility, U'_{ht}, as well as an effect due to the possibility of using collateral to secure a loan. The value of the second source is proportional to credit tightness θ_t. In other words, smaller θ_t reduces the loan size and as a result the benefit from using house as a collateral falls.

We assume that the loans market is perfectly competitive and that the interest rate is at the zero bound, $r_t = 0$. Using the particular functional form of utility (1) and the assumption of zero interest rate, $r_t = 0$, assuming a constant and normalizing unity of housing $h_t = 1$, we transform equation (8) to the following form

$$\eta + \beta E_t [q_{t+1}] = \left(1 - \theta_t \left(1 - \beta E_t \left[\frac{P_t Y_t}{P_{t+1} Y_{t+1}}\right]\right)\right) q_t \tag{9}$$

where q_t is relative housing expenditure, defined as $q_t = \frac{Q_t h_t}{P_t c_t}$.

Finally, the first order condition with respect to L_t defines labor supply

$$-\frac{U_L'(C_t, L_t)}{U_C'(C_t, L_t)} = \lambda L_t^v Y_t = \frac{W_t}{P_t}. \tag{10}$$

2.2 Final good producers

We assume that final goods are imperfect substitutes and that consumption is defined over Dixit-Stiglitz (1977) basket of goods, $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$. The average price-level, P_t, is known to be $P_t = \left[\int_0^1 p_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$. The demand for each good is given by $Y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} Y_t$, where $p_t(i)$ is the nominal price of the final good produced in industry i and Y_t denotes aggregate demand. Each good is produced according to a linear technology using labor as the only input $Y_t(i) = L_t(i)$. There is an economy-wide labor market so that all firms pay the same wage for the same labor, $w_t(i) = w_t, \ \forall i$. All households provide the same share of labor to all firms. So that the total labor supply in (1) is defined as $L_t = \int_0^1 L_t(i) di$, which in combination with production function and demand relates output to labor income. $L_t = \int L_t(i) di = Y_t \int \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di = Y_t \Delta_t$, where Δ_t is the measure of price dispersion: $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$. The real wage, $w_t = \frac{W_t}{P_t}$, then is

$$w_t = \lambda Y_t^{v+1} \Delta_t^v. \tag{11}$$

2.2.1 Representative firm's price setting

We will model price stickiness according to Calvo (1983). Each period a fixed proportion of firms adjust prices. Those firms choose the nominal price which maximizes their expected profit given that they have to charge the same price in k periods time with probability α^k. The real profit can be written as $\Pi(i) = \frac{p_t(i)}{P_t} Y_t(i) - \phi_t w_t L_t(i)$, where ϕ_t is a cost-push shock. We assume that firms are price takers and cannot affect any aggregate variables. Let p_t' denote the choice of nominal price by a firm that is permitted to re-price in period t. Then the firm's objective is to chose p_t' to maximize the following sum

$$\max E_t \sum_{k=0}^{\infty} \alpha^k \beta^k \left[\frac{p_t'}{P_{t+k}} \left(\frac{p_t'}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} - \phi_t w_t \left(\frac{p_t'}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right]. \tag{12}$$

The first order condition implies

$$\left(\frac{p_t'}{P_t} \right) = \frac{\left(\frac{\varepsilon}{\varepsilon - 1} \right) E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\phi_{t+k} w_{t+k} Y_{t+k} (P_t/P_{t+k})^{-\varepsilon} \right]}{E_t \sum_{k=0}^{\infty} (\alpha\beta)^k (P_t/P_{t+k})^{1-\varepsilon} Y_{t+k}}. \tag{13}$$

It is useful to introduce new variables, X_t and Z_t, for the discounted expected real revenue and costs of the firm. We define them as $X_t = E_t \sum_{k=0}^{\infty} (\alpha\beta)^k (P_t/P_{t+k})^{1-\varepsilon} Y_{t+k}$, $Z_t = E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\phi_{t+k} w_{t+k} Y_{t+k} (P_t/P_{t+k})^{-\varepsilon} \right]$. The price index will evolve according to the following law of motion, $P_t = \left[(1-\alpha) p_t^{1-\varepsilon} + \alpha P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$, which can be rewritten as $\frac{p_t'}{P_t} = \left[\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1-\alpha} \right]^{\frac{1}{1-\varepsilon}}$. Formula (13) then can be represented in VAR form as a block of three equations $X_t \left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1} Z_t$, $X_t = Y_t + a\beta E_t \pi_{t+1}^{\varepsilon-1} X_{t+1}$ and $Z_t = \phi_t w_t Y_t + a\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}$. Finally, because relative prices of the firms that do not change their prices in period t fall by the rate of inflation, the law of motion for the measure of price dispersion is $\Delta_t = \alpha \Delta_{t-1} \pi_t^{\varepsilon} + (1-\alpha) \left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a} \right)^{-\frac{\varepsilon}{1-\varepsilon}}$.

2.3 Government's optimisation problem

Policy maker maximizes household's utility function with awareness that house supply is constant and normalized to 1, $h_t = 1$,

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \left(\log Y_t - \frac{Y_t^{v+1} \Delta_t^{v+1}}{v+1} \right), \tag{14}$$

subject to a set of constraints imposed by private agents' behavior:

$$
\begin{aligned}
\eta + \beta E_t\left[q_{t+1}\right] &= \left(1 - \theta_t\left[1 - \beta E_t\left(1 + r_t\right)\left(\frac{Y_t}{\pi_{t+1} Y_{t+1}}\right)\right]\right) q_t & (15)\\
X_t &= Y_t + a\beta E_t \pi_{t+1}^{\varepsilon-1} X_{t+1} & (16)\\
Z_t &= \phi_t \lambda Y_t^{v+2} \Delta_t^v + a\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1} & (17)\\
X_t \left(\frac{1 - \alpha\pi_t^{\varepsilon-1}}{1-a}\right)^{\frac{1}{1-\varepsilon}} &= \frac{\varepsilon}{\varepsilon - 1} Z_t & (18)\\
\Delta_t &= \alpha \Delta_{t-1} \pi_t^{\varepsilon} + (1-\alpha)\left(\frac{1 - \alpha\pi_t^{\varepsilon-1}}{1-a}\right)^{\frac{\varepsilon}{\varepsilon-1}} & (19)\\
\frac{M_t}{P_t Y_t} &= 1 - \theta_t q_t & (20)\\
T_t &= M_t - M_{t-1} & (21)\\
\pi_t &= \frac{P_t}{P_{t-1}}, & (22)
\end{aligned}
$$

In the appendix we show that optimal steady state is achieved under price stability, that is inflation is stable ($\pi = 1$). Moreover, similarly to Midrigan and Philippon (2011), we obtain that steady state output, Y, does not depend on the credit constraint $\overline{\theta}$. However value of $\overline{\theta}$ will positively affect relative housing expenditure, $\overline{q} = \frac{\eta}{\left(1-\overline{\theta}\right)(1-\beta)}$, and therefore equilibrium real house price $\frac{Q}{P} = \overline{q}Y$. It will also define the broad money multiplier, $m_t = \frac{M_t + B_t}{M_t}$. Since broad money, $M_t + B_t$, equals to the total expenditure, we can compute the money multiplier from (20)

$$
m_t = \frac{M_t + B_t}{M_t} = \frac{P_t Y_t}{M_t} = \frac{1}{1 - \theta_t q_t}. \tag{23}
$$

This positive relation between the money multiplier, m_t, and the credit constraint, θ_t, and the relative collateral value, q_t, will drive all our results.

2.4 Log-linearized equations

To make our model tractable and comparable to standard New Keynesian version, we log linearise equations (15)-(22) around the zero inflation steady state[3].

$$
\begin{aligned}
\beta\widehat{q}_{t+1} &= \widehat{q}_t(1-\overline{\theta}(1-\beta(1+r))) - \widehat{\theta}_t\overline{\theta}(1-\beta(1+r)) + r\beta\overline{\theta}\widehat{r}_t + \beta\overline{\theta}(1+r)(\widehat{Y}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{Y}_{t+1}); && (24)\\
\widehat{X}_t &= (1-a\beta)\widehat{Y}_t + a\beta E_t\left(\widehat{X}_{t+1} + (\varepsilon-1)\widehat{\pi}_{t+1}\right); && (25)\\
\widehat{Z}_t &= (1-\alpha\beta)\left[(v+2)\widehat{Y}_t + \widehat{\phi}_t\right] + \alpha\beta E_t\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{Z}_{t+1}\right); && (26)\\
\widehat{Z}_t &= \widehat{X}_t + \frac{a}{1-a}\widehat{\pi}_t; && (27)\\
\widehat{\Delta}_t &= \alpha\widehat{\Delta}_{t-1}; && (28)\\
\widehat{M}_t &= \widehat{P}_t + \widehat{Y}_t - \frac{\overline{\theta}\overline{q}}{1-\overline{\theta}\overline{q}}\left(\widehat{\theta}_t + \widehat{q}_t\right); && (29)\\
\widehat{M}_t &= \widehat{M}_{t-1} + \widehat{T}_t; && (30)\\
\widehat{\pi}_t &= \widehat{P}_t - \widehat{P}_{t-1}. && (31)
\end{aligned}
$$

2.4.1 Social welfare

Applying the Benigno and Woodford (2012) method to the non-linear problem (14, 15-22) we receive a pure quadratic approximation to the social objective. As we can see, it consists of the squares of output and inflation gaps similar to Benigno and Woodford (2005),

$$
U_t = -\frac{1}{2}E_t\sum_{s=1}^{\infty}\beta^s(\alpha_C\left(\widehat{Y}_{t+s} + \alpha_\phi\widehat{\phi}_{t+s}\right)^2 + \alpha_\pi\widehat{\pi}^2_{t+s}) + O3 + tip. \qquad (32)
$$

Here α_C and α_π are the policy maker's preferences towards the output gap and inflation respectively, $-\alpha_\phi\widehat{\phi}_t$ is the target level of output, which is in inverse relation to the cost push shock, *tip* denotes terms that are independent of policy maker's choices. Coefficients α_C, α_π and α_ϕ are all positive and computed in appendix.

[3] All variables with hats here are expressed in terms of percentage deviations from steady state.

2.4.2 Private sector behavior constraints

Equations (25)-(27) can be combined to form a New Keynesian Phillips Curve (33). Equation (28) shows that the relative price dispersion term is of second order importance and can be ignored. Finally, (29)-(31) can be combined into (35), which relates monetary policy instrument, T_t, to inflation and output. Therefore, we receive a reduced system of three equations

$$\widehat{\pi}_t = \frac{1-a}{a}(1-\alpha\beta)\left[(v+2)\widehat{Y}_t+\widehat{\phi}_t\right]+\beta E_t\widehat{\pi}_{t+1}; \tag{33}$$

$$\beta\widehat{q}_{t+1} = \widehat{q}_t(1-\overline{\theta}(1-\beta(1+r)))-\widehat{\theta}_t\overline{\theta}(1-\beta(1+r))+r\beta\overline{\theta}\widehat{r}_t+\beta\overline{\theta}(1+r)(\widehat{Y}_t-E_t\widehat{\pi}_{t+1}-E_t\widehat{Y}_{t+1}); \tag{34}$$

$$\widehat{\pi}_t = \widehat{Y}_{t-1}-\widehat{Y}_t+\frac{\overline{\theta}\overline{q}}{1-\overline{\theta}\overline{q}}\left(\widehat{\theta}_t-\widehat{\theta}_{t-1}+\widehat{q}_t-\widehat{q}_{t-1}\right)+\widehat{T}_t. \tag{35}$$

Expression (35) is the essence of the paper: whatever is the target for inflation and output dynamics, one cannot neglect the fluctuations in relative house value, $\widehat{q}_t$, or credit availability, $\widehat{\theta}_t$. In other words, for given dynamics of $\widehat{\pi}_t$ and $\widehat{Y}_t$, monetary policy $\widehat{T}_t$, should be adjusted to the shock in the credit constraint and the change in relative house expenditure ($\widehat{q}_t$).

Our objective is to find the first order approximation to the optimal policy reaction function. We will allow two shocks to perturb our economy: a cost-push shock, $\widehat{\phi}_t$, and a credit shock, $\widehat{\theta}_t$. We assume that $\widehat{\phi}_t$ and $\widehat{\theta}_t$ follow two independent $AR(1)$ processes

$$\widehat{\theta}_t = \rho_\theta\widehat{\theta}_{t-1}+\xi_{\theta t}; \tag{36}$$

$$\widehat{\phi}_t = \rho_\phi\widehat{\phi}_{t-1}+\xi_{\phi t}. \tag{37}$$

The linear approximation to optimal policy can be found by maximizing the second order approximation to social welfare (32) subject to linear constraints (33)-(35).

3 Optimal monetary policy

3.1 Reaction to credit shock

To understand more clearly the optimal reaction of the policy to the credit shock, we first consider the case when the price markup is constant, that is $\widehat{\phi}_t = 0$. If credit shock is the only source of instability, government can achieve zero losses perfectly stabilizing both output and inflation. We formalize this statement in Proposition 1.

Proposition 1 *In the absence of the cost push shock, credit market contraction can be perfectly neutralized. Indeed, the policy maker can achieve perfect price and output stabilization, that is* $\widehat{\pi}_t = 0$ *and* $\widehat{Y}_t = 0$.

Proof. *If* $\widehat{\phi}_t = 0$, *output and price stability are not in contradiction with system of constraints (33)-(35).* ■

Note, complete price and output stabilization delivers maximum value of social welfare (32). Hence corresponding policy is optimal and the optimal monetary policy rule in this case follows from equation (35) if one set inflation and output deviations to zero.

$$\widehat{T}_t = -\frac{\overline{\theta}\overline{q}}{1-\overline{\theta}\overline{q}}\left(\widehat{\theta}_t - \widehat{\theta}_{t-1} + \widehat{q}_t - \widehat{q}_{t-1}\right). \tag{38}$$

As a result, we obtain an example where credit constraint, $\widehat{\theta}_t$, and collateral price, $\widehat{q}_t$, are not targeted directly by the government, however, they are the only arguments in the government reaction function. This is exactly the case discussed in Svensson (2009). The social planner does not care about financial sector per se, but since it affects inflation and output volatilities, policy maker has to consider the change in financial environment when it implements its monetary policy.

The optimal monetary policy rule (38) has straightforward interpretation. Recalling collateral constraint (3) and taking into consideration price and output stability we may write it as

$$\widehat{T}_t = k\left(\widehat{B}_{t+1} - \widehat{B}_t\right), \tag{39}$$

where we define $k = \frac{\overline{\theta}\overline{q}}{1-\overline{\theta}\overline{q}}$. Coefficient k has an important economic meaning. In the steady state, collateral constraint (3) implies

$$\overline{\theta}\overline{q}PY = B, \tag{40}$$

while cash-in-advance constraint (2) implies

$$PY = \overline{M} + \overline{\theta}\overline{q}PY. \tag{41}$$

Combining (40) with (41) one can compute the debt to money ratio

$$B = kM.$$

Therefore k is the marginal effect on loans of a change in base money. In other words, a 1 dollar expansion of the monetary base will create k dollars of loans: $k = \frac{dB}{dM}$.

Equation (39) tells us how much central bank should expand its monetary base. The expansion should be just enough to offset the reduction of debt capacity.

3.2 Cost of inactive government

In this section we will numerically assess the value of monetary policy. For that purpose we will compare optimal policy generating (46) with policy which neglects changes in the credit market. Our alternative policy is

$$\widehat{T}_t = 0. \tag{42}$$

If a negative credit shock hits the economy and the government does not provide any monetary response to that shock, $\widehat{T}_t = 0$, then one would expect both deflation and a significant fall in GDP.

Figure 1. Inflation and output:

Impulse response to negative credit shock, $\widehat{T}_t = 0$.

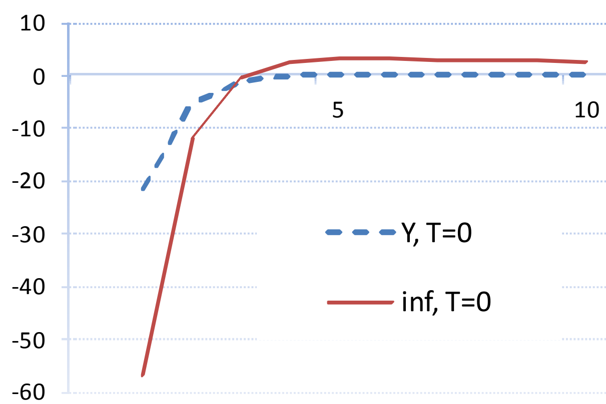


Figure 1 shows that a 1% drop in the loan to collateral value ratio reduces output by 0.2%. If our model is reasonably calibrated, then a 20% drop in the mortgage LTV ratio will result in a 4% fall in GDP in the absence of quantitative easing. It would be even more damaging for consumer prices. In the absence of monetary transfers the model economy will experience a 12% deflation.

3.3 Credit Shocks and the Money Multiplier

It is well known that the money multiplier fell dramatically after the recent financial crisis. Now the monetary authorities have to expand the monetary base to a much larger extent in order to achieve the same expansion of broad money. The importance of the money multiplier is discussed in Goodhart (2009), who criticises macroeconomics literature for ignoring the money multiplier and for failing to model it formally. That criticism is not entirely fair, since the behavior of the money multiplier was a popular research topic in 1990s. See for example Bernanke (1983), Bernanke and Blinder (1988), Beenstock (1989), and more recently, Freeman and Kydland (2000). However, as the money multiplier was relatively stable for over 20 years, it became a concern of second order importance. Although the model we consider is very simple, it manages to identify two variables which may explain fluctuation in the money multiplier as it is computed in equation (23). First there is θ_t, household's borrowing constraint. If we simply consider mortgage contract offered before the crisis, the loan to value ratio was up to 110% in the UK. After the crisis it fell to 90% or by almost 20%.

The second variable, q_t, is defined as

$$q_t = \frac{Q_t h_t}{P_t Y_t}. \tag{43}$$

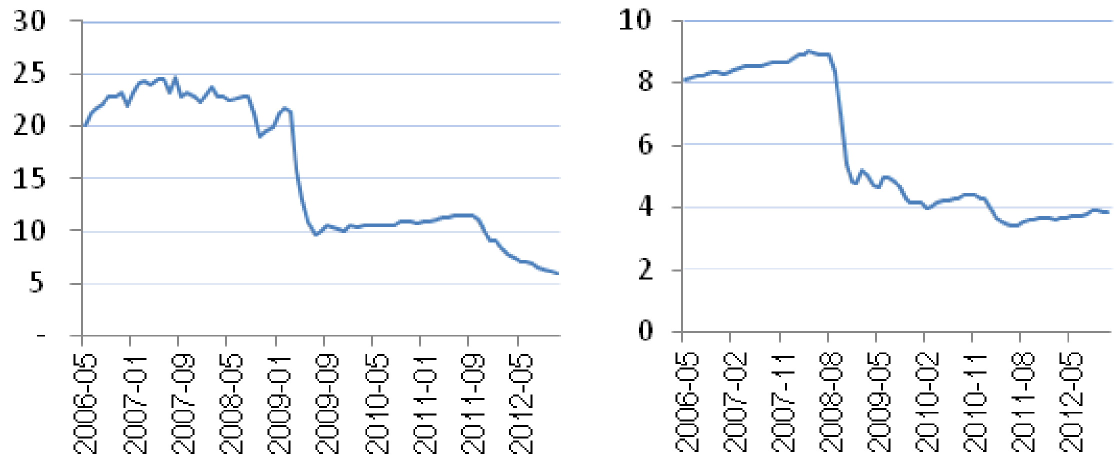
In a relatively stable economy, where h and Y do not change, the proxy for q will be the real price for collateral. If we refer to the mortgage market, the collateral are houses and the real house price index will be a proxy for q. Therefore, the fall of house prices should reduce the money multiplier. As the money multiplier is significant for the transmission of monetary policy (Bernanke and Blinder, 1988; Goodhart, 2009; Abrams, 2011), its fluctuation definitely should be taken into account when monetary policy is designed. As house prices and the loan to value ratio affect the money multiplier, they can not be neglected by the monetary authorities too.

As we have noted before, the money multiplier experienced a significant fall after the last financial crisis. Figure 2 shows the dynamics of the M2 multiplier in the USA and the M4 multiplier in the UK.

Figure 2. Money multiplier after the Crisis

UK: M4/MO

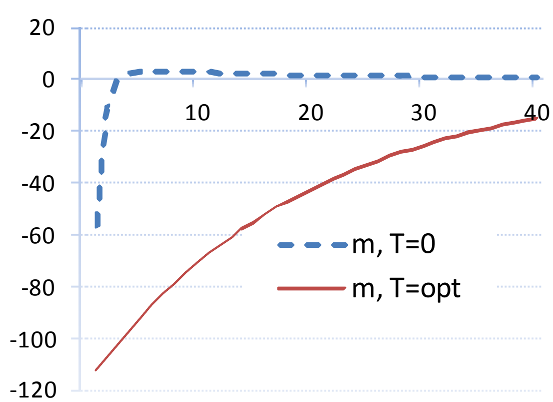
USA: M2 multiplier



Source: FRED and BoE database

The slump of the money multiplier is consistent with our model. Although optimal monetary policy can stabilize output and price fluctuations, it causes an even stronger and much more persistent decrease in the money multiplier than a policy of inaction. Both policies imply a dramatic fall in the money multiplier as we can see from Figure 3, but the policy of stabilisation almost doubles the size of the fall and causes a much slower recovery.

Figure 3. Money multiplier IRFs to negative credit shock



We draw Figure 3 based on linear approximation to (23)

$$\widehat{m_t} = k(\widehat{\theta}_t + \widehat{q}_t). \tag{44}$$

It is easy to notice that the optimal policy rule in absence of cost-push shock (38), implies that optimal transfers should be equal negative change of money multiplier

$$\widehat{T}_t = -(\widehat{m}_t - \widehat{m}_{t-1}). \tag{45}$$

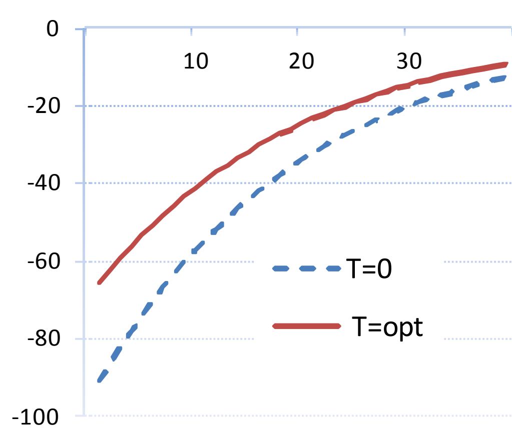
Therefore, Figure 3 shows that the optimal policy should perform a large expansion immediately after reduction in LTV and then conduct gradual and modest contraction in all consequent periods.

3.4 Credit shock and house prices

It is very intuitive that relative price of collateral should react to the worsening in the *loan to value ratio*. The value of collateral in our model has two components. The first one comes directly in the utility function (as housing, for example). The second one is indirect and associated with the use of collateral for borrowing purposes. The bigger is the LTV, θ_t, the larger is the indirect component of the collateral value and, therefore, the higher is the price of collateral. Formally we can observe it from equation (8). That is why the negative shock to θ_t should result in falling house prices.

Lower prices for collateral, in turn, further reduce the amount of available credit. As a consequence, households have less money to finance their consumption and purchase additional housing units. Figure 4 shows how house prices react to the tightening in the household's borrowing constraint in two different cases. The first case is when government implements the optimal policy rule. The second case is when it keeps the monetary base constant, $\widehat{T}_t = 0$. When a negative credit market shock hits the economy, house prices decline in both cases, but optimal policy helps to reduce the fall by 20% approximately.

Figure 4: House prices IRFs to negative credit shock.



3.5 Optimal policy with cost-push shock

Now we will consider an economy with a cost-push shock. The optimal policy in this case generates the same dynamics for the output gap and inflation as the basic New Keynesian model and is presented in Proposition 2.

Proposition 2 *Optimal policy implies the following dynamics inflation dynamics*

$$\widehat{\pi}_t = \rho\left(\widehat{Y}_{t-1} - \widehat{Y}_t - \alpha_\phi\left(\widehat{\phi}_{t-1} - \widehat{\phi}_t\right)\right), \tag{46}$$

where $\rho = \frac{\alpha_C}{\alpha_\pi}\frac{a}{(1-\alpha\beta)(v+2)(1-a)}$.

Proof. *Provided in Appendix.* ■

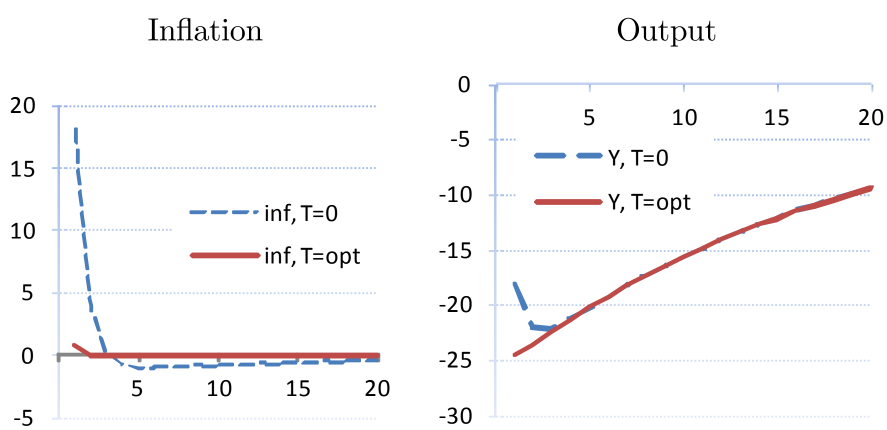
Proposition 2 shows that optimal inflation dynamics (46) should be the same as in the basic new Keynesian model of Clarida, Gali and Gertler (2000), which does not featured credit constraint. However, to achieve these dynamics monetary policy has to react the change in credit conditions

$$\widehat{T}_t = -\widehat{\pi}_t - \left(\widehat{Y}_t - \widehat{Y}_{t-1}\right) - \frac{\overline{\theta}\overline{q}}{1-\overline{\theta}\overline{q}}\left(\widehat{\theta}_t - \widehat{\theta}_{t-1} + \widehat{q}_t - \widehat{q}_{t-1}\right). \tag{47}$$

3.5.1 Cost-push shock, house prices and inflation

As expected, the optimal policy stabilizes inflation. In Figure 5 we provide impulse responses to cost push shock.

Figure 5: Cost-push shock IRFs



When the loan to value ratio is constant and only cost-push shocks hit the economy, there is a trade off between inflation and house price stability. The policy of inaction in the absence of credit shocks implies zero house inflation. Indeed, consider dynamics (34)-(35) with $\widehat{T}_t = 0$, $\widehat{\theta}_t = 0$.

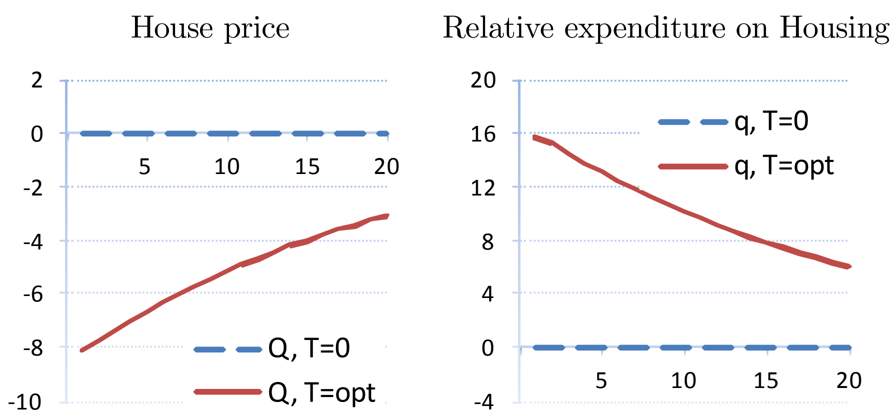
Proposition 3 *If $\widehat{\theta}_t = 0$ policy $\widehat{T}_t = 0$ results in house price stability and the stability of the relative collateral value, $\widehat{q}_t = 0$.*

Proof. *Provided in Appendix.* ■

Figure 5 shows that in the absence of credit shock, stabilization of house prices will result in higher volatility of the CPI inflation. That result is consistent with the findings of Iacoviello (2005).

It is interesting to see that the money multiplier, which is proportional to relative housing expenditure, increases with the cost push shock, when optimal policy is implemented. However, house prices declines in this case, but at smaller extent than the decline in output.

Figure 6: Cost-push shock IRFs



4 Loan to value ratio

We have shown that government ought to react to the loan to value ratio when it stabilizes the economy. In that discussion we considered θ_t to be exogenously given. Perhaps one of the most important questions is to identify the factors which explain the fluctuation in θ_t.

Endogenising the loan to value ratio can have a number of very important policy implication. For example, if LTV ratio equals the effective recovery rate of mortgages, so that direct lending to households compromises the balance sheet of central bank. In that case Help to Buy and Start Up loans will result in budget losses and Funding for Loans scheme in this case could result in yet more non-performing loans on the central bank's balance. We are not aware of any model which can assess these consequences of it[4].

[4] In our model, the central bank simply increases their liabilities in a form of outstanding cash without any back up on the asset side.

However, although the recovery rate is highly correlated with default risk (Mora 2012), there are some other explanatory factors which can be influenced by government.

4.1 Expected collateral inflation

One of the explanatory variables may be expected house price inflation as in Iacoviello (2005). That can easily be modelled by substitution of (36) with (48)

$$\theta_t = \delta E_t \left(\widehat{Q}_{t+1} - \widehat{Q}_t \right) + u_t \tag{48}$$

where $\delta < 1$, and u_t is a persistent shock unrelated to expected changes in house prices.

Shocks u_t in this case can be regarded as a shock to expected future house prices and that is another factor for consideration of monetary authorities. In some cases expectation shocks to expectation does not reflect the changes in fundamentals. Then the central bank will not compromise its balance sheet by buying collateral and keep them for a longer time period until negative shock dies out or be offset by a favorable shock.

In the modified model, we received a very similar impulse response functions to unexpected change in u_t and still we found that optimal policy can completely stabilize output and inflation when credit shock affects the economy. Similarly, in response to cost-plush shocks, policy $\widehat{T}_t = 0$ stabilizes house prices but causes positive and relatively large response in consumer prices.

4.2 Liquidity, transaction costs and the value of collateral

An increase of the collateral value can be a good way to enlarge the money multiplier. The attractiveness of collateral may increase with liquidity. A positive relation between the liquidity of collateral and availability of funding is discussed in Brunnermeier and Pedersen (2009). Securitization of collateral is one way to enhance liquidity. Different types of securitization were used for American housing market. Thus, according to Frame and White (2005), Federal Home Loan Mortgage Corporation-commonly known as Freddie Mac - was created to support mortgage markets by securitizing mortgages. That arrangement worked successfully for at least 30 years before the last crisis. Funding for Lending scheme is in line with that reasoning. The possibility of collateral swap for T-Bills should increase the value of collateral which can be used by commercial bank to secure liquidity.

However, collateral securitization could also add to the risk and even generate additional moral hazard problems (Ashcraft and Schuermann, 2008). In this context Freddie Mac arrangements were safer than new measures proposed by UK. The mortgages illegible for securitization with Freddie Mac were usually required 20% downpayment, while Funding for Lending Scheme does not specify the quality of the loans which can be used in the scheme (Bank of England ,2012). British "Help to Buy" could be even riskier, such the Government will "loan up to 20% of the value of your new build home and "mortgage guarantee" where lenders will be incentivised to make more mortgages available for people with small deposits." (HM Treasury, 2013). It could mean that the government intends to provide for risk margin charged by lender as an insurance against construction risks.

Apart from the expected value of collateral, the loan to value ratio should depend on the recovery rate for non-performing loans. The recovery rate negatively depends on the transaction costs associated with selling of the repossessed assets. Any taxes collected during that process would negatively contribute to LTV. One straightforward recommendation can be to abolish stamp duties for repossessed properties.

The other way to increase the value of collateral is to encourage the construction and consumption of housing. According to Frame and White (2005), for that purpose the USA government use tax deduction of mortgage interest and accelerated depreciation on rental housing.

4.3 Stability and credit constraint

The optimal policy in a simple framework without cost-push shock can be reduced to two equations

$$\beta E_t \widehat{q}_{t+1} = \widehat{q}_t(1-\overline{\theta}(1-\beta)) - \widehat{\theta}_t\overline{\theta}(1-\beta)); \quad (49)$$

$$\widehat{\theta}_{t+1} = \rho_\theta \widehat{\theta}_t - \xi_{\theta t+1}. \quad (50)$$

Blanchard and Kahn (1980) formulated necessary conditions for dynamic linear R.E. system to have a unique solution. It states that there must be the same number of eigenvalues larger than 1 in modulus as there are forward looking variables. To satisfy this condition, following relation is necessary and sufficient, $\frac{1-\overline{\theta}(1-\beta)}{\beta} > 1$, which is true if and only if $\overline{\theta} < 1$.

In case $\overline{\theta} > 1$ we will have indeterminacy and sunspot equilibrium. Practically, $\overline{\theta} > 1$, was observed in $2006 - 2007$, when new mortgages were available with up to 110% loan to value ratio. Such high LTV might have been partly responsible for the house bubble and the consequential financial crisis. Thus, according to Korteweg and Sorensen (2012) LTV significantly contributed to the probability of foreclosure sales. In this light, Hong Kong Monetary Authority (2011) suggestion to use LTV as a policy tool of the macroprudential regulation looks very reasonable.

5 Conclusion

We have shown that contrary to some other findings presented in the literature (such as Bernanke and Gertler 2001, Iacoviello 2005), a simple stylized macro model could yield the results which are in favor of including credit market parameter into the optimal monetary policy rule. In particular, monetary authorities should adjust their rule to unfavorable changes in loan conditions, such as a fall in the loan to value ratio or the relative price of collateral, i.e. the real house price index.

We derived our results in a model where social welfare consists of output and inflation gaps. Therefore, credit market variables are not a part of the government direct objective. However, as mentioned in Svensson (2009), credit capacities affect output and inflation through the household's constraints. Lower credit capacities reduce demand causing deflation and because prices are sticky, output also falls. We have shown that an exogenous decrease in the loan to value ratio can be offset with expansionary monetary policy in such way that credit tightening will neither affect output nor the consumer price index.

We connected our results to the money multiplier which is the most important variable in propagation of monetary policy as discussed in Bernanke and Blinder (1988) and recently in Abrams (2011). Indeed, it is very intuitive that monetary base expansion should be larger when the money multiplier falls. And since the money multiplier depends on the loan to value ratio and real house prices, optimal monetary policy should react to their fluctuations. Finally, we have shown that optimal policy generates a large and persistent fall in the money multiplier in response to a credit shock.

Although our model is helpful in providing some justification for Quantitative Easing and explaining the fall in the money multiplier, there is a number of important extensions that should be addressed. Firstly, the volatility of the loan to value ratio requires economic explanation. That would allow for a better assessment of a number of currently proposed or adopted policy measures. Secondly, the assets of central bank are not modelled directly and, therefore it is impossible to see how the risk taken onto the central bank balance sheet will affect the economy. If assets are risky, the central bank will face difficulties when it decides to implement monetary contraction. Finally, the money multiplier *per se* does not generate any value in our model, and the steady state output value does not depend on any financial variables. This is not the case according to King and Levine (1993) and Freeman and Kydland (2000) who found that total borrowing by the non-financial sector positively affects economic growth.

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6 Appendix

6.1 First order conditions for household optimization

The household maximises the expected discounted sum of future utility (1) subject to constraints (6-7). The corresponding Lagrangian is:

$$\begin{aligned}
\mathcal{L} &= E_t \sum_{t=0}^{\infty} \beta^t [\left(\log C_t - \frac{L_t^{v+1}}{1+v} + \eta \log h_t \right) \\
&\quad -\mu_t (P_t C_t + Q_t(h_t - h_{t-1}) - \theta_t Q_t h_t - M_t) \\
&\quad -\Lambda_t (-M_{t+1} + W_t L_t + \Pi_t + T_t - (1+r)\,\theta_t Q_t h_t)].
\end{aligned}$$

Where μ_t and Λ_t are Lagrange multipliers. The first order conditions with respect to consumption (C_t), housing quantity (h_t) and money (M_{t+1}) are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = U_c^{'}(C_t) - \mu_t P_t = 0; \tag{51}$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = U_h^{'} - \mu_t Q_t + \beta \mu_{t+1} Q_{t+1} + \mu_t \theta_t Q_t + \Lambda_t\,(1+r)\,\theta_t Q_t = 0; \tag{52}$$

$$\frac{\partial \mathcal{L}}{\partial M_{t+1}} = \beta \mu_{t+1} + \Lambda_t = 0. \tag{53}$$

Combining (51) and (53) we express Lagrange multipliers Λ_t and μ_t. After this substitution, equation (52) becomes

$$U_h^{'} - U_c^{'}(C_t)\frac{Q_t}{P_t} + \beta U_c^{'}(C_{t+1})\frac{Q_{t+1}}{P_{t+1}} + U_c^{'}(C_t)\frac{Q_t}{P_t}\theta_t - \beta \frac{U_c^{'}(C_{t+1})}{P_{t+1}}\,(1+r)\,\theta_t Q_t = 0. \tag{54}$$

This is the same as (8) in the main text.

6.2 Optimal steady state

Following Benigno and Woodford (2012) we will find the best steady state for the optimal commitment policy from a timeless perspective. The policy maker will maximize household utility (14) subject to constraints (15-22). It is easy to see that constraints (15) and (20-22) are not binding.

Therefore, we write a Lagrangian to reduced form with constraints (16-19).

$$
\begin{aligned}
\mathcal{L}_t &= E_t \sum_{t=0}^{\infty} \beta^t \{ \log Y_t - \lambda \frac{Y_t^{v+1} \Delta_t^{v+1}}{v+1} & (55)\\
&+ \Lambda_{2t} (-X_t + Y_t + a\beta E_t \pi_{t+1}^{\varepsilon-1} X_{t+1}) & (56)\\
&+ \Lambda_{3t} \left(-Z_t + \phi_t \lambda Y_t^{v+2} \Delta_t^v + a\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1} \right) & (57)\\
&+ \Lambda_{4t} \left[X_t \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{1}{1-\varepsilon}} - \frac{\varepsilon}{\varepsilon - 1} Z_t \right] & (58)\\
&+ \Lambda_{5t} \left[-\Delta_t + \alpha \Delta_{t-1} \pi_t^{\varepsilon} + (1-\alpha) \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right] \}. & (59)
\end{aligned}
$$

The first order conditions are

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial \pi_t} \pi_t &= +(\varepsilon - 1) \Lambda_{2t-1} a \pi_t^{\varepsilon-1} X_t \\
&+ \varepsilon \Lambda_{3t-1} a \pi_t^{\varepsilon-1} Z_t + \Lambda_{4t} X_t \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{1}{1-\varepsilon}} \frac{\alpha \pi_t^{\varepsilon-1}}{1 - \alpha \pi_t^{\varepsilon-1}} \\
&+ \Lambda_{5t} \varepsilon \Delta_{t-1} \alpha \left(\pi_t^{\varepsilon} - \pi_t^{\varepsilon-1} \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{1}{\varepsilon-1}} \right); & (60)\\
\frac{\partial \mathcal{L}_t}{\partial Y_t} Y_t &= 1 - \lambda Y_t^{v+1} \Delta_t^{v+1} + \Lambda_{2t} Y_t + \Lambda_{3t} \phi_t \lambda Y_t^{v+2} \Delta_t^v (v+2); & (61)\\
\frac{\partial \mathcal{L}_t}{\partial \Delta_t} &= -\lambda Y_t^{v+1} \Delta_t^v + \Lambda_{3t} v \phi_t \lambda Y_t^{v+2} \Delta_t^{v-1} - \Lambda_{5t} + \beta \alpha \Lambda_{5t+1} \pi_t^{\varepsilon}; & (62)\\
\frac{\partial \mathcal{L}_t}{\partial X_t} &= -\Lambda_{2t} + a \pi_t^{\varepsilon-1} \Lambda_{2t-1} + \Lambda_{4t} \left(\frac{1 - \alpha \pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{1}{1-\varepsilon}}; & (63)\\
\frac{\partial \mathcal{L}_t}{\partial Z_t} &= -\Lambda_{3t} + a \pi_t^{\varepsilon} \Lambda_{3t-1} - \Lambda_{4t} \frac{\varepsilon}{\varepsilon - 1}; & (64)
\end{aligned}
$$

We can easily verify that prices are stable in steady state, that is $\pi = 1$. From constraints (15-22) and first order conditions (60-64) we compute the steady state values for endogenous variables and Lagrange multipliers

$$
\begin{array}{ll}
1 = \frac{\varepsilon}{\varepsilon-1} \phi \lambda Y^{v+1}; & \Lambda_2 = \frac{1 - \lambda Y^{v+1}}{Y(v+1)}; \\
X = \frac{Y}{1 - a\beta}; & \Lambda_3 = -\Lambda_2 \frac{\varepsilon}{\varepsilon - 1}; \\
Z = \frac{\varepsilon - 1}{\varepsilon} X; & \Lambda_4 = \Lambda_2 (1-a); \\
\Delta = 1; & \Lambda_5 (1 - \beta\alpha)(v+1) = -\lambda Y^{v+1} - v.
\end{array}
\qquad (65)
$$

6.3 Linear approximation to the constraints of private behavior

We start with the log linear approximation to constraints (15-22) around the optimal steady state. As shown in Benigno and Woodford (2005) constraint (19) implies $\widehat{\Delta}_t = \alpha\widehat{\Delta}_{t-1} + O2$, and log deviation of the relative price dispersions is of second order importance when price stability is optimal. Therefore log linearisation of (15-18)

$$\beta\widehat{q}_{t+1} = \widehat{q}_t(1-\theta+\beta\theta) - (\beta-1)\theta\widehat{\theta}_t + \beta\theta\left(\widehat{Y}_t - E_t\widehat{Y}_{t+1} - E_t\widehat{\pi}_{t+1}\right); \tag{66}$$

$$\widehat{X}_t = (1-a\beta)\widehat{Y}_t + a\beta E_t\left(\widehat{X}_{t+1} + (\varepsilon-1)\widehat{\pi}_{t+1}\right); \tag{67}$$

$$\widehat{Z}_t = (1-\alpha\beta)\left[(v+2)\widehat{Y}_t + \widehat{\phi}_t\right] + \alpha\beta E_t\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{Z}_{t+1}\right); \tag{68}$$

$$\widehat{Z}_t = \widehat{X}_t + \frac{a}{1-a}\widehat{\pi}_t. \tag{69}$$

Here we will show derivation of the Phillips Curve. Firstly, combine equations (68) and (69):

$$\widehat{X}_t + \frac{a}{1-a}\widehat{\pi}_t = (1-\alpha\beta)\left[(v+2)\widehat{Y}_t + \widehat{\phi}_t\right] + \alpha\beta\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{X}_{t+1} + \frac{a}{1-a}\widehat{\pi}_{t+1}\right). \tag{70}$$

Then we subtract expression (67) and simplify to obtain the New Keynesian Phillips curve

$$\widehat{\pi}_t = \frac{1-a}{a}(1-\alpha\beta)\left[(v+2)\widehat{Y}_t + \widehat{\phi}_t\right] + \beta E_t\widehat{\pi}_{t+1}. \tag{71}$$

6.4 Second order approximation

Applying the Benigno and Woodford (2012) algorithm we will get the social welfare function which consists of the sum of squares of the output and inflation gaps. In particular, Benigno and Woodford (2012) show that the second order approximation to social welfare can be computed as a sum of pure second order terms.

$$U_t = E_t\sum_{t=0}^{\infty}\beta^t u_t = E_t\sum_{t=0}^{\infty}\beta^t\left[S(u_t) + \sum_i \Lambda_i S(F_i)\right], \tag{72}$$

where F_i is the dynamic constraint imposed by household behavior and Λ_i is the value of the corresponding Lagrange multiplier in steady state. Furthermore, $S(\cdot)$ is a functional defined on twice differentiable functions of multiple arguments $F(X_t)$, $X_t = [X_{1t}, ...X_{nt}]$ as following

$$S(F(X_t)) = \widehat{X}_t' X' \nabla^2 F(X) X \widehat{X}_t = \sum_{jk} \frac{\partial^2 F(X)}{\partial X_j \partial X_k} X_k X_j \widehat{X}_{kt} \widehat{X}_{jt},$$

where $\widehat{X}_{kt}$ is the log deviation of variable X_{kt} from its steady state value X_k. To implement that algorithm we need to apply functional S to constraints (16-19) since all the other constraints are not binding and corresponding Lagrange multipliers have zero values in the optimal steady state.

$$
\begin{aligned}
S_u &= S(\log Y_t - \lambda \frac{Y_t^{v+1} \Delta_t^{v+1}}{v+1}) = -\widehat{Y}_t^2 - v\lambda Y^{v+1} \widehat{Y}_t^2; && (73) \\
S_2 &= S(-X_t + Y_t + a\beta E_t \pi_{t+1}^{\varepsilon-1} X_{t+1}) = a\beta(\varepsilon-1)(\varepsilon-2) X E_t \widehat{\pi}_{t+1}^2 + 2a\beta(\varepsilon-1) X E_t \widehat{\pi}_{t+1} \widehat{X}_{t+1}; && (74) \\
S_3 &= S(-Z_t + \phi_t \lambda Y_t^{v+2} \Delta_t^v + a\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}) \\
&= (v+2)(v+1) \phi\lambda Y^{v+2} \widehat{Y}_t^2 + 2(v+2)\phi\lambda Y^{v+2} \widehat{Y}_t \phi_t + 2a\beta Z\varepsilon \widehat{\pi}_{t+1} \widehat{Z}_{t+1} + a\beta Z\varepsilon(\varepsilon-1) \widehat{\pi}_{t+1}^2; && (75) \\
S_4 &= S\left(X_t \left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{1}{1-\varepsilon}} - \frac{\varepsilon}{\varepsilon-1} Z_t \right) \\
&= 2\frac{a}{1-a} X \widehat{\pi}_t \widehat{X}_t + \frac{a}{1-a} X \left(\varepsilon \frac{a}{1-a} + \varepsilon - 2 \right) \widehat{\pi}_{t+1}^2; && (76) \\
S_5 &= S\left(-\Delta_t + \alpha \Delta_{t-1} \pi_t^{\varepsilon} + (1-\alpha) \left(\frac{1-\alpha\pi_t^{\varepsilon-1}}{1-a} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right) \\
&= \frac{a}{1-a} \left((1-a)\,\varepsilon(\varepsilon-1) + \varepsilon \frac{a}{1-a} + (\varepsilon-2) \right) \widehat{\pi}_t^2. && (77)
\end{aligned}
$$

Using steady state values (65) we can compute the welfare approximation (72)

$$W = S_u + \Lambda_2 S_2 + \Lambda_3 S_3 + \Lambda_4 S_4 + \Lambda_5 S_5. \tag{78}$$

First we simplify S_3. We will use (69) to substitute for $\widehat{\pi}_{t+1} \widehat{Z}_{t+1}$ term:

$$\widehat{Z}_{t+1} \widehat{\pi}_{t+1} = \widehat{X}_{t+1} \widehat{\pi}_{t+1} + \frac{a}{1-a} \widehat{\pi}_{t+1}^2 + O3.$$

$$\begin{aligned}
S_u &= -\left(1+v\lambda Y^{v+1}\right)\widehat{Y}_t^2; && (79)\\
\Lambda_2 S_2 &= \Lambda_2 a\beta(\varepsilon-1)\frac{Y}{1-a\beta}\left((\varepsilon-2)E_t\widehat{\pi}_{t+1}^2+2E_t\widehat{\pi}_{t+1}\widehat{X}_{t+1}\right); && (80)\\
\Lambda_3 S_3 &= -\Lambda_2\left(v+2\right)\left(v+1\right)Y\widehat{Y}_t^2+-\Lambda_2 2(v+2)Y\widehat{Y}_t\widehat{\phi}_t;\\
&\quad -\Lambda_2 2a\beta\frac{Y}{1-a\beta}\varepsilon\left(\widehat{X}_{t+1}\widehat{\pi}_{t+1}+\frac{a}{1-a}\widehat{\pi}_{t+1}^2\right)-\Lambda_2 a\beta\frac{Y}{1-a\beta}\varepsilon(\varepsilon-1)\widehat{\pi}_{t+1}^2; && (81)\\
\Lambda_4 S_4 &= 2\Lambda_2 a\frac{Y}{1-a\beta}\widehat{\pi}_t\widehat{X}_t+\Lambda_2 a\frac{Y}{1-a\beta}\left(\varepsilon\frac{a}{1-a}+\varepsilon-2\right)\widehat{\pi}_{t+1}^2; && (82)\\
\Lambda_5 S_5 &= -\frac{a}{1-a}\frac{\lambda Y^{v+1}+v}{\left(1-\beta\alpha\right)\left(v+1\right)}\left(\left(1-a\right)\varepsilon(\varepsilon-1)+\varepsilon\frac{a}{1-a}+(\varepsilon-2)\right)\widehat{\pi}_t^2; && (83)
\end{aligned}$$

Now we use that for any dynamic variable x_t,

$$\sum_{t=0}^{+\infty}\beta^t x_{t+1}=\frac{1}{\beta}\sum_{t=0}^{+\infty}\beta^t x_t-\frac{1}{\beta}x_0=\frac{1}{\beta}\sum_{t=0}^{+\infty}\beta^t x_t+tip$$

where x_0 is considered a "precommitted " variable which can not be changed because of commitment and therefore, it is regarded as "term independent of policy (tip)".

We finally can compute the welfare approximation (72).

$$U_t+O3=E_t\sum_{t=0}^{+\infty}\beta^t W=-E_t\sum_{t=0}^{+\infty}\beta^t\left[\alpha_C\left(\widehat{Y}_t+\alpha_\phi\widehat{\phi}_t\right)^2+\alpha_\pi\widehat{\pi}_t^2\right], \tag{84}$$

where parameters are defined as follows

$$\begin{aligned}
\alpha_C &= \left(1+v-2\lambda Y^{v+1}\right);\\
\alpha_\phi &= \frac{1-\lambda Y^{v+1}}{1+v-2\lambda Y^{v+1}}\frac{v+2}{v+1};\\
1 &= \frac{\varepsilon}{\varepsilon-1}\phi\lambda Y^{v+1};\\
\alpha_\pi &= \frac{a\left[\left(1-\lambda Y^{v+1}\right)\varepsilon+\left(\lambda Y^{v+1}+v\right)\left(\left(1-a\right)\varepsilon(\varepsilon-1)+\frac{\varepsilon-2+2a}{1-a}\right)\right]}{\left(1-a\right)\left(1-\beta\alpha\right)\left(v+1\right)}.
\end{aligned}$$

6.5 Solution to social planner LQ problem

The social planner maximises (84) subject to constraint (71) only, since all the other constraints are non-binding,

$$J_t = -\frac{1}{2}E_t\sum_{t=1}^{\infty}\beta^t\left[\alpha_C\left(\widehat{Y}_t-\alpha_\phi\widehat{\phi}_t\right)^2+\alpha_\pi\widehat{\pi}_t^2+\lambda_t\left(-\widehat{\pi}_t+\frac{1-a}{a}(1-\alpha\beta)\left[(v+2)\widehat{Y}_t+\widehat{\phi}_t\right]+\beta\widehat{\pi}_{t+1}\right)\right]. \tag{85}$$

The first order conditions imply the optimal inflation dynamics

$$\widehat{\pi}_t = \rho\left(\widehat{Y}_{t-1}-\widehat{Y}_t-\alpha_\phi\left(\widehat{\phi}_{t-1}-\widehat{\phi}_t\right)\right), \tag{86}$$

where $\rho = \frac{\alpha_C}{\alpha_\pi}\frac{a}{(1-\alpha\beta)(v+2)(1-a)}$.

6.6 Proof of Proposition 3

Proposition 3 *If* $\widehat{\theta}_t = 0$ *policy of inaction,* $\widehat{T}_t = 0$*, result in house price stability and the stability of the relative collateral value.*

Proof. Consider dynamics (34)-(35) with $\widehat{T}_t = 0$, $\widehat{\theta}_t = 0$.

$$\widehat{\pi}_t = \frac{1-a}{a}(1-\alpha\beta)\left[(v+2)\widehat{Y}_t+\widehat{\phi}_t\right]+\beta E_t\widehat{\pi}_{t+1}; \tag{87}$$

$$\beta E_t\widehat{q}_{t+1} = \widehat{q}_t(1-\overline{\theta}(1-\beta))+\beta\overline{\theta}(\widehat{Y}_t-E_t\widehat{\pi}_{t+1}-E_t\widehat{Y}_{t+1}); \tag{88}$$

$$\widehat{\pi}_t = \widehat{Y}_{t-1}-\widehat{Y}_t+\frac{\overline{\theta}\overline{q}}{1-\overline{\theta}\overline{q}}\left(\widehat{q}_t-\widehat{q}_{t-1}\right). \tag{89}$$

Plugging Equation (89) with one lead into (88) we would get that dynamic for relative house expenditure does not depend on shock or any other variable in the system.

$$\beta E_t\widehat{q}_{t+1} = \widehat{q}_t(1-\overline{\theta}(1-\beta))-\beta\overline{\theta}\frac{\overline{\theta}\overline{q}}{1-\overline{\theta}\overline{q}}\left(E_t\widehat{q}_{t+1}-\widehat{q}_t\right),$$

which implies complete stability, $\widehat{q}_t = 0$. Moreover, by definition , $Q_t = q_tP_tY_t$ inflation of house prices is

$$\widehat{\pi}_{Qt} := \widehat{Q}_t-\widehat{Q}_{t-1} = \widehat{\pi}_t+\widehat{Y}_t-\widehat{Y}_{t-1}+\left(\widehat{q}_t-\widehat{q}_{t-1}\right).$$

Combining it with (89) and $\widehat{q}_t = 0$, we get zero house price inflation in every period. ■

6.7 Parameter's values

We use the following values of parameters: $\overline{q} = 0.5$, $v = 2$, $\beta = 0.95$, $\overline{\theta} = 0.8$, $a = 0.5$, $\varepsilon = 6$, $\phi = 1.1$, $\rho_\theta = 0.95$, $\rho_\phi = 0.95$.