Neural Network Controller

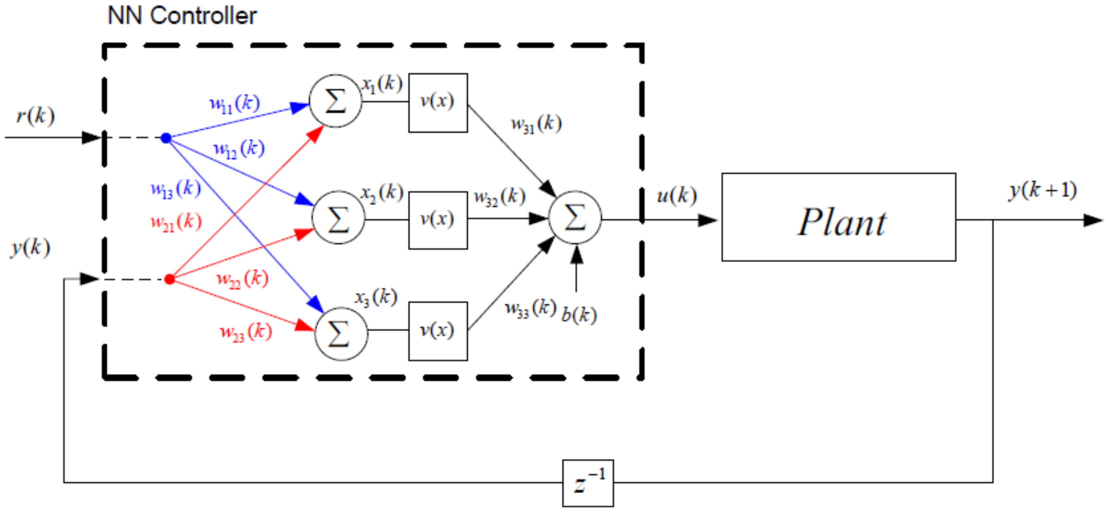


Fig.1 Neural Network Structure

Neural Network structure can be used as a controller directly as in fig.1. The performance of the controller is going to be evaluated on the following nonlinear time-varying plant :

$$y(k+1) = \frac{1.2(1-0.8\,e^{-0.1k})\ y(k)}{1+\ y^2(k)} + u(k) \quad \textbf{(1)}$$

The initial values of weights in NN controller are initialized as in Table 1.

Table 1. Symbols and Initial values

Symbol	Value	Description
$W(0) = \begin{bmatrix} w_{11}(0) & w_{12}(0) & w_{13}(0) \\ w_{21}(0) & w_{22}(0) & w_{23}(0) \\ w_{31}(0) & w_{32}(0) & w_{33}(0) \end{bmatrix}$ $b(0)$	$W(0) = \begin{bmatrix} -1.7502 & -0.8314 & -1.1564 \\ -0.2857 & -0.9792 & -0.5336 \\ -2.0026 & 0.9642 & 0.5201 \end{bmatrix}$ $b(0) = 10^{-5}$	Weights in input and hidden layer
$u(0)$	$u(0) = 0$	Control Signal
$\eta(k)$	$\eta(k) = 0.01$	Learning Rate
t_s	$t_s = 0.01$	Sampling Time

The system jacobian information is approximated using the following equation:

$$\frac{\partial y(k+1)}{\partial u(k)} = \begin{cases} 1 & , \; if \quad u(k)-u(k-1) \cong 0 \\ \dfrac{y(k+1)-y(k)}{u(k)-u(k-1)} & , \; otherwise \end{cases} \qquad \textbf{(2)}$$

Utilize tan-sigmoid function ($v(x) = \dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$) as activation functions in hidden layer.

minimize the following objective function:

$$J(e_{tr}(k)) = \frac{1}{2} e_{tr}^{\;2}(k) \; \textbf{(3)}$$

where

$$e_{tr}(k) = r(k) - y(k)$$

Reference Signal:

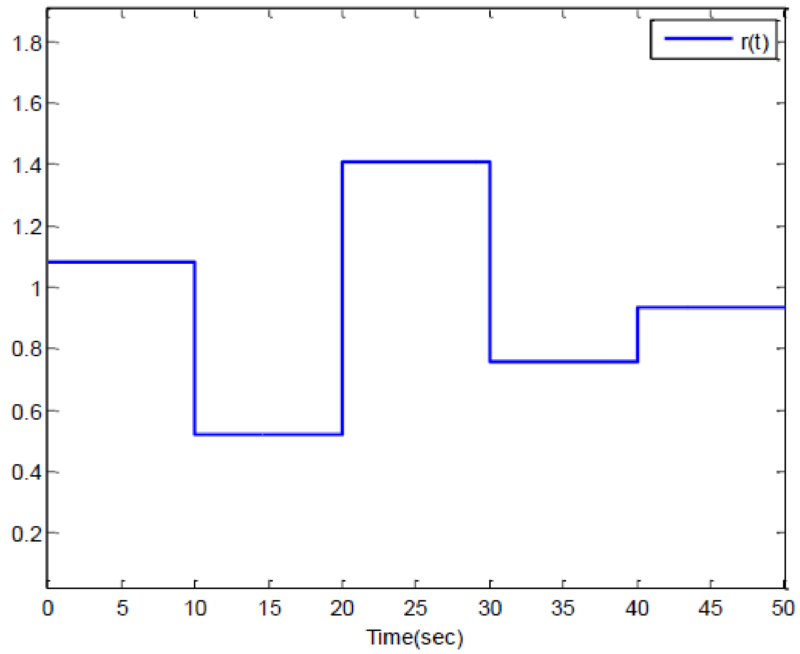


Fig.2 Reference Signal

Explanation of the code:

$etr\ (k) = r(k) - y(k)$. The back propagation algorithm is given below. It is used to update weights in order to find best PID-NN coefficients:

$$\frac{\partial J}{\partial w_{ij}} = \frac{\partial J}{\partial e}\frac{\partial e}{\partial y}\frac{\partial y}{\partial v_j}\frac{\partial v_j}{\partial w_{ij}}$$

$$\frac{\partial y}{\partial v_j} = \varphi_j{}'(v_j)$$

$$v(x)\ is\ the\ activation\ func\ which\ is\ \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{\partial J}{\partial e} = e$$

$$\frac{\partial e}{\partial y} = -1$$

$$\frac{\partial v_j}{\partial w_{ij}} = y_{Hj}$$

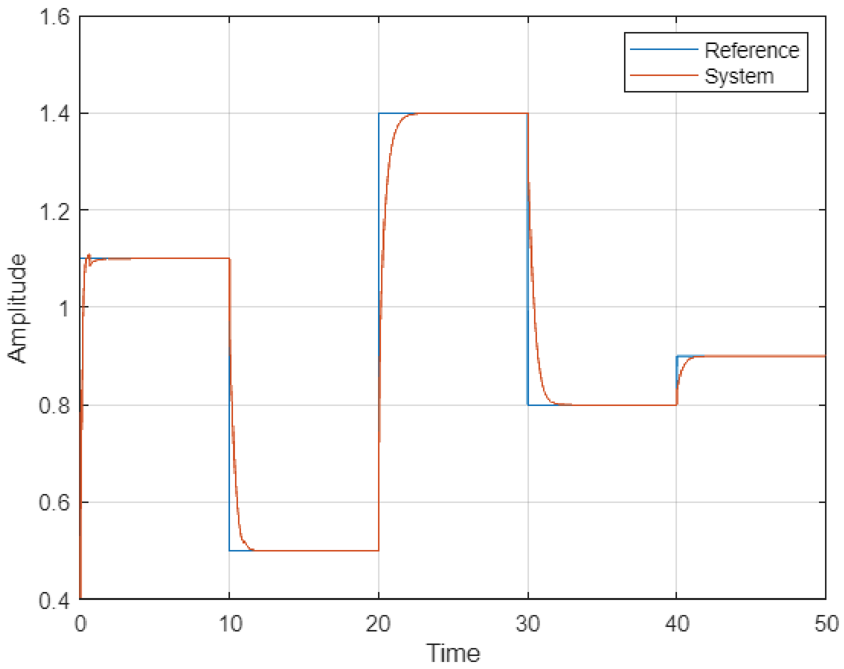
$$after\ the\ values\ are\ substituted\ we\ can\ obtain\ new\ w\ by\ this\ eq.$$

$$w_{ij} = (k+1) = w_{ij}(k) + \ \eta\frac{\partial J}{\partial w_{ij}}$$

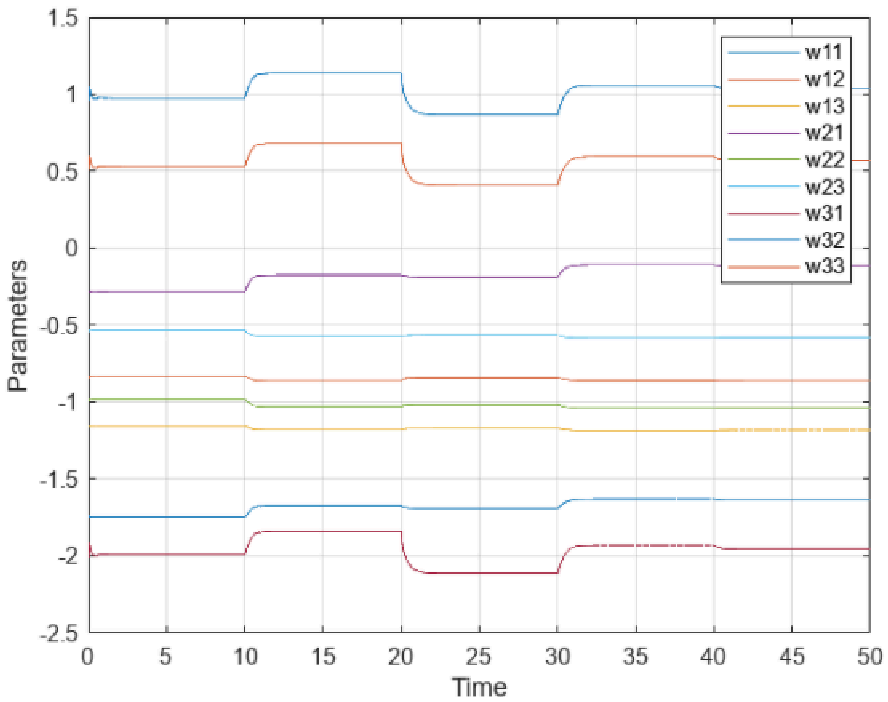
$$y(k+1) = \frac{1.2(1-0.8e^{-0.1k})y(k)}{1+y^2(k)} + u(k)$$

$$Gradient\ descent\ is\ used\ for\ cost\ function\ to\ be\ below\ the\ threshold.$$

Reference and system output signals:



Weight signals:



Controller signal: