EE415: Introduction to Medical Imaging Term Project - Final Report

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***Abstract*— In this term project, X-Ray Computed Tomography is simulated using MATLAB. The computed tomography, projection, back projection, and filtering steps are modeled by MATLAB source codes, using projection and back projection algorithms, e.g., Radon Transform, and filters and windows used in signal processing with the help of Fourier's techniques, e.g., Ram-Lak Filter and Gaussian Window. The projection and back projection results are plotted. The projection plots are derived by setting different beam numbers and angle step sizes. The back projection images are derived with different filters, e.g., Gaussian windowed filter, Ram-Lak filter, beam numbers, etc.**

In this project, the main projection and back projection algorithm is based on the Radon Transform, and Fourier Transform modules on MATLAB, such as the Fast Fourier Transform and Inverse Fast Fourier Transform, are used.

***Index Terms*—Backprojection, computed tomography, filtering, projection, simulation**

I. INTRODUCTION

A. Projection and Backprojection Algorithms

THE X-ray Computed Tomography includes, the projection of an object is the observation of the attenuation coefficients in the object, by using Radon Transform of it. This feature is defined by a set of line integrals, in which the Dirac Delta functions may be used. [1]

The projection function is defined by $p_\theta(t)$, whereas the attenuation coefficient is defined by $\mu(x, y)$.

Backprojection algorithms are used to construct the attenuation coefficients from the projections. There are methods such as

- Filtered Backprojection with Fourier Slice Theorem

Uses Fourier transformations to filter the obtained image by Radon Transform. The filtering operation is the convolution of the backprojection signal with the impulse response.

- Feldkamp-Davis-Kress Reconstruction

Utilizes an approximation, including cosine weighting and line filtering, by using a ramp filter to create the 3D reconstruction.

- Algebraic Reconstruction Technique

The summation of the line segments times the attenuation coefficients for every pixel, derived via the linear equations.

- Iterative Least Squares Reconstruction Technique

Least square solutions are used in this technique.

- Simultaneous Iterative Reconstruction Technique
- Statistical Reconstruction Technique [2]

Bayes' Rule and log likelihood function are used to obtain maximum-a-posteriori estimation is used.

B. Short History of X-Ray Imaging

X-Ray imaging started by the discovery of X-Rays, in 1895, by Wilhem Conrad Röntgen, while he was working with a cathode-ray tube. After a few months, medical radiographies became popular, being started to be used in battlefields, to find the bullets. X-Ray imaging became industrialized in the beginning of 1930's, by the efforts of General Electric Company. [3]

X-Ray Computed Tomography is invented by Allan Cormack, in 1963; which brought him Nobel Physics Award in 1979, with Hounsfield; who reinvented this phenomenon.

The first-generation CT scanners were having only one single detector, with parallel beams, which can translate and rotate. Then, the second ones were using detector arrays, with fan beams. Third-generation CT scanners can only rotate, by using the beam technique used in second generation. The fourth generation CT scanners use a stationary circular detector, rotating independently. [1]

II. THEORY AND ALGORITHM

In projection, the Radon Transform is used, which can be written as follows:

$$p_\theta(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(x, y)\delta(x\cos(\theta) + y\sin(\theta) - t)dxdy$$

(1)

An individual procession of x and y takes place in this integral.

In backprojection, the algorithm can be written as follows:

1. Getting the Discrete Fourier Transform of the data, which is gotten from the projection.
2. Using a filter, which is already designed before.
3. Taking the Inverse Discrete Fourier Transform of the filtered projection.
4. Back projection of the filtered projection data, by Radon Transform.
5. Plottting the reconstructed images.

The filtered back-projection algorithm can be briefly stated with the integral equations.

$$f_b(x, y) = \int_0^\pi \int_{-\infty}^{\infty} p_\theta(t)\delta(x\cos(\theta) + y\sin(\theta) - t)dtd\theta$$

(2)

Eq. 2 can be written for the filtered back-projection algorithm. Then, the Fourier Slice Theorem can be utilized to replace $p_\theta(t)$:

$$p_\theta(t) = \int F(\rho, \theta)e^{j2\pi\rho t}d\rho$$

(3)

Plugging Eq. 3 to Eq.2 would give

$$f_b(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} p_\theta(t)[\int F(\rho,\theta)e^{j2\pi\rho t}d\rho]\delta(xcos(\theta)+ysin(\theta)-t)dtd\theta$$
(4)

$$f_b(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} p_\theta(t)[\int F(\rho,\theta)e^{j2\pi\rho(xcos(\theta)+ysin(\theta))}\, d\rho d\theta$$
(5)

As this expression is very similar to 2D IFT expression of $F(\rho,\theta)$, which can be seen in Equation 6:

$$f(x,y) = \int_0^{2\pi}\int_0^{\infty} F(\rho,\theta)e^{j2\pi\rho(xcos(\theta)+ysin(\theta))}\rho d\rho d\theta$$
(6)

The back projected image can be written, with changing the limits, as follows:

$$f_b(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} \frac{F(\rho,\theta)}{|\rho|}e^{j2\pi\rho(xcos(\theta)+ysin(\theta))}|\rho|d\rho d\theta$$
(7)

Returning to the $f(x,y)$, to fully cover the filtered backprojection:

$$f(x,y) = \int_0^{\pi}\int_0^{\infty} F(\rho,\theta)|p|e^{j2\pi\rho(xcos(\theta)+ysin(\theta))}d\rho d\theta$$
(8)

Using the Dirac Delta function again, replacing $e^{j2\pi\rho(xcos(\theta)+ysin(\theta))}$:

$$f(x,y) = \int_0^{\pi}\int_0^{\infty} F(\rho,\theta)|p|[e^{j2\pi\rho t}\delta(xcos(\theta)+ysin(\theta)-t)dt]d\rho d\theta$$
(9)

$$f(x,y) = \int_0^{\pi}\int_0^{\infty} F(\rho,\theta)|p|[e^{j2\pi\rho t}\delta(xcos(\theta)+ysin(\theta)-t)d\rho]dtd\theta$$
(10)

The tenth equation would yield nothing but Eq. 11

$$f(x,y) = \int_0^{\pi}\int_0^{\infty} F_1^{-1}\{F_1\{p_\theta(t)\}\cdot|p|\}\delta(xcos(\theta)+ysin(\theta)-t)dtd\theta$$
(11)

The algorithm can be written briefly as above. [1]

III. Results and Discussion

A. Projection

For the projection stage, two images are used; one of which is a simple image (Figure 1) whereas the other one is a complex image (Figure 2).

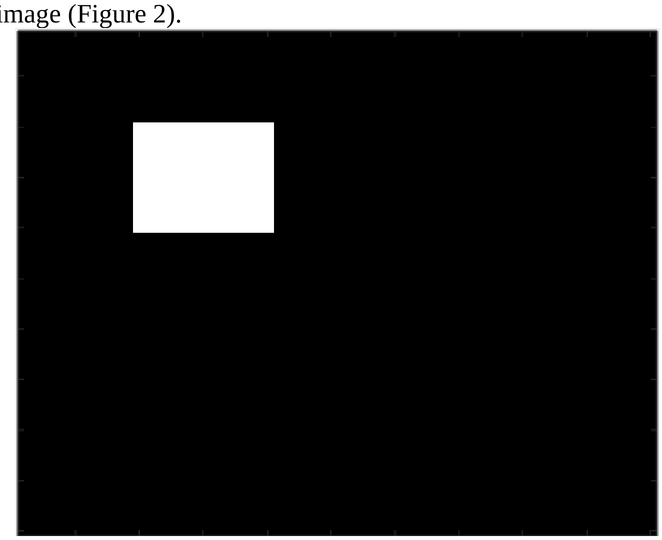

Fig. 1. Square image used as a phantom.

Fig. 2. Lena image used as another phantom.

Then, the projection plots, for 12 different angles uniformly chosen between 0 and 180 degrees, with 180 steps, and 100 beams are drawn. Figure 3 shows the square image projections whereas Figure 4 shows the complex image projections.

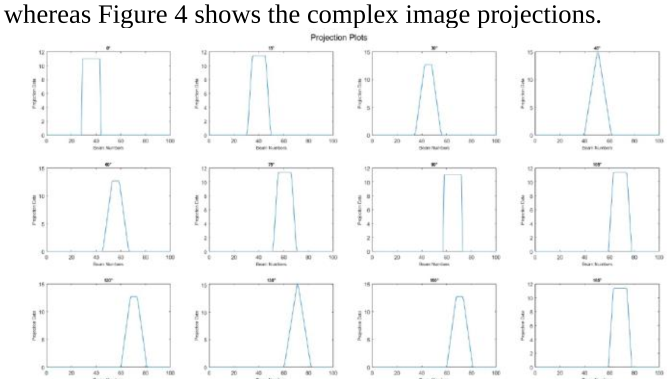

Fig. 3. Square image projection plots.

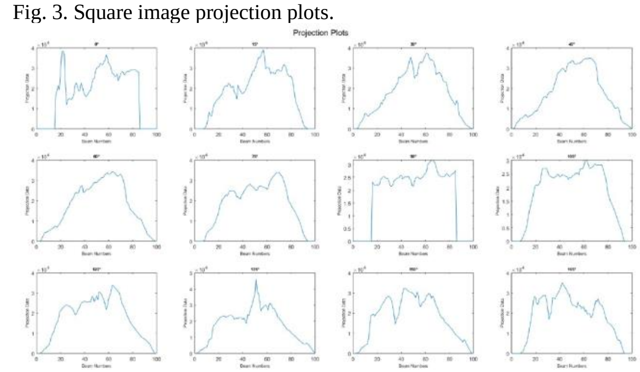

Fig. 4. Lena image projection plots.

B. Backprojection without Filtering

Setting 100 beams and 180 degree step sizes, as it is set before in the projection stage, for the square image and Lena image, on Figures 5 and 6.

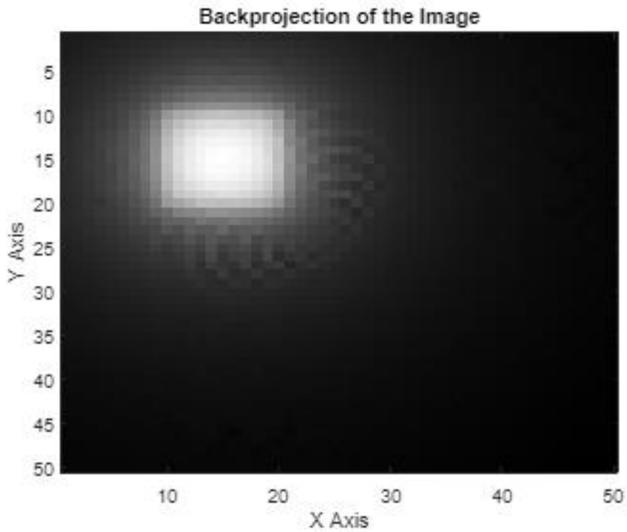

Fig. 5. Square image, backprojected without any filter.

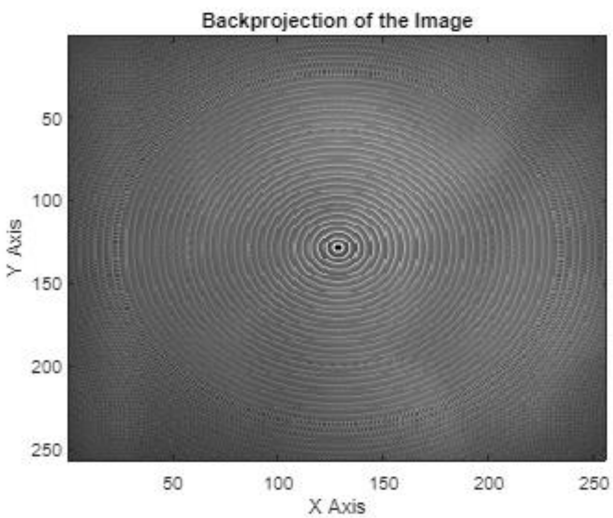

Fig. 6. Lena image, backprojected without any filter.

C. Backprojection with Filtering

By only using Ram-Lak Filter, the results can be seen on Figures 7 and 8.

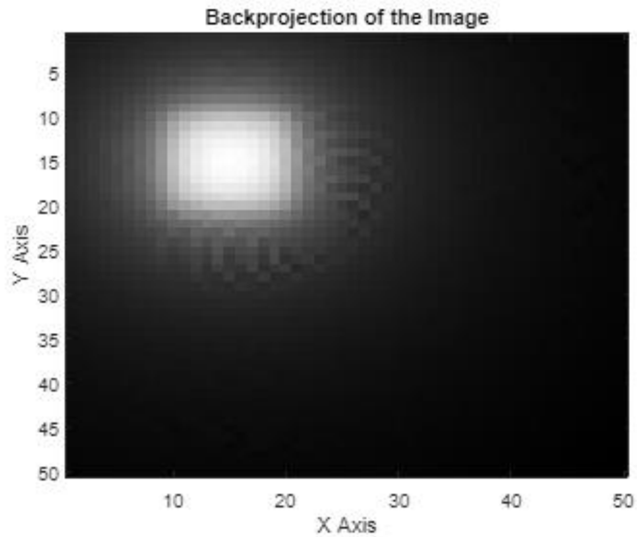

Fig. 7. Square image, backprojected without any filter.

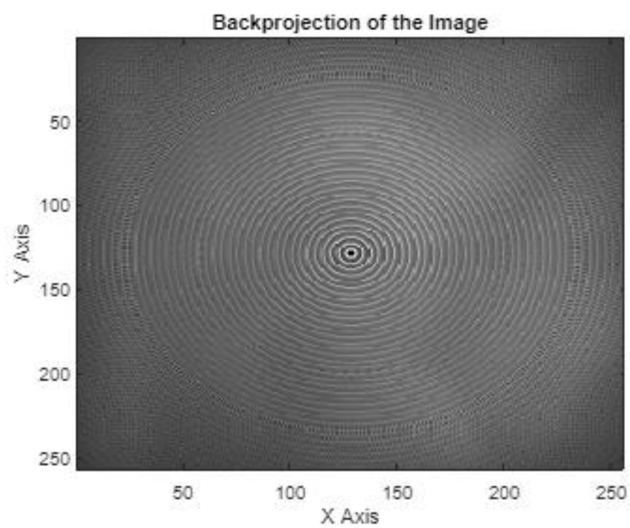

Fig. 8. Lena image, backprojected without any filter.

D. Filtered Backprojection with Windowing

Three methods; triangular, Hamming and Gaussian windows are used.

1) Triangular Windowing

By using a triangular window, the results can be seen on Figures 9 and 10.

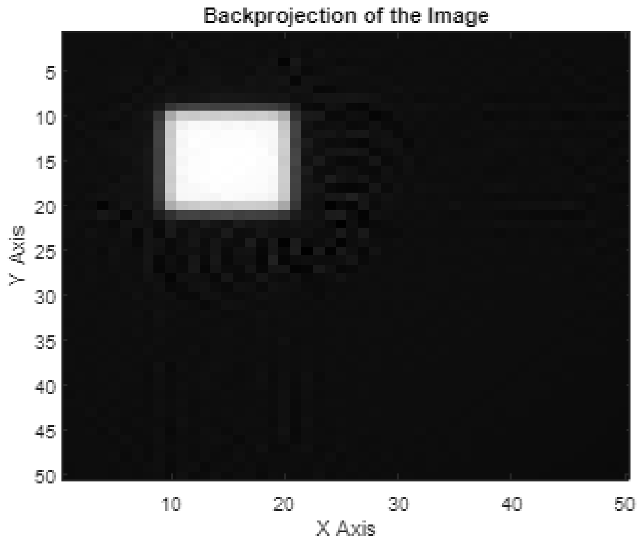

Fig. 9. Square image, backprojected with a filter and the triangular window.

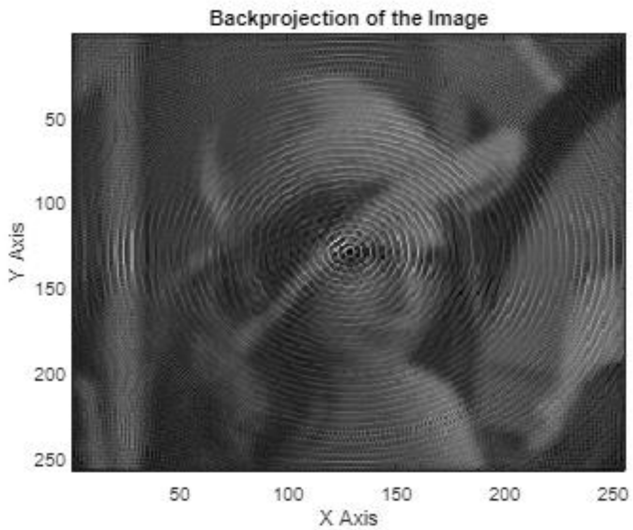

Fig. 10. Lena image, backprojected with a filter and the triangular window

2) Hamming Windowing

By using a Hamming window, the results can be seen on Figures 11 and 12.

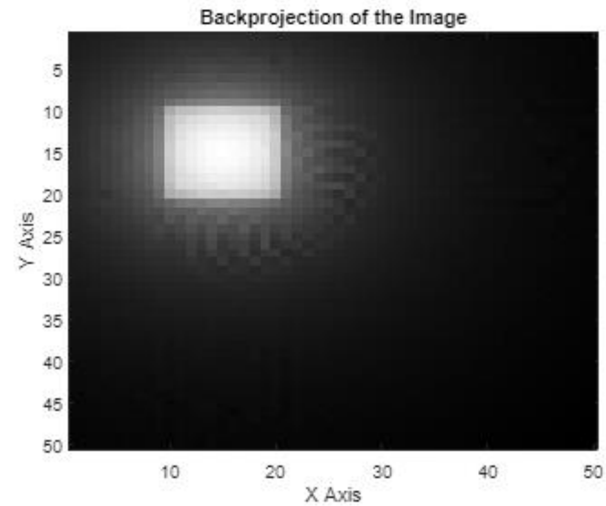

Fig. 11. Square image, backprojected with a filter and with Hamming window

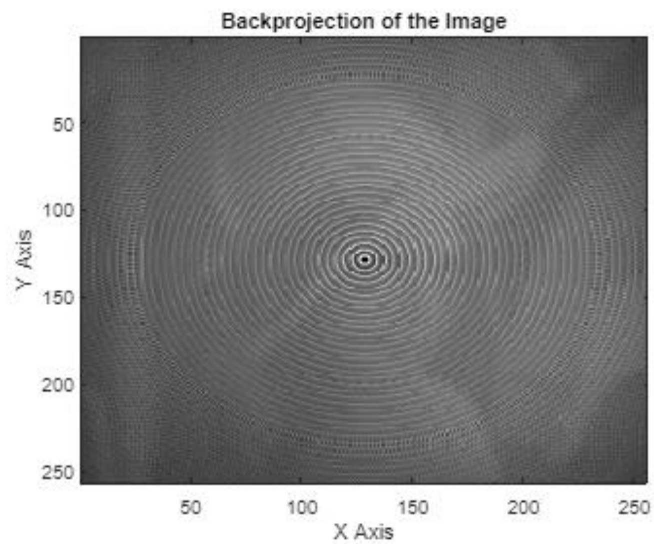

Fig. 12. Lena image, backprojected with a filter and with Hamming window

3) Gaussian Windowing

By using a triangular window, the results can be seen on Figures 13 and 14.

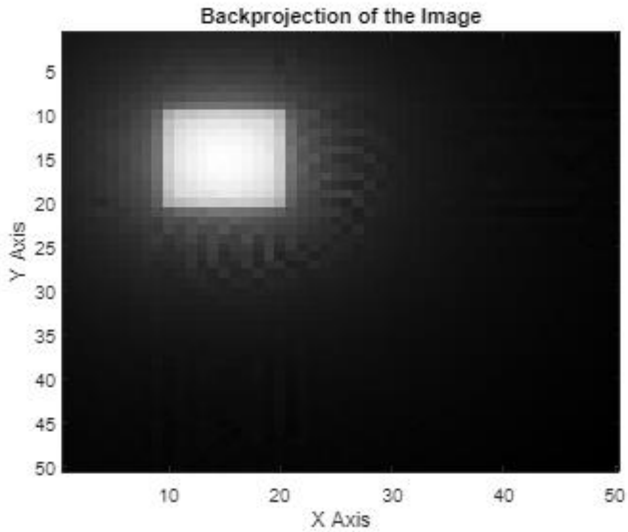

Fig. 13. Square image, backprojected with a filter and with Hamming window

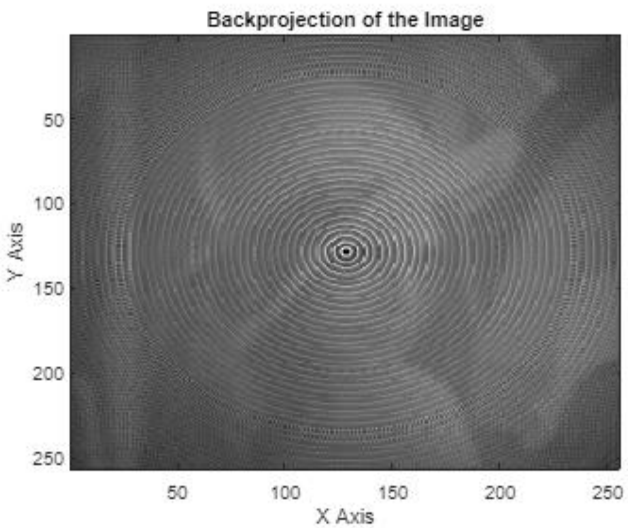

Fig. 14. Lena image, backprojected with a filter and with Hamming window

It is seen that the projection plots converted to backprojection images would definitely need a filtering since both of the images are blurred, and even the square image displays that it has more area dedicated to the square.

Using Ram-Lak filter (without windowing) would yield a sharper image with considerable amount of contrast, letting the user to differentiate the parts of the image, especially in Lena image.

The triangular window is the best option for sharper images; whilst Hamming and Gaussian windows are not sharping the images as much as this window. One could easily say that Gaussian filter is slightly better than the Hamming one.

IV. Conclusion

X-Ray Computed Tomography, whose roots date back to more than one century, is simulated only by using a computer and an a programming tool named MATLAB. Different phantoms are used for comparing the projection results with given beam and angle numbers, and backprojection results are obtained either the lack of filtering or usage of filtering. Moreover, utilization of windowing is shown by comparing the results of different windows.

V. Appendix

In the final form of the codes, the filtering stages and windowing stages are edited.

- `fftshift` command of the MATLAB added to the stages, in the `Filter_Ram_Lak` function. The added code lines can be seen in the code sample.

```
function [Filtered_Projection] =
Filter_Ram_Lak(Projection_of_the_Image)
%Getting the image by taking the .mat form.
DFT_of_Projection =
fft2(Projection_of_the_Image); %Getting the
2D FFT of the projection.
[Number_of_Angles, Number_of_Beams] =
size(Projection_of_the_Image);  %Taking the
size of the projection of the image.
Filtered_DFT_of_Projection =
zeros(size(Projection_of_the_Image));
%Adjusting the filtered DFT matrix.
Filter_Size = Number_of_Beams - 1;
Ram_Lak_Filter = [0 1:(Filter_Size/2)
flip(1:(Filter_Size/2)) 0];
Ram_Lak_Filter = Ram_Lak_Filter /
sum(Ram_Lak_Filter);
Shifted_Ram_Lak_Filter =
fftshift(Ram_Lak_Filter);
for i = 1:Number_of_Angles
    Filtered_DFT_of_Projection(i,:) =
DFT_of_Projection(i,:) .*
Shifted_Ram_Lak_Filter; %Using the
triangular filter.
end
IFFT_of_Filtered_DFT_of_Projection =
real(ifft2(Filtered_DFT_of_Projection));
%Getting the IFFT of the filtered FFT.
Filtered_Projection =
IFFT_of_Filtered_DFT_of_Projection;
end
```

- In the usage of windows, `Filter_Ram_Lak` is combined with the window functions such as `Window_Gauss`, `Window_Triangular`, `Window_Hamming`.

With this combination, a comparison is made by looking at the previous results and the current ones, which are already in the report.

For the previous version of Hamming windowed results, at Figure 15 and 16,

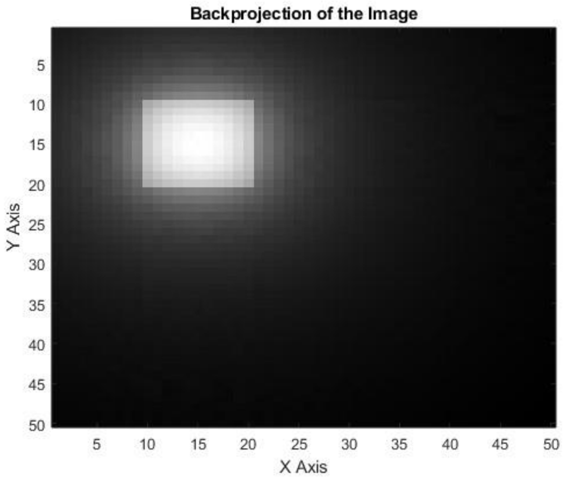

Fig. 15. Square image, backprojected with a filter and with the previous Hamming window

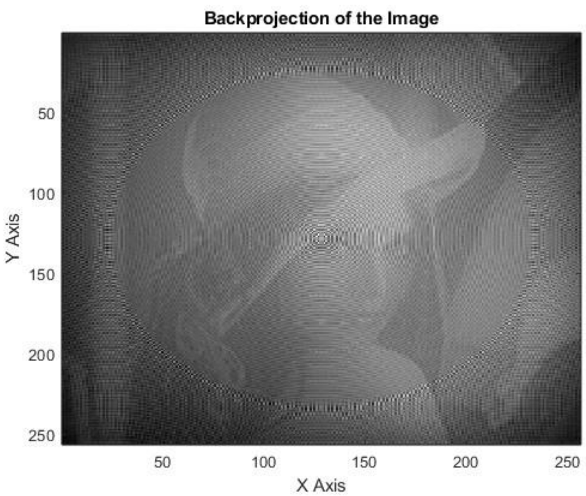

Fig. 16. Lena image, backprojected with a filter and with the previous Hamming window

It could be seen that the backprojection of the square image might seem more contrast than Figure 11; however, the backprojection of Lena image is highly contrasted and overbright with respect to Figure 12.

For the previous version of Gaussian windowed results, at Figure 15 and 16,

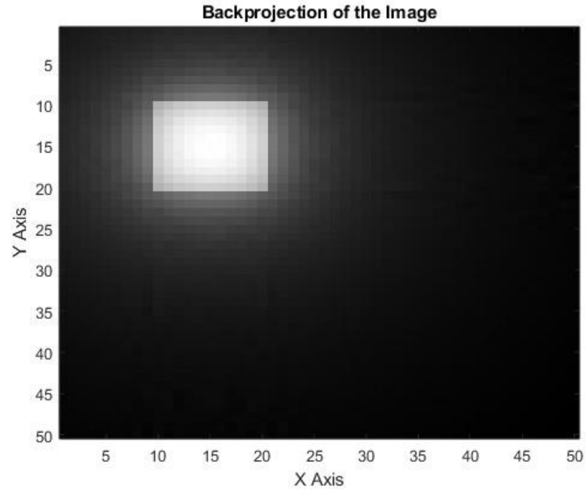

Fig. 17. Square image, backprojected with a filter and with the previous Gaussian window

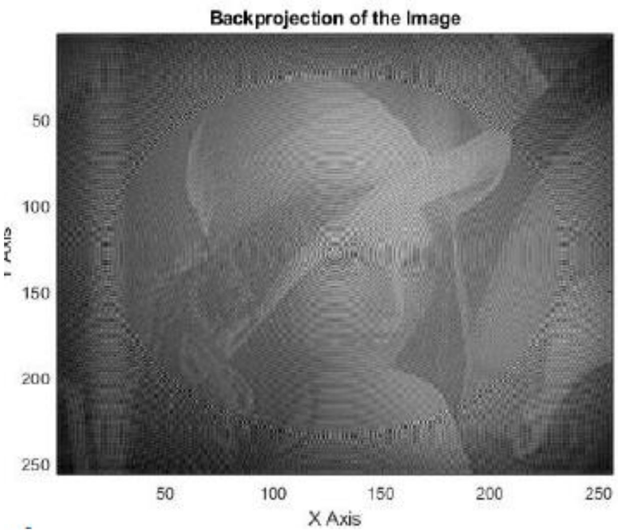

Fig. 18. Lena image, backprojected with a filter and with the previous Gaussian window

The similar discussions can be made for this window, especially by comparing Figure 14 and Figure 18.

References

[1] N. G. Gencer, Computed Tomography, Ankara: METU, 2023.

[2] J. Hornegger, "CT Image Reconstruction Basics," [Online]. Available: https://www5.cs.fau.de/fileadmin/persons/MaierAndreas/maier/Hornegger16-CRB.pdf. [Accessed January 5 2024].

[3] Iowa State University, "History of Radiography," [Online]. Available: https://www.nde-ed.org/NDETechniques/Radiography/interactionradmat.xhtml. [Accessed January 5 2024].