11) 8.1.20 Remark: This problem is asking you to compute $\int_{\partial D} F \cdot ds$ and $(\nabla \times F) \cdot k$ and explain why $\int_{\partial D} F \cdot ds \neq \iint_D (\nabla \times F) \cdot k \, dA$ does not violate Green's Theorem.

Let $P(x,y) = -y/(x^2+y^2)$ and $Q(x,y) = x/(x^2+y^2)$

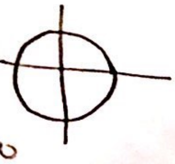
Assuming D is the unit disc, investigate why greens theorem fails for this $P + Q$.

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0 \end{vmatrix} \begin{matrix} i & j \\ \partial x & \partial y \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{matrix} \quad i(0-0) - j(0-0) + k\left(\frac{\partial}{\partial x}\frac{x}{x^2+y^2} + \frac{\partial}{\partial y}\frac{y}{x^2+y^2}\right)$$

$$= \frac{\cancel{x^2} + \cancel{y^2} - 2x^2 + x^2 + \cancel{y^2} - 2y^2}{(x^2+y^2)^2} k = \vec{0} k$$

So $\nabla \times \vec{F} = 0$ so $\iint_D (\nabla \times F) \cdot k \, dA = 0$

$\int_{\partial D} F \cdot ds$



can deform to any e so unit circle

so $\int_{\partial D} \vec{F} \cdot dr = 2\pi$

(from Lecture "More on Green's Theorem EX1")

$\oint_{C^+} F \cdot d\vec{r} = 2\pi N \leftarrow$ # of times curve C wraps around origin

for $F = \frac{-y}{x^2+y^2} i + \frac{x}{x^2+y^2} j$

$2\pi \neq 0$ BUT!

Unit disk D contains the origin where $P(x,y)$ and $Q(x,y)$ aren't defined, and for Green's Theorem to be applicable F must be "nicely defined on D" so the conditions aren't satisfied so Green's theorem isn't violated

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