7a $D = [-1,1] \times [-1,1]$, $P(x,y) = x$, $Q(x,y) = y$

$$\iint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy \quad (1)$$

path btwn $(-1,1)$ + $(-1,-1)$ C_1 is $\left.\begin{matrix} x = -1 \\ y = -2+t \end{matrix}\right\}$ $t = [0,1]$

$\langle x,y \rangle = \langle -1, 1 \rangle + t\langle 0,-2 \rangle$
$\langle x,y \rangle$ $\langle -1, -2t+1 \rangle$

$\left.\begin{matrix} x = -1 \\ y = -2t+1 \end{matrix}\right\}$ $t = [0,1]$

path btwn $(-1,-1)$ + $(1,-1)$ is C_2

$\langle x,y \rangle = \langle -1,-1 \rangle + (t-1)\langle 2,0 \rangle$
$\langle x,y \rangle = \langle 2t-3, -1 \rangle$

$\left.\begin{matrix} x = 2t-3 \\ y = -1 \end{matrix}\right\}$ $t = [1,2]$

path btwn $(1,-1)$ + $(1,1)$ is C_3

$\langle x,y \rangle$ $\langle 1,-1 \rangle + (t-2)\langle 0,2 \rangle$
$\langle x,y \rangle = \langle 1, 2t-5 \rangle$

$\left.\begin{matrix} x = 1 \\ y = 2t-5 \end{matrix}\right\}$ $t = [2,3]$

path btwn $(1,1)$ + $(-1,1)$ is C_4

$\langle x,y \rangle = \langle 1,1 \rangle + (t-3)\langle -2,0 \rangle$
$\langle x,y \rangle = \langle -2t+7, 1 \rangle$

$\left.\begin{matrix} x = -2t+7 \\ y = 1 \end{matrix}\right\}$ $t = [3,4]$

path	$x(t)$	$y(t)$	t	$x'(t)$	$y'(t)$
C_1	-1	$-2t+1$	$0 \le t \le 1$	0	-2
C_2	$2t-3$	-1	$1 \le t \le 2$	2	0
C_3	1	$2t-5$	$2 \le t \le 3$	0	2
C_4	$-2t+7$	-1	$3 \le t \le 4$	-2	0

(2)

$$\int_{\partial D} P\,dx + Q\,dy = \int_{C_1} P\,dx + Q\,dy + \int_{C_2} P\,dx + Q\,dy + \int_{C_3} P\,dx + Q\,dy + \int_{C_4} P\,dx + Q\,dy$$

$$\int_{C_1} P\,dx + Q\,dy = \int_{C_1} x\,dx + y\,dy \quad \int_0^1 -1(0) + (-2t+1)(-2)\,dt$$

$$= -2\left[\frac{-2t^2}{2} + t\right]_0^1 = 0$$

$$\int_{C_2} P\,dx + Q\,dy \quad \int_{C_2} x\,dx + y\,dy = \int_1^2 2(2t-3) + 0(-1)\,dt$$

$$= 2\left[\frac{2t^2}{2} - 3t\right]_1^2 = 0$$

$$\int_{C_3} P\,dx + Q\,dy = \int_2^3 1(0) + (2t-5)(2)\,dt = 2\left[\frac{2t^2}{2} - 5t\right]_2^3 = 0$$

$$\int_{C_4} = \int_3^4 (-2t+7)(-2) + (-1)(0)\,dt \quad 2\left[\frac{2t^2}{2} + 7t\right]_3^4 = 0$$

$$\int_{\partial D} P\,dx + Q\,dy = 0$$

$$\iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dx\,dy = \int_{-1}^{1}\int_{-1}^{1} \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dx\,dy$$

$$\int_{-1}^{1}\int_{-1}^{1} \left(\frac{d(y)}{dx} - \frac{d(x)}{dy}\right) dx\,dy = 0$$

both sides equal 0 so Green's Theorem is valid for the above integral

7b $D = [0, \pi/2] \times [0, \pi/2]$, $P(x,y) = \sin x$, $Q(x,y) = \cos y$

$$\iint_C P\,dx + Q\,dy = \iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dx\,dy \quad (1)$$

$C_1 : (0,0) \rightarrow (\pi/2, 0)$

$\langle x, y\rangle = \langle 0,0\rangle + t\langle \pi/2, 0\rangle$
$\langle x, y\rangle = \langle \pi t/2, 0\rangle$

$\left.\begin{aligned} x &= \pi t/2 \\ y &= 0 \end{aligned}\right\} t = [0,1]$

$C_2 : (\pi/2, 0) \rightarrow (\pi/2, \pi/2)$

$\langle x, y\rangle = \langle \pi/2, 0\rangle + (t-1)\langle \pi/2, \pi/2\rangle$
$\langle x, y\rangle = \langle \pi/2, \pi(t-1)/2\rangle$

$\left.\begin{aligned} x &= \pi/2 \\ y &= \pi(t-1)/2 \end{aligned}\right\} t = [1,2]$

$C_3 : (\pi/2, \pi/2) \longrightarrow (0, \pi/2)$

$\langle x, y\rangle = \langle \pi/2, \pi/2\rangle + (t-2)\langle 0, \pi/2\rangle$
$\langle x, y\rangle = \langle \pi(-t+3)/2, \pi/2\rangle$

$\left.\begin{aligned} x &= \pi(-t+3)/2 \\ y &= \pi/2 \end{aligned}\right\} t = [2,3]$

$C_4: (0, \pi/2) \to (0,0)$

$\langle x,y \rangle = \langle 0, \pi/2 \rangle + (t-3)\langle 0,0 \rangle$ $\quad$ $\left.\begin{aligned} x &= 0 \\ y &= \pi(-t+4)/2 \end{aligned}\right\} t = [3,4]$

$\langle x,y \rangle = \langle 0, \pi(-t+4)/2 \rangle$

$x(t)$	$y(t)$	t	$x'(t)$	$y'(t)$
$\pi t/2$	0	$0 \le t \le 1$	$\pi/2$	0
$\pi/2$	$\pi(t-1)/2$	$1 \le t \le 2$	0	$\pi/2$
$\pi(-t+3)/2$	$\pi/2$	$2 \le t \le 3$	$-\pi/2$	0
0	$\pi(-t+4)/2$	$3 \le t \le 4$	0	$-\pi/2$

$$\int_{\partial D} P\,dx + Q\,dy = \int_{C_1} P\,dx + Q\,dy + \int_{C_2} P\,dx + Q\,dy + \int_{C_3} P\,dx + Q\,dy + \int_{C_4} P\,dx + Q\,dy$$

$$\int_{C_1} P\,dx + Q\,dy = \int_0^1 \sin(\pi t/2)(\pi/2) + \cos(0)(0)\,dt$$

$$= \frac{\pi}{2}\left[\frac{-\cos(\pi t/2)}{\pi/2}\right]_0^1 = 1$$

$$\int_{C_2} P\,dx + Q\,dy = \int_1^2 \sin(\pi/2)(0) + \cos(\pi(t-1)/2)(\pi/2)\,dt$$

$$= \left[\sin\frac{\pi}{2}(t-1)\right]_1^2 = 1$$

$$\int_{C_3} P\,dx + Q\,dy = \int_2^3 \sin(\pi(-t+3)/2)(-\pi/2) + \cos(\pi/2)(0)\,dt$$

$$= -\left[\cos\frac{\pi}{2}(-t+3)\right]_2^3 = 1$$

$$\int_{C_4} P\,dx + Q\,dy = \int_3^4 \sin(0)(0) + \cos(\pi(-t+4)/2)(-\pi/2)\,dt$$

$$= -\frac{\pi}{2}\left[\frac{\sin \pi/2(-t+4)}{-\pi/2}\right]_3^4 = -1$$

$$\int_{\partial D} P\,dx + Q\,dy = 1+1-1-1 = 0$$

Green Theorem is VALID for the integral