4a $\iint_S \vec{F}\cdot d\vec{S} \qquad \vec{F}(x,y,z) = x\hat{i} + y\hat{j} + z^2\hat{k}$

$\phi(u,v) = (2\sin u, 3\cos u, v)$

$0 \le u \le 2\pi \qquad 0 \le v \le 1$

$\vec{T}_u = \dfrac{d((\phi(u,v)))}{du} = 2\cos u\,\hat{i} - 3\sin u\,\hat{j}$

$\vec{T}_v = \dfrac{d((\phi(u,v)))}{dv} = \hat{k}$

$\vec{T}_u \times \vec{T}_v = ((2\cos u\,\hat{i} - 3\sin u\,\hat{j}) \times (\hat{k})) = -3\sin u\,\hat{i} - 2\cos u\,\hat{j}$

$\vec{F}\cdot(\vec{T}_u \times \vec{T}_v) = (x\hat{i} + y\hat{j} + z^2\hat{k})\cdot(-3\sin u\,\hat{i} - 2\cos u\,\hat{j})$
$= -6\sin^2 u - 6\cos^2 u$
$= -6$

$\iint_S \vec{F}\cdot d\vec{S} = \iint_\phi \vec{F}\cdot d\vec{S} = \iint_D \vec{F}\cdot(\vec{T}_u \times \vec{T}_v)\,du\,dv = \int_0^1 \int_0^{2\pi} -6\,du\,dv$

$\int_0^1 -6\,[u]_0^{2\pi}\,dv = [-12\pi v]_0^1 = \boxed{-12\pi}$

4b $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ $\quad \vec{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$

$S = x^2 + y^2 + z^2 = 16, z \geq 0$

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \iint_D (\nabla \times \vec{F}) \cdot (\vec{T_x} \times \vec{T_y})\, dy\, dx$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ x^2 + y - 4 & 3xy & 2xz + z^2 \end{vmatrix}$$

$= \hat{i}\,(d/dy(2xz + z^2) - d/dz(3xy))$
$- \hat{j}\,(d/dx(2xz + z^2) - d/dz(x^2 + y - 4))$
$+ \hat{k}\,(d/dx(3xy) - d/dy(x^2 + y - 4))$

$= \hat{i}\,(0 - 0) - \hat{j}\,(2z) + \hat{k}\,(3y - 1)$

$0\hat{i} - 2z\hat{j} + (3y - 1)\hat{k} = (0, -2z, 3y - 1)$

$x^2 + y^2 + z^2 = 16 \rightarrow z = \pm\sqrt{16 - x^2 - y^2}$, $z \geq 0$

$z = \sqrt{16 - x^2 - y^2}$

$T(x, y, z) = (x, y, \sqrt{16 - x^2 - y^2})$, $-4 \leq x \leq 4$, $-\sqrt{16 - x^2} \leq y \leq \sqrt{16 - x^2}$

$T_x(x, y) = d/dx\,(x, y, \sqrt{16 - x^2 - y^2}) = \left(1, 0, \frac{-x}{\sqrt{16 - x^2 - y^2}}\right)$

$T_y(x, y) = d/dy\,(x, y, \sqrt{16 - x^2 - y^2}) = \left(0, 1, \frac{-y}{\sqrt{16 - x^2 - y^2}}\right)$

$$\vec{T_x} \times \vec{T_y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & \frac{-x}{\sqrt{16-x^2-y^2}} \\ 0 & 1 & \frac{-y}{\sqrt{16-x^2-y^2}} \end{vmatrix}$$

$$= \left(\frac{x}{\sqrt{16-x^2-y^2}}, \frac{y}{\sqrt{16-x^2-y^2}}, 1 \right)$$

$$(\nabla \times F) \cdot (\vec{T_x} \times \vec{T_y}) = (0, -2z, 3y-1) \cdot \left(\frac{x}{\sqrt{16-x^2-y^2}}, \frac{y}{\sqrt{16-x^2-y^2}} \right)$$

$*$ sub
$z = \sqrt{16-x^2-y^2}$

$$= \left(0 + \frac{-2zy}{\sqrt{16-x^2-y^2}} + 3y - 1 \right) = \frac{-2zy}{\sqrt{16-x^2-y^2}} + 3y - 1$$

$$= -\frac{2(\sqrt{16-x^2-y^2})y}{\sqrt{16-x^2-y^2}} + 3y - 1 = -2y + 3y - 1 = y - 1$$

$$\iint_S (\nabla \times \vec{T}) \, d\vec{S} = \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} (y-1) \, dy \, dx = \int_{-4}^{4} \left[\frac{y^2}{2} - y \right]_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} dx$$

$$= \int_{-4}^{4} -2\sqrt{16-x^2} \, dx = -2 \int_{-4}^{4} \sqrt{4^2 - x^2} \, dx = -2 \left[\frac{x\sqrt{16-x^2}}{2} + \frac{16}{2} \sin^{-1}\left(\frac{x}{4}\right) \right]_{-4}^{4}$$

$$= -2 \left[8\left[\frac{\pi}{2}\right] + 8\left[\frac{\pi}{2}\right] \right) = -2(8\pi) = \boxed{-16\pi}$$