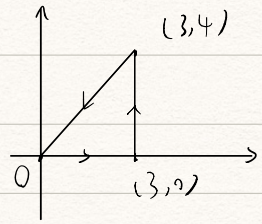
8:



$$D = \{ 0 \le x \le 3, \quad 0 \le y \le \tfrac{4}{3}x \}$$

$$\therefore \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dy\, dx$$

$$= \int_0^3 \int_0^{\frac{4}{3}x} (10y - 8y)\, dy\, dx$$

$$= \int_0^3 \frac{16}{9} x^2\, dx$$

$$= \frac{16}{27} x^3 \Big|_0^3$$

$$= 16$$

9. Using divergence theorem:

$$\int_{\partial D} \vec{F} \cdot \vec{n} \cdot ds = \iint_D \operatorname{div} \vec{F} \cdot dA$$

Note that $\operatorname{div} \vec{F} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} = 0$

$$\therefore \iint_D \operatorname{div} \vec{F} \cdot dA = 0$$

10. a): The area of Region D is

$$A(D) = \frac{1}{2} \int_C x\,dy - y\,dx$$

since $\begin{cases} x = R\cos t \\ y = R\sin t \end{cases} \quad (0 \le t \le 2\pi)$

Hence, $A(D) = \dfrac{1}{2} \displaystyle\int_C [R\cos t \cdot R\cos t - R\sin t \cdot (-R\sin t)]\, dt$

$$= \frac{1}{2} \int_0^{2\pi} R^2 \cdot dt$$

$$= \frac{1}{2} \cdot 2\pi R^2$$
$$= \pi R^2 .$$

b): since $A = \frac{1}{2} \int_C x\,dy - y\,dx$

We let $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$$\therefore \quad A = \frac{1}{2} \int_C (r^2\cos^2\theta + r^2\sin^2\theta)\, d\theta$$
$$= \frac{1}{2} \int_C r^2\, d\theta$$