how to deal with v-dependent frictional forces

if not all forces come from a potential, can pile up those that do come from a potential in 'L', and those that don't in 'Q'

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$$

L − conservative

Q − dissipative

Example: v-dependent friction

x-component $F_{fx} = -k_x v_x$

consider the "Rayleigh's dissipation function"

$$\mathcal{F} = \frac{1}{2}\sum_i \left(k_{x_i} v_{x_i}^2 + k_{y_i} v_{y_i}^2 + k_{z_i} v_{z_i}^2\right)$$

$$F_{fx_i} = -\frac{\partial}{\partial v_{x_i}} \mathcal{F} \rightarrow \bar{F}_{f_i} = -\nabla_{v_i} \mathcal{F}$$

generalized force

$$Q_j = \sum_i \bar{F}_{f_i} \cdot \frac{\partial \bar{r}_i}{\partial q_j} = -\sum_i \bar{\nabla}_{v_i} \mathcal{F} \cdot \frac{\partial \bar{r}_i}{\partial q_j}$$

$$= -\sum_i \bar{\nabla}_{v_i} \mathcal{F} \cdot \frac{\partial \dot{\bar{r}}_i}{\partial \dot{q}_j}$$

$$\boxed{Q_j = -\frac{\partial}{\partial \dot{q}_j} \mathcal{F}}$$

equations of motion are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial \mathcal{F}}{\partial \dot{q}_j} = 0$$

when solving for the eqns of motion, now we have to provide 'curly F' in addition to 'L'