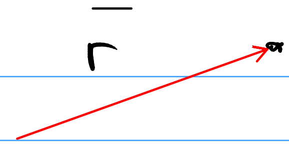
Mechanics of a single particle



$$\bar v = \frac{d\bar r}{dt}$$

$$\bar p = m\bar v$$

$$\bar F = \frac{d\bar p}{dt} = \dot{\bar p}$$

Newton's 2nd law

m const $\quad \bar F = \frac{d}{dt}(m\bar v) = m\frac{d\bar v}{dt} = m\bar a$

2nd order differential equation

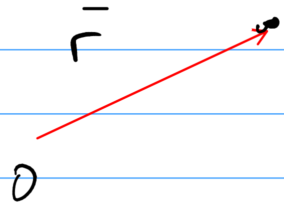
this is valid in intertial (Galilean) reference frames

Conservation Theorem:

$\boxed{\text{If the total force = 0, then liner momentum is conserved}}$

$\dot{\bar p} = 0 \qquad \bar p = \text{const}$

Angular momentum



$$\bar L = \bar r \times \bar p$$

$$\bar N = \bar r \times \bar F$$

$$= \bar r \times \frac{d}{dt}(m\bar v)$$

$$\frac{d}{dt}(\bar r \times m\bar v) = \underbrace{\cancel{\bar v \times m\bar v}}_{0} + \bar r \times \frac{d}{dt} m\bar v$$

$$\bar N = \frac{d}{dt}(\bar r \times m\bar v) = \frac{d}{dt}\bar L = \dot{\bar L}$$

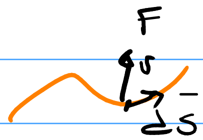
Conservation Theorem:

> If the total torque = 0, then the angular momentum is conserved.

$$\dot{L} = 0 \qquad \bar{L} = \text{const}$$

Work done by the external force

$$W_{12} = \int_1^2 \bar{F} \cdot d\bar{s}$$



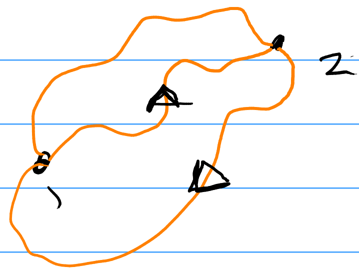
$$\int \bar{F} \cdot d\bar{s} = m \int \frac{d\bar{v}}{dt} \cdot \bar{v}\, dt = \frac{m}{2} \int_1^2 \frac{dv^2}{dt}\, dt$$

$$W_{12} = \frac{m}{2}\left(v_2^2 - v_1^2\right) \equiv T_2 - T_1 \qquad (1)$$

$T = \frac{1}{2} m v^2$

*If the force is conservative: work is independent of the path

$$\oint \bar{F} \cdot d\bar{s} = 0$$



for friction, $\bar{F} \cdot d\bar{s} < 0$

there is no cancellation -> closed path integral is not zero, thus it is not conservative.

It can be proven that F also satisfies, $\bar{F} = -\nabla V(\bar{r})$

potential energy V(r)

b/c F is derivative of V, then V is defined up to a constant.

$$\bar{F} \cdot d\bar{s} = -dV$$

$$W_{12} = +\int \bar{F} \cdot d\bar{s} = -\int_1^2 dV = V_1 - V_2 \qquad (2)$$

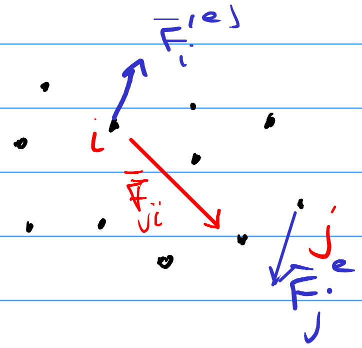
$$T_2 - T_1 = V_1 - V_2$$

$$\boxed{T_2 + V_2 = T_1 + V_1}$$

Final energy = initial energy

Energy is conserved!

IF the force is time-dependent, (1) is true, but (2) will not. Thus, energy is not conserved.



Mechanics of a system of particles

$$\sum_j \bar{F}_{ji} + \bar{F}_i^{(e)} = \dot{\bar{P}}_i$$

force on i due to j

external force on i

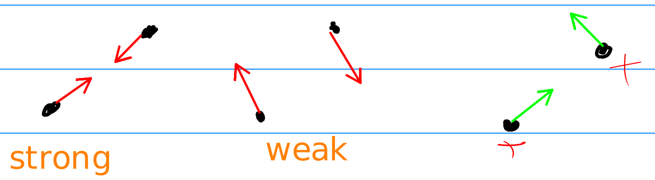
$$\sum_i \dot{\bar{P}}_i = \frac{d^2}{dt^2} \sum_i m_i \bar{r}_i = \sum_i \bar{F}_i^{(e)} + \sum_{\substack{ij \\ i \neq j}} \bar{F}_{ji} \quad (4) \quad (3)$$

$$\sum_{\substack{ij \\ i\neq j}} \to \sum_{ij}{}'$$

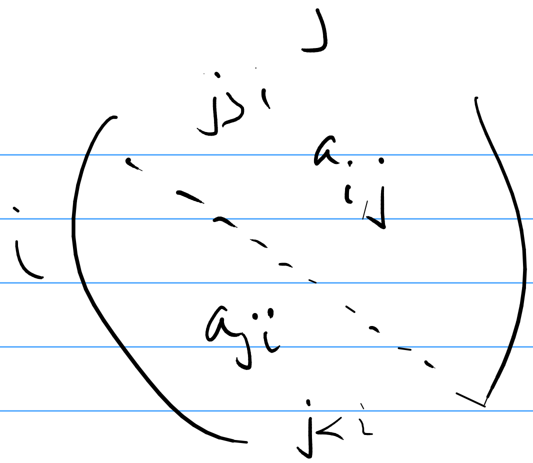
$$(3) \quad \sum_{ij}{}' \bar{F}_{ji} = \sum_{j>i} \underbrace{\left(\bar{F}_{ji} + \bar{F}_{ij}\right)}_{0}$$

by Newton's 3rd law, the internal forces are equal but opposite.

"weak law of action and reaction"



$$\sum_{ij} a_{ij}$$



$$\sum_{j>i} (a_{ij} + a_{ji})$$

$$\bar{R} = \frac{\sum m_i \bar{r}_i}{\sum m_i} = \frac{1}{M} \sum m_i \bar{r}_i$$

$$\sum m_i \bar{r}_i = M\bar{R}$$

substituting

$$M \frac{d^2}{dt^2} \bar{R} = \sum_i \bar{F}_i^{(e)} = \bar{F}^{(e)}$$

center of mass moves as if total external force would act on the mass concentrated at the center of mass.

$$\bar{P} = \sum_i m_i \frac{d\bar{r}_i}{dt} = M \frac{d\bar{R}}{dt} \rightarrow \boxed{\dot{\bar{P}} = \bar{F}^{(e)}}$$

in the absence of external forces, the TOTAL linear momentum is conserved

Total Angular Momentum

$$\frac{d\bar{L}}{dt} = \bar{N}^{(e)}$$

if there are no external torques, then the total angular momentum is conserved

for this to happen, we need to invoke "strong law of action and reaction"