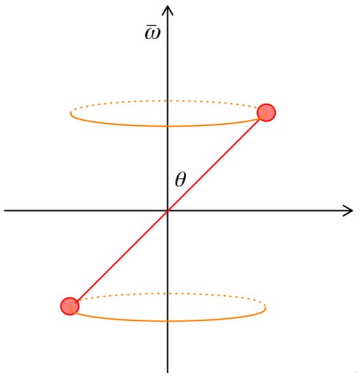
(Due Friday, Nov. 13)

Problems

Solve the following problems from Goldstein, 3rd edition.

- (5.18) A bar of negligible weight and length *l* has equal mass points *m* at the two ends. The bar is made to rotate uniformly about an axis passing through the center of the bar and making an angle θ with the bar.



  1. (a) Using Euler's equations, find the components along the principal axes <u>of the bar</u> (body set, non-inertial) of the torque driving the bar.

     (b) Show that the torque is perpendicular to both, the angular velocity and to a vector parallel to the bar.

  2. (a) Using the fundamental torque equation (1.26)

$$\frac{d\mathbf{L}}{dt} = \mathbf{N}$$

     find the components of the torque along axes <u>fixed in space</u> (inertial).

     (b) Show that these components of the torque are consistent in magnitude and direction with those found in part 1 above. In particular, show that 1(b) still holds.
     Hint for 2: calculate **L** *explicitly and then take the time derivative. Use as origin the center of the bar.*

- (5.23) An automobile is started from rest with one of its doors initially open and making a right angle with the car. If the hinges of the door are toward the front of the car, the door will slam shut as the automobile picks up speed.
  - Obtain a formula for the time needed for the door to close if the acceleration *a* is constant, and the center of mass is at a distance *b* from the hinges. Leave your answer in terms of a definite integral involving trigonometric functions.
    Hints: Notice that this is a body rotating around a fixed point. Carefully place your origin and define your Euler angles. Euler's equations should give you the equations of motion for the angles. Also, when integrating, convert a second time derivative of the angle in terms of a first time derivative by using the same trick used for the constrained bead on a wire (chapter 2, example from lecture).
  - Show that if *a* is $0.3\ m/s^2$ and the door is a uniform rectangle $1.2\ m$ wide, the time will be approximately $3.04\ s$.
    Hints: The integral from the previous part can be converted into an elliptic integral of the first kind. Once you do that, realize that the argument is a singular value that can be used to convert that integral in terms of gamma functions.[1]

1 https://mathworld.wolfram.com/EllipticIntegraloftheFirstKind.html, https://mathworld.wolfram.com/EllipticIntegralSingularValue.html, https://mathworld.wolfram.com/EllipticLambdaFunction.html