$$W_{12} = \sum_i \int_1^2 \bar{F}_i \cdot d\bar{s}_i = \sum_i \int_1^2 m_i \dot{\bar{v}}_i \cdot \bar{v}_i \, dt$$

$$= \sum_i \int_1^2 m_i \frac{1}{2} \frac{d}{dt} (\bar{v}_i \cdot \bar{v}_i) \, dt$$

$$= \sum_i \int_1^2 d\left(\frac{1}{2} m_i v_i^2\right) = T_2 - T_1$$

where $T = \frac{1}{2} \sum_i m_i v_i^2$

to write the kinetic energy in terms of the CM

$$\bar{v}_i = \bar{v}_i' + \bar{v}$$

substituting

$$T = \frac{1}{2} \sum_i m_i (\bar{v}_i' + \bar{v}) \cdot (\bar{v}_i' + \bar{v})$$

$$= \frac{1}{2} \sum_i m_i v_i'^2 + \frac{1}{2} \sum_i m_i v^2 + \underbrace{\sum_i m_i \bar{v}_i' \cdot \bar{v}}_{\bar{v} \cdot \frac{d}{dt} \cancel{\sum_i m_i \bar{r}_i'} \to 0}$$

So

$$T = \frac{1}{2} M v^2 + \frac{1}{2} \sum_i m_i v_i'^2$$

$$\boxed{\text{total kinetic energy is the sum of the kinetic energy as if all mass is at the CM plus that one about the CM}}$$

total force on particle i

$$\bar{F}_i = \bar{F}_i^{(e)} + \sum_j \bar{F}_{ji}$$

$$W_{12} = \sum_i \int_1^2 \bar{F}_i \cdot d\bar{s}_i = \sum_i \int_1^2 \overset{(1)}{\bar{F}_i^{(e)} \cdot d\bar{s}_i} + \overset{(2)}{\sum_{ij}{}' \int_1^2 \bar{F}_{ij} \cdot d\bar{s}_i}$$

if the external forces are derivable from a potential,

(1) $$\sum_i \int_1^2 \bar{F}_i^{(e)} \cdot d\bar{s}_i = -\sum_i \int_1^2 \nabla_i V_i \cdot d\bar{s}_i = -\sum_i V_i \Big|_1^2$$

if the internal forces are also conservative, then they are derivable from potentials.
To satisfy the strong law of action/reaction, we need only consider a potential that is a function of the *distance* between particles.

$$V_{ij} = V_{ij}(|\bar{r}_i - \bar{r}_j|)$$

satisfies the week form

$$\bar{F}_{ji} = -\nabla_i V_{ij} = +\nabla_j V_{ij} = -\bar{F}_{ij}$$

satisfies the strong form

$$\nabla V_{ij}(|\bar{r}_i - \bar{r}_j|) = (\bar{r}_i - \bar{r}_j) f$$

(2) $$\sum_{ij}{}' \int_1^2 \bar{F}_{ji} \cdot d\bar{s}_i = -\sum_{j>i} \int_1^2 (\nabla_i V_{ij} \cdot d\bar{s}_i + \nabla_j V_{ij} \cdot d\bar{s}_j)$$

$$\nabla_i V_{ij} = \nabla_{ij} V_{ij} = -\nabla_j V_{ij}$$

$$d\bar{s}_i - d\bar{s}_j = d\bar{r}_i - d\bar{r}_j = d\bar{r}_{ij}$$

(2) $$= -\sum_{j>i} \int_1^2 \nabla_{ij} V_{ij} \cdot d\bar{r}_{ij}$$

$$= -\frac{1}{2} \sum_{ij}' \int \nabla_{ij} V_{ij} \cdot d\bar{r}_{ij} = -\frac{1}{2} \sum_{ij}' V_{ij} \Big|_1^2$$

adding both terms of the work from external and internal forces, we obtain

$$W_{12} = -\sum_i V_i \Big|_1^2 - \frac{1}{2} \sum_{ij}' V_{ij} \Big|_1^2$$

we can define a total potential energy of the system

$$V = \sum_i V_i + \frac{1}{2} \sum_{ij}' V_{ij}$$

energy T + V is conserved

Internal Energy: In a "rigid body" this will be a constant (interpaticle distances do not change) and can be ignored.