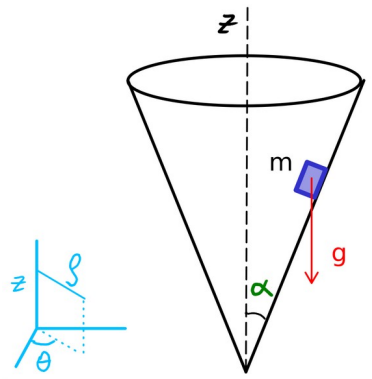
Problems

Solve problem 1 and only **two** other problems.

1. A particle of mass *m* is constrained to move under gravity inside the frictionless inner surface of a cone of half-angle α, as shown in the figure.
   - Write expressions for the kinetic and potential energy of the particle. *Hint: use cylindrical coordinates.*
   - Write an expression for the Lagrangian, and make sure that it is in terms of independent generalized coordinates.
   - Given an initial velocity $v_o\hat{\theta}$, the particle performs circular motion with radius ρ_o. Find the relationship between $v_o, \rho_o,$ and α for this to happen.



2. A spring of rest length l_o (no tension) is connected to a support at one end and has a mass *m* attached at the other. Neglect the mass of the spring and assume that the motion is confined to a vertical plane. Also, assume that the spring only stretches without bending but it can swing in the plane.
   - Using the angular displacement of the mass from the vertical and the length that the string has stretched from its rest length (hanging with the mass *m*), find the Lagrangian.
   - Using the Lagrangian, determine the equations of motion.
   - (*3 pts extra credit*) Starting form the Lagrangian, find equations of motion for very small stretching and angular displacements.
3. Assuming near circularity of the orbits of the Earth around the Sun, and Moon around the Earth,
   - derive an expression for the ratio of mass of the Sun to that of Earth in terms of only the lengths of the year, the lunar period, and the radii of Earth's orbit and the Moon's orbit.
   - (*3 pts extra credit*) Find a numerical estimate of this ratio using 27.3 days for the lunar month, $1.49\times10^8\ km$ for the mean radii of Earth's orbit, and $3.8\times10^5\ km$ for the Moon's orbit.
4. A wagon wheel with spokes is mounted on a vertical axis so it is free to rotate in the horizontal plane. The wheel is rotating with an angular speed ω. A bug initially at the center of the wheel crawls out on one of the spokes with a velocity *v* holding on to the spoke with a coefficient of friction μ.
   - Find an expression of the distance from the center that the bug can crawl along the spoke before it starts to slip.
   - (*3 pts extra credit*) If $\omega = 3.0\,radians/s$, $v = 0.5\,cm/s$, and $\mu = 0.30$, compare the numerical value of the distance you found above to the distance at which the magnitudes of the centrifugal and Coriolis forces are the same.