

Short answers:

#8: a pulsar emits magnetic dipole radiation because it has a rotating magnetic dipole that is misaligned from the axis of its angular velocity. (i.e. the dipole axis is at some inclination angle from the spin axis) Like an accelerated charge, an accelerated magnetic moment radiates power (and the magnetic moment accelerates, or decelerates, in time due to the change in rotational kinetic energy that manifests as a change in the spin period)

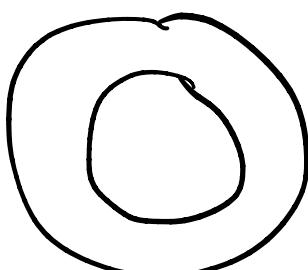
#2 refraction increases w/ shorter wavelengths since they are slowed down more and thus bend more. Blue light has a shorter wavelength than red light so it refracts at a greater angle

#5 a wave will propagate in waveguide if it has a higher frequency than the cutoff frequency. So for $\omega = 5c/R$ to propagate, waveguide must have a cutoff frequency w/

(TM)	X_{mn}	$m=0$	$m=1$	$m=2$
$n=1$	2.4	3.83	5.14	
$n=2$	5.52	7.02	8.42	
$n=3$	8.65	10.17	11.62	

$X_{mn} < 5$. So only TM_{01} and TM_{11} have low enough cutoff frequencies ($\omega = 2.4c/R$ and $\omega = 3.83c/R$ respectively)

#4 waveguide can't be hollow if it can support TEM modes, an example is a cable



#6 magnetic reconnection is the breaking and reconnecting of oppositely directed magnetic field lines - this reconnection converts the magnetic field energy to kinetic and thermal energy in a plasma. An example is the auroras that happen at Earth's polar regions - solar winds disturb Earth's magnetic field so reconnection in the plasma get accelerated along field lines to the polar regions

long answers

#2

a. for TE, $\psi = H_z$ and need $\frac{\partial \psi}{\partial n} = 0$ at $x=0$ and a and $y=0$ and b

so wave eq is $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2) \psi = 0$

w/ solution $\Psi_{mn}(x,y) = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$

$$\text{and } \gamma^2 = \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$$

$$\text{also have } H_t = \pm \frac{i k}{\gamma^2} D_t \psi$$

for TE₁₀:

$$H_z = H_0 \cos\left(\frac{\pi x}{a}\right) e^{i(kz - ut)}$$

$$\begin{aligned} H_x &= \pm \frac{i k}{\gamma^2} D_x \psi \\ &= -\frac{i k a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) e^{i(kz - ut)} \end{aligned}$$

$$E_t = \pm z \times H_t \hat{z}$$

$$E_y = i \frac{w a \mu}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) e^{i(kz - ut)}$$

$$\text{where } k = k_{1,0} = \sqrt{\mu \epsilon} \sqrt{w^2 - w_{1,0}^2}$$

$$w_{1,0} = \frac{\pi}{\sqrt{\mu \epsilon}} \sqrt{\frac{1}{a^2} + \frac{0}{b^2}} = \frac{\pi}{a \sqrt{\mu \epsilon}}$$

b. need to find cutoff frequency, $a=3b$

$$\omega_{mn} = \frac{\pi mn}{\sqrt{\mu\epsilon}} = \frac{\pi}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \\ = \frac{\pi}{\sqrt{\mu\epsilon}} \sqrt{\frac{m^2}{9b^2} + \frac{n^2}{b^2}} = \frac{\pi}{\sqrt{\mu\epsilon}} \sqrt{\frac{m^2 + 9n^2}{9b^2}}$$

$$\omega = \frac{3.5\pi}{\sqrt{\mu\epsilon}a} = \frac{3.5\pi}{\sqrt{\mu\epsilon}(3b)}$$

need m, n such that $\omega_{mn} < \omega$

$$\text{so } \frac{\pi}{\sqrt{\mu\epsilon}} \sqrt{\frac{m^2 + 9n^2}{9b^2}} < \frac{3.5\pi}{(3b)}$$

$$\frac{m^2 + 9n^2}{9b^2} < \frac{10.5}{9b^2}$$

$$m^2 + 9n^2 < 10.5$$

so possible
TE modes are
 $TE_{01}, TE_{10}, TE_{11}$
and TE_{20}

$$m, n = 0, 1 \quad 9 < 10.5 \quad \checkmark$$

$$m, n = 0, 2 \quad 36 < 10.5 \quad \times$$

$$m, n = 1, 0 \quad 1 < 10.5 \quad \checkmark$$

$$m, n = 1, 1 \quad 10 < 10.5 \quad \checkmark$$

$$m, n = 1, 2 \quad 37 < 10.5 \quad \times$$

$$m, n = 2, 0 \quad 4 < 10.5 \quad \checkmark$$

$$m, n = 2, 1 \quad 11 < 10.5 \quad \times$$

$$c. \quad V_p = \frac{\omega}{k_x} = \frac{1}{\sqrt{\mu\epsilon}} \frac{1}{\sqrt{1-(\omega/\omega_0)^2}}$$

lowest mode from b is TE₁₀ w/ $\omega_0 = \frac{1}{a\sqrt{\mu\epsilon}}$
 use $\omega = \frac{3.5\pi}{\sqrt{\mu\epsilon}a}$

$$\text{so } \frac{\omega/\omega_0}{\omega} = \frac{1}{3.5}$$

then $V_p = \frac{1}{\sqrt{\mu\epsilon}} \frac{1}{\sqrt{1-(1/3.5)^2}}$

$$V_g = \frac{\sqrt{1-(\omega/\omega_0)^2}}{\sqrt{\mu\epsilon}} = \frac{\sqrt{1-(1/3.5)^2}}{\sqrt{\mu\epsilon}} = V_g$$

d. wave number must have correction terms

$$k'_\lambda = k_\lambda^{(0)} + \alpha_\lambda + i\beta_\lambda$$

so all equations in a) must have k' where k was

#3 linear antenna

a) total charge is 0 but we have a +q and -q at opposite ends of antenna \rightarrow oscillatory electric dipole

$$\text{so } \vec{P} = q d \hat{z}$$

$$I = \frac{dq}{dt} \quad \text{so} \quad dq = I dt$$

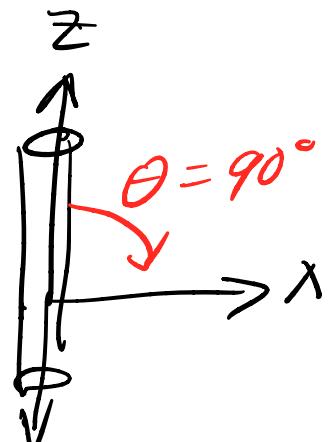
$$q = \int I dt$$

$$= \int I_0 e^{-i\omega t} dt$$

$$q = \frac{I_0}{-i\omega} e^{-i\omega t} = \frac{i I_0 e^{-i\omega t}}{\omega}$$

$$\boxed{\vec{P} = \frac{idI_0}{\omega} e^{-i\omega t} \hat{z}}$$

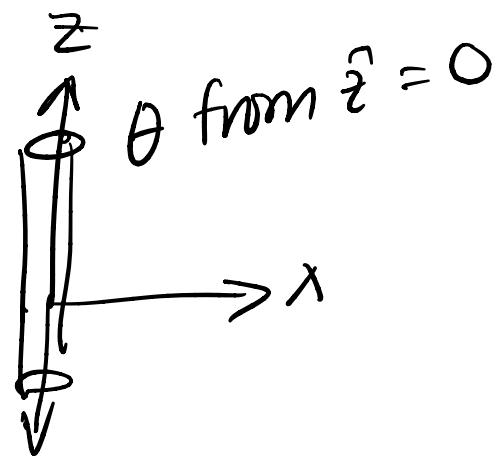
$$\begin{aligned} b) \frac{dP}{d\Omega} &= \frac{c^2 Z_0 k^4}{32\pi^2} |\vec{P}|^2 \sin^2 \theta \\ &= \frac{c^2 Z_0 k^4}{32\pi^2} \left| \frac{idZ_0}{\omega} e^{-i\omega t} \right|^2 \\ &= \frac{c^2 d^2 Z_0 k^4}{32\pi^2 \omega^2} I_0^2 e^{-2i\omega t} \end{aligned}$$



c)

$$\frac{dP}{dr} = \frac{c^2 Z_0 k^4}{32\pi^2} |\vec{P}|^2 \sin^2 \theta$$

$$= 0$$



$$d) A(x) = -\frac{iM_0 w}{4\pi} \vec{P} \frac{e^{ikr}}{r}$$

$$= -\frac{iM_0 w}{4\pi} \frac{e^{ikr}}{r} \cdot \frac{idI_0}{w} e^{-iut} \hat{z}$$

$$= \frac{M_0 dI_0}{4\pi} \frac{e^{ikr}}{r} e^{-iut} \hat{z}$$

$$H = \frac{1}{M_0} \nabla \times A = -\frac{iw}{4\pi} D \left(\frac{e^{ikr}}{r} \right) \times \vec{P}$$

$$= -\frac{iw}{4\pi} \left(\frac{ik e^{ikr}}{r} - \frac{e^{ikr}}{r} \right) \hat{n} \times \vec{P}$$

$$= \frac{w}{4\pi} \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \hat{n} \times \left(\frac{idI_0}{w} e^{-iut} \hat{z} \right)$$

$$= \frac{idI_0}{4\pi} \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) e^{-iut} \hat{n} \times \hat{z}$$

since $kr \gg 1$ in far zone,

$$H = \frac{idI_0}{4\pi} \frac{e^{ikr}}{r} e^{-iut} \hat{n} \times \hat{z}$$