Short answers:

#8: a pulsar emits magnetic dipole radiation because it has a rotating magnetic dipole that is misaligned from the axis of its angular velocity. (i.e. the dipole axis is at some inclination angle from the spin axis) Like an accelerated charge, an accelerated magnetic moment radiates power (and the magnetic moment accelerates, or deaccelerates, in time due to the change in rotational kinetic energy that manifests as a change in the spin period)

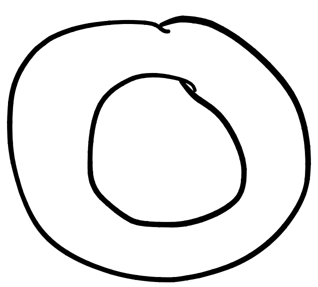
#2 refraction increases w/ shorter wavelengths since they are slowed down more and thus bend more. Blue light has a shorter wavelength than red light so it refracts at a greater angle

#5 a wave will propagate in waveguide if it has a r higher frequency than the cutoff frequency. So for $\omega = 5c/R$ to propagate, waveguide must have a cutoff frequency w/ $X_{mn} < 5$. So only TM_{01} and TM_{11} have low enough cutoff frequencies ($\omega = 2.4c/R$ and $\omega = 3.83c/R$ respectively)

(TM)

X_{mn}	m=0	m=1	m=2
n=1	2.4	3.83	5.14
n=2	5.52	7.02	8.42
n=3	8.65	10.17	11.62

#4 waveguide can't be hollow if it can support TEM modes, an example is a cable



#6 magnetic reconnection is the breaking and reconnecting of oppositely directed magnetic field lines - this reconnection converts the magnetic field energy to kinetic and thermal energy in a plasma. An example is the auroras that happen at Earth's polar regions - solar winds disturb Earth's magnetic field so reconnections in the plasma get accelerated along field lines to the polar regions

Long answers

#2

a. for TE, $\psi = H_z$ and need $\frac{\partial \psi}{\partial n} = 0$ at $x=0$ and a and $y=0$ and b

so wave eq is $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2\right)\psi = 0$

w/ solution $\psi_{mn}(x,y) = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$

and $\gamma_{mn}^2 = \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)$

also have $H_t = \pm \frac{ik}{\gamma^2} \nabla_t \psi$

for TE_{10}:

$H_z = H_0 \cos\left(\frac{\pi x}{a}\right) e^{i(kz - \omega t)}$

$H_x = \pm \frac{ik}{\gamma^2} \nabla_x \psi$

$= -\frac{ika}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) e^{i(kz-\omega t)}$

$E_t = \pm Z \times H_t \hat{z}$

$E_y = \frac{i\omega a \mu}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) e^{i(kz-\omega t)}$

where $k = k_{1,0} = \sqrt{\mu\varepsilon}\sqrt{\omega^2 - \omega_{10}^2}$

$\omega_{10} = \frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{\frac{1}{a^2} + \frac{0}{b^2}} = \frac{\pi}{a\sqrt{\mu\varepsilon}}$

b. need to find cutoff frequency, $a = 3b$

$$\omega_{mn} = \frac{\gamma_{mn}}{\sqrt{\mu\varepsilon}} = \frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}$$

$$= \frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2}{9b^2} + \frac{n^2}{b^2}} = \frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2 + 9n^2}{9b^2}}$$

$$\omega = \frac{3.5\pi}{\sqrt{\mu\varepsilon}\,a} = \frac{3.5\pi}{\sqrt{\mu\varepsilon}\,(3b)}$$

need m, n such that $\omega_{mn} < \omega$

so $\pi\sqrt{\frac{m^2+9n^2}{9b^2}} < \frac{3.5\pi}{(3b)}$

$$\frac{m^2+9n^2}{9b^2} < \frac{10.5}{9b^2}$$

$$m^2 + 9n^2 < 10.5$$

$m, n = 0, 1$	$9 < 10.5$	✓
$m, n = 0, 2$	$36 < 10.5$	X
$m, n = 1, 0$	$1 < 10.5$	✓
$m, n = 1, 1$	$10 < 10.5$	✓
$m, n = 1, 2$	$37 < 10.5$	X
$m, n = 2, 0$	$4 < 10.5$	✓
$m, n = 2, 1$	$11 < 10.5$	X

$\boxed{\text{so possible TE modes are } TE_{01},\ TE_{10},\ TE_{11} \text{ and } TE_{20}}$

c. $V_p = \frac{\omega}{k_x} = \frac{1}{\sqrt{\mu\varepsilon}} \frac{1}{\sqrt{1-(\omega_\lambda/\omega)^2}}$

lowest mode from b is TE_{10} w/ $\omega_\lambda = \frac{\pi}{a\sqrt{\mu\varepsilon}}$

use $\omega = \frac{3.5\pi}{\sqrt{\mu\varepsilon}\, a}$

so $\frac{\omega_\lambda}{\omega} = \frac{1}{3.5}$

then $\boxed{V_p = \frac{1}{\sqrt{\mu\varepsilon}} \frac{1}{\sqrt{1-(1/3.5)^2}}}$

$$V_g = \frac{\sqrt{1-(\omega_\lambda/\omega)^2}}{\sqrt{\mu\varepsilon}} = \boxed{\frac{\sqrt{1-(1/3.5)^2}}{\sqrt{\mu\varepsilon}} = V_g}$$

d. wavenumber must have correction terms

$k'_\lambda = k_\lambda^{(0)} + \alpha_\lambda + i\beta_\lambda$

so all equations in a) must have k' where k was

#3 linear antenna

a) total charge is 0 but we have a +q and −q at opposite ends of antenna → oscillatory electric dipole

so $\vec{p} = q d \hat{z}$

$I = \frac{dq}{dt}$ so $dq = I\,dt$

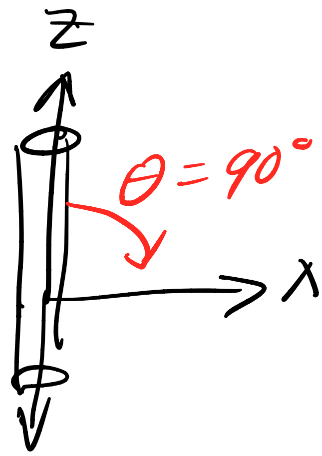
$$q = \int I\,dt$$

$$= \int I_0 e^{-i\omega t}\,dt$$

$$q = \frac{I_0}{-i\omega} e^{-i\omega t} = \frac{i I_0}{\omega} e^{-i\omega t}$$

$$\boxed{\vec{p} = \frac{i d I_0}{\omega} e^{-i\omega t}\,\hat{z}}$$

b)



$$\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^4}{32\pi^2} |\vec{p}|^2 \underbrace{\sin^2\theta}_{1}$$

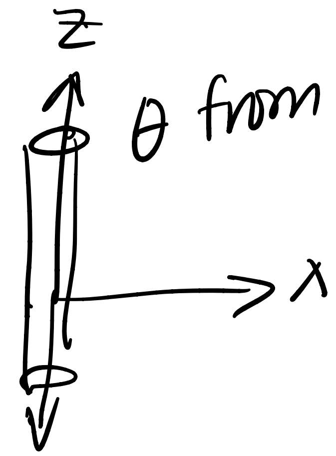
$$= \frac{c^2 Z_0 k^4}{32\pi^2} \left| \frac{i d I_0}{\omega} e^{-i\omega t} \right|^2$$

$$\boxed{= \frac{c^2 d^2 Z_0 k^4}{32\pi^2 \omega^2} I_0^2 e^{-2i\omega t}}$$

c)

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^4}{32\pi^2} |\vec{p}|^2 \underbrace{\sin^2\theta}_{0}$$

$$\boxed{= 0}$$



θ from $\hat{z} = 0$

d) $$A(x) = -\frac{i\mu_0 \omega}{4\pi} \vec{p} \frac{e^{ikr}}{r}$$

$$= -\frac{i\mu_0 \cancel{\omega}}{4\pi} \frac{e^{ikr}}{r} \cdot \frac{i d I_0}{\cancel{\omega}} e^{-i\omega t} \hat{z}$$

$$\boxed{= \frac{\mu_0 d I_0}{4\pi} \frac{e^{ikr}}{r} e^{-i\omega t} \hat{z}}$$

$$H = \frac{1}{\mu_0} \nabla \times A = \frac{-i\omega}{4\pi} \nabla\left(\frac{e^{ikr}}{r}\right) \times \vec{p}$$

$$= \frac{-i\omega}{4\pi}\left(\frac{ik e^{ikr}}{r} - \frac{e^{ikr}}{r}\right) \hat{n} \times \vec{p}$$

$$= \frac{\omega}{4\pi} \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr}\right) \hat{n} \times \left(\frac{i d I_0}{\omega} e^{-i\omega t} \hat{z}\right)$$

$$= \frac{i d I_0}{4\pi} \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr}\right) e^{-i\omega t} \hat{n} \times \hat{z}$$

since $kr \gg 1$ in far zone,

$$\boxed{H = \frac{i d I_0}{4\pi} \frac{e^{ikr}}{r} e^{-i\omega t} \hat{n} \times \hat{z}}$$

$$E = \frac{i Z_0}{k} (\nabla \times H)$$

$$\boxed{= \frac{i Z_0}{k} \nabla \left(\frac{i d I_0}{4\pi} e^{ikr} e^{-i\omega t} \right) (\hat{n} \times (\hat{n} \times \hat{z}))}$$

e) $\Phi(\vec{x}) = \frac{\vec{p} \cdot \vec{x}}{4\pi\varepsilon_0 r^3} = \frac{i d I_0}{\omega} e^{-i\omega t} \cdot \frac{\vec{x}}{4\pi\varepsilon_0 r^3}$

near zone: $r >> d$

so $\boxed{\Phi(\vec{x}) \to 0}$