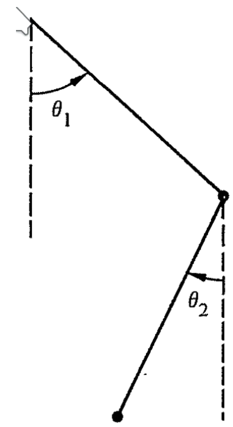
(Due Tuesday, Nov. 24)

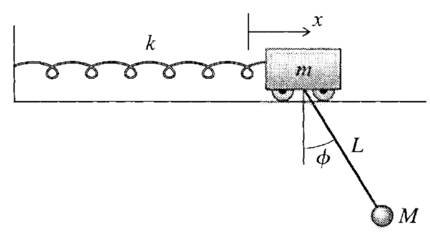
Problems

Solve the following problems.

- Consider the double pendulum shown in the figure with equal lengths, but not equal masses.
  - Obtain the normal modes of vibration (eigenfrequencies and eigenvectors) for small angular displacements. Check that your expressions yield the known results for the case of equal masses.
  - If the pendula are set in motion by pulling the upper mass slightly away from the vertical and then releasing it, show that subsequent motion is such that at regular intervals one pendulum is at rest while the other has its maximum amplitude. This is the familiar phenomenon of "beats."



- A pendulum is suspended from a cart that can oscillate on the end of a spring, as shown in the figure.
  - Obtain the normal modes of vibration (eigenfrequencies and eigenvectors) for small angular displacements.
  - Approximate the eigenfrequencies for the case of $m \gg M$. Explain your result.



- Consider a system with kinetic $T=\frac{1}{2}m(\dot{x}_1^2+\dot{x}_2^2)$ and potential $V=\frac{1}{2}V_{ij}x_i x_j$ energies with $V_{11}=V_{22}=0$ and $V_{12}=V_{21}=b>0$.
  - Obtain the eigenvalues and normal modes.
  - Using equation (6.35) write general expressions for $x_1(t)$ and $x_2(t)$ substituting in the values for a_{ik} and ω_k.
  - Find *normal coordinates* by making use of equation (6.41) and solving for the ζ_j. Your result should show that one of these normal coordinates gives purely oscillatory behavior while the other one gives an unbounded behavior.