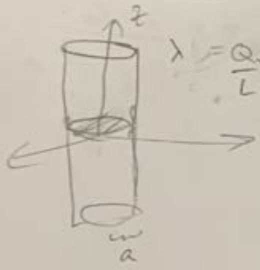
(1)



$\lambda = \frac{Q_{enc}}{L}$ then $Q_{encl} = \lambda L$

$\int dA = 2\pi a L$

a) $\oint E\,dA = \frac{Q_{encl}}{\varepsilon_0} = E \oint dA = E\,(2\pi a L)$

$\frac{\lambda L}{\varepsilon_0} = E\, 2\pi a L$

$$\boxed{E(r) = \frac{\lambda}{2\pi\varepsilon_0 a} = \frac{Q}{2\pi\varepsilon_0 L a}}$$

b) potential outside?

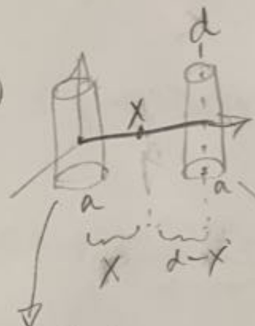
$V = \frac{Q_{enc}}{4\pi\varepsilon_0 a} = \frac{\lambda L}{4\pi\varepsilon_0 a}$

$V = \int_\infty^R dV = -\int_\infty^R E\,dR$

$= -\int \frac{\lambda}{2\pi\varepsilon_0 a}\,dR = \boxed{\frac{-\lambda}{2\pi\varepsilon_0 a} R = -\frac{QR}{2\pi\varepsilon_0 L a}}$

for $R > a$

c)



$E = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x} + \frac{1}{d-x}\right)\hat{x}$

take dl to be a to $d-a$? (distance between surfaces)

$E = \frac{-\lambda}{2\pi\varepsilon_0 (d-x)}(-\hat{x}) = \frac{\lambda\,\hat{x}}{2\pi\varepsilon_0 (d-x)}$

$E = \frac{\lambda\,\hat{x}}{2\pi\varepsilon_0 x}$

$V = \int E\,dl = \int_a^{d-a} \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x} + \frac{1}{d-x}\right)dx = \frac{\lambda}{2\pi\varepsilon_0}\left[\ln x - \ln(d-x)\right]_a^{d-a}$

$= \frac{\lambda}{2\pi\varepsilon_0}\left[\ln(d-a) - \ln(d-(d-a)) - \ln(a) - \ln(d-(d-a))\right]$

$\cancel{d} - \cancel{d} + a$

$= \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{\ln(d-a)}{\ln a} - \frac{\ln(a)}{\ln(d-a)}\right] = \frac{\lambda}{2\pi\varepsilon_0}\ln\left[\frac{d-a}{a}\right]$

$$= \boxed{\frac{Q}{2\pi\varepsilon_0 L}\ln\left[\frac{d-a}{a}\right] = V}$$

$\cancel{V = \int_0^x \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{d-r}\right)dr =}$

d) $Q = C\Delta V$ so $C = \frac{Q}{\Delta V} = \frac{\cancel{Q} \cdot 2\pi\varepsilon_0 L}{\cancel{Q}\ln\left[\frac{a}{d-a}\right]} = \frac{2\pi\varepsilon_0 L}{\ln\left[\frac{a}{d-a}\right]}$

Capacitance per unit length = $\boxed{\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left[\frac{a}{d-a}\right]}}$

(2)

a)

we know $\nabla'^2 G(\vec{x},\vec{x}') = -4\pi\delta(x-x')$

in polar: $\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}$

$G(\rho,\phi,\rho',\phi')$

then $x-x'$ is in ρ and ϕ

polar: $-4\pi\frac{\delta(\rho-\rho')\delta(\phi-\phi')}{\rho}$

$\nabla'^2 G(x,x') = -4\pi\delta(x-x')$

$$\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\rho'\frac{\partial}{\partial\rho'} + \frac{1}{\rho'^2}\frac{\partial^2}{\partial\phi^2}\right)G(\rho,\phi,\rho',\phi') = -\frac{4\pi}{\rho}\delta(\rho-\rho')\delta(\phi-\phi')$$

$\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} = 2\pi\delta(\phi-\phi')$ so $4\pi\delta(\phi-\phi') = 2\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}$

$$\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\rho'\frac{\partial}{\partial\rho'} + \frac{1}{\rho'^2}\frac{\partial^2}{\partial\phi^2}\right)G(\rho,\phi,\rho',\phi') = -\frac{\delta(\rho-\rho')}{\rho}\cdot 2\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}$$

$$\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\rho'\frac{\partial}{\partial\rho'} + \frac{1}{\rho'^2}\frac{\partial^2}{\partial\phi^2}\right)\sum_{m=-\infty}^{\infty} g_m(\rho,\rho')e^{im(\phi-\phi')} = -\frac{2\delta(\rho-\rho')}{\rho}\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}$$

so $\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\rho'\frac{\partial}{\partial\rho'} + \frac{1}{\rho'^2}\frac{\partial^2}{\partial\phi^2}\right)g_m(\rho,\rho')\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} = -\frac{2\delta(\rho-\rho')}{\rho}\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}$

$$\frac{\partial}{\partial \phi'} \sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} = im\, e^{im(\phi-\phi')}$$

$$\frac{\partial^2}{\partial \phi'^2} \sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} = (im)^2\, e^{im(\phi-\phi')}$$

$$= -m^2$$

$$\left(\frac{1}{\rho'} \frac{\partial}{\partial \rho'} \rho' \frac{\partial}{\partial \rho'} - \frac{m^2}{\rho'^2}\right) g_m(\rho,\rho') \underbrace{\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}} = -\frac{2\delta(\rho-\rho')}{\rho} \underbrace{\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}}$$

$$\boxed{\left(\frac{1}{\rho'} \frac{\partial}{\partial \rho'} \rho' \frac{\partial}{\partial \rho'} - \frac{m^2}{\rho'^2}\right) g_m(\rho,\rho') = -\frac{2\delta(\rho-\rho')}{\rho}}$$

b) Dirichlet conditions $\rightarrow G(\rho,\rho',\phi,\phi')$ vanishes at surface

so at $\rho = a$ and b, $G = 0$

$$\boxed{G(a,\rho',\phi,\phi') = 0 = G(b,\rho',\phi,\phi')}$$

c)