* Do either question 1 or question 3!

PHY 517 Spring 2020 Final

Name:

1. Estimate the ground-state energy of a one-dimensional simple harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

using the variational method. For the trial function, because it should be symmetric around and peaked at $x=0$, guess a normalized Gaussian where β is to be optimized:

$$\langle x|\tilde{0}\rangle = \left(\frac{\beta}{\pi}\right)^{1/4} e^{-\beta x^2/2}$$

Clearly indicate what value of β is optimal. You may use the following integrals:

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \quad (a > 0)$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}} \quad (a > 0)$$

2. For two spin ½ particles, ignoring orbital angular momentum, the singlet state is

$$|\, s = 0, m = 0\rangle = \frac{1}{\sqrt{2}} (|+,-\rangle - |-,+\rangle)$$

Verify by explicitly rotating this state about the y-axis by angle θ that it is rotationally invariant.

3. Consider the 1D Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 + \lambda x^4$. Treating the last term as a perturbation, compute the first order energy shift for the unperturbed state $|n\rangle$. Comment on why we shouldn't apply perturbation theory for large n.

4. A spin ½ particle is in an orbital angular momentum $l = 2$ state. Initially, it is in the maximum state

$$|\, j = \frac{5}{2}, m = \frac{5}{2}\rangle = |\, m_1 = 2, m_2 = \frac{1}{2}\rangle$$

Use the lowering operator on both sides to obtain the next state down. Calculate the expectation values of J_z, L_z and S_z for this new state.