Clarification on how to split a double sum into sums of "lower" and "upper" triangle regions of a matrix representation.

Consider the unrestricted double sum (the upper range of 3 is an example, it can be any number):

$$\sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij} = a_{11} + a_{12} + a_{13} + a_{21} + a_{22} + a_{23} + a_{31} + a_{32} + a_{33}$$

If we place the elements in the shape and form of a matrix, we can label some of them as part of the "upper" and other as part of the "lower" triangle of the matrix.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Note that this is only a representation, not an equivalence. The double sum is not really a matrix!

Now regroup the sum by the elements "identity":

$$\sum_{ij} a_{ij} = a_{12} + a_{13} + a_{23}$$ "upper" triangle

$$+ a_{21} + a_{31} + a_{32}$$ "lower" triangle

$$+ a_{11} + a_{22} + a_{33}$$ diagonal

Which in sum notation, can be written as (please verify by explicit index substitution):

$$\sum_{ij} a_{ij} = \sum_{i=1}^{3} \sum_{j>i}^{3} a_{ij} + \sum_{i=1}^{3} \sum_{j>i}^{3} a_{ji} + \sum_{i=1}^{3} a_{ii}$$

Obtaining the expression used in class.

$$\sum_{ij} a_{ij} = \sum_{j>i} (a_{ij} + a_{ji}) + \sum_{i} a_{ii}$$

shorthand notation for a double sum

The context used in class was that the aij's represent forces,

$$a_{ij} = F_{ij}$$

and b/c the particles cannot do force on themselves,

$$F_{ii} = 0$$

obtaining,

$$\sum_{ij} \bar{F}_{ji} = \sum_{j>i} (\bar{F}_{ji} + \bar{F}_{ij})$$

For the case of forces obeying the "weak" law of action/reaction, then this sum is equal to zero.