Midterm

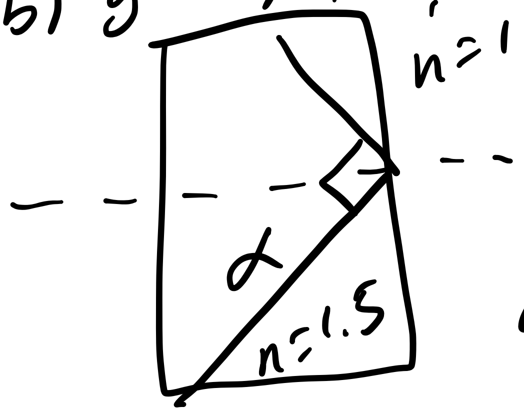
#2

a) $E(y,t) = E_0 e^{i(\vec{k}\vec{y}-\omega t)}\hat{x}$

$B(y,t) = -\frac{E_0}{c} e^{i(\vec{k}\vec{y}-\omega t)}\hat{z}$

since $k = k\hat{y}$ and $\hat{y}\times\hat{x} = -\hat{z}$

b) going from glass to air



$n \sin\alpha = n' \sin\beta$

want reflected wave to only perpendicular comp i.e. angle between reflected light and plane of incidence is $90° = \beta$

$\alpha = \sin^{-1}\left(\frac{1}{1.5}\sin 90\right)$ $\alpha = 41.81°$ $\sim 45°$

in order for incident wave to be broken up into parallel + perp components, $\boxed{\alpha \text{ must be } 45°}$

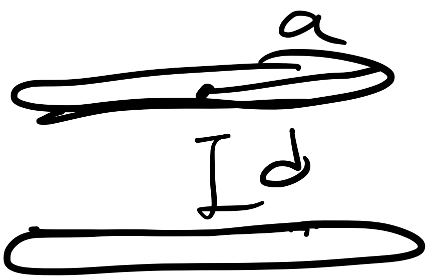
c) for $\alpha = 45°$, the transmitted/refracted wave will have parallel polarization

d) find Brewster angle — no reflection

$\theta_B = \tan^{-1}\left[\frac{n'}{n}\right] = \tan^{-1}\left[\frac{1}{1.5}\right] = 33.69°$

#1 $E=\frac{V(t)}{d}\hat{z}$

a) $H=\frac{1}{\mu_0}B = \frac{1}{\mu}\frac{E}{c} = \frac{V(t)}{\mu_0 c d}\hat{z}$



$$\nabla\times H = \frac{\partial D}{\partial t} = \frac{d}{dt}(\varepsilon_0 E)$$
$$= \frac{d}{dt}\left(\varepsilon_0 \frac{V(t)}{d}\right)$$

in cylindrical:
ρ, ϕ, z

$$\nabla = \frac{\partial}{\partial\phi} + \frac{1}{\rho}\frac{\partial}{\partial\phi} + \frac{\partial}{\partial z}$$

$$\nabla\times H = \frac{\varepsilon_0}{d}\frac{d}{dt}(V(t))$$

$$\frac{\partial}{\partial\phi}+\frac{1}{\rho}\frac{\partial}{\partial\phi}+\frac{\partial}{\partial z}(H) = \frac{\varepsilon_0}{d}\frac{dV(t)}{dt}$$

$$\boxed{H = \frac{\rho}{2}\frac{\varepsilon_0}{d}\frac{dV(t)}{dt}\hat{\phi}}$$

b) show $P=\frac{d}{dt}\left(\frac{1}{2}CV^2\right)$

$B=\frac{\mu_0 I}{2\pi a}\hat{\theta}$ $\quad$ $E=\frac{Q}{\mu_0\varepsilon_0\pi a^2}\hat{k}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}\Big|_{r=a} = \frac{Q}{\mu_0\varepsilon_0\pi a^2}\hat{k}\times\frac{\mu_0 I}{2\pi a}\hat{\theta}$$

$\hat{k}\times\hat{\theta} = -\hat{r}$ $\quad$ $= \frac{QI}{2\varepsilon_0\pi^2 a^3}(-\hat{r})$

power $= \oint_s \vec{S}\cdot da =$

$$= \oint_{cyl}\frac{QI}{2\varepsilon_0\pi^2 a^3}\hat{r}\cdot d$$

$$P = -\frac{QI}{2\pi^2 a^3 \varepsilon_0} (2\pi a d) = \frac{-QId}{\pi a^2 \varepsilon_0}$$

$$= -QIC$$

$$= -Q\left(\frac{dQ}{dt}\right) C$$

Since $V = \frac{Q}{C}$, $QC = CV^2$ $\boxed{= \frac{d}{dt}\left(\frac{CV^2}{2}\right)}$

c) we know $U = \int P \, dt = \int \frac{QId}{\pi a^2 \varepsilon_0} dt$

$I = dQ/dt$ $$= \int \frac{Q\left(\frac{dQ}{\cancel{dt}}\right) d}{\pi a^2 \varepsilon_0} \cancel{dt}$$

$$= \int \frac{Qd}{\pi a^2 \varepsilon_0} dQ = \frac{Q^2 d}{2\pi a^2 \varepsilon_0}$$

and $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 (\pi a^2)}{d}$

so $U = \frac{Q^2}{2C} = \frac{CV^2}{2}$ so $P = \frac{dU}{dt}$

$$\boxed{= \frac{d}{dt}\left(\frac{CV^2}{2}\right)}$$