

The Shifting Sands of Safety

Comparing Bayesian and MLE GARCH Models for Government Bond Volatility in an Era of Economic Turbulence

written by
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1 Abstract

Government bonds - particularly US Treasuries and German Bunds - have long been regarded as safe-haven assets for risk-averse investors across industries and geographies. Institutions with long-term liabilities, such as life insurance companies, have traditionally relied on these instruments to **hedge future payouts**, benefitting from their low volatility and minimal default risk. However, growing concerns over the U.S. economy, the global ramifications of tariffs introduced during President Trump's administration, and the steady rise in government debt have prompted financial institutions to reassess the risk profiles of these assets.

Volatility modeling has always played a central role in financial forecasting, asset pricing, and - more critically in recent times - risk management. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model remains a cornerstone in this domain. While GARCH parameters are commonly estimated via Maximum Likelihood Estimation (MLE), the heightened market uncertainty invites renewed interest in alternative approaches. Bayesian methods offer a compelling framework: they not only model parameter uncertainty but also allow for the evaluation of entire posterior distributions, enabling a more comprehensive understanding of model risk.

This project aims to compare MLE and Bayesian estimation methods for GARCH models in the context of government bond markets, with a particular focus on recent volatility driven by trade tensions and shifting macroeconomic conditions.

2 Task description

- *Objective:* Compare MLE and Bayesian estimation methods for GARCH models in the context of government bond volatility.
- *Data Focus:* US Treasuries, German Bunds and other related government securities.

- *Contextual Frame*: Analyze how recent macroeconomic events (e.g., US tariffs, rising debt) have influenced volatility in traditionally safe assets.
- *Methodology*:
 - Implement and calibrate GARCH models using both MLE and Bayesian approaches.
 - Evaluate differences in volatility estimation, parameter uncertainty, and predictive performance using part of the recent market turmoil as a test data set.
- *Outcome*: Assess the - theoretical and practical - suitability of Bayesian methods for risk management under heightened uncertainty.

3 Research Questions and Hypotheses

Building on our motivation to reassess volatility modeling for government bonds in an environment of heightened economic uncertainty, this project aims to address (at least some of) the following research questions:

- How do Maximum Likelihood Estimation (MLE) and Bayesian approaches differ in their estimation of GARCH model parameters, particularly with regard to uncertainty quantification and computational efficiency?
- Which estimation method yields superior model fit and forecasting performance when applied to US Treasuries and German Bunds during periods of increased volatility?
- To what extent does the choice of estimation method influence key model diagnostics (eg model stability)?
- How can custom priors be designed and incorporated into the Bayesian framework to better reflect economic context and improve model performance?

4 Methods and Data

To address the research questions, this project will use the following methodological structure:

- **Theoretical Framework**: We begin with a review of the GARCH model family, provide a comprehensive literature review of current research in volatility modeling on the one hand, economic modeling on the other side as well as its intersection and present the mathematical underpinnings of both Maximum Likelihood Estimation (MLE) and Bayesian estimation techniques.
- **Data**: The empirical analysis will focus on daily yield data for US Treasuries and German Bunds as well as other selected developed markets government bonds, covering periods of heightened volatility, particularly during and after the enactment of US tariffs. Data will be sourced from

publicly available financial databases such as the Federal Reserve, Bundesbank, and/or FRED (depending on availability).

- **Model Estimation:** We will estimate a set of GARCH(p,q) models using both approaches:
 - **MLE-based estimation** will be implemented via the rugarch package in R (R Core Team, 2025).
 - **Bayesian estimation** will be performed using the bayesGARCH package in R.
 - Additional graphs, analyses and descriptive statistics will be generated/computed using both R and Python along with several packages.
- **Model Comparison and Evaluation:** We will assess and compare model performance using:
 - **In-sample diagnostics:** AIC, BIC, and residual tests (e.g., Ljung–Box test for autocorrelation, ARCH-LM test for conditional heteroskedasticity).
 - **Out-of-sample forecasting performance:** Mean Squared Error (MSE) of volatility forecasts.

5 Time frame

Date	Task/Meeting/Deadline	Completed
23th of March 2025	First meetup of the team along with a basic setup	✓
2nd of May 2025	Upload of first proposal	✓
End of May 2025	Finished data cleaning and preparation	
End of June 2025	Preliminary results and presentation	
Mid July 2025	First draft of paper	
Beginning of August 2025	Finish paper and submission	

6 Expected results

We expect both MLE and the Bayesian approach to produce reasonable GARCH models, but Bayesian modeling approach to better handle *six sigma events* and provide more information on model and parameter uncertainty. In the end, the practicability of Bayesian models often depend on computational efficiency and run time statistics.

Note: The following references are being used as base literature for our research.

References

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