

Tuesday, 5 December 2023

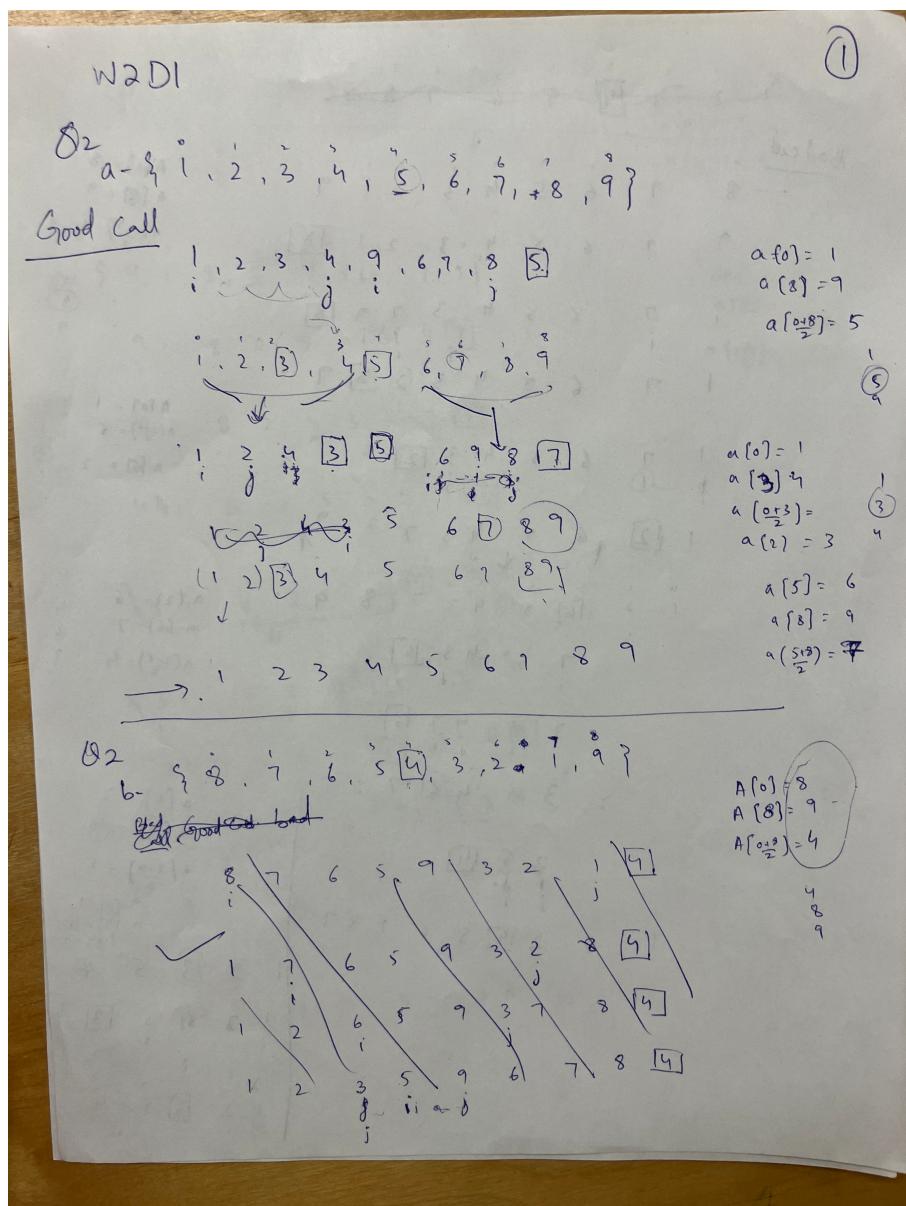
## Title

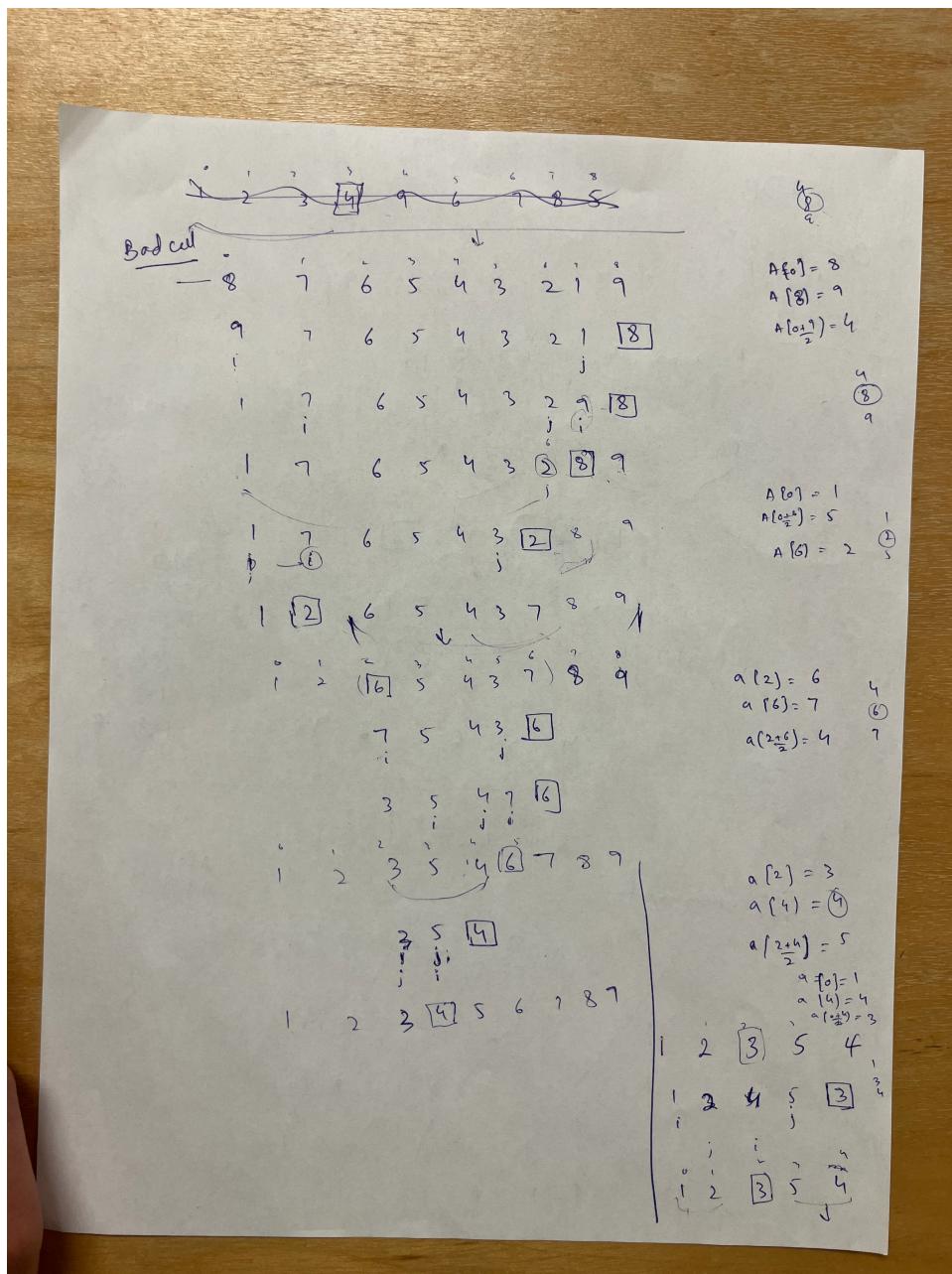
---

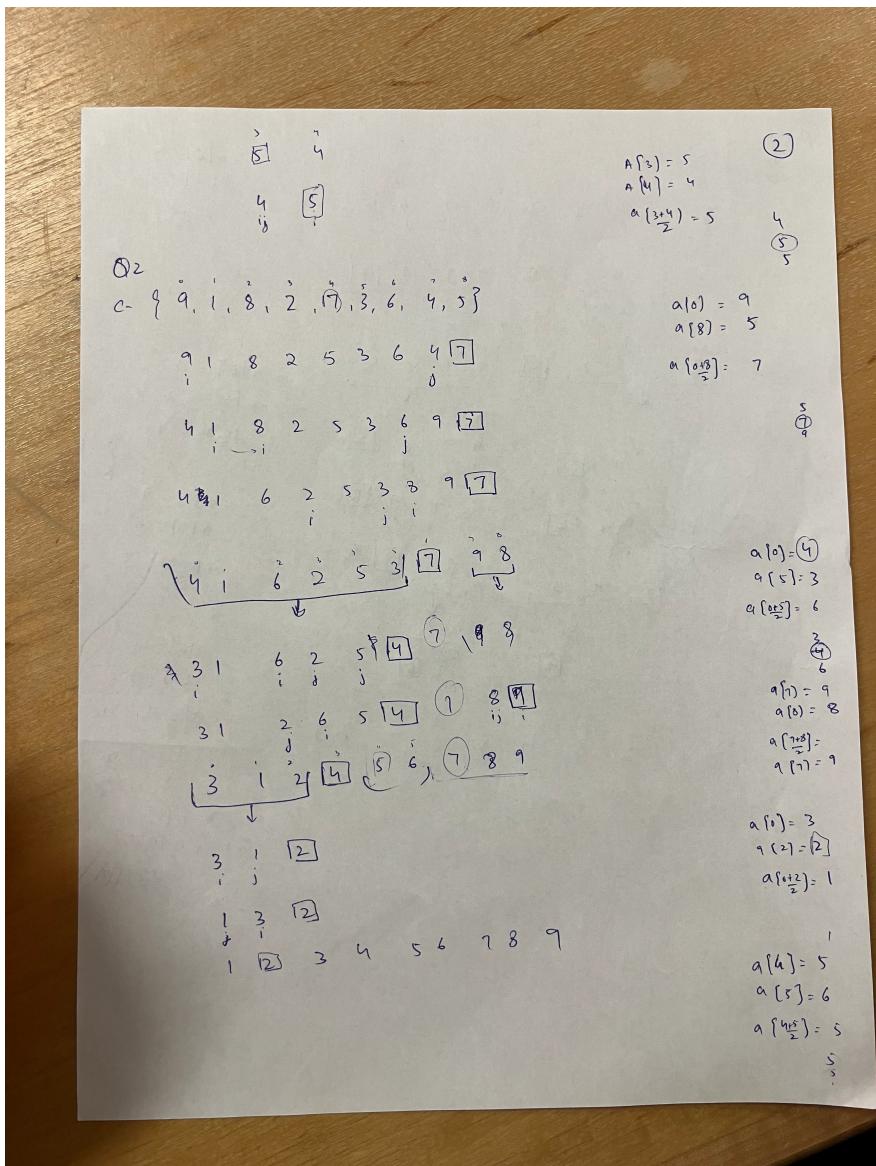
### Question 1.

You can find question 1 answer in the provided .java files attached to this lab

### Question 2.







02  
d-

$\{ \overset{\circ}{5}, 1, \overset{2}{4}, \overset{3}{2}, \overset{4}{3}, \overset{5}{9}, \overset{6}{7}, \overset{7}{6}, \overset{8}{8} \}$

8 1 4 2 3 9 7 6  $\boxed{5}$

$$a[0] = 5$$

$$a[8] = 8$$

$$a[\frac{0+8}{2}] = 3$$

3 1 4 2 8 9 7 6  $\boxed{5}$

$\frac{3}{8}$

$\boxed{3} \ 1 \ 4 \ 2 \ 5 \ \boxed{5} \ \boxed{9} \ 7 \ 6 \ 8$

$$a[0] = 3$$

$$a[3] = 2$$

$$a[\frac{0+3}{2}] = 1 \quad \frac{1}{3}$$

3 1 4 2  $\boxed{5}$  9 7 6  $\boxed{8}$

$$a[5] = 9$$

$$a[8] = 8$$

$$a[\frac{5+8}{2}] = 7$$

1 3 4  $\boxed{12}$  6 7 9  $\boxed{8}$

1  $\boxed{12}$  4 3 5 6 7  $\boxed{8}$  9

$$a[2] = 4$$

$$a[3] = 3$$

$$a[\frac{2+3}{2}] = 4$$

$\frac{4}{7}$

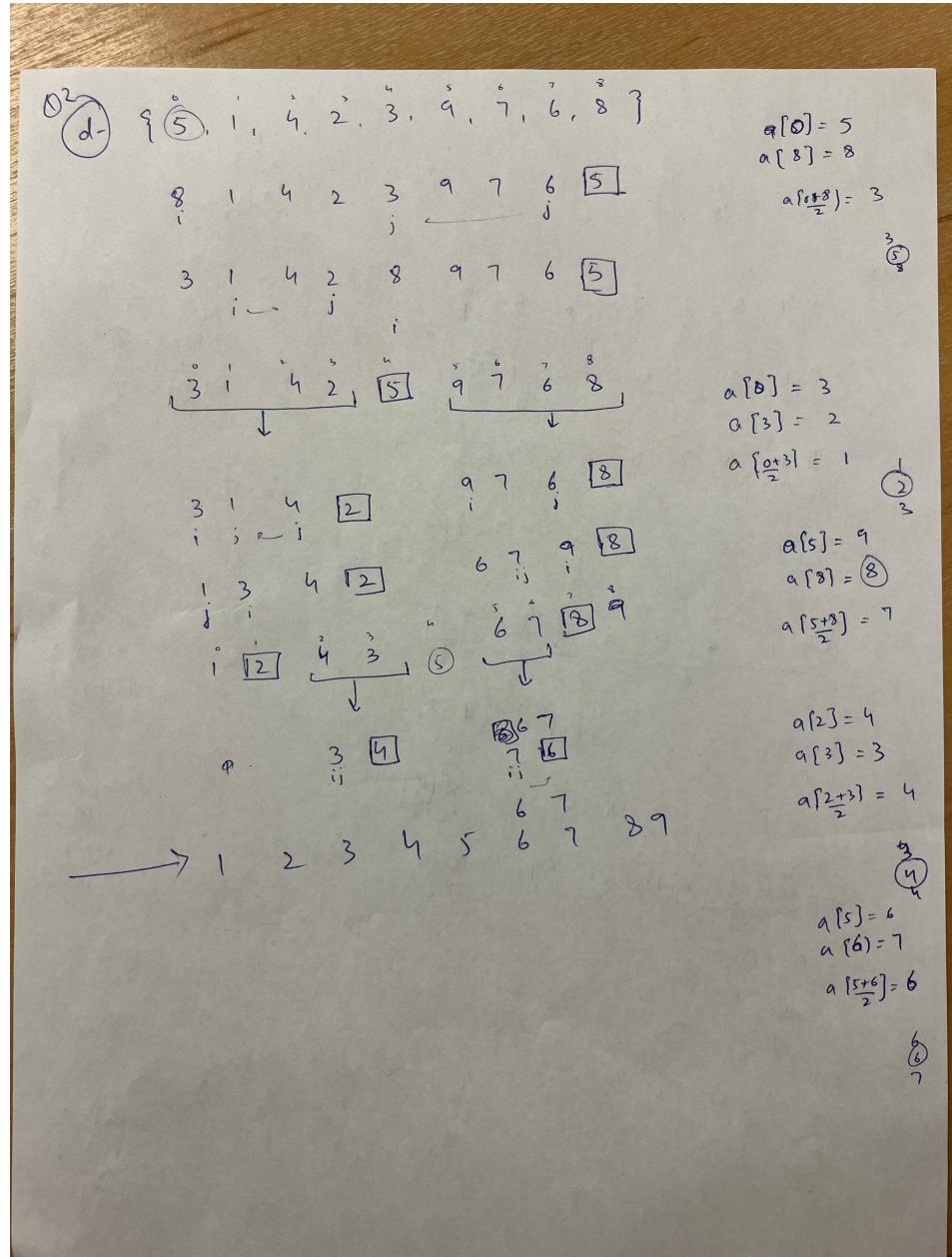
$\rightarrow 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9$

$$a[5] = 6$$

$$a[6] = 7$$

$$a[\frac{5+6}{2}] = 6$$

$\frac{6}{7}$



## Question 3

W2D1

Q3. Quick Select.

(i)

a-  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$K = 5$

$L = a[0] = 1, a[8] = 9, a\left[\frac{0+8}{2}\right] = 5$

$1, \boxed{5}, 9$   
So

$\rightarrow 1, 2, 3, 4, \boxed{5}, 6, 7, 8, 9$

$\rightarrow \begin{matrix} i & 2 & 3 & 4 & 9 & 6 & 7 & 8 & j \\ i & - & - & - & i & - & j & - & \end{matrix}$

$\rightarrow 1 \quad 2 \quad 3 \quad 4 \quad \boxed{5} \quad 6 \quad 7 \quad 8 \quad 9$   
 $|L| = 4 \quad |E| = 1 \quad |G| = 4$

$|U| = 4 < K = 5 \leq 5 \lceil (|L| + |E|) \rceil$

b-  $\{8, 7, 6, 5, 4, 3, 2, 1, 9\}$

$K = 3$  By median of three

$L = a[0] = 8, a[8] = 9, a\left[\frac{0+8}{2}\right] = 4 \quad 1, \boxed{8}, 9$

$\rightarrow \begin{matrix} i & 7 & 6 & 5 & 4 & 3 & 2 & j & \boxed{8} \\ i & - & - & - & i & - & j & \end{matrix}$

$\rightarrow \begin{matrix} 1 & ? & 6 & 5 & 4 & 3 & 2 & 9 & \boxed{8} \\ i & - & - & - & i & - & j & \end{matrix}$

$\rightarrow \begin{matrix} 1 & 2 & 6 & 5 & 4 & 3 & 7 & 9 & \boxed{8} \\ i & - & - & - & i & - & i & \end{matrix}$

Q3

W2 D1

(2)

b-  $\{ 8, 7, 6, 5, 4, 3, 2, 1, 9 \}$   $k=3$

Bad Pivot = 7

$$\begin{matrix} 8 & 9 & 6 & 5 & 4 & 3 & 2 & 1 & \boxed{7} \\ i & & & & & & & & j \end{matrix}$$

$$\begin{matrix} 1 & 9 & 6 & 5 & 4 & 3 & 2 & 8 & \boxed{7} \\ i & & & & & & & & j \end{matrix}$$

$$\begin{matrix} 1 & 2 & 6 & 5 & 4 & 3 & 9 & 8 & \boxed{7} \\ i & & j & & & & & & \end{matrix}$$

$$\begin{matrix} 1 & 2 & 6 & 5 & 4 & 3 & \boxed{7} & 8 & 9 \\ & & & & & & & & \\ K & \leq & 14 & & & & & & \end{matrix}$$

~~Good~~.  $\{ 1, 2, 6, 5, 4, 3 \}$   $k=3$

Good Pivot = 3

$$\begin{matrix} 1 & 2 & 6 & 5 & 4 & \boxed{3} \\ i & \xrightarrow{i} & i & & & j \end{matrix}$$

$$\begin{matrix} 1 & 2 & \boxed{3} & 5 & 4 & 6 \\ |L|=2 & |E|=1 & \frac{|E|}{|L|}=3 \end{matrix}$$

$$|L| < K \leq |L| + |G|$$

$$Q^3 \\ C^- \quad [9, 1 8 2 7 3 6 4 5]$$

found  $K=8$

Bad pivot  $\underline{\underline{4}} = 1$

$$\begin{matrix} 9 & 5 & 8 & 2 & 7 & 3 & 6 & 4 & \boxed{1} \\ i & j & & & & & & & \end{matrix}$$

no smaller value for  $j$

$$\boxed{1} \ 5 \ 8 \ 2 \ 7 \ 3 \ 6 \ 4 \ 9$$

$$|L|=0 \quad |E|=1 \quad |G|=8$$

$$K > |L| + |E|$$

$$5 \ 8 \ 2 \ 7 \ 3 \ 6 \ 4 \ 9 \quad K=7$$

Good Pivot  $= 4$

$$\begin{matrix} 5 & 8 & 2 & 7 & 3 & 6 & 9 & \boxed{4} \\ i & & & j \leftarrow j & & & & \end{matrix}$$

$$3 \ 8 \ 2 \ 7 \ 5 \ 6 \ 9 \ \boxed{4}$$

$$3 \ 2 \ 8 \ 7 \ 5 \ 6 \ 9 \ \boxed{4}$$

$$3 \ 2 \ \boxed{4} \ 7 \ 5 \ 6 \ 9 \ 8$$

$$|L|=2 \quad |E|=1 \quad |G|=5$$

$$K > |L| + |G|$$

$$7 \ 5 \ 6 \ 9 \ 8 \quad K=4$$

Good pivot = 7

$$\begin{matrix} 8 & 5 & 6 & 9 & \boxed{7} \\ i & & j \leftarrow j & & \end{matrix}$$

$$\begin{matrix} 6 & 5 & 8 & 9 & \boxed{7} \\ ij \rightarrow i & & & & \end{matrix}$$

$$6 \ 5 \ \boxed{7} \ 9 \ 8$$

WLDI

$$6 \ 5 \ \boxed{7} \ 9 \ 8 \\ |L|=2 \quad |E|=1 \quad |G|=2$$

(3)

$$K > |L| + |E|$$

$$\boxed{9} \ \boxed{8} \quad K=1$$

$$\text{Pivot} = 9$$

$$\begin{matrix} 8 & \boxed{9} \\ ij & , i \end{matrix}$$

$$8 \ \boxed{9} \\ |L|=1 \quad |E|=1 \quad |G|=0$$

$$K \leq |L|$$

$$\text{So } \boxed{8}$$

---

$$d - \{5, 1, 4, 2, 3, 9, 7, 6, 8\} = k = 5$$

$$\text{Good Pivot} = 5$$

$$\begin{matrix} 8 & 1 & 4 & 2 & 3 & 9 & 7 & 6 & \boxed{5} \\ i & & & & j & \leftarrow & & & \end{matrix}$$

$$\begin{matrix} 3 & 1 & 4 & 2 & 8 & 9 & 7 & 6 & \boxed{5} \\ i & & & j & , i \end{matrix}$$

$$\begin{matrix} 3 & 1 & 4 & 2 & \boxed{5} & 9 & 7 & 6 & 8 \\ |L|=4 & |E|=1 & & & & 6=141 \end{matrix}$$

$$|L| < K \leq |L| + |E|$$

$$\boxed{5}$$

## Question 4

Let us redefine “Good Self Call” and “Bad Self Call”

Good self-call: the sizes of L and G are each less than  $2n/3$  (normal division)

Bad self-call: one of L and G has size greater than or equal to  $2n/3$ .

(a) Repeat the calculations shown in Slides 15, 16 and 17. (You need not draw pictures).

(b) Are you able to derive the same results in Slides 16 and 17? If not, why?]

Since we define a good self-call less than  $2n/3$ , so, taking an example from the slide 15:

$$A = \{7\ 2\ 9\ 4\ 3\ 7\ 6\ 1\}$$

$$L = 2\ 4\ 3\ 1$$

$$G = 7\ 9\ 7\ 1$$

Where  $L < (2*8/3 = 5)$  and  $G < (2*8/3 = 5)$ . Hence, it's still a good self-call.

Let's check how many elements are processed in each level of a recursion tree:

$$n + n*(2/3) + n*(2/3)^2 + n*(2/3)^3 \dots + 0 = n/(1-(2/3)) = 3n \text{ which is } O(n)$$

Hence, it showed the same possible recursion depth