Thursday, 30 November 2023

Lab 2

Question 1. Comparing Algorithms. Problem: Find the THIRD largest in an array.

Algorithm 1 : Idea – Use three loops one after another. First will find Max. Second will find Second Max,
Third will find third max. Note that it is possible First max == second Max == Third Max as in
7, 20, 18, 4, 20, 19, 20, 3.
and your program should return 20 in this case.
Algorithm 2 : Idea – Use one loop. Maintain three variable max, preMax and prePreMax such that max
will have the maximum value, preMax will have the second largest and prePreMax will have the third
largest value.

In this lab, for both algorithms you will
(a) write the pseudo code. (Must follow the notations and conventions used in today's Lecture)
(b) determine the worst-case time complexity by counting as in Slide 15 Lesson 2.
(c) Perform an empirical time comparison by implementing using Java similar to what you did in
W1D1.
(d) Draw a chart to compare algorithms.

a)

Algorithm algorithm1(array)
Input: array of integers
Output: third largest value

```
if length(array) is 0:
   return Integer.MIN_VALUE;

maxIndex ← -1
secondMaxIndex ← -1

max ← Integer.MIN_VALUE
for i ← 0 to length(array) - 1 do:
   if array[i] > max then:
      max ← array[i]
      maxIndex ← i

max ← Integer.MIN_VALUE
for i ← 0 to length(array) - 1 do:
   if i ≠ maxIndex and array[i] > max then:
```

```
            max ← array[i]
            secondMaxIndex ← i

    max ← Integer.MIN_VALUE
    for i ← 0 to length(array) - 1 do:
        if i ≠ maxIndex and i ≠ secondMaxIndex and array[i] > max then:
            max ← array[i]

    return max
```

Algorithm algorithm2(array)
Input: array of integers
Output: third largest value

```
if length(array) is 0:
        return Integer.MIN_VALUE

    max ← Integer.MIN_VALUE
    preMax ← Integer.MIN_VALUE
    prePreMax ← Integer.MIN_VALUE

    for i ← 0 to length(array) - 1 do:

        number ← array[I]

        if number > max then:
            prePreMax ← preMax
            preMax ← max
            max ← number
        else if number > preMax then:
            prePreMax ← preMax
            preMax ← number
        else if number > prePreMax then:
            prePreMax ← number

    return prePreMax
```

b)

Analyzing Algorithm1:

There are 3 loops:

```
// n assignments of I
// n times of incrementing i
// n comparisons made on i along with traversing
for i ← 0 to length(array) - 1 do:
        // n times array[i] has been accessed
       // n comparisons made
        if array[i] > max then:
            // n times array[i] has been accessed
            // n assignments of max
            max ← array[i]
            // n assignments of maxIndex
```

```
        maxIndex ← i
Total for the loop: 8n
```

Other 2 loops are the same except they have 1 and 2 additional comparisons on each element respectively:

```
max ← Integer.MIN_VALUE
for i ← 0 to length(array) - 1 do:
    if i ≠ maxIndex and array[i] > max then:
        max ← array[i]
        secondMaxIndex ← I
Total: 9n since 1 more comparison (i ≠ maxIndex) was added
```

```
max ← Integer.MIN_VALUE
for i ← 0 to length(array) - 1 do:
    if i ≠ maxIndex and i ≠ secondMaxIndex and array[i] > max then:
        max ← array[i]
```

Total: 9n since 1 more comparison (ii ≠ secondMaxIndex) was added but no index assignment in this loop

Grand Total Time complexity for the worst case of the algorithm is: 8n+9n+9n+c=26n where c represents number of constant operations

Analyzing Algorithm 2:

There is one loop:

```
// n assignments of I
// n times of incrementing i
// n comparisons made on i along with traversing
for i ← 0 to length(array) - 1 do:
    // array[I] will be accessed n times
    // n assignments
    number ← array[I]

    // n comparisons may be made
    // 3n assignments may be made
    if number > max then:
        prePreMax ← preMax
        preMax ← max
        max ← number
    // n-1 comparisons may be made
    // 2(n-1) assignments may be made
    else if number > preMax then:
        prePreMax ← preMax
        preMax ← number
    // n-2 comparisons may be made
    // n-2 assignments may be made
    else if number > prePreMax then:
        prePreMax ← number
```

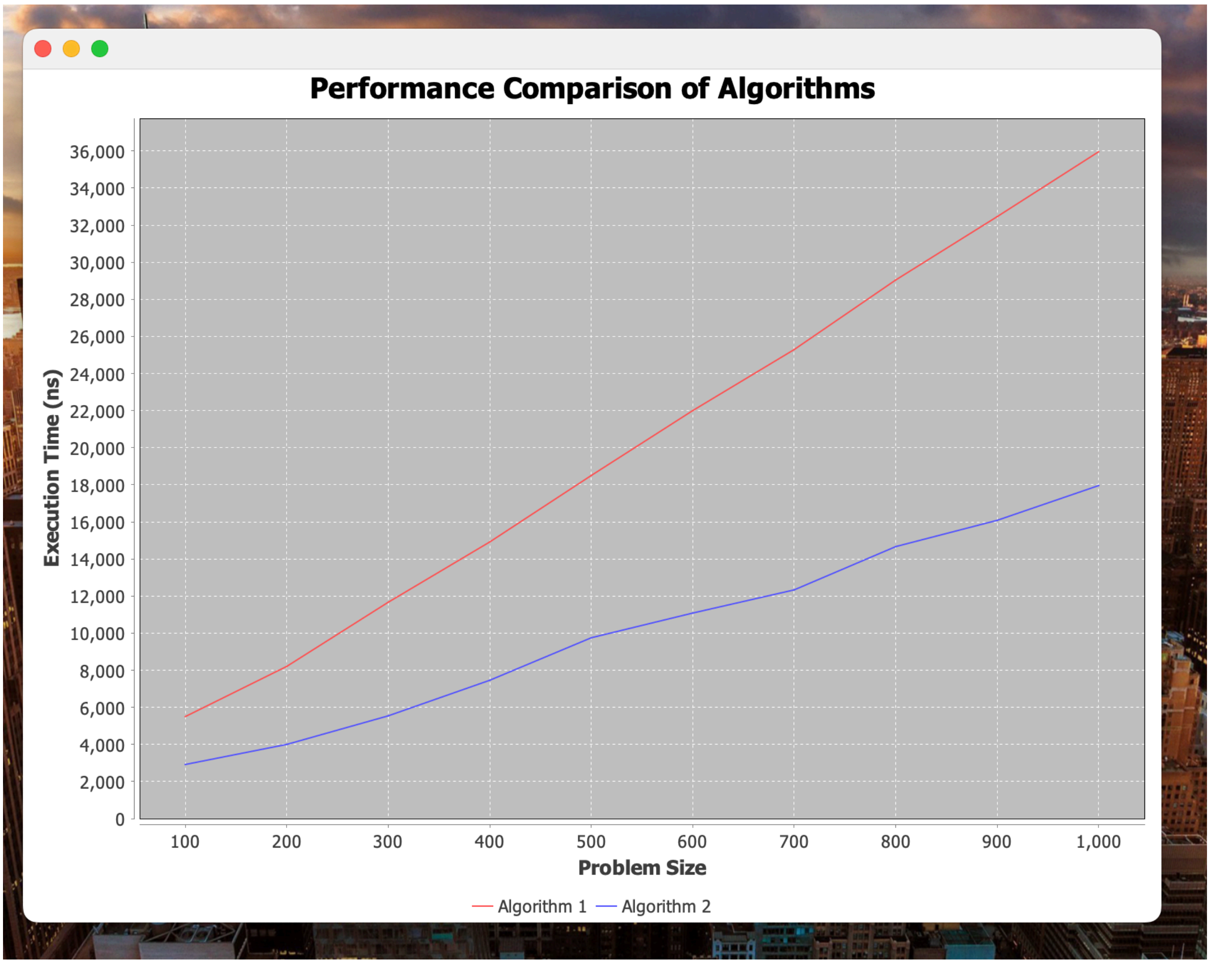
return prePreMax

Grand Total Time complexity for the worst case of the algorithm is: 13n + c where c represents number of constant operations

c) You can find Algorithm.java file along with Main.java file. You can validate Algorithm.java with basic input included without any additional library. However, to run Main.java, you will need to install JFreeChart through maven since the library was used to draw the graph along with Swing. Although, the screenshot of the graph is available at d) below.

d) Comparison graph:

Neglecting constant operations, we could evaluate an approximate ratio of time complexities between algorithms as (26n)/(13n) = 2 which is also represented on the graph



Question 2. Consider the following functions to determine the relationships that exist among the complexity classes they belong.

10,1	O(1)
log(logn)	O(log(logn))
ln n	O(ln n)
logn	O(logn)
n^(1/k) (k>3)	O(n^(1/k) (k>3))
n^(1/3)	O(n^(1/3))
n^(1/2)	O(n^(1/2))
n^(1/3)logn	O(n^(1/3)logn)
n^(1/2)logn	O(n^(1/2)logn)
nlogn, logn^n	O(nlogn)
n^2	O(n^2)
n^3	O(n^3)
n^k (k>3)	O(n^k) (k>3)
2^n	O(2^n)
3^n	O(3^n)
n!	O(n!)
n^n	O(n^n)