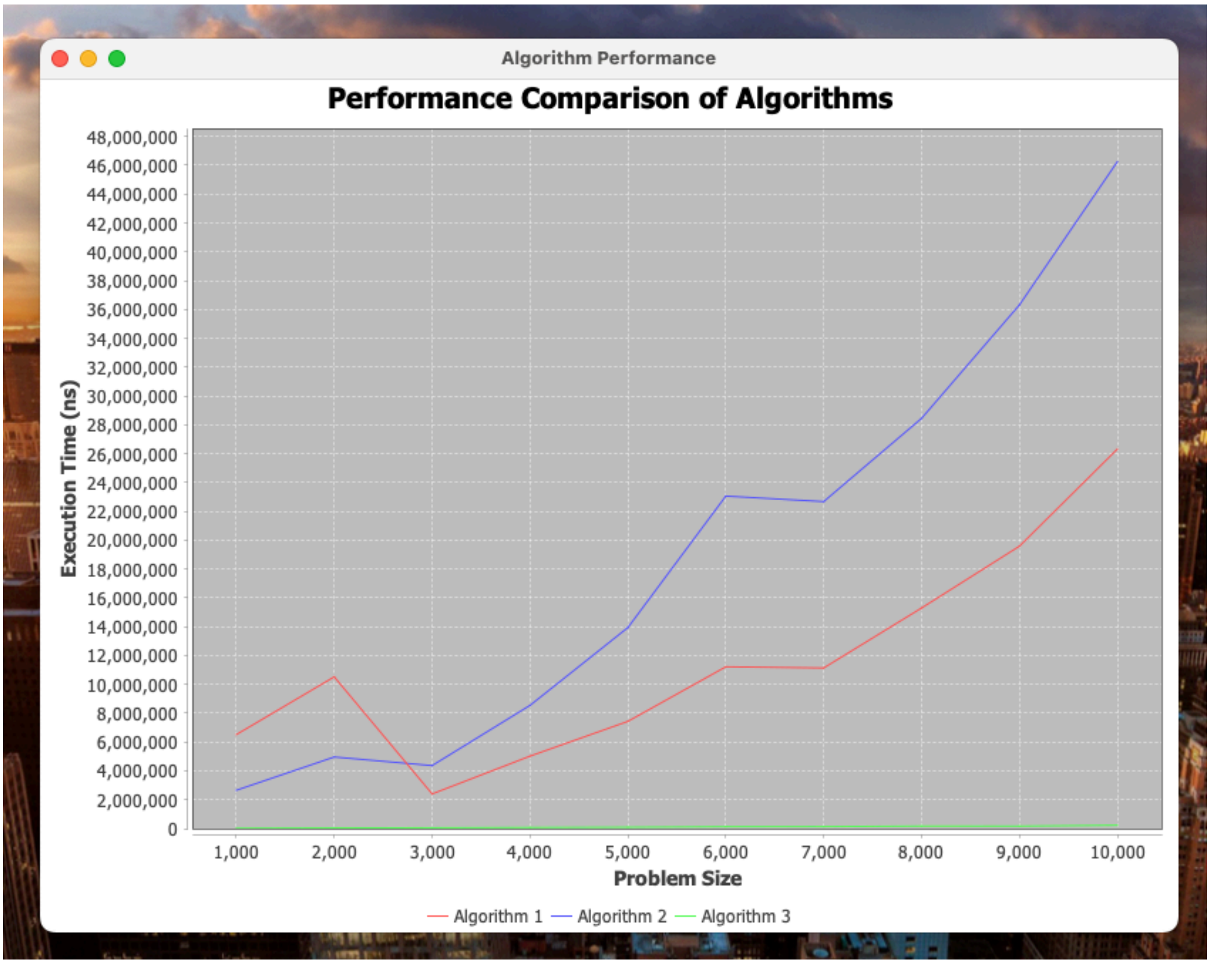
Thursday, 30 November 2023

# Title

---

## Question 1:




From this graph we can see

- Algorithm 3 executes linearly since it has O(n) time complexity
- Algorithm 1 and Algorithm 2 have curves with similar tendency, however, Algorithm 1 is more performant with larger array sets since it extracts all even numbers in a separate array and O(n^2) time complexity is applied to a smaller subset of an array whereas the same time complexity is applied for a whole array in case of Algorithm 2.

## Question2

Let $F(n)$ denote the nth Fibonacci number. Prove $F(n) > (4/3)^n$ for $n > 4$.

Hints: Use the fact $F(n) = F(n-1) + F(n-2)$. Since you are using two values, you must prove the two base

cases: $n = 5$ and $n = 6$.

So, let’s prove it applying the base cases:

$F(5) = 5$, and $(4/3)^5 = 128/243 > 5$. True.

For $n = 6$:

$F(6) = 8$, and $(4/3)^6 = 256/729 > 8$. True.

Making an induction hypothesis we get following:

Assume $n=k$ and $k>=5$ so then $F(k) > (4/3)^k$.

Applying an induction step:

We want to prove that if $n=k+1$ then $F(k+1) > (4/3)^k+1$. Recalling the fibonacci numbers we get:

$F(k+1) = F(k) + F(k-1)$

By the induction hypothesis, we know that:

$F(k) > (4/3)^k$

$F(k - 1) > (4/3)^{(k - 1)}$

Then we can write following statement:

$F(k+1) > (4/3)^{(k+1)}$, therefore

$(4/3)^k + (4/3)^{(k - 1)} > (4/3)^{(k+1)}$

$((4/3)^{(k-1)})*((4/3)+1) > (4/3)^{(k+1)}$

$((4/3)^{(k-1)})*(7/3) > (4/3)^{(k+1)}$

$(7/3) > ((4/3)^{(k+1)})*(4/3^{(1-k)})$

$(7/3) > 16/9$. True

Therefore, we have proven that $F(k + 1) > (4/3)^{(k + 1)}$.