

The Dynamic Effects of Changes to Japanese Immigration Policy *

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Abstract

This paper uses a single-sector dynamic stochastic general equilibrium (DSGE) model with heterogeneous households to analyze Japanese immigration policy. We examine the effects on output, consumption, factor prices, and utility. We do this for both steady states and for transition paths. We find that: 1) aggregate output, investment and consumption in Japan are likely to rise with any sort of loosening of immigration restrictions. 2) Allowing more skilled immigration generates greater aggregate changes. 3) Increasing (decreasing) skilled immigration relative to unskilled immigration drives down (up) skilled wages, consumption and utility. 4) The effects of changes to immigration policy are small when compared to the changes that occur naturally due to business cycle fluctuations. As a result, the effects are likely to be difficult to distinguish when the policies are put into place.

keywords: labor migration, factor mobility, dynamic general equilibrium, Japan, immigration

JEL classification: F15, F22, F42

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1 Introduction and Literature Review

Immigration issues create tough political problems for policy makers. Whether or not to allow workers from low wage countries to migrate to high wage countries is a source of constant domestic and international political debate. Western Europeans struggle with the optimal number of workers from Eastern Europe, North Africa, and the Middle East. Americans confront issues of immigration from Mexico and other parts of Latin America, as well as from China and other countries in Asia.

By comparison, immigration issues do not loom so large in Japan. Nonetheless, Japan's aging population and low birthrates have led to debate in Japanese policy circles on the wisdom of allowing foreign workers into the country. Such immigration would help Japanese firms and capital owners but hurt Japanese workers.

Japan has, until very recently, strictly limited immigration. Currently Japanese immigration law favors skilled workers and those with Japanese ancestry. This is at least partly because of concerns of possible links between non-assimilation of low wage workers and crime. There is no consensus at the current time on whether immigration restrictions should be eased or not. Advocates of the status quo argue that available jobs can largely be filled by Japanese workers.¹ Advocates of increased immigration argue that Japan's demographics demand an increase in immigration to fill job openings and support an increasingly older population.

In June, 2018 the Japanese government announced plans to allow increased employment of foreign workers. While avoiding permanent immigra-

¹For example, see [Ogawa \(2004\)](#).

tion, the change will allow foreigners to live and work in Japan for up to five years. The intent of the change is to boost employment in relatively unskilled jobs that Japanese workers do not take. These workers are expected to make up for employment shortfalls in nursing care, construction, agriculture, lodging and shipbuilding².

Some observers of Japan's immigration policy argue that, rather than increases or decreases in immigration quotas, the government needs to focus on consistent enforcement of a simple set of immigration rules.³

In this paper, we examine the effects of broad changes to immigration policy in Japan. We build and calibrate a single-sector dynamic stochastic general equilibrium (DSGE) model and consider changes in the supply of both unskilled and skilled labor. We are interested in the effects these policy changes will induce on the welfare of existing domestic workers and on aggregate output, consumption, and other key measures of economic activity.

We find that immigration raises overall consumption per capita (including the immigrants as a class) only if the share of skilled labor in immigration is sufficiently high. This finding seems to validate the government's policy of preferential treatment to skilled workers. We also show that the per capita consumption in Japan will rise even with purely unskilled immigration. Since Japanese residents are also direct or indirect owners of capital, the increased productivity of capital with greater numbers of workers offsets the losses due to diminishing marginal product of labor. We show that even hypothetical households which own no capital may be better or worse off, depending on whether they supply the type of labor which is most abundantly allowed to

²See [Shigeta \(2018\)](#)

³See [Kuwahara \(2004\)](#) and [Tezuka \(2004\)](#).

immigrate.

This paper is not the first to examine these issues using formal models. [Goto \(1998\)](#) builds and calibrates a small open computable general equilibrium (CGE) model for Japan. He groups goods into three categories: exportable, importable, and non-traded. Rather than aggregating into a single final good, he allows each of these to enter the utility function separately. Since the model is not explicitly dynamic, he holds capital in each production sector constant. Labor, however, is homogeneous and mobile across sectors. Goto examines the effects of several shocks having to do with changes in trade and international prices. His most interesting result is that small amounts of labor immigration reduce welfare, while sufficiently large amounts may improve welfare.

[Choi \(2004\)](#) builds a static general equilibrium model of the South Korean economy. His model is similar in spirit to ours but has important differences. He allows for imperfect competition in intermediate goods which are produced using sectorally-mobile capital and skilled labor that is specific to that particular intermediate good. Final goods are perfectly competitive and produced with capital and unskilled labor. Choi focuses on the welfare effects of easing immigration restrictions and is concerned primarily with behavior in the short run as a result of business cycle movements. He reports the effects of various business cycle shocks to the economy on welfare and wage inequality. The result most relevant is that an immigration biased towards skilled labor yields the greatest welfare gains.

[Shimasawa and Oguro \(2010\)](#) construct a multi-sector CGE model with overlapping generations and consider the dynamic effects of both increased

immigration and an increase in the consumption tax for Japan. They find "...that substantially increased inflows of working-age immigrants would alleviate the need for future fiscal reform and also help to dramatically reduce the public pension burden on the working generations." Their model includes demographic dynamics within each of the sixteen countries in the model and derives a world general equilibrium, rather than making a small open economy assumption with a single country.

In contrast, this paper uses the tools of DGE modeling. It is similar in spirit to [Phillips \(2012\)](#), which modeled immigration in South Korea. That model, though, considered many intermediate sectors and did not allow for international borrowing and lending.

In this paper, we focus on the long-run transition to a new steady state equilibrium. Business cycle movements are important only because they add uncertainty and volatility to this transition. By incorporating these shocks, though, we are able to present not only impulse responses of key variables to immigration shocks but also derive confidence bands about these responses. (We could also create confidence bands by varying parameters within their confidence bands.)

Section [2](#) below presents the model and shows how it can be rendered stationary and suitable for finding a steady state. In Section [3](#), we discuss calibration of the model. In Section [4](#), we discuss possible policy changes and resulting steady states. Our policies differ in the mix of skilled and unskilled workers that are allowed to immigrate. We explain the technique for finding linear approximations of the policy functions that govern the dynamics of our simulated model. We simulate the various policy options and derive

both smooth transition paths as well as paths with confidence bands for the key variables considered. Section 5 concludes the paper.

2 The Model

We construct a small open economy single sector DSGE model. Our model allows for a single traded final good (Y) which is used for consumption (C) and investment in capital goods. This final good is produced using capital (K) and two types of labor: skilled (L_S) and unskilled (L_U). Both types of labor are supplied in fixed endowments. Capital is traded and accumulates optimally over time. Households may save or borrow internationally and trade balances every period.

2.1 Households

There are four types of households; Skilled and unskilled domestic households, and skilled and unskilled immigrant households. Domestic households have access to savings and credit markets, while immigrant households do not and must consume whatever they earn from wages each period.

Each period, the domestic skilled household maximizes utility, supplies capital and labor, and saves by holding physical capital and savings bonds. They solve the Bellman equation below, subject to the budget constraint in (2.1).

$$V(K_{SD}, B_{SD}; \Theta) = \max_{K'_{SD}, B'_{SD}} L_{SD} \frac{C_{SD}^{1-\gamma} - 1}{1-\gamma} + \beta E\{V(B'_{SD}; \Theta')\}$$

$$C_{SD} = w_S + (1 + r - \delta) \frac{K_{SD}}{L_{SD}} + (1 + s_D) \frac{B_{SD}}{L_{SD}} - \frac{K'_{SD}}{L_{SD}} - \frac{B'_{SD}}{L_{SD}} \quad (2.1)$$

In these equations K is holdings of physical capital, B is holdings of real financial assets, Θ is an information set which includes: w_S , the wage for skilled labor, r the gross return on capital, and s_S , the return on savings for a skilled worker. L_{SD} is the total number skilled domestic workers C_{SD} is consumption per household member and per capital utility is $U(C_{SD})$. β is the subjective discount factor, δ is the rate of capital depreciation. A prime denotes next period's value.

The Euler equations are:

$$C_{SD}^{-\gamma} = \beta E\{C'_{SD}{}^{-\gamma}(1 + r' - \delta)\} \quad (2.2)$$

$$C_{SD}^{-\gamma} = \beta E\{C'_{SD}{}^{-\gamma}(1 + s'_S)\} \quad (2.3)$$

The equivalent equations for the unskilled domestic household are shown below.

$$C_{UD} = w_S + (1 + r - \delta) \frac{K_{UD}}{L_{UD}} + (1 + s_D) \frac{B_{UD}}{L_{UD}} - \frac{K'_{UD}}{L_{UD}} - \frac{B'_{UD}}{L_{UD}} \quad (2.4)$$

$$C_{UD}^{-\gamma} = \beta E\{C'_{UD}{}^{-\gamma}(1 + r' - \delta)\} \quad (2.5)$$

$$C_{UD}^{-\gamma} = \beta E\{C'_{UD}{}^{-\gamma}(1 + s'_S)\} \quad (2.6)$$

Immigrant households are hand-to-mouth consumers and their behavior is

described by the two budget constraints below.

$$C_{SI} = w_S \quad (2.7)$$

$$C_{UI} = w_U \quad (2.8)$$

2.2 Firms

Final good producers maximize profit from hiring capital and both types of labor and selling the final good, as shown in the profit equation below.

$$\text{Max}_{K, N_S, N_U} \Pi = \left[aK^{\frac{b-1}{b}} + (1-a)W^{\frac{b-1}{b}} \right]^{\frac{b}{b-1}} - rK - w_S N_S - w_U N_U$$

with N_S and N_U denoting the demands for skilled and unskilled labor and,

$$W = e^{z+gt} \left[c(fN_S)^{\frac{d-1}{d}} + (1-c)N_U^{\frac{d-1}{d}} \right]^{\frac{d}{d-1}} \quad (2.9)$$

We use a CES production function that has nested in it a CES labor aggregate. Here, W is a composite labor input and technology is labor augmenting. f is the relative productivity of skilled labor over unskilled labor.

The first-order conditions reduce to equations (2.10) through (2.13).

$$Y = \left[aK^{\frac{b-1}{b}} + (1-a)W^{\frac{b-1}{b}} \right]^{\frac{b}{b-1}} \quad (2.10)$$

$$r = a\left(\frac{Y}{K}\right)^{1/b} \quad (2.11)$$

$$w_S = f(1-a)\left(\frac{Y}{W}\right)^{1/b} c\left(\frac{W}{e^{gt}N_S}\right)^{1/d} \quad (2.12)$$

$$w_U = (1-a)\left(\frac{Y}{W}\right)^{1/b} (1-c)\left(\frac{W}{e^{gt}N_U}\right)^{1/d} \quad (2.13)$$

2.3 Financial Sector

Banks extend loans or accept deposits from domestic households. These funds are used to purchase foreign capital which returns a rate of s . As in [Schmitt-Grohe and Uribe \(2003\)](#), we assume that banks limit the amount of credit available by charging a higher interest rate the more a household has borrowed and offering a lower rate the more it has saved relative to some target amount. We let this target vary according to household labor income. This gives us a unique allocation of assets between the two types of domestic households. We show this with the following returns functions.

$$s_S = \bar{s}^* - \nu \phi_S e^{-gt} (B_{SD} + B_{UD}) \quad (2.14)$$

$$\phi_S = \frac{w_S}{\frac{1}{H}(w_S L_{SD} + w_U L_{UD})} \quad (2.15)$$

$$s_U = \bar{s}^* - \nu \phi_U e^{-gt} (B_{SD} + B_{UD}) \quad (2.16)$$

$$\phi_U = \frac{w_U}{\frac{1}{H}(w_S L_{SD} + w_U L_{UD})} \quad (2.17)$$

where \bar{s}^* is a fixed foreign rate of return.

2.4 Market Clearing

All markets must clear and this imposes additional restrictions on the model. Physical capital and labor are not traded internationally.

The capital markets clear as shown below.

$$K_{SD} + K_{UD} = K \quad (2.18)$$

$$B_{SD} + B_{UD} = B \quad (2.19)$$

The labor market clearing conditions are that labor demand equal the number of workers times the time each works.

$$N_S = L_{SD} + L_{SI} \quad (2.20)$$

$$N_U = L_{UD} + L_{UI} \quad (2.21)$$

Balanced trade gives (2.22).

$$X + sB + (B - B') = 0 \quad (2.22)$$

The first two terms are the current account and the last term is the capital account.

Equations (2.1) through (2.22) are a system of dynamic equations that define the behavior of the economy over time.

2.5 A Stationary Version

We can render the model stationary by redefining variables. Equation (2.10) shows that technology is growing with a trend growth rate of g . Hence we can transform all growing non-utility variables ($K, B, F, w_U, w_S, Y, C, X$) by dividing them by e^{gt} . We denote transformed variables by placing a carat over them.

This transformed system of equations is given in Appendix A2 by (A.1) through (A.22).

3 Calibration

Equations (A.1) through (A.22) are a stationary system. We can find the steady state of this system by replacing the variables in these equations with their steady state values. Equations (A.1) through (A.18) can be used as definitions. This system cannot be easily solved algebraically, so we solve it numerically instead. The steady state values will be functions of the parameter values chosen. Our parameters are: β , δ , γ , g , a , b , c , d , f , L_{SD} , L_{UD} , L_{SI} , L_{UI} , \bar{s} and ν . We explain our choice of parameter values below.

The annual depreciation rate δ is set to 11.31%, the average of the implied rate consistent with a capital stock measure constructed by the perpetual inventory method from IMF real investment data and a direct measure of Japanese capital from the Federal Reserve Economic Database. We use the period 1995 to 2014.

γ is the coefficient of relative risk aversion. We set this to 2.5, a value often used in the business cycle literature.

The annual growth rate of technology, g , is set to zero, which is the average value for the growth of TFP in Japan over the period 2000 to 2014.

The average international real interest rate (\bar{s}^*) is set to .6043%. This is the average value of the real return on five-year inflation-indexed bond in U.S. for the period 2003 to 2016. We use an annual subjective discount factor (β) of .99.

We calculate a by taking the capital share in national income for Japan in 1980. c is the share of skilled worker's total compensation in total labor compensation for 2014. This data come from Japanese Ministry of Health, Labor and Welfare. The parameters b and d , elasticities of substitution, are chosen based on the findings of Pessoa et al. (2005) and Behar (2010), respectively.

f is the relative effectiveness of skilled workers. We choose this by taking the ratio of total compensation per skilled worker and dividing it by the total compensation per unskilled worker. This value is 3.24 in the 2014 data. $f = 1.817$ sets the steady state ratio of skilled to unskilled wages to this value.

We set the total number of workers in the baseline to one by normalization. Relative amounts of unskilled labor and skilled labor (L_{SD} and L_{UD}) are obtained using data from the Japanese Ministry of Health, Labor and Welfare for 2014. We use the percent of the workforce ages 15 or older with an advanced education⁴. This percentage is 23.25%. The baseline values of L_{SI} and L_{UI} are set to values that yield the same skilled to unskilled ratio as the rest of the population and that give a foreign population of 1.24%. These values change as immigration policy changes.

The parameter ν is set to .81 so that the ratio of \bar{B} to \bar{Y} equals the average value of -6.51% over the period 1980 - 2015,

The values of all baseline parameters are reported in Table 1.

⁴Defined as graduates of universities, versus graduates from junior high schools, senior high schools and higher professional schools or junior colleges.

Table 1: Summary of Baseline Parameter Values

β	0.9985	a	0.38	L_{SD}	0.2296
δ	0.0283	b	0.7	L_{UD}	0.7580
g	0	c	0.4952	L_{SI}	0.0029
ν	0.81	d	2.0	L_{UI}	0.0095
f	1.187	\bar{s}^*	0.001511	γ	2.5

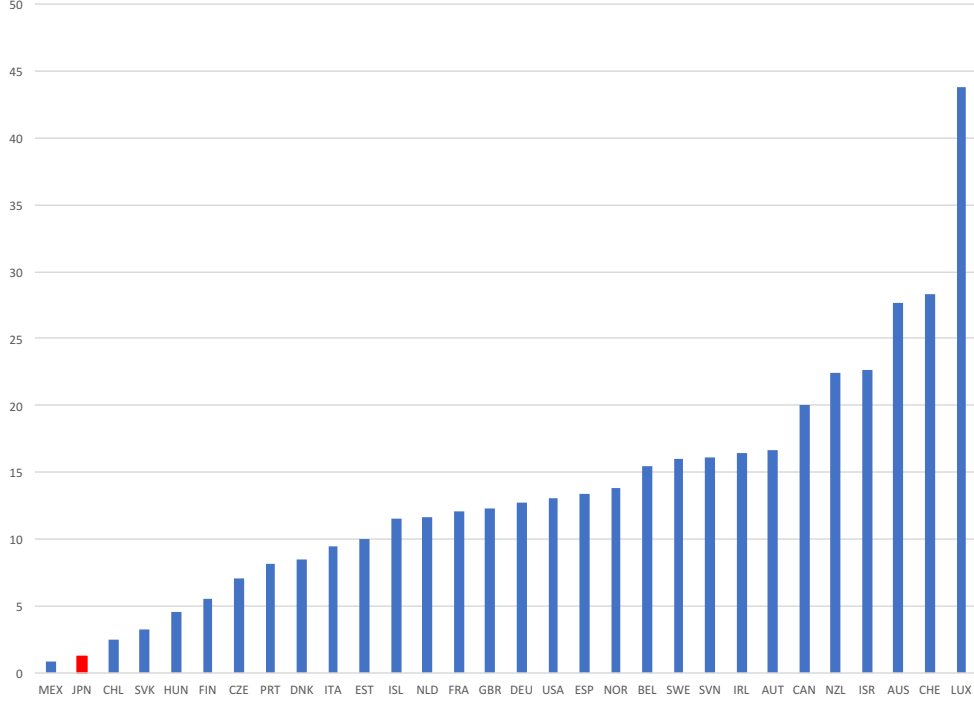
4 Policy Experiments

We consider relaxation of immigration constraints by imagining policies that allow the labor endowment of the economy to rise by some fixed percent. We view foreign and domestic labor as perfect substitutes as long as the labor is of the same type. A policy maker can choose to relax or constrain immigration and alter the domestic supplies of labor. The policy maker can target a particular type of labor and leave endowments of the other type unchanged.

As Figure 1 shows, the percentage of foreign residents to the total population is quite low in Japan compared to other developed countries. This number was 1.24% in 2013 and is around 2% as of 2018. By contrast, it was 13.08% for the United States, 12.04% in France, 12.78% for Germany and 12.26% in Great Britain. We consider a change in immigration policy that raises the percentages from their current values to 5%. This is about half of the normal value for other large developed economies. This corresponds to new immigration equal to 3.76% of the existing population. The policies we consider differ only in the mix of labor types allowed to immigrate.

- i. We first consider a case (UN) where only unskilled labor is allowed to immigrate.

Figure 1: Foreign Population as a Percentage of the Total Population, 2000



Sources: OECD and Statistics Japan

- ii. Secondly, we consider a case (PR) where both skilled and unskilled labor are allowed to immigrate in the same proportions of the current labor force.
- iii. Third, we consider a case (EQ) where both types of labor can immigrate, but skilled labor is given a priority. We allow equal numbers of workers of both types to enter the country, but since there are more unskilled workers in the workforce already, this leads to smaller percentage increases for unskilled labor.
- iv. A fourth scenario (SK) is to allow only skilled labor into a country.

4.1 Steady States

The steady state values for the baseline case and for the four different immigration cases are presented in Table 2. Several interesting patterns emerge from this table. First, greater increases in skilled immigration lead to greater increases in capital and output. The ranking from lowest to highest is: 1) unskilled only, 2) proportional, 3) skilled only, and 4) equal.

Second, as the mix of immigration moves from unskilled to skilled labor, skilled wages fall and unskilled wages rise, as one would expect. This is because L_S and L_U are substitutes in production. This also causes consumption by skilled labor to fall, while that of unskilled labor rises as skilled immigration becomes more predominant. This is true for both domestic workers and foreign workers.

Finally, welfare (as measured by utility levels) rises for unskilled workers and falls for skilled workers as the mix of new immigrants becomes more skilled. Again this is for both domestic and foreign households.

All these results are for the steady state, to which the economy will trend in the long-run. However, the long-run can be very far in the future and policy makers may well be interested in changes in output, consumption and wages along the transition path to this new steady state.

We now turn to these transition paths.

Table 2: Steady State Utility Values
5% Immigrants

Variable	UN	PR	EQ	SK
K	2.821%	3.787%	5.395%	6.665%
Y	2.821%	3.787%	5.395%	6.665%
w_S	1.401%	-0.040%	-2.087%	-4.179%
w_U	-0.995%	0.029%	1.576%	3.279%
C_{SD}	1.715%	0.392%	-1.486%	-3.440%
C_{UD}	-1.276%	-0.328%	1.112%	2.734%
C_{SI}	1.401%	-0.040%	-2.087%	-4.179%
C_{UI}	-0.995%	0.029%	1.576%	3.279%
U_{SD}	0.0048	0.0011	-0.0043	-0.0102
U_{UD}	-0.0220	-0.0056	0.0186	0.0448
U_{SI}	0.0040	-0.0001	-0.0063	-0.0129
U_{UI}	-0.0172	0.0005	0.0263	0.0536

BL - baseline

UN - unskilled only immigration

PR - proportional immigration

EQ - Equal amounts of immigration

SK - skilled only immigration.

Note: bold text indicates the highest level of utility or the largest percent increase across policies.

4.2 Transition Paths

We approximate the policy functions for our model using standard linearization techniques like those from Uhlig (1999) and Christiano (2002). Our endogenous state variables are the set of capital stocks and of savings held by domestic households. $\mathbf{X}_t = \{K_{SD}, K_{UD}, B_{SD}, B_{UD}\}$. Our set of exogenous state variables is denoted \mathbf{Z}_t and can include any number of stochastic shocks we might wish to include.

Linearization leads to a policy function of the following form, which can

be used to simulate the model over time if a law of motion for \mathbf{Z}_t is specified.

$$\mathbf{X}_{t+1} - \bar{\mathbf{X}} = \mathbf{P}(\mathbf{X}_t - \bar{\mathbf{X}}) + \mathbf{Q}(\mathbf{Z}_t - \bar{\mathbf{Z}}) \quad (4.1)$$

In our case \mathbf{P} is a 4×4 matrix, since \mathbf{X}_t is a 4×1 vector. \mathbf{Q} will be a $4 \times n_z$ matrix, where n_z is the number of terms in \mathbf{Z}_t .

4.2.1 Expected Transition Paths

To examine the transition of our model economy from the current steady state to a new one, we assume our economy is initially in the baseline steady state, and the economy will remain there until something changes. When policy is changed in some period t_0 , the economy will have a new steady state. From this point in time on, the economy will slowly converge to the new steady state.

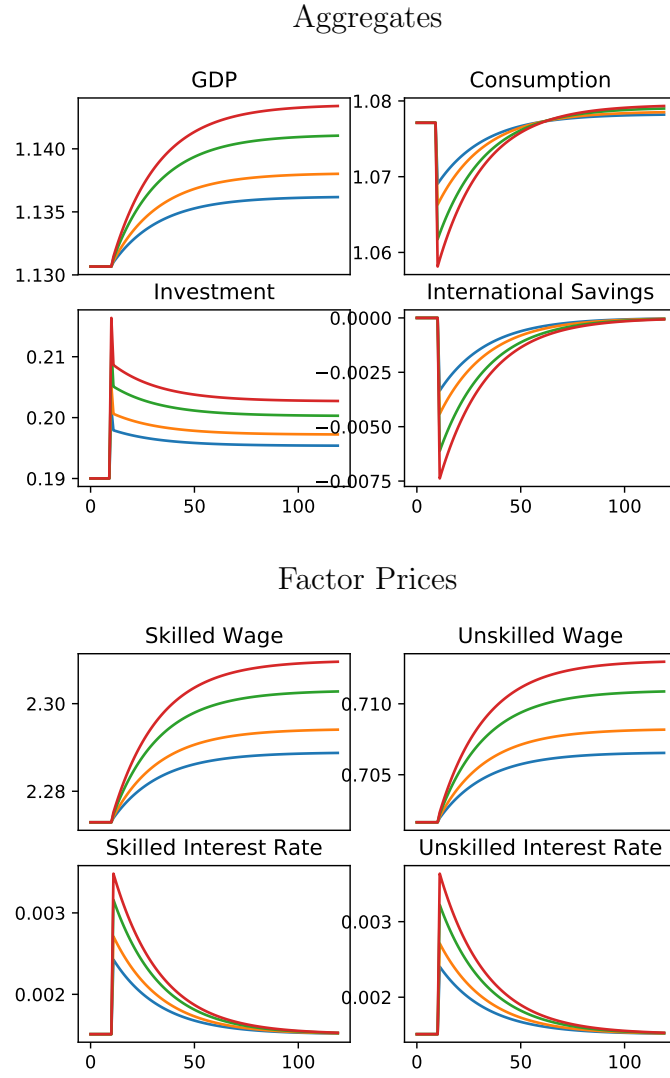
We simulate this using equation (4.1) and use zero for \mathbf{Z}_t each period. We consider a change in policy that occurs in period 10. Since we start off in the steady state under the first policy, we remain there until period 10. The simulation then converges over time from period 10 onward to the new steady state associated with the changed immigration policy.

Figure 2 panel 1 shows the transition paths of GDP, private consumption, investment, and the international financial account over each scenario. Panel 2 shows the transition paths of factor prices. Panel 3 shows the transition paths of agent utilities.

We find the same general patterns along the transition paths as we do in the steady states. That is, the larger effects come from allowing greater

proportions of skilled immigrants.

Figure 2: Transition Paths for Aggregate Variables
5% Immigrants Policy



UN - blue, PR - orange, EQ - green, SK - red

4.2.2 Transition Paths with Aggregate Shocks

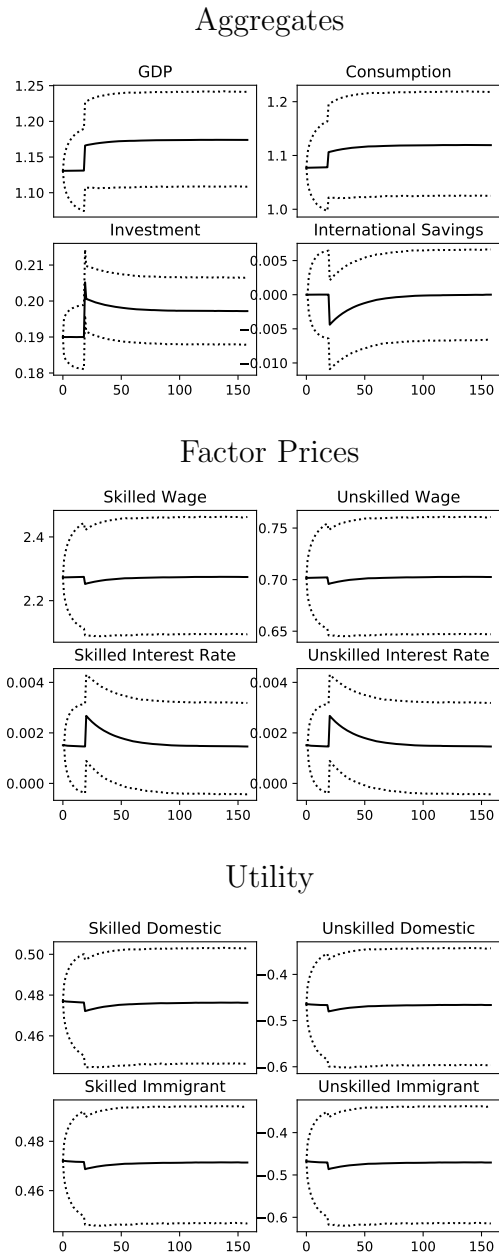
Figure 2 shows the path of variables in the face of an unexpected policy change with no aggregate shocks. It is useful for comparison purposes to see how large the variable changes caused by policy are relative to the changes due to natural fluctuations in the business cycle. To examine this question we simulate our model with a series of random draws to the \mathbf{Z}_t process in equation (4.1) (which corresponds to z in section 2).

We consider a change in immigration policy that occurs suddenly and unexpectedly in five years. We simulate our model economy for 40 years and perform 100,000 simulations. For each period in our 100,000 simulations we sort each variable in the model from highest to lowest across all the simulations. We then take the 5% and 95% values along with the averages over the simulations and plot these for selected variables in Figure 3.

That figure shows the results for a proportional increase in skilled and unskilled immigration; the policy most closely in line with the Japanese government's recent proposal. Similar figures for our other three policies are presented in Appendix A3.

All these figures illustrate a common theme; the effects of immigration reform, even when implemented suddenly and unexpectedly, are small relative to the movement due to the business cycle. Only in the case of investment does the expected path (solid line) even approach the original 90% confidence bands when the reform occurs. We conclude that the effects of loosening immigration restrictions are likely to be unnoticeable against this backdrop. This is particularly true for wages and utility.

Figure 3: Transition Paths for Aggregate Variables
with 90% Confidence Bands
5% Proportional Immigration



5 Conclusion

In this paper we constructed a single sector dynamic general equilibrium model for a trading economy in order to examine the dynamic effect of changes to immigration policy. We solved and calibrated our model using data from Japan. We considered four immigration policies, relating to the mix of skilled and unskilled immigration allowed: unskilled immigrants only, proportional increases in both types, equal increases, and skilled immigrants only. We then examined the effects of these policies on output, consumption, factor prices and utility. This was done for both the new steady state and for the time-path leading to that steady state.

From these policy experiments, we learn the following:

- 1) Overall aggregate output, investment and consumption in Japan are likely to rise with any sort of loosening of immigration restrictions.
- 2) Allowing more skilled immigration generates greater aggregate changes.
- 3) Increasing (decreasing) skilled immigration relative to unskilled immigration drives down (up) skilled wages, consumption and utility.
- 4) The effects of changes to immigration policy are small when compared to the changes that occur naturally due to business cycle fluctuations. As a result, the effects are likely to be difficult to distinguish when the policies are put into place. This particularly likely if the policies are eased in slowly over time.

APPENDICES

A1 Derivation of Factor Price Equations

In this appendix we derive equations (2.10) - (2.13) and (A.6) - (A.10).

Recall that:

$$\begin{aligned}\Pi &= \left[aK^{\frac{b-1}{b}} + (1-a)W^{\frac{b-1}{b}} \right]^{\frac{b}{b-1}} - rK - w_UN_U - w_SN_S \\ W &= \left[c(e^{gt}fN_S)^{\frac{d-1}{d}} + (1-c)(e^{gt}N_U)^{\frac{d-1}{d}} \right]^{\frac{d}{d-1}}\end{aligned}$$

Define the following as equation (2.10).

$$Y = (aK^{\frac{b-1}{b}} + (1-a)W^{\frac{b-1}{b}})^{\frac{b}{b-1}}$$

Differentiating Π with respect to K and setting this to zero:

$$\begin{aligned}\frac{b}{b-1} \left[aK^{\frac{b-1}{b}} + (1-a)W^{\frac{b-1}{b}} \right]^{\frac{b}{b-1}-1} \frac{b-1}{b} aK^{\frac{b-1}{b}-1} - r &= 0 \\ \left[aK^{\frac{b-1}{b}} + (1-a)W^{\frac{b-1}{b}} \right]^{\frac{1}{b-1}} aK^{-\frac{1}{b}} - r &= 0 \\ Y^{\frac{1}{b}} aK^{-\frac{1}{b}} - r &= 0 \\ r &= a \left(\frac{Y}{K} \right)^{1/b} \\ r &= a \left(\frac{Ye^{gt}}{Ke^{gt}} \right)^{1/b} \\ r &= a \left(\frac{\hat{Y}}{\hat{K}} \right)^{1/b}\end{aligned} \tag{A.1}$$

Differentiating Π with respect to N_U and setting this to zero:

$$\begin{aligned}
& \frac{b}{b-1} \left[aK^{\frac{b-1}{b}} + (1-a)W^{\frac{b-1}{b}} \right]^{\frac{b}{b-1}-1} \frac{b-1}{b} (1-a)W^{\frac{b-1}{b}-1} \frac{\partial W}{\partial N_U} - w_U = 0 \\
& (1-a) \left(\frac{Y}{W} \right)^{1/b} \frac{d}{d-1} \left[c(e^{gt}fN_S)^{\frac{d-1}{d}} + (1-c)(e^{gt}N_U)^{\frac{d-1}{d}} \right]^{\frac{d}{d-1}-1} \\
& \times \frac{d-1}{d} (1-c)(e^{gt}N_U)^{\frac{d-1}{d}-1} e^{gt}f - w_U = 0 \\
& (1-a) \left(\frac{Y}{W} \right)^{1/b} \left[c(e^{gt}fN_S)^{\frac{d-1}{d}} + (1-c)(e^{gt}N_U)^{\frac{d-1}{d}} \right]^{\frac{1}{d-1}} \\
& \times (1-c)(e^{gt}N_U)^{-\frac{1}{d}} e^{gt}f - w_U = 0 \\
& (1-a) \left(\frac{Y}{W} \right)^{1/b} (1-c) \left(\frac{W}{e^{gt}N_U} \right)^{1/d} e^{gt}f - w_U = 0 \\
& \frac{w_U}{e^{gt}} = f(1-a) \left(\frac{Y}{W} \right)^{1/b} (1-c) \left(\frac{W}{e^{gt}N_U} \right)^{1/d} = 0 \\
& \frac{w_U}{e^{gt}} = f(1-a) \left(\frac{Ye^{gt}}{We^{gt}} \right)^{1/b} (1-c) \left(\frac{W}{e^{gt}N_U} \right)^{1/d} = 0 \\
& \hat{w}_U = f(1-a) \left(\frac{\hat{Y}}{\hat{W}} \right)^{1/b} (1-c) \left(\frac{\hat{W}}{N_U} \right)^{1/d} = 0
\end{aligned} \tag{A.2}$$

Similarly, differentiating Π with respect to N_S and setting this to zero yields:

$$\begin{aligned}
& \frac{w_S}{e^{gt}} = (1-a) \left(\frac{Y}{W} \right)^{1/b} c \left(\frac{W}{e^{gt}L_S} \right)^{1/d} = 0 \\
& \hat{w}_S = (1-a) \left(\frac{\hat{Y}}{\hat{W}} \right)^{1/b} c \left(\frac{\hat{W}}{L_S} \right)^{1/d} = 0
\end{aligned} \tag{A.3}$$

A2 The Stationary Model

$$\hat{K}_{SD} + \hat{K}_{UD} = \hat{K} \quad (\text{A.1})$$

$$\hat{B}_{SD} + \hat{B}_{UD} = \hat{B} \quad (\text{A.2})$$

$$N_S = L_{SD} + L_{SI} \quad (\text{A.3})$$

$$N_U = L_{UD} + L_{UI} \quad (\text{A.4})$$

$$\hat{W} = e^z \left(c(fN_S)^{\frac{d-1}{d}} + (1-c)N_U^{\frac{d-1}{d}} \right)^{\frac{d}{d-1}} \quad (\text{A.5})$$

$$\hat{Y} = \left(a\hat{K}^{\frac{b-1}{b}} + (1-a)\hat{W}^{\frac{b-1}{b}} \right)^{\frac{b}{b-1}} \quad (\text{A.6})$$

$$\hat{X} = \hat{B}'(1+g) - (1+s)\hat{B} \quad (\text{A.7})$$

$$r = a\left(\frac{\hat{Y}}{\hat{K}}\right)^{1/b} \quad (\text{A.8})$$

$$\hat{w}_S = f(1-a)\left(\frac{\hat{Y}}{\hat{W}}\right)^{1/b} c\left(\frac{\hat{W}}{N_S}\right)^{1/d} \quad (\text{A.9})$$

$$\hat{w}_U = (1-a)\left(\frac{\hat{Y}}{\hat{W}}\right)^{1/b} (1-c)\left(\frac{\hat{W}}{N_U}\right)^{1/d} \quad (\text{A.10})$$

$$\phi_S = \frac{\hat{w}_S}{\frac{1}{H}(\hat{w}_S L_{SD} + \hat{w}_U L_{UD})} \quad (\text{A.11})$$

$$\phi_U = \frac{\hat{w}_U}{\frac{1}{H}(\hat{w}_S L_{SD} + \hat{w}_U L_{UD})} \quad (\text{A.12})$$

$$s_S = s - \nu\phi_S(\hat{B}_{SD} + \hat{B}_{UD}) \quad (\text{A.13})$$

$$s_U = s - \nu\phi_U(\hat{B}_{SD} + \hat{B}_{UD}) \quad (\text{A.14})$$

$$\hat{C}_{SD} = \hat{w}_S + (1+r-\delta)\frac{\hat{K}_{SD}}{L_{SD}} - \frac{\hat{K}'_{SD}}{L_{UD}}(1+g) \quad (\text{A.15})$$

$$\hat{C}_{UD} = \hat{w}_U + (1 + r - \delta) \frac{\hat{K}_{UD}}{L_{UD}} - \frac{\hat{K}'_{UD}}{L_{UD}}(1 + g) \quad (\text{A.16})$$

$$\hat{C}_{SI} = \hat{w}_S \quad (\text{A.17})$$

$$\hat{C}_{UID} = \hat{w}_U \quad (\text{A.18})$$

$$\hat{C}_{SD}^{-\gamma} = \beta E\{[\hat{C}'_{SD}(1 + g)]^{-\gamma}(1 + r' - \delta)\} \quad (\text{A.19})$$

$$\hat{C}_{UD}^{-\gamma} = \beta E\{[\hat{C}'_{UD}(1 + g)]^{-\gamma}(1 + r' - \delta)\} \quad (\text{A.20})$$

$$\hat{C}_{SD}^{-\gamma} = \beta E\{[\hat{C}'_{SD}(1 + g)]^{-\gamma}(1 + s'_S)\} \quad (\text{A.21})$$

$$\hat{C}_{UD}^{-\gamma} = \beta E\{[\hat{C}'_{UD}(1 + g)]^{-\gamma}(1 + s'_S)\} \quad (\text{A.22})$$

A3 Confidence Bands for Remaining Policies

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Figure 1: Transition Paths for Aggregate Variables
with 90% Confidence Bands
5% Unskilled Immigration

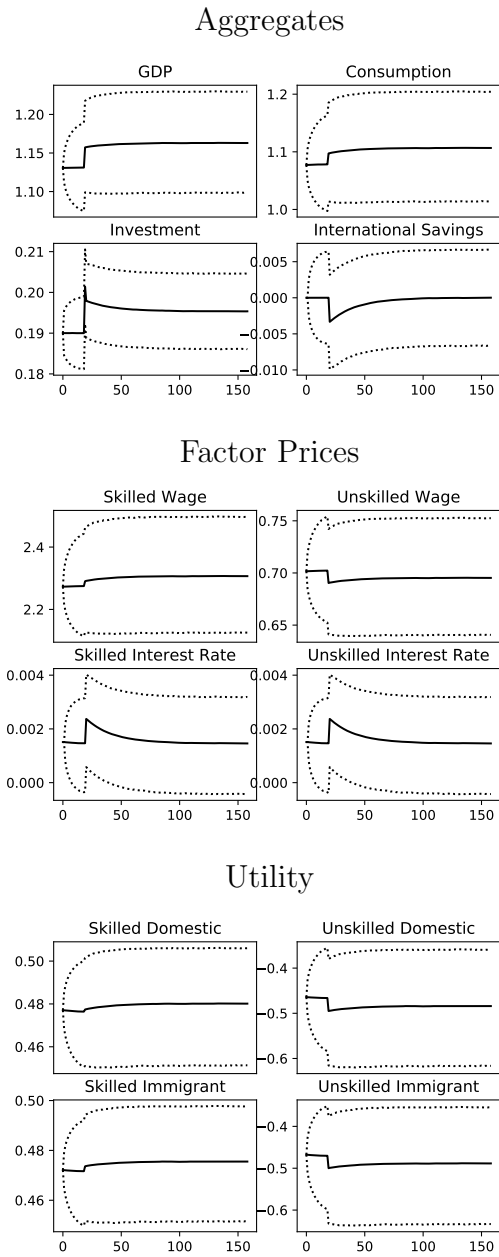


Figure 2: Transition Paths for Aggregate Variables
with 90% Confidence Bands
5% Equal Immigration



Figure 3: Transition Paths for Aggregate Variables
with 90% Confidence Bands
5% Skilled Immigration

