A Model of Sleep, Leisure and Work over the Business Cycle *

James H. Cardon † Eric R. Eide ‡ Kerk L. Phillips §
Mark H. Showalter ¶

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Abstract

This paper uses a single-sector dynamic stochastic general equilibrium (DSGE) model to analyze time use patterns over the business cycle. We solve and simulate a model of a utility maximizing consumer subject to a penalty function based on the biological model for sleep. We calibrate the model with data from the American Time Use Survey. We find that sleep is countercyclical with the business cycle: sleep increases as economic activity declines. We also show that our model provides a reasonable estimate of observed sleep behavior over the period from 2003 through 2016. We find that the opportunity cost of sleep over this period is around $9 per hour.

keywords: sleep, opportunity cost, business cycle, dynamic stochastic general equilibrium

JEL classification: E37, E39, I19, J22

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†Brigham Young University, Department of Economics, 172 FOB, Provo, Utah 84602, (801) 422-5735, james_cardon@byu.edu.

‡Brigham Young University, Department of Economics, 183 FOB, Provo, Utah 84602, (801) 422-4883, eide@byu.edu.

§Brigham Young University, Department of Economics, 166 FOB, Provo, Utah 84602, (801) 422-5928, kerkphil@gmail.com.

¶Brigham Young University, Department of Economics, 179 FOB, Provo, Utah 84602, (801) 422-4645 showalter@byu.edu.

1 Introduction

Sleep is a critical biological function for all known life forms (Walker, 2017). Similar to other essential biological functions like eating and breathing, insufficient sleep can have serious biological consequences, including death (Carskadon and Dement, 2011). Despite its biological necessity, sleep does involve tradeoffs: a sleeping animal is at risk of predators and is not finding food for survival. Likewise, a sleeping human generates neither economic nor social activity. Given that there exists a substantial element of choice on timing and quantity of sleep, putting sleep into an economic choice framework is a natural exercise and can illuminate interesting and important interactions between the biological aspects of sleep and the broad set of other activities of individuals and society.

The first systematic evaluation of sleep through an economic lens was Biddle and Hamermesh (1990). They model sleep as an input to utility and as a factor in labor force productivity. However, their framework ignores the biological underpinnings of sleep–the necessity of sleep acts as a constraint, not just an input. More recent work includes evaluation of sleep and household production (Asgeirsdottir and Zoega, 2011), sleep and wages (Gibson and Shrader, 2018), and sleep and educational performance (Hafner et al., 2017; Eide and Showalter, 2012).

Our paper adds to the literature on the economics of sleep using a new model that blends insights from the large scientific literature on the biology of sleep with the classic insights from economics on optimization and choice behavior. We apply the model to an evaluation of the effects of business cycle on observed sleep behavior. Our framework is a single-sector dynamic stochastic general equilibrium (DSGE) model where we solve and simulate a model of a utility-maximizing consumer subject to a biologically-based penalty function.

An evaluation of sleep and the business cycle is related to the work by Ruhm (2000), who uses unemployment and mortality data to show that health improves when the economy declines. Sleep is a potential contributor of this observed link between health and the business cycle. There is some empirical work evaluating the relationship between sleep and the business cycle, with most finding that sleep is countercyclical (Barnes et al. 2016, Antillon et al. 2014, and Maclean and Hill 2017); but Asgeirsdottir and Olafsson (2015) finds no effect.

One difficulty with the work to date on sleep and the business cycle is that the research offers no way to evaluate the economic value of the sleep gain/loss due to business cycle variations. Suppose we accept that sleep tends to increase during economic downturns. Is this increase in sleep sufficient to fully compensate for the lost economic output from the downturn? Without an economic framework for evaluating the tradeoffs, it is impossible to say.

The DSGE model used in this paper explicitly accounts for the biological constraints associated with the need to sleep, but which also accounts for individual choice in the timing and amount of sleep and relates that choice to the business cycle. We solve and simulate a model of a utility maximizing consumer subject to a penalty function based on the biological model for sleep. The model is calibrated with time-diary data from the American Time Use Survey. We analyze time use patterns over the business cycle and find that sleep is countercyclical with the business cycle: sleep increases as economic activity declines. But the value of the additional sleep does not fully offset the loss in utility from the decline in economic activity. We also show that our model provides a reasonable estimate of observed sleep behavior over the period from 2003 to 2016.

The outline of the paper is as follows: Section 2 shows how we derived our sleep utility function from optimal choice of sleep over a circadian cycle. Section 3 presents the utility maximization and market equilibrium model. Section 4 describes the calibration exercise and results. Section 5 gives a final discussion of the results.

2 Modeling the Utility of Sleep

In this section we introduce our fundamental model of sleep choice. The primary mathematical model of sleep combines two biological processes, the circadian cycle and 'sleep pressure' (also referred to as homeostasis [1]). The circadian cycle is a roughly 24-hour pattern of alertness and fatigue that also leads to daily cyclical patterns of body temperature, hunger, metabolic rate, etc. The circadian cycle interacts with sleep pressure, where sleep pressure is determined by the accumulation in the brain of the chemical adenosine. Adenosine concentration in the brain increases the longer a person is awake, with a corresponding increase in the physical desire to sleep. This combination of the circadian cycle and sleep pressure, typically referred to as the two-process model, creates a deterministic prediction for sleep patterns; asleep when the circadian cycle indicates it is time to do so and awake simi-

[1] see, Skeldon et al. (2014).

larly.

Since this model is purely mechanical and does not allow an agent conscious choice on the timing of sleep, it is not amendable for incorporation into our model where we explicitly consider opportunity costs of sleep.

2.1 Two-Process Model

We take our discussion and notation of this model from Skeldon et al. (2014).

There is a baseline circadian cycle, which we will denote y_t. This is a weighted sum of sine waves of various frequencies and will fluctuate between -1 and +1. There are upper and lower bounds related to this cycle. We denote these H_t^u and H_t^l.

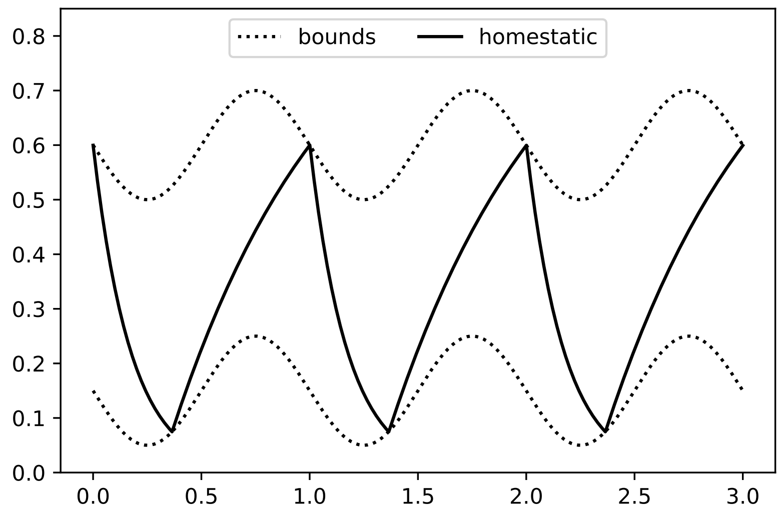
$$\begin{aligned} H_{t+1}^u &= \bar{H}_t^u + a y_t \\ H_{t+1}^l &= \bar{H}_t^l + a y_t \\ \bar{H}^u &> \bar{H}^l \end{aligned} \tag{2.1}$$

The homeostatic process (i.e. sleep pressure) which determines sleep versus waking is denoted H_t. If awake, it rises toward an upper asymptote of μ and if sleeping it falls toward a lower asymptote of zero.

$$H_{t+1} = \begin{cases} H_t e^{-1/\chi_S} & \text{if sleeping} \\ \mu + (H_t - \mu) e^{-1/\chi_W} & \text{if waking} \end{cases} \tag{2.2}$$

If the individual is awake and H_t is rising over time it will eventually reach the upper bound of H_t^u. At this point the individual begins sleep and H_t falls until it eventually hits the lower bound, and the individual wakes up. This is illustrated in Figure 1.

Figure 1: Two-Process Model



2.2 Our Sleep Model

In our model we use the same circadian cycle, y_t. Sleep utility is defined as the difference between this cycle and and index of sleep stock we denote d_t. This corresponds roughly to the negative of the homeostatic process in the previous model. We model d_t as follows.

$$Z_{t+1} = \begin{cases} Z_t(1-\delta)+1 & \text{if sleeping} \\ Z_t(1-\delta) & \text{if waking} \end{cases} \tag{2.3}$$

$$d_t = \phi Z_t + \xi$$

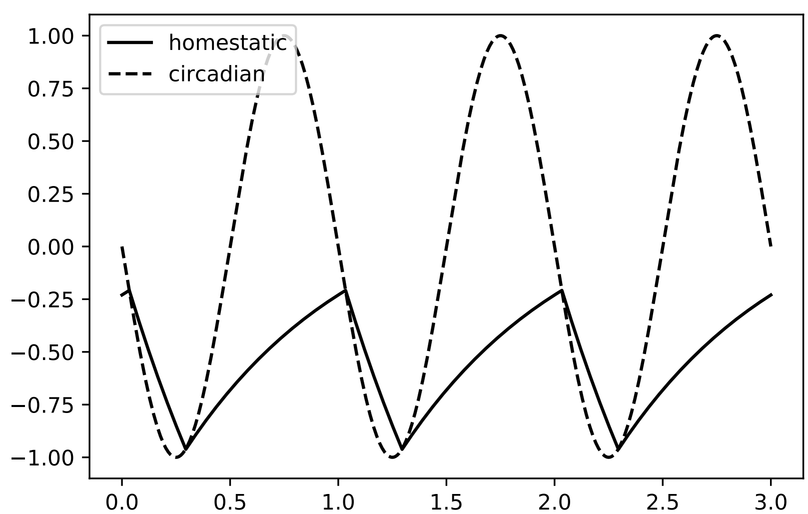
where Z_t is the stock of sleep

Utility is given by:

$$v_t = \begin{cases} -(y_t + d_t) & \text{if sleeping} \\ y_t + d_t & \text{if waking} \end{cases} \tag{2.4}$$

If $y_t = -d_t$ the individual is indifferent between sleeping and waking. If $-d_t < y_t$ the individual will be awake and d_t will be falling (or $-d_t$ will be rising). Eventually $-d_t$ crosses the circadian cycle and the individual chooses to sleep. From here on $-d_t$ falls until it crosses the cycle again and the individual wakes up. This is illustrated in Figure 2.

Figure 2: Our Utility Model



2.3 Choosing Sleep on a Daily Basis

We model the sleep choice as occurring once a day. Agents observe their stock of sleep at the beginning of the period along with other variables that describe their economic state. They then choose how long to sleep during the coming day and the optimal sleeping and waking times. They do this will full knowledge of how their sleep stock and economic conditions will be affected the next day by these choices.

If d_t were constant, unconstrained waking and sleeping times would be unaffected by the stock of sleep. This is unlikely to be the case in reality. Raising d_t results in less sleep and lowering it results in more sleep. Also, d_t must lie between -1 and 1 in order for the individual to avoid sleeping or waking constantly. We make d_t a function of Z_t, subject to limiting constraints.

$$\begin{aligned} d_t &= d(Z_t) \\ d(0) &= \underline{d} < 0 \\ \lim_{Z \to \infty} d(Z) &= \bar{d} > 0 \end{aligned} \tag{2.5}$$

We assume that the stock of sleep is updated at the end of each period to reflect depreciation and addition of new sleep time. This means we modify equation (2.3) to that shown below.

$$Z_{t+1} = (1 - \delta)Z_t + S_t \tag{2.6}$$

where S_t denotes time spent sleeping during period t.

Utility in our model comes from two sources. First there is sleep utility, which comes from conforming to the circadian cycle. We denote this as v. The second source is utility from economic conditions.

We assume the circadian cycle, y_t, follows a simple sine wave and can be written as follows.

$$y(t) = -\cos(t\tfrac{\pi}{12}) \tag{2.7}$$

for $0 \leq t \leq 24$.

As above, we interpret utility as the difference between this cycle and the current level of sleep pressure, $-d_t$, but hold d_t constant over the day. This value will rise or fall tomorrow based on today's total sleep. If waking at time t the utility generated is $y_t - (-d_t)$. If sleeping utility is $-d_t - y_t$.

In the absence of any constraint the household would choose to be awake when the cycle is positive and asleep when it is negative. The optimal choices for a_t and s_t are the roots of this cycle and are given by:

$$a^* = 12 - \frac{12}{\pi}\cos^{-1}(d_t) \tag{2.8}$$

$$s^* = 24 + \frac{12}{\pi}\cos^{-1}(d_t) \tag{2.9}$$

The utility of being awake between times a_t and s_t is given by:

$$\begin{aligned} v_w &= \int_{a_t}^{s_t} \left(-\cos(t\tfrac{\pi}{12}) + d_t\right) dt \\ &= -\tfrac{12}{\pi}\sin(s_t\tfrac{\pi}{12}) + \tfrac{12}{\pi}\sin(a_t\tfrac{\pi}{12}) + d_t(s_t - a_t) \end{aligned} \tag{2.10}$$

The utility of being asleep for a day will include that from time 0 to a_t

and that from s_t to 24.

$$\begin{aligned} v_s =& \int_0^{a_t} \left(\cos(t\tfrac{\pi}{12}) - d_t\right) dt + \int_{s_t}^{24} \left(\cos(t\tfrac{\pi}{12}) - d_t\right) dt \\ =& \int_0^{a_t} \cos(t\tfrac{\pi}{12}) dt + \int_{s_t}^{24} \cos(t\tfrac{\pi}{12}) dt \\ & - d_t \left(\int_0^{a_t} 1 \; dt + \int_{s_t}^{24} 1 \; dt \right) \\ =& \tfrac{12}{\pi} \sin(a_t \tfrac{\pi}{12}) - \tfrac{12}{\pi} \sin(0 \tfrac{\pi}{12}) \\ & + \tfrac{12}{\pi} \sin(24 \tfrac{\pi}{12}) - \tfrac{12}{\pi} \sin(s_t \tfrac{\pi}{12}) \\ & - d_t(24 - s_t + a_t - 0) \\ =& \tfrac{12}{\pi} \sin(a_t \tfrac{\pi}{12}) - \tfrac{12}{\pi} \sin(s_t \tfrac{\pi}{12}) - d_t(24 - s_t + a_t) \end{aligned} \tag{2.11}$$

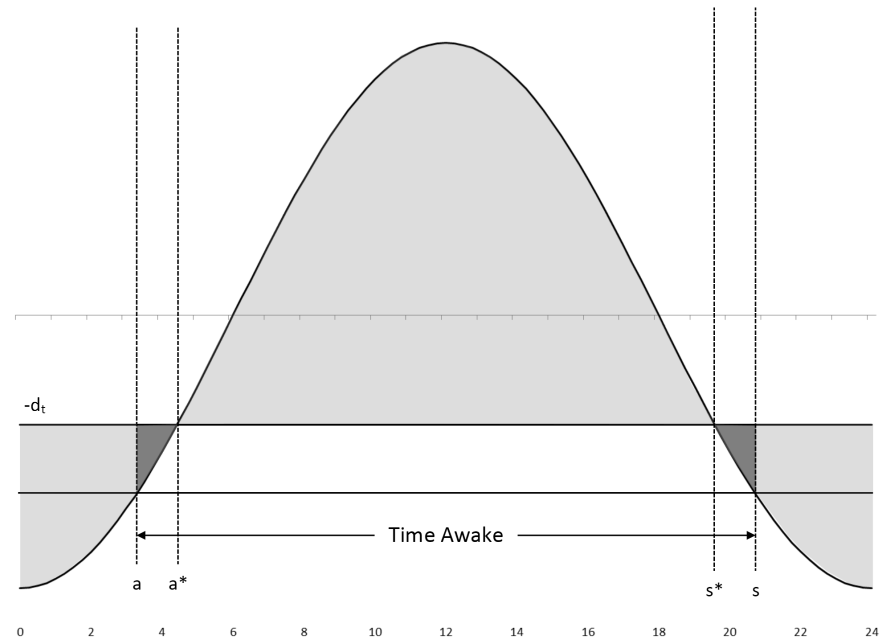
Hence, the function for sleep utility, $v(a_t, s_t, d_t)$, can be written as a sum of the above terms.

$$\begin{aligned} v(a_t, s_t, d_t) =& \tfrac{12}{\pi} \sin(a_t \tfrac{\pi}{12}) - \tfrac{12}{\pi} \sin(s_t \tfrac{\pi}{12}) + d_t(s_t - a_t) \\ & \tfrac{12}{\pi} \sin(a_t \tfrac{\pi}{12}) - \tfrac{12}{\pi} \sin(s_t \tfrac{\pi}{12}) - d_t(24 - s_t + a_t) \\ =& \tfrac{24}{\pi} \left[\sin(a_t \tfrac{\pi}{12}) - \sin(s_t \tfrac{\pi}{12}) \right] + d_t(2s_t - 2a_t - 24) \end{aligned} \tag{2.12}$$

The choice of S_t necessitates optimal choices of a waking time, a_t, and a sleeping time, s_t. Given our circadian cycle with a symmetric sine wave that has a minimum at time $t = 0$, it will always be optimal to choose a waking time of $a_t = \frac{1}{2} S_t$ and a sleeping time of $s_t = 24 - \frac{1}{2} S_t$.

This is illustrated in Figure 3, where positive utility is shown as a light shade, and negative utility is shown as a darker one. The sleep and wake times chosen in the presence of opportunity costs are a and b, while the times that would be chosen if there were no opportunity cost are a^* and b^*.

Figure 3: Optimal Timing of Sleep over the Circadian Cycle



Substituting these optimal sleep times into equation (2.12) [2] gives us the sleep utility function we will use in the next section.

$$\begin{aligned} v(S_t, d_t) =& \frac{24}{\pi}\left[\sin(\tfrac{1}{2}S_t\tfrac{\pi}{12}) - \sin([24 - \tfrac{1}{2}S_t]\tfrac{\pi}{12})\right] + d_t(24 - 2S_t) \\ =& \frac{48}{\pi}\sin(S_t\tfrac{\pi}{24}) + d_t(24 - 2S_t) \end{aligned} \tag{2.13}$$

3 A Consumption Utility Model

3.1 The Household's Problem

We want to create a model that incorporates the biological constraint represented by the two-process model within the optimization framework of a

[2] Recall that $\sin(2\pi - x) = -\sin(x)$

standard economic model. Consider the following dynamic program for a household:

$$
\begin{aligned}
V(K_t;\Theta_t)& \\
&= \max_{K_{t+1},N_t,S_t} \frac{C_t^{1-\gamma}-1}{1-\gamma} + \chi_L \frac{L_t^{1-\lambda}-1}{1-\lambda} \\
&\quad + \chi_s \left[\tfrac{48}{\pi}\sin(S_t \tfrac{\pi}{24}) + d_t(24-2S_t)\right] \\
&\quad + \beta E\{V(K_{t+1};\Theta_{t+1})\}
\end{aligned}
$$

$$C_t = w_t b(S_t) N_t + (1+r_t-\delta)K_t - K_{t+1} \tag{3.1}$$

$$L_t = 24 - N_t - S_t \tag{3.2}$$

where K_t is the household's stock of capital, Θ_t is an information set which includes factor prices (w_t, r_t), N_t is the time spent working, S_t is the time spent sleeping, L_t is the time spent in leisure, C_t is its consumption of goods and services, γ is a constant relative risk aversion (CRRA) parameter for consumption utility, λ is the CRRA parameter for leisure utility, χ_L is the utility weight on leisure, χ_S is the utility weight on sleep, $d_t = d(S_t)$ is a measure of sleep pressure, $b_t = b(S_t)$ is the effectiveness of the worker, and δ is the depreciation rate for physical capital.

We choose the following functional forms for $d(S_t)$ and $b(S_t)$:

$$d_t = d(S_t) = \frac{2}{\pi}\tan^{-1}(\phi S_t + \xi) \tag{3.3}$$

$$b_t = b(S_t) = \kappa S_t^{\eta} \tag{3.4}$$

These are chosen so that sleep pressure is bounded both above and below and lies in the same range as the circadian process. And so that sleep exhibits diminishing marginal product for labor effectiveness.

The first-order conditions from this problem are[3]:

$$
\begin{aligned}
&C_t^{-\gamma} + \beta E\{V_K(t+1)\} = 0 \\
&C_t^{-\gamma} w_t b_t + \chi_L L_t^{-\lambda}(-1) = 0 \\
&C_t^{-\gamma} w_t N_t \eta \kappa S_t^{\eta-1} + \chi_L L_t^{-\lambda}(-1) \\
&+ \chi_s \left[\frac{2\phi}{\pi}\frac{1}{1+S_t^2}(24-2S_t) + 2\cos(S_t\tfrac{\pi}{24}) - 2d(S_t)\right] = 0
\end{aligned}
$$

The envelope conditions are:

$$V_K(t) = C_t^{-\gamma}(1 + r_t - \delta_K)$$

These can be combined to give the following Euler equations:

$$C_t^{-\gamma} = \beta E\{C_{t+1}^{-\gamma})(1 + r_{t+1} - \delta_K)\} \tag{3.5}$$

$$C_t^{-\gamma} w_t b_t = \chi_L L_t^{-\lambda} \tag{3.6}$$

$$\chi_L L_t^{-\lambda} = C_t^{-\gamma} w_t N_t \eta \kappa S_t^{\eta-1} + \chi_s \left[2\cos(S_t\tfrac{\pi}{24}) - 2d(Z_t) + \frac{2\phi}{\pi}\frac{24-2S_t}{1+S_t^2}\right] \tag{3.7}$$

3.2 Firm's Problem and Law of Motion

Each period a large number of firms spring into existence. They hire labor and capital and produce final goods. The production function and profit

[3] Recall that $\frac{d\tan^{-1}(x)}{dx} = \frac{1}{1+x^2}$

maximizing condition are given below.

$$Y_t = K_t^{\alpha}(e^{z_t} b_t N_t)^{1-\alpha} \tag{3.8}$$

$$w_t = (1-\alpha)\frac{Y_t}{b_t N_t} \tag{3.9}$$

$$r_t = \alpha \frac{Y_t}{K_t} \tag{3.10}$$

where α is the Cobb-Douglas share of capital in total income and z_t is an exogenous stochastic process for labor productivity with the following law of motion.

$$z_t = \rho z_{t-1} + \varepsilon_t;\ \varepsilon_t \sim iid(0, \sigma^2) \tag{3.11}$$

3.3 Summary of the Model

There are three markets in the model, a goods market, a capital market, and a labor market. By Walras' Law we need consider only two of these and we choose the capital and labor markets. These markets clear by our choice of notation where K_t stands for both the supply and demand for capital, and N_t stands for both the supply and demand for labor.

Hence, our model is described by equations (3.1) - (3.11). This is eleven equations which determine the behavior of the following eleven variables over time: $\{C_t, L_t, d_t, b_t, K_t, N_t, S_t, Y_t, w_t, r_t, z_t\}$

K_t is our sole endogenous state variable. N_t and S_t are implicitly defined co-state variables. z_t is the only exogenous state variable. The remainder of the variables can be solved for as explicit function of these four.

3.4 Finding the Dynamic Solution

The law of motion is:

$$z_t = \rho z_{t-1} + \varepsilon_t;\ \varepsilon_t \sim iid(0, \sigma^2) \tag{3.12}$$

Rewriting the definitions we get the following as functions of K_t, N_t and S_t in various periods.

$$L_t = 24 - N_t - S_t \tag{3.13}$$

$$d_t = \frac{1}{\pi}\tan^{-1}(\phi S_t + \xi) \tag{3.14}$$

$$b_t = \kappa S_t^{\eta} \tag{3.15}$$

$$Y_t = K_t^{\alpha}(e^{z_t} b_t N_t)^{1-\alpha} \tag{3.16}$$

$$w_t = (1-\alpha)\frac{Y_t}{b_t N_t} \tag{3.17}$$

$$r_t = \alpha\frac{Y_t}{K_t} \tag{3.18}$$

$$C_t = w_t b(S_t) N_t + (1 + r_t - \delta)K_t - K_{t+1} \tag{3.19}$$

The dynamic equations implicitly defining N_t and S_t are:

$$C_t^{-\gamma} w_t b_t = \chi_L L_t^{-\lambda} \tag{3.20}$$

$$\chi_L L_t^{-\lambda} = C_t^{-\gamma} w_t N_t \eta\kappa S_t^{\eta-1} + \chi_s\left[2\cos(S_t\tfrac{\pi}{24}) - 2d_t + \frac{2\phi}{\pi}\frac{24 - 2S_t}{1 + S_t^2}\right] \tag{3.21}$$

While the dynamic equation defining K_t over time is:

$$C_t^{-\gamma} = \beta E\{C_{t+1}^{-\gamma})(1 + r_{t+1} - \delta_K)\} \tag{3.22}$$

We need to find a policy function and jump function of the forms shown below.

$$\mathbf{X}_t = \Phi\left(\mathbf{X}_{t-1}, \mathbf{Z}_t\right) \tag{3.23}$$

$$\mathbf{Y}_t = \Lambda\left(\mathbf{X}_{t-1}, \mathbf{Z}_t\right) \tag{3.24}$$

$$\mathbf{X}_t \equiv \begin{bmatrix} K_{t+1} \end{bmatrix} \tag{3.25}$$

$$\mathbf{Y}_t \equiv \begin{bmatrix} N_t \\ S_t \end{bmatrix} \tag{3.26}$$

$$\mathbf{Z}_t \equiv \begin{bmatrix} z_t \end{bmatrix} \tag{3.27}$$

3.5 Numerical Solution

We linearize the policy and jump functions using the method laid out in Uhlig (1999). This gives us policy and jump functions of the following form.

$$K_{t+1} = P(K_t - \bar{K}) + Qz_t + \bar{K} \tag{3.28}$$

$$\begin{bmatrix} N_t \\ S_t \end{bmatrix} = \begin{bmatrix} R_N \\ R_S \end{bmatrix} (K_t - \bar{K}) + \begin{bmatrix} S_N \\ S_S \end{bmatrix} z_t + \begin{bmatrix} \bar{N} \\ \bar{S} \end{bmatrix} \tag{3.29}$$

where P, Q, R_N, R_S, S_N and S_S are scalar values that are derived from the derivatives of equations (3.20) - (3.22) with respect to values of N, S, K ans z in various periods and evaluated at the steady state, and $\bar{K}$ is the steady state value of K.

We calibrate the model as follows.

First, α is the share of capital in GDP and is set to 0.33.

The quarterly subjective discount factor is β and is set to 0.99.

We set coefficient of relative risk aversion (γ) to 1.5 and the Frisch elasticity for labor supply ($\frac{1}{\theta}$) to one.

The quarterly rate of depreciation, δ is set to 0.01, which is consistent with the implied depreciation rates that come from comparing the annual U.S. capital stock numbers with the NIPA investment numbers for the period 2003 - 2014.

We choose χ_L and χ_S to set the values of $\bar{N}$ and $\bar{S}$ to 4.81 hours and 8.97 hours. These values are 0.1777 and -0.0103, respectively.

We set κ in the $b(S_t)$ function to 1. We choose η in the function to hit the highest level of sleep during the last recession, as explained below and illustrated in Figure 7. That value is $\eta = 0.73$.

Finally, we set ρ to 0.87, to generate an autocorrelation close to 0.8663 for Y, and we set σ to 0.0044 to match the volatility of quarterly HP filtered GDP data over the same sample.

We simulate our model over a sample of 1,000,000 observations and calculate key statistical moments which are reported in Table 2. The table shows that our model does quite well in matching volatility. The model cyclicalities of consumption and investment are a bit too low, while that for wages is much too high. Similarly the persistence of consumption is much too low and the persistence of wages is a bit too high. Overall, in the context of such a simple model with only productivity shocks as a source of fluctuations, the model does very well.

They show that a positive productivity shock leads to an increase in work, with a corresponding decrease in sleep and leisure. The effect is immediate,

Table 1: Parameter Values

symbol	concept	value
α	capital share in national income	0.33
β	subjective discount factor	0.99
γ	curvature of consumption utility	1.5
λ	curvature of leisure utility	1
χ_L	utility weight on leisure	0.1777
χ_S	utility weight on sleep	-0.0103
δ	depreciation rate	0.01
ϕ	response of sleep pressure to stock of sleep	1
ξ	sleep pressure shifter	0
κ	response of effectiveness to stock of sleep	1
η	curvature of effectiveness function	0.73
ρ	autocorrelation of productivity shocks	0.87
σ	standard deviation of productivity shocks	0.0044

but relatively small, with a half-minute per day decrease in sleep which would imply about a half-hour of sleep per quarter. Due to the persistence in the technology shock, the effect for hours–sleep, work, and leisure–is also relatively persistent.

Impulse response functions for our three time-use categories to a productivity shock are shown in Figure 4

Table 2: Key Business Cycle Moments
Bold values are targeted moments

Data
HP Filtered Data 1960:1 - 2016:4

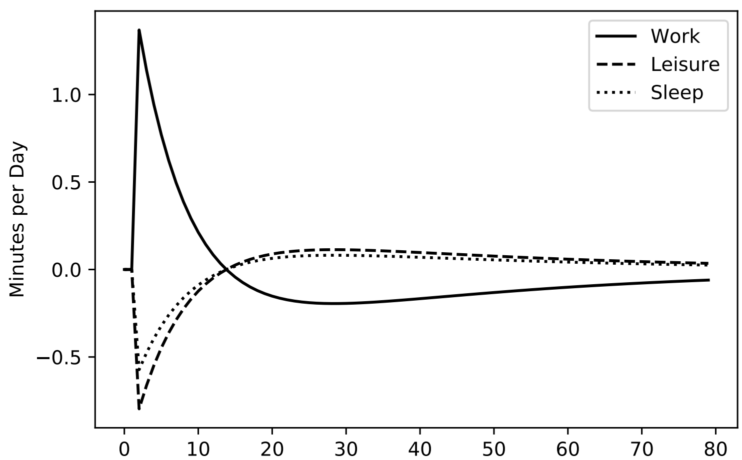
	standard deviations	standard deviations relative to GDP	correlations with GDP	autocorrelations
$\ln Y$	**0.0147**	1.0000	1.0000	**0.8663**
$\ln C$	0.0120	0.8184	0.8738	0.8779
$\ln I$	0.0662	4.5090	0.9042	0.8307
$\ln N$	0.0091	0.6200	0.7650	0.7743
$\ln w$	0.0104	0.7087	0.3848	0.7288

We use average weekly hours in manufacturing as our N series

Model
1,000,000 observations

	standard deviations	standard deviations relative to GDP	correlations with GDP	autocorrelations
$\ln Y$	**0.0147**	1.0000	1.0000	**0.8780**
$\ln C$	0.0106	0.7221	0.5793	0.3442
$\ln I$	0.0733	5.0003	0.7965	0.8492
$\ln N$	0.0087	0.5914	0.7968	0.8498
$\ln w$	0.0099	0.6786	0.9007	0.9435

Figure 4: Impulse Responses for N, L and S to a $+\sigma$ shock to z (measured in minutes relative to the steady state)



4 Taking the Model to the Data

4.1 Stylized Facts about Sleep over the Business Cycle

We use data from the American Time Use Survey (ATUS) to get quarterly observations of hours spent in four categories: sleep, work, leisure and home production. We average the data each quarter over three classes of individuals: full-time workers, part-time workers, and non-workers (unemployed or outside the labor force).

The ATUS is sponsored by the Bureau of Labor Statistics and provides nationally representative estimates of how, where, and with whom Americans spend their time. It is a stratified random sample that covers all civilian, non-institutionalized residents living in the U.S. who are at least 15 years old. The ATUS data files include information collected from over 180,000 interviews conducted from 2003 to 2016.

The ATUS respondents are sampled from the group of households in the outgoing rotation of the Current Population Survey. The ATUS is conducted by phone interview with a single person randomly selected from each household. The respondent is asked to account for their activities for the 24 hour period that starts at 4 a.m. the day before the interview and ends at 4 a.m. on the day of the interview. Start and end times of each activity are included as well as where and with whom the activity occurred.

Our sample includes all respondents age 15 and older, both employed and unemployed, over both weekends and weekdays. Note that the ATUS oversamples for weekends - about half of the data comes from people surveyed about weekend time use. Our data spans the period from the 1st quarter of

2003 to final quarter of 2016 with a total of 181,335 observations (average of 3238 per quarter).

The quarterly observations of hours spent in sleep, work, leisure and home production are defined as follows:

Sleep: Time spent sleeping (including naps) and time spent in bed trying to sleep. Average hours of sleep average is 8.81.

Work: Working at main job or part-time job, security procedures relating to work, all other activities done as part of job, job searching, education, including taking class, non-athletic extracurricular activities, homework and research. Average hours of work is 3.16. It is useful to remember that this is a weighted average across full-time, part-time, and unemployed individuals.

Leisure: Grooming and personal care activities, eating, drinking, relaxing, socializing, sports, exercise, recreation, church attendance/religious involvement, volunteer activities, phone calls to family, friends, and neighbors, travel time not relating to sleep, work, or housework. Average hours of leisure is 8.04.

Home production: Taking care of yard, pets or vehicles, cooking meals, caring for household members, shopping, using professional or personal care services, like going to the bank, using government services, such as social services, telephone calls that aren't to family, friends, or neighbors. Average hours of home production is 3.78.

These data show very little cyclical variation, though there is seasonal movement. Borowczyk-Martins and Lale (2016) show that movements in work hours are driven overwhelmingly by movements between full-time and part-time work, rather than by variation in hours worked for these two types of workers. The samples for our three categories of workers in the ATUS show little or no variation in the fractions of individuals within each category over the 2003 - 2017 period. Accordingly, we use Current Population Survey (CPS) data on the numbers of full-time and part-time workers along with unemployed workers and individuals not in the labor force to construct weights for three types of workers: full-time, part-time, and not working. We then take a weighted average of the ATUS hours in sleep, work, leisure and home production using these weights. This gives us an average for the representative agent in our model.

Our model does not include a home production activity. And it is not clear whether these activities should be included in work or leisure. For simplicity, we assign half of the home production time to each of these two categories.

4.2 Predicted Sleep Patterns 2003 - 2016

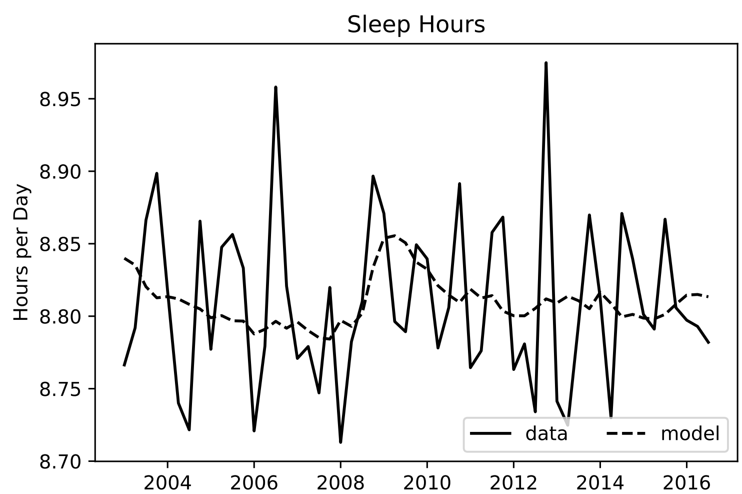
We can use our model to predict the amount of time a typical worker sleeps. We do so by finding the history of technology shocks (z_t) that generates the observed pattern for GDP. Recall that our policy and jump functions for K and N takes the forms shown in (3.29). Similarly, GDP, wages and interest rates are given by (3.8) - (3.10). Finally, b_t is given by (3.15). Combining these equations gives us the following function which implicitly defines z_t as a function of Y_t and K_t.

$$Y_t = K_t^{\alpha} \left(e^{z_t} \kappa \left[\bar{S} + R_S(K_t - \bar{K}) + S_S z_t \right]^{\eta} \left[\bar{N} + R_N(K_t - \bar{K}) + S_N z_t \right] \right)^{1-\alpha} \tag{4.1}$$

We construct empirical Y_t and K_t series by removing a linear trend from the U.S. data for GDP and the real capital stock. Each period we numerically solve (4.1) for the value of z_t given our data on K_t and Y_t. Once we have z_t, we can use equation (3.29) to construct model predictions for N_t and S_t.

Figures 5 and 6 show these predictions where the data and model are filtered with the Hodrick and Prescott (1997) or HP filter:

Figure 5: Predicted and Actual Sleep Hours per Day: 2003 - 2016
HP Filter



As these figures show there is a lot a high frequency movement in the data. Some of this is seasonal variation in sleep, work and leisure. Some of this is undoubtedly due to noise from sample size and nature of the ATUS data collection.

To filter out the high-frequency noise as well as trends in the time-series we replicate these figures using the Christiano and Fitzgerald (2003) or CF filter.

Figure 6: Predicted and Actual Hours per Worker: 2003 - 2016
HP Filter

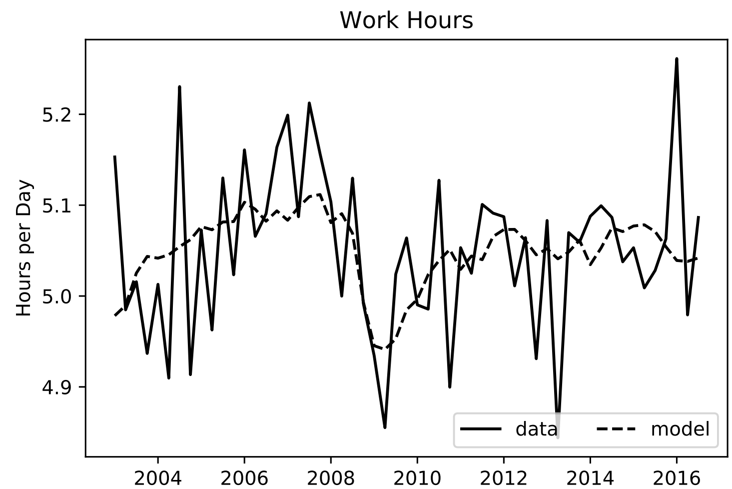


Figure 7: Predicted and Actual Sleep Hours per Day: 2003 - 2016
CF Filter

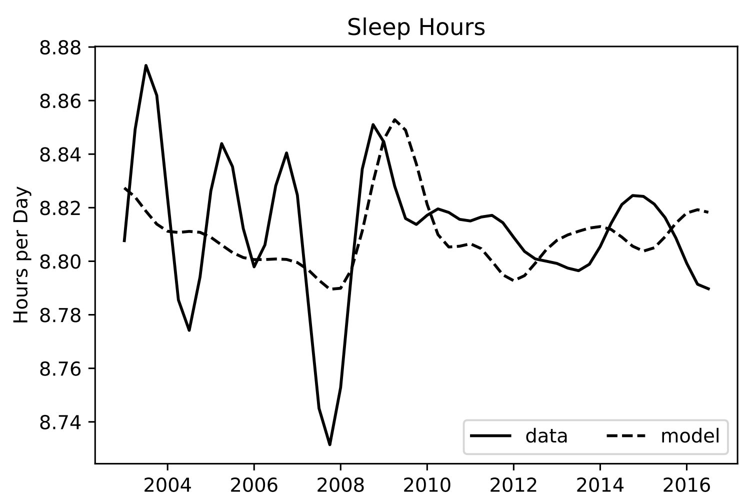
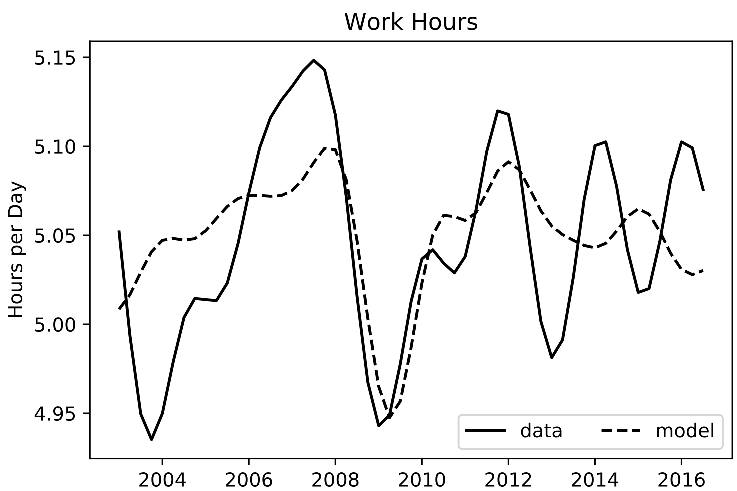


Figure 8: Predicted and Actual Hours per Worker: 2003 - 2016
CF Filter



In summary, after filtering out the high frequency movements in the data, the model does reasonably well predicting movements in work hours. For predicted sleep, the amplitude of the predictions is small relative to the variation found in the data. Relative to GDP and the other national-level data, the time-diary sleep data is much higher variation and is based on self-reports of sleep which is less than ideal and might be responsible for some of the difference in the results. But the broad patterns in sleep (particularly with regard to the 2007-2009 recession) do seem to be captured by the model.

4.3 Compensating Consumption

One useful result of modeling the relationship between sleep and the business cycle in an economic framework is that it allows us to put the value of sleep in terms of an opportunity cost. For example, we can answer the following question: If the representative agent had slept the steady state number of hours rather than actual amount, how much extra consumption would be needed to keep utility unchanged?

To evaluate this question recall that the utility function from Section 3.1 denotes the various sources of utility as follows:

$$u_t^C = \frac{C_t^{1-\gamma} - 1}{1-\gamma} \tag{4.2}$$

$$u_t^L = \chi_L \frac{L_t^{1-\lambda} - 1}{1-\lambda} \tag{4.3}$$

$$u_t^S = \chi_s \left[\tfrac{48}{\pi} \sin(S_t \tfrac{\pi}{24}) + d_t(24 - 2S_t)\right] \tag{4.4}$$

Total utility is the sum of these three values, $u_t = u_t^C + u_t^L + u_t^S$. The difference in sleep utility from the steady state value will be denoted $\Delta u_t^S = u_t^S - \bar{u}^S$. If the agent slept $\bar{S}$, while consumption and leisure remained unchanged utility would be $u_t - \Delta u_t^S$. For utility to remain unchanged, consumption must change. We denote this new level as C_t^* and the corresponding consumption utility as u_t^{C*}. For total utility to remain unchanged we have the following equation.

$$u_t^{C*} - u_t^C + \Delta u_t^S = 0$$

Rewriting this using (4.2) gives us C_t^*.

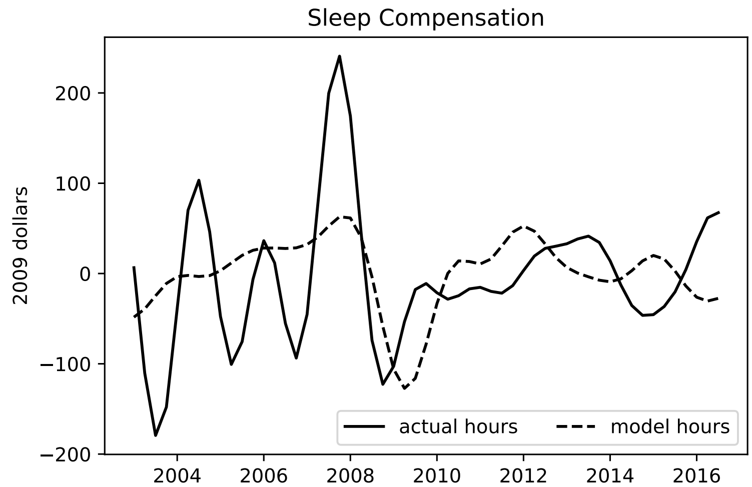
$$C_t^* = \left[(1-\gamma)(u_t^C - \Delta u_t^S)\right]^{\frac{1}{1-\gamma}} \tag{4.5}$$

We have both model and actual data for sleep, so equation (4.4) gives us Δu_t^S. We also have model data for consumption which, using (4.2) and (4.5) gives us C_t^*. We take the percent difference between this value and the C_t generated by the model to get a percent consumption variation. We then multiply this percentage by the actual real per capita personal consumption data as reported by the U.S. Bureau of Economic Analysis for the same sample period. This gives us a real consumption compensation for the change in sleep hours (measured in 2009 dollars). We plot this in Figure 9.

The results from Figure 9 give a peak compensation of about \$100 (per annum). This matches with an effect of about 0.03 hours (2 minutes) per day from Figure 7. This implies an estimated value of sleep of about \$9 per hour [4].

[4] \$100 per year/(0.03 hours per day × 365 days per year)

Figure 9: Compensating Consumption in 2009 Dollars: 2003 - 2016



5 Conclusion

In this paper we add to the literature on the economics of sleep by using a new model that blends insights from the scientific literature on the biology of sleep with insights from economics on optimization and choice behavior. Specifically, we simulate a DSGE model in which consumers balance the biological need to sleep with other choices (consumption, work and leisure) and relate these choices to the business cycle. Similar to Ruhms (2000) finding on health and the business cycle, we find that sleep is countercyclical, that is, sleep increases during economic downturns.

A benefit of using an economic model to understand the relationship between sleep and the business cycle is that it allows us to put a dollar value on sleep. We use our model to answer the question: If the representative agent had slept the steady state number of hours rather than the observed amount, how much extra consumption would be needed to keep utility unchanged? The answer to this question implied by our model is $9 per hour. It is worth highlighting that our estimate of the dollar value of sleep is more complex than just the opportunity cost of foregone wages. It incorporates the full opportunity cost of time, including leisure activities, as well a biological constraint. This would not be possible to compute with a purely empirical approach and requires an economic model.

There are some caveats in this research that are worth noting. One is that our sleep variable is retrospective and self-reported from a point in time based on the ATUS time diary survey. It therefore likely has measurement error and is a noisy measure of steady state sleep. Furthermore, our model was calibrated from the ATUS time diary data, and so any contamination in our variables would have affected the calibration of our economic model. Better and more accurate data would improve the predictive power of the model. Finally, while our model is ideally suited to modeling sleep timing and duration within a single day, in order to model the business cycle with quarterly observations we model consumers as choosing sleep, work and leisure for an entire quarter and use quarterly averages of these variables.

Our model that integrates economic optimization and choice with the biological constraints of sleep opens other fruitful avenues of research. For example, it may inform markets for products that modify sleep through keeping people awake or helping them go to sleep.

The data and programs used in this paper are publicly available at the following repository on Github - https://github.com/kerkphil/SleepOverBC.

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