1 Model

1.1 Households

Bonds exist with maturities from 1 to I periods into the future. The price in period t of a bond delivering 1 unit of real consumption in period $t+i$ is denoted q_{it}. The quantity of bonds held in period t, coming due in period $t+i$ is denoted b_{it}. Bonds are contracts structured so that the bond pays one unit of the final output good when the maturity goes to zero, i.e. $q_{it} = 1$.

Households solve the following dynamic program:

$$
\begin{aligned}
V(k_{t-1}, \{b_{i-1,t-1}\}_{i=1}^{I}; \Theta_t) = \max_{x_t, \{b^{i,t+1}\}_{i=1}^{I}, l_t} & \left[c_t^{\gamma} + \psi(1-l_t)^{\gamma}\right]^{\frac{\phi}{\gamma}} \\
& + \beta E\left\{V(k_t, \{b_{it}\}_{i=1}^{I}; \Theta_{t+1})\right\}
\end{aligned}
$$

$$
\begin{aligned}
c_t = {} & w_t l_t - \tau_t^l(w_t l_t - ded_t) + tr_t + r_t k_{t-1} + cr_t + \sum_{i=1}^{I} q_{it} b_{i-1,t-1} \\
& - k_t - \sum_{i=1}^{I} q_{it} b_{it} - x_t - \tau_t^k \left(r_t k_{t-1} - x_t \theta_t - A_t\right)
\end{aligned} \tag{1.1}
$$

$$
A_t \equiv \sum_{v=0}^{\infty} \delta_g (1-\delta_g)^v (1-\theta_{t-v}) \frac{P_{t-v} x_{t-v}}{P_t} \tag{1.2}
$$

$$
k_t = k_{t-1}(1-\delta) + e^{\omega_t}\left[1 - S\left(\frac{x_t}{x_{t-1}}\right)\right] x_t \tag{1.3}
$$

where Θ_t is a set of variables known to the household.

There is one Euler equation which implicitly defines x_t.

There is another Euler equation which implicitly defines l_t.

There are I Euler equations which implicitly define the $\{b_{it}\}_{i=1}^{I}$.

$$u_{c,t}q_{it} = \beta E\left\{u_{c,t+1}q_{i-1,b+1}\right\} \tag{1.4}$$

$$u_{ct} \equiv \phi[c_t^\gamma + \psi(1-l_t)^\gamma]^{\frac{\phi-\gamma}{\gamma}} c_t^{\gamma-1} \tag{1.5}$$

1.2 Market Clearing

Markets for bonds of all maturities must clear each period. Demand for bonds of maturity i are denoted b_{it} above. Supply of bonds by the government will be denoted B_{it}. Narket clearing gives:

$$b_{it} = B_{it};\ \forall i, t \tag{1.6}$$

The laws of motion for the supplies of bonds are as follows. Each period the government issues a new set of assets of maturities 1 through I, we denote these as B_{it}^N Assets of maturity 0 are retired. This gives:

$$B_{It} = B_{It}^N \tag{1.7}$$

$$B_{it} = B_{i+1,t-1} + B_{it}^N;\ \text{for } 1 \leq i \leq I-1 \tag{1.8}$$

If the longest maturity is 6 periods and if the government only ever issues bonds of maturites 1 and 6, then we have $I = 6$ and $B_{it}^N = 0$ if $1 < i < 6$. The remaining B_{1t}^N and B_{6t}^N will follow some exogenous law of motion that refelcts government policy.

Once we know the exogenous behavior of the supplies from above, the I Euler equations from equation (1.5) now impliicty define the market clearing prices, $\{q_{it}\}_{i=1}^{I}$, as functions of the state variables. Hence, like l_t, bond prices are "jump" variables.

The yield curve can be derived from the bond prices by calculating yield-to-maturity for simple bonds.

$$
\begin{aligned}
q_{it} &= \left(\frac{1}{1+r_{it}}\right)^{i} \\
r_{it} &= q_{it}^{-\frac{1}{i}} - 1
\end{aligned}
\tag{1.9}
$$