Fiscal Multipliers and The Term Structure of Government Debt
PRELIMINARY

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Abstract

Recent empirical and theoretical work in public finance has focused on the size of fiscal multipliers and how they vary under different states of an economy. However, nearly all existing theoretical research relies on a simplifying assumption that interest bearing government debt is a short lived, one period asset. I vary the term structure of government debt in a standard DSGE model and allow the government to purchase a multi-period debt instrument. I observe the effect of different debt term structures on long-run and short-run fiscal multipliers and I also explore the response of private investment and interest rates to fiscal innovations.

Keywords: DSGE Models; Fiscal Policy; Asset Pricing; Dynamic Scoring

JEL Codes: C32; C51; C52

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1 Introduction and Literature Review

In order to capture the effect of government debt on an economy, a structural macroeconomic model should follow a general equilibrium approach. Such a model would determine not only the market clearing price level for goods consumed (publicly and privately) in the economy, but interest rates at which loanable funds markets for private and public debt would clear. Fiscal policy analysis is particularly suited to dynamic stochastic general equilibrium models (DSGE) due to their ability to account for the real effects of future expectations. In this class of stochastic models unanticipated and anticipated fiscal innovations have different real effects on the model economy[1] The ability of agents in these models to correctly form expectations given a known distribution of random shocks to the economy (i.e., rational expectations) can also produce risk premia for long term and short term assets, or in other words, an upward sloping yield curve.[2] I combine and extend existing analyses of fiscal multipliers and interest rate term structure within DSGE models to explore the effect of the term structure of government debt on fiscal multipliers.

Recent work by Greenwood et al. [10] considers the optimal choice of government debt maturity given the trade-offs of rollover risk and the flooding of short term debt markets, or crowding out. Their theoretical framework is a simple three period economy, however, and offers little intuition on the general equilibrium dynamics that a fiscal policy shock might generate. Work by Leeper et al. [12] considers fiscal policy in a model of the US economy with high and low return debt in a fixed government portfolio. The high return debt, however, has the same maturity as the low return debt and the dynamics of interest to my paper (e.g., the easing of the short-term government budget constraint under fiscal expansion due to later repayment of debt) are not captured by this specification. Aside from these papers, there is little consideration of what the term structure of government debt implies for fiscal policy.

My model includes random variables in the processes describing federal taxes, federal government consumption, transfer payments and monetary policy. Decision makers in the model anticipate the values of future federal tax and spending variables based on a stochastic process. That is, individuals and businesses think tax rates may be higher or lower than current tax rates, but they are uncertain which outcome will prevail. The model contains two public debt instruments, a short term asset and a long term asset. In particular, the model allows for the government to issue a specified fraction of new debt on a long term basis. This fraction will be the key parameter to generating each of the term structure experiments in the paper.

2 The Model

My model is a quarterly model of the US economy. The model is based on the neoclassical growth framework[18] and incorporates new Keynesian price frictions and adjustment costs. Households in the model supply labor and capital to firms. Firms produce investment,

[1] See work by Leeper et al. [13].

[2] Work by Van Binsbergen et al. [20] shows that an upward sloping yield curve can be produced within a DSGE model given a high degree of risk aversion.

consumption and housing goods. Because the firms exist in a monopolistically competitive market with sticky prices, the model produces persistent price growth or inflation. This sticky price feature combined with adjustment costs on the investment good comprise a common set of rigidities for a new Keynesian DSGE model[6]. The real economy is closed in the model. Therefore, all goods produced by firms are consumed in the model and all goods consumed or invested are produced by firms in the model. Thus, the model cannot consider international flows of capital goods or services. A central monetary authority sets the nominal interest rate according to a prescribed rule which all other households and firms are aware of. The model also contains a government or fiscal authority that purchases goods and services, makes a transfer payment, and levies taxes. When government expenditures exceed government revenues the government issues long term and short term debt to households. The model is a medium scale new Keynesian model similar to Fernandez-Villaverde and Rubio-Ramirez (2008) [9], Traum and Yang (2011) [19] and Rudd and Edge (2010) [8].

2.1 Households

There exists a continuum of infinitely lived households indexed by $j \in [0, 1]$. Households in the interval $[\mu, 1]$ are able to access capital markets and purchase one period ($b_t^{'}$) or six period ($b_t^{''}$) government bonds.[3] Households in the interval $[0, \mu]$ are not able to access capital markets or hold bonds. All households have constant relative risk aversion preferences over the consumption good, c_t^r, and leisure, $(1 - l_t^r)$, which are additively separable. Their intertemporal preferences are also additively separable and discounted geometrically. Households which are able to access capital and bond markets, or Ricardians, face the following optimization problem:

$$\max_{c_t^r, x_t, l_t^r, b_t, k_t} E_t \sum_{t=0}^{\infty} \beta^t \left[c_t^{\gamma} + \psi(1 - l_t)^{\gamma}\right]^{\frac{\phi}{\gamma}}$$

s.t.

$$c_t^r + x_t + b_t^{r'} + b_t^{r''} = \frac{R_{t-1}^{'}}{\Pi_t} b_{t-1}^{r'} + \frac{R_{t-6}^{''} P_t}{P_{t-6}} b_{t-6}^{r''} + w_t^* l_t^r - \tau_t^{l,r}(w_t^* l_t^r - ded_t^r) + tr_t + r_t^{*k} k_{t-1} + cr_t^r$$
$$-\tau_t^k \left(r_t^{*k} k_{t-1} - x_t \theta_t - \sum_{v=0}^{\infty} \delta_g (1 - \delta_g)^v (1 - \theta_{t-v}) \frac{P_{t-v} x_{t-v}}{P_t} \right) \quad (1)$$

$$k_t = k_{t-1}(1 - \delta) + exp(\omega_t) \left(1 - S\left(\frac{x_t}{x_{t-1}}\right)\right) x_t \quad (2)$$

Investment, x_t, is transformed into capital via an adjustment cost function. The function S is continuous and behaves such that $S(1) = 0$, $S'(1) > 0$ and $S''(1) > 0$. Each period the Ricardian household is able to deduct a fraction, θ_t, of investment from their capital tax liability. The remaining fraction of the nominal amount of investment is deducted via a geometric depreciation schedule in later periods. Temporary changes to the fraction of new

[3] It can be shown, without loss of generality, that n period debt would be specified in a similar manner.

capital which can be deducted from Ricardian households'tax liabilities temporarily alter the cost of capital and lead to short lived investment responses. In the case of a permanent change to this fraction households will change investment instantly and permanently with little change from short run to long run. In the case of a temporary change households are constrained by investment adjustment costs and, depending on the magnitude of the shock, choose not to respond.

The variable ω_t represents investment productivity which follows the process:

$$\omega_t = \rho_\omega \omega_{t-1} + \upsilon_t^\omega$$

Non-Ricardian households optimize the same preference structure. with their choice of consumption and labor subject to the following constraint:

$$c_t^n = l_t^n w_t^* - \tau_t^{l,n}(l_t^n w_t^* - ded_t^n) + tr_t + cr_t^n \tag{3}$$

2.2 Firms

We assume that there exists a continuum of intermediate goods and their respective firms $i \in [0, 1]$. There also exists a zero profit final good packer with a constant elasticity of substitution production function over all intermediate goods. If the packer chooses the lowest cost combination of intermediate goods given prices, $p_t(i)$, to produce an amount of output, y_t^d, then the packer's demand function for intermediate good i must be:

$$y_t(i) = \left(\frac{p_t(i)}{p_t}\right)^{-\epsilon} y_t^d$$

Intermediate goods producers have Cobb-Douglas production functions:

$$y_t(i) = A_t(k_{t-1}(i))^\alpha (l_t^d(i))^{1-\alpha} - \phi$$

Where $k_{t-1}(i)$ is the capital rented by the firm and $l_t^d(i)$ is the amount of labor rented by the firm. And the aggregate productivity shock follows the process:

$$log(A_t) = \rho_A log(A_{t-1}) + \upsilon_t^A$$

The parameter ϕ represents fixed costs and is typically set so that economic profits are zero in steady state. Intermediate good producers face a two stage problem. First, given rental rates and a level of output they must decide the lowest cost combination of capital and labor. Equating marginal product to marginal cost ratios yields the equation:

$$k_{t-1}(i) = \frac{\alpha}{1-\alpha} \frac{w_t^*}{r_t^*} l_t^d(i) \tag{4}$$

Combining this equation with the production function, which has constant returns to scale, one can then derive the marginal cost of output:

$$mc_t(i) = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{1}{\alpha} \frac{(w_t^*)^{1-\alpha}(r_t^*)^\alpha}{A_t} \tag{5}$$

Intermdiate goods firms, like labor unions, are given probability θ_p that they will not be able to set their wages in a given period. These firms will then maximize profits through the choice of price for their goods subject to the demand of the labor packer. Firms will discount future profits using, λ_t^r, from the Ricardian households. Their problem can be represented as:

$$\max_{p_{it}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^\tau \frac{\lambda_{t+\tau}^r}{\lambda_t^r} \prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \left(\frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y_{it+\tau}^d$$

s.t.

$$y_{it+\tau} = \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} \right)^{-\epsilon} y_{t+\tau}^d$$

Since all firms face the same demand from the final good packer we assume a symmetric equilibrium where all firms able to set price at time t choose p_t^*. The first order condition to the producer problem equates two infinite sums. These sums can be redefined using the following helper variables:

$$g_t^1 = \lambda_t mc_t y_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{-\epsilon} g_{t+1}^1 \tag{6}$$

$$g_t^2 = \lambda_t \Pi_t^* y_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{1-\epsilon} \frac{\Pi_t^*}{\Pi_t} g_{t+1}^2 \tag{7}$$

Where Π_t^* represents the ratio $\frac{p_t^*}{p_t}$. Now we can express our first order condition as:

$$g_t^1 = g_t^2 \tag{8}$$

It can be shown that the aggregate intermediate good price evolves according to:

$$1 = \theta_p p_{t-1}^{1-\epsilon} \left(\frac{\Pi_{t-1}^{\chi_p}}{\Pi_t} \right)^{1-\eta} + (1-\theta_p)(\Pi_t^*)^{1-\epsilon} \tag{9}$$

2.3 Aggregation

Since we assume complete markets the invidual Ricardian household variables will be identical across Ricardian households. Aggregating these variables we find:

$$c_t^r = \int_{\mu}^{1} c_{jt} dj$$

$$c_t^r = \int_{0}^{\mu} c_{jt} dj$$

$$C_t = \int_{0}^{1} c_{jt} dj = (1-\mu) c_t^r + \mu c_t^n$$

$$X_t = \int_{\mu}^{1} x_{jt} dj = (1-\mu) x_t^r$$

$$B_t^{''} = \int_\mu^1 b_{jt}^{''} dj = (1-\mu) b_t^{r''}$$

$$B_t^{'} = \int_\mu^1 b_{jt}^{'} dj = (1-\mu) b_t^{r'}$$

$$K_t = \int_\mu^1 k_{jt} dj = (1-\mu) k_t^r$$

The government budget constraint is then:

$$tr_t + g_t + \left(\frac{R_{t-6}^{''} P_t}{P_{t-6}} b_{t-6}^{r''} + \frac{R_{t-1}^{'}}{\Pi_t} b_{t-1}^{r'} \right) (1-\mu) = (b_t^{r'} + b_t^{r''})(1-\mu) + \tau_t^{l,r}(w_t^* l_t^r - ded_t^r)(1-\mu)$$
$$+\tau_t^{l,n}(w_t^* l_t^n - ded_t^n)(1-\mu) + \tau_t^k (r_t^{*k} k_{t-1} - x_t \theta_t - Liab_t)(1-\mu) - cr_t^r(1-\mu) - cr_t^n \mu (10)$$

The monetary authority sets short term interest rates according to the following rule:

$$\frac{R_t^{'}}{R_{ss}} = \left(\frac{R_{t-1}^{'}}{R_{ss}} \right)^{\gamma_r} \left(\frac{\Pi_t}{\Pi_{ss}} \right)^{\gamma_\pi (1-\gamma_r)} \left(\frac{y_t}{y_{ss}} \right)^{\gamma_y (1-\gamma_r)} \upsilon_t^m \qquad (11)$$

The government follows a fixed portfolio rule;

$$b_t^{r'} = b_t^{r''} \gamma_b \qquad (12)$$

To close the model we must specify a resource constraint as follows:

$$y_t^d = C_t + g_t + X_t \qquad (13)$$

2.4 Fiscal Shocks

The model assumes that tax rates follow a stochastic process which reverts to a steady state value as time goes to infinity. The expected marginal tax rate of labor income at time t, found in equation 14, is determined by a weighted average, ρ_{τ^l}, of its prior value, τ_{t-1}^l, and its steady state value τ_{ss}^l subject to a shock, $\upsilon_t^{\tau^l}$.

$$\tau_t^l = \rho_{\tau^l} \tau_{t-1}^l + (1-\rho_{\tau^l})(\tau_{ss}^l) + \upsilon_t^{\tau^l} \qquad (14)$$

Similarly, both the depreciation allowance, θ_t, and the marginal tax rate on capital, τ_t^k, follow a stochastic process.

$$\tau_t^k = \rho_{\tau^k} \tau_{t-1}^k + (1-\rho_{\tau^k})(\tau_{ss}^k) + \upsilon_t^{\tau^k}$$

$$\theta_t = \rho_\theta \theta_{t-1} + (1-\rho_\theta)(\theta_{ss}) + \upsilon_t^\theta$$

Both the government consumption and transfer processes follow similar rules to those of the tax variables. However, both g_t and tr_t include a fiscal correction term which reduces each variable when last period's debt rises above steady state. This correction is needed to ensure that the path of debt can be solved for (i.e., is not explosive). The parameter is set to be as close to zero as possible.

$$g_t = \rho_g g_{t-1} + (1 - \rho_g)(g_{ss} - \phi_g(b_{t-1} - \hat{b})) + \upsilon_t^g$$

$$tr_t = \rho_{tr} tr_{t-1} + (1 - \rho_{tr})(tr_{ss} - \phi_{tr}(b_{t-1} - \hat{b})) + \upsilon_t^{tr}$$

3 Results

4 Conclusion

A Auxiliary variables

In order to track the promised payments the model must specify the last six payments due to bond holders. They are carried as state variables in the following manner.

$$pmt_t^0 = b_t^{r''} r_t'' \tag{15}$$

$$pmt_t^1 = pmt_{t-1}^0 / \Pi_t \tag{16}$$

$$pmt_t^2 = pmt_{t-1}^1 / \Pi_t \tag{17}$$

$$pmt_t^3 = pmt_{t-1}^2 / \Pi_t \tag{18}$$

$$pmt_t^4 = pmt_{t-1}^3 / \Pi_t \tag{19}$$

$$pmt_t^5 = pmt_{t-1}^4 / \Pi_t \tag{20}$$

The government budget constraint is then:

$$\begin{aligned} tr_t + g_t + \left(\frac{pmt_{t-1}^5}{\pi_t} + \frac{R'_{t-1}}{\Pi_t} b_{t-1}^{r'} \right)(1-\mu) &= (b_t^{r'} + b_t^{r''})(1-\mu) + \tau_t^{l,r}(w_t^* l_t^r - ded_t^r)(1-\mu) \\ &+ \tau_t^{l,n}(w_t^* l_t^n - ded_t^n)(1-\mu) + \tau_t^k(r_t^{*k} k_{t-1} - x_t \theta_t - Liab_t)(1-\mu) - cr_t^r(1-\mu) - cr_t^n \mu \end{aligned} \tag{21}$$

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