# Analyzing indonesian rice farms

# Jonas Kernebeck, Alexander Flick, Felix Lehner

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#### 1 Introduction

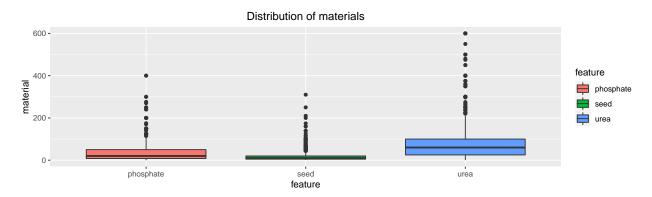
The present data set includes production data for 171 indonesian rice farms. The dataframe contains the following variables:

variable	description	expressions
id	unique identifier for a farm	unique id
$_{ m time}$	unique identifier for a specific growing season	1 - 6
size	total production area in hectares	0.01 - 5.322
status	status of property rights	"owner", "share", "mixed"
varieties	rice seed varietes	"trad", "high", "mixed"
bimas	bimas-status of the farmers	"no", "yes", "mixed"
seed	seed in kilogram	1 - 1250  kg
urea	urea in kilogram	1 - 1250  kg
phosphate	phosphate in kilogram	0 - 700 kg
pesticide	pesticide cost in Rupiah	0 - 62600 r
pseed	price of seed in Rupiah per kg	40 - 375  r/kg
purea	price of urea in Rupiah per kg	50 - 100  r/kg
pphosph	price of phosphate in Rupiah per kg	60 - 120  r/kg
hiredlabor	hired labor in hours	1 - 4536 h
famlabor	family labor in hours	1 - 1526 h
totlabor	total labor (excluding harvest labor)	1 - 4774 h
wage	labor wage in Rupiah per hour	30 - 175.35 r/h
goutput	gross output of rice in kg	42 - 20960 kg
noutput	gross output minus harvesting cost	42 - 17610 kg
price	price of rough rice in Rupiah per kg	50 - 190  r/kg
region	region of the farm	unique region

As present in the table, the data set consists of 16 numeric variables and 4 categorical variables. The target variable for the regression modeling will be goutput, what represents the gross output of rice in kg for the respective rice farm. In the following some explorative data analysis will be made to get to get a first impression of the distribution of the individual variables.

#### 1.1 Numerical Variables

The following figure shows boxplots for the used materials and the prices paid for the materials of the respective rice farms. The boxplots for the materials show, that the distribution of all materials is rightskewed. The spread width of seed is the lowest, followed by phosphate and urea. Therefore urea also has the highest variance with 16166 followed by phosphate with 2264 and seed with 2048. The distribution of urea indicates that rice farms in Indonesia may use urea very different, caused by e.g. the bimas-status. The bimas program is a rice intensification program by the government to support local rice production by providing high-yield rice seeds as well as technical assistance. If we look at the prices for phosphate pphosph and urea purea, we can see a slight left-skewed distribution with low variance (75 for purea and 86 for pphosph). In contrast to that, the prices for seeds scatter much. The distribution of pseed is strongly right-skewed as well as the distribution for the rice price price. The price for the rice also scatters, but less than pseed. The two prices have a correlation of 0.67. Of course, the price of seeds affects the selling price of rice. The prices may fluctuate due to seasonal or regional factors and have an impact on each other. of labor hours is also slightly skewed to the right. Overall, the dispersion is lowest for the famlabor. For hiredlabor and totlabor we have a similar spread, but totlabor has a higher level overall. This is caused by the *hiredlabor* which is a subset of *totlabor*.



# Distribution of material prices feature price phosphate price rice price seed price urea

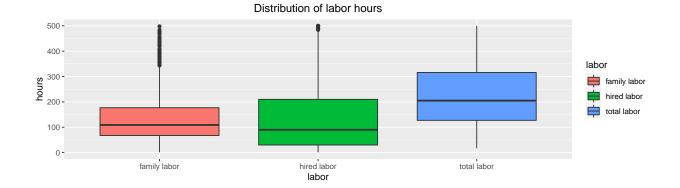
price seed

price urea

rupiah\_per\_kg - 000 -

100 -

price phosphate

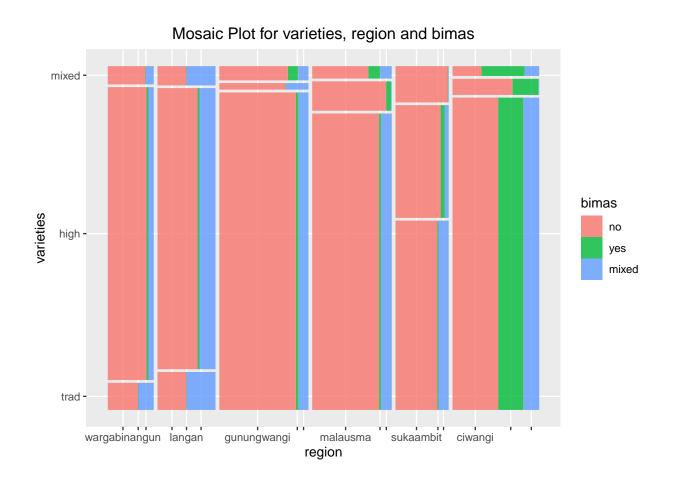


feature

price rice

### 1.1 Categorical Variables

The following mosaic plot shows the distribution of of the categorical variables varietes, region and bimas. Overall, all regions are roughly equally represented in the data set. We can detect, that most of the farmers with the bimas status yes and mixed are located in the region ciwangi. The distribution of the different varieties is strongly dependent on the region. While the high varieties have the biggest share in the regions wargabinangun and langan, the traditional varieties are dominating the regions gunungwangi, malausma and ciwangi. The mixed varieties are only used slightly in all regions.



To test wheter the categorical variables have impact on our target variable *goutput*, one- and two-sided anovas are performed. The results of these are summarized in the following table:

formula	F-value	p-value	significant
region varieties	22.981 $11.764$	< 2e-16 8.94e-06	yes yes
bimas	14.817	4.57e-07	yes

formula	F-value	p-value	significant
region+varietes	3.847	3.96e-05	yes
region + bimas	5.651	2.94e-08	yes
varieties+bimas	0.791	0.531	no
region+varieties+bimas	0.860	0.580	no

The anova outputs show, that all of the categorical variables have a significant effect on *goutput*. The null hypothesis, that the mean of *goutput* is the same across the groups is rejected. The results of the two-sided anovas also show a significant interaction effect on *goutput*. While the interaction effect from the *region* with *varieties* and *bimas* is significant, the interaction effect of *varieties* and *bimas* and the interaction effect of all three variables is not.

#### 1.3 Variable selection and transformation

The performance of the regression modeling is highly dependent of the variable selection and transformation. Therefore a suitable choice is very important. The variable *noutput* is a linear transformation of *goutput* as it represents *goutput* decreased by the harvesting costs. Therefore it is not used for the modeling because it would violate the multicollinearity assumption.

The variable *size* also correlates *strongly* with the target variable. This can be intuitively explained by the fact that a larger rice field naturally always produces a higher yield. Since the variables *seed*, *urea*, *phosphate* and *pesticide* are dependent on size, they are transformed into per-hectare sizes by dividing them with the respective hectare size of the farm. The *size* variable is not used for further modeling.

The variables famlabor and hiredlabor are subsets of the variable totlabor and are therefore transformed into the share of totlabor by dividing them with the amount of totlabor. The variable totlabor is after that transformed to a per-hectare size by dividing it with the size. The variable wage follows a bimodal distribution. Therefore it is transformed into a binary variable, which indicates if the respective value is over or under 100.

#### 1.4 Model evaluation

The data set will be splitted in 60-20-20 parts, where 60% of the data is used for training the model, and 20% for testing and validating respectively. In the modeling part, also cross-validation is used. To evaluate the models and compare them, different metrics will be used. The numeric metrics used are the MSE, which stands for the mean squared error and the AIC, which stands for the Akaike information criterion. Beside these metrics, also graphical analysis plots like a residual plots are used for evaluation.

3 Outliers most likely due to typos for observation 110, 947 and 1004

## 2 First Model

The first model to present and evaluate is called Lasso Regression.

#### 2.1 Lasso Regression

As in most of the regression types, it minimizes the residual sum of squares (short: RSS). But in addition to that a penalty term is included in the formula which shrinks some parameters coefficient estimates to zero.

$$RSS + \lambda \sum_{j=1}^{p} \left| \widehat{\beta}_{j} \right|$$

,where  $\lambda$  is a hyperparameter, p are the parameters and  $\beta$  the parameter coefficient estimates. The sum of the coefficients starts at j=1 because the  $\beta_0$  is no parameter coefficient estimate, it is the bias of the model. Furthermore the absolute values of the estimates are calculated and summarized. As you can see from the equation, the lasso regression turns into a simple linear regression if the  $\lambda$  is zero.

The goal and intention of the lasso regression is to create a sparse model which makes it easier to interpret.

#### 2.2 Feature Selection

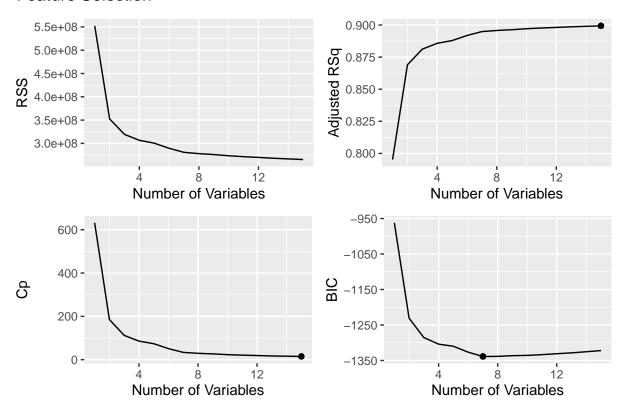
Before starting with the actual regression, we can investigate which of the features could be important for predicting the goutput of the data. This step is helpful as the number of features in the dataset is over twenty.

The method used for selecting the variables is the forward selection. The package leaps provides therefore a function regsubsets for linear models. It takes the target variable and the features of the dataset as input and gives some hints which variables could fit best to predict the goutput. The used algorithm optimizes the Mallows  $C_p$  statistics which is related to the AIC. (James,p.79)

There are more input parameters available for the function, i.e. the maximum size of a subset, the weight vector, the number of the best subsets or the method of variable selection (i.e. forward selection, backward selection, etc.).

The following graphics show the results of the analysis.

## **Feature Selection**



**Explanation:** The graphic show the the RSS, the adjusted  $R^2$ , the  $C_p$  statistics and the BIC of the models. The metrics help to identify the overall best models of the problem. Each of the metrics show that in general a 3-variable model could be enough as the metrics are getting slightly better as the number of variables increase. Just for the BIC it can be seen that after 7 variables the metric gets worse. By investigating the output of the regsubsets, it shows which of the variables are selected to give the best results.

The best 3-variable model is selecting size, phosphate and totlabor as the best performing variables. These variables are also in every other greater model. Therefore they are remembered when searching for a optimal model for the lasso regression. Also some other variables like pesticide, variety, wage and region are likely to have an influence on the model.

#### 2.3 Preprocessing

As a specific package for the lasso regression, the glmnet package, is used, the data needs to be preprocessed. One important step therefore is to transform the qualitative variables like factor variables to dummy variables so the model can use them. The method is relatively simple by creating extra features with binary values. model.matrix is doing this transformation automatically by creating a design matrix out of the data frame. It also needs the information which variables to transform for which an expression is needed. This gives the opportunity to select the wished variables and also to do mathematical transformation like logarithm or polynomial conversion before applying and getting the model matrix for the model. In addition to that a scaling option is implemented to test if the model is better when scaling the data.

#### 2.4 Training and Evaluation

As the input features are transformed properly the lasso regression can be trained. After training the function return a list of models. This is because of the hyperparameter  $\lambda$ . This allows us to fine tune the model and improve its performance. To do this, a simple 10-fold-cross-validation on the training data is applied which is also included in the package glmnet. But before starting the validation a grid of possible lambdas is prepared to have the possibility to change the spectrum of the lambda parameter. Afterwards a plot is obtained showing the mean cross-validated error depending on the  $\lambda$ . The dotted lines in the plot are the  $\lambda$  which minimizes the mean cross-validated error the most and the  $\lambda$  which gives most regularized model such that the cross-validated error is within one standard error. In this task the  $\lambda$  with the minimum mean cross-validated error is chosen.

To evaluate the selected model on the validation data with the chosen  $\lambda$  the metrics MSE, BIC, AIC, AICc (modification of AIC as a correction for small sample sizes) and  $R^2$  are used.

As for the glmnet package no known implemented function is found the metrics BIC,AIC, AICc and  $\mathbb{R}^2$  are implemented therefore manually. The formulas used are:

$$BIC = \chi^2 + k \ln(n)$$

with

$$\mathbf{X}^2$$
 = Null deviance – Residual deviance

and k as the number of parameters estimated by the model and n the number of observations.

$$AIC = -\chi^2 + 2k$$

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}$$

and

$$R^{2} = 1 - \frac{\text{sum squared regression (SSR)}}{\text{total sum of squares (SST)}}$$
$$= 1 - \frac{\sum (y_{i} - \hat{y}_{i})^{2}}{\sum (y_{i} - \bar{y})^{2}}$$

where y are the actual values,  $\hat{y}$  the predicted values and  $\bar{y}$  the mean value of the y.

#### 2.5 Model Selection

After setting the environment and the criterions of the training and evaluating of models, the best model can be selected out of the possible feature subsets. As indicated at the beginning of the Feature Selection section, the variable size, totlabor and phosphate are used to start with. However, this is also done sequentially to obtain the metrics of each model. For the other variables mentioned, this preselection is continued. Additional mathematical transformations of the variables are also considered and included. In order to provide as much variation as possible, the variables are also compared with their size-scaled correspondents. The results of the evaluation are stored in a table and assessed.

Table 3: Lasso comparison for variable selection

var	df	MSE.train	MSE.val	aic	aicc	bic	r_sq
size+seed log(seed) seed	2 2	0.1675 0.1789	0.1244 0.1466	-505.5 -498.5		-496.6 -489.6	0.8753 0.8530

Table 3: Lasso comparison for variable selection (continued)

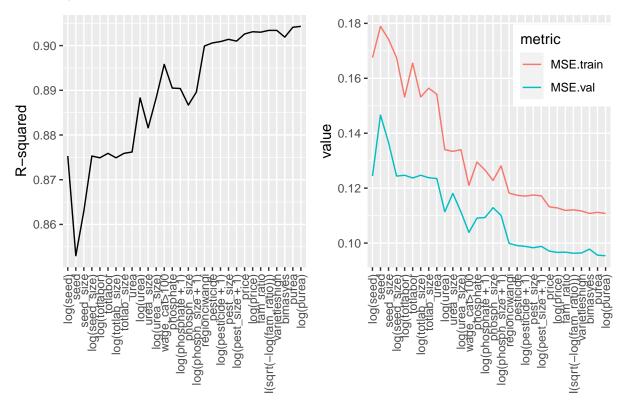
var	df	MSE.train	MSE.val	aic	aicc	bic	r_sq
seed_size	2	0.1742	0.1366	-501.4	-501.3	-492.5	0.8630
$\log(\text{seed\_size})$	2	0.1675	0.1244	-505.5	-505.4	-496.6	0.8753
size+seed+totlabor							
log(totlabor)	3	0.1532	0.1247	-512.2	-512.2	-499.0	0.8749
totlabor	3	0.1652	0.1247 $0.1237$	-512.2	-512.2	-491.4	0.8749
log(totlab size)	3	0.1633 $0.1532$	0.1237 $0.1247$	-512.2	-504.0 $-512.2$	-499.0	0.8749
totlab size	3	0.1564	0.1238	-510.3	-510.3	-497.0	0.8759
		0.2002	0.2200	0_0.0	0_0.0	20110	0.0,00
size+seed+totlabor+ure		0.1540	0.1005	<b>500</b> 6	<b>5</b> 00 0	401.0	0.0700
urea	4	0.1542	0.1235	-509.6	-509.6	-491.9	0.8762
$\log(\text{urea})$	4	0.1340	0.1114	-522.1	-522.0	-504.4	0.8883
urea_size	4	0.1334	0.1181	-522.4	-522.3	-504.7	0.8816
$\log(\text{urea\_size})$	4	0.1340	0.1114	-522.1	-522.0	-504.4	0.8883
size + seed + totlabor + urc	e <b>a+v</b>	vage_cat					
$wage\_cat>100$	5	0.1210	0.1039	-528.0	-527.9	-505.9	0.8958
size+seed+totlabor+urc	aa⊥r	hosphate					
phosphate	εα <del>, μ</del> 5	0.1295	0.1091	-522.8	-522.7	-500.7	0.8905
log(phosphate + 1)	5	0.1266	0.1093	-524.6	-524.5	-502.5	0.8904
phosph_size	5	0.1228	0.1129	-526.9	-526.8	-504.8	0.8867
$\log(\text{phosph size} + 1)$	5	0.1281	0.1101	-523.7	-523.6	-501.6	0.8896
,				0_0.1	0_0.0	00210	0.0000
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			_	<b>510</b> 0	<b>510.4</b>	455.0	0.0000
regionciwangi	10	0.1182	0.0999	-519.8	-519.4	-475.6	0.8999
${ m size+seed+totlabor+urc}$	e <b>a+v</b>	vage_cat+r	egion+pe	sticide			
pesticide	11	0.1174	0.0991	-518.2	-517.8	-469.6	0.9006
$\log(\text{pesticide} + 1)$	11	0.1171	0.0988	-518.4	-518.0	-469.8	0.9009
nost sies	11	0.1175	0.0983	-518.1	-517.7	-469.5	0.9014
$\operatorname{pest\_size}$						100.0	
$ \frac{\text{pest\_size}}{\log(\text{pest\_size} + 1)} $	11	0.1172	0.0988	-518.4	-517.9	-469.7	0.9010
$\log(\text{pest\_size} + 1)$					0 = 1.10		
log(pest_size + 1) size+seed+totlabor+ure		$wage\_cat+r$	egion+pe		0 = 1.10	-469.7	0.9010
log(pest_size + 1)  size+seed+totlabor+ure  price	ea+v			${ m sticide}+$	-price		
log(pest_size + 1)  size+seed+totlabor+ure  price log(price)	e <b>a+v</b> 10 10	vage_cat+r 0.1132 0.1128	egion+pe 0.0971 0.0966	sticide+ -522.8 -523.0	-price -522.5 -522.7	-469.7 -478.6 -478.8	0.9010 0.9026 0.9031
log(pest_size + 1)  size+seed+totlabor+ure price log(price)  size+seed+totlabor+ure	ea+w 10 10	vage_cat+r 0.1132 0.1128 vage_cat+r	egion+pe 0.0971 0.0966 egion+pe	sticide+ -522.8 -523.0 sticide+	-price -522.5 -522.7 -price+	-469.7 -478.6 -478.8 fam_rat	0.9010 0.9026 0.9031 <b>io</b>
log(pest_size + 1)  size+seed+totlabor+ure price log(price)  size+seed+totlabor+ure fam_ratio	ea+v 10 10 10 ea+v 12	vage_cat+r 0.1132 0.1128 vage_cat+r 0.1119	egion+pe 0.0971 0.0966 egion+pe 0.0967	sticide+ -522.8 -523.0 sticide+ -519.6	-price -522.5 -522.7 -price+5	-469.7 -478.6 -478.8 fam_rat -466.5	0.9010 0.9026 0.9031 <b>io</b> 0.9030
log(pest_size + 1)  size+seed+totlabor+ure price log(price)  size+seed+totlabor+ure	ea+w 10 10	vage_cat+r 0.1132 0.1128 vage_cat+r	egion+pe 0.0971 0.0966 egion+pe	sticide+ -522.8 -523.0 sticide+	-price -522.5 -522.7 -price+	-469.7 -478.6 -478.8 fam_rat	0.9010 0.9026 0.9031 <b>io</b>
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log(pest_size + 1)  size+seed+totlabor+ure price log(price)  size+seed+totlabor+ure fam_ratio I(sqrt(-log(fam_ratio)))  size+seed+totlabor+ure varietieshigh  size+seed+totlabor+ure	ea+v 10 10 ea+v 12 11 ea+v 14	vage_cat+r 0.1132 0.1128 vage_cat+r 0.1119 0.1121 vage_cat+r 0.1117 vage_cat+r 0.1108	egion+pe 0.0967 0.0966 egion+pe 0.0963 egion+pe 0.0964 egion+pe 0.0978	sticide+ -522.8 -523.0 sticide+ -519.6 -521.5 sticide+ -515.7 sticide+ -512.3	-price -522.5 -522.7 -price+1 -519.1 -521.0 -price+1 -515.0 -price+1	-469.7  -478.6 -478.8  fam_rat -466.5 -472.8  varieties -453.9  varieties -441.6	0.9010 0.9026 0.9031 io 0.9030 0.9034 +bimas 0.9019

Table columns:

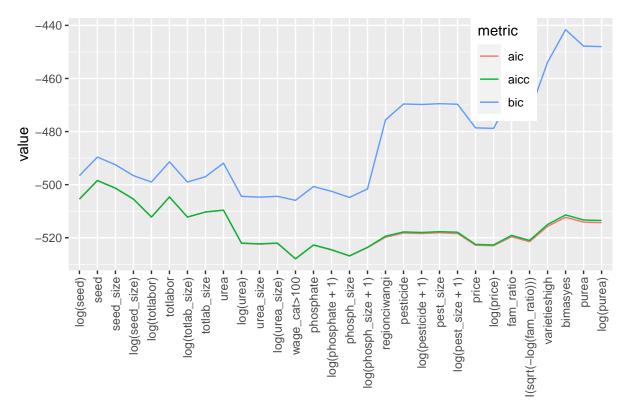
- var: the variable that is added
- df: degrees of freedom
- MSE.train: MSE on training data
- MSE.val: MSE on validation data
- aic: Akaike information criterion
- aicc: Akaike information criterion + penalty term
- bic: Bayesian information criterion
- $r_sq: R-squared$

As described at the feature selection, the RSS and the  $R^2$  are improving with the increase of features selected. However the other metrics (AIC,BIC and AICc) are behaving like the feature selection predicts. By increasing the number of features after approximately 6 the metrics are generally getting worse. This means that the number of features does not justify the improvement of the RSS and the  $R^2$ . The penalty for the more complex model is higher than it has a positive effect. Therefore the smaller model is preferred.

## Model performance



## Model metrics



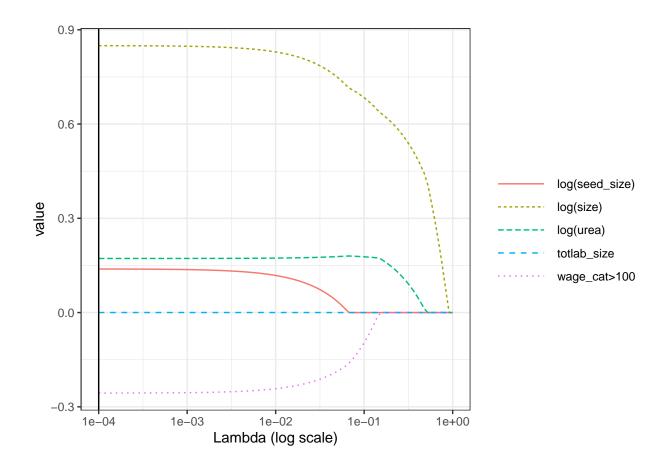
The best model to be chosen is:

$$log(g) = \beta_1 * log(s) + \beta_2 * log(e) + \beta_3 * t + \beta_4 * log(u) + \beta_5 * w + \beta_0$$

where:

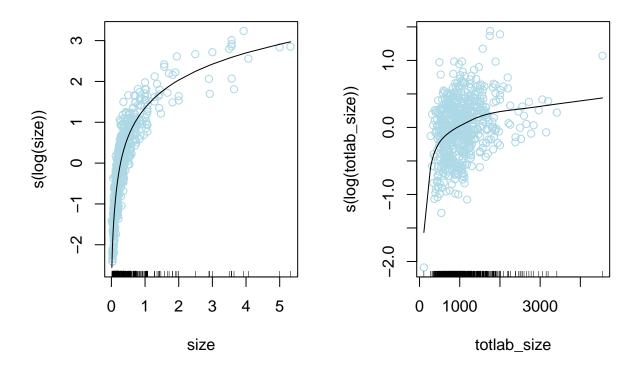
- g: goutput
- s: size
- e: seed
- t: totallabor
- u: urea
- w: wage\_cat
- $\beta_i$ : coefficients and intercept

The best model also allows to look at its lambdas and the coefficients.



#### 3 Second Model

## GAM (Generalized Additive Model)



**Explanation:** The left-hand panel indicates that holding totlab\_size fixed, goutput increases with size. The right-hand panel indicates that holding size fixed, goutput increases drasticall with the increased proportion of labour per size up to a proportion of 500 h / hectar and then flattens out.

## **GAM Comparison**

table for model selection: The following table summarizes the variable selection process for the final GAM model. Starting from the top of the table we add for each grouped row in the table another variable with different representations of the variable for example checking whether log transformation yields any better results than using the variable as is. The results are mainly compared by using the p-value from the model summary, which is telling if the variable is significant important. We will use a p-value of 5% for the evaluation of variable significance. To compare models we will use the deviance. In special cases additional anova tests are performed to compare larger models with smaller ones. In addition we list other metrics like MSE on the training and validation data as well as AIC. For each grouped row, the last model is the chosen model for further analysis.

Forward selection (from James p. 79): We begin with a model, that contains the variable with highest correlation to our dependent variable goutput, which is size. We then add different representations of one variable and add those to the model, which result in the lowest deviance for the new two-variable model. This approach is continued until all variables have been tried out. Forward selection is a greedy approach, and might include variables early that later become redundant.

			1						
var	df	MSE.train	MSE.val	dev	aic	p_val_p	p_val_np	df_np	
size									
$\overline{s(size)}$	609	924954	535892	126.3	783.5	0.0000	0.0000	3	
$s(\log(\text{size}))$	609	928878	546896	107.5	684.3	0.0000	0.0016	3	
size+labour									
s(totlab_size)	605	794773	540756	95.1	617.3	0.0000	0.0001	3	
$s(\log(\text{totlab\_size}))$	605	796454	532193	93.7	608.2	0.0000	0.0140	3	
size+labour+urea									
s(urea_size)	601	785027	554113	77.6	500.3	0.0000	0.0031	3	
s(log(urea_size))	601	785928	543432	77.7	501.3	0.0000	0.0000	3	
size+labour+urea+phos	phor						,		
s(phosph_size)	597	640702	494023	71.7	460.0	0.0000	0.2846	3	
$s(\log(\text{phosph\_size} + 1))$	597	637849	493065	71.7	459.9	0.0000	0.0000	3	
size+labour+urea+phos	phor-	+seed	I				,		
$s(\log(\text{seed\_size}))$	593	643332	475996	70.7	459.2	0.0077	0.1453	3	
$s(seed\_size)$	593	646291	475982	70.6	458.1	0.0245	0.0851	3	
size+labour+urea+phos	phor-	+seed+pest	icide			,	,		
s(pest_size)	589	609355	448837	68.9	451.8	0.0006	0.2926	3	
$s(pest\_size, df = 1)$	592	619311	450226	69.4	449.4	0.0010	0.0024	0	
size+labour+urea+phos	phor-	+seed $+$ pest	icide+pric	e	•				
s(price)	585	490980	374465	64.6	419.7	0.0007	0.0000	3	
size+labour+urea+phos	phor-	+seed $+$ pest	icide+pric	e+fam	ily_lab	our			
s(fam_ratio)	581	486748	367829	63.9	421.6	0.1542	0.2932	3	
size+labour+urea+phos	phor-	+seed $+$ pest	icide+pric	e+pric	e_info				
$\overline{s(pseed)}$	581	478646	372191	63.1	413.3	0.9013	0.0013	3	
s(pphosph)	581	456162	356139	59.4	376.0	0.0000	0.0000	3	
s(pphosph)+s(purea)	577	442872	360534	58.9	378.7	0.0711	0.0108	3	
size+labour+urea+phos	phor-	+seed $+$ pest	icide+pric	e+pric	e_info-	+wage			
$\overline{s(wage)}$	577	412261	352955	57.5	364.1	0.0000	0.2517	3	
wage_cat>100	580	423262	358715	58.0	363.5	0.0000	NA	NA	
size+labour+urea+phos	${\bf size+labour+urea+phosphor+seed+pesticide+price+price\_info+wage+categorical}$								
bimas	578	427986	373423	57.0	357.0	0.0041	NA	NA	
bimas+varieties	576	412347	351919	56.6	356.5	0.1174	NA	NA	
bimas+status	576	412929	372755	56.8	359.1	0.3627	NA	NA	
bimas+region	573	420793	367757	56.4	360.4	0.1572	NA	NA	
size+labour+urea+phos	phor-							xcluded	
s(purea)	574	411251	384322	56.2	356.9	0.0367	0.0019	3	
s(purea)+varieties	572	395042	361190	55.8	356.2	0.1137	NA	NA	

#### Table columns:

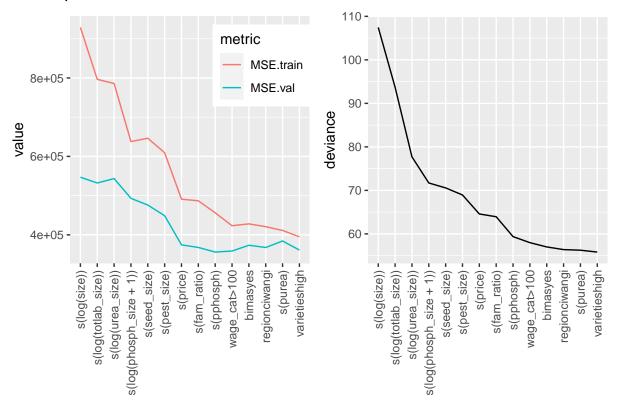
- var: the variable that is added
- df: degrees of freedom
- MSE.train: MSE on training data
- MSE.val: MSE on validation data
- dev: residual deviance (goodness of fit)
- aic: Akaike information criterion
- p\_val\_p: parametric p-value for last added variable
- p\_val\_np: non-parametric p-value for last added variable
- df\_np: non-parametric degrees of freedom for last added variable

#### Variable selection explanation:

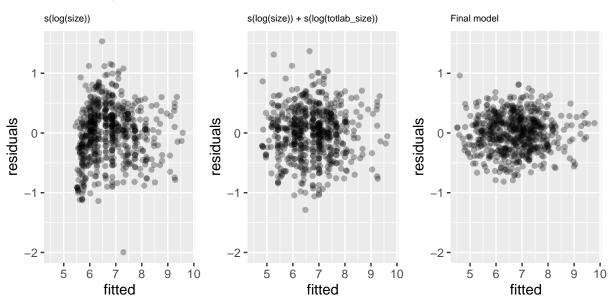
- size, labour, and urea: we will use logarithm of these variables, because of a smaller deviance
- phosphor: this variable has lots of zeros as values. So we can not use directly log transformation because log of 0 is -inf.  $\log(x+1)$  transformation is the best way to avoid errors created by log transformation and is widely used among data scientists. So we will use this approach. We will use logarithm of phosph\_size, because of a smaller deviance.
- seed: we will use seed as is, because of a smaller deviance.
- pesticide: non-parametric p-value is > 0.05 so we decrease df to achieve more suitable fit.
- price, family labour: Both variables are significant important for the model.
- material prices:
  - pseed: p-value > 0.05, thus we do not use this variable
  - pphosph: p-value < 0.05
  - pphosph + purea: p-value < 0.05, but the anova test comparing smaller including only pphosph with larger model pphosp + purea yields a p-value of
- wage:
- categorical variables:
  - bimas and bimas+region: p-value > 0.05,
  - bimas+varieties and bimas+status: p-value < 0.05
- final model: we check again for excluded variables after some more variables have been added if the results change. purea has a p-value of 0.05 and performing an anova test for varieties we can see that there is a significant difference between the two models.

```
## Analysis of Deviance Table
## Model 1: log(goutput) ~ s(log(size)) + s(log(totlab_size)) + s(log(urea_size)) +
##
       s(log(phosph_size + 1)) + s(seed_size) + s(pest_size) + s(price) +
       s(pphosph) + wage_cat + bimas + s(purea)
##
## Model 2: log(goutput) ~ s(log(size)) + s(log(totlab_size)) + s(log(urea_size)) +
##
       s(log(phosph_size + 1)) + s(seed_size) + s(pest_size) + s(price) +
       s(pphosph) + wage_cat + bimas + s(purea) + varieties
##
    Resid. Df Resid. Dev Df Deviance Pr(>Chi)
##
## 1
           574
                   56.247
                   55.826 2 0.42118
## 2
           572
                                        0.1156
```

# Model performance



# Residual comparison



#### Final GAM

```
## log(goutput) ~ s(log(size)) + s(log(totlab_size)) + s(log(urea_size)) +
## s(log(phosph_size + 1)) + s(seed_size) + s(pest_size) + s(price) +
## s(pphosph) + wage_cat + bimas + s(purea) + varieties
```

**R-squared** (**R**<sup>2</sup>): is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable or variables in a regression model. Essentially, an R-Squared value of 0.9 would indicate that 90% of the variance of the dependent variable being studied is explained by the variance of the independent variable.

$$\begin{split} R^2 &= 1 - \frac{\textit{Unexplained Variation}}{\textit{Total Variation}} \\ R^2 &= 0.9102197 \end{split}$$

Adjusted R-Squared: The adjusted R-squared compares the descriptive power of regression models that include diverse numbers of predictors. Every predictor added to a model increases R-squared and never decreases it. Thus, a model with more terms may seem to have a better fit just for the fact that it has more terms, while the adjusted R-squared compensates for the addition of variables and only increases if the new term enhances the model above what would be obtained by probability and decreases when a predictor enhances the model less than what is predicted by chance.

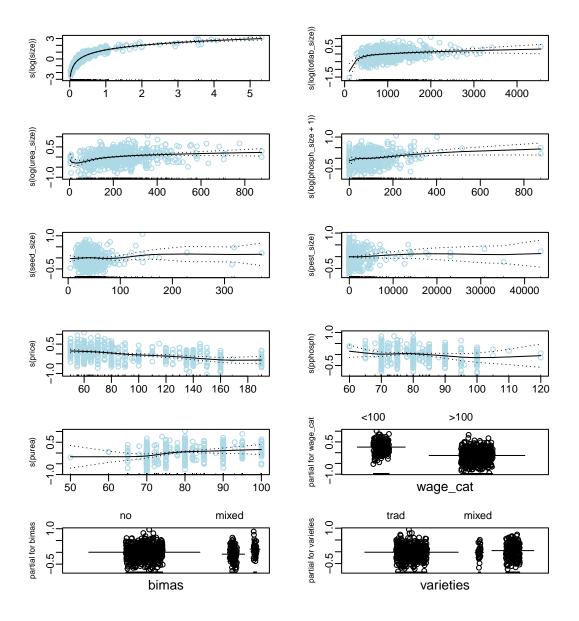
Source: https://www.investopedia.com/terms/r/r-squared.asp https://www.researchgate.net/post/How-can-I-get-the-adjusted-r-squared-value-of-GAM-model

$$adjR^2 = 1 - \frac{(1-R^2)*(n-1)}{n-p-1}$$

n = total sample size p = number of predictors

$$adjR^2 = 0.9079677$$

#### Final GAM visualization



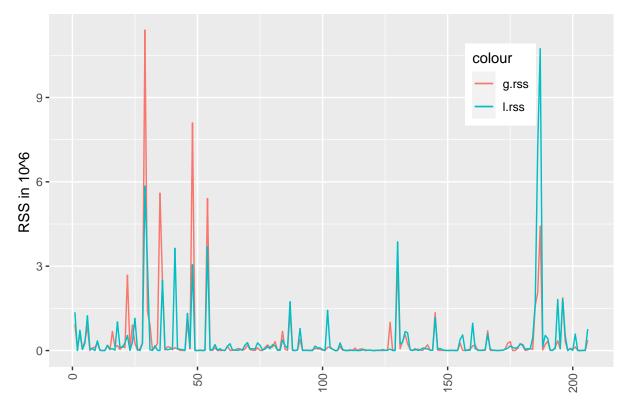
# 4 Comparison

After the best model of Lasso Regression and the Generalised Additive Model has been determined by validation, a comparison of these is undertaken.

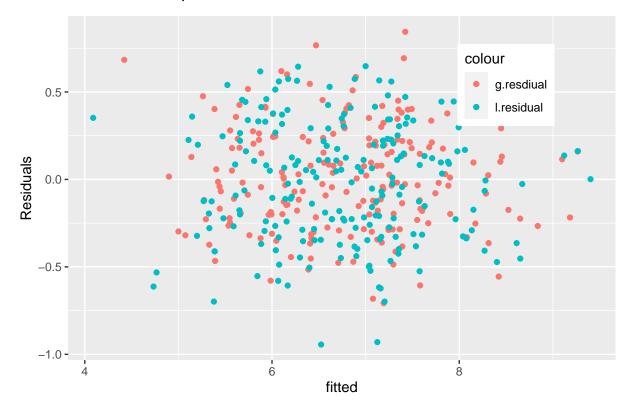
The test data set that was put away at the beginning is used for this. Again, familiar metrics as in the previous sections are used for evaluation. Among others, the number of degrees of freedom, MSE and the  $R^2$ . In addition, the residuals sum of squares and the residuals are compared to see how good the prediction of the model is.

Metric	Lasso	GAM
df	609	572
$R^2$	0.878	0.875
MSE	$3.8238397 \times 10^5$	$3.5825297 \times 10^5$

# Model RSS Comparison



## Model Residual Comparison



It can be seen from the tables that both models perform equally well. The Lasso model performs a little better with the  $\mathbb{R}^2$ , but this should not be a problem. However, the Lasso model has more degrees of freedom than the other model, which is due to the fact that fewer predictor variables are needed than in the GAM model. However, the GAM model performs better on the MSE.

With regard to the graphs, there is also no significant difference between the models. It can be seen, however, that in the RSS plot all two models have similar false predictions at certain points in the test data set, since the peaks are at the same points. This is probably due to extreme values in the data set.

It can be concluded that both models can be used for predicting the output of the data set. It is up to the user to decide which model to use. If a simple model is needed, even for explanation, the Lasso model is preferred, because the GAM model is more complex and is used for non-linear complex relationships in the data set.