

WIA2005 Algorithm Design & Analysis
Semester 2
Tutorial 1

1. The following is an insertion sort algorithm.

```
def InsertionSort(A):  
1  for j in range(1, len(A), 1):  
2    key = A[j]  
3    # insert A[j] into the sorted sequence A[1..j-1]  
4    i = j - 1  
5    while (i >= 0 and A[i] > key):  
6      A[i+1] = A[i]  
7      i = i - 1  
8    A[i+1] = key
```

Illustrate the insertion sort operation on array A = 41, 51, 69, 36, 51, 68.

41, (51), 69, 36, 51, 68

j=1 key=51 i=0 tak lalu while (!41>51) A[i+1]=51

41, 51, (69), 36, 51, 68

j=2 key=69 i=1 tak lalu while (!51>69) A[i+1]=69

41, 51, 69, (36), 51, 68

j=3 key=36 i=2 lalu while (69>36)
A[i+1]=36
A[i]=69
letak no. besar ke tmpt no. kecil
new A[i+1]=69
i=1

41, 51, 69, 69, 51, 68 key=36
A[i+1]=69
A[i]=51
letak no. besar ke tempat no. kecil
new A[i+1]=51
i=0

41, 51, 51, 69, 51, 68 key=36
A[i+1]=51
A[i]=41
letak no. besar ke tempat no. kecil
new A[i+1]=41

i=-1

keluar loop

A[i+1]=36

36,41,51,69,(51),68

j=4 key=51

i=3 lalu while (69>51)

36,41,51,69,(51),68

A[i+1]=51//i+1 will always refer to the key value

A[i]=69

new A[i+1]=69

i=2

keluar dari loop since (51=51)

A[i+1]=51//overwrite 69

36,41,51,51,69,(68)

j=5 key=68

i=4 lalu while (69>68)

36,41,51,51,69,(68)

A[i+1]=68

A[i]=69

new A[i+1]=69//copy A[i] value

i=3

36,41,51,51,69,69 key=68

keluar dari loop since (51<68)//lagi kecil dari the key

A[i+1]=68

36,41,51,51,68,69

2. Modify the insertion sort algorithm to sort array into decreasing order.

```
def InsertionSort(A):  
    for j in range(1,len(A),1):  
        key=A[j]  
        i=j-1  
        while (i>=0 and A[i]<key):  
            A[i+1]=A[i]  
            i=i-1  
        A[i+1]=key  
    return A
```

```
print(InsertionSort(A))
```

3. Write a pseudocode for linear search for the following requirement:

Input: A sequence of n numbers $A = \langle a_1, a_2, \dots, a_n \rangle$ and a value v .

Output: An index i such that $v = A[i]$ or the special value NIL if v does not appear in A .

```

Declare empty array List[]
For i=0 to Array.length
    If A[i] ==v
        List.add(A[i])

```

```

If List.length>0
    return list
else
    return NIL

```

```

for i=0 to length(A)
    if A[i]==v
        return i
return NIL

```

4. Express the function $n^3 / 1000 - 100n^2 - 100n + 3$ in terms of Θ -notation.

$$\Theta(e) = \Theta(\max(n^3 / 1000, -100n^2, -100n, 3)).$$

$$\Theta(1) < \Theta(n) < \Theta(n^2) < \Theta(n^3)$$

Then

$$\Theta(e) = \Theta(\max(n^3 / 1000, -100n^2, -100n, 3)) = \Theta(n^3)$$

5. For the following pairs of functions, $f(n)$ and $g(n)$, determine if they belong to Case 1: $f(n) = O(g(n))$ or Case 2: $g(n) = O(f(n))$. Formally justify your answer.

- a. $f(n) = 3n + 2$, $g(n) = n$
Case 2: $g(n) = O(f(n))$
 because $f(n) \leq cg(n)$ for $n > n_0$
 For example for $n=1000$
 $f(n) = 302$ while $g(n) = 100$

- b. $f(n) = (n^2 - n)/2$, $g(n) = 6n$
Case 2: $g(n) = O(f(n))$
 because $O_f(\max(n^2, n)) = O_f(n^2)$
 $O_g(n) < O_f(n^2)$ for $n > n_0$

- c. $f(n) = n + 2\sqrt{n}$, $g(n) = n^2$
Case 1: $f(n) = O(g(n))$
 because $O_f(\max(n, \sqrt{n})) = O_f(n)$
 $O_g(n^2) > O_f(n)$ for $n > n_0$

- d. $f(n) = n^2 + 3n + 4$, $g(n) = n^3$

Case 1: $f(n) = O(g(n))$

because $O_f(\max(n^2, n)) = O_f(n^2)$
 $O_g(n^3) > O_f(n^2)$ for $n > n_0$

6. Given the iterative function below (in Java), calculate their time complexity.

```
a. function1 () {
    for (int i = 1; i <= n; i++) {
        printf("Hello world");
    }
}
```

$T(n) = O(n)$

```
b. function2 () {
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            printf("Hello world");
        }
    }
}
```

$T(n) = O(n^2)$

```
c. function3 () {
    for (int i = 1; i^2 <= n; i++) {
        printf("Hello world");
    }
}
```

$T(n) = O(n^{0.5})$

```
d. function4 () {
    for (int i = 1; i <= n; i = i*2) {
        printf("Hello world");
    }
}
```

$T(n) = O(\log_2(n))$

```
e. function3 () {
    for (int i = n/2; i <= n; i++) {
        for (int j = 1; j <= n/2; j = 2*j) {
            for (int k = 1; k <= n; k = k*2) {
                printf("Hello world");
            }
        }
    }
}
```

$T(n) = O(n[\log_2(n)]^2) = O(n[\log_2^2(n)])$