Relative Positioning Estimation using Double-Difference Carrier Phase Measurements with Integer Ambiguity Resolution

Kerry Sun

***Abstract*—This final report demonstrates how the relative position between two static locations is estimated using carrier phase measurements. Specifically, Double-difference method is utilized along with three different integer ambiguity methods. These three methods are Geometry-Free approach, Batch Least Square rounded float solution, and LAMBDA method. Centimeter-level accuracy was indeed obtained for estimating relative position using carrier phase measurements. LAMBDA method was found to be most accurate for resolving the integer ambiguity problem.**

***Keywords*—GPS, Carrier-Phase, LAMBDA, integer ambiguity.**

I. Introduction

GPS was initially designed for positioning using code phase measurements. Positioning with centimeter-level accuracy was not foreseen by the designers of the system [1]. Carrier phase positioning, also called Real-Time Kinematic (RTK) positioning, is a precise GPS-based positioning method. The techniques of precise relative positioning with GPS carrier phase measurements was first demonstrated by Counselman and colleagues at MIT and Draper Laboratory in the late 1970s. In this approach, determination of the range signal can be resolved in the centimeter level. There are many important applications such as geological surveying, agriculture mapping and aerial refueling that need centimeter level GPS positioning. Even though carrier phase positioning is still not widely used due to lack of robustness, it can become more relevant when new satellite navigation system are in use (e.g. Beidou plans to be operational by 2020 and Galileo plans to be fully deployed by 2020).

This paper demonstrates the relative position of two static locations located near Minneapolis through the carrier phase relative positioning technique. Specifically, Double-difference method will be used for calculating the relative position. Centimeter-level accuracy shall be obtained through this method. The technical difficulty arises when resolving the integer ambiguities. This problem, usually called integer ambiguity resolution problem, is solved using three different methods: Geometry-Free approach, Batch Least Square rounded float solution and LAMBDA method. The details of Double-difference method along with all the integer ambiguity resolution methods will be discussed in the Technical Approach section.

II. Technical Approach

Two static locations were chosen for this project. The data is discussed in Section A. The relative position between them was estimated using Double-difference method with carrier phase measurements. The estimation then was compared to the true relative position. The basic idea of Double-difference method is to use the carrier phase double-difference measurements to cancel out clock bias, ionospheric and tropospheric terms, and obtain a linear equation that only includes the unknown relative position vector and the integer ambiguities. We will also have to estimate the integer ambiguities associated with pairs of satellites and receivers in order to estimate the relative position. Double-difference method and integer ambiguity estimation will be described in details in Section B and C.

A. Data Collection

All the data was collected from the MnCORS website (Minnesota Department of Transportation). MnCORS provides known static antenna locations with its associated GPS observation and navigation data. All the data files are in the RINEX (Receiver Independent Exchange Format) format. The RINEX version 2.11 was used for the observation file and 2.10 was used for the navigation file. The RINEX observation file provides pseudorange, carrier-phase, signal to noise ratio, etc. of the associated GPS satellites. The RINEX navigation file provides Almanac parameters for calculating the GPS locations. Fig. 1 shows what a typical RINEX observation file looks like.

```
     2.11           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
VRSNet 1.4                              14-NOV-16 00:01     PGM / RUN BY / DATE
                                                            COMMENT
                                                            COMMENT
                                                            COMMENT
Egan                                                        MARKER NAME
Eagan - MVTS Bus Gar                                        MARKER NUMBER
MNDOT               MNDOT                                   OBSERVER / AGENCY
5237K52372          TRIMBLE NETR9       Nav 5.1 / Boot 5.02 REC # / TYPE / VERS
31051273            TRM55971.00     NONE                    ANT # / TYPE
  -253009.9462 -4524288.7850  4473855.2146                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1     0                                          WAVELENGTH FACT L1/2
     8    C1    L1    S1    P1    P2    C2    L2    S2      # / TYPES OF OBSERV
    15.000                                                  INTERVAL
     0                                                      RCV CLOCK OFFS APPL
    17                                                      LEAP SECONDS
    55                                                      # OF SATELLITES
  2016    11    14    00    00   00.0000000     GPS         TIME OF FIRST OBS
                                                            END OF HEADER
 16 11 14  0  0  0.0000000  0 18G01G03G10G11G12G14G22G25G26G31G32R02 0.000000000
                                R03R11R12R13R18R19
  22663862.180  119099439.789 6        44.900                   22663868.980
  22663868.801   92804805.701 7        29.900
  23911307.023  125654805.472 5        39.600                   23911313.402
  23911312.586   97912847.552 7        26.400
  22669962.438  119131512.380 6        47.000                   22669967.652
  22669967.430   92829805.878 8        34.400
```

Fig. 1: An example of RINEX 2.11 Observation File.

For this project, the two chosen antennas are located at Eagan and Apple Valley. These two locations were chosen because its relative distance is the shortest among all available antennas. The relative distance between these 2 locations is about 10.43 km. RTK positioning requires the use of a base or reference receiver, typically within a range of 10 to 20 kilometers. Hence, these 2 locations are suitable for the project. These 2 locations' information is listed in Table I. Fig. 2 also shows their locations graphically. The data for two location

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were collected on Novermber 14, 2016. Note the observation data were formatted at a 15-sec epoch in the RINEX file.

TABLE I: Two receiver information.

Station	Latitude	Longitude	Altitude
Eagan	N 44 49' 35.59276"	W 93 12' 02.85931"	222.687 m
Apple Valley	N 44 43' 58.51370"	W 93 12' 34.93819"	280.459 m

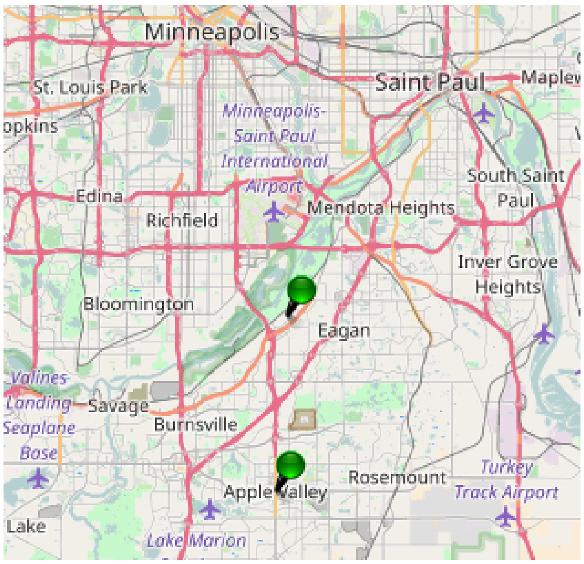

Fig. 2: Two antenna locations on a Map.

All the data navigation and observation data were parsed out in MATLAB before implementing the estimation method. Only observation data C1 (carrier phase measurements using 1575.42 MHz frequency) and L1 (pseudorange measurements using 1575.42 MHz frequency) were used for the purpose of this project.

B. *Double-difference Method*

1) Carrier Phase Measurements: A measurement more precise than code ranging is the phase of the carrier signal received from a satellite. It is the difference between the phases of the receiver-generated carrier signal and the carrier signal received from a satellite at the instant of the measurement. Dropping the explicit reference to measurement epoch, the equation for carrier phase measurement is given as [1]:

$$\phi = \lambda^{-1}[r - I_\phi + T_\phi] + f \cdot \delta t_{ur} + N + \varepsilon_\phi \tag{1}$$

The carrier phase measurments are extremely precise, but are encumbered with integer ambiguity term N.

2) Single-difference Measurements: Before talking about double-difference measurements, single-difference measurement should be introduced first. Carrier phase measurements in Eq. (1) contain nuisance term such as clock bias δt_{ur}, integer ambiguity N, ionospheric I_ϕ and tropospheric T_ϕ error, etc. Single-difference method is used to remove the GPS satellite clock bias and atmospheric error terms [1]. Difference of measurements of two user receivers (u and r) from the GPS satellite k yields:

$$\begin{aligned}\phi_{ur}^{(k)} &= \phi_u^{(k)} - \phi_r^{(k)} \\ &= \lambda^{-1}[r_{ur}^{(k)} - I_{ur}^{(k)} + T_{ur}^{(k)}] \\ &\quad + f \cdot \delta t_{ur} + N_{ur}^{(k)} + \varepsilon_{\phi,ur}^{(k)}\end{aligned} \tag{2}$$

where (k) denotes the measurement from k-th satellite and $(*)_{ur} = (*)_u - (*)_r$ If the distance between the user and the reference station is short (100 km baseline [1]), the residual ionospheric, tropospheric, and ephemeris errors in Eq. (2) would be small in comparison with the typical errors due to receiver noise and multi-path. In fact, we can reduce Eq. (2) to Eq. (3)

$$\phi_{ur}^{(k)} = \lambda^{-1} r_{ur}^{(k)} + f \cdot \delta t_{ur} + N_{ur}^{(k)} + \varepsilon_{\phi,ur}^{k} \tag{3}$$

All the measurements in carrier phase are in terms of cycles.

3) Double-difference Measurements: The relative clock bias in the single-difference measurement is the parameter also needed to be removed. Taking the difference of the single-difference measurements from two different GPS satellites results in double-difference measurement given:

$$\begin{aligned}\phi_{ur}^{(kl)} &= \phi_{ur}^{(k)} - \phi_{ur}^{(l)} \\ &= \lambda^{-1} r_{ur}^{(kl)} + N_{ur}^{(kl)} + \varepsilon_{\phi,ur}^{(kl)}\end{aligned} \tag{4}$$

in Eq. (4), $r_{ur}^{(kl)}$ should be related to the relative position $\boldsymbol{x_{ur}}$ shown in Eq. (5). The corresponding measurement geometry is shown in Fig. 3.

$$r_{ur}^{(kl)} = (r_u^{(k)} - r_r^{(k)}) - (r_u^{(l)} - r_r^{(k)}) = -(\mathbf{1}_r^{(k)} - \mathbf{1}_r^{(l)}) \cdot \mathbf{x}_{ur} \tag{5}$$

In order to estimate $\boldsymbol{x_{ur}}$, $N_{ur}^{(kl)}$ need to be estimated; this is the ambiguity related to the pair of satellites and receivers. Using GPS satellite 1 as reference and assuming K satellites are in view, there are (K-1) double differences which can be written in vector-matrix notation as below:

$$\begin{bmatrix} \phi_{ur}^{(2l)} \\ \phi_{ur}^{(3l)} \\ \vdots \\ \phi_{ur}^{(Kl)} \end{bmatrix} = \lambda^{-1} \begin{bmatrix} -(\mathbf{1}_r^{(2)} - \mathbf{1}_r^{(1)})^T \\ -(\mathbf{1}_r^{(3)} - \mathbf{1}_r^{(1)})^T \\ \vdots \\ -(\mathbf{1}_r^{(K)} - \mathbf{1}_r^{(1)})^T \end{bmatrix} \mathbf{x}_{ur} + \begin{bmatrix} N_{ur}^{(21)} \\ N_{ur}^{(31)} \\ \vdots \\ N_{ur}^{(K1)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\phi,ur}^{(21)} \\ \varepsilon_{\phi,ur}^{(31)} \\ \vdots \\ \varepsilon_{\phi,ur}^{(K1)} \end{bmatrix} \tag{6}$$

The stacked Eq. (6) can be solved if the $N_{ur}^{(kl)}$ is estimated prior to estimating $\boldsymbol{x_{ur}}$. Also, $\boldsymbol{x_{ur}}$ and $N_{ur}^{(kl)}$ can be solved simultaneously using least square estimation, if we have multiple epochs data.

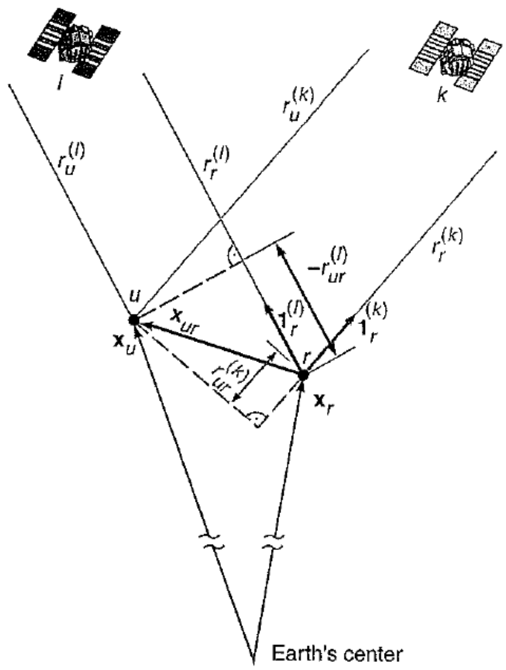

Fig. 3: Geometry of the double-difference measurements [1].

C. Integer Ambiguity Estimation

Three different methods were used to determine the integer ambiguities: Geometry-Free approach, Batched Least Square rounded float solution, and LAMBDA method.

1) Geometry-Free Approach: Geometry-free approach resolves integer ambiguity one at a time. It deals with measurements from one pair of satellites at a time. If (K-1) pairs of satellites are available, then it would resolve (K-1) integers at a time. Specifically, suppose if we have double-difference code and carrier phase measurements using L1 from K^{th} pair of satellites as the following (the superscript (kl) is dropped for simplicity):

$$\phi_{L1} = \frac{r}{\lambda_{L1}} + N_{L1} + \varepsilon_{\phi,L1} \tag{7}$$

$$\rho_{L1} = r + \varepsilon_{\rho,L1} \tag{8}$$

then an estimate of N_{L1} of K^{th} pair can be resolved as:

$$\hat{N}_{L1} = \left[\phi_{L1} - \frac{\rho_{L1}}{\lambda_{L1}}\right]_{roundoff} \tag{9}$$

The theoretical standard deviation of the estimate $\hat{N}_{L1}$ is about 5 cycles [1], which isn't that good. However, $\hat{N}_{L1}$ does not change as long as there is no loss of luck or cycle slips. The uncertainty of $\hat{N}_{L1}$ can be reduced by averaging over a sequence of estimates and rounding off to the nearest integer. Note that since the measurement errors are highly correlated over time, it will take a long stretch of clean data to get a good estimate. For example, the integer estimation error should be reduced to one-half cycle if we average uncorrelated measurements from over 100 epochs [1].

2) Batch least Square Rounded Float Solution: Estimation of integer ambiguities in a batch as if they were real values can be obtained given a time series of carrier phase measurements. Such estimates are called float solutions. Rounding off the float solutions give the fixed solutions or integer estimates.

Specifically, starting with the double-difference Eq. (4), it can be further changed into a simpler form by regrouping terms:

$$y_{ur}^{kl} = \mathbf{g}^{kl} \cdot \delta\mathbf{x} + N_{ur}^{kl} + \varepsilon_{\phi,ur}^{kl} \tag{10}$$

where $y_{ur}^{kl} = \phi_{ur}^{kl} - \lambda^{-1} r_0^{kl}$ and $\mathbf{g}^{kl} = \lambda^{-1}(\mathbf{1}_r^k - \mathbf{1}_r^l)$. Combining $K-1$ measurements for one epoch, the equation can be written the following:

$$\mathbf{y} = \mathbf{G}\delta\mathbf{x} + \mathbf{N} + \varepsilon_\phi \tag{11}$$

Now if we take the measurement for $n \cdot 15$ seconds for one batch, the equations can be written as:

$$\begin{bmatrix} \mathbf{y}(i) \\ \mathbf{y}(i+1) \\ \vdots \\ \mathbf{y}(i+n) \end{bmatrix} = \begin{bmatrix} \mathbf{G}(i) \\ \mathbf{G}(i+1) \\ \vdots \\ \mathbf{G}(i+n) \end{bmatrix} \delta\mathbf{x} + \begin{bmatrix} \mathbf{N} \\ \mathbf{N} \\ \vdots \\ \mathbf{N} \end{bmatrix} + \begin{bmatrix} \varepsilon_\phi(i) \\ \varepsilon_\phi(i+1) \\ \vdots \\ \varepsilon_\phi(i+n) \end{bmatrix} \tag{12}$$

Since the form of the float solution is:

$$float\, solution = \begin{bmatrix} \hat{\delta\mathbf{x}} \\ \hat{\mathbf{N}} \end{bmatrix} \tag{13}$$

Eq. (12) can be further simplified as:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_n \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{G}}_1 & 0 & 0 & \cdots & \mathbf{I} \\ 0 & \hat{\mathbf{G}}_2 & 0 & \cdots & \mathbf{I} \\ \vdots & \vdots & \vdots & \ddots & \mathbf{I} \\ 0 & 0 & 0 & \hat{\mathbf{G}}_n & \mathbf{I} \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \mathbf{N} \end{bmatrix} + \varepsilon_\phi \tag{14}$$

where I is $(K-1)\times(K-1)$ identity matrix; $\mathbf{G}_i$ is $(K-1)\times 3$ matrix; $\mathbf{y}_i$ is $(K-1)\times 1$ matrix and n is equal to the number of epoch taken. Finally, the relative position $\hat{\delta\mathbf{x}}$ and the integer $\hat{\mathbf{N}}$ are simultaneously solved by the linear least square estimation:

$$\begin{bmatrix} \hat{\delta\mathbf{x}} \\ \hat{\mathbf{N}} \end{bmatrix} = (H'H)^{-1}H'Y \tag{15}$$

where H is the square matrix in Eq. (14) and Y is the matrix on left-hand side in Eq. (14). The least square solution can be thought as to minimize the cost function:

$$c(\delta\mathbf{x}, \mathbf{N}) = ||\mathbf{y} - \mathbf{G}\delta\mathbf{x} - \mathbf{A}\mathbf{N}||^2 \tag{16}$$

where A is just the stack of $\mathbf{I}$ in H. The float solution $\hat{\mathbf{N}}$ then can be rounded off to obtain the estimated integers. If the float solution appears to be approaching integer values, it would offer an indication that the estimates are good. In reality, convergence of the float solution estimates to integers tends to be slow because measurements taken within several seconds of each other tend to be highly correlated [1], so simply rounding off float estimates can be an error-prone process.

3) LAMBDA method: LAMBDA (Least-squares AMBiguity De-correlation Adjustment method) is a unique method to correctly estimate the value of the integer ambiguities in the double difference measurements. This method is broken in 3 sequential parts: 1) Obtaining float solutions, 2) Search for nearest N, and 3) Obtaining fixed solution ([1], [2], [3], [4]).

The float solution is obtained using the same approach described above, except there is a weighting matrix $\mathbf{W}$ now inside of the cost function in Eq. (16). The revised cost function is:

$$\begin{aligned} c_{\mathbf{W}}(\delta\mathbf{x}, \mathbf{N}) &= ||\mathbf{y} - \mathbf{G}\delta\mathbf{x} - \mathbf{A}\mathbf{N}||^2_{\mathbf{W}} \\ &= (\mathbf{y} - \mathbf{G}\delta\mathbf{x} - \mathbf{A}\mathbf{N})^T\mathbf{W}(\mathbf{y} - \mathbf{G}\delta\mathbf{x} - \mathbf{A}\mathbf{N}) \end{aligned} \tag{17}$$

where $\mathbf{W}$ is equal to the inverse of the noise covariance matrix $\mathbf{\Sigma}_{dd}^{-1}$. The float solution therefore will be solved as a weighted least square estimation. The covariance matrix of a pair of double differences is given by [1]:

$$\mathbf{\Sigma}_{dd} = 2\sigma_\phi^2 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{18}$$

where σ_ϕ is the standard deviation of carrier phase measurement noise ($\sigma(\varepsilon_\phi) = 0.025$ cycles (5mm) [1]). The double differences are correlated even if the original measurements are not. For K satellites, the $(K-1)\times(K-1)$ covariance matrix would have 2's on the main diagonal and 1's in all off-diagonal positions. Least square estimation through Cholesky decomposition would yield the float solution and covariance matrix of integer $\mathbf{\Sigma}_{\hat{\boldsymbol{N}}}$.

Search for the nearest integer is the heart of LAMBDA method. The main goal is to find the value of N which minimizes the cost function:

$$c(\boldsymbol{N}) = (\boldsymbol{N} - \hat{\boldsymbol{N}})^T \mathbf{\Sigma}_{\hat{\boldsymbol{N}}}{}^{-1}(\boldsymbol{N} - \hat{\boldsymbol{N}}) \tag{19}$$

where $\hat{\boldsymbol{N}}$ is the float solution from step 1. This cost function can be thought as a constant-cost ellipsoid search space, and we need to find the integer vectors $\boldsymbol{N}$ to minmize the cost. If the weight matrix $\boldsymbol{W_N}$, which is equal to $\mathbf{\Sigma}_{\hat{\boldsymbol{N}}}{}^{-1}$, is diagonal, then the best estimate of an integer ambiguity is the corresponding float estimate rounded off to the nearest integer. However, $\boldsymbol{W_N}$ is generally not diagonal. This problem is resolved by the means of Z-transformation [4]; the ambiguities are decorrelated prior to the integer estimation. The integer minimization problem is then attacked by a discrete search over an ellipsoidal region.

Intuitively, the shape and orientation of the ellipsoid are governed by the variance covariance matrix of the ambiguities. The decorrelation realizes an ellipsoid that is very much sphere-like. Hence it can searched through very efficiently [4]. Another important aspect of LAMBDA method is the size of the ellipsoid can be controlled prior to the search using the volume function. The volume gives an indication of the number of candidates contained in the ellipsoid [4].

The decorrelation or Z-transformation was done using LDL decomposition (variant of Cholesky Decomposition) [4] in Matlab. The discrete search algorithm FI71 is based on the sequential conditional integer estimation. This search algorithm is list in section 4.5 in [4]. Due to the complexity of this algorithm, some sub-functions listed in [2] were used for writing the search algorithm.

As the search is made on the transformed ambiguities $\boldsymbol{Z}$, a back transformation is needed ($\boldsymbol{N} = \boldsymbol{Z^{-1}M}$). It provides the integer least-square estimate for the vector of original double difference ambiguities $\boldsymbol{N}$. Finally, fixed solution $\delta\mathbf{x}$ can be calculated from the value $\boldsymbol{N}$ using Eq. (12).

III. RESULTS

A relative position estimation was simulated between 2 static locations: Eagan and Apple Valley stations. The simulation was carried out for 40 minutes from the beginning of November 14, 2016 (from 00 hr 00 min 00 sec to 00 hr 40 min 00 sec). The simulation time could have been carried out longer, but it wouldn't give much more useful information. 7 common GPS satellites were observed by Eagan and Apple Valley stations and used for this 40-min duration. They were G1, G3, G10, G12, G14, G22, and G25. Fig. 4 shows the elevation of those 7 satellites. The elevation was deemed to be acceptable since they are all above minimum 5° elevation requirement [1]. Since the G14 has the highest elevation, it was used as reference for the double-difference method to improve accuracy.

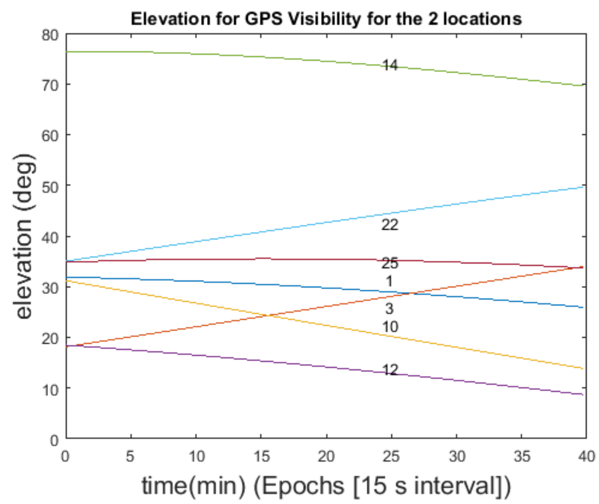

Fig. 4: Elevations of the 7 GPS satellites during 40 mins.

Fig. 5 shows different DOPS for GPS visibility during the 40 minutes period. These DOPS values look reasonable comparing to the typical values [1]. They also don't vary much during the 40 minutes period, which is good for accuracy.

Fig. 6 shows a relative position estimation comparison among 3 different integer ambiguity methods. The black horizontal line is the true relative distance between Eagan and Apple Valley stations. The true relative distance was determined to be 10.43 km as mentioned in section A of Technical Approach. It can be seen that Geometry-Free approach was better than rounded float solution for this case. LAMBDA method gave the best estimation as expected.

The order of accuracy among 3 different integer ambiguity methods was a little bit unexpected; Batch Least Square should provide a better solution than Geometry-Free because it uses all the data in one batch. One possible explanation why Geometry-Free was more accurate in this case was that the code and carrier measurements were very accurate from those 2 static locations. The accuracy of Geometry-Free approach depend on solely on the measurements [1]. The other possible explanation is Batch Least Square estimates were not accurate for this case due to a couple of low elevation GPS satellites (G12 and G10 after the first 25 minutes). Those low

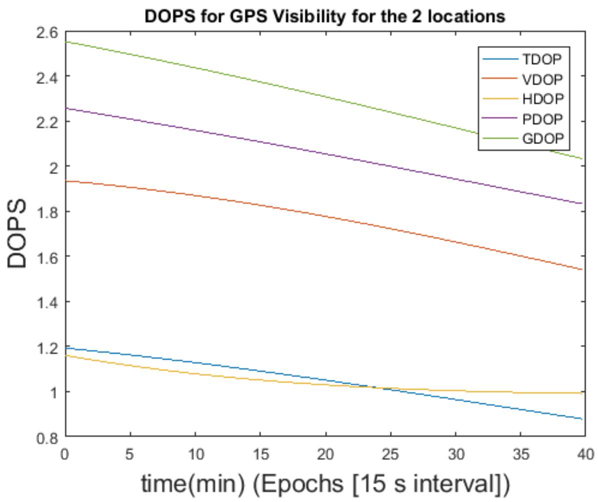

Fig. 5: Different DOPS for GPS Visibility during 40 mins for the 2 chosen locations.

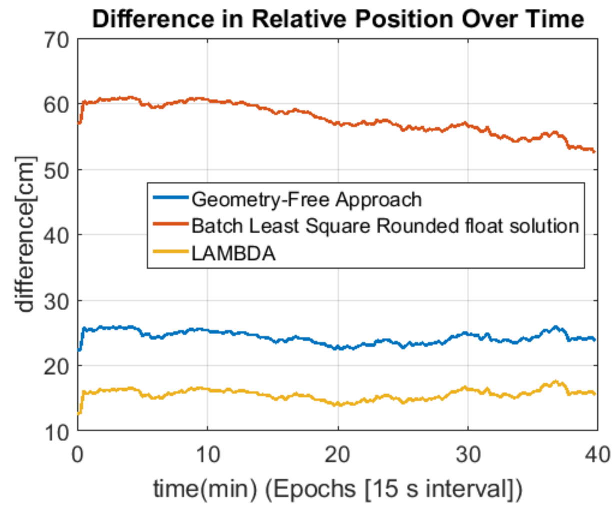

Fig. 7: Difference in Relative Position Over Time Comparison.

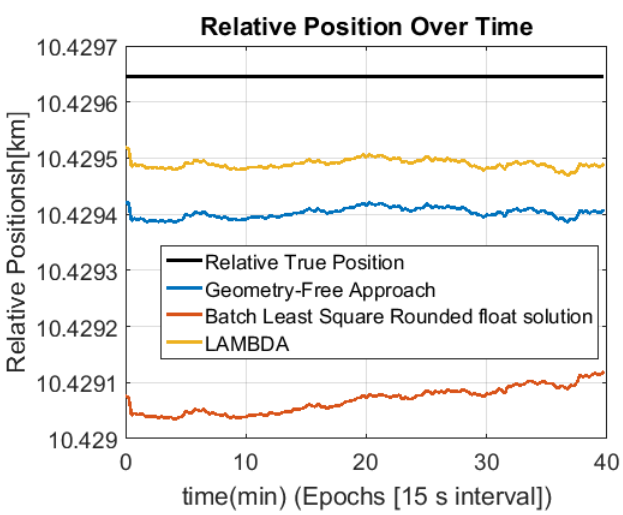

Fig. 6: Relative Position Over Time Comparison.

TABLE II: 6 estimated integer using 3 different methods.

Pair of Satellites	N_{round}	N_{free}	N_{LAMBDA}
1^{st}	6	0	0
2^{nd}	-20	-26	-26
3^{rd}	-7	-1	-1
4^{th}	4	8	7
5^{th}	29	25	25
6^{th}	-14	-13	-13

elevation satellites might have caused the geometry matrix $\boldsymbol{G}$ close to be rank-deficient. Note the LAMBDA solution was generated using the Geometry-Free integer solution instead of the float solution as baseline since it was more accurate for this particular case.

Fig. 7 plots the difference between the true relative position and three different estimated relative positions over time. The average difference between the true and estimated relative positions were 57.67 cm, 24.29 cm and 15.56 cm for Batch Least Square, Geometry-Free and LAMBDA methods respectively.

These numbers are reasonable; they showed centimeter accuracy indeed can be obtained by carrier phase measurements. Note Batch Least Square relative position has the worst performance and seems to drift over time. This bad performance was also reflected from integer calculated from 3 different methods shown in Table II. If N_{LAMBDA} was used to represent the most accuracy integers, we can see N_{round} (calculated simply from rounding off the float solution) are pretty off for the first 5 pairs.

IV. Conclusion

A relative position between two close static stations (the relative distance was about 10.43km) was estimated using carrier phase measurements. The estimation method is called Double-difference method. Double-difference method removes lock bias, ionospheric and tropospheric terms and obtains a linear model that only includes the unknown relative position vector and integer ambiguities. Integer ambiguities can be solved before estimating the relative position or simultaneously with the relative position. Three different integer ambiguities methods were presented and used to resolve the integers: Geometry-Free Approach, Batch Least Square rounded float solution and LAMBDA method. LAMBDA method was found to be the most accurate method for resolving the integers. The simulation obtained a 15.56 cm difference between the true relative position and the average estimated relative position using LAMBDA method. This simulation indeed showed centimeter-level accuracy can be obtained using carrier phase measurements. Note the estimated relative position from Batch Least Square was not accurate as expected. The possible explanation was there were a couple of low elevation GPS satellites during the 40 minute segment.

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