AEM 8423 Final Project Report

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Abstract

This final report presents an implementation of uncertainty modeling using convex optimization techniques for an Ultrastick25e UAV aircraft from the AEM UAV lab. Specifically, a numerical algorithm was implemented in Matlab utilizing convex optimization to create an additive uncertainty model for a longitudinal short period linear dynamic model. The algorithm was implemented in the frequency domain, and also discusses the feasibility of uncertainty modeling in the time domain. The simulation showed that using convex optimization can provide a lower uncertainty weight than the average for the nominal model. Although uncertainty modeling in the time domain is deemed to be unnecessary, we can validate the uncertainty model in the time domain using convex optimization methods.

1 Introduction

1.1 Uncertainty Modeling in the Frequency Domain

Modeling uncertainty using convex optimization in the frequency domain has been used for many applications such as aircrafts and hard disk drives [1, 2]. Traditionally in the frequency domain, a collection of frequency responses will be generated from input/output experiments over a common grid of frequencies. The input will be a basic sinusoidal frequency sweep. The uncertainty set should cover the collection of experimental data for each frequency response. This project will focus on only one type of uncertainty sets: additive input uncertainty. The input uncertainty set defined from Eq. 1 in [1] as the following,

$$S_A := \{G_0 + W_L \Delta W_R : \Delta \in \mathbb{RL}_\infty{}^{n\times m}\} \tag{1}$$

The objective is to find the smallest set that covers the collection of experimental data shown in the Eq. 2 from [1].

$$\begin{gathered}\min_{G_0, W_L, W_R} \int_0^\infty h(W_L, W_R, \omega) d\omega \\ subject\ \ to: S_A\ \ covers\ \ \{G_k\}_{k=1,\dots,K}\end{gathered} \tag{2}$$

where the function h is a measure for the size of the uncertainty set at the frequency ω shown in Eq. 3. The Γ in Eq. 3 can be chosen to be identity matrix I.

$$h(W_L, W_R, \omega) := Tr[W_L(j\omega)^* \Gamma_L W_L(j\omega)] + Tr[W_R(j\omega)\Gamma_R W_R(j\omega)^*] \tag{3}$$

Eq. 2 can be turned into a finite-dimensional SDP convex optimization problem shown in Eq. 4.

$$\min_{G_0,L,R} \sum_{f=1}^{F} Tr[\Gamma_L L + \Gamma_R R](j\omega_f)$$
$$subject \;\; to:$$
$$\begin{bmatrix} L & G_k - G_0 \\ (G_k - G_0)^* & R \end{bmatrix} (j\omega_f) \geq 0 \tag{4}$$
$$f = 1, ..., F \;\; and \;\; k = 1, ..., K$$

where the new SDP variables $L = W_L W_L^*$ and $R = W_R^* W_R$. The decision variables L, R, and G_0 in Eq. 4 are complex at each frequency gridpoint. This optimization is therefore decoupled into F smaller SDP problems; one for each frequency gridpoint.

1.2 Longitudinal Motion Mathematical Model

A stable longitudinal short period model was investigated. The linear state-space approximation for these dynamics and measurements is

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -\frac{\bar{q}_o S}{m V_o} C_{L_\alpha} & 1 - \frac{\bar{q}_o S \bar{c}}{2 m {V_o}^2} C_{L_q} \\ \frac{\bar{q}_o S \bar{c}}{I_y} C_{M_\alpha} & \frac{\bar{q}_o S \bar{c}^2}{2 V_o I_y} C_{M_q} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -\frac{\bar{q}_o S}{m V_o} C_{L_{\delta_e}} \\ \frac{\bar{q}_o S \bar{c}}{I_y} C_{M_{\delta_e}} \end{bmatrix} \delta_e \tag{5}$$

$$\begin{bmatrix} \alpha \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \delta_e \tag{6}$$

Airspeed, dynamic pressure, and mass properties inside the matrices are constant nominal values at the initial trim condition. The aerodynamic parameters are set to be the uncertain variables. They are listed in the Table 1. We will look at how those aerodynamic parameter uncertainties affect the dynamic states angle of attack α and pitch rate q, and how we model an additive uncertainty for each state. The uncertainty models can be useful when creating a robust controller.

Table 1: Parameter Values for the Ultrastick25e.

Parameter	True Value	Uncertainty Bound
C_{L_α}	4.5800	[3.5800, 5.5800]
C_{L_q}	6.1639	[4.1639, 8.1639]
$C_{L_{\delta e}}$	0.0983	[0.0583, 0.1383]
C_{M_α}	-0.7230	[-1.2230, -0.2230]
C_{M_q}	-13.5664	[-16.5664, -10.5664]
$C_{M_{\delta e}}$	-0.8488	[-1.0488, -0.6488]

2 Simulation

A simulation was carried out in the frequency domain using the numerical algorithm outlined in the Section III of [1]. All the SDPs were solved using freely available CVX Matlab toolbox. The linearized nominal plant matrices A, B, C, D came from the Eq. 5 and 6. L and R in Eq. 4 are the optimization variables. R was set to be 1 since the simulation was treated as a SISO system for each state.

Experimental data $\{G_k\}_{k=1}^{K}$ was simulated by taking $K = 20$ randoms samples of the 6 aerodynamic parameters with a frequency containing 500 frequency points from $\omega_1 = 10^{-1}$ rad/sec to $\omega_{500} = 10^2$ rad/sec.

Fig. 1 upper plot shows the 20 random nominal model (blue dotted lines) for angle of attack α as well as the two nominal models created by the LMI constraints (green) and the average (red) of the 20 models respectively. The lower plot shows the smallest additive uncertainty weight required to cover all of the experimental data using the LMI and the average. You can see the the uncertainty weight for the nominal model created by the optimization method is slightly lower than the uncertainty weight for the average system, which indicates the convex optimization method provides a better uncertainty modeling. Notice the difference only comes in at the lower frequency range, concluding that the angle of attack does not change much when the elevator surface is moving very fast.

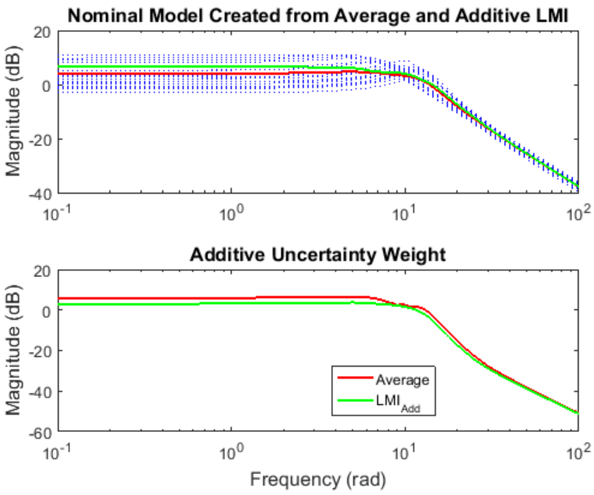

Figure 1: The upper plot is the Nominal Model, $G_0(j\omega)$ for the output α from the additive uncertainty optimization method and averaging of the simulated data. The lower plot is the smallest additive uncertainty weight, W_L to cover all of the experimental data.

Fig. 2 shows the similar results for the pitch rate q. Again, the uncertainty modeled by convex optimization is better than the average. Notice the magnitude of q varies a lot with uncertainty in the parameters in comparison to α at the lower frequency range. This is expected since the pitch rate usually has a faster rate than angle of attack and the parameters in q is much larger (see C_{L_q} and C_{M_q} value in Table 1).

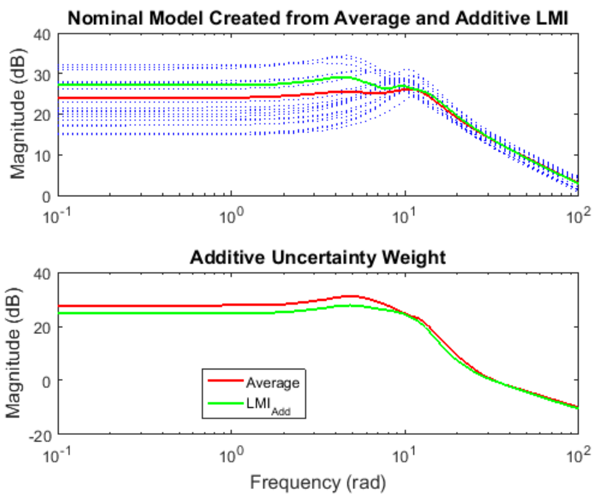

Figure 2: The upper plot is the Nominal Model, $G_0(j\omega)$ for the output q from the additive uncertainty optimization method and averaging of the simulated data. The lower plot is the smallest additive uncertainty weight, W_L to cover all of the experimental data.

For a SISO system, the MLI contraints find a complex number at each frequency gridpoint that will require a circle with smallest radius to cover all the data points. This is demonstrated in Fig. 3, which shows the a scatter plot of the simulated data, two nominal data points and the smallest circle centered at each of the nominal data points required to cover all the data points at $\omega = 290$ rad/sec. The nominal model derived from the LMI constraint requires an uncertainty weight with smaller magnitude (smaller red dotted circle) to cover the frequency response data, which result in a controller with better performance.

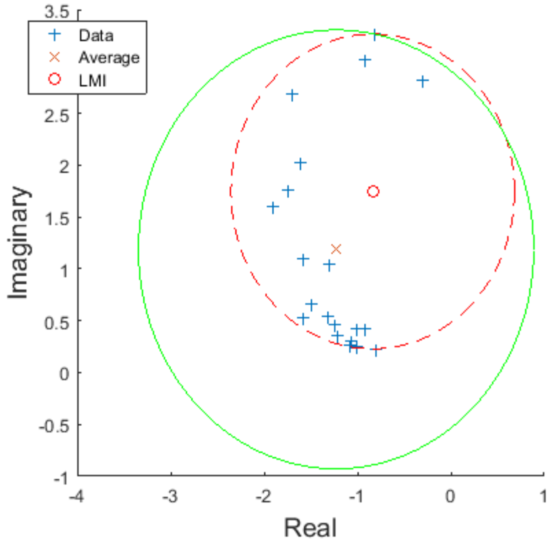

Figure 3: The data points and nominal points created by LMI method and Average method for α at $\omega = 290$ rad/sec.

3 Theoretical Discussion

Although there exists approaches to construct uncertainty models from time domain data in the literature (see [**?**, 3]), there isn't uncertainty modeling in the time domain based on my literature review. I have concluded a couple of possible reasons. First, uncertainty usually is expressed in the form of state space or transfer function, and it is analyzed in the frequency domain to emphasize specific frequency bands. For example, if we have a 5Hz actuator, we would like to know if the uncertainty model is satisfied at 5Hz. Second, it is not clear how we can convert the optimization problem in Eq. 2 into a time domain integral. Looking at Eq. 3, one can see the Trace of the weighting function would not have any physical meaning if it's presented in the time domain.

However, there is literature about validating uncertainty models in a time-domain approach using convex optimization technique [4]. Consider the same additive dynamic uncertainty with the perfect measurement in the following form:

$$y = G_0 u + W_L \Delta W_R u \quad with \quad \Delta \in \mathbb{RL_\infty}^{n\times m} \tag{7}$$

Here, W_L and W_R are stable and stably invertible weighting functions that are same as the ones in Eq. 1. In the [4] we have the following Theorem:

Theorem: consider the uncertainty model in Eq. 7, and suppose the applied input is $u = (u_0, u_1, \cdots, u_{l-1})$; $u_i \in \mathbb{R}^m$ with $u_0 \neq 0$ and the observed output is $y = (y_0, y_1, \cdots, y_{l-1})$; $y_i \in \mathbb{R}^n$. Define the sequences

$$\begin{aligned} \hat{u} = (\hat{u}_0, \hat{u}_1, \cdots, \hat{u}_{l-1}) = \pi_l W(u) \\ \hat{y} = (\hat{y}_0, \hat{y}_1, \cdots, \hat{y}_{l-1}) = y - \pi_l G_0(u) \end{aligned} \tag{8}$$

Suppose $\Delta = \Delta_{LTI-2}$. Then, the above uncertainty model is not invalidated by the observed input-output data if and only if

$$\bar{\sigma}[\hat{Y}(\hat{U}^T\hat{U})^{-\frac{1}{2}}] \leq \gamma \tag{9}$$

where $\hat{U}$ and $\hat{Y}$ are associated block Toeplitz matrices from from $\hat{u}$ and $\hat{y}$ respectively.

This theorem shows that we can check whether the uncertainty model is valid or not. If the uncertainty model is valid, it would also provide an upper bound for the input and output relation. Note this method can only test if the model is **invalidated** by the input-output data. In other words, if a particular uncertainty model is consistent with the data, we can only conclude the model is not contradicted by the given data. This does not mean that the model is a correct description of the physical system.

4 Conclusion

Uncertainty modeling in the frequency domain with convex optimization techniques was implemented using existing algorithm. Simulation results showed that the convex optimization method is very useful when we are looking for an improved uncertainty bound. Even though the computational time greatly increased in Matlab, it is still acceptable, given fast processors in the modern computers. Uncertainty modeling in the time domain was concluded to be not necessary. Rather, it is applicable to do a model validation in the time domain to see if the uncertainty model is consistent with the data.

References

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