Reliable Air Data Solutions for Small Unmanned Aircraft Systems

Final Oral Exam

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ACKNOWLEDGEMENTS

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COLLEAGUES AND FRIENDS

FAMILY

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- MN DRIVE Initiative
- Sentera LLC
- Legislature of the State of Minnesota

ROAD TO FINAL GOAL

FINAL GOAL

Air Data System for UAS

- **Fault Tolerant**
- **Safe and Reliable**
- **Certifiable**

Air Data Software

- Calibration (Chapter 3-4)
- Air Data Estimation (Chapter 5)
- Fault Detection & Isolation (Chapter 6)

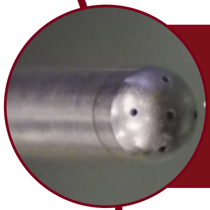
Requirements

- Accuracy
- Continuity
- Integrity
- Availability

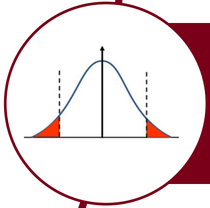
Reliable Air Data Solution for Small Unmanned Aircraft Systems (UAS)

Overview

Introduction (Chapter 1-2)



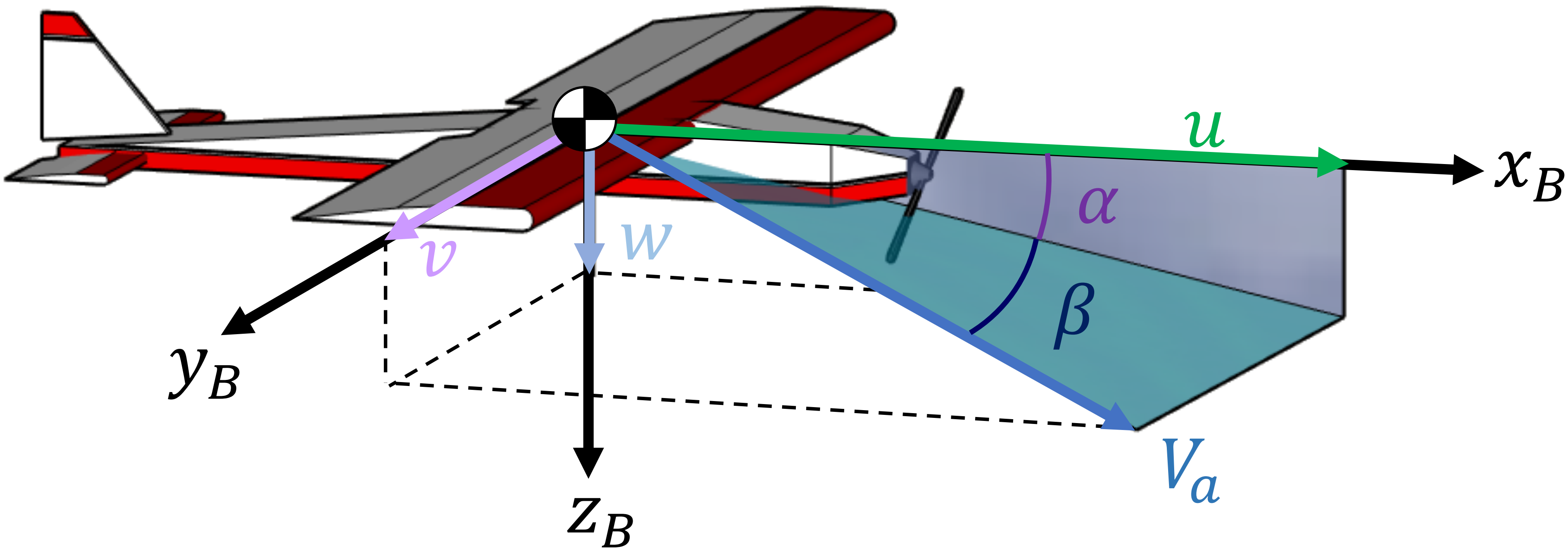
Air Data Calibration (Chapter 3-4)



Air Data Fault Detection and Isolation (Chapter 6)

Conclusion and Future Outlook (Chapter 7)

AIR DATA SYSTEM (ADS)



V_a: airspeed
α: angle of attack
β: angle of sideslip

ADS provide measurements needed for the **safe** operation of an aircraft.

Air Data System Requirements Examples

	Commercial Aircraft (Part 25, FAR)	Unmanned Air Carrier (Part 135)
Sensor	**# of Sensor Requirement**	
Pitot Static System	≥ 2	1
Angle Vane System	≥ 2	Optional
Parameter	**Accuracy Requirement**	
Airspeed V_a	±4 knots (V_a > 100 knots)	5% or ±10 knots
Angle-of-Attack (AoA)	±0.25 deg	±2 deg
Sideslip	±0.25 deg	5%
Sensor	**Functional Requirement**	
Single AoA Sensor	$< 10^{-3}$ failure/hr	Unspecified
Combined 2 AoA sensors	$< 10^{-7}$ failure/hr	Unspecified

FAR: Federal Aviation Regulation
Part 25: Airworthiness Standards: Transport Category Airplanes
Part 135: Air Carrier and Operator Certification

Motivation: Alternative Strategies for ADS

Hardware Redundancy (HR)

Boeing 787

Analytical Redundancy (AR)

Ultra Stick 120

Issues with Hardware Redundancy

- Expensive, Common Mode Failure
- Size, Weight, and Power (SWAP) Limitations

Solution: Analytical Redundancy (AR)

Analytically provide synthetic quantities via mathematical models and indirect sensing information

KEY QUESTIONS

Boeing 787

Ultra Stick 120

1. **Can analytical redundancy be used to provide alternative reliable estimates of air data?**
   - In-situ Calibration [1] [2]
   - Model-free Synthetic Air Data System [3][4]

2. **Can analytical methods be used to improve safety from requirement point of view?**
   - Air Data fault detection and Isolation [5][6]

[1] Sun, K and Gebre-Egziabher, D., "A Two-Stage Batch Algorithm for Nonlinear Static Parameter Estimation", AIAA Journal of GCD, 2020
[2] Sun, K., Regan, C. D., and Gebre-Egziabher, D., "A GNSS/IMU-Based 5-Hole Pitot Tube Calibration Algorithm", AIAA Scitech, 2019
[3] Sun, K., Regan, C. D., and Gebre-Egziabher, D., "Observability and Performance Analysis of Model-Free Synthetic Air Data Estimators", Journal of Aircraft, 2019
[4] Sun, K., Regan, C. D., and Gebre-Egziabher, D., "GNSS/INS Based Estimation of Air Data and Wind Vector using Flight Maneuvers," IEEE/ION PLANS, 2018
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OVERVIEW

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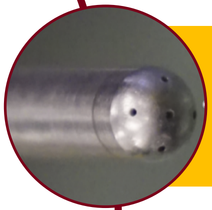
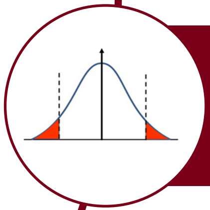


Air Data Calibration (Chapter 3-4)



Air Data Fault Detection and Isolation (Chapter 6)

Conclusion and Future Outlook (Chapter 7)

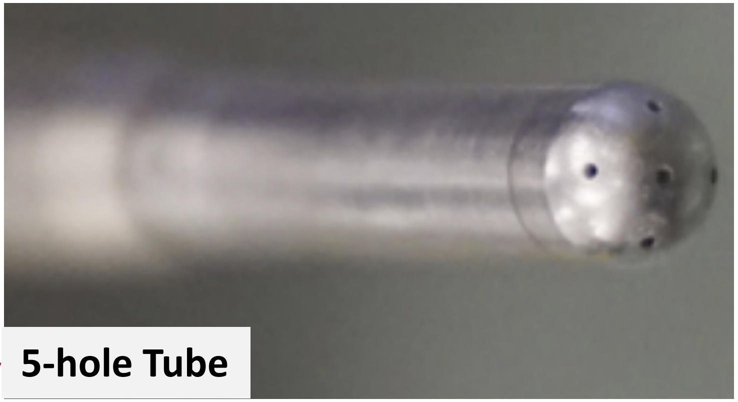
Air Data Calibration

Objective

Air data sensors on small UAS need to be calibrated in-situ to account for installation error, upwash, and wind effect

Challenges

- No direct ground true reference
- Dominating wind
- Sensor noise



5-hole Tube

Pitot Static Tube

Angle Vane

Ultra Stick 120e Testbed

Air Data Calibration Methods

Existing Method	Requirement	Advantage	Disadvantage	Examples
Wind tunnel testing	Wind tunnel	Accurate at the sensor level	Inaccessible; Installation error	N/A
In-flight Calibration	Air data Nose Boom (true reference)	Account for all errors (static and time-varying)	Insufficient data; Developed for large sized aircraft	Kalman filtering; Recursive Least Square (RLS)
Post-flight Calibration	Air data Nose Boom (true reference)	Can process large amount of data	Periodic update; Developed for large sized aircraft	Least Square (LS); Maximum Likelihood Estimation (MLE)

Main Issue

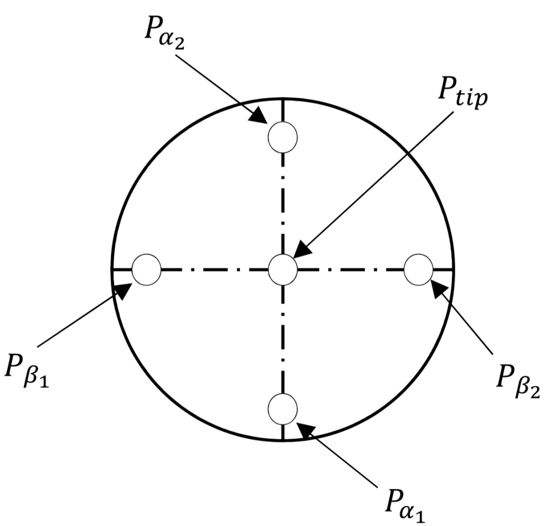
	Traditional Aircraft	Small UAV
Wind effect	10 % of airspeed	Up to **50%** of airspeed

How do we calibrate air data sensors that can accommodate for wind?

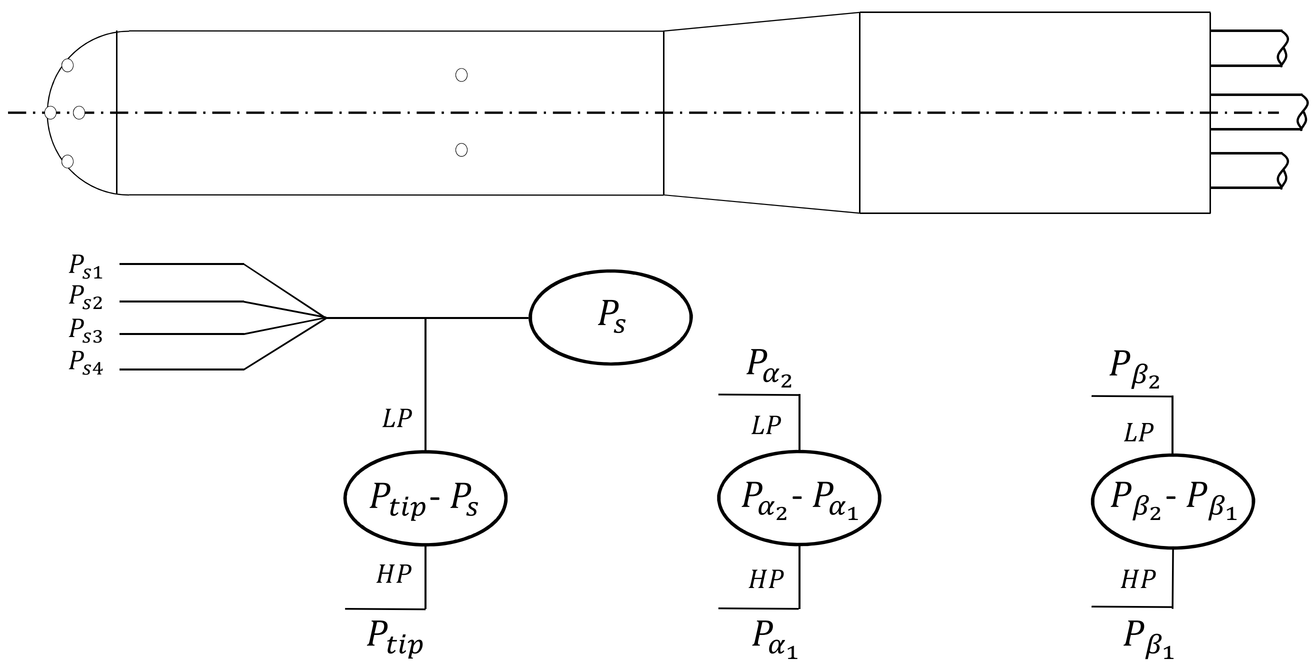
Air Data Sensor - Five Hole Pitot Tube

- 5-Hole probe provides (α, β, V_a)
- Post-flight calibration using GPS, IMUs and wind triangle kinematic equation

Pressure reading Configuration



Collins Aerospace Mini 5-hole probe



Collins Aerospace

Initial Calibration Attempt

Step 1: Wind tunnel testing $\alpha = \frac{P_{\alpha 1} - P_{\alpha 2}}{K_\alpha (P_t - P_s)}, \beta = \frac{P_{\beta 1} - P_{\beta 2}}{K_\beta (P_t - P_s)}$

Sensitivity Coefficient	Wind Tunnel Testing	Manufacturer
K_α	0.0825	0.079
K_β	0.0853	0.079

Step 2: Measurement modeling

$$V_a = (1 + \lambda_{V_a}) \sqrt{\frac{2(P_t - P_s)}{\rho}} + b_{V_a}$$

$$\alpha = (1 + \lambda_\alpha) \frac{P_{\alpha 1} - P_{\alpha 2}}{K_\alpha (P_t - P_s)} + b_\alpha$$

$$\beta = (1 + \lambda_\beta) \frac{P_{\beta 1} - P_{\beta 2}}{K_\beta (P_t - P_s)} + b_\beta$$

$$\Rightarrow \quad V_{a,s} = \begin{bmatrix} V_a \cos\alpha \cos\beta \\ V_a \sin\beta \\ V_a \sin\alpha \cos\beta \end{bmatrix}$$

$$V^n = C_b^n V_{a,cg} + W^n = C_b^n \left[C(\epsilon_\phi) V_{a,s} - [\omega]_\times \boldsymbol{r} \right] + W^n$$

Step 3: Solve the unknown through nonlinear parameter optimization

Inconsistent, non-sensical solution!

Proposed: Two Stage Estimator

Measurement Model

$$\boldsymbol{z_k} = \boldsymbol{h}_k(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{\xi}) + \boldsymbol{v_k}$$

$$\boldsymbol{z} = \boldsymbol{C}\boldsymbol{y} + \boldsymbol{b} + \boldsymbol{v}$$

$$\boldsymbol{z_k} = \boldsymbol{A}(\boldsymbol{\xi}_2)\boldsymbol{\xi}_1 + \boldsymbol{b}(\boldsymbol{\xi}_2) + \boldsymbol{v_k}$$

Flowchart

Stage 1

Linear Least Square

$$\arg min_{\xi_{1p}} \frac{1}{2}\sum_{k=1}^{N} \|\mathbf{z}_k - \mathbf{A}(\xi_{2p})\xi_{1p} - \mathbf{b}(\xi_{2p})\|_{\mathbf{I}}^2$$

Stage 2

Nonlinear Optimization

$$\arg min_{\xi_2, R} \frac{1}{2}\sum_{k=1}^{N} \|\mathbf{z}_k - \mathbf{A}(\xi_2)\xi_{1p} - \mathbf{b}(\xi_2)\|_{\mathbf{R}}^2 + \frac{N}{2}\ln|\mathbf{R}|$$

$$\xi_1^*, \xi_2^*, \mathbf{R}$$

Advantages

1. Canonical Form (e.g., Mag. Calibration, Data Compatibility)
2. Systematic Approach

Two Stage Estimator - Formulation

Measurement Model

$$z_k = h_k(x_k, u_k, \xi) + v_k$$

How do you estimate ξ?

Canonical Form

$$z_k = A(\xi_2)\xi_1 + b(\xi_2) + v_k$$

$$\mathrm{E}\{v_k\} = 0, \mathrm{E}\{v_k v_k^T\} = \mathbf{R}$$

Cost Function

$$\xi^* = \arg\min_{\xi \in \xi_{limit}} J(\xi)$$

$$J(\xi) = J(\xi_1, \xi_2) = \frac{1}{2}\sum_{k=1}^{N} \|\mathbf{z}_k - \mathbf{A}(\xi_2)\xi_1 - \mathbf{b}(\xi_2)\|_{\mathbf{R}}^2 + \frac{N}{2}\ln|\mathbf{R}|$$

Two Stage Estimator – Procedure (1 of 2)

Objective

$$\arg min_{\xi_1,\xi_2} \ \frac{1}{2}\sum_{k=1}^{N}\|\mathbf{z}_k - \mathbf{A}(\boldsymbol{\xi}_2)\boldsymbol{\xi}_1 - \mathbf{b}(\boldsymbol{\xi}_2)\|_{\mathbf{R}}^2 + \frac{N}{2}\ln|\mathbf{R}|$$

First Stage (linear least square)

Solve for $\boldsymbol{\xi}_{1p}$

$$\arg min_{\xi_{1p}} \ \frac{1}{2}\sum_{k=1}^{N}\left\|\mathbf{z}_k - \mathbf{A}(\boldsymbol{\xi}_{2p})\boldsymbol{\xi}_{1p} - \mathbf{b}(\boldsymbol{\xi}_{2p})\right\|_{\mathbf{I}}^2$$

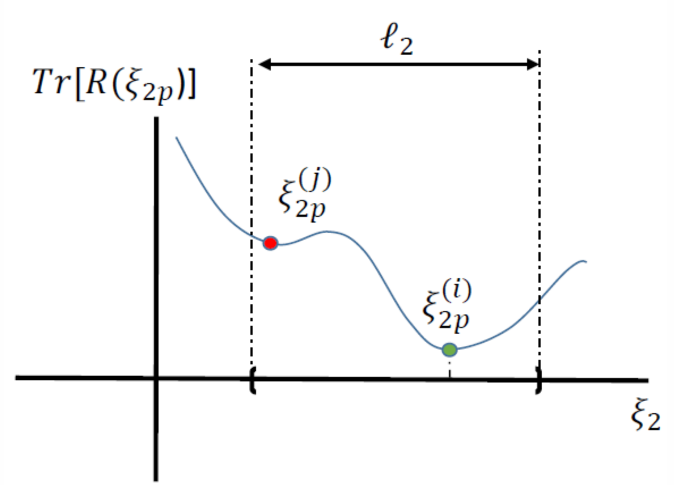
For this, an appropriate $\boldsymbol{\xi}_{2p}$ is selected

$$\arg\min_{\xi_{2p}} \mathrm{Tr}\left[\mathbf{R}(\xi_{2p})\right] = \arg\min_{\xi_{2p}} \sum_{k=1}^{N} \mathrm{Tr}\left(\boldsymbol{v}_k \boldsymbol{v}_k^T\right)$$

where $\boldsymbol{v}_k = \mathbf{z}_k - \mathbf{A}(\xi_{2p})\hat{\xi}_{1p} - \mathbf{b}(\xi_{2p})$

and $\hat{\xi}_{1p} = (\mathcal{A}^T\mathcal{A})^{-1}\mathcal{A}^T(\mathbf{Z} - \mathcal{B})$

for $\xi_{2p} \in S_{\xi_2}$



Two Stage Estimator – Procedure (2 of 2)

Objective

$$\arg min_{\xi_1,\xi_2} \ \frac{1}{2}\sum_{k=1}^{N}\|\mathbf{z}_k - \mathbf{A}(\boldsymbol{\xi}_2)\boldsymbol{\xi}_1 - \mathbf{b}(\boldsymbol{\xi}_2)\|_{\mathbf{R}}^2 + \frac{N}{2}\ln|\mathbf{R}|$$

Second Stage (nonlinear optimization)

Use $\boldsymbol{\xi}_{1p}$, $\boldsymbol{\xi}_{2p}$, $\boldsymbol{R}(\boldsymbol{\xi}_{2p})$ as initial conditions to solve for $\boldsymbol{\xi}_2^*$ and $\mathbf{R}$

$$\arg min_{\xi_2,\boldsymbol{R}} \ \frac{1}{2}\sum_{k=1}^{N}\left\|\mathbf{z}_k - \mathbf{A}(\boldsymbol{\xi}_2)\boldsymbol{\xi}_{1p} - \mathbf{b}(\boldsymbol{\xi}_2)\right\|_{\mathbf{R}}^2 + \frac{N}{2}\ln|\mathbf{R}|$$

Solve $\boldsymbol{\xi}_1^*$ using weighted linear least squares

Two Stage Estimator - Properties

1. Bounded from **above and below**:

$$J(\xi_1^*, \xi_{2p}) - E \le J(\xi_1^*, \xi_2^*) \le J(\xi_1^*, \xi_{2p})$$

2. ***E*** is upper **bounded** and function of $\boldsymbol{\xi_{2p}}$:

$$E = \max_{\xi_1,\xi_2} \frac{1}{2}\sum_{k=1}^{N}\left(\left\|\left[\mathbf{A}(\xi_2) - \mathbf{A}\left(\xi_{2p}\right)\right]\xi_1\right\|^2 + \left\|\mathbf{b}(\xi_2) - \mathbf{b}\left(\xi_{2p}\right)\right\|^2\right)$$

$$= \frac{1}{2}\sum_{k=1}^{N}\left(\max_{\xi_1,\xi_2}\left\|\left[\mathbf{A}(\xi_2) - \mathbf{A}\left(\xi_{2p}\right)\right]\xi_1\right\|\right)^2 + \frac{1}{2}\sum_{k=1}^{N}\left(\max_{\xi_1,\xi_2}\left\|\mathbf{b}(\xi_2) - \mathbf{b}\left(\xi_{2p}\right)\right\|\right)^2$$

$$= \frac{1}{2}\sum_{k=1}^{N}(E_1^2 + E_2^2) \le \frac{N}{2}(L_{\boldsymbol{A}}^2\ell_2^2\ell_1^2 + L_{\boldsymbol{b}}^2\ell_2^2) = \frac{N}{2}\ell_2^2(L_{\boldsymbol{A}}^2\ell_1^2 + L_{\boldsymbol{b}}^2)$$

$$E \le \frac{N}{2}\ell_2^2\left(L_A^2\ell_1^2 + L_b^2\right)$$

where

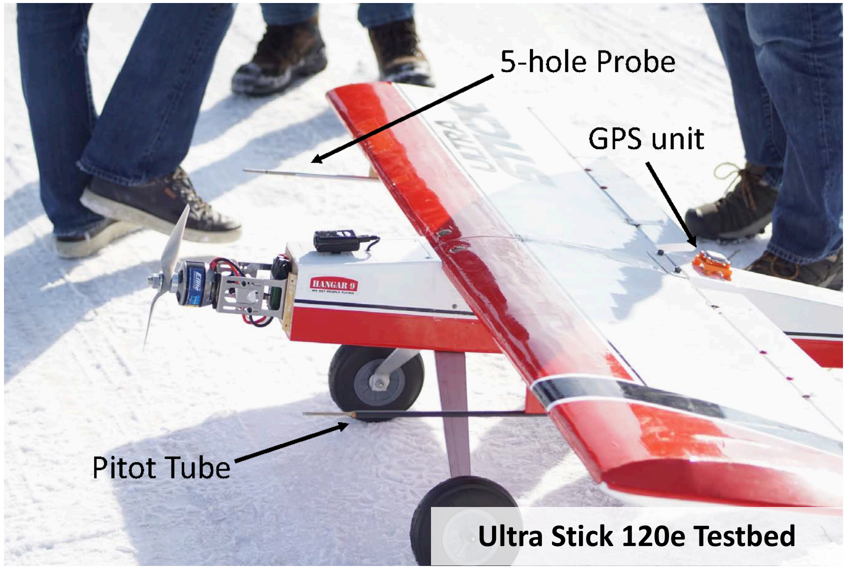
$$E_1 \triangleq \max_{\xi_1,\xi_2}\left\|\left[\mathbf{A}(\xi_2) - \mathbf{A}\left(\xi_{2p}\right)\right]\xi_1\right\| \le \max_{\xi_1,\xi_2}\left\|\mathbf{A}(\xi_2) - \mathbf{A}\left(\xi_{2p}\right)\right\|\|\xi_1\|$$

$$\le \max_{\xi_2}\left\|\mathbf{A}(\xi_2) - \mathbf{A}\left(\xi_{2p}\right)\right\|\ell_1 \le L_{\boldsymbol{A}}\left\|\xi_2^* - \xi_{2p}\right\|\ell_1$$

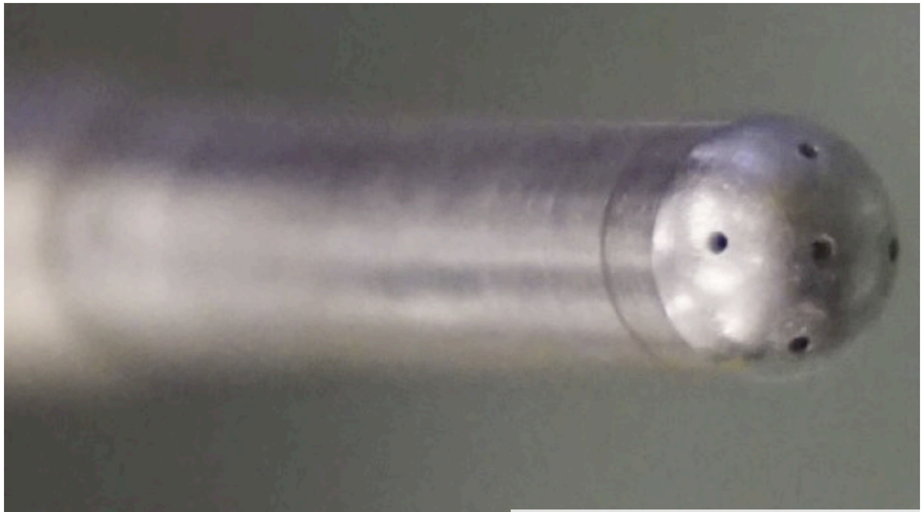
$$\le L_{\boldsymbol{A}}\ell_2\ell_1$$

$$E_2 \triangleq \max_{\xi_1,\xi_2}\left\|\mathbf{b}(\xi_2) - \mathbf{b}\left(\xi_{2p}\right)\right\| \le L_{\boldsymbol{b}}\left\|\xi_2^* - \xi_{2p}\right\| \le L_{\boldsymbol{b}}\ell_2$$

Flight Test Example - 5 Hole Air Data Calibration



Ultra Stick 120e Testbed

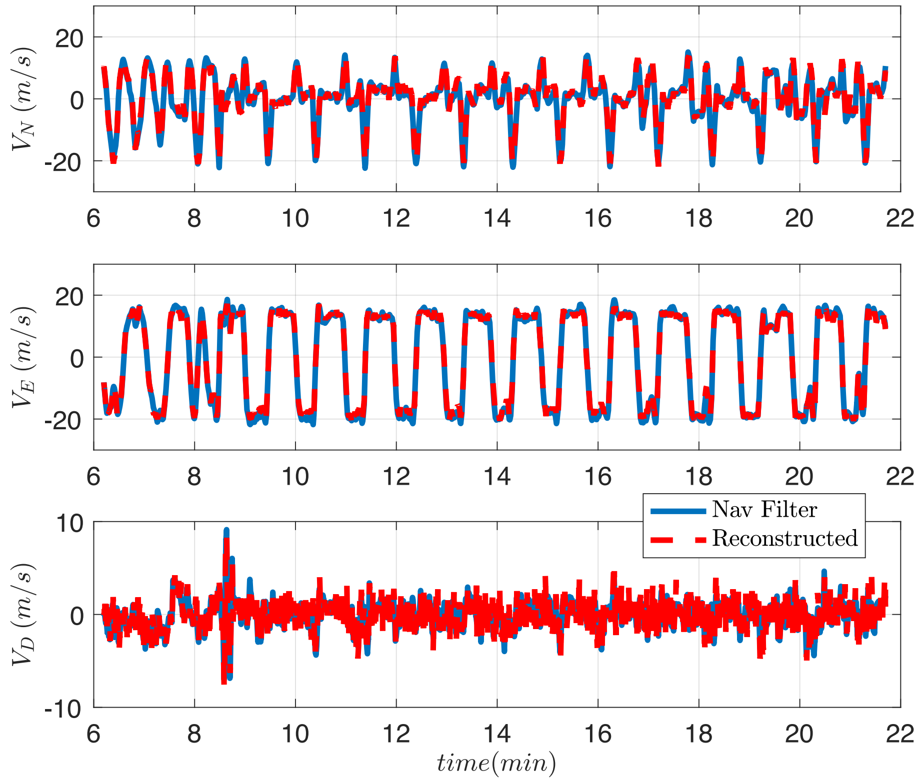


5 Hole Probe

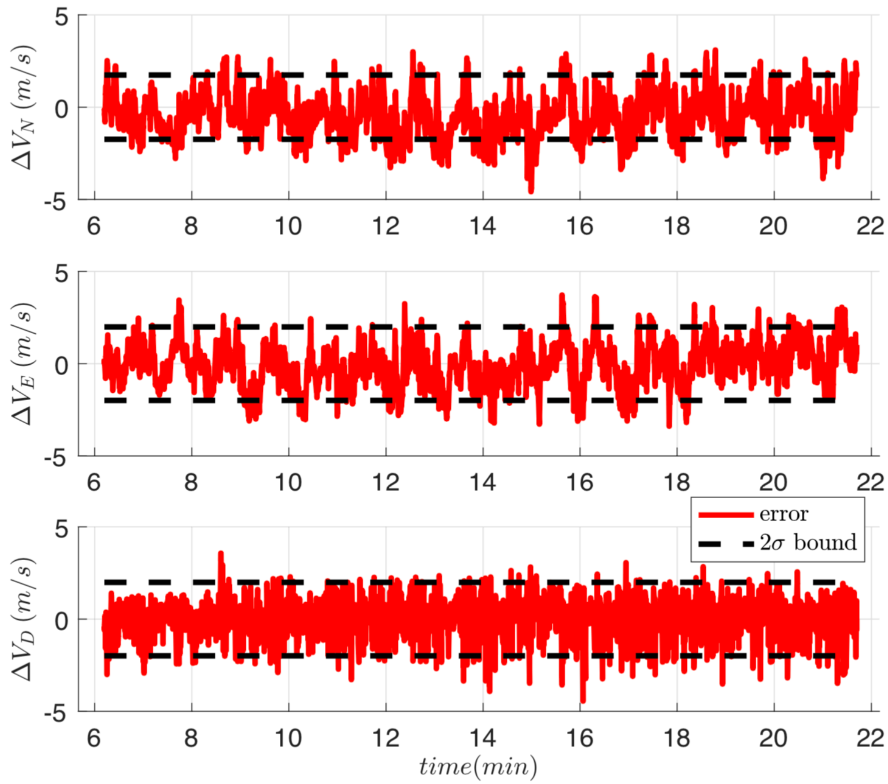
State	$\mathbf{x} = [p \quad q \quad r \quad b_{g_x} \quad b_{g_y} \quad b_{g_z} \quad \phi \quad \theta \quad \psi]^T$
Input	$\mathbf{u} = [P_{\Delta_\alpha} \quad P_{\Delta_\beta} \quad P_t \quad P_s]^T$
Output	$\mathbf{z} = [V_N \quad V_E \quad V_D]^T$
Estimated Parameters	$\boldsymbol{\xi} = [\lambda_{V_a} \quad b_{V_a} \quad \lambda_\alpha \quad b_\alpha \quad \lambda_\beta \quad b_\beta \quad \epsilon_\phi \quad W_N \quad W_E \quad W_D]^T$
Input Design for Observability	Wind circle, chirps, doublet, multi-sine, pushover-pullup

Flight Test Example - Results

Reconstructed vs. Navigation Solution



Error and its 2 sigma bounds

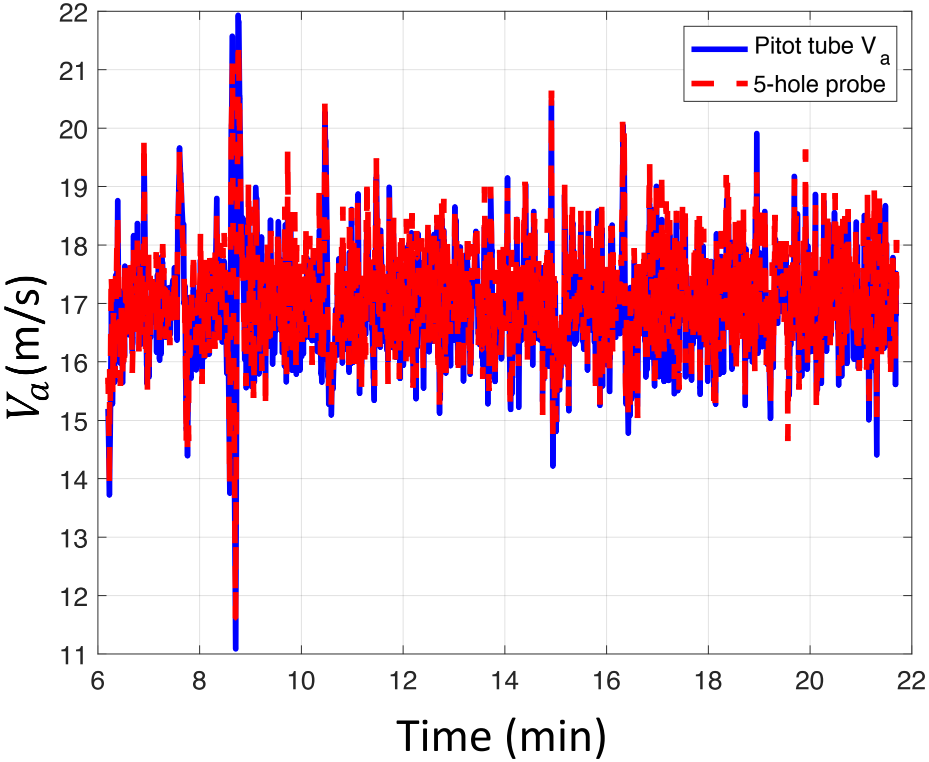


	V_N	V_E	V_D
Root Mean Square Error (m/s)	1.2564	1.1028	0.8321

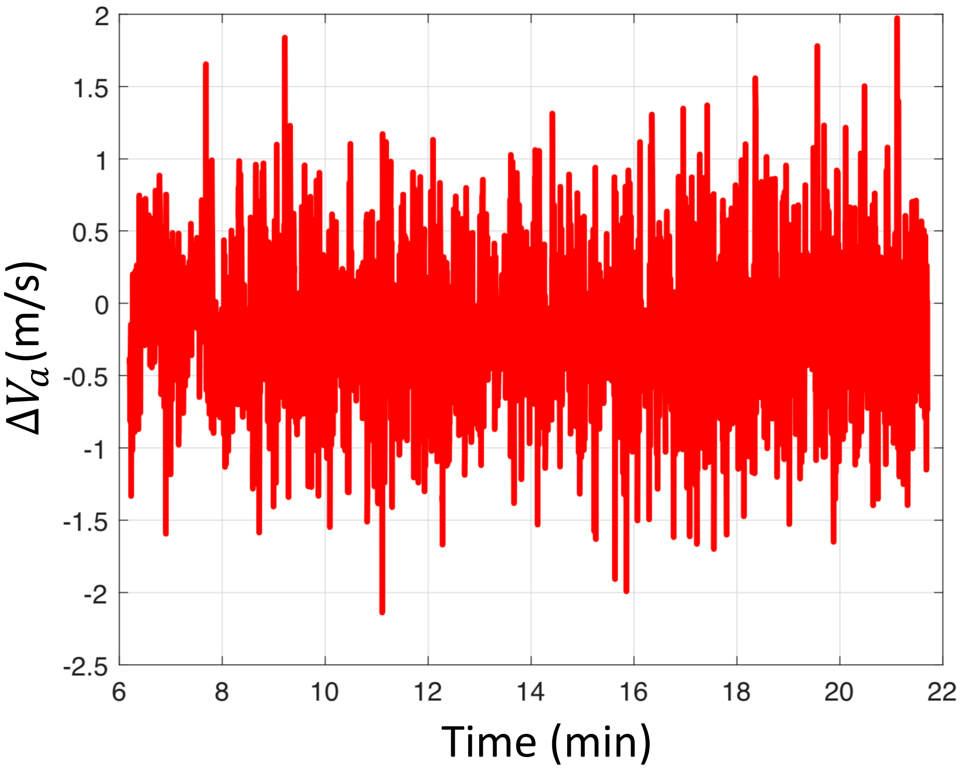
Inertial velocity error is small and bounded

Flight Test Example - Results

Pitot Tube vs. 5-hole probe



Error between Pitot Tube vs. 5-hole probe



Calibration results are compatible with the Pitot tube

OVERVIEW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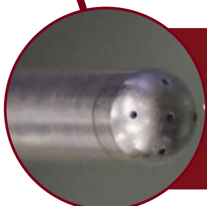
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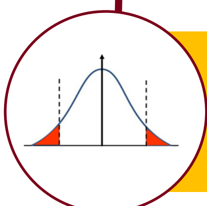
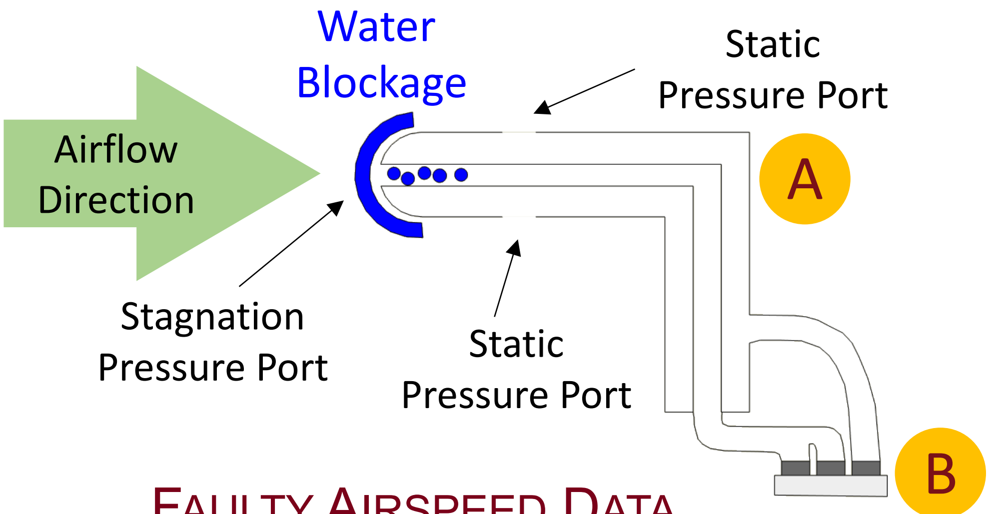
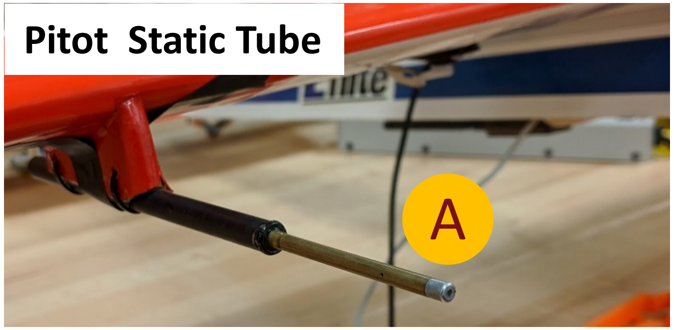

Air Data Fault Detection and Isolation (Chapter 6)

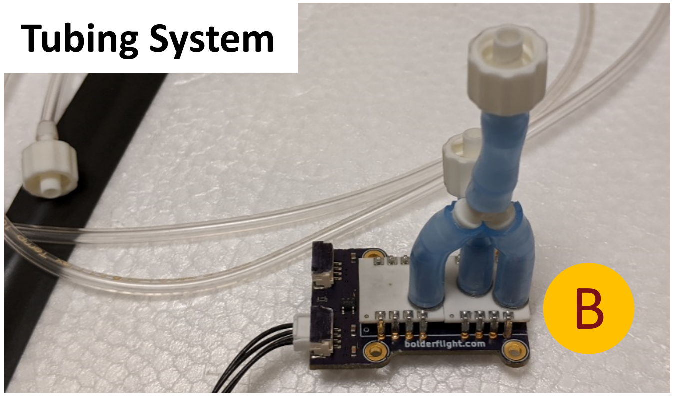

Conclusion and Future Outlook (Chapter 7)

Intro: Air Data System on small UAS

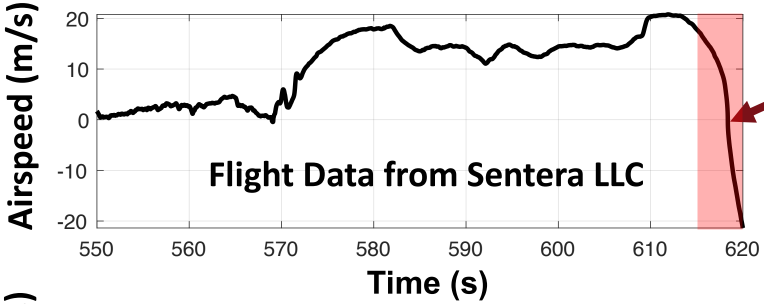
Pitot Tube Under Foggy Conditions



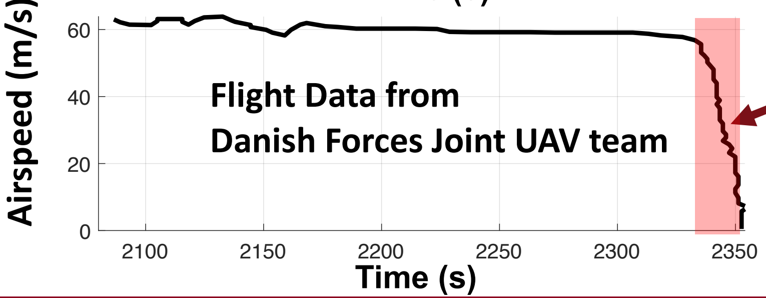




Faulty Airspeed Data



Slope: 2.5 m/s^2



Slope: 3.0 m/s^2

Air data Fault Detection and Isolation (FDI)

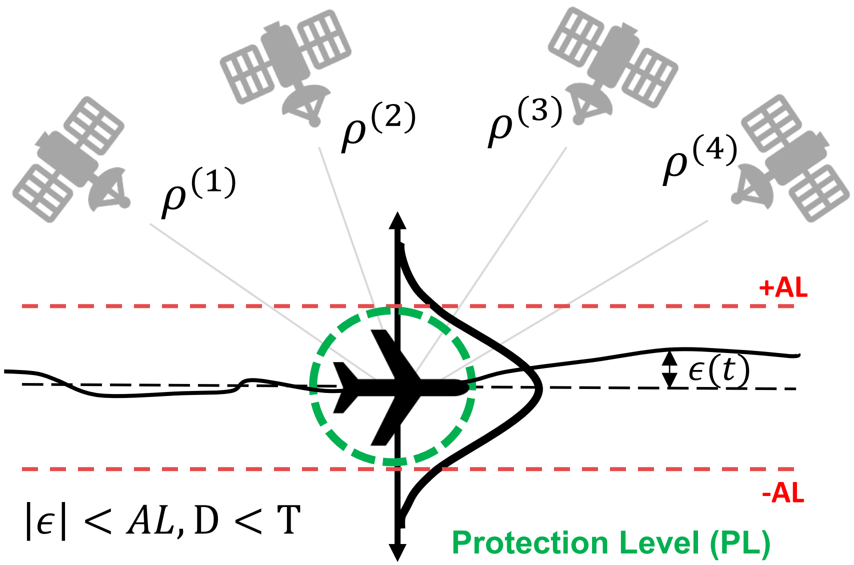
Prior Work

Model-Based	Model-Free	Data-Driven
Utilize dynamic model to estimate fault, reject disturbances, and provide robustness to model errors	Filtering approach (e.g., EKF, UKF) and it relies on kinematic relationships to reject bad measurement via statistical test	Data dependent input-output models (e.g., neural net, auto-regressive) to reject inconsistent measurement
E.g.: Freeman (2013) - H_∞ robust filter to estimate and detect Pitot tube blockage fault	E.g.: Lu (2016) – Three step UKF to detect and isolate faults	E.g.: Fravolini (2019) – data driven approach to select regressors for the input-output modeling and minimize false alarms

- Existing methods focus on algorithm
- Proposed solution: Integrity Monitoring framework, which focuses on system requirement

Integrity Monitoring (IM) Framework

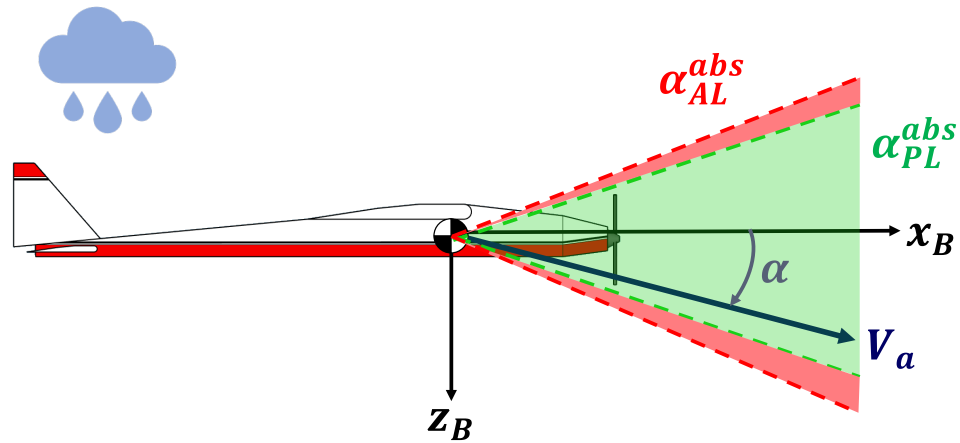
GNSS Applications



$|\epsilon| < AL, \mathrm{D} < \mathrm{T}$

$PL < AL$

Air Data System



$|\alpha| < \alpha_{AL}^{abs}, D < T$

$\alpha_{PL}^{abs} < \alpha_{AL}^{abs}$

Note: Threshold $\boldsymbol{T}$ and Protection Level $\boldsymbol{PL}$ are derived from system requirements.

D: Detection function/test statistics, $\boldsymbol{AL}$: alert limit

Challenges with ADS
Observability, nonlinearity, limited measurements, sensor accuracy

GNSS: Global Navigation Satellite System

IM – System Requirement

"High-Level" Performance Metrics:

- **Accuracy** – estimation error
- **Integrity** – correctness of the estimate, be able to alert
- **Continuity** – performing its function without interruptions
- **Availability** – the percentage of time that the services of the system are usable by the navigator

Fault Detection   Fault Isolation

Integrity Risk: $\mathrm{I} = \boxed{P(MD)} + \boxed{P(MI|DF)P(DF)}$

Continuity Risk: $\mathrm{C} = \boxed{P(FA)} + \boxed{P(NI|DF)P(DF)}$

MD = Missed Detection, DF = Detected Failure
MI = Mis−Identified failure,NI = Non−Isolable failure
FA = False Alarm

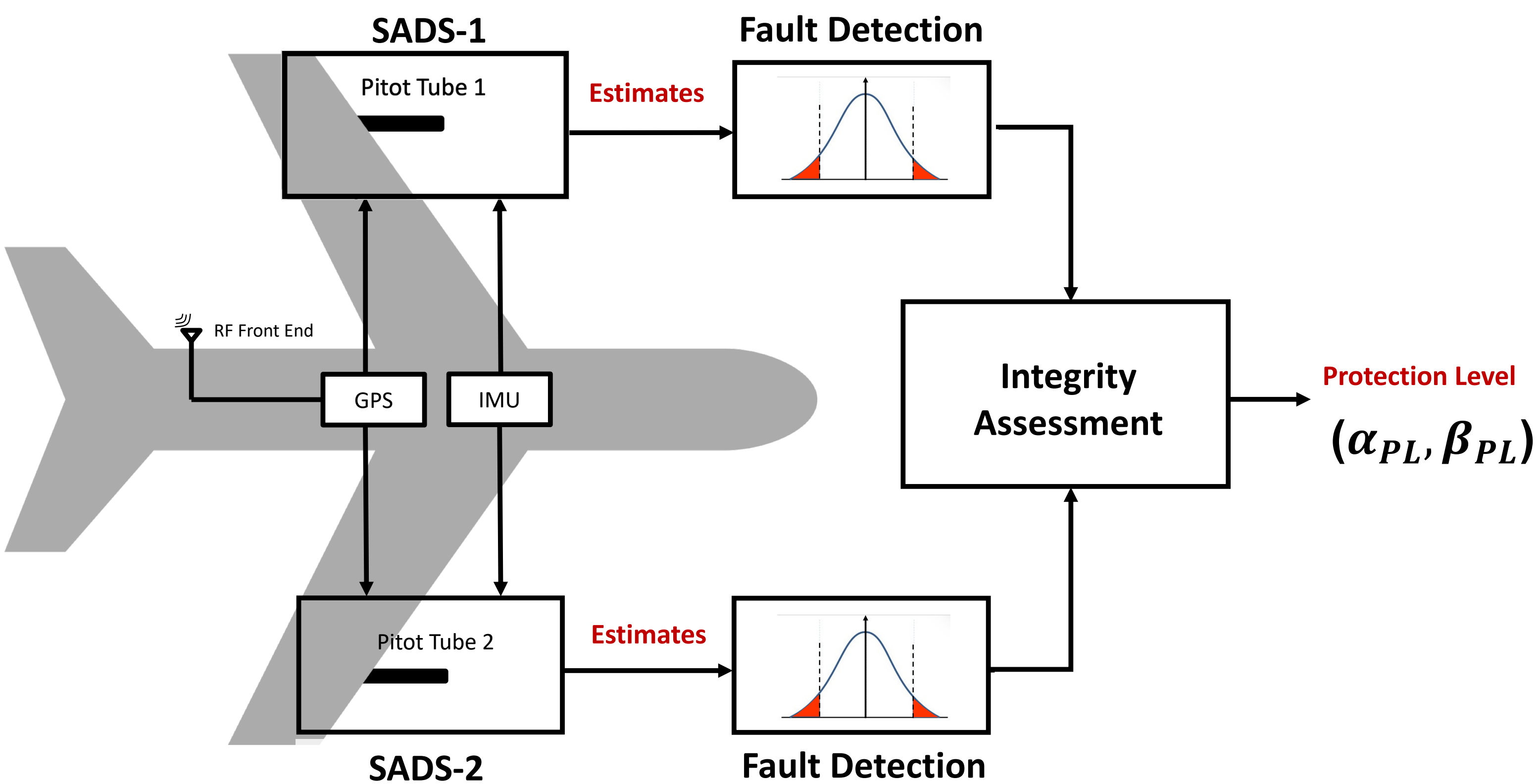
GNSS requirements example for civil aviation [ICAO, 2006]

Typical Operation	Accuracy (95%)		Integrity				Continuity	Availability
	Horiz.	Vert.	$P_{int}(I)$	HAL	VAL	TTA		
En-route	3.7km	N/A	10^{-7}/h	7.4 km (oceanic) 3.7 km (continental)	N/A	5min	$1-10^{-4}/h$ to $1-10^{-8}/h$	0.99 to 0.99999
Category I Precision Approach	16m	6m to 4m	$2\cdot10^{-7}$ in any approach	40m	35m to 10m	6s	$1-10^{-6}$ per 15s	0.99 to 0.99999

PROPOSED AIR DATA FDI DESIGN

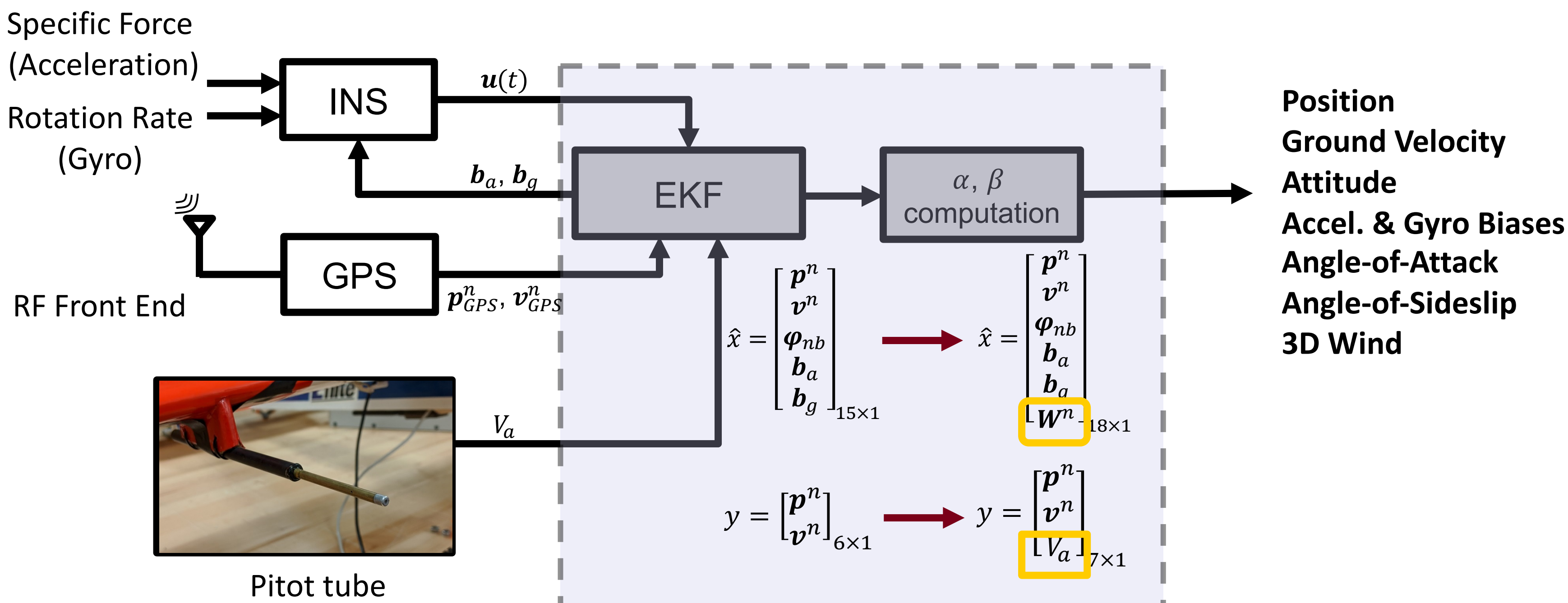
Dual Pitot Tube ADS with IM

1. Two independent Synthetic Air Data System (SADS)
2. Fault Detectability against Pitot tube WB fault
3. Isolation is achieved by independent fault software



SADS: Synthetic Air Data System

Model-Free SADS Estimator (Chapter 5)



SADS: Synthetic Air Data System

Observability Conditions

1. Airplane is accelerating (enabled by heading change)
2. Wind is quasi-static (slow time varying)

Sun, K., Regan, C. D., and Gebre-Egziabher, D., "Observability and Performance Analysis of Model-Free Synthetic Air Data Estimators", Journal of Aircraft, 2019

Fault Detection Design Via IM

Procedure

1. Define

a. Performance Requirement (P_{MD}, P_{FA})

b. Fault Mode and Fault Profile Modeling

2. Design

a. Threshold $\boldsymbol{T}$ via Detector Operating Characteristic (DOC) Curves

b. Detection Function Selection **D**

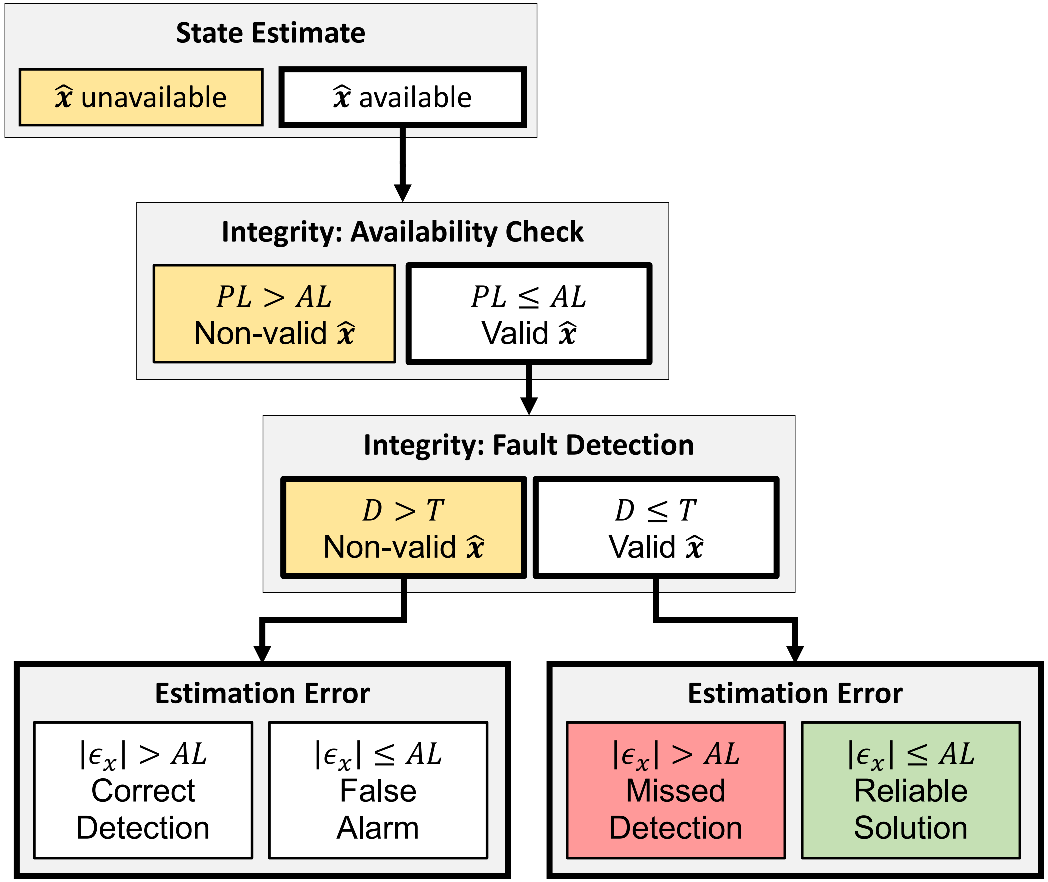
c. Protection Level (**PL**) Calculations

3. Performance Evaluation

a. Detectability (**D**)

b. PL vs. Alert Limit (**AL**)

Example



Fault Modeling and Requirement Definition (1)

Define

A. Performance Requirement

Requirement	Description
Integrity	$I_{req} \approx P_{MD} = P(\text{Pitot tube fault not sensed} \mid \text{Pitot tube has failed})$
Continuity	$C_{req} \approx P_{FA} = P(\text{issuing an alarm} \mid \text{no Pitot tube failure})$
Minimum Detectable Error (MDE)	Measure for size of the error can be detected with certain (P_{FA}, P_{MD})

B. Fault Mode and Fault Profile Modeling

Measurement Model Equation

$$V_a = \|\boldsymbol{V}^n - \boldsymbol{W}^n\|_2 + f_{V_a} + n_{V_a}$$

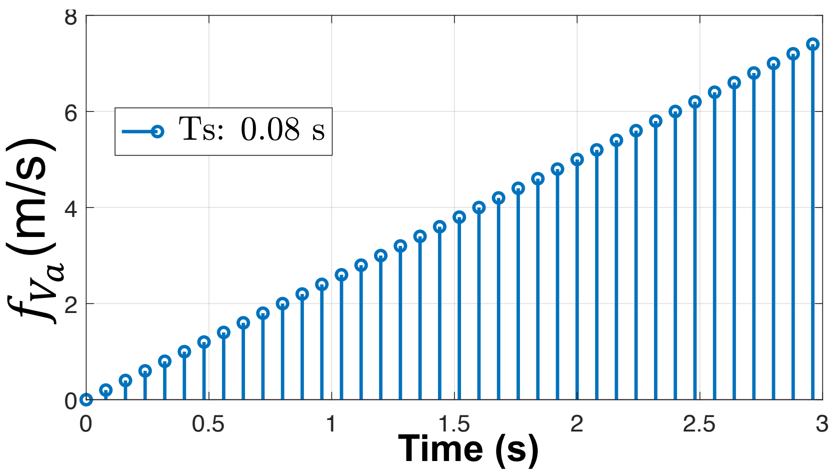
Maximum Allowable Error (MAE)

$$MAE = \bar{V}_a - V_{a,stall} = 17.5 - 10 = 7.5\, m/s$$

Maximum Detection Time

$$\tau_{max} = \frac{\bar{V}_a - V_{a,stall}}{slope} = \frac{17.5 - 10}{2.5} = 3s$$

Fault Profile Modeling

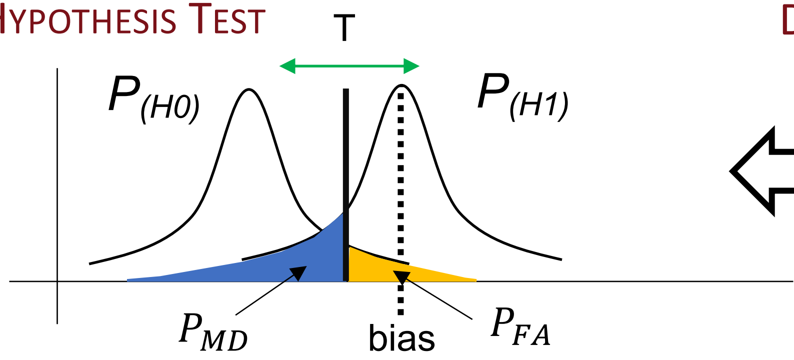


$$\boldsymbol{MDE_{req}(P_{FA}, P_{MD}) \le AE \le MAE(\overline{V_a}, V_{a,stall})}$$

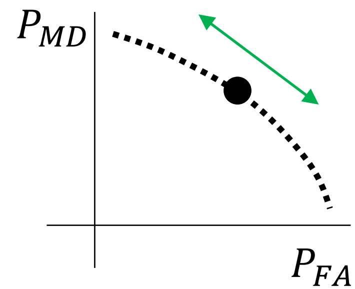
Tradeoff Analysis – Via Doc curves (2a)

Objective: Determine Threshold $\boldsymbol{T}$ and Minimum Detectable Error requirement MDE_{req}

Dual Hypothesis Test



Detector Operating Characteristic (DOC) curves



DOC generation equation

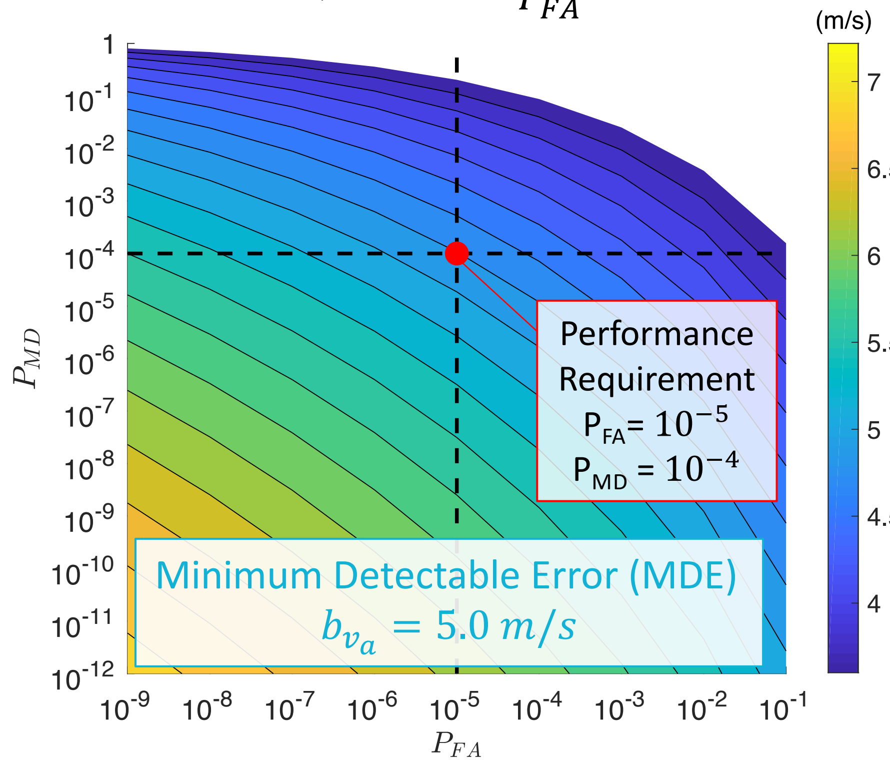
$$P_{MD} = F_{\chi^2(\lambda)}(T, Df) = F_{\chi^2(\lambda)}(F_{\chi^2}^{-1}(1 - P_{FA}, Df), Df)$$

WHERE

$$\overline{MDE} \triangleq \sqrt{\lambda} = \sqrt{\sum_{j=k-q+1}^{k} \frac{f_{V_{aj}}^2}{\sigma_{V_{aj}}^2}} \;\; \Rightarrow \;\; MDE_{req} \triangleq \mathrm{f}_{V_{ak}}$$

Requirement (P_{MD}, P_{FA})	Allowable (airframe)
MDE_{req} = 5 m/s	AE = 7.5 m/s
τ_{req} = 2 s	τ_{allow} = 3 s

Df: Degree-of-freedom. q: window size



DETECTION FUNCTION (2B)

GIVEN Dynamic Model $\mathrm{x}_k = \Phi_{k-1}\mathrm{x}_{k-1} + \mathrm{G}_{k-1}\mathrm{w}_{k-1}$

Measurement Model $\mathrm{z}_k = \mathrm{H}_k x_k + \mathrm{v}_k + \mathrm{f}_k$

CONVENTIONAL DETECTION FUNCTION **D**

- Kalman Filter Innovation test $\gamma_k = z_k - \mathrm{H}_k \hat{x}_k^-$

$$\boldsymbol{D}_{\boldsymbol{\gamma},k}|H_0 = \frac{1}{q}\sum_{j=k-q+1}^{k} \gamma_j^T S_j^{-1}\gamma_j \sim\chi^2(m)$$

- Residual test: $r_k = z_k - \mathrm{H}_k \hat{x}_k^+$

$$\boldsymbol{D}_{r,k}|H_0 = r_k^{\mathbf{T}}\mathbf{W}r_k = z_k^{\mathbf{T}}\mathbf{W}(\mathbf{I} - \mathrm{H_k H_k}^*)z_k \sim\chi^2(m-n)$$ **(if m > n)**

PROPOSED (SEQUENTIAL RESIDUAL)

$$\boldsymbol{D}_{\boldsymbol{r},k}|H_0 = \boldsymbol{r}_{k-q:k}^T \Sigma^{-1}\boldsymbol{r}_{k-q:k} \sim\chi^2(mq-n)$$

$$\mathbf{r}_{k-q:k} = \begin{bmatrix} \mathbf{r}_{k-q} \\ \mathbf{r}_{k-q+1} \\ \vdots \\ \mathbf{r}_k \end{bmatrix} \triangleq (\mathbf{I}_{mq\times mq} - \boxed{\mathcal{O}_{k-q:k}}\mathcal{O}_{k-q:k}^*)\mathbf{Z}_{k-q:k}$$ **Observability-Aware**

$$\mathbf{\Sigma} = \mathbf{I}_{q\times q}\otimes\mathbf{R} + \mathbf{Q}_{w,k-q:k}(\mathbf{I}_{q\times q}\otimes\mathbf{R}_w)\mathbf{Q}_{w,k-q:k}^T = \boldsymbol{W}^{-1}$$

m: number of measurement, n: number of state, q: window size, k: current time step

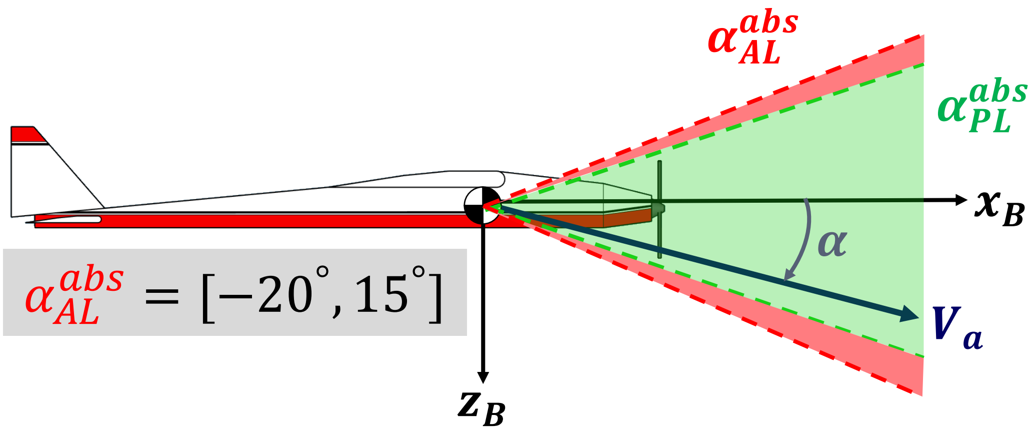
Protection Level Calculations (2c)

Objective: Provide statistical bound on ϵ to protect against fault

Protection Level (PL): Guaranteed upper bound of estimation error (i.e., $PL = K\sigma$)

Alert Limit (AL): the maximum error that can be tolerated (i.e., $PL < AL$)

Mathematically: $P(|\epsilon| > PL|H_i) = P(|\epsilon| > K\sigma|H_i) \leq P_{MD}$



	H_0(Fault Free)	H_1(Faulted)
α_{PL}	$K_{\alpha,0}\sigma_\alpha$	$\frac{\sigma_\alpha}{\sqrt{D_k}}\sqrt{\lambda_U} + K_{\alpha,1}\sigma_\alpha$

K: Inflation Factor, σ: standard deviation, α_{PL}^{abs}: absolute Alert Limit ($\alpha_{PL}^{abs} = \alpha + \alpha_{PL}$)

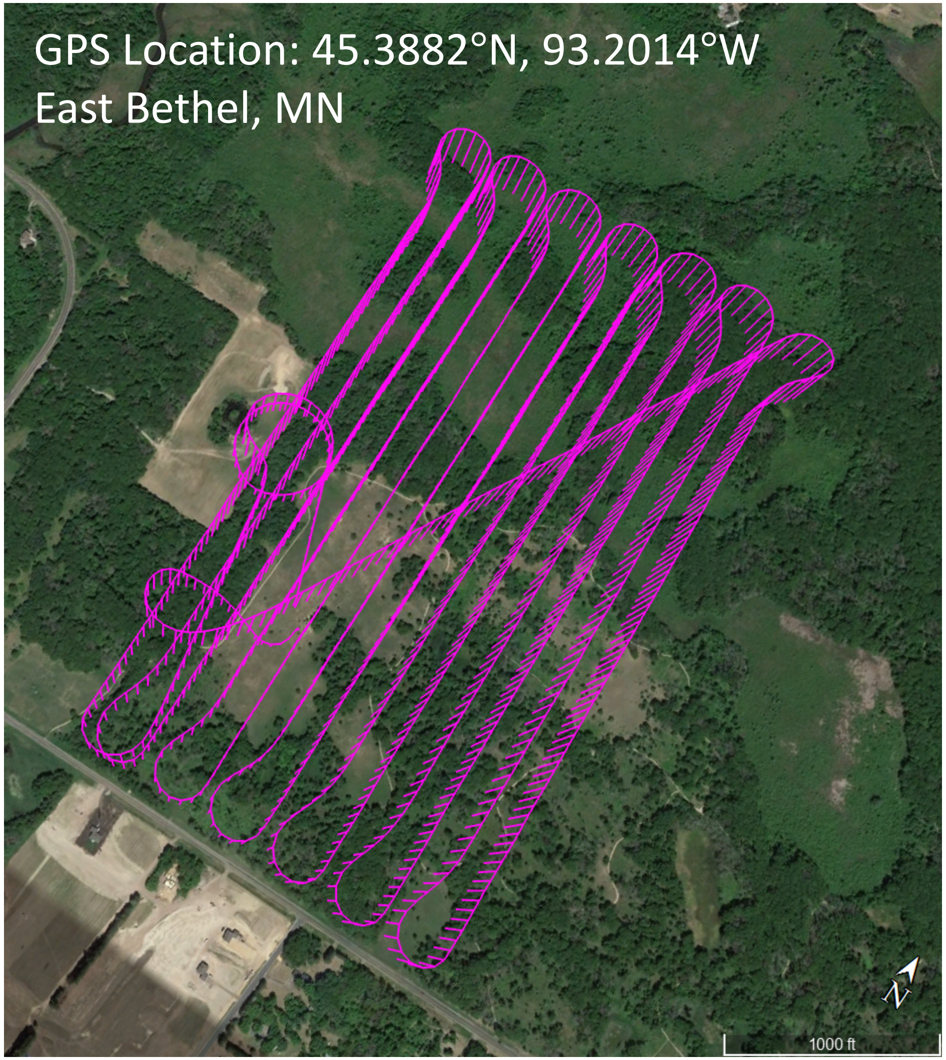
Flight Data testing (3)

Sentera PHX

Eagle Tree Pitot Tube

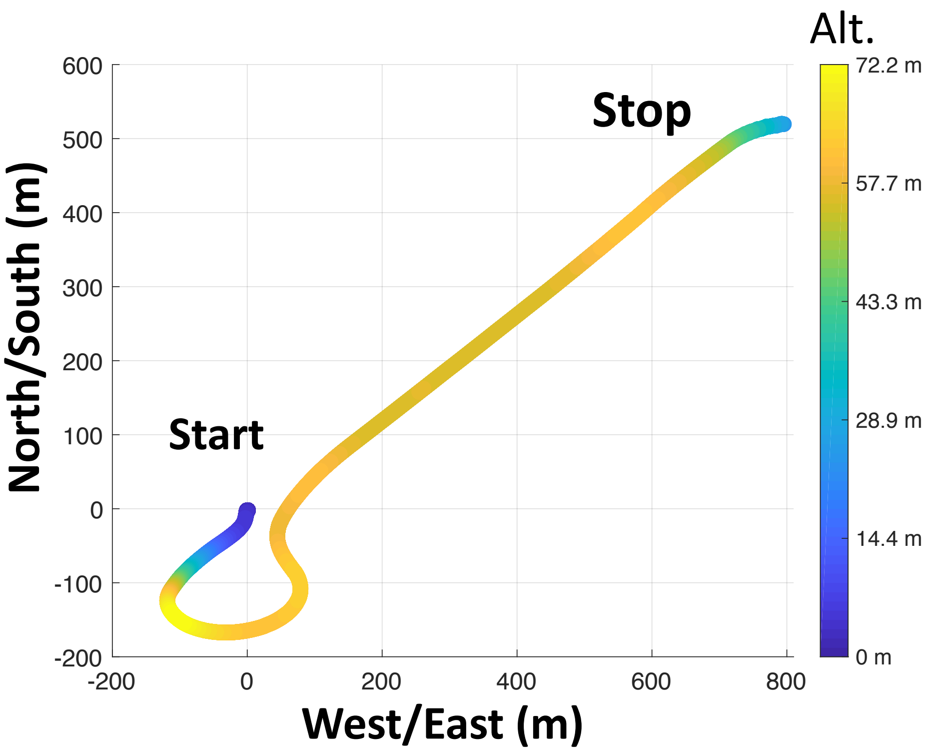
Performance Specifications	
Takeoff weight	4.2 lbs (1.9 kg)
Wingspan	4.5 ft (1.4 m)
Max speed	45 kts (20.6 m/s)

Agricultural Inspection

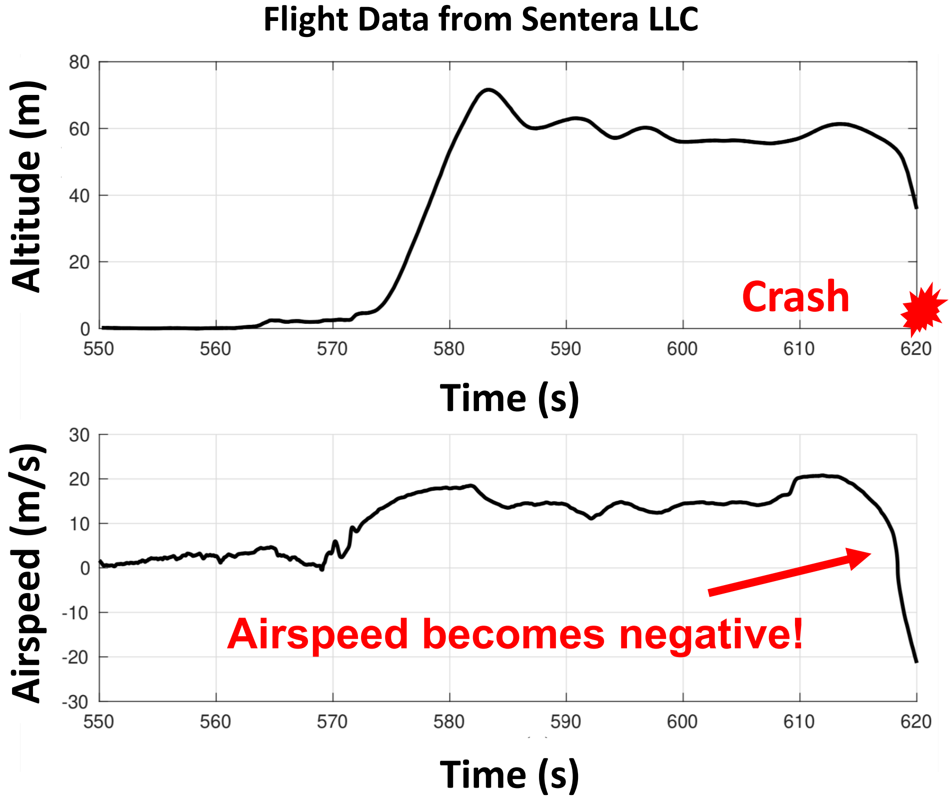


Performance Evaluation - Flight Data Testing

Flight Trajectory



Altitude and Airspeed



Flight Information	
Takeoff	Around 570 s
Crash	Around 623 s

Performance Evaluation- Detector (3a)

Additional Detector/Test Statistic

$$D_{r,k}^{GMA}|H_0 = \left(r_{k-q:k}^{GMA}\right)^T \Sigma^{-1} r_{k-q:k}^{GMA}$$

Where $r_{k-q:k}^{GMA} = \left[\mu^q r_{k-q}^T \cdots \mu r_{k-1}^T\ r_k^T\right]^T$
and μ is the discounting factor

GMA: Geometric Moving Average

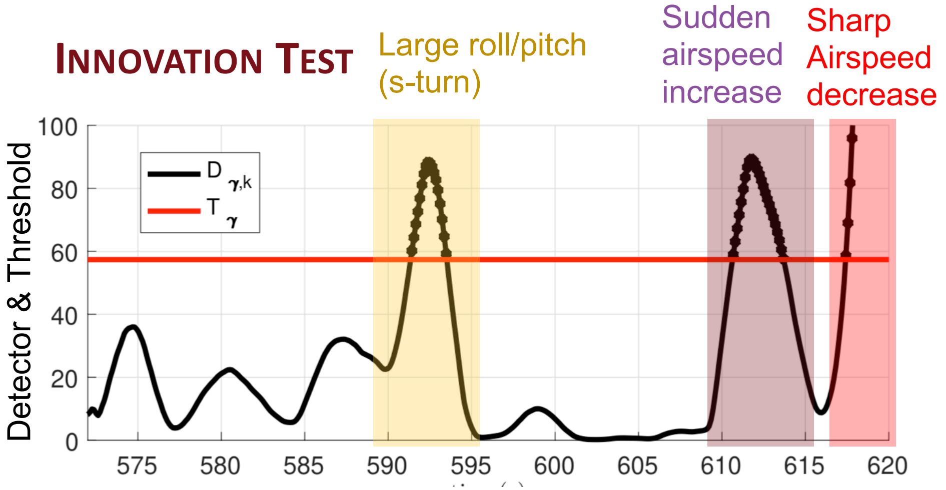
Faulty Airspeed Estimation

Parameter	Estimated Onset	Estimated Detection
Time (s)	616	618
Airspeed (m/s)	15	10*

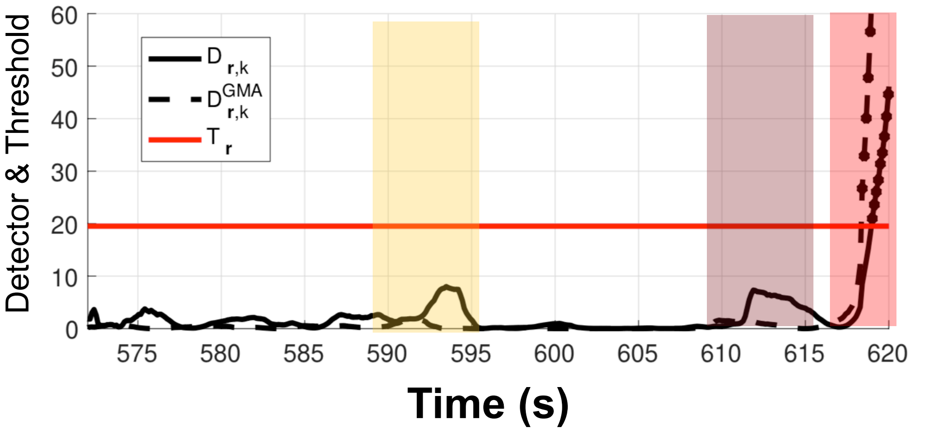
$*$: The actual record airspeed is -10 m/s

Result

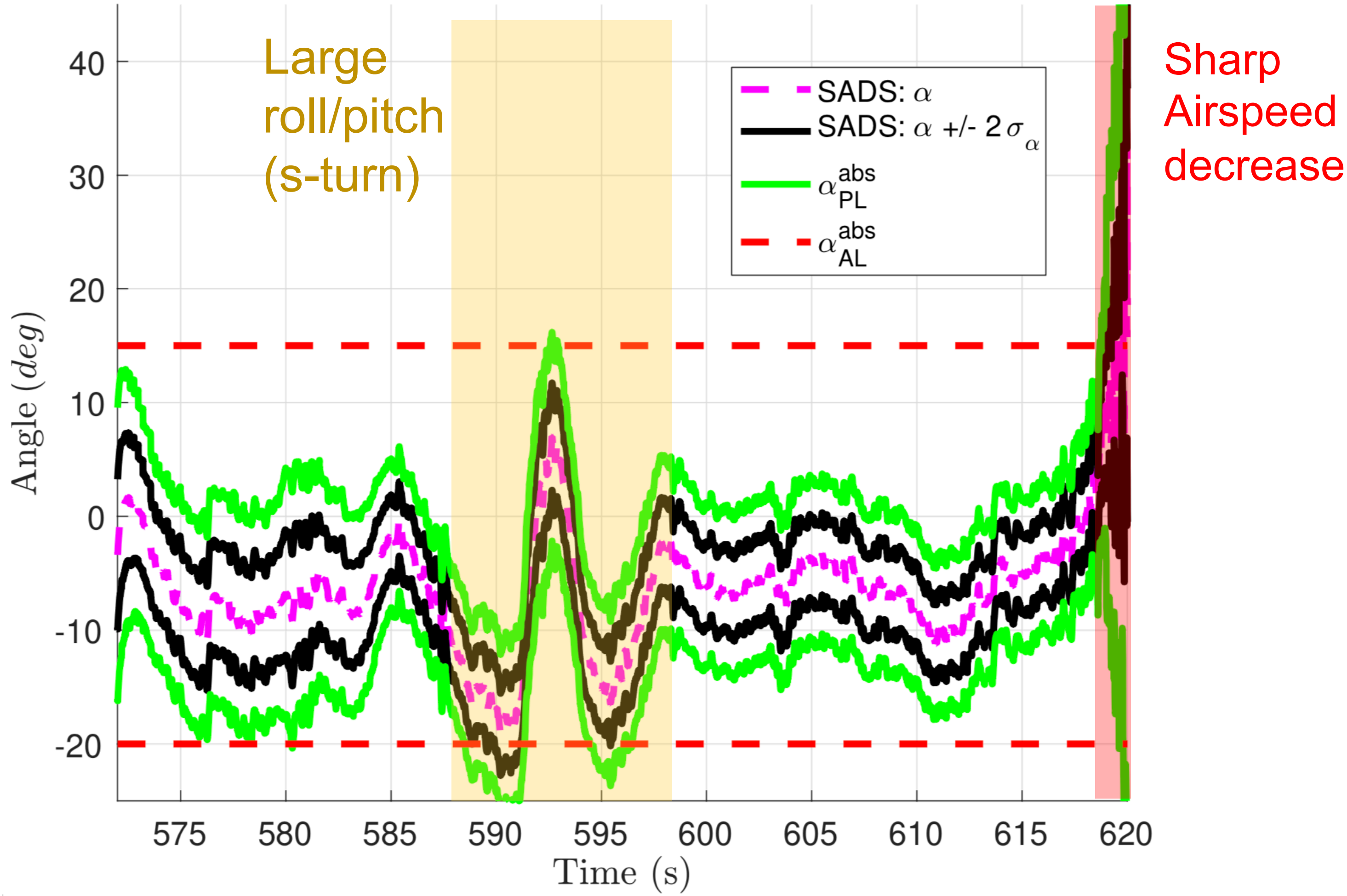
Innovation Test



Residual-based Test



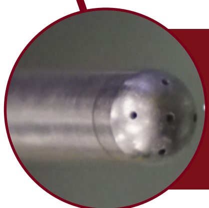
Protection Level of angle-of-attack (3B)



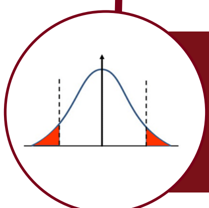
Loss of Integrity due to aggressive s-turn and faulty Pitot tube

OVERVIEW

Introduction (Chapter 1-2)



Air Data Calibration (Chapter 3-4)

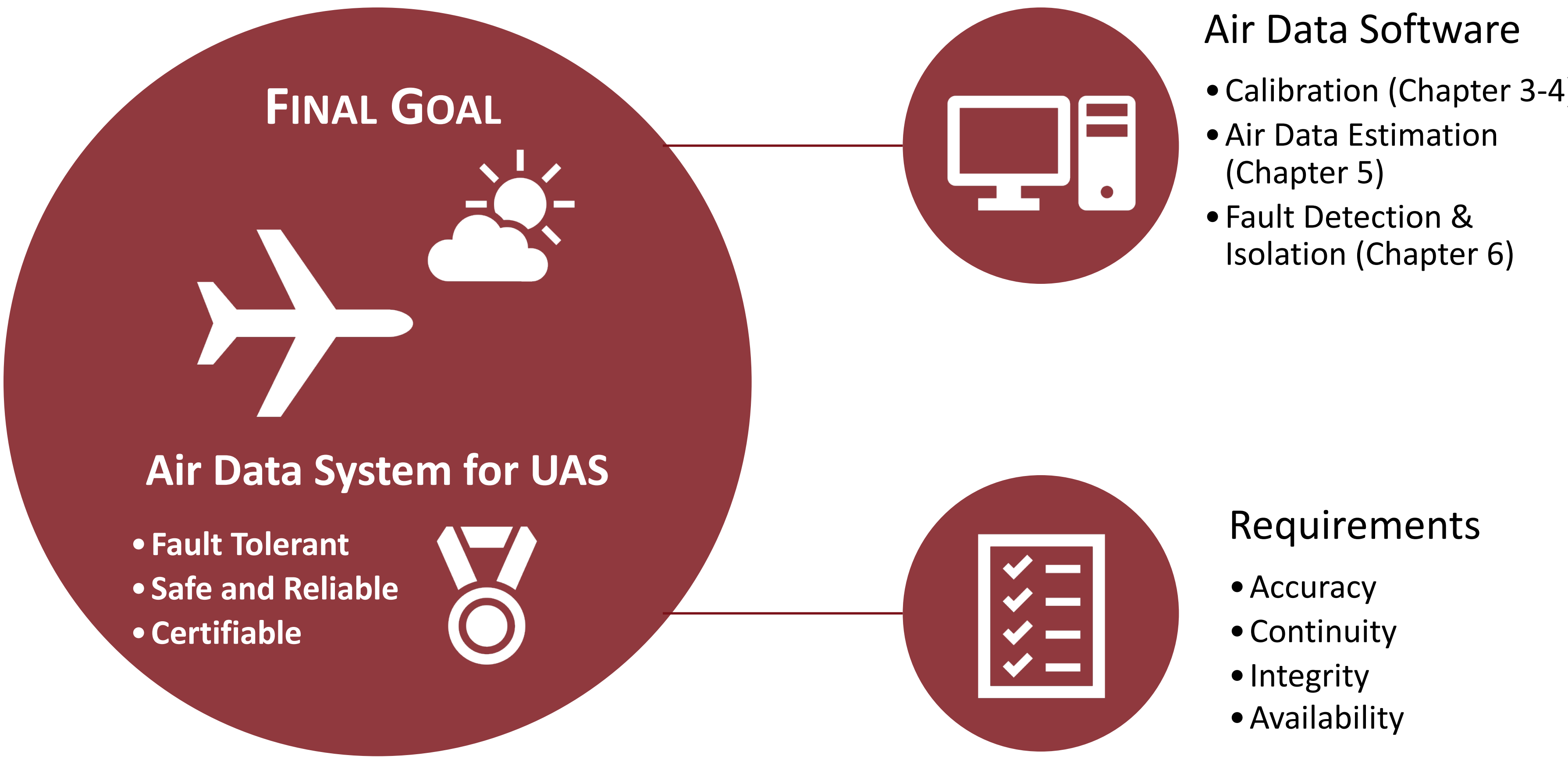


Air Data Fault Detection and Isolation (Chapter 6)

Conclusion and Future Outlook (Chapter 7)

CONCLUSION

- Presented two aspects of air data solution to improve reliability of ADS on small UAS
  - Air data calibration
  - Air data fault detection and isolation



Future work

Theoretical work on Observability

- Observability of SADS
- Existing tools are <u>not</u> sufficient
- Extension of Observability using differential geometry, Lie groups

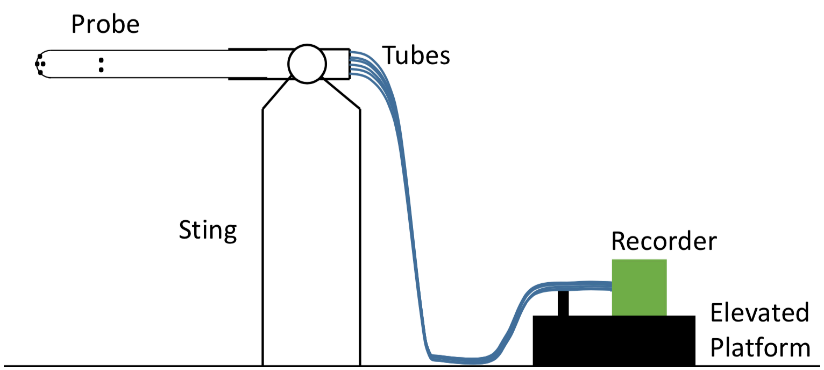
Experimental work on fault mode analysis (FMEA)

- Fault mode study
- Improve measurement modeling
- Realistic Requirement

Other Possible fault modes for 5-hole probe

Fault Mode	Failure Rate	Effect
Partial Water Blockage	?	?
Shape Degradation	?	?
Foreign Object	?	?

Wind tunnel probe setup

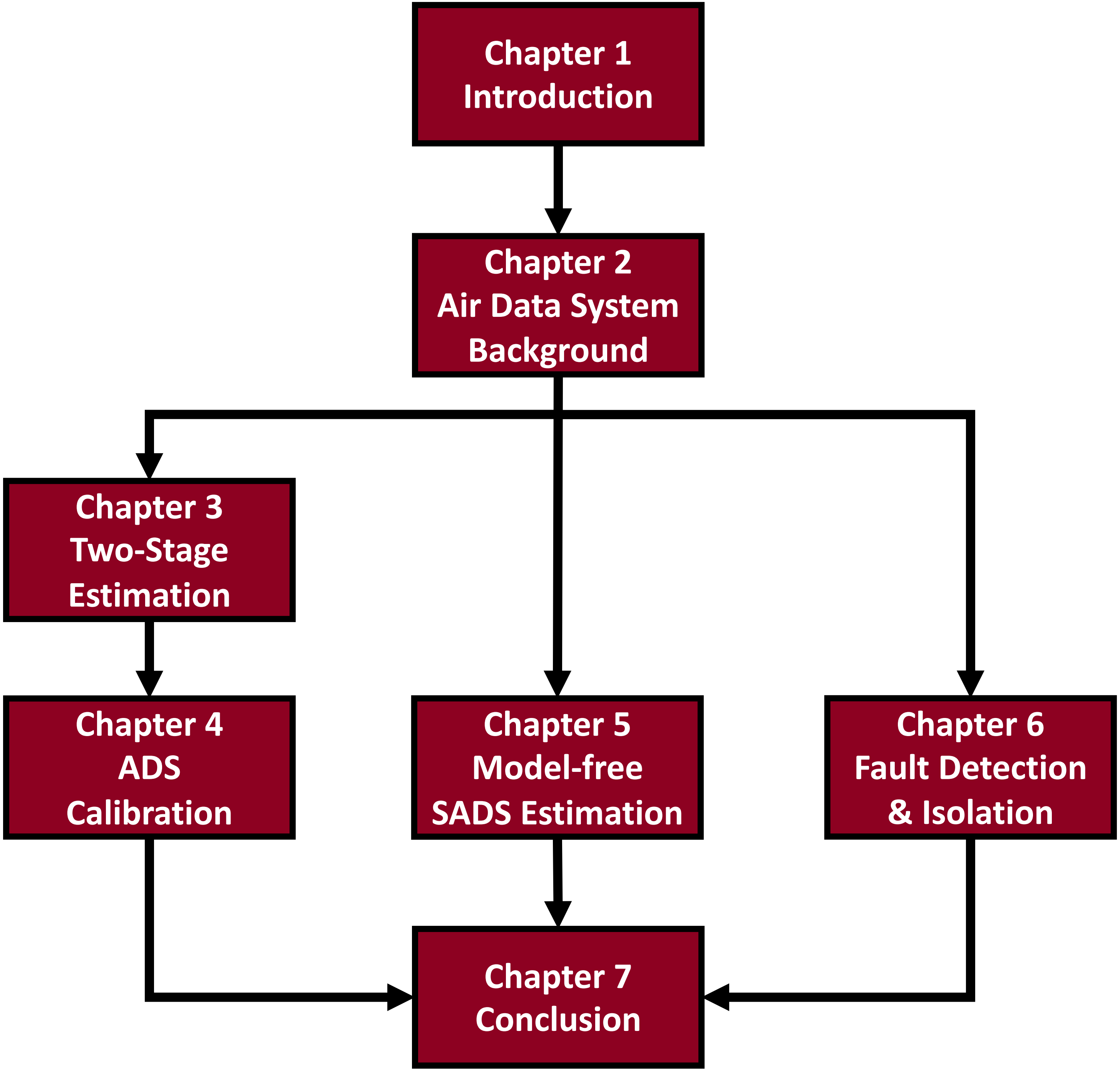


FMEA: Failure Mode and Effects Analysis

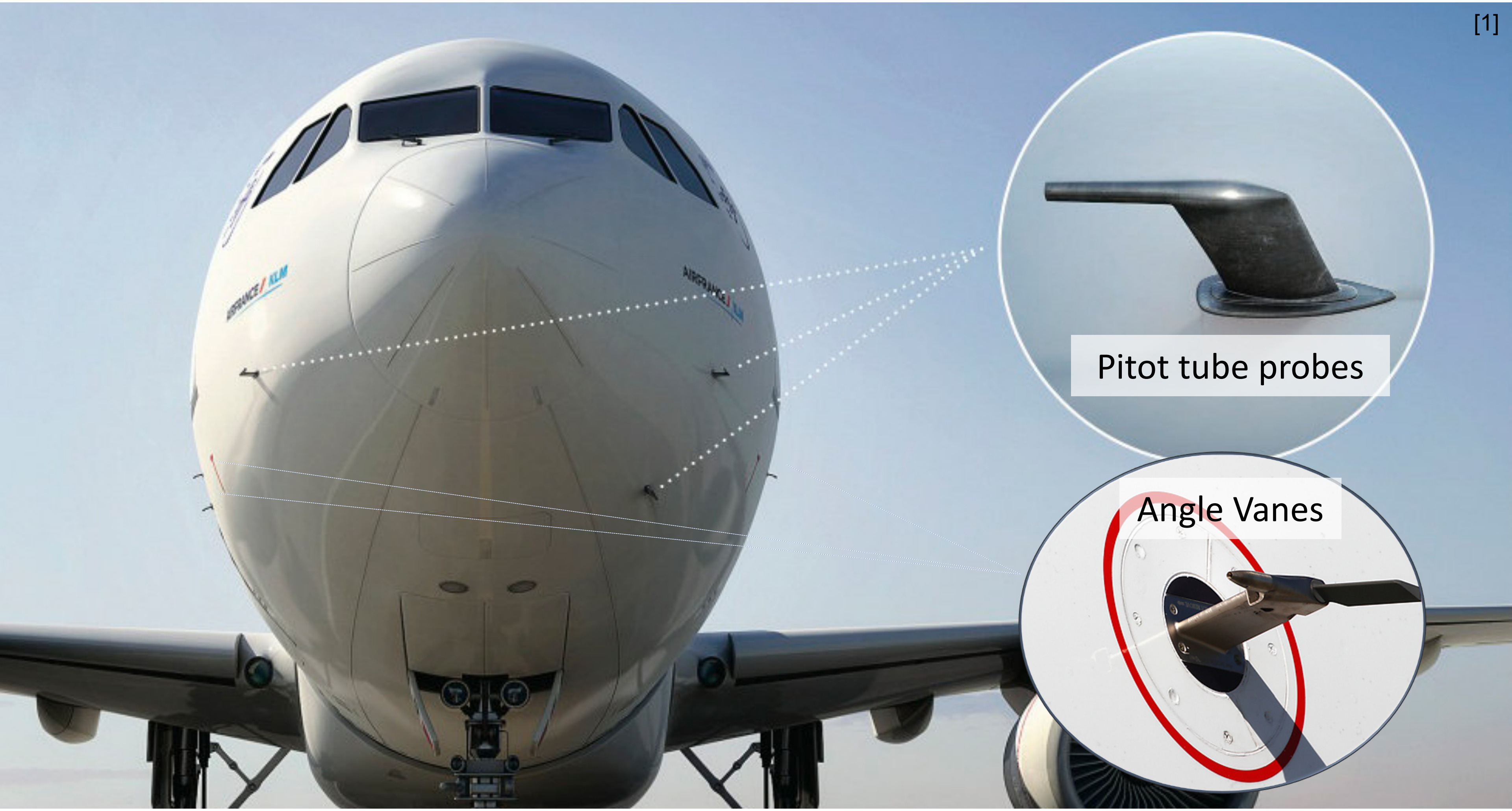
THANKS

Advisor: Demoz Gebre-Egziabher

Backup Slides

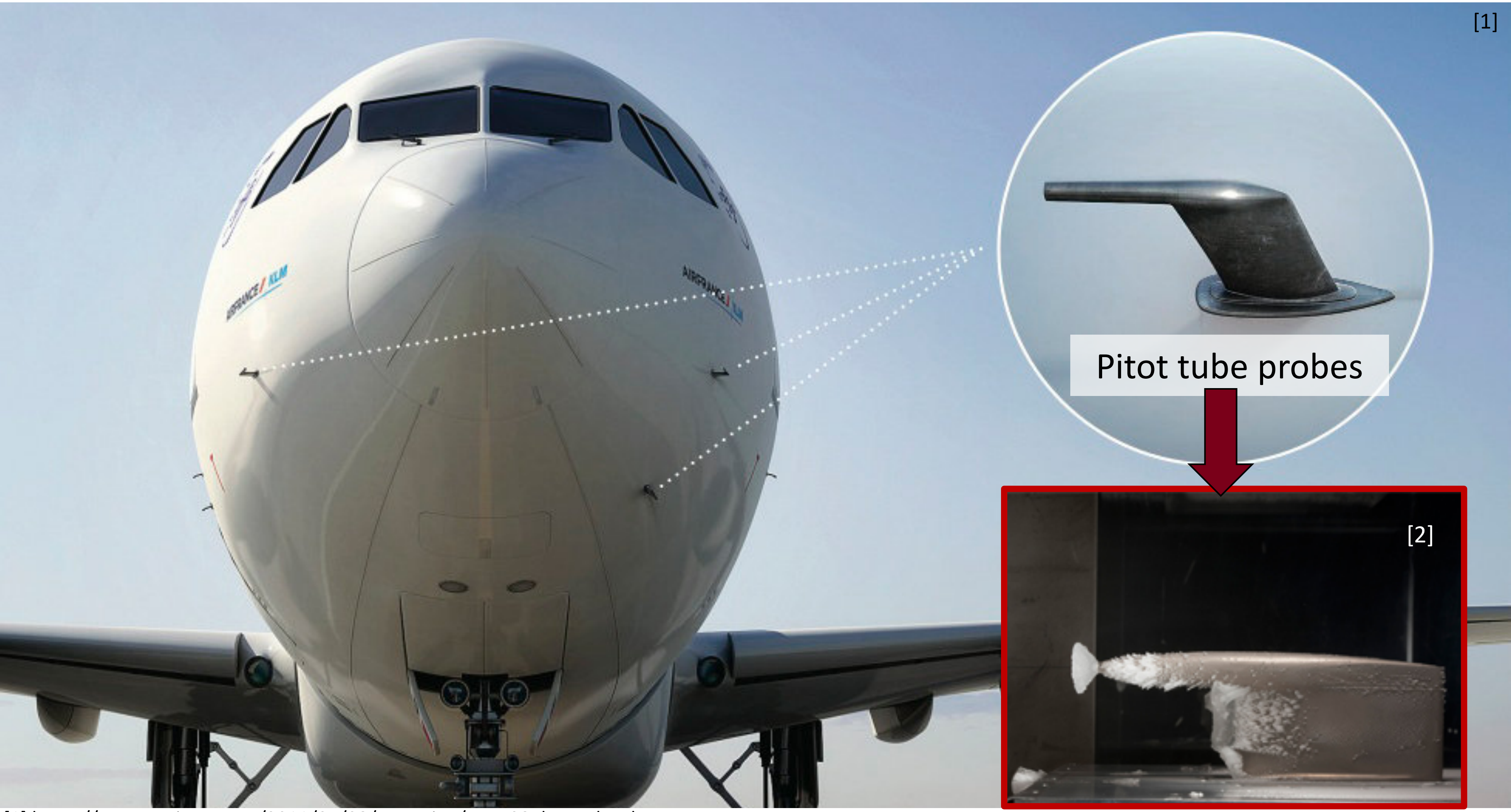

Chapter 1
Introduction
Chapter 2
Air Data System
Background
Chapter 3
Two-Stage
Estimation
Chapter 4
ADS
Calibration
Chapter 5
Model-free
SADS Estimation
Chapter 6
Fault Detection
& Isolation
Chapter 7
Conclusion

Reliability via Hardware Redundancy



[1] https://www.nytimes.com/2011/05/08/magazine/mag-08Plane-t.html

LIMITATIONS OF PHYSICAL REDUNDANCY
AIR FRANCE FLIGHT 447 | CRASH IN 2009



[1] https://www.nytimes.com/2011/05/08/magazine/mag-08Plane-t.html
[2] https://www.nrc-cnrc.gc.ca/eng/solutions/facilities/wind_tunnel/altitude_icing.html

Indonesia Lion Air (Oct 2019)
Ethiopian Airlines (March 2019)

Modern Drones/UAVs

Single Air Data System Design

Size, Weight, and Power (SWAP) issue

Proposed: Two Stage Estimator

Measurement Model

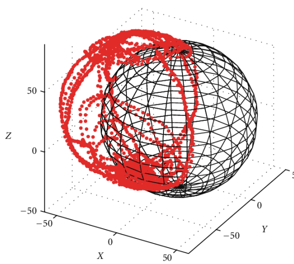
$$z_k = h_k(x_k, u_k, \xi) + v_k$$

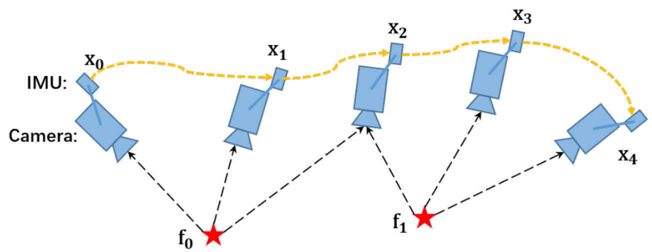
$$z = Cy + b + v$$

$$z_k = A(\xi_2)\xi_1 + b(\xi_2) + v_k$$

Applications

1. Magnetometer Calibration
2. Flight Data Compatibility
3. IMU-Camera Calibration





Advantages

1. Canonical Form
2. Systematic Approach

Two Stage Estimator Flowchart

Select appropriate ξ_{2p}

Stage 1

Linear Least Square

$$\arg min_{\xi_{1p}} \; \frac{1}{2}\sum_{k=1}^{N} \left\| \mathbf{z}_k - \mathbf{A}(\xi_{2p})\xi_{1p} - \mathbf{b}(\xi_{2p}) \right\|_{\mathbf{I}}^{2}$$

Initial Condition

$\xi_{1p},\ \xi_{2p},\ R(\xi_{2p})$

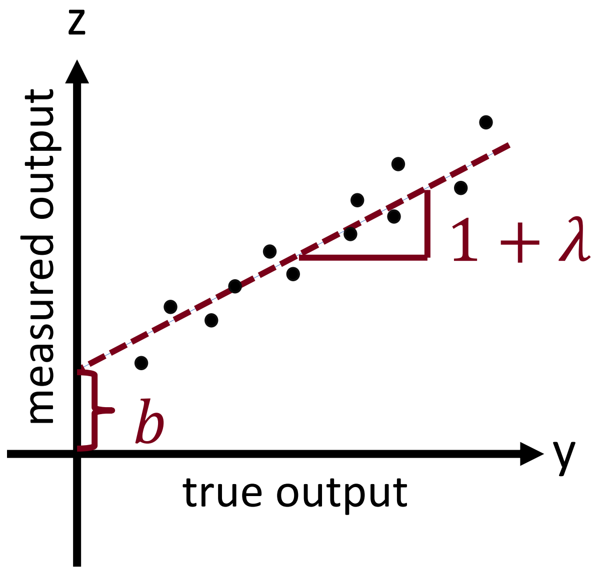
Stage 2

Nonlinear Optimization

$$\arg min_{\xi_2, R} \; \frac{1}{2}\sum_{k=1}^{N} \left\| \mathbf{z}_k - \mathbf{A}(\xi_2)\xi_{1p} - \mathbf{b}(\xi_2) \right\|_{\mathbf{R}}^{2} + \frac{N}{2}\ln|\mathbf{R}|$$

$\xi_1^*, \xi_2^*, \mathbf{R}$

Two Stage Estimator - Motivation



$$z_k = (1+\lambda)y_k + b + v_k$$

Collect points: (y_k, z_k)

$$\underbrace{\begin{bmatrix} y_1 & 1 \\ \vdots & \vdots \\ y_N & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1+\lambda \\ b \end{bmatrix}}_{\xi} = \underbrace{\begin{bmatrix} z_1 \\ \vdots \\ z_N \end{bmatrix}}_{b}$$

Least square solution:

$$\xi = (A^T A)^{-1} A^T b$$

Problem Statement

What if $\boldsymbol{z} = \boldsymbol{C}\boldsymbol{y} + \boldsymbol{b} + \boldsymbol{v}$

where $\boldsymbol{C}(\boldsymbol{\xi}_s) = \boldsymbol{C}_1\boldsymbol{C}_2\boldsymbol{C}_3 \cdots$

$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\xi}_t)$

How do you estimate $\boldsymbol{\xi} = \{\boldsymbol{\xi}_s, \boldsymbol{\xi}_t, \boldsymbol{b}\}$ *and* $\boldsymbol{R}$?

Flight Test Example - Formulation

Measurement Model $\boldsymbol{z_k = h_k(x_k, u_k, \xi) + v_k}$

Wind Triangle Equation: $V^n = C_b^n V_{a,cg} + W^n = C_b^n[C(\epsilon_\phi)V_{a,s} - [\omega]_\times \boldsymbol{r}] + W^n$

5-hole Error Modeling:

$$V_a = (1 + \lambda_{V_a})\sqrt{\frac{2(P_t - P_s)}{\rho}} + b_{V_a}$$

$$\alpha = (1 + \lambda_\alpha)\frac{P_{\alpha 1} - P_{\alpha 2}}{K_\alpha(P_t - P_s)} + b_\alpha$$

$$\beta = (1 + \lambda_\beta)\frac{P_{\beta 1} - P_{\beta 2}}{K_\beta(P_t - P_s)} + b_\beta$$

Canonical Form $\boldsymbol{z_k = A(\xi_2)\xi_1 + b(\xi_2) + v_k}$

$$z_k = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}_k = \underbrace{\begin{bmatrix} F\sqrt{\frac{2(P_t - P_s)}{\rho}} & F & I_3 \end{bmatrix}}_{\mathbf{A}(\mathbf{x},\mathbf{u},\boldsymbol{\xi}_2)} \underbrace{\begin{bmatrix} \lambda_{V_a} \\ b_{V_a} \\ W_N \\ W_E \\ W_D \end{bmatrix}}_{\boldsymbol{\xi}_1} + \underbrace{F\sqrt{\frac{2(P_t - P_s)}{\rho}} - C_b^n[\omega]\times\mathbf{r}}_{\mathbf{b}(\mathbf{x},\mathbf{u},\boldsymbol{\xi}_2)} + \underbrace{\begin{bmatrix} v_{V_N} \\ v_{V_E} \\ v_{V_D} \end{bmatrix}_k}_{\mathbf{v_k}}$$

where $F = C_b^n C(\epsilon_\phi)\begin{bmatrix} cos\alpha cos\beta \\ sin\beta \\ sin\alpha cos\beta \end{bmatrix}$

Flight Test Example - Results

Input design and time specification

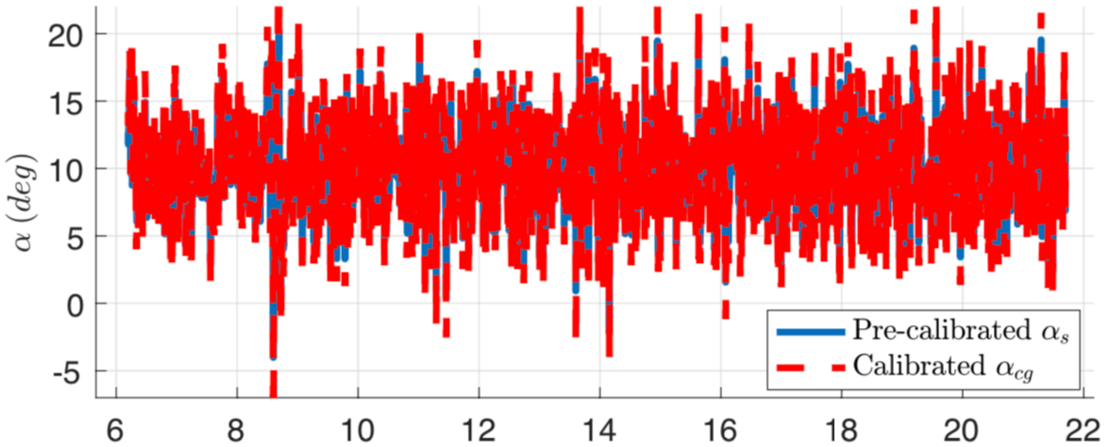
Maneuver Type	Time (sec)	Usage
Wind Circle 1	[384, 408.2]	Stage 1
Wind Circle 2	[411, 438.3]	Stage 1
POPU	[510.9, 530]	Stage 1
Multisine 1	[576, 596]	Stage 2
Multisine 2	[690, 711]	Stage 2
Multisine 3	[752, 772]	Stage 2
Multisine 4	[810, 830]	Stage 2
Pitch Chirp 1	[867, 887]	Stage 2
Yaw Chirp	[980.080, 1013.595]	Stage 2
Pitch Chirp 2	[1041, 1061]	Stage 2
Rudder Doublet	[1113, 1115]	Stage 2

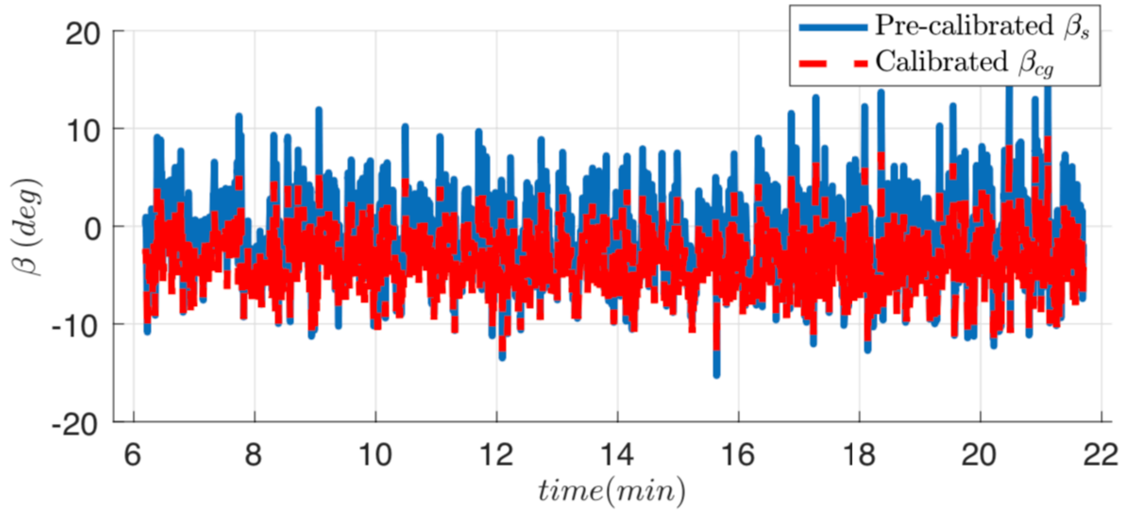
Flight Test Example - Results

Parameter estimate, standard deviation, and constraint setting

Parameter	Two-stage (Standard deviation)	Unit	Constraint used
λ_{V_a}	-0.1748 (0.0739)	--	[-0.5, 0.5]
b_{V_a}	4.3553 (1.2672)	m/s	[-5, 5]
λ_α	0.2982 (0.3228)	--	[-0.5, 0.5]
b_α	-2.4854 (0.5603)	deg	[-5, 5]
λ_β	-0.2673 (0.2379)	--	[-0.5, 0.5]
b_β	-1.2980 (0.5164)	deg	[-5, 5]
ϵ_ϕ	-7.9068 (15.4641)	deg	[-15,15]
W_N	-3.8038 (0.8704)	m/s	[-6, 6]
W_E	-2.4137 (0.9956)	m/s	[-6, 6]
W_D	-0.7168 (0.9952)	m/s	[-2, 2]

Flight Test Example - Results

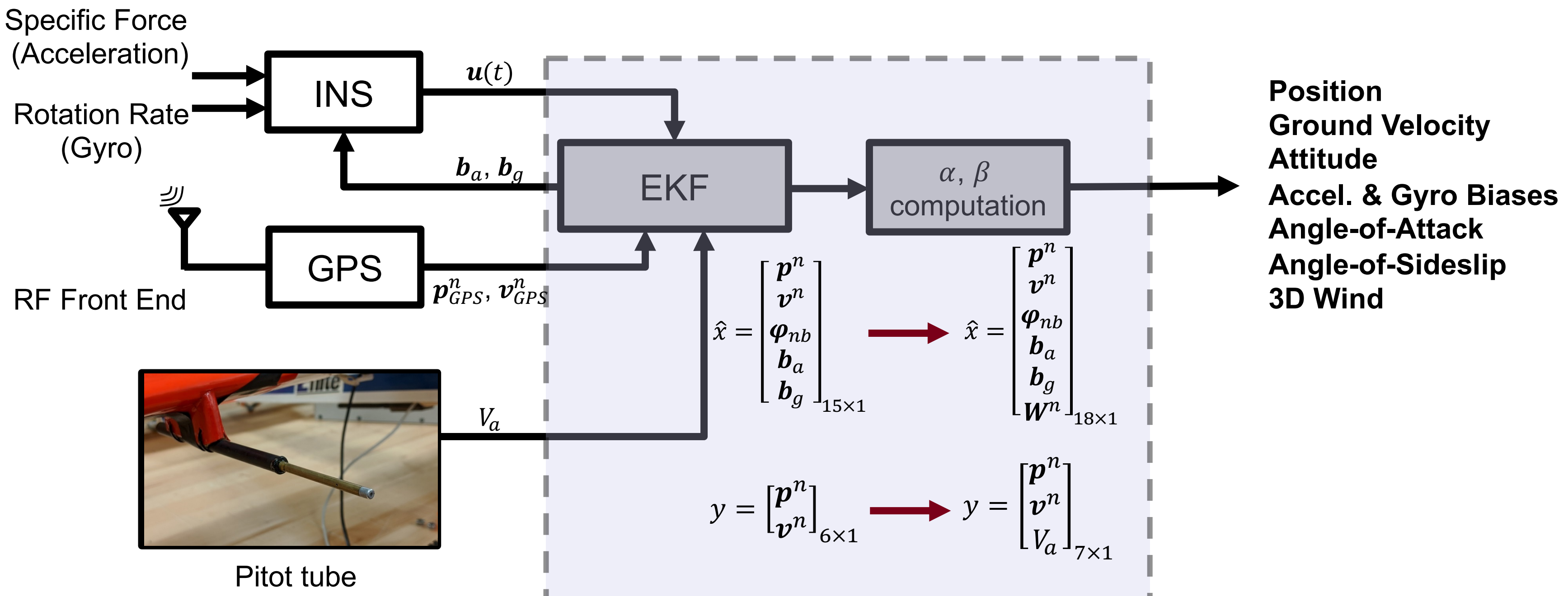




Calibrated 5-hole probe vs. Pitot tube

Model-Free SADS Filter Architecture

SADS: Synthetic Air Data System



groundspeed **airspeed** wind

$$\begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} = C_b^n \begin{bmatrix} V_a \cos\alpha\cos\beta \\ V_a \sin\beta \\ V_a \sin\alpha\sin\beta \end{bmatrix} + \begin{bmatrix} W_N \\ W_E \\ W_D \end{bmatrix}$$

5 unknown
(α, β, W_N, W_E, W_D)

$$V_g^n = C_b^n V_a^b + W^n$$

n: navigation frame
b: body frame

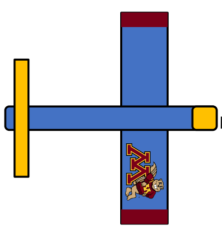
Observability Analysis

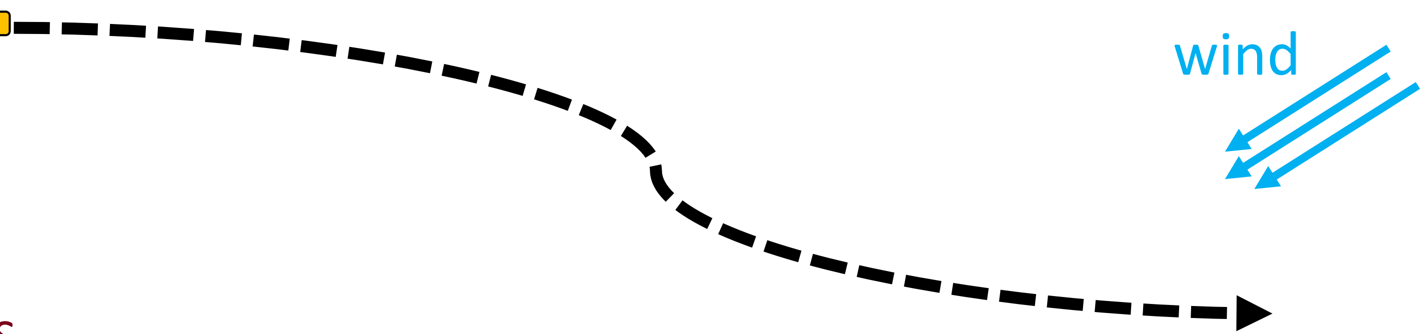
System Description (mathematical model) → **<u>Question 1</u> Linear Time Varying (LTV) observability** → **<u>Question 2</u> Degree of observability**

Solution to Question 1

Observability Gramian: $\mathcal{G}_d = H_0^T H_0 + \sum_{k=1}^{n} \Phi_{k-1,0}^T H_k^T H_k \Phi_{k-1,0}$

Rank Test: $rank(\mathcal{G}_d)$





Conditions

1. Airplane is accelerating (enabled by heading change)
2. Wind is quasi-static (slow time varying)

$$\delta \dot{W}_i = -\frac{1}{\tau_{w_i}} \delta W_i + \eta_{w_i}$$

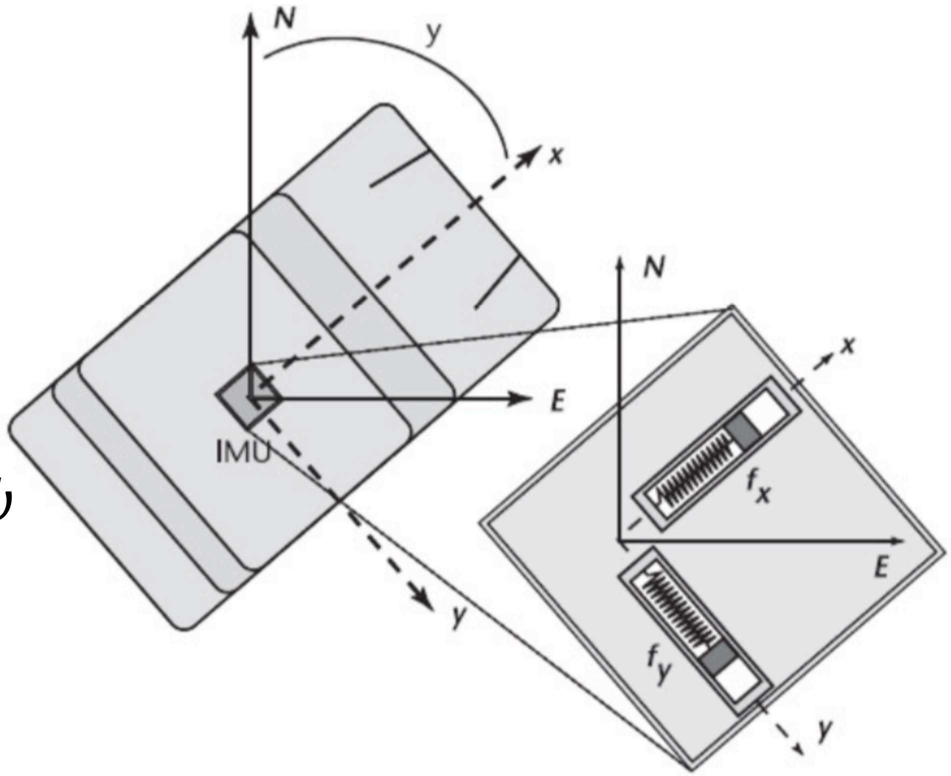
Observability of Yaw in GNSS/INS (1)

Assume: perfect accelerometer, but biased gyro and no noise

$$a_N = \dot{v}_N = f_x cos(\psi) - f_y sin(\psi)$$
$$a_E = \dot{v}_E = f_y sin(\psi) + f_y cos(\psi)$$

$$\hat{\psi} = \psi + \delta\psi = \psi + \int_{t_0}^{t} b_{g_z} d\tau$$

$$\hat{a}_N = a_N - [f_y cos\psi + f_x sin\psi]\delta\psi$$
$$\hat{a}_E = a_E + [f_y sin\psi + f_x cos\psi]\delta\psi$$



If f_x and f_y are zero, then $\delta\psi$ will NOT be reflected in position and velocity estimates, hence heading is unobservable

Observability of Yaw in GNSS/INS (2)

Assume: perfect accelerometer, but biased gyro and no noise

$$\frac{d}{dt}\begin{pmatrix}\delta V_N\\ \delta V_E\\ \delta V_D\end{pmatrix}=\begin{bmatrix}0 & \hat{f}_D & -\hat{f}_E\\ -\hat{f}_D & 0 & \hat{f}_N\\ \hat{f}_E & -\hat{f}_N & 0\end{bmatrix}\begin{pmatrix}\delta\phi\\ \delta\theta\\ \delta\psi\end{pmatrix}$$

- Yaw error ($\delta\psi$) can only affect velocity error when the specific force in east and north direction is nonzero.
- Similar statement can be made for the other two angle errors when accelerometer bias is non-zero.

OBSERVABILITY ANALYSIS

System Description (mathematical model) → **Question 1: Linear Time Varying (LTV) observability** → **Question 2: Degree of observability**

Groundspeed = **Airspeed** + **Wind**

$$\mathbf{V}_g^n = \mathbf{C}_b^n \mathbf{V}_a^b + \mathbf{W}^n$$

$$\underbrace{\begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \boldsymbol{y}_3 \\ \vdots \\ \boldsymbol{y}_n \end{bmatrix}}_{\mathcal{Y}} = \underbrace{\begin{bmatrix} \boldsymbol{\Psi}_1 & \boldsymbol{\Omega}_1 & \mathbf{0}_{3\times2} & \cdots & \cdots & \mathbf{0}_{3\times2} \\ \boldsymbol{\Psi}_2 & \mathbf{0}_{3\times2} & \boldsymbol{\Omega}_2 & \mathbf{0}_{3\times2} & \cdots & \mathbf{0}_{3\times2} \\ \boldsymbol{\Psi}_3 & \mathbf{0}_{3\times2} & \mathbf{0}_{3\times2} & \boldsymbol{\Omega}_3 & \cdots & \mathbf{0}_{3\times2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{\Psi}_n & \cdots & \cdots & \cdots & \cdots & \boldsymbol{\Omega}_n \end{bmatrix}}_{\mathcal{H}} \underbrace{\begin{bmatrix} \boldsymbol{W}^n \\ \hline \alpha_1 \\ \beta_1 \\ \hline \alpha_2 \\ \beta_2 \\ \hline \alpha_3 \\ \beta_3 \\ \hline \vdots \\ \alpha_n \\ \beta_n \end{bmatrix}}_{\mathcal{X}}$$

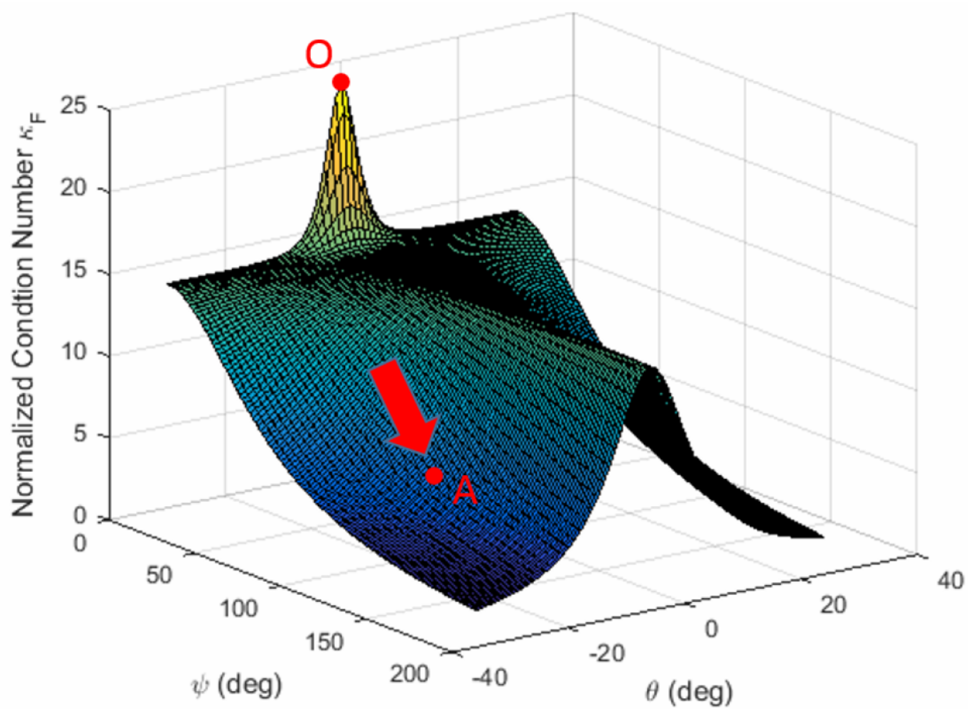
$$\boldsymbol{y}_k = [\, u_g(k) - V_a(k) \quad v_g(k) \quad w_g(k) \,]^T$$
$$\boldsymbol{\Omega} = \boldsymbol{\Omega}(V_a)$$
$$\boldsymbol{\Psi} = C_n^b$$

$$\kappa(\mathcal{G}_d) = \|\mathcal{G}_d\| \cdot \|\mathcal{G}_d{}^{-1}\|$$

$$\arg\min \kappa(\mathcal{G}_d)$$

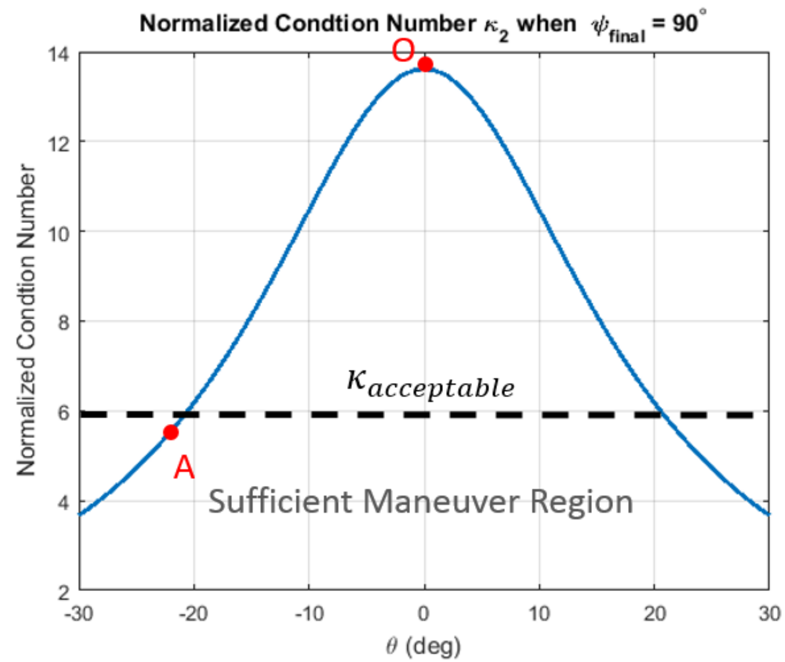
Maneuver	V_a	ψ	θ	ϕ
		2-D case		
1	Constant	Constant	Varying	— —
2	Constant	Varying	Constant	— —
3	Constant	Constant	— —	Varying
4	Varying	Constant	Varying	— —
5	Varying	Varying	Constant	— —
6	Varying	Constant	— —	Varying
		3-D case		
1	Constant	Varying	Varying	— —
2	Varying	Varying	Varying	— —
3	Varying	Varying	Varying	Varying

OBSERVABILITY ENHANCEMENT

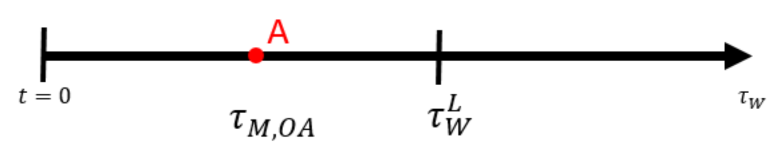


Normalized κ_F $(\mathcal{G}_d)$ over ψ *and* θ

Condition 1

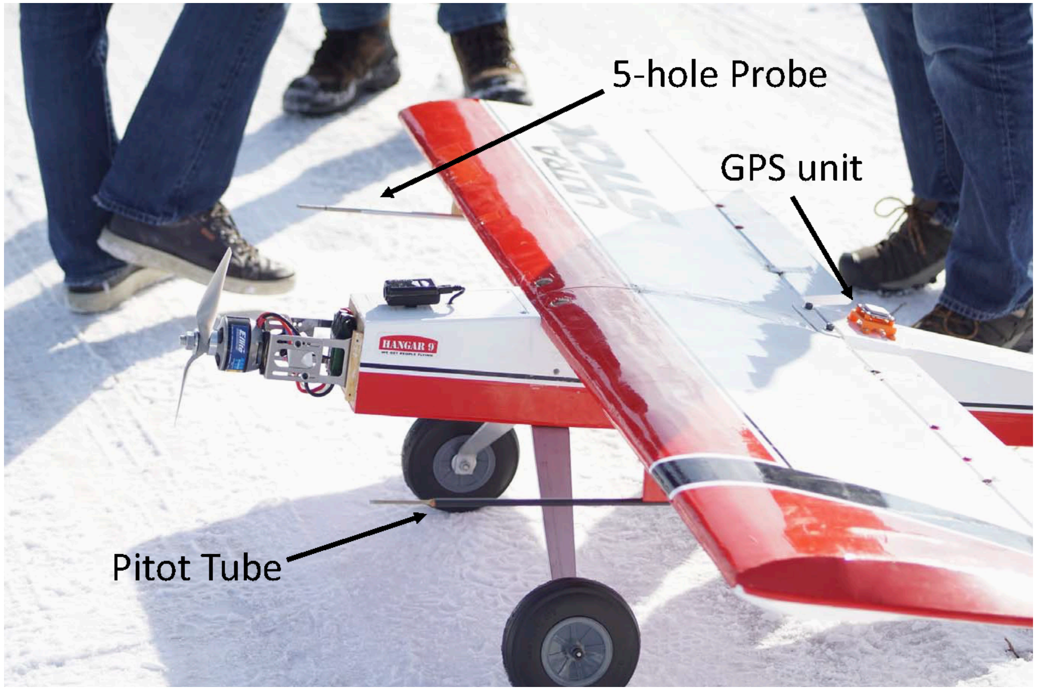


Condition 2



Acceptable maneuver condition: 1. $\boldsymbol{\kappa}_{(\cdot)} \leq \boldsymbol{\kappa}_{accept}$ 2. $\boldsymbol{\tau}_{M,(\cdot)} \leq \boldsymbol{\tau}_W^L$

FLIGHT TESTING VALIDATION

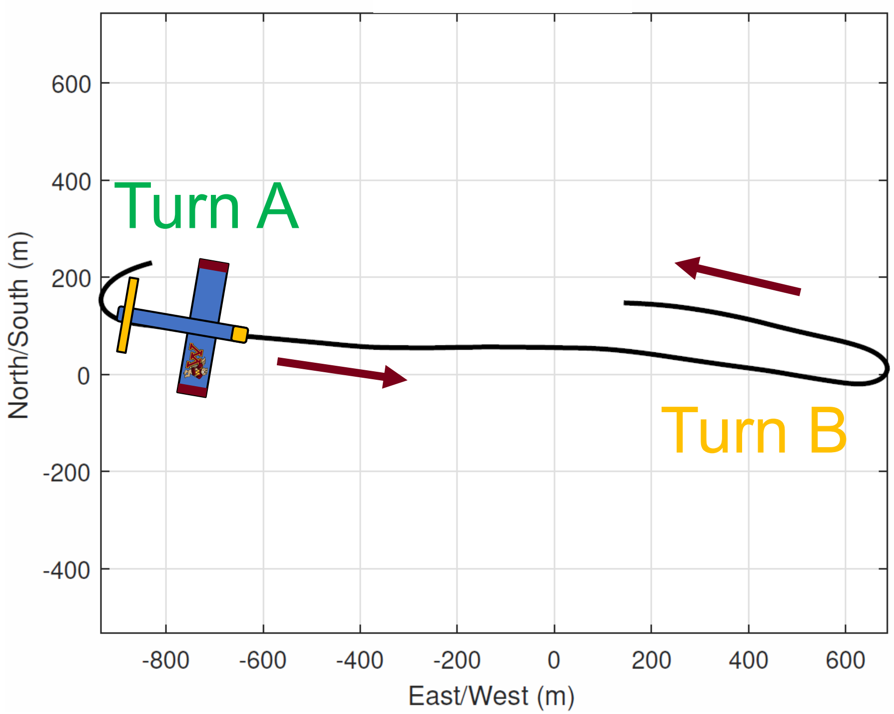


Ultra Stick 120e Testbed

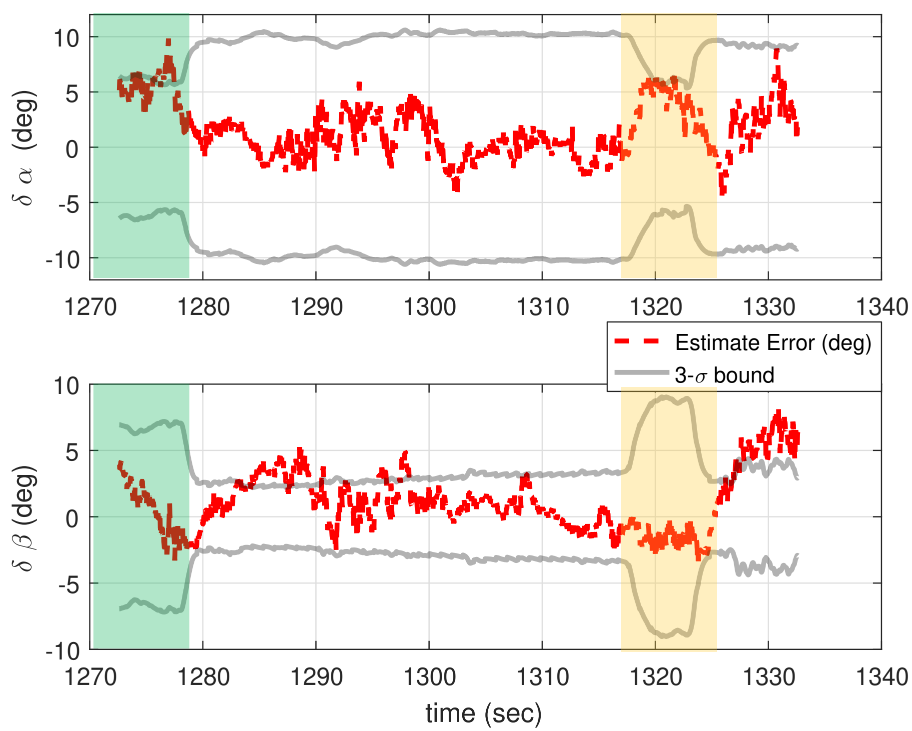
University of Minnesota Wind Turbine

Flight Test Results

Trajectory



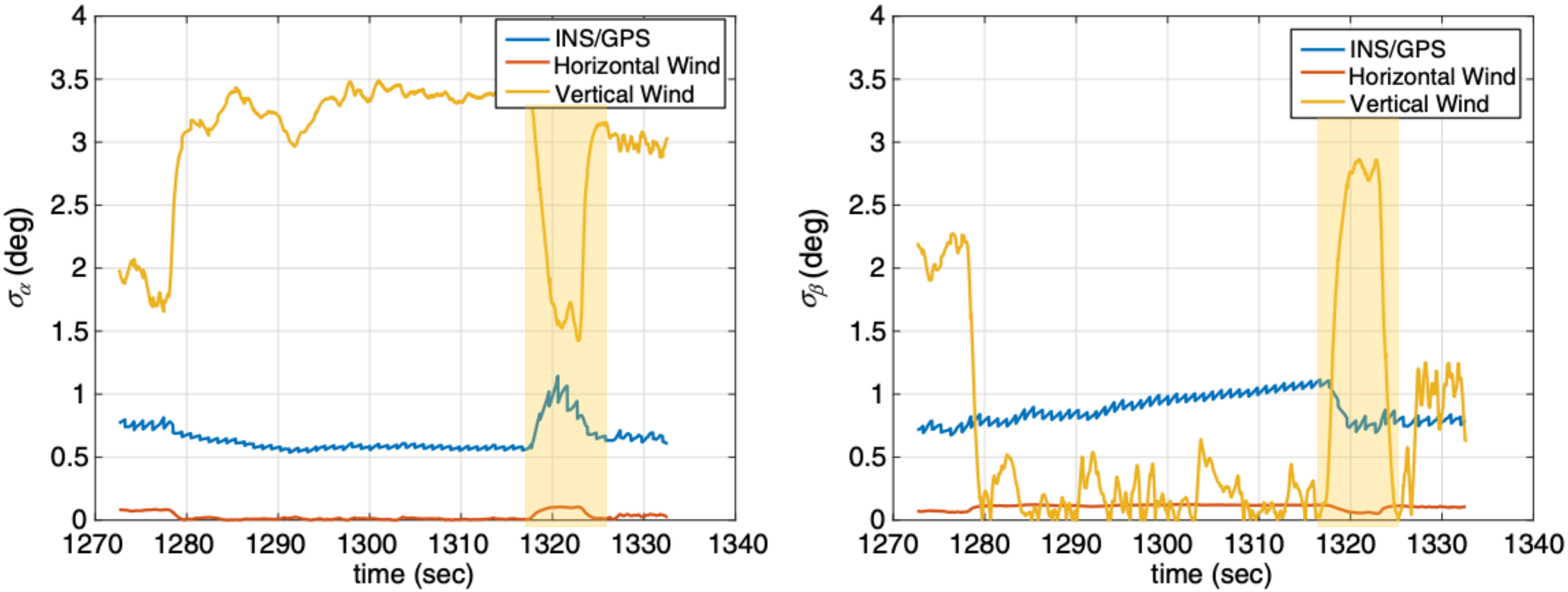
α and β estimation Error and 3-σ (99%) bounds



Accuracy over the flight region: $\sigma_\alpha = 3.07^\circ, \sigma_\beta = 1.28^\circ$

Flight test results- covariance Analysis

Break down of the effect on $\sigma_\alpha, \sigma_\beta$



Dominant Uncertainty	σ_α	σ_β
σ_{w_D}	Decrease	Increase

PERFORMANCE LIMITATION – AIRSPEED

- Increasing airspeed can potentially increase the accuracy of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$
- Small UAVs operate around 12 – 30 m/s, which inherently limits the performance of the estimator

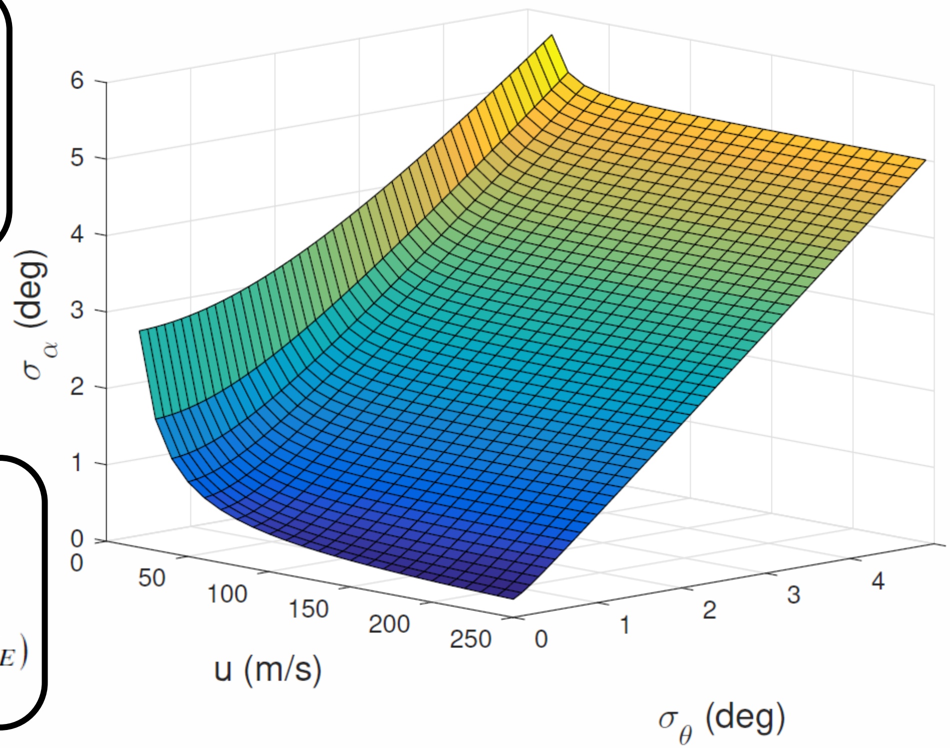
$$\delta\alpha = -\frac{w}{u^2 + w^2}\delta u + \frac{u}{u^2 + w^2}\delta w$$

$$\delta\beta = -\frac{uv}{V_a^2\sqrt{u^2 + w^2}}\delta u + \frac{u^2 + v^2}{V_a^2\sqrt{u^2 + w^2}}\delta v - \frac{vw}{V_a^2\sqrt{u^2 + w^2}}\delta w$$

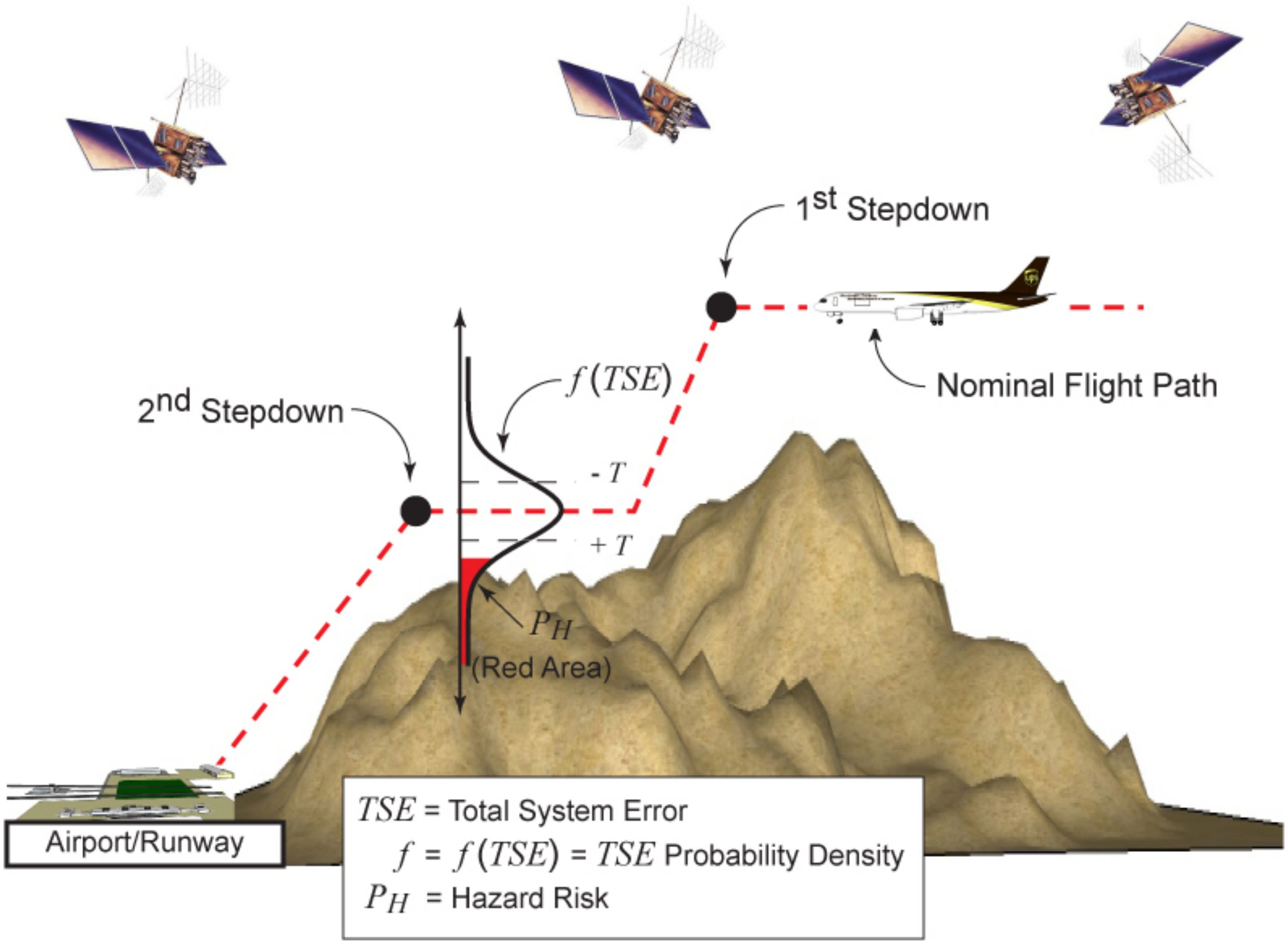
$$if\ u \gg v\ and\ u \gg w$$

$$\delta\alpha \approx \frac{1}{u}\delta w = \delta\theta + \frac{1}{u}\left(cos\theta\delta V_N - cos\theta\delta W_N - sin\theta\delta V_D + sin\theta\delta W_D\right)$$

$$\delta\beta \approx \frac{1}{u}\delta v = -\delta\psi + \frac{1}{u}\left(-sin\psi\delta V_N + sin\psi\delta W_N + cos\psi\delta V_E - cos\psi\delta W_E\right)$$



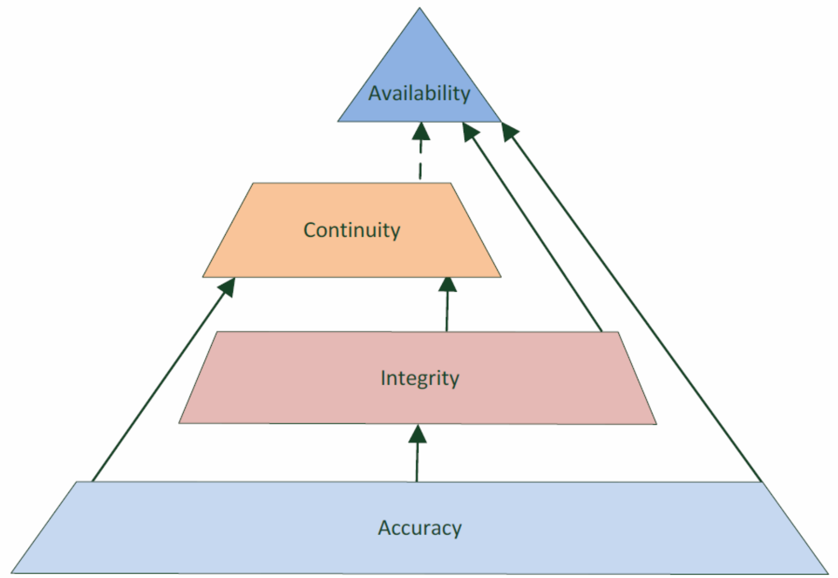
Safe Navigation



SAFE NAVIGATION REQUIREMENT

"HIGH-LEVEL" PERFORMANCE METRICS:

- **Availability** – – the percentage of time that the services of the system are usable by the navigator
- **Continuity** – performing its function without interruptions
- **Integrity** – correctness of the position, be able to alert
- **Accuracy** – position error



INTEGRITY RISK

Fault Detection | Fault Isolation

$$\mathrm{I} = P(MD) + P(MI|DF)P(DF)$$

CONTINUITY RISK

$$\mathrm{C} = P(FA) + P(NI|DF)P(DF)$$

MD = Missed Detection
DF = Detected Failure
MI = Mis−Identified failure
NI = Non−Isolable failure
FA = False Alarm

Assume no Isolation

Integrity Risk

$$\mathrm{I} = P(MD) + P(MI|DF)P(DF) = \mathrm{P(MD)}$$

Continuity Risk

$$\mathrm{C} = P(FA) + P(NI|DF)P(DF) = \mathrm{P(FA)} + \mathrm{P(DF)} \approx P(FA)$$

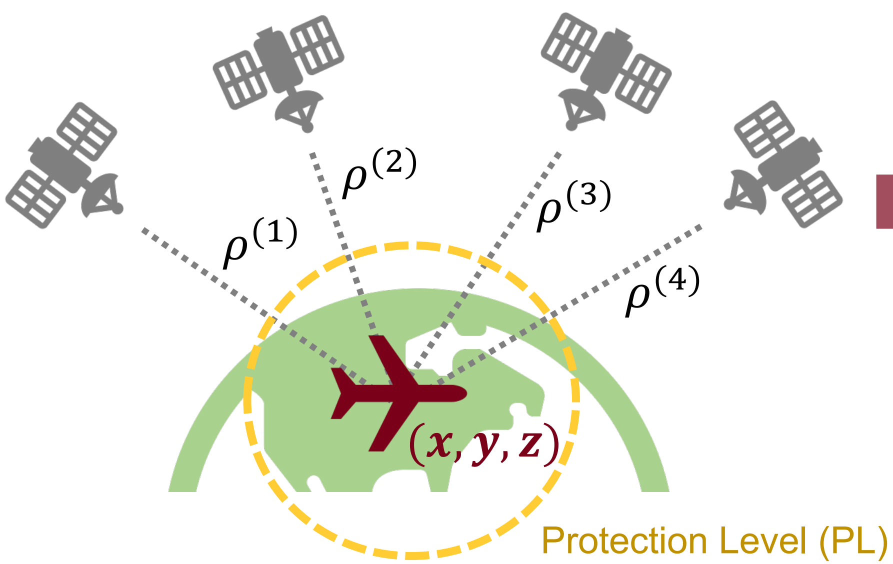
Where $P(MI|DF) = 0, P(NI|DF) = 1, P(DF) \approx 0$

MD = Missed Detection
DF = Detected Failure
MI = Mis−Identified failure
NI = Non−Isolable failure
FA = False Alarm

Example - GNSS Integrity Monitoring

Measurement Model

$$\rho^{(n)} = \sqrt{(x^{(n)} - x)^2 + (y^{(n)} - y)^2 + (z^{(n)} - z)^2} + b$$



Linearization

$$\underbrace{\begin{bmatrix} \delta\rho^{(1)} \\ \delta\rho^{(2)} \\ \delta\rho^{(3)} \\ \vdots \\ \delta\rho^{(K)} \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} -\frac{(x^{(1)}-x_0)}{\rho_0^{(1)}} & -\frac{(y^{(1)}-y_0)}{\rho_0^{(1)}} & -\frac{(z^{(2)}-z_0)}{\rho_0^{(1)}} & 1 \\ -\frac{(x^{(2)}-x_0)}{\rho_0^{(2)}} & -\frac{(y^{(2)}-y_0)}{\rho_0^{(2)}} & -\frac{(z^{(2)}-z_0)}{\rho_0^{(2)}} & 1 \\ -\frac{(x^{(3)}-x_0)}{\rho_0^{(3)}} & -\frac{(y^{(3)}-y_0)}{\rho_0^{(3)}} & -\frac{(z^{(2)}-z_0)}{\rho_0^{(3)}} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{(x^{(K)}-x_0)}{\rho_0^{(K)}} & -\frac{(y^{(K)}-y_0)}{\rho_0^{(K)}} & -\frac{(z^{(2)}-z_0)}{\rho_0^{(K)}} & 1 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} \left.\begin{matrix} \delta x \\ \delta y \\ \delta z \end{matrix}\right\} \delta \mathbf{p} \\ \delta b \end{bmatrix}}_{x}$$

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v} + \mathbf{b}$$

$\mathbf{v} \sim \mathbf{N}(\mathbf{0}, \mathbf{R})$

$\mathbf{b}$: bias (fault)

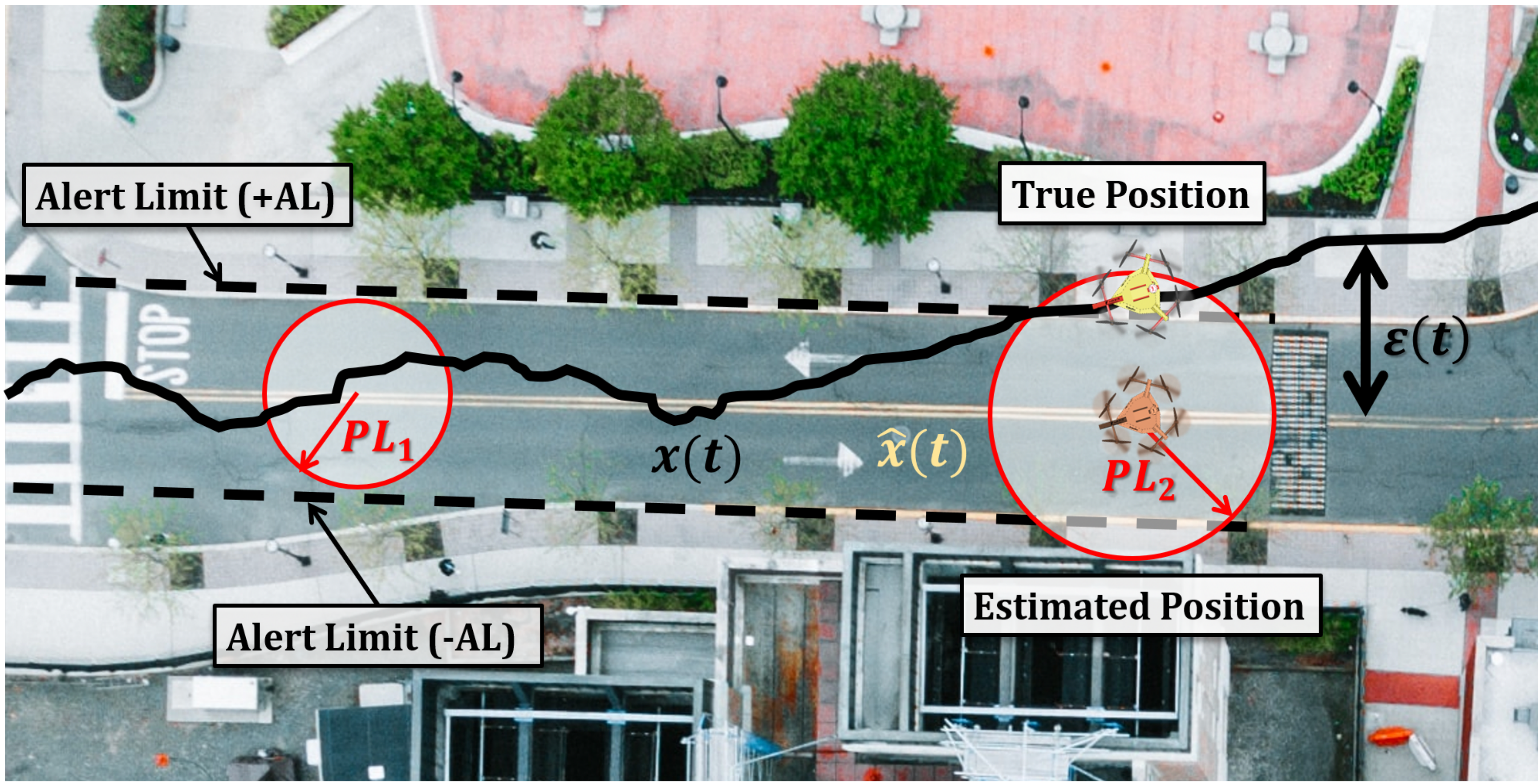
Dual Hypotheses

$$\boldsymbol{D} = \mathbf{z}^{\mathrm{T}}\mathbf{W}(\mathbf{I} - \mathbf{H}\mathbf{H}^{*})\mathbf{z} \begin{cases} \boldsymbol{D} \le \boldsymbol{T}: \text{No fault is present } (\mathbf{H}_0) \\ \boldsymbol{D} > \boldsymbol{T}: \text{ A fault is present } (\mathbf{H}_1) \end{cases}$$

$\boldsymbol{D}$: detection function, $\boldsymbol{T}$: threshold, $\boldsymbol{W} = \boldsymbol{R}^{-1}$: weight

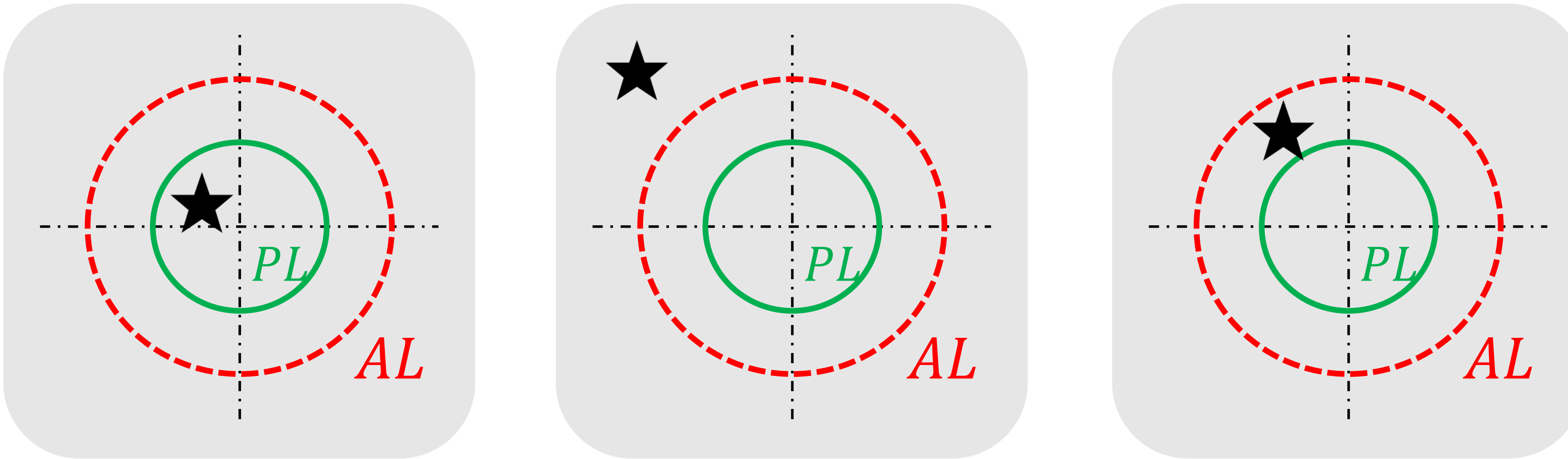
Protection Levels

- Protect the user from experiencing position errors that are greater than the Alert Limit (AL)
- Provide ***Protection Levels (PL)*** for the the vehicle

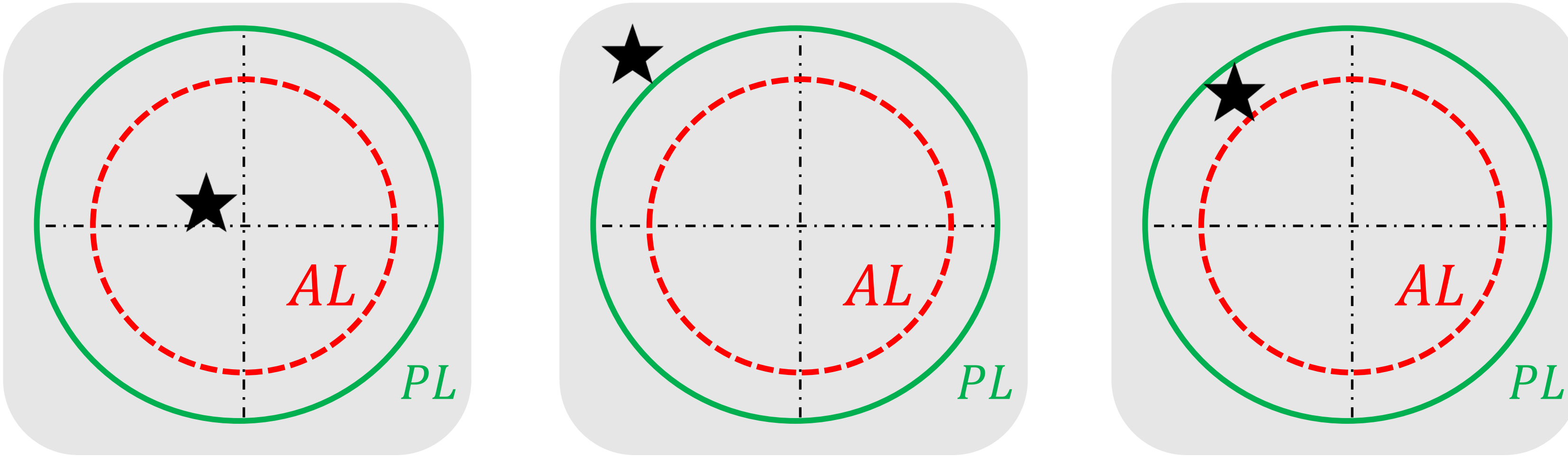


POSSIBLE CASES OF ESTIMATE & BOUNDS

Available

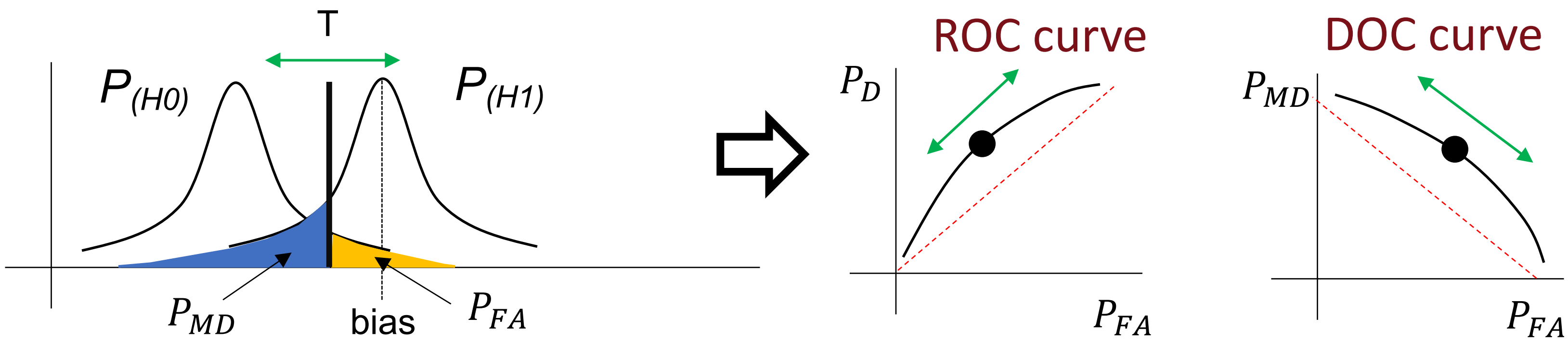


Not Available



TRADEOFF ANALYSIS – VIA DOC CURVES (2A)

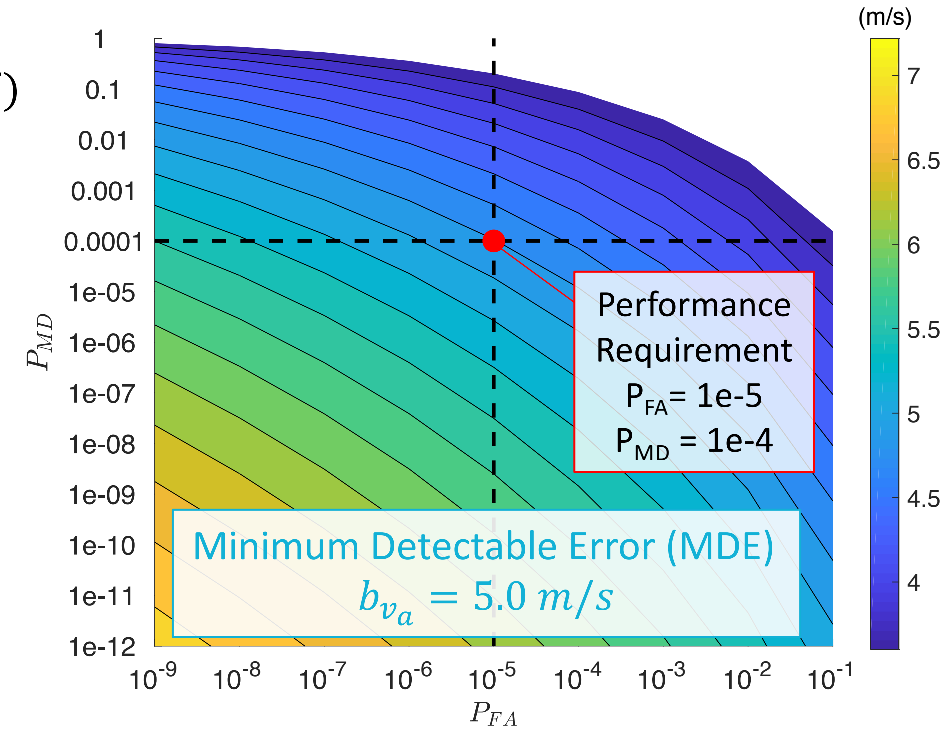
Objective: determine Threshold $\boldsymbol{T}$ and Minimum Detectable Error requirement MDE_{req}



$$P_{MD} = F_{\chi^2(\lambda)}(T, Df) = F_{\chi^2(\lambda)}(F_{\chi^2}^{-1}(1 - P_{FA}, Df), Df)$$

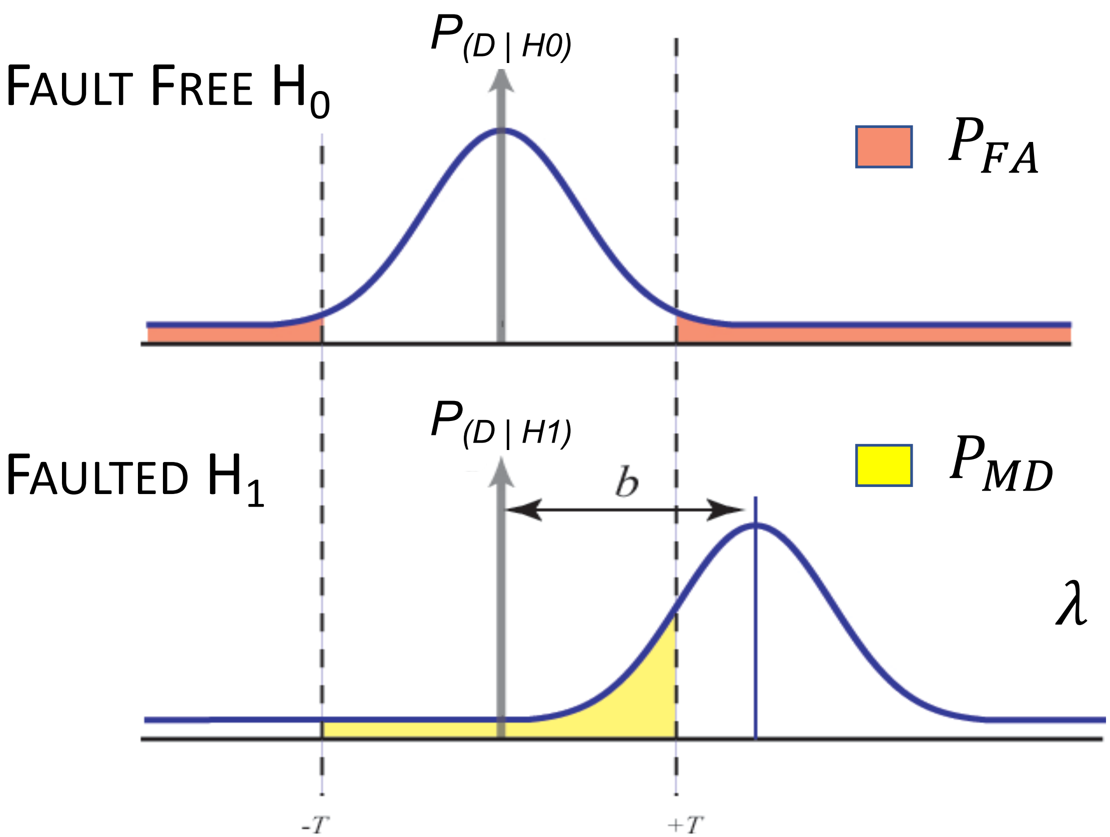
$$\overline{MDE} \triangleq \sqrt{\lambda} = \sqrt{\sum_{j=k-q+1}^{k} \frac{f_{V_{aj}}^2}{\sigma_{V_{aj}}^2}} \qquad MDE_{req} \triangleq \mathrm{f}_{V_{ak}}$$

Requirement (P_{MD}, P_{FA})	**Allowable (airframe)**
MDE_{req} =5m/s	AE = 7.5m/s
τ_{req}= 2s	τ_{allow} = 3s



Df: Degree-of-freedom. q: window size

Tradeoff of P_{FA} and P_{MD}



P_{FA} = Probability of False Alarm
P_{MD} = Probability of Missed Detection
P_D = Probability of Detection
b = Range bias
T = Threshold

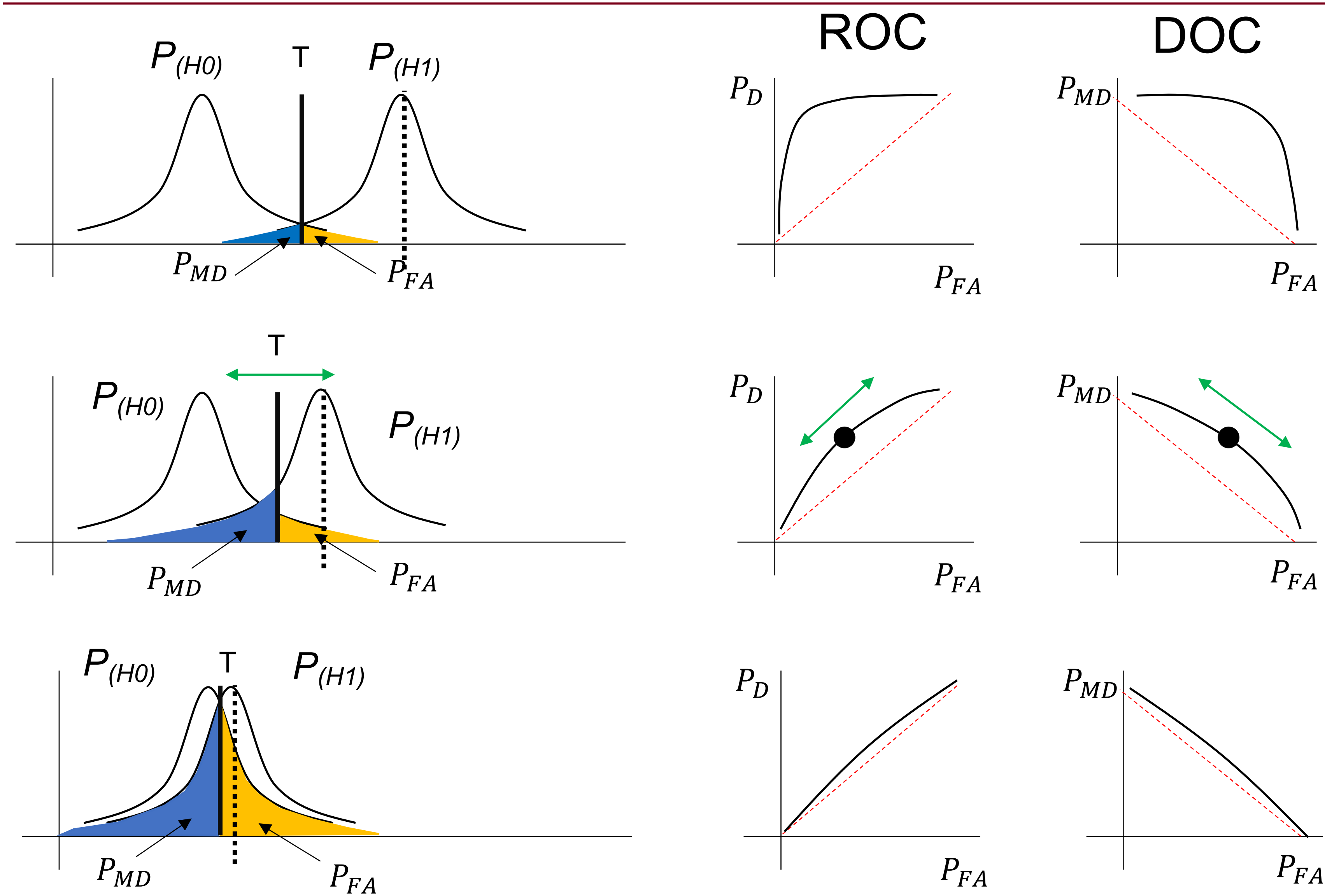
Associate Equation:

$$\chi^2_{P_{MD},n-m,\lambda} = \chi^2_{1-P_{FA},n-m}$$

$$\lambda = b^2 = \left(n_{1-\frac{\alpha_0}{2}} + n_{1-P_{MD}}\right)^2$$

Threshold	Integrity	Continuity
Small / Tight	Increases (lower P_{MD})	Decreases (higher P_{FA})
Large / Loose	Decreases (higher P_{MD})	Increases (lower P_{FA})

ROC/DOC curves Illustration



Performance Evaluation- Detector

Detector/Test Statistic

#1: $\boldsymbol{D}_{\boldsymbol{\gamma},k}|H_0 = \frac{1}{q}\sum_{j=k-q+1}^{k}\gamma_j^T S_j^{-1}\gamma_j$

Where $\gamma_k = z_k - \mathrm{H}_k\hat{x}_k^-$

#2: $\boldsymbol{D}_{\boldsymbol{r},k}|H_0 = \boldsymbol{r}_{k-q:k}^T\Sigma^{-1}\boldsymbol{r}_{k-q:k}$

Where $r_k = z_k - \mathrm{H}_k\hat{x}_k^+$

#3: $\boldsymbol{D}_{\boldsymbol{r},\boldsymbol{k}}^{\boldsymbol{GMA}}|H_0 = \left(\boldsymbol{r}_{k-q:k}^{GMA}\right)^T\Sigma^{-1}\boldsymbol{r}_{k-q:k}^{GMA}$

Where $\boldsymbol{r}_{k-q:k}^{GMA} = \left[\mu^q\boldsymbol{r}_{k-q}^T \cdots \mu\boldsymbol{r}_{k-1}^T\,\boldsymbol{r}_k^T\right]^T$

Faulty Airspeed Estimation

Assume: $t_{detect} = 618\ s$

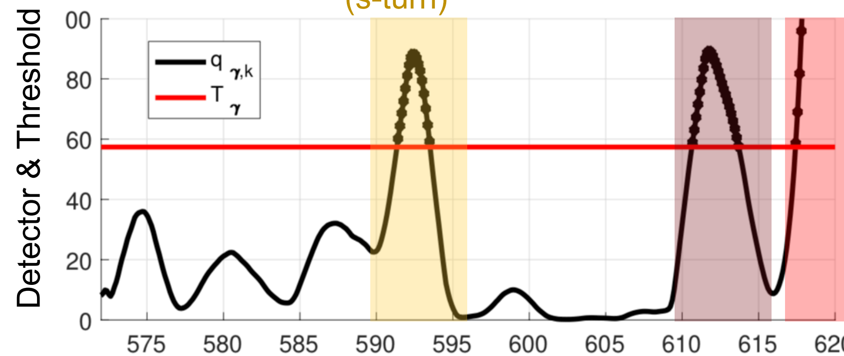
Then:

$t_{fault} = t_{detect} - \tau_{req} = 618 - 2 = 616\ s$

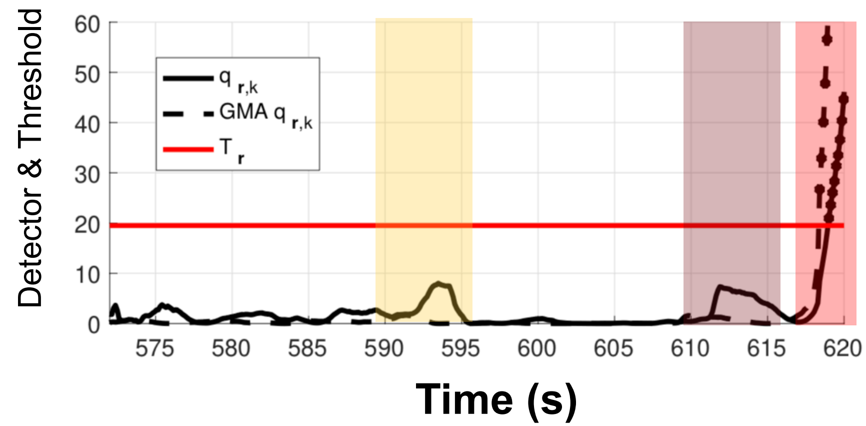
$\Longrightarrow V_{a,0} = 15\ m/s$

$\Longrightarrow V_{a,1} = 15 - MDE_{req} = 15 - 5 = 10\ m/s$

Innovation Test

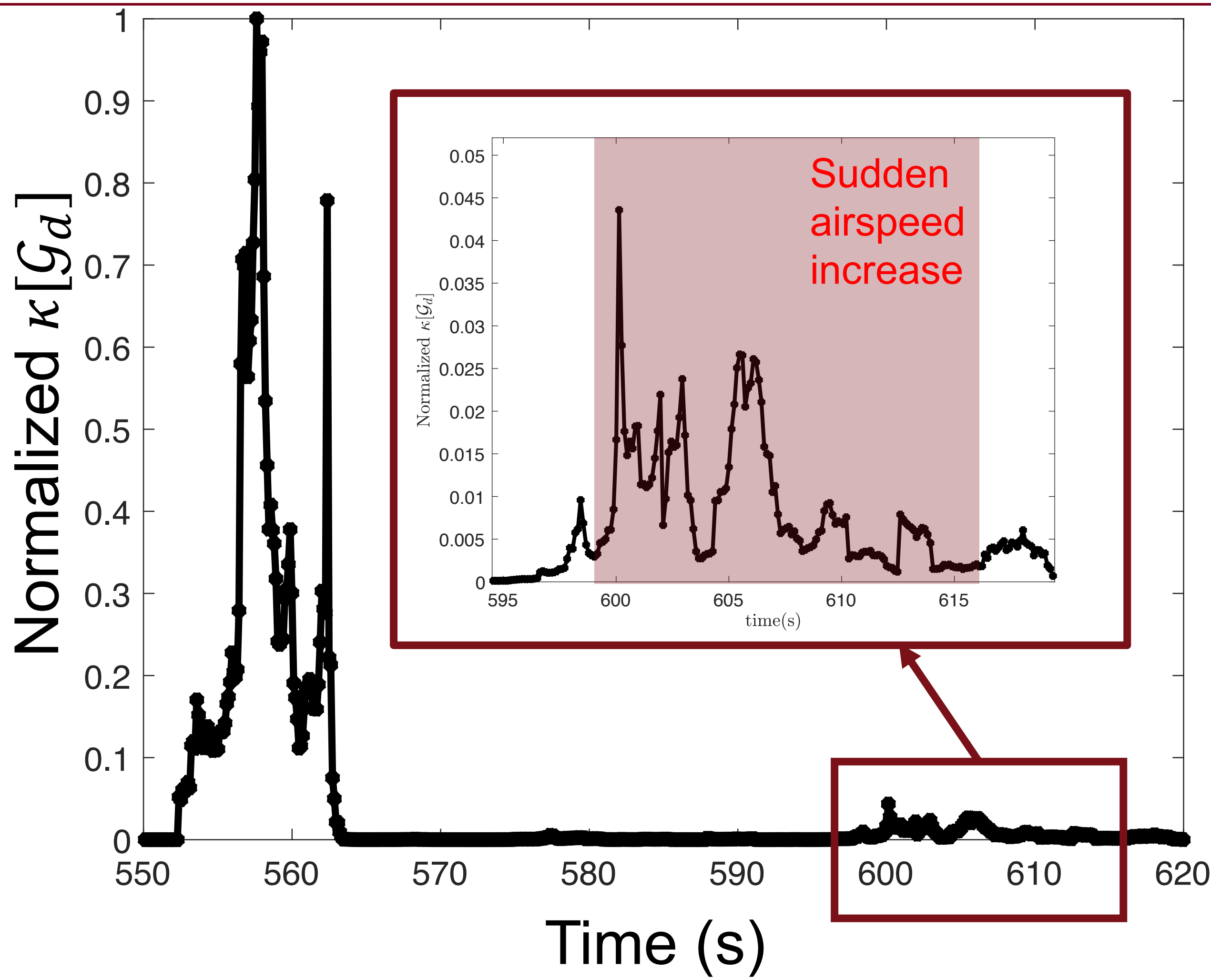


Residual-based Test



Parameter	Estimated Start	Estimated Detection	Actual Recorded
Time (s)	616	618	618
Airspeed (m/s)	15	10	-10

OBSERVABILITY MONITORING



Monitoring Observability can potentially help rule out false alarm

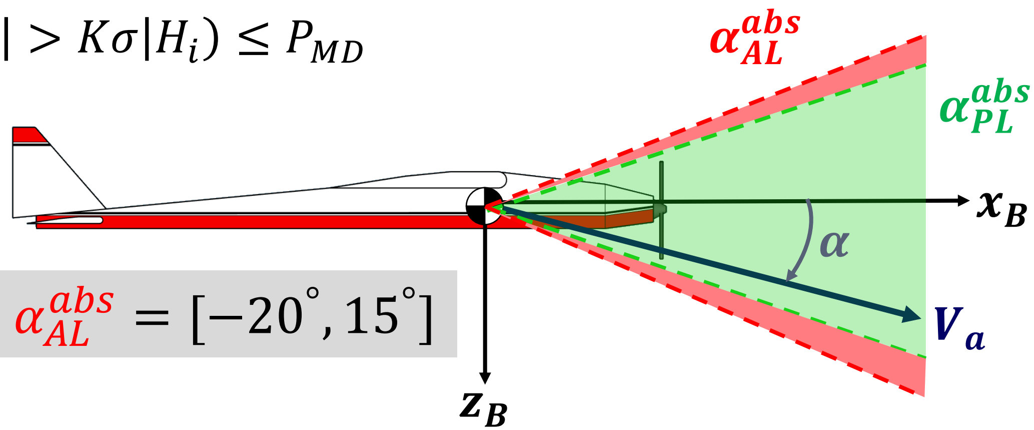
PROTECTION LEVEL CALCULATIONS (2C)

OBJECTIVE: Provide some guarantee on ϵ

Protection Level (PL): Guaranteed upper bound of estimation error (i.e., $PL = K\sigma$)

Alert Limit (AL): the maximum error that can be tolerated (i.e., PL < AL)

Mathematically: $P(|\epsilon| > PL|H_i) = P(|\epsilon| > K\sigma|H_i) \leq P_{MD}$



$$\alpha_{AL}^{abs} = [-20^{\circ}, 15^{\circ}]$$

INFLATION FACTOR K CALCULATION:

$$P(|\epsilon_\alpha| > \alpha_{PL}|H_0)$$
$$= 2P\left(\frac{\epsilon_\alpha}{\sigma} < \frac{\alpha_{PL}}{\sigma}|H_0\right)$$
$$\leq 2P\left(\frac{\epsilon_\alpha}{\sigma} < \frac{\alpha_{PL}}{\sigma}\right)P(H_0)$$
$$\leq 2P\left(\frac{\epsilon_\alpha}{\sigma} < \frac{\alpha_{PL}}{\sigma}\right)$$
$$= 2Q\left(\frac{\alpha_{PL}}{\sigma}\right) = P_{MD} \Longrightarrow \frac{\alpha_{PL}}{\sigma} = \mathrm{Q}^{-1}\left(\frac{P_{MD}}{2}\right) \Longrightarrow \boxed{K_{\alpha,0} = \mathrm{Q}^{-1}\left(\frac{P_{MD}}{2}\right)}$$

ABSOLUTE PL: $P(|\epsilon_\alpha| > \alpha_{PL}|H_0) = P(|\epsilon_\alpha| + \hat{\alpha} > \alpha_{PL} + \hat{\alpha}|H_0) = P\left(\alpha > \alpha_{PL}^{abs}\middle|H_0\right)$

	H_0(Fault Free)	H_1(Faulted)
$\boldsymbol{\alpha_{PL}}$	$K_{\alpha,0}\sigma_\alpha$	$\frac{\sigma_\alpha}{\sqrt{D_k}}\sqrt{\lambda_U} + K_{\alpha,1}\sigma_\alpha$

Q: right hand cdf of a normal standard distribution

AL, PL AND INTEGRITY REQUIREMENT

$$P(|x-\hat{x}| \geq AL, y \in R) \leq 2Q\left(\frac{AL}{\sigma^{(0)}}\right) + \sum_{k=1}^{K} Q\left(\frac{AL-T}{\sigma^{(k)}}\right) p_k \leq P_{HMI} = I$$

$$2Q\left(\frac{PL}{\sigma^{(0)}}\right) + \sum_{k=1}^{K} Q\left(\frac{PL-T}{\sigma^{(k)}}\right) p_k = P_{HMI} = I$$

p_k: threat model (failure rate)
σ: nominal error model
T: threshold (false alarm)
P_{HMI}: integrity requirement (probability of hazard misleading information

Protection level calculation (H1)

$$\lambda = \mathbb{E}\left[\mathbf{r}_{k-q:k}^{T}\boldsymbol{\Sigma}^{-1}\mathbf{r}_{k-q:k}\right]$$

$$\lambda = \mathbf{E}_7\mathbf{F}^T\mathbf{E}_7^T\boldsymbol{\Sigma}^{-1}(\mathbf{I} - \boldsymbol{\mathcal{O}}\boldsymbol{\mathcal{O}}^*)\mathbf{E}_7^T\mathbf{F}\mathbf{E}_7$$

$$\leq \mathbf{E}_7\mathbf{F}^T\mathbf{E}_7^T\boldsymbol{\Sigma}^{-1}\mathbf{E}_7^T\mathbf{F}\mathbf{E}_7$$

$$\leq \mathbf{E}_7\mathbf{F}^T\mathbf{E}_7^T\boldsymbol{R}^{-1}\mathbf{E}_7^T\mathbf{F}\mathbf{E}_7$$

$$\leq \frac{f_{V_a}^2}{\sigma_{V_a}^2}q = \lambda_U$$

$$\overline{MDE} \triangleq \sqrt{\lambda}$$

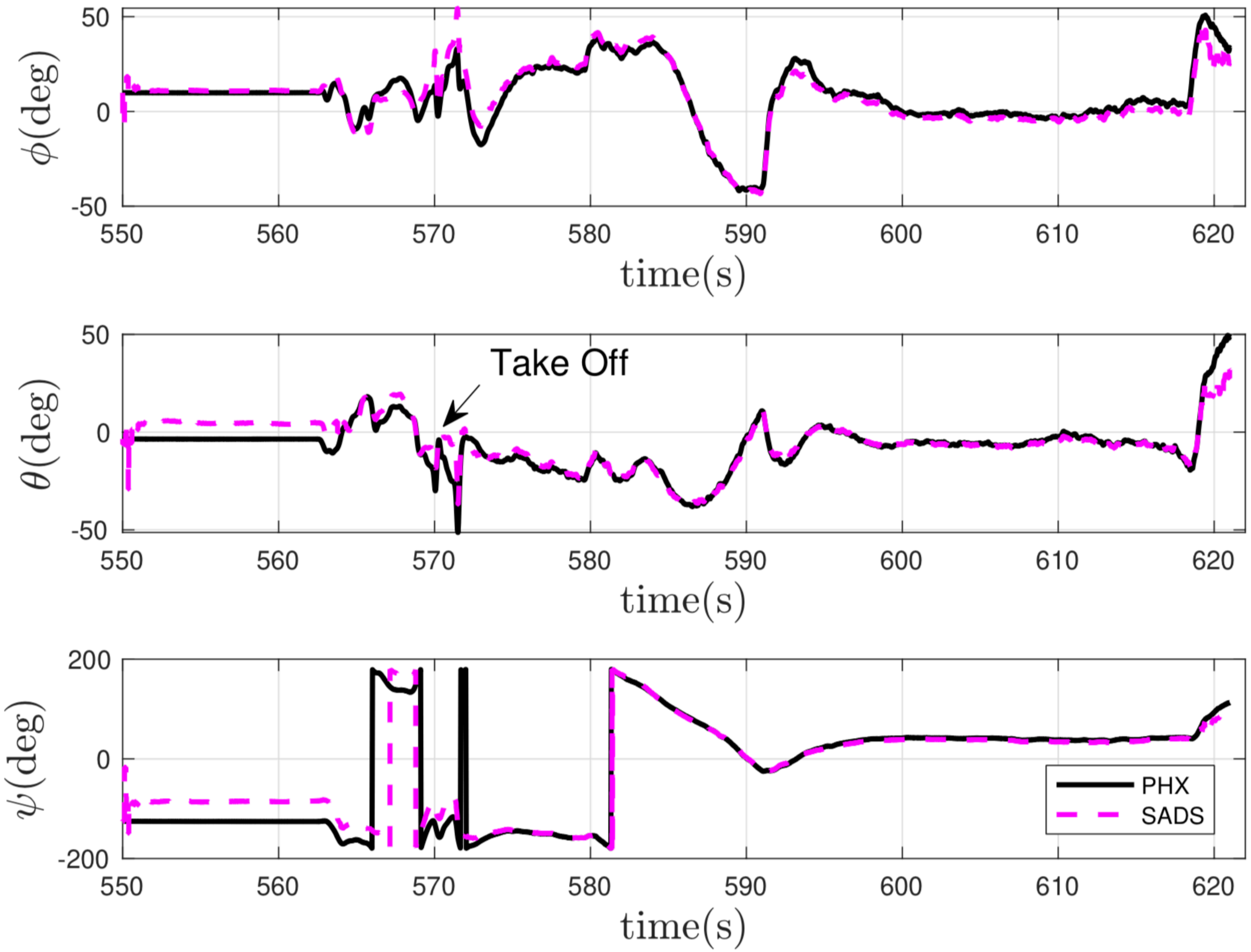
$$\boldsymbol{D}_{\boldsymbol{r},k} = \boldsymbol{r}_{k-q:k}^{T}\Sigma^{-1}\boldsymbol{r}_{k-q:k}$$

$$\frac{\sigma_\alpha}{\sqrt{\boldsymbol{D}_{\boldsymbol{r},k}}} = \frac{\sigma_{\alpha,U}}{\sqrt{\lambda_U}}$$

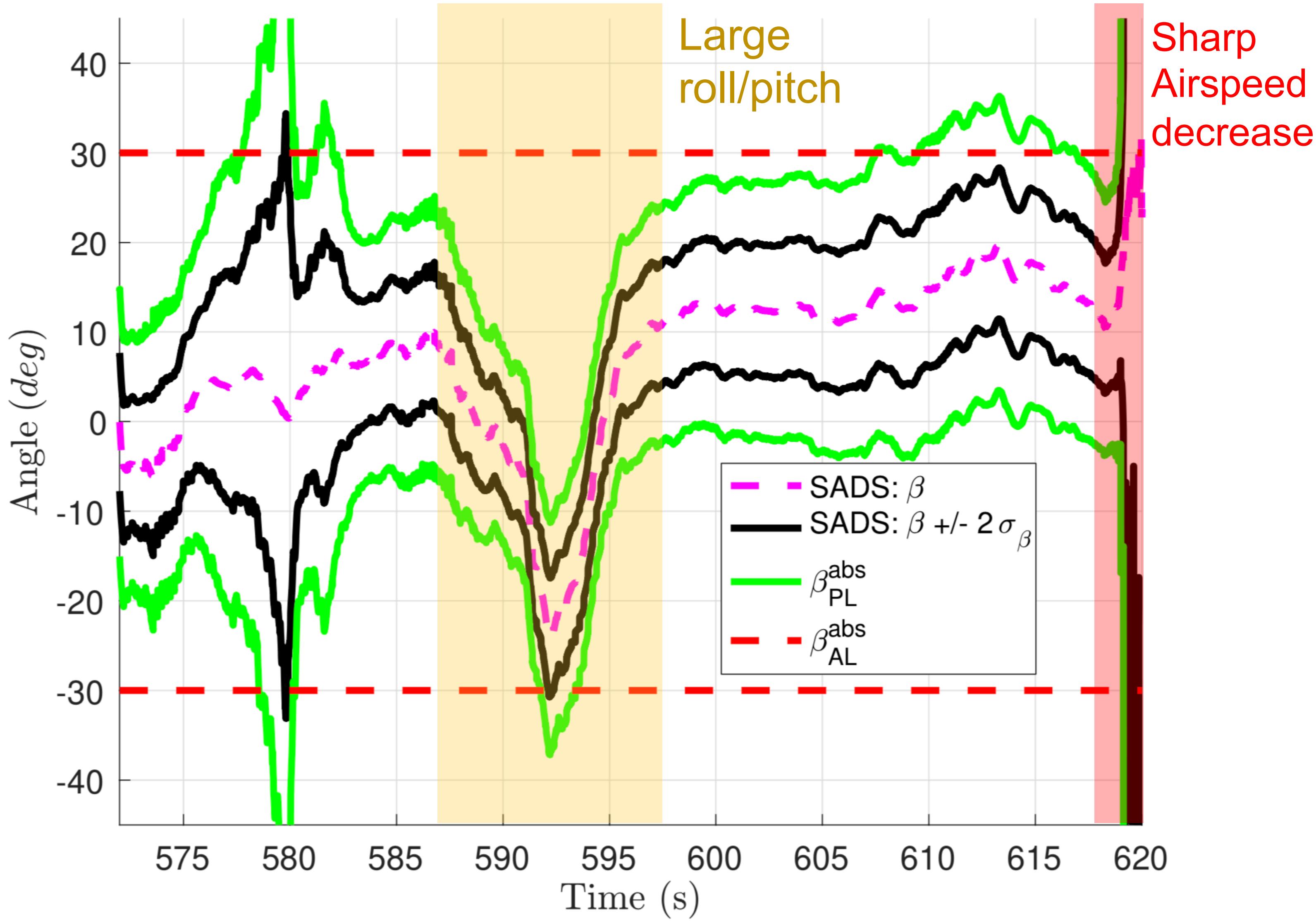
$$\Downarrow$$

$$\sigma_{\alpha,U} = \frac{\sigma_\alpha}{\sqrt{\boldsymbol{D}_{\boldsymbol{r},k}}}\sqrt{\lambda_U}$$

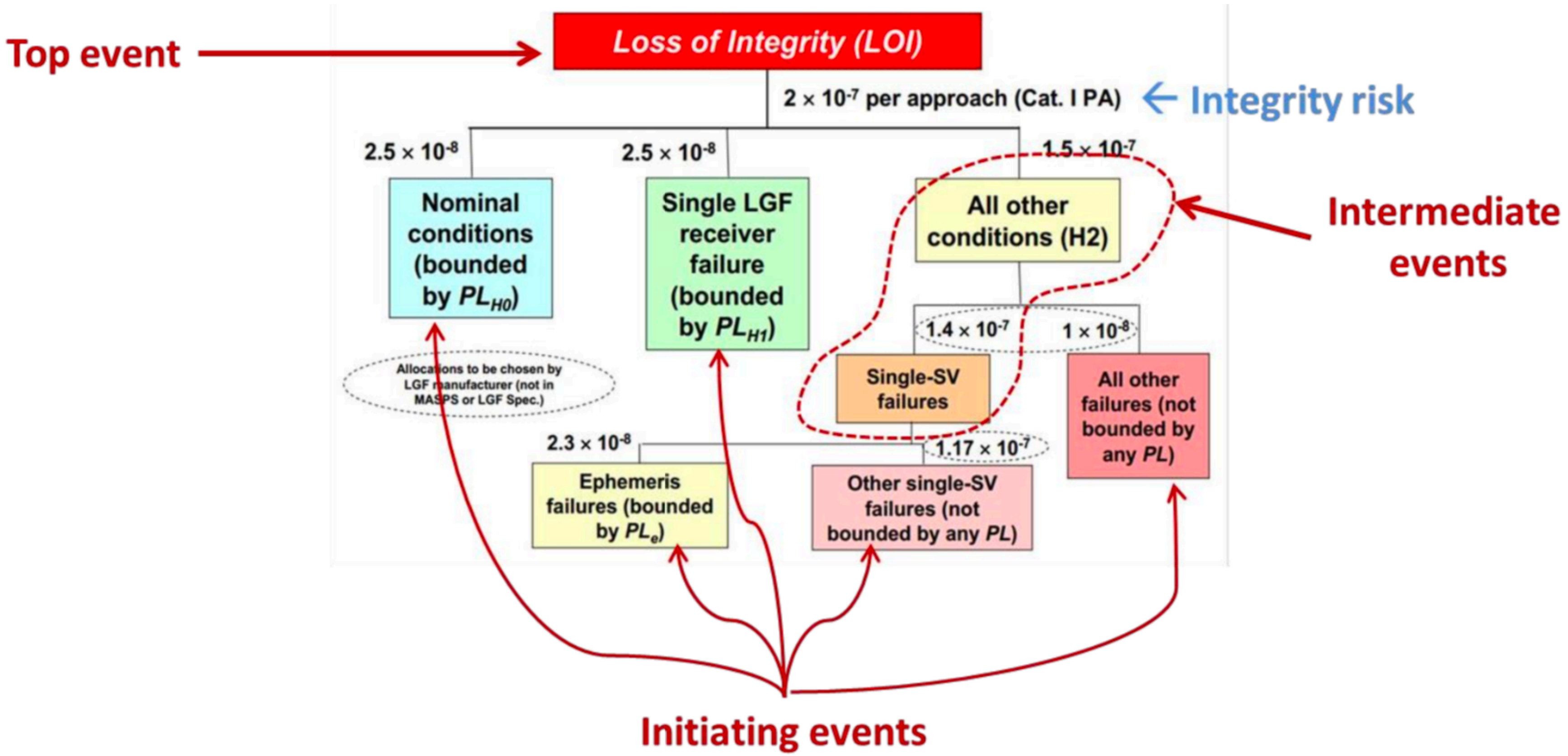
Flight test attitude



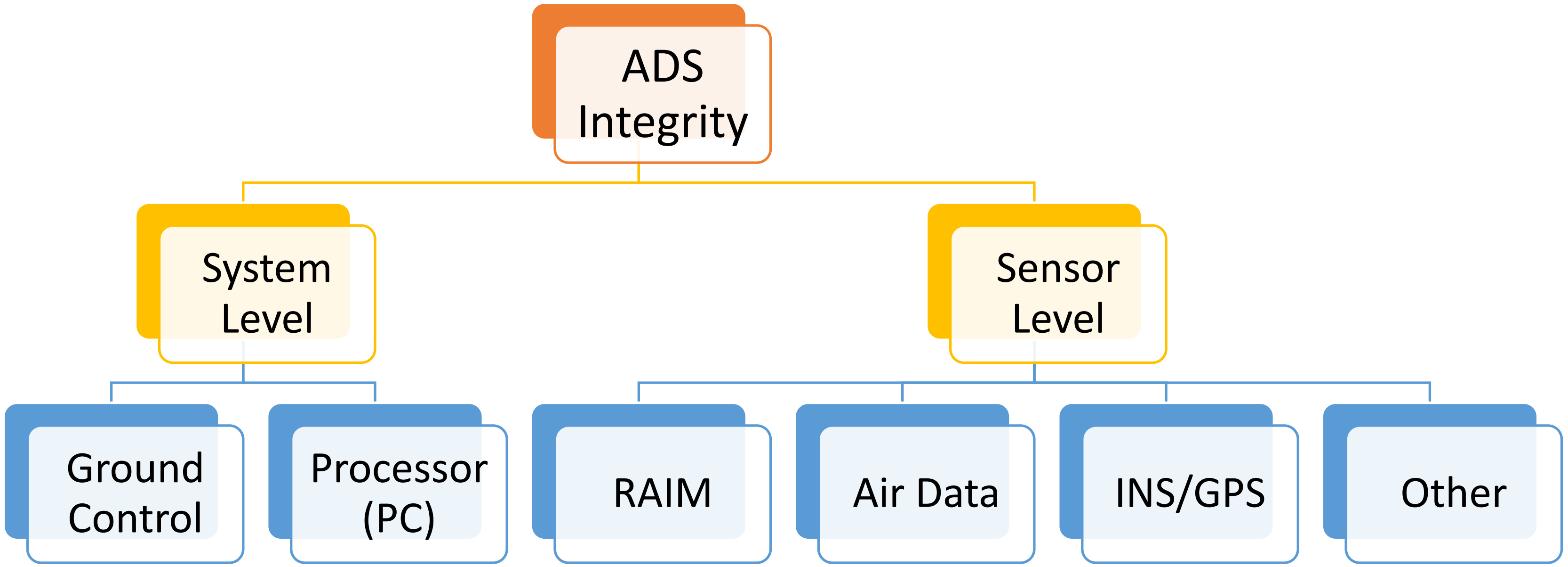
Protection Level of Sideslip



INTEGRITY FAULT TREE EXAMPLE



ADS INTEGRITY



OBSERVABILITY

Linear System

$$\dot{x}(t) = Ax(t) + Bu(t), x \in \mathbb{R}^n$$
$$y(t) = Cx(t) + Du(t), y \in \mathbb{R}^m$$

LTI Observability:

- Observable if and only if $\mathcal{O} = [C; CA; \dots; CA^{n-1}] = n$

LTV Observability:

- Observe if rank(W_o)$= n$ where $W_o = \int_0^\infty e^{A^T\tau} C^\tau C e^{A\tau} d\tau,$

Degree of LTV Observability:

- Usually measured by condition number, trace, etc.
  - e.g. $\kappa(W_o)$ is a measure of state observability

Stochastic Observability:

$$\sigma_{max}(P_k) < T$$

GEOMETRIC CONTROL

Nonlinear system

$$\dot{x}(t) = f(x(t)) + \sum_{j=1} g_j(x(t))\, u_j(t), x \in \mathbb{M}^n$$
$$y(t) = h(x(t)) + \sum_{j=1} g_j(x(t))\, u_j(t), y \in \mathbb{M}^m$$

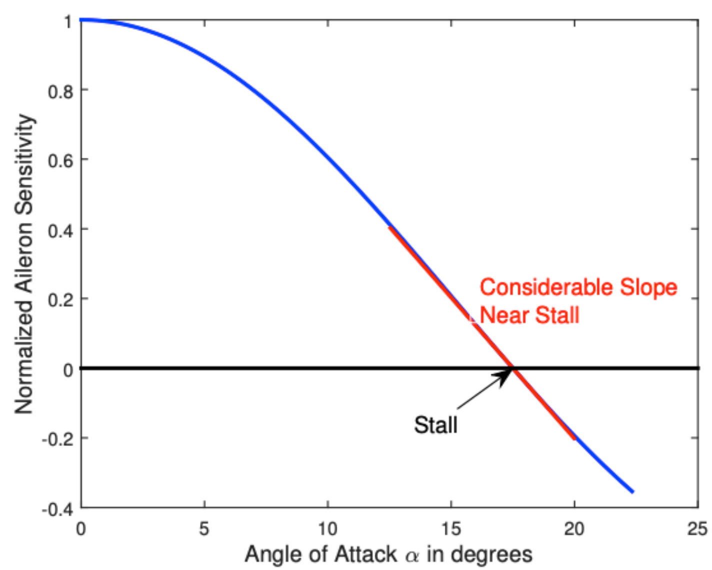
Using Lie bracket $[g_i, g_k] = \frac{\partial g_k}{\partial x} g_j - \frac{\partial g_j}{\partial x} g_k$ to recover nonlinear controllability (unactuated direction)

Ex. Nonlinear Interactions of Flight Control

Old: $C_L = C_{L_a} u_a$

New: $C_L = \mathrm{k} \frac{\partial C_{L_a}}{\partial \alpha} u_{[g_e, g_a]}$

Can we flip this and find similar "new" measurement for observability?



(a) $C_{\mathcal{L}_a}$ versus α.

1

1.Taha (2020)

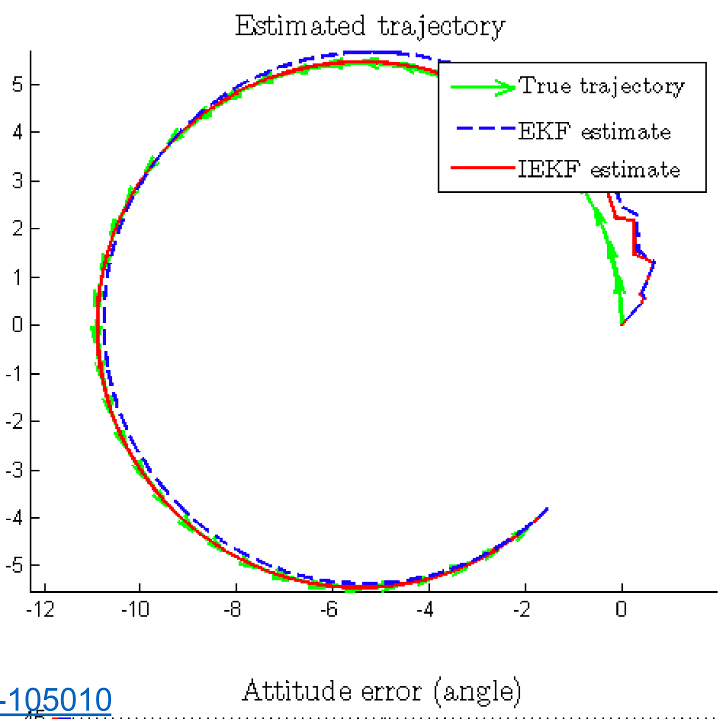
DIFFERENTIAL GEOMETRY IN KALMAN FILTERING

- Invariant Extended Kalman Filtering (since 2008)
  - Used for nonlinear systems possessing symmetries (invariances)
  - Benefit: converge on a bigger set of trajectories than equilibrium points, which leads to a better convergence
  - Uses differential geometry
  - Applications: Lie group-based attitude estimation

Ex. 2D Wheel robot

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2$$

$$\frac{d}{dt}\xi_t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \omega_t \\ -u_t & -\omega_t & 0 \end{pmatrix} \xi_t .$$



https://arxiv.org/pdf/1410.1465.pdf
https://www.annualreviews.org/doi/pdf/10.1146/annurev-control-060117-105010

Proposed Research Questions

- Notion of Degree of controllability - Is there a inconsistency between degree of observability and controllability?
- Connection between Geometric control and Invariant Kalman Filtering via differential geometry