Homework 3

Monte Carlo Simulation

Consider a non-dividend-paying stock with current stock price S_0 =\$50, volatility σ =0.3, strike price K=\$52, time to maturity T=2 years, interest rate r=5%.

To simulate the path followed by S, we can divide the life of the derivative into N short intervals of length Δt and simulate the stock price in a risk-neutral world at each time

$$S(t+\Delta t) = S(t)\exp\left[\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right]$$

where ε is a random sample from a normal distribution with mean zero and standard deviation of 1.0.

(50%)

I. Standard Error and Number of Trials

(a) Please use Monte Carlo simulation to compute the European put option prices by sampling 100 random paths (NP=100). For each sample path, you can obtain a payoff of the put option at maturity. Next compute the estimate of the value of put option by averaging sample payoffs of these 100 sample paths (NP=100). Discount this estimate at the risk-free rate. This is the estimate of the put option value of your experiment.

(b) Compute and report the standard error of the estimated put option value as well as the 95% confidence interval. Please also plot a histogram of the terminal value of stock prices. Please set the number of bins to 100.

(c) Next, change the number of sample paths (NP) to 10,000 and 1,000,000, redo the experiment, and report your results. Please still keep the number of bins of histogram as 100 in (b). Compare your results with the option value of the Black-Scholes formula (see section 15.8) and briefly explain your findings.

(50%)

II. Standard Deviation of Estimated Option Values

(d) Repeat the experiment in (a) for 500 times (NE), and compute the mean and standard deviation of the estimated put option prices of these 500 experiments.

(e) Redo (d) and increase the number of sample paths (NP) of each experiment in (a) to 10,000 and 1,000,000 (still repeat the experiment for 500 times (NE)).

* Please report the first 30 results (the estimates of European put values in (a)) under these three alternative settings, namely, (i) 100 (ii) 10,000 (iii) and 1,000,000 sample paths. Next, summarize in a table the means and standard deviations of the put option values under these three settings. Briefly explain your results.

*Note that in doing European call or put option valuation, it is **not** necessary to sample a whole path for *S*. The payoff at time *T* only depends on the value of *S* at time *T*. Therefore, you can jump straight from the value of *S* at time zero to its terminal value at time *T* as follows:

$$S(T) = S(0)\exp\left[\left(r - \frac{\sigma^2}{2}\right)T + \sigma\varepsilon\sqrt{T}\right]$$

This can make your program much faster in simulation!

Matlab function and syntax:

1. mean(): Average or mean value of arrays
2. std(): standard deviation

*You have to submit your homework and **programs by e3**. Your computer program is part of this assignment.