Homework 4
Binomial Trees

I. Basics

(a) Derivation of (Eq.13.2 and Eq.13.3) (15%)

In the binomial model, suppose that the initial stock price is S_0, and the life of the option is T. S_0 can either move up from S_0 to a new level, $S_0 u$, where $u > 1$, or down to a new level, $S_0 d$, where $0 < d < 1$. Suppose the payoff from option is f_u in the up state, and is f_d in the down state. Denote the risk-free rate by *r*.

Please construct a riskless portfolio in a one-step tree and show **<u>in detail</u>** that $f = e^{-rT}[pf_u + (1-p)f_d]$ where $p = \dfrac{e^{rT} - d}{u - d}$

(b) (10%) End-of-Chapter exercise 21.7.

II. Computing Option Prices Using Binomial Model

Consider a non-dividend-paying stock with current stock price S_0 =\$50, volatility σ =0.3, strike price K=\$52, time to maturity T=2 years, interest rate r=5%.

Please use binomial model to price European put options. You may refer to the materials on page 475 of the textbook. Consider the following three alternative settings of time steps: Δt =1 month (12*T steps); 1 week (52*T steps); and 1 day (252*T steps).

(a) (5%) First compute the up step size *u*, the down step size *d*, and the probability of up move *p* under these three settings.

(b) (30%) Use binomial model to compute the put option prices under these three settings. Report your results and compare them with that of the Black-Scholes formula. Briefly explain your findings.

(c) (10%) Change the number of time steps from 1 to 2 to 3 all the way to 252. Plot your results as well as the Black-Scholes closed form solution. Briefly explain your findings.

(d) (10%) For 6, 12, and 52 time steps, compute the terminal stock prices as well as their corresponding probabilities. Plot the terminal stock price distribution. Briefly explain your findings.

(e) (20%) Modify your program in (b) to compute the American put option values. Report your result.

Matlab function and syntax:

1. To plot terminal stock distribution, you may use matlab function plot().

    e.g.

    plot(ST,Prob,'-o');

    where ST is a vector of terminal stock prices and Prob is a vector of their corresponding probabilities. '-o' is the LineSpec option that specifies the line type, marker symbol and color: LineStyle is Solid line ('-') and marker type is circle ('o').

2. nchoosek(): Binomial coefficient or all combinations

*You have to submit your homework and **programs by e3**. Your computer program is part of this assignment.