Forcing Firms to Talk: Financial Disclosure Regulation and Externalities

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We analyze a model of voluntary disclosure by firms and the desirability of disclosure regulation. In our model disclosure is costly, it has private and social value, and its precision is endogenous. We show that (i) a convexity in the value of disclosure can lead to a discontinuity in the disclosure policy; (ii) the Nash equilibrium of a voluntary disclosure game is often socially inefficient; (iii) regulation that requires a minimal precision level sometimes but not always improves welfare; (iii) the same is true for subsidies that change the perceived cost of disclosures; and (iv) neither regulation method dominates the other.

Should firms be required to disclose information? If they should, what and how much information should they be required to disclose? Although there has been much debate on disclosure regulation, it is apparent that there is no universal agreement on what disclosure regulation should be and actual requirements vary dramatically across countries. The debate has taken on added significance as regulators in a number of emerging financial markets have wrestled with the problem of setting appropriate disclosure requirements, and as markets in different countries attempt to harmonize their requirements.

It is often argued that strict disclosure requirements lead to liquid and efficient markets in financial securities and reduce the cost of capital for firms.[1] The question arises: If disclosure is good, why don't firms do it voluntarily? Regulation should not be necessary if disclosure is in the firm's best interest. The need for disclosure regulation is further brought into question

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[1] For example, Arthur Levitt, the Chairman of the U.S. Securities and Exchange Commission said recently, "high quality accounting standards result in greater investor confidence, which improves liquidity, reduces capital costs, and makes fair market prices possible." (Remarks by Arthur Levitt, Inter-American Development Bank, Washington, D.C., September 29, 1997.)

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by the well-known "unraveling" result of Ross (1979), Grossman (1981), and Milgrom (1981), whereby lack of disclosure is taken to be bad news, forcing the informed party to reveal its information in equilibrium. In this case again, regulation that requires that certain information be disclosed seems to be redundant.

Full voluntary disclosure, however, rarely seems to occur in reality, and firms typically do not disclose more than regulation requires. One possible reason for the lack of full disclosure is that disclosure is costly to firms. First, there may be a direct cost associated with producing and disseminating information. In particular, information may need to be disclosed or certified by third parties such as accounting firms. Second, since disclosure reveals information to competitors or others who interact strategically with the firm, it may cause the firm to lose competitive advantage or bargaining power in various contexts. However, as pointed out in Fishman and Hagerty (1998), even if disclosure is costly, it does not imply that disclosure regulation is desirable. It is quite possible that firms' disclosure policies are socially optimal given the cost of disclosure.[2]

In this article we develop a model in which disclosure regulation that requires or encourages more precise information to be disclosed by firms may have a justifiable role. The source of the gains to regulation is an externality that causes individual firms to internalize less than fully the social value of the information they release.[3] Our model's basic assumptions are that

(i) firms' values are correlated and the disclosures made by one firm are used by investors in valuing other firms;
(ii) disclosure of information is costly, and this cost increases in the precision of the disclosed information;
(iii) information asymmetries between firms and investors reduce firm value.

These assumptions seem reasonable. First, correlation in firms' values, as well as the impact of one firm's disclosure on the valuation and market prices of others [assumption (i)] has been documented extensively and arises when values depend on common economic factors and especially when firms are

[2] A related point, made in Diamond (1985), is that disclosure can be desirable if it changes investors' incentives for costly private information production. [The notion that investors overproduce information in the sense that they duplicate each other's costly efforts has been discussed also in Coffee (1984).] In Diamond's (1985) model firms optimally disclose information which investors can potentially acquire at a cost to the point where agents do not have incentives to acquire private information. Disclosure regulation is not needed in this model. It is also possible that firms overinvest in disclosure, in which case requiring even more disclosure would be socially undesirable. For a model where there is overinvestment in disclosure see Jovanovic (1982).

[3] The model we develop is most closely related to the suggestion in Easterbrook and Fischel (1991) concerning the possibility that correlation in firm values can play a role in justifying disclosure regulation. For other discussions of possible rationales for financial disclosure regulation see Baiman (1980), Foster (1980), Coffee (1984), Beaver (1988, Chapter 7), Dye (1990), Mahoney (1995), Fishman and Hagerty (1999), and Rock (1998).

in the same or related industries. For example, information about comparable firms is often used in valuation by analysts and investment bankers. The accounting literature on information transfers [see, e.g., Foster (1981) and Freeman and Tse (1992)] has demonstrated that disclosures made by one firm often affect the stock price of other firms.

Assumption (ii), that disclosure is costly to firms, has already been discussed above. It is reasonable that the cost of the disclosure rises with its precision (in our model, cost will be linear in precision.) Finally, many models, including in particular Myers and Majluf (1984), have shown the existence of costs stemming from information asymmetries between a firm's management and outside investors. These costs include losses due to underinvestment and costs associated with illiquidity in the market for the firm's stock. In the model of this article we will assume that disclosure has value since it increases the chances that a transaction that enhances the value of the firm, such as the sale of the firm to someone who values it more than its current holders, will occur.[4]

It is important to emphasize that the focus in this article is on the government regulation of the *amount* or type of information firms disclose and not on regulations that require that any information disclosed be truthful. In fact, we will assume, as is often done in the literature on disclosure, that all disclosed information is truthful. This is a reasonable assumption when antifraud laws are rigorously enforced. Also, disclosures are often made by third parties, such as accounting firms, who are not directly affected by the content of the disclosure and for whom the reputation for truthfulness is valuable.[5]

We first analyze the disclosure policy of a single, isolated firm and determine the optimal amount of information that the firm should disclose. We find that there are certain convexities or economies of scale in the value of the disclosure, which give rise to interesting and important discontinuities in the choice of precision as various parameters change. For example, it is possible that no disclosure is made for a range of small prior precision levels, but that at a certain threshold it is optimal to disclose a lot of information. This is due to the fact that for certain parameters of our model a small amount of information does not have much value, and information is only valuable if enough of it is disclosed.

In our model there is no need for disclosure regulation in the case of a single firm. The disclosure policy that is optimal for the firm is also socially optimal, because we assume that the firm captures all the surplus generated in the transaction. Regulation becomes an issue only when there are multiple

[4] Shavell (1994) makes the distinction between information that has social value and information that does not. In his model the cost of information is random and disclosure regulation takes the form of forcing agents who acquire information to disclose it.

[5] For a detailed discussion of antifraud laws in the context of security regulations see Easterbrook and Fischel (1991).

firms with correlated values and an externality arises in the disclosure process. We build on the analysis of the disclosure policy for one firm to analyze the Nash equilibria of a game of voluntary disclosure with multiple firms. We show that when there is symmetry across firms, there is either one or two Nash equilibria. If there are two equilibria, one of them always involves no disclosure at all and one involves some disclosure by all firms. When firms differ in their parameters, it is also possible that a Nash equilibrium (in pure strategies) does not exist.

In comparing the Nash equilibria with the social optimum we find that the Nash equilibrium outcome is often inefficient, because the disclosure decision of each firm does not take into account the informational spillovers that occur when this disclosure is used to value other firms. Both overinvestment and underinvestment in disclosure are possible. This inefficiency occurs when firm values are sufficiently correlated and becomes more pronounced as the correlation increases. These results suggest that there is scope for disclosure regulation to improve welfare in such cases. The gains from disclosure regulation can be especially significant when there are multiple equilibria in our game, since one equilibrium often dominates the other, and regulation can eliminate the equilibrium in which no disclosure occurs. If no equilibrium exists, then regulation can restore the existence of an equilibrium.

We proceed to examine in more detail ways in which regulation might improve the outcome of the voluntary disclosure game. Clearly if the regulator can specifically set a (possibly different) required precision level for each firm, the socially optimal outcome can be obtained. Since achieving the social optimum in this way requires that the regulator possess detailed information about each firm and its interaction with others and also, if firms differ, requires that the regulator draws up a specific standard for each firm, such regulation is probably not practical. We therefore focus on cases where a disclosure requirement is uniformly applied to all firms. We show that while in some cases a uniform disclosure requirement can be welfare improving (and in symmetric cases it can lead to the social optimum), there are cases where there is no single minimal precision that can be uniformly imposed on firms that leads to an improvement in welfare. This can occur even in situations where the Nash equilibrium absent regulation is very inefficient. It may therefore be useful for regulators to attempt to design disclosure regulation for different types of firms, e.g., differentiated by size or industry.

Another method of regulation we consider involves firms receiving a subsidy that reduces the perceived cost of information disclosure. We show that such subsidies can be useful, and in some cases can lead to a welfare improvement over the best that can be achieved through requiring a uniform precision level. However, there are also cases where subsidies are incapable of improving welfare. In comparing the use of subsidies to regulations requiring a minimal precision level for disclosures, we find that neither method dominates the other in all cases.

Our results point to a potential benefit of disclosure regulation and they indicate when regulation is most useful. However, they also suggest that it may not be easy to set the precise regulation so that a gain definitely occurs. First, there are cases where regulation of any sort is potentially harmful or unnecessary. Second, even when there is scope for regulation to improve welfare, the optimal regulation is quite sensitive to the parameters and setting regulations improperly can be costly. Clearly the results point to the potential benefit of trying to target regulation to different types of firms.

Most of the literature on disclosure regulation focuses on the case of a single firm and does not consider interactions between firms. In addition to Foster (1980) and Easterbrook and Fischel (1991), who discuss externalities of the type we model but do not develop formal models, we are aware of three articles which consider multiple firms in the context of disclosure. Dye (1990) develops a model with multiple firms whose values are correlated and whose owners choose a level of precision for their disclosure. The main difference between his article and ours is that Dye (1990) focuses on risk-sharing considerations, while our model assumes risk neutrality and focuses on the adverse selection problem between investors and the firm. In Dye's model entrepreneurs must sell the firm to investors and disclosure affects the risk allocation between them. While the model and the results are different, Dye also finds cases in which the voluntary disclosure outcome is socially optimal and others in which this is not the case.

Two other articles that involve multiple firms, although in very different contexts than ours, are Dye and Sridhar (1995) and Barth, Clinch, and Shibano (1998). Dye and Sridhar (1995) consider a sequential model in which a firm's decision to observe information is dependent on whether other firms observe information. Firms' cash flows, however, are assumed to be independent. This article does not address the issue of regulation. Barth, Clinch, and Shibano (1998) discuss a rational expectations equilibrium model with two risky assets and asymmetric information. The model focuses on the effects of harmonization of accounting principles on the portfolio choices and on the incentives to collect information by individual investors.

This article is organized as follows. The next section presents the basic model with one firm. Section 2 analyzes the disclosure policy in the case of a single firm. In Section 3 we present a voluntary disclosure model with multiple firms and discuss the Nash equilibria of the disclosure game. Section 4 considers the social welfare properties of the Nash equilibria. Section 5 illustrates the potential impact of disclosure regulation by requiring a minimal precision or by setting subsidies for disclosure. Finally, we offer some concluding remarks in Section 6.

1. A Voluntary Disclosure Model with One Firm

We consider a firm whose "retention value" is denoted by $\tilde{v}$. This represents the value of the firm to its current owners under the status quo. We assume that a value-increasing transaction is available to the firm. If the transaction occurs, it increases the firm's value to $\tilde{v} + \delta$, where $\delta > 0$. The parameter δ represents the net gains from the transaction, and we assume that it is known to all agents. (Since we will assume risk neutrality, δ could be also random, as long as there is no asymmetric information regarding its value.) For simplicity, we will think of the transaction as the sale of the firm to a buyer who values it more than its current owners, so that δ represents the extra gains to the buyer. Other interpretations are consistent with our analysis, although some may lead to a somewhat more complex model. For example, the source of the gains could be risk sharing [e.g., in the case of an initial public offering (IPO)], or the additional value of a new, positive net present value (NPV) project that the firm can undertake as in Myers and Majluf (1984).[6]

Absent any other information, investors assess $\tilde{v}$ to be normally distributed with a variance equal to $1/\omega$, so that the prior precision of $\tilde{v}$ is ω. The firm can, at a cost, disclose a signal $\tilde{s}$ about the value of $\tilde{v}$, where $\tilde{s} = \tilde{v} + \tilde{\eta}$ and $\tilde{\eta}$ is an independently, normally distributed noise term with mean zero and variance $1/h$. We assume that the firm can choose the precision of the information that is disclosed, and the cost of disclosure is linear in the precision. Specifically, the cost of a signal with precision h is γh, where $\gamma > 0$.[7]

As discussed in the introduction, there are various interpretations for the cost of the disclosure, some or all of which might be relevant in a particular case. The cost can represent a payment to an accounting firm for verifying information that the firm has and would like to disclose. It can also represent the cost of producing new information.[8] The disclosure cost may also be related to loss of competitive advantage or bargaining power which the firm suffers as a result of the disclosure.

We assume that $\tilde{v}$ is known to the current owners of the firm, but that it is not possible for the firm to credibly disclose this except through the disclosure mechanism described above. The firm's disclosure policy, that is, the choice of precision for the disclosed signal, is assumed to be determined independent of the value of $\tilde{v}$. Either this decision must be made before $\tilde{v}$ is

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[6] A model in which the firm needs to issue a fixed number of shares in order to gain δ and the market price of these shares is determined endogenously maps easily into our analysis with slight modifications. The Myers and Majluf (1984) model in which the firm needs to raise a fixed amount of money in order to gain δ and the number of shares it issues is also endogenous and depends on the market's assessment is somewhat more complicated and raises some additional issues to those we discuss. A full analysis of such a model, as well as one in which benefits are due to risk sharing, is left for future research.

[7] Preliminary analysis suggests that many of our results do not change in any important way if the cost function is nonlinear, for example, quadratic. This is discussed in more detail in Section 2.

[8] The model can be extended so that the firm does not observe $\tilde{v}$ perfectly and finds the signal informative given its information. Suppose the firm observes a noisy signal whose precision is b. If the firm discloses information, it generates new information in the form of another signal, which is conditionally independent of the firm's original information (given the true value $\tilde{v}$). Now the firm's forecast of $\tilde{v}$ is based on the two signals, while investors only use the disclosed signal. The model we use corresponds to the extreme case where b is infinite, but otherwise the model can be analyzed similarly and our results do not change in any important way.

known to the firm or, equivalently, the firm is committed to the precision and cannot change it given the realization of $\tilde{v}$. This is a reasonable assumption if setting up the disclosure mechanism is costly and time consuming, as would be the case when contracting with an accounting firm to produce a report or to certify the disclosure made by the firm.[9]

Although the firm cannot make its choice of precision depend on $\tilde{v}$, the knowledge of $\tilde{v}$ will be used by the firm's owners to determine whether to engage in a transaction in which the gain of δ will be realized, that is, whether to sell the firm. This is not a trivial decision. Since the disclosed signal is imperfect, it is possible that the price of the firm conditional on it being sold, which is characterized below, is lower than the retention value $\tilde{v}$. In such a case the firm will be retained by its current owners and no sale will take place. This is obviously a source of potential inefficiency, since there are always positive gains from trade, measured by δ, in this model. The inefficiency arises because the firm's value for the transaction is assessed given the imperfect disclosure.

We assume that the firm captures all the gains from the sale if the sale occurs. This assumption implies that the firm chooses the socially optimal precision level so that there is no role for regulation in the case of one firm. If the firm only captures a fraction of the gains from the sale, then most of our conclusions will be either unchanged or strengthened, because the firm's choice of precision will only internalize its portion of the benefits from the sale. This will result in even more scope for the regulation of the choice of precision.

Given a disclosure of $\tilde{s}$ with precision h, we let p be the price or market value of the firm *assuming that it participates in the transaction, that is, that the firm is sold*. This price is determined as if the outside investors are risk neutral.[10] Investors form expectations concerning $\tilde{v}$ given both the signal $\tilde{s}$ that has been disclosed and the fact that the sale is taking place. Obviously the owners of the firm will be willing to sell if and only if $\tilde{v} \leq p$, where p is the market value of the firm upon the sale taking place. It follows that p must satisfy

$$p = \mathrm{E}(\tilde{v} \mid \tilde{s}, \tilde{v} \leq p) + \delta. \tag{1}$$

[9] In the literature, the disclosure decision (whether to disclose the true value or not) is often made conditional on the value [see, e.g., Grossman (1981), Jovanovic (1982), and Fishman and Hagerty (1999)]. Allowing the precision choice to depend on $\tilde{v}$ in our model would lead to a complex signaling game in which the choice of precision, if observable, signals the value even prior to the disclosure of the signal itself. Preliminary analysis of some simplified models of this sort suggests that in many such models there is no pure strategy equilibrium and that the equilibria, if they exist, have some unintuitive properties.

[10] This assumption is not quite as restrictive as it may initially seem in our context. For example, if the information signal being disclosed concerns a nonsystematic risk term in the return distribution of the firm, then well-diversified investors will effectively be risk neutral with respect to the transaction. Later we will discuss disclosure policies of several firms when values across firms are correlated. The risk neutrality assumption can still be appropriate in these situations as long as the factor introducing the correlations among the firms is not priced.

Note that there is an underpricing that occurs because firms for which $\tilde{v} > p$ are not sold, and so the average firm that is sold is of lower value relative to the average firm based only on the disclosure. Note also that δ is important in determining the extent of this underpricing, because with a higher δ fewer types of $\tilde{v}$ will withdraw and the adverse selection problem will be less severe. If the disclosed signal is imperfect, then the existence of a price satisfying Equation (1) requires that $\delta > 0$, since $p < \mathrm{E}(\tilde{v} \mid \tilde{s}, \tilde{v} \leq p)$. With $\delta = 0$ we see a complete market failure due to adverse selection.

2. The Optimal Disclosure Policy for One Firm

In this section we analyze the choice of precision for the disclosed signal made by a single firm in isolation. (This analysis applies to a firm whose value is uncorrelated with the values of all other firms.) We start by characterizing p, the market price of the firm given that it is sold. Let $\mathrm{E}(\tilde{v} \mid \tilde{s})$ be the expectation of $\tilde{v}$ conditional on $\tilde{s}$ and define $\tilde{\epsilon} = \tilde{v} - \mathrm{E}(\tilde{v} \mid \tilde{s})$. Thus $\tilde{\epsilon}$ is the forecast error made when $\tilde{v}$ is predicted using only $\tilde{s}$, and it is independent of $\tilde{s}$ and observable to the firm. It is easy to show that, given the parametric assumptions made above and the properties of the normal distribution, the variance of $\tilde{\epsilon}$ is $(\omega+h)^{-1}$. It will be convenient for our analysis to define $z = \omega + h$, the precision of $\tilde{\epsilon}$. This is the posterior precision obtained when information with precision h is disclosed if the prior precision is ω. Finally, define $x = p - \mathrm{E}(\tilde{v} \mid \tilde{s})$. Then we can rewrite Equation (1) as

$$\mathrm{E}(\tilde{v} \mid \tilde{s}) + x = \mathrm{E}\Big(\mathrm{E}(\tilde{v} \mid \tilde{s}) + \tilde{\epsilon} \,\Big|\, \tilde{s}, \mathrm{E}(\tilde{v} \mid \tilde{s}) + \tilde{\epsilon} \leq \mathrm{E}(\tilde{v} \mid \tilde{s}) + x\Big) + \delta, \tag{2}$$

which simplifies to[11]

$$x = \mathrm{E}(\tilde{\epsilon} \mid \tilde{\epsilon} \leq x) + \delta. \tag{3}$$

Thus the transaction takes place (the firm is sold) if and only if $\tilde{\epsilon} \leq x$. Since $\tilde{v}$ and $\tilde{s}$ are by assumption jointly normally distributed, the forecast error $\tilde{\epsilon}$ is normally distributed. The conditional expectation in Equation (3) is therefore the expectation of a truncated normal variable [see Johnston, Kotz, and Balakrishnan (1994)], and so the cutoff point x must satisfy

$$x = -\frac{\phi(x\sqrt{\omega+h})}{\Phi(x\sqrt{\omega+h})\sqrt{\omega+h}} + \delta = -\frac{\phi(x\sqrt{z})}{\Phi(x\sqrt{z})\sqrt{z}} + \delta, \tag{4}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are, respectively, the density and cumulative distribution functions for the standard normal distribution.

[11] Because the distribution of $\tilde{\epsilon}$ is independent of $\tilde{s}$ and because δ is a constant, x is a constant that does not depend on the realization of $\tilde{s}$.

In choosing the precision level h, the firm faces the following trade-off: on the cost side, a unit of precision costs γ. Now the sale of the firm leads to a gain of δ, which is realized with a probability equal to $\Phi(x\sqrt{z})$. The value of the disclosure is due to the fact that it increases the probability that the sale occurs. Formally, the precision $h \geq 0$ (equivalently, posterior precision $z \geq \omega$) is chosen to maximize the following objective function:

$$\delta\Phi\Big(x(\omega+h)\sqrt{\omega+h}\Big) - \gamma h = B(z) - \gamma(z-\omega), \tag{5}$$

where $z = \omega + h$, $B(z) = \delta\Phi(x(z)\sqrt{z})$, and $x(z) = x(\omega + h)$ satisfies Equation (4) for every z.

The objective function given in Equation (5) is comprised of the "benefit" function, $B(z) = \delta\Phi(x(z)\sqrt{z})$, which measures the value of having posterior precision of z (i.e., disclosing information with precision $h = z - \omega$), less the cost function γh. The choice of an optimal precision for the disclosed signal obviously depends on the properties of the benefit function $B(\cdot)$, which in turn depends on the properties of $x(\cdot)$. While we do not have a closed form solution for $x(\cdot)$, we will be able characterize the optimal precision choice and how it varies with the model's parameters.

First, note that from its definition and from Equation (4) it follows immediately that $x(\cdot)$ is continuous and bounded above by δ. The following is proved in the appendix:

Lemma 1. *$x(z)\sqrt{z}$ is increasing in z (and therefore in h).*

Thus the higher the precision of the information, the higher the probability that the sale occurs and δ is realized by the firm. It follows that the benefit function $B(\cdot)$ is increasing. Since $B(\cdot)$ is differentiable and bounded above by δ, it must be that $B'(z)$ vanishes as z (or h) goes to infinity. Thus $B(z)$ is concave for large values of z. The benefit function is not concave throughout its domain; however, as the following result shows, it is actually convex near $z = 0$. The proof is in the appendix.

Proposition 1. $\lim_{z\to 0} B'(z) = 0$. *Since $B(z)$ is increasing in z it follows that for every δ there exists $\bar{z}$ such that $B(z)$ is convex for all $z < \bar{z}$.*

This result has some very important consequences for our model. It implies that for certain parameters of our model there are "economies of scale" in the value of information, whereby a small amount of information is not very valuable and will not be disclosed, and only information whose precision exceeds a certain threshold of precision may be disclosed.[12] As we will see,

[12] This may seem similar to the nonconcavity result in Radner and Stiglitz (1972). However, our setting is quite different and it does not seem that a mapping can be made from their result, which postulates a single person taking an action given information, to our model, where information has value through the market price mechanism in an adverse selection environment.

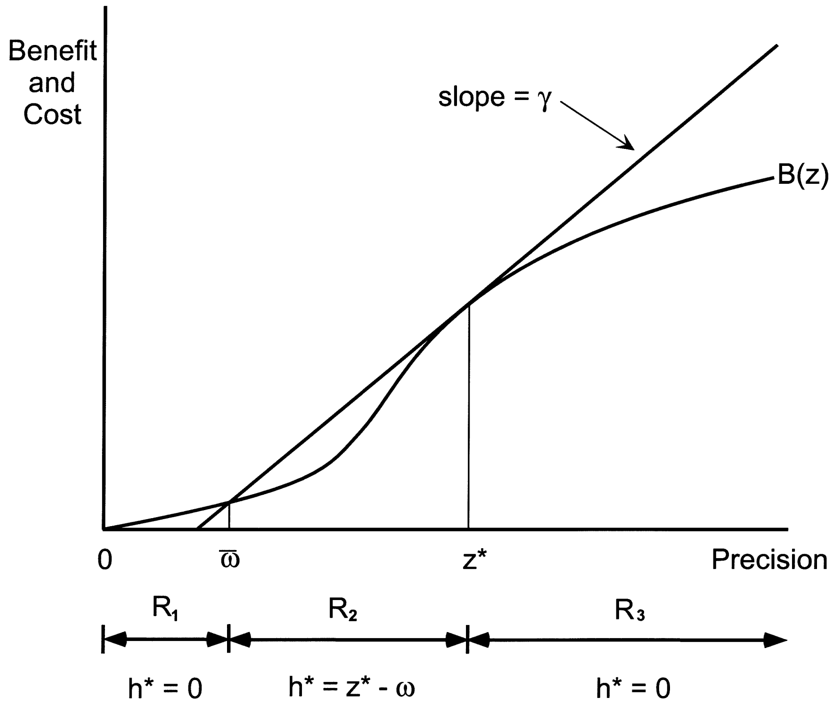

Figure 1
The general shape of the benefit function $B(z)$, where $z = \omega + h$ is the total conditional precision following the disclosure, ω is the prior precision and h is the precision of the disclosed signal. The benefit function is convex when z is sufficiently small and becomes concave for large values of z. Also plotted is the straight line with slope γ (the cost per unit precision) that is tangent to $B(z)$ at its concave part. The tangency point and the intersection of this line with $B(z)$ define three regions. In region R_2 the optimal disclosure policy is to bring the total precision to the value $z^\star$. In the other regions it is optimal not to make a disclosure at all, that is, the optimal precision for the disclosure is zero.

this creates possible discontinuities in the relations between the optimal precision and the various parameters of the model. We will provide an intuitive explanation and a discussion of the robustness of this result after we complete the characterization of the optimal precision choice.

For any δ the benefit function $B(z)$ has the shape depicted in Figure 1.[13] Note that in addition to the properties discussed above, $B(z)$ has a unique inflection point, at which its first derivative is maximized. The following result characterizes the optimal precision.

Proposition 2. *Let $z^\star$ be such that*

(i) $B'(z^\star) = \gamma$,
(ii) $B''(z^\star) < 0$.

If there is no value $z^\star$ satisfying these conditions, then define $z^\star = 0$.

[13] The value of $B(z)$ for $\delta \neq 1$ is related to its value for $\delta = 1$ via a change of variable from z to z/δ^2. This can be seen because the definition of $x(\cdot)$ depends on δ only through $\delta\sqrt{z}$.

Let $\overline{\omega}$ be such that $B(\overline{\omega}) = B(z^\star) - \gamma(z^\star - \overline{\omega})$ if the solution to this equation is positive, and define $\overline{\omega} = 0$ otherwise.

The optimal precision, $h^\star$, which maximizes Equation (5)*, is given by*

$$h^\star = \begin{cases} z^\star - \omega, & \text{if } \overline{\omega} \leq \omega < z^\star \\ 0, & \text{if } \omega \leq \overline{\omega} \quad \text{or} \quad \omega \geq z^\star. \end{cases}$$

This result gives the optimal precision for the disclosed signal for every possible value of the prior precision ω. First, if the prior precision falls in region R_2 in Figure 1, then it is optimal for the firm to increase total precision to $z^\star$ (i.e., disclosing a signal with precision $z^\star - \omega$), since at all precision levels between the prior and $z^\star$, the total benefit to increasing precision exceeds the total cost. However, when the prior precision is greater than $z^\star$, that is, the prior level falls in region R_3 of Figure 1, no additional information is disclosed by the firm since the marginal cost exceeds the marginal benefit for all precision levels above $z^\star$. Finally, if the prior falls in region R_1 of Figure 1, the total benefit of disclosing information up to level $z^\star$ (or any level below or above $z^\star$) is less than or equal to the cost of doing so. Thus it is optimal to not disclose any information.[14]

Some of the discontinuities created by the nonconcavity of $B(\cdot)$ are illustrated in Figure 2. Figure 2A plots the optimal chosen precision as a function of the prior precision ω, assuming $\delta = 0.22$ and $\gamma = 0.01$. When $\omega \leq \overline{\omega} \approx 1.689$, the optimal precision is zero (no disclosure). This is due to the convexity of the benefit function for low values of z and will be discussed in more detail below. Then the optimal precision jumps discontinuously to about 7.89 and decreases linearly, reaching zero again for $\omega = z^\star \approx 9.576$ and remaining at zero thereafter. Intuitively the value of additional information is very low when the prior information is already very precise.

In Figure 2B we plot the optimal precision as a function of the benefit parameter δ assuming $\omega = 1$ and $\gamma = 0.01$. Note that if δ is sufficiently small the optimal precision is zero. Intuitively, for these parameters the sale is unlikely to happen in any case, and the benefit of the transaction is small when it does occur, so the value of the disclosure is zero. The discontinuity is again due to the convexity of the benefit function, which is discussed below. When δ is very large, the sale is likely to occur even without a disclosure, and so the value of information is again small.[15]

[14] Note that in the special case where the prior is $\overline{\omega}$, the firm is indifferent between disclosing no information and disclosing information with precision $z^\star - \overline{\omega}$. Also, in Figure 1 the parameters γ and δ are such that all three regions R_1, R_2, and R_3 are nonempty. There are, however, values of γ and δ for which either R_1 is empty or both R_1 and R_2 are empty. The proposition covers these cases as well.

[15] The comparative statics related to changes in the cost parameter γ are straightforward and can be seen by changing the slope of the appropriate line in Figure 1. First, the optimal precision is generally decreasing in γ. For a fixed value of δ and ω, as γ goes to zero, the optimal precision goes to infinity. On the other hand, as γ grows, the optimal precision eventually becomes zero. Because of the shape of the benefit function, for relatively low values of the prior precision ω there is a discontinuity in the relation between γ and the optimal precision. The optimal precision drops to zero discontinuously when γ is equal to the slope of $B(z)$ at its inflection point. For larger cost parameters the optimal precision remains zero.

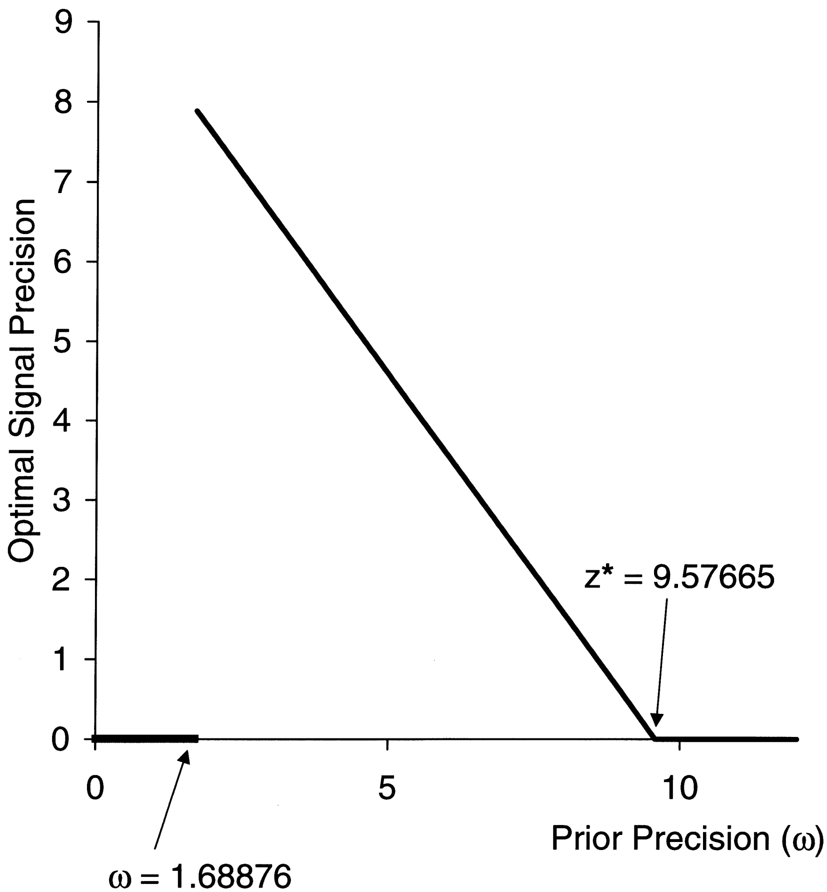

Figure 2A
Figure 2A plots the optimal precision of the disclosed signal $h^\star$ (given in Proposition 2) as a function of the prior precision ω. This is done for an example with $\delta = 0.22$ and $\gamma = 0.01$, where δ measures the gains obtained from the transaction and γ is the cost per unit precision of the disclosed signal. For these parameters $\overline{\omega} = 1.68876$ and $z^\star = 9.57665$, where $\overline{\omega}$ and $z^\star$ are defined in Proposition 2. Note the discontinuity at $w = \overline{\omega}$.

One might suspect that the convexity of the benefit function is tied to our specific parametric assumptions, particularly normal distributions, or to the linearity of the cost function. However, it turns out that the intuition behind this is quite general and similar results obtain in many related models. To understand this consider the following simple version of our model: Suppose the retention value $\tilde{v}$ of the firm can be either 10 or zero, with prior probability of .5. The signal that may be disclosed can take one of two values, say H or L, and more precise signals have a higher conditional probability of H given that the true value is 10. The cost of this informa-

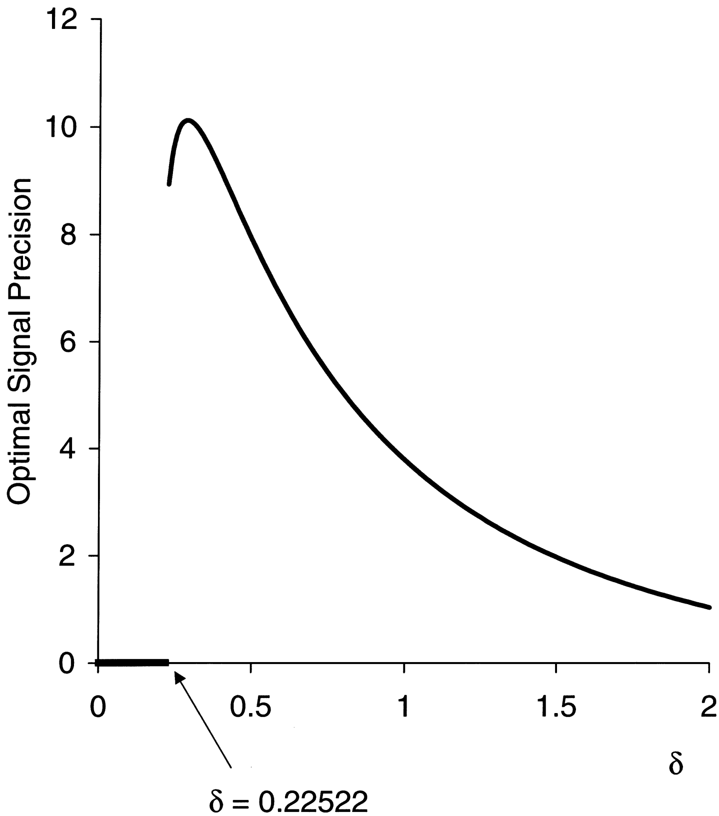

Figure 2B
Figure 2B plots the optimal precision of the disclosed signal as a function of δ. The figure shows that there is a discontinuity in this function as well, which occurs at $\delta = 0.22522$.

tion is a function of this "precision" level. Now suppose the gains from trade parameter δ is equal to 3. Clearly in this case if no information is disclosed, then only the firm with value of zero is sold at a price of 3. This follows because if investors conjecture that both types of firms are sold then the market price will be $5 + 3 = 8$, and the firm whose value is 10 will withdraw.

In this example, if firms collect information, then clearly the high-value firm is sold only if the conditional expectation of $\tilde{v}$, given the signal (and a willingness to sell) is at least 7, leading to a total price of at least 10. Information that does not raise the conditional expectation beyond this cut-off point with sufficient likelihood is useless since it will not induce the firm whose value is 10 to sell. Thus the value of information is zero for low precision levels and becomes positive only for sufficiently precise information.

The situation in our model is similar, although more complicated, than the simple example above, and the intuition is the same.[16]

4. A Game of Voluntary Disclosure

In the last section we considered the disclosure decisions made by an isolated firm. This provides the foundation for the discussion in this section of the case in which multiple firms make disclosure decisions and each firm's disclosure potentially affects the other firms. We start by extending the model of the previous section to include two firms whose values may be correlated. Let the retention value of firm i (i = 1 or 2) be denoted by $\tilde{v}_i$. If $\tilde{v}_i$ were common knowledge, then firm i can be sold for $\tilde{v}_i + \delta_i$, where $\delta_i > 0$. Prior to any disclosure, $\tilde{v}_1$ and $\tilde{v}_2$ are normally distributed with the variance of $\tilde{v}_i$ equal to ω_i^{-1}; the correlation coefficient between $\tilde{v}_1$ and $\tilde{v}_2$ is ρ. Firm i can disclose a signal $\tilde{s}_i = \tilde{v}_i + \tilde{\eta}_i$, where $\tilde{\eta}_i$ is independent of $\tilde{\eta}_j$ for $j \neq i$ and independent of $\tilde{v}_i$, $i = 1, 2$. If signal $\tilde{s}_i$ has a precision h_i, then the cost incurred by firm i is $\gamma_i h_i$.

We assume that the two firms engage in the following game of voluntary disclosure. First, firms choose precision levels for their signals simultaneously. This is done before $\tilde{v}_i$ is known to firm i, so that the precision level is not a function of $\tilde{v}_i$. Second, signals from the appropriate distributions are publicly disclosed. Each firm i then makes the decision whether to participate in the transaction or not, given the value $\tilde{v}_i$ and the market price of the firm p_i which is characterized below.

In determining the market price of each firm, market participants observe both disclosed signals.[17] The market price p_i of firm i (conditional on it being sold) is equal to

$$p_i = \mathrm{E}(\tilde{v}_i \mid \tilde{s}_1, \tilde{s}_2, \tilde{v}_i \leq p_i) + \delta_i. \tag{6}$$

Note that while the disclosed signals of both firms are assumed to be publicly observable and to affect both prices in Equation (6), it is assumed that

[16] It should be noted that although the linearity of the cost function simplifies our analysis and the characterization of the solution, Figure 1 illustrates that similar results could obtain even if the cost function were convex. One possible difference from the case of a linear cost function is that when the cost function is strictly convex, say $C(\cdot)$, then $\omega + h(\omega)$ might be in the convex range of $B(\cdot)$, in which case $h'(\omega) = -B''(\cdot)/(B''(\cdot) - C''(\cdot))$ might be positive for positive values of ω. In this case higher prior precision may lead to a choice of higher signal precision, which is not possible in the linear cost case except starting at zero precision. (Once a positive precision is chosen, the optimal precision is decreasing in the prior precision.) Some of our other results might change, but the convexity of $B(\cdot)$ is still important for the characterization of the equilibrium in the disclosure game. In particular, the result that more precision might be provided in the game than when firms act independently continues to hold.

[17] While this model is static and assumes simultaneous disclosure decisions, we view it as a reduced form model to a situation in which disclosure is more cumulative and occurs over time. (See Section 6 for a discussion of dynamic issues.) If the timing of the disclosure is also endogenous and part of the game between firms, then new issues arise as in Dye and Sridhar (1995), for example.

the price of each firm does not depend on whether the other firm is sold or not.[18]

In order to derive the reaction functions of the firms, we first calculate the conditional variance or precision of firms' values given the two disclosed signals. It can be shown using the theory of bivariate normal distributions that the posterior precision of $\tilde{v}_i$ (i.e., the inverse of the conditional variance of $\tilde{v}_i$) given $\tilde{s}_1$ and $\tilde{s}_2$ is equal to

$$h_i + g_i(h_j), \quad \text{where} \quad g_i(h_j) = \frac{\omega_i(\omega_j + h_j)}{\omega_j + h_j(1-\rho^2)}. \tag{7}$$

The function $g_i(h_j)$ gives the precision for predicting $\tilde{v}_i$ given firm j's disclosure $\tilde{s}_j$, but "prior to" firm i's disclosure. As is intuitive, when $\rho = 0$, $\tilde{s}_j$ is uninformative about $\tilde{v}_i$ for $j \neq i$, and indeed $g_i(h_j) = \omega_i$ and the conditional precision of $\tilde{v}_i$ given both signals is $h_i + \omega_i$. If $\rho \neq 0$, then $g_i(h_j)$ is larger than ω_i, since $\tilde{s}_j$ provides some information about $\tilde{v}_i$. If $|\rho| = 1$ and $\omega_i = \omega_j$, then $g_i(h_j) = \omega_i + h_j$, since information about one firm is the same as information about the other. In this case the posterior precision is $\omega_i + h_i + h_j$. Note also that $g_i(0) = \omega_i$, that is, if firm j discloses no information, then the precision of firm i given firm j's disclosure is ω_i. Finally, $g_i(h_j)$ depends on the correlation coefficient ρ only through its absolute value $|\rho|$, but not its sign. This will be true for all our results. For simplicity we will treat the correlation coefficient as if it were positive from now on, but all our results hold for correlation coefficients with the same absolute value.

From Equation (7) it follows that, taking the choice of precision by firm j as given, firm i is faced with a decision such as analyzed in the previous section, with the prior precision being equal to $g_i(h_j)$. The reaction function $h_i^r(h_j)$ can now be derived by applying Proposition 2. This gives

$$h_i^r(h_j) = \begin{cases} z_i^\star - g_i(h_j), & \text{if } \overline{\omega}_i < g_i(h_j) < z_i^\star \\ 0, & \text{if } g_i(h_j) \leq \overline{\omega}_i \text{ or } g_i(h_j) \geq z_i^\star, \end{cases}$$

where $z_i^\star$ and $\overline{\omega}_i$ are defined as in Proposition 2.

A Nash equilibrium (in pure strategies) is a pair (h_1^N, h_2^N) such that $h_i^r(h_j^N) = h_i^N$ for $i \neq j$. The next result characterizes parameters for which there exists an equilibrium with positive precisions for the disclosed signals and solves for the equilibrium precisions. We will then discuss the equilibria further and give some examples.

[18] This assumption simplifies the analysis. An interpretation of the assumption is that the decisions by each of the firms as to whether the transaction will take place are made simultaneously, and cannot be made contingent on the transaction of the other firm. If transactions occur sequentially, then our analysis applies to the first firm to decide about the transaction. For the second firm the analysis is different, because additional information can be gleaned from whether the first firm has chosen to go ahead with the sale. Nevertheless, the externality we study, whereby the disclosure of each firm affects the outcome of the other, is still present.

Proposition 3. *A Nash equilibrium in which both chosen precisions are positive exists if and only if the following conditions hold:*

(i) $z_i^\star > 0$ *for* $i = 1, 2$, *and*
(ii) $\overline{\omega}_i < g_i(h_j^N) < z_i^\star$ *for* $i = 1, 2$ *and* $j \neq i$, *where*

$$h_i^N = \frac{z_i^\star}{2} - \frac{\omega_i}{1-\rho^2} + \left(\frac{(z_i^\star)^2}{4} + \frac{\rho^2 \omega_i \omega_j z_i^\star}{(1-\rho^2)^2 z_j^\star} \right)^{1/2}.$$

In this case the equilibrium precisions are given by h_1^N *and* h_2^N.

Proof. Assume that (h_1^e, h_2^e) is an equilibrium with both h_1^e and h_2^e positive. Then from Equation (7), (h_1^e, h_2^e) must solve

$$h_1^e = z_1^\star - \frac{\omega_1(\omega_2 + h_2^e)}{\omega_2 + h_2^e(1-\rho^2)}$$
$$h_2^e = z_2^\star - \frac{\omega_2(\omega_1 + h_1^e)}{\omega_1 + h_1^e(1-\rho^2)}.$$

These two (nonlinear) equations are solved at $(h_1^e, h_2^e) = (h_1^N, h_2^N)$ as defined above. Conditions (i) and (ii) guarantee that the precision for each firm conditional only on the other firm's (equilibrium) signal is in the region where the firm optimally chooses a positive precision for the disclosure according to Proposition 2. ■

An equilibrium in which both chosen precisions are zero obviously exists when $z_i^\star = 0$ for $i = 1, 2$, so that it is optimal for each firm not to disclose any information no matter what the other firm does. More generally, such an equilibrium exists if $h_i^r(0) = 0$ for $i = 1, 2$. In such cases no information is disclosed. As we show below, an equilibrium with no disclosure by either firm always exists if both reaction functions are discontinuous.

We have shown in the last section that the optimal precision can be a discontinuous function of the prior precision. Since the effective prior precision of firm i given firm j's precision, $g_i(h_j)$, is increasing in h_j, it is not surprising that the reaction function $h_i^r(h_j)$ may be discontinuous. The following examples show that the possible discontinuity of the reaction functions can lead to multiple equilibria or (in asymmetric cases) to the possible nonexistence of an equilibrium.

First, we illustrate the types of equilibria that can occur in a symmetric case. Suppose $\omega_1 = \omega_2 = 1$, $\gamma_1 = \gamma_2 = 0.01$, $\delta_1 = \delta_2 = 0.22$, and $\rho = 0.8$. Note that the parameters of each firm are the same as those used in Figure 2A, and so we know that $\overline{\omega}_i \approx 1.689$ and $z_i^\star \approx 9.57$. We see in Figure 3 that the reaction functions are discontinuous and that there are two Nash equilibria. In one equilibrium both firms choose zero precision, in the other they both

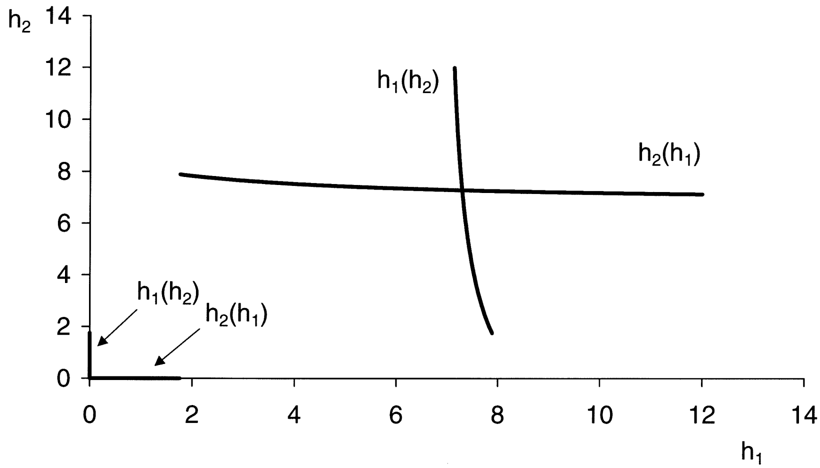

Figure 3
The reaction functions and Nash equilibria of the disclosure game assuming there are two identical firms $i = 1, 2$ with $\delta_i = 0.22$, $\omega_i = 1$, $\gamma_i = 0.01$, and $\rho = 0.8$, where for firm i, δ_i is the parameter measuring the gains from the transaction, γ_i is the cost per unit precision of the disclosed signal, ω_i is the prior precision, and ρ is the correlation coefficient between the firms' values. For these parameters the reaction functions are discontinuous and there are two Nash equilibria. One equilibrium involves both firms choosing zero precision and the other involves both firms choosing precision equal to about 7.3.

choose precision of approximately 7.3.[19] Note that since $\omega_i < \overline{\omega}_i$, if there is no disclosure by firm j, or if firm values are uncorrelated, it is optimal for firm i not to disclose. Thus $h_1^r(0) = h_2^r(0) = 0$, and there is an equilibrium with no disclosure. However, with a correlation coefficient of 0.8, the disclosure by firm j of a signal with sufficient precision brings the effective prior precision of firm i to a level above the critical point $\overline{\omega}_i$, and so firm i as well finds it optimal to choose a positive precision for its disclosure. This gives rise to the equilibrium with the positive precisions, where each firm chooses a precision of about 7.3. Note that in this equilibrium significantly *more* information is disclosed than would be the case if firm values were uncorrelated and there was no interaction between their disclosure decisions.

Since $g_i(\cdot)$ is at least weakly increasing, our results in Section 2 imply that if the reaction functions are continuous, then they are at least weakly decreasing. Thus if reaction functions are continuous, there is a unique equilibrium, and in equilibrium the choice of precision of each firm is always lower than it would be if firm values were uncorrelated. As the example in Figure 3 shows, this is not true when the reaction functions are discontinuous. Note that in the symmetric case there always exists at least one equilibrium. This follows because in this case either both reaction functions are continuous

[19] Unlike the graph of the optimal precision choice as a function of the prior in Figure 2A, the reaction functions are nonlinear in their continuous segment. This is because the precision of each firm given the other firm's signal, $g_i(h_j)$ is a nonlinear function of h_j for $0 < |\rho| < 1$.

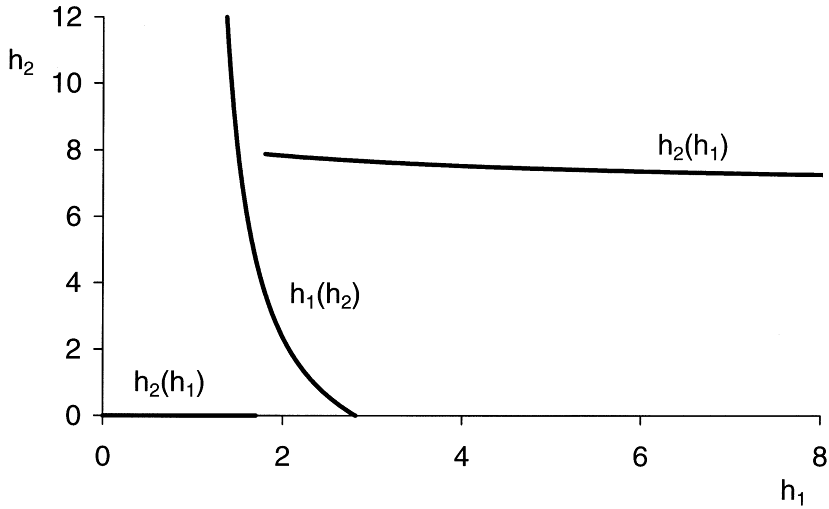

Figure 4
An example where no (pure strategy) Nash equilibrium for the disclosure game exists. The figure plots the reaction functions of firm 1 and firm 2 when the model's parameters are given by $\delta_1 = 0.7$, $\delta_2 = 0.22$, $\gamma_1 = 0.04$, $\gamma_2 = 0.01$, $\omega_1 = \omega_2 = 1$, and $\rho = 0.8$, where for firm i, δ_i is the parameter measuring the gains from the transaction, γ_i is the cost per unit precision of the disclosed signal, ω_i is the prior precision, and where ρ is the correlation coefficient between the firms' values. Note that firm 1's reaction function is continuous while firm 2's is discontinuous.

(and at least weakly decreasing) or both are discontinuous and have a shape similar to that in Figure 3. In the former case there is a unique equilibrium, and in the latter there are two equilibria.

The following example shows that nonexistence of an equilibrium is possible in asymmetric cases. The parameters for firm 2 are still given by $\delta_2 = 0.22$ and $\gamma_2 = 0.01$, but firm 1 has $\delta_1 = 0.7$ and $\gamma_1 = 0.04$. Thus the sale is more valuable and for any given precision level is more likely for firm 1, while firm 2 has a lower cost of disclosure. We continue to assume that the correlation between the firms' values is 0.8. The reaction functions for this example are plotted in Figure 4. As can be seen in the figure, one reaction function, $h_2^r(\cdot)$, is continuous while the other, $h_1^r(\cdot)$, is discontinuous and these functions do not intersect. Thus there is no (pure strategy) Nash equilibrium in this case.[20]

It should be noted that our model can be extended to the case of more than two firms in a straightforward manner. To see this, consider the symmetric case with $N + 1$ firms. Suppose for simplicity that firm values are given by the following simple one-factor model,

$$\tilde{v}_i = \tilde{\theta} + \tilde{\zeta}_i, \tag{8}$$

[20] It is possible that a mixed strategy Nash equilibrium exists, but we do not consider this possibility here.

where the variance of the common $\tilde{\theta}$ is $1/t$ and the variance of $\tilde{\zeta}$ is $1/d$. (That is, t and d are the precisions of $\tilde{\theta}$ and $\tilde{\zeta}$, respectively.) If firm i chooses precision h and all firms $j \neq i$ choose precision g for their disclosures, then the posterior precision of $\tilde{v}_i$ given all the signals is given by

$$h + \frac{d(gt + dt + dgN)}{gt + d(t + g(N+1) + d)}. \tag{9}$$

This can be used to analyze the Nash equilibrium of the game with $N+1$ firms in a way similar to our discussion above, and one can solve for the symmetric Nash equilibrium (or equilibria). Again, there will typically be either one or two equilibria, depending on whether the reaction functions are continuous or not.

5. The Social Welfare Properties of the Nash Equilibria

In this section we discuss the social welfare properties of the Nash equilibria and the potential role of financial disclosure regulation in our model. First, note that since there are built-in gains from trade in our model, the first best outcome is that the sale of the firm always occurs. However, this can only emerge if investors know $\tilde{v}_i$ perfectly, which would be the case if disclosure is costless or if the regulator knows $\tilde{v}_i$ and can costlessly disclose them. We assume that neither is the case and take as given the cost structure and the technology for disclosure that is assumed in our model.

The socially optimal precision levels are those that maximize the total benefit, net of the cost, of the disclosure by all firms.[21] Formally, with N firms, $h_1, h_2, \ldots, h_N$ are chosen to maximize

$$\sum_{i=1}^{N} B_i\big(z_i(h_1, h_2, \ldots, h_N)\big) - \sum_{i=1}^{N} \gamma_i h_i, \tag{10}$$

where $z_i(h_1, h_2, \ldots, h_N)$ is the posterior precision of v_i when firm i makes a disclosure with precision h_i, and, as in Equation (5), $B_i(z_i) = \delta_i \Phi(x(z_i)\sqrt{z_i})$, with $x(\cdot)$ satisfying Equation (4).

For the case of one firm this is exactly the objective function analyzed in Section 2. For the case of multiple firms, the presence of the externality that arises when firm values are correlated suggests that in general the Nash equilibrium outcome may differ from the social optimum. Below we consider the magnitude of this efficiency loss and examine how the Nash equilibrium precisions differ from the socially optimal precisions. This will allow us to consider the potential role of disclosure regulation.

[21] We assume that all costs associated with the disclosure are deadweight costs and therefore social costs. The welfare analysis would be different if some of the perceived costs were, for example, due to loss of competitive advantage. However, in this case there is likely to be an even greater divergence between the voluntary disclosure policies of firms and the social optimum.

Our discussion in this section will be developed through a series of examples that will illustrate the relevant issues. In most of these examples we assume for simplicity that $\omega_1 = \omega_2 = 1$. First, Figure 5A presents the chosen precisions in the Nash equilibria and in the social optimum for an example with two firms in which $\delta_1 = \delta_2 = 0.22$ and $g_1 = \gamma_2 = 0.01$, as a function of the correlation coefficient ρ. (Note that the parameters for each firm are the same as in Figure 2A and that the Nash equilibria in this example with $\rho = 0.8$ are illustrated in Figure 3.) We see that for all values of ρ there is a Nash equilibrium in which neither firm discloses any information. For relatively high correlation levels (starting at about $\rho = 0.68$) there is an

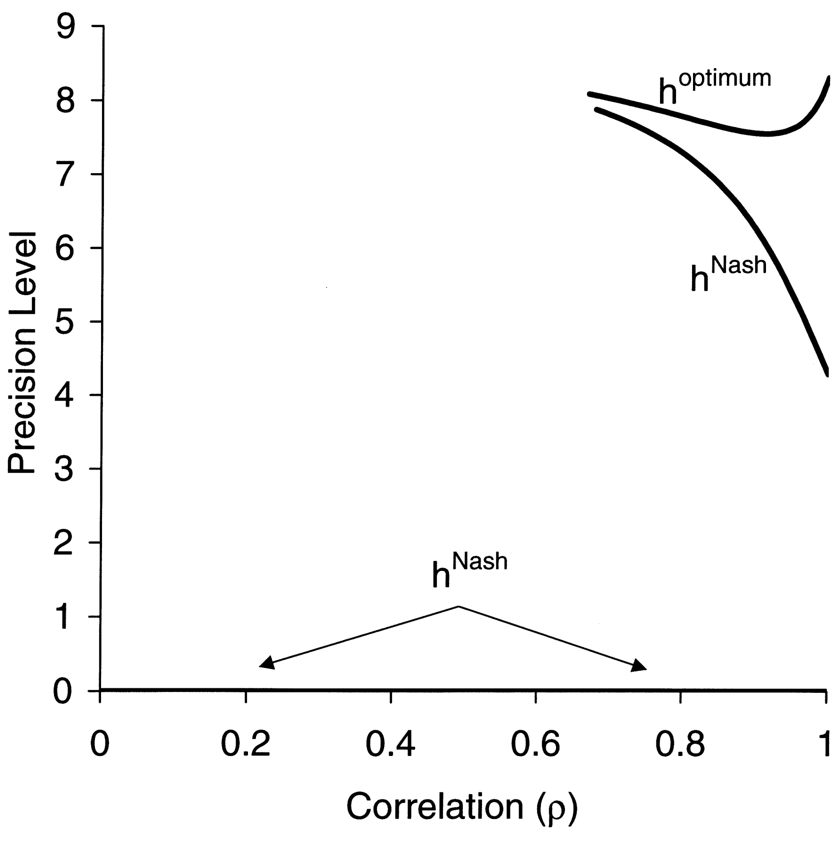

Figure 5A
Figure 5A plots the socially optimal and the Nash equilibrium precision levels chosen by each of two identical firms $i = 1, 2$ when $\delta_i = 0.22$, $\gamma_i = 0.01$, and $\omega_i = 1$ for each value of ρ, the correlation coefficient between firm values. As in Figure 3, δ_i is the parameter measuring the gains from the transaction, γ_i is the cost per unit precision of the disclosed signal, and ω_i is the prior precision for firm i. For every ρ there exists a Nash equilibrium with no disclosure, and for sufficiently high ρ there is also an equilibrium with positive disclosure levels. The socially optimal precision level of disclosure is zero for low correlations and then jumps discontinuously to a positive level. It is then always above the Nash equilibrium precision levels.

additional equilibrium in which the chosen precisions are positive. We also see that the socially optimal precision level makes a discontinuous jump from zero to a positive and relatively high level at about $\rho = 0.67$. This discontinuity should not be surprising in light of our discussion in Section 2 that is illustrated in Figure 2A, since there are convexities in the social welfare function derived from the sum of the individual benefit functions less the total cost.[22]

It is interesting to examine the Nash equilibrium and the socially optimal precision levels when the correlation between firm values is very high. Note that as firm values become highly correlated, each firm's disclosure is more informative about the other firm's value. Consider first the socially optimal precisions. We see that initially after the optimal precision becomes positive, it declines slightly as correlation increases, but eventually as the correlation approaches unity the optimal precision increases. To understand this, note that there are two effects influencing the optimal precision. On one hand, with higher correlation each unit of precision in the signal of one firm increases the prior precision of the other firm by more, lowering the additional precision needed by the other firm to achieve the same posterior precision. On the other hand, as the correlation increases, information disclosure becomes more socially efficient in the sense that for the same cost more overall information can be disclosed. The first effect is stronger initially, causing the optimal precision to decline, but as correlation approaches one, the large "bang for the buck" associated with disclosure causes the optimal precision to increase. By contrast, Figure 5A shows that when there exists a Nash equilibrium with positive precisions (i.e., for relatively high correlation levels), the equilibrium precision decreases monotonically in the correlation coefficient. This is because in the Nash equilibrium the second effect discussed above is not taken into consideration.

In Figure 5B we show the attained value in the social optimum and the Nash equilibria in this example. First, note that both the socially optimal value and the value in the Nash equilibrium (with positive precisions) are increasing in the correlation coefficient. This is intuitive and is due to the spillover of information, which is more pronounced when correlation is high. Although there is a discontinuity in the optimal precisions, the attained value is continuous in the correlation levels both in the social optimum and in both types of Nash equilibria. Since the optimal as well as the Nash precision is zero for relatively low correlations, there is no difference between the attained values, but for high correlations the Nash equilibrium with zero precisions leads to a much worse outcome than the social optimum. The Nash equilibrium with positive precisions obviously attains a smaller value

[22] Note that the lowest correlation coefficient for which the socially optimal precisions are positive (about 0.67) is slightly smaller than the lowest correlation coefficient for which there is a Nash equilibrium with positive precisions (about 0.68).

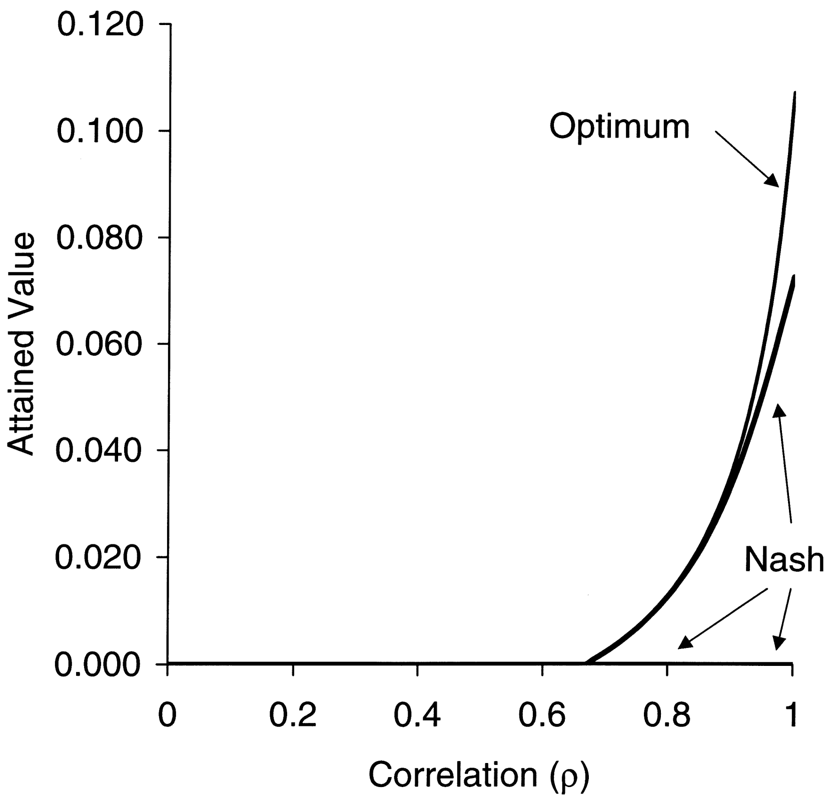

Figure 5B
Figure 5B shows the attained value of the social welfare function for the same parameters.

than to social optimum (since the Nash precisions are different from the optimal precisions), but its attained value is not very far from the socially optimal value.

In the example above there is no role for disclosure regulation if firm values are not highly correlated (if $\rho < 0.67$), because in that case the unique Nash equilibrium (zero precisions) leads to the socially optimal outcome. If firm values are more highly correlated ($\rho \geq 0.67$), however, then there is a role for regulation. For example, if by regulation the firms were required to disclose a signal with at least a prespecified positive precision (e.g., 7), then the equilibrium with zero precisions, which leads to a particularly poor outcome, would be eliminated and the outcome would be much closer to the social optimum.

It is interesting to examine social welfare when there are more than two firms in the market, since obviously the number of firms in reality is quite large, and because the free-rider problem that arises from the fact that each firm's disclosure provides information about other firms becomes more severe

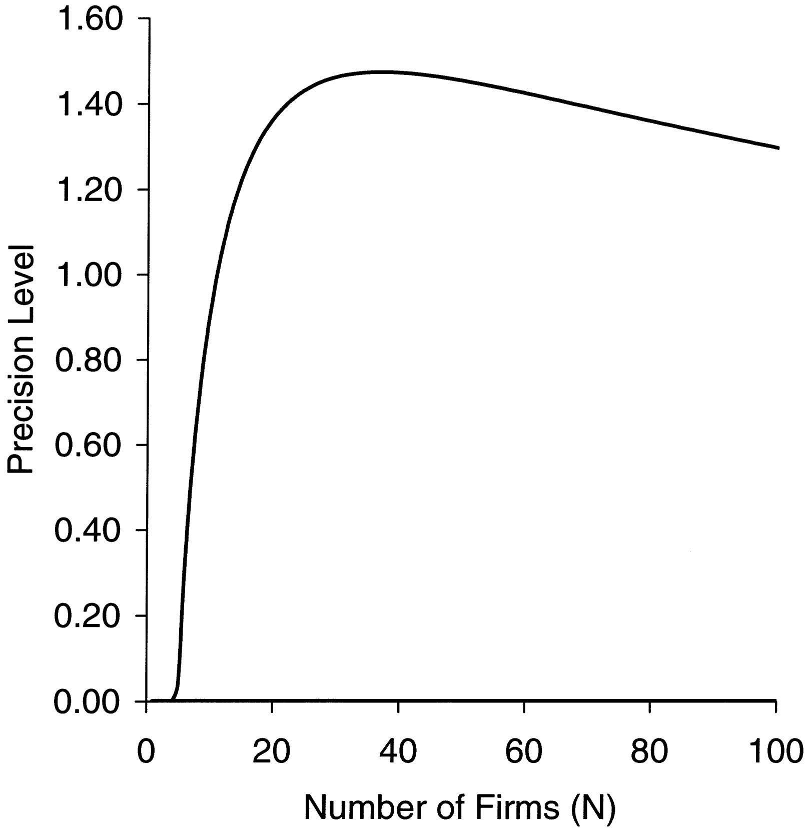

Figure 6A
Figure 6A plots the Nash equilibrium and socially optimal precision levels as a function of the number of firms N in a model with N identical firms. The parameters are given by $\delta_i = 0.2$ and $\gamma_i = 0.01$, where for firm i, δ_i is the parameter measuring the gains from the transaction and γ_i is the cost per unit precision of the disclosed signal. The firms' values are given by the one-factor model in Equation (8) with $t = 50$ (the precision of the common factor) and $d = 10$ (the precision of the firm-specific component). The unique Nash equilibrium for all N involves no disclosure, but the socially optimal outcome involves disclosure for $N > 4$.

as the number of firms increases. Figures 6A and 6B show the precision levels and the total value, respectively, in an example with many firms, as discussed at the end of the previous section. In this example $\delta_i = 0.20$, $\gamma_i = 0.01$, $t = 50$, and $d = 10$, where t is the precision of the common factor and d is the precision of the firm-specific component of the value. Note that the pairwise correlation coefficient is equal to $d/(t + d) = 1/6$.

It can be shown that in this example $z_i^\star = 0$ for each i and N.[23] Thus the unique Nash equilibrium has each firm choosing zero precision, so that there

[23] Note that this is in contrast to the case with one firm in which $\delta = 0.22$, depicted in Figure 2A, where $z^\star$ is positive. With $\delta = 0.20$ rather than 0.22, the optimal precision is zero for all ω.

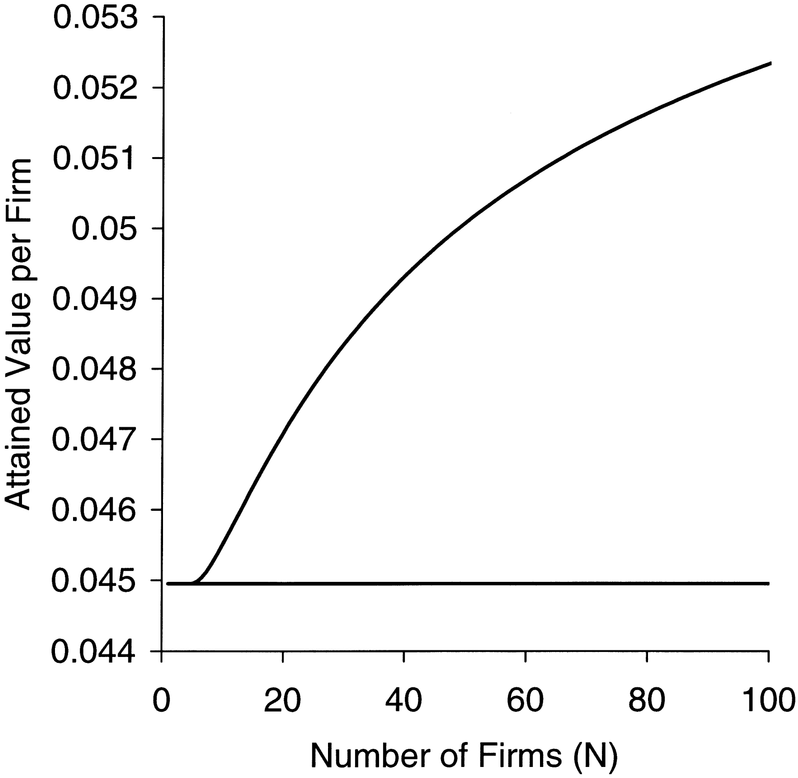

Figure 6B
Figure 6B plots the attained social welfare in the Nash equilibrium and in the social optimum as a function of N and shows that the Nash equilibrium is increasingly inefficient as N grows.

is no disclosure at all. However, Figure 6A shows that the socially optimal precision level is positive if the number of firms exceeds 4. Intuitively, when there are very few firms the spillover of information associated with disclosure is not sufficiently significant and so no information is disclosed. When the number of firms is large enough there is a benefit to each firm disclosing relatively precise information, which is useful for valuing all firms. Eventually, as the number of firms grows, the same amount of information can be more efficiently disclosed, with each firm disclosing a less precise signal, which is more cost effective. Thus the optimal precision declines.

Figure 6B shows that in this example with many firms, the optimal social value is increasing as the number of firms grows. (The plotted value is on a per firm basis; obviously the total value increases even more.) This is intuitive, since the information each firm can disclose provides value to many other firms, value which is not internalized in the noncooperative Nash equilibrium of the disclosure game. In this example there is clearly a role for regulation which would require a positive minimal precision level for firms. Such regulation would force firms to disclose more than they would in the Nash equilibrium in which no information is disclosed. Since the free-rider problem is severe, regulation can help in achieving a much better outcome.

In the two examples so far, firms were assumed to be symmetric. If symmetry does not hold, then new issues arise if regulation cannot be specialized to different firms. This is illustrated by the following example. Suppose that $\delta_1 = 0.8$, $\gamma_1 = 0.09$, $\delta_2 = 0.2$, and $\gamma_2 = 0.01$. Thus firm 1 has a relatively high gain from being sold but also a relatively high cost of disclosure relative to firm 2. Figure 7 shows the precision levels and the net value per firm in the social optimum and in the Nash equilibrium. We see (in Figure 7A)

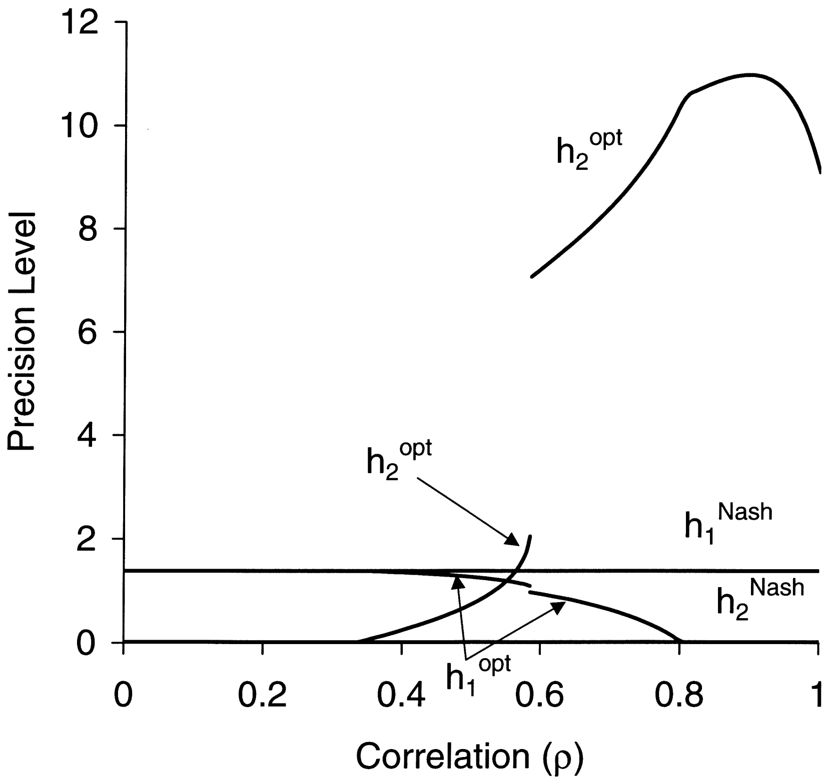

Figure 7A

Figure 7A plots the Nash equilibrium and socially optimal precision levels for each value of ρ, the correlation coefficient between the two firm values, for an example with two asymmetric firms. The parameters in this example are $\delta_1 = 0.8$, $\delta_2 = 0.2$, $\gamma_1 = 0.09$, $\gamma_2 = 0.01$, and $\omega_1 = \omega_2 = 1$, where for firm i, δ_i is the parameter measuring the gains from the transaction, γ_i is the cost per unit precision of the disclosed signal, and ω_i is the prior precision. For low values of ρ both the Nash equilibrium and the socially optimal disclosure policies involve firm 2 disclosing zero and firm 1 disclosing a signal with positive precision. While the Nash equilibrium is the same for all correlation levels, the social optimum involves firm 1 making a much more precise disclosure, with firm 2 eventually disclosing nothing when correlation levels are high.

that for all correlation levels there is a unique Nash equilibrium in which firm 2 discloses no information (chooses zero precision) and firm 1 chooses a positive precision level of about 1.384. Thus in the Nash equilibrium, the firm with the higher cost and higher gain parameters discloses the information, while the firm with the lower cost and lower gain parameters does not disclose any.[24]

Now consider the socially optimal precisions in this example, also shown in Figure 7A. Note first that for low values of correlation the optimal precisions are identical to the Nash equilibrium precisions. (This is true up to a correlation coefficient of about 0.34.) For higher correlations, however, it is socially optimal for firm 2 to start disclosing information with positive and relatively high precision, and eventually for firm 1 to reduce its precision choice. As the correlation coefficient grows it becomes more efficient for firm 2, which has the lower costs, to make disclosures, since the disclosure provides information to value firm 1 as well. This yields a high attained social value because of the large value of δ_1.

Note also that the socially optimal precision level for firm 2 is actually decreasing when the correlation coefficient becomes close to 1. This is due to the fact that as the correlation gets to be quite high, it is again more cost effective to disclose somewhat less precise information, leading still to sufficient benefit because of the "economies of scope" in the inference. Finally, note that there is a discontinuity in the optimal precisions at a correlation of about 0.58. Figure 7B confirms that the total value obtained in the Nash equilibrium is the same as the social optimum for low correlation levels, but that it is significantly below the socially optimal level for high values of the correlation coefficient. This is consistent with the above discussion.

In the example in Figure 7, regulation again does not have a role if the correlation between the firm values is relatively low. For high correlation levels there is scope for regulation to improve the situation, since the Nash equilibrium outcome is significantly worse than the social optimum. However, if a uniform precision level for the disclosed signal is imposed on both firms, then it is not clear that regulation can improve the situation, since the socially efficient outcome has one firm disclosing no information and the other disclosing relatively precise information. This is discussed in more detail below.

6. The Effect of Disclosure Regulation

Having shown in the previous section that the Nash equilibria of the disclosure game are often socially inefficient, we now examine more closely the impact that disclosure regulation might have on the outcome of the game.

[24] The fact that this equilibrium is insensitive to the correlation coefficient can be understood by noting that for all correlation levels, firm 2 chooses zero precision. As a result there is no interaction between the firms. Thus firm 1 acts as if it were in isolation and the correlation coefficient does not enter its decision.

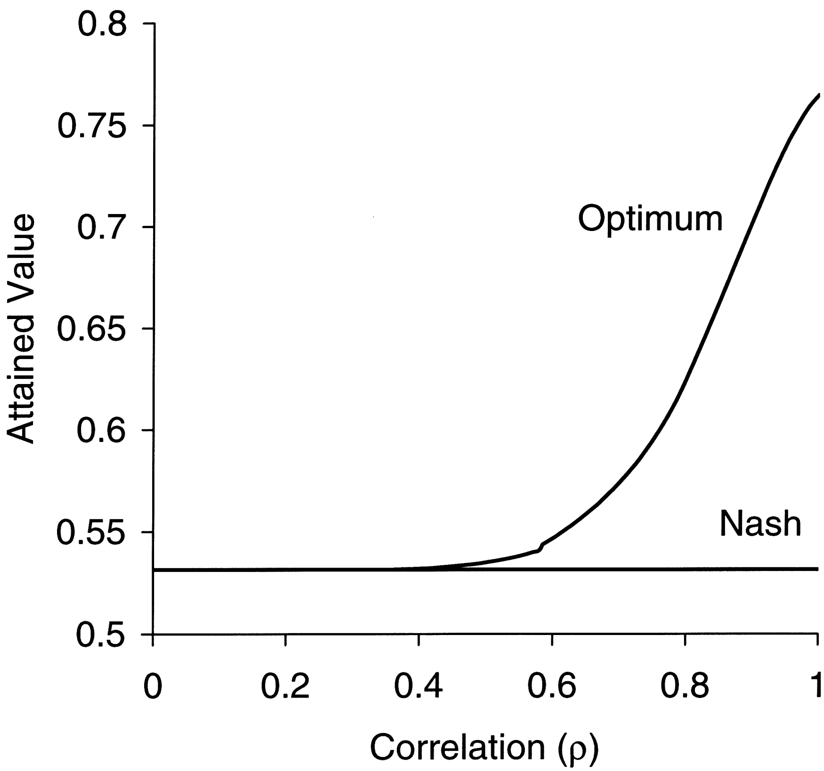

Figure 7B
Figure 7B plots the attained social welfare levels for this example, showing that the Nash equilibrium is increasingly inefficient as correlation between the firms' values gets larger.

We will again present a series of examples in which the welfare properties of the Nash equilibria of the disclosure game with regulation will be examined. For simplicity we assume that $\omega_i = 1$ for all i.

We start by examining the noncooperative Nash equilibrium of the game under a constraint on the precisions, that is, firms' precisions are required to be at least as large as a prespecified level set by regulators. We assume that this minimal level must be the same for all firms. First consider the last example of the previous section, where $\delta_1 = 0.8$, $\delta_2 = 0.2$, $\gamma_1 = 0.09$, and $\gamma_2 = 0.01$, and assume $\rho = 0.9$. The socially optimal precision levels for these parameters are $h_1^\star = 0$ and $h_2^\star = 10.98$. Figure 8A shows the chosen precision levels of the two firms as a function of the required (i.e., minimal) precision level for this example. We see that when there is no regulation (the required minimal precision level is zero), only firm 1 discloses information. As the required precision level rises, firm 2 discloses information at the minimal precision level and firm 1 correspondingly reduces its chosen precision

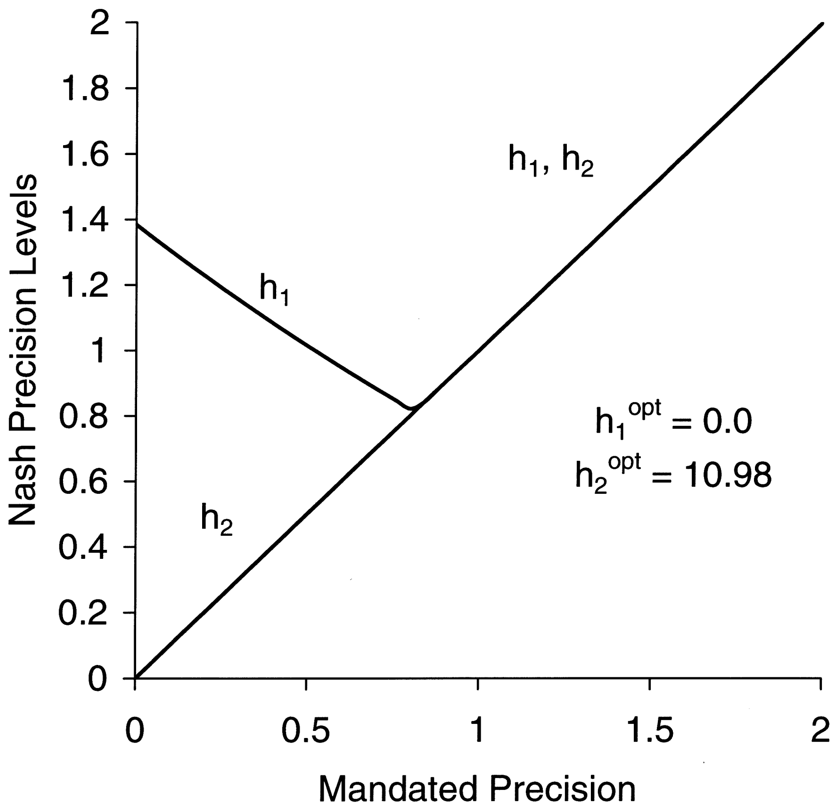

Figure 8A
For the same parameters as Figure 7, and fixing the correlation coefficient at $\rho = 0.9$, Figure 8A shows the Nash equilibrium precisions in a game in which firms are constrained to choose a precision level at least as high as a given mandated precision. This is shown for a range of possible minimal precision levels.

level. The constraint is binding on firm 2 but not on firm 1. Eventually, when the minimal precision is large enough, the constraint is binding on both firms and both disclose information at the minimal required level.

In Figure 8B we see the effect of various levels of regulation on the values obtained in equilibrium for this example and how they relate to the social optimum. It is clear that regulation can improve welfare in this case. For all the positive values of required precision shown, the social value exceeds the value attained in the Nash equilibrium without regulation and it is highest (although not equal to the first best) when the minimal precision is set at about 1.2. Intuitively, the inefficiency that occurs in this example when there is no regulation stems largely from the fact that the "wrong" firm is disclosing information. By imposing a minimal precision level, the more cost-effective firm 2, which is not disclosing any information in the Nash equilibrium, is

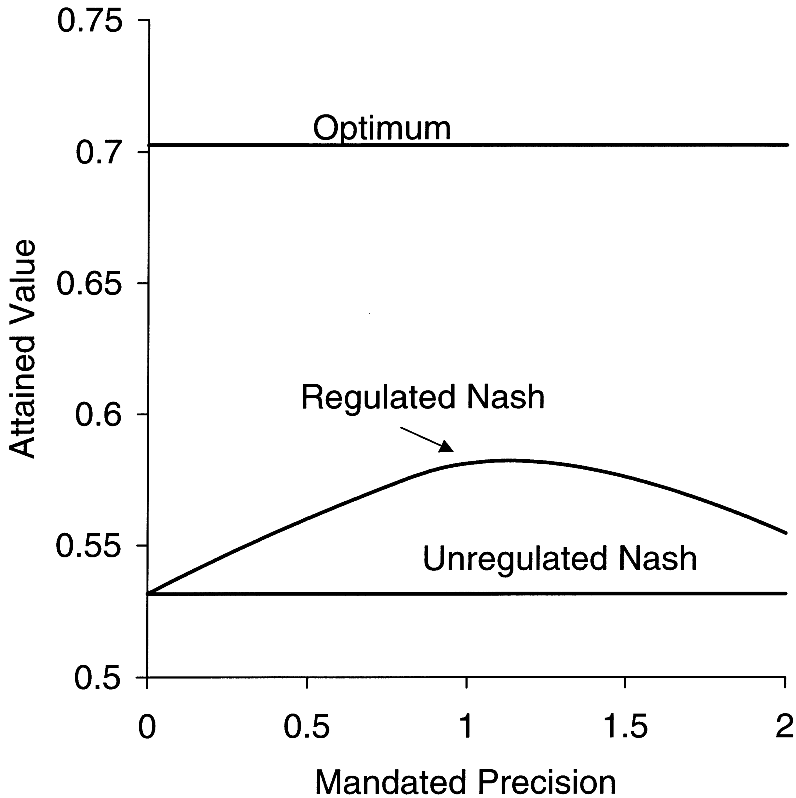

Figure 8B
Figure 8B plots the attained social welfare levels in each case, showing that for appropriately chosen levels of minimal precision, this type of regulation can improve welfare in this example.

forced to disclose, while the less efficient firm 1 reduces its precision. This leads to an improvement in the attained social value.

It is interesting to note that while regulation improves the total welfare, it is not the case that both firms are made better off by it. As is intuitive, it can be shown that firm 2, for whom the constraint on precision is always binding, is slightly worse off, while firm 1 is better off when a minimal precision is required. Thus, without transfer payments, an improvement cannot be accomplished through firms voluntarily committing to the minimal precision (say, through a trade organization).

The above example shows that setting a uniform required precision can improve social value even in some asymmetric cases. The next example shows that such regulation does not always lead to an improvement. Suppose the parameters are $\delta_1 = 0.7$, $\delta_2 = 0.2$, $\gamma_1 = 0.005$, $\gamma_2 = 0.01$, and $\rho = 0.99$. In this example firm 2 has the lower cost and the higher gain from the sale. It can be shown that for these parameters the socially optimal precision levels are $h_1 = 0$ and $h_2 \approx 14.27$. In Figure 9A we see that the Nash equilibrium precisions with no regulation have firm 1 disclosing no information (as in the social optimum) and firm 2 disclosing information with precision of about 7.67.

The effects of various levels of regulation on the attained value are shown in Figure 9B. We see that any positive required minimum precision level lowers social welfare. Intuitively, unlike the previous example, here the inefficiency of the Nash equilibrium without regulation does not stem from the wrong firm doing the disclosure (indeed, it is socially optimal for firm 1 not

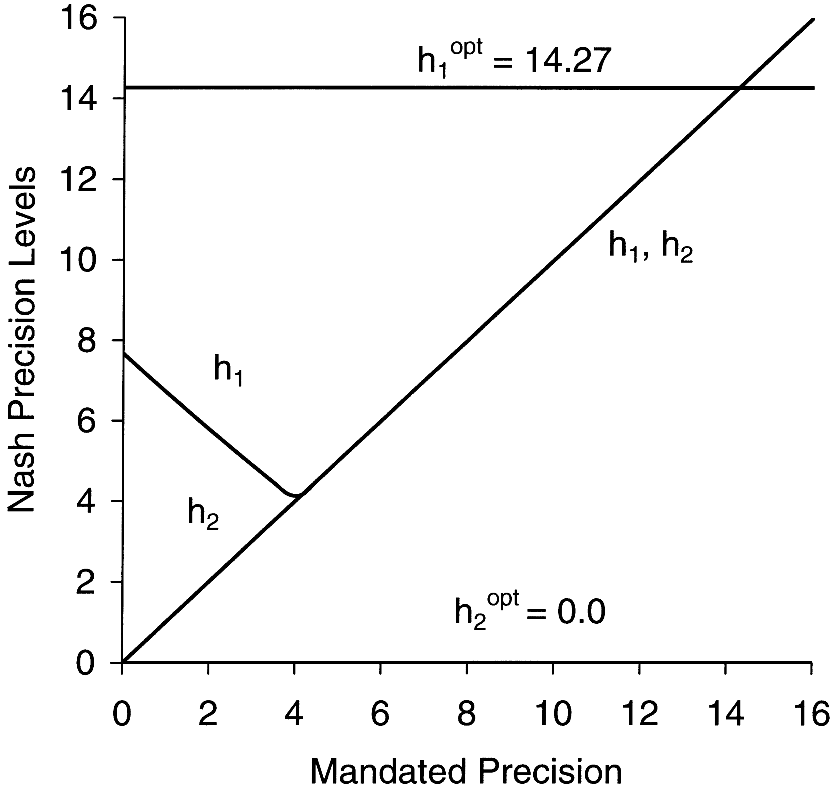

Figure 9A
An example in which setting any minimal precision level for the disclosure does not improve welfare. The parameters for this example are $\delta_1 = 0.7$, $\delta_2 = 0.2$, $\gamma_1 = 0.005$, $\gamma_2 = 0.01$, $\omega_1 = \omega_2 = 1$, and $\rho = 0.99$, where for firm i, δ_i is the parameter measuring the gains from the transaction, γ_i is the cost per unit precision of the disclosed signal and ω_i is the prior precision, and where ρ is the correlation coefficient between firms' values. Figure 9A shows the socially optimal precision levels and the Nash precisions for various levels of minimal precision constraint (with zero being the case of no regulation). In both the Nash equilibrium without regulation and in the social optimum, firm 2 does not disclose while firm 1 does, but the Nash equilibrium precision chosen by firm 1 is significantly lower than the social optimum.

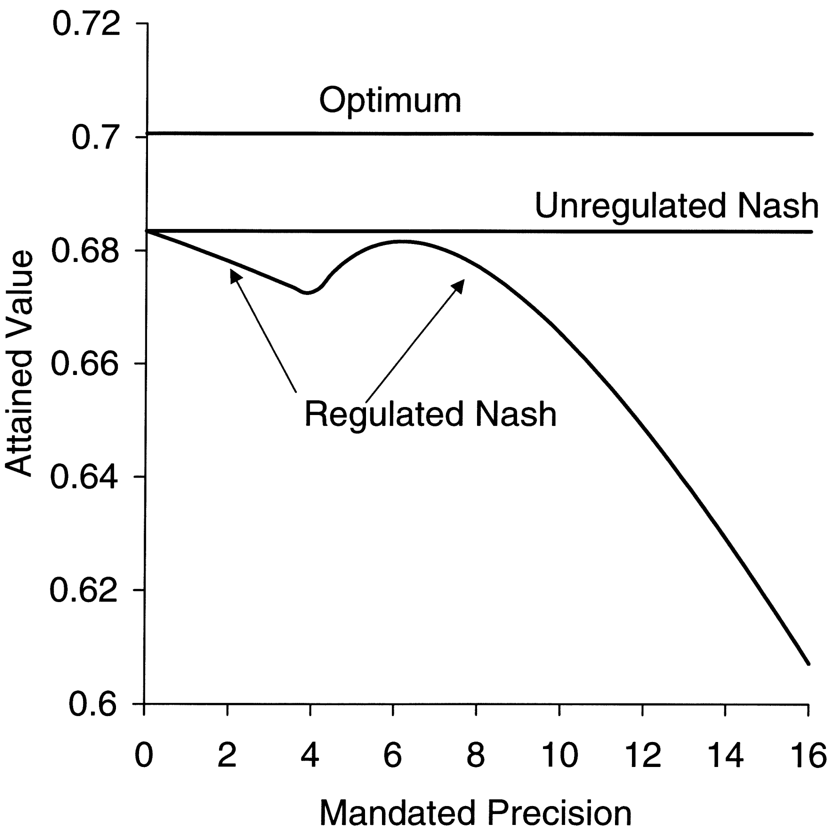

Figure 9B
Figure 9B, which plots the attained social value under the different regimes, shows that setting a positive minimal precision level for the disclosure always lowers social welfare relative to the case of no regulation.

to disclose any information), but rather from the fact that firm 2 underinvests in disclosure relative to the social optimum. Requiring a minimal precision level forces firm 1, for whom the cost of disclosure is high, to invest in disclosure, and this reduces the total value. Thus regulation that imposes a minimal precision level on both firms cannot address the type of inefficiency that occurs in this example.

The two examples above show that requiring the same minimal level of precision for the disclosure might be useful, but it may also reduce social welfare even when the Nash equilibrium is quite inefficient. We now consider another form of intervention that uses subsidies to reduce the perceived cost of disclosure. Specifically, a *uniform* subsidy level of y reduces the cost per unit precision by y for each firm and is funded by taxes or other assess-

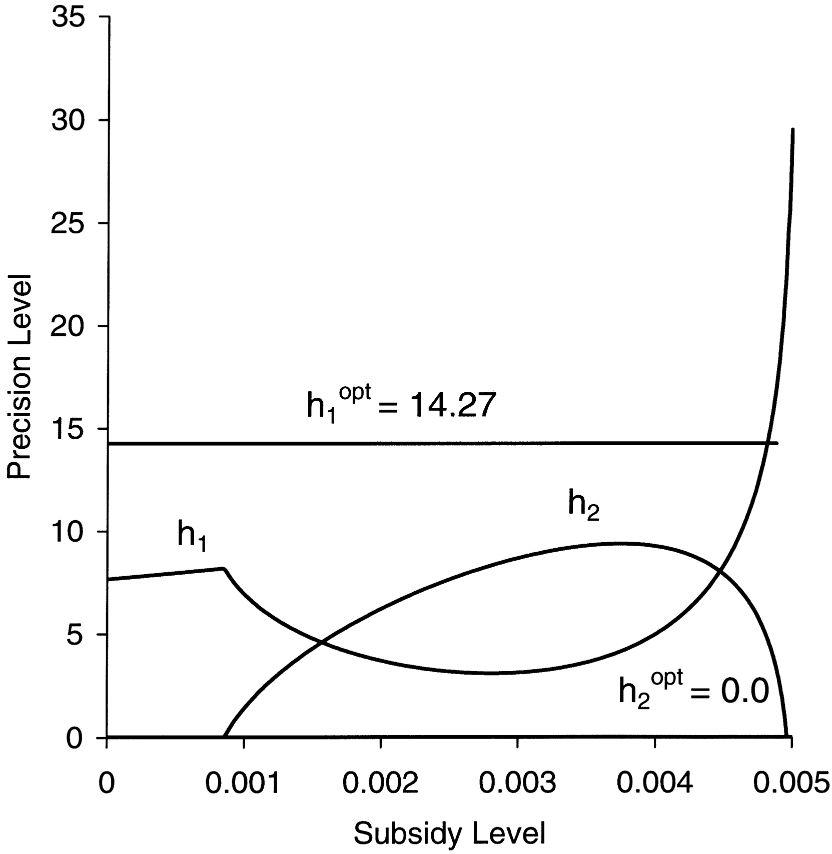

Figure 10A
For the same parameters as in Figure 9, the figure plots results obtained with various levels of subsidy given to decrease the perceived cost of the disclosure to both firms. Figure 10A shows the precision levels in the Nash equilibrium with a range of subsidy levels, as well as the socially optimal precisions.

ments.[25] This obviously alters the game played by the firms and changes the Nash equilibria. We will examine the difference in the chosen precisions and the effect such subsidies have on the attained values relative to the social optimum. In calculating the total attained value under the subsidy we will use the Nash equilibrium precisions for the game where costs are reduced by the subsidy, but we calculate the actual cost of these precisions using the true per unit cost γ.

[25] We do not consider here the cost of any distortions that might occur because of the taxation. In principle, subsidies can be financed by a lump sum tax that covers the cost of the subsidy and leads to no distortion. Since lump sum taxes are generally viewed as being difficult or impossible to implement, some distortion due to taxation is likely to occur in practice.

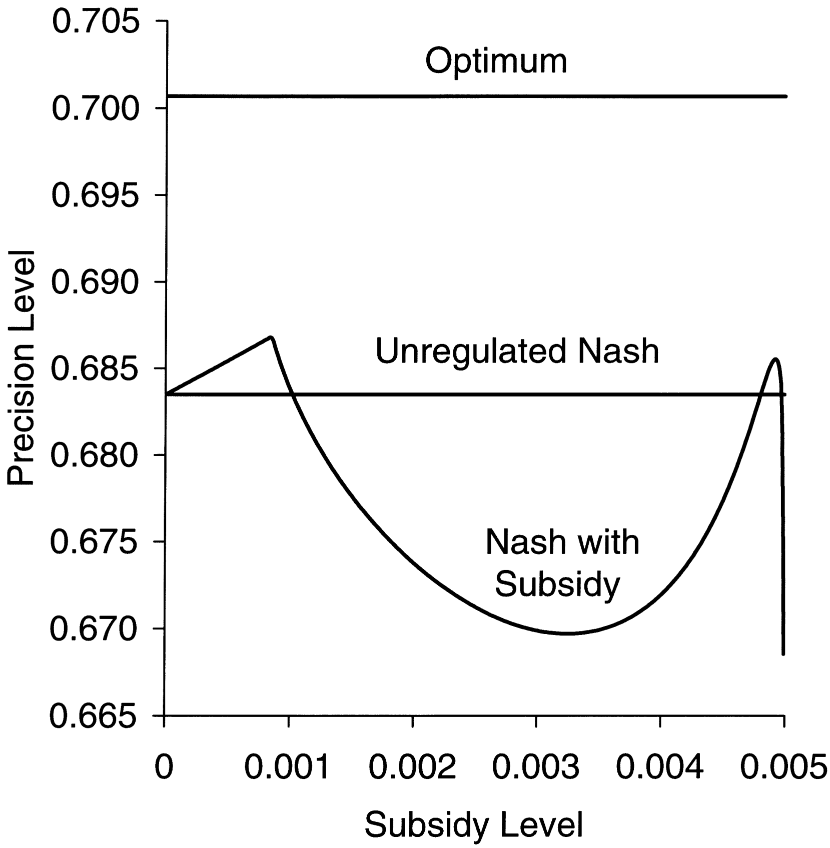

Figure 10B
Figure 10B plots the attained social value in the various cases. We see that in this case a small subsidy can improve welfare relative to the benchmark case of no regulation.

Consider the effect of subsidies in the example of Figure 9, where regulation that requires a positive precision level never improves welfare. Recall that in this example firm 1 discloses too little information in the Nash equilibrium. Figure 10A shows the equilibrium precision levels of the two firms as a function of the subsidy level, and shows how they compare with the socially optimal precisions. Figure 10B displays the attained values realized in the Nash equilibrium of the game with subsidies and compares these with the value achieved in the social optimum. We see that there are subsidy levels which improve welfare relative to the unregulated Nash equilibrium. For example, when the subsidy level is small, firm 1 discloses slightly more precise information than it does without the subsidy, and firm 2 still does not disclose any information in equilibrium. This is an improvement in the outcome. For intermediate values of the subsidy, the outcome is worse than the equilibrium without subsidies, because the cost is reduced enough for firm 2 that it too produces information, sometimes leading firm 1 to reduce its precision. As the subsidy level gets close to 0.005, the cost of disclosure for firm 1 goes to zero. As long as it is not too close to zero, an improvement can again be obtained, because again firm 1 produces more precise information and firm 2 reduces its precision. If the subsidy is very close to 0.005, firm 1 eventually produces too much information, and again the outcome is inferior to the Nash equilibrium without regulation.

It can be shown that, if chosen appropriately, subsidies can also improve welfare in the example in Figure 8. In fact, in this example subsidies can bring the attained value quite close to the social optimum. More generally, however, it is not always the case that subsidies are useful. It can be shown that in some cases regulation by requiring a uniform precision level can actually improve social welfare, while (uniform) subsidies cannot.[26] Thus it is not possible, based on our model, for us to recommend one of these methods of intervention over another.[27]

7. Concluding Remarks

In this article we have analyzed a model of voluntary disclosure by firms in financial markets. Our model focuses on externalities that arise when firm values are correlated and as a result the disclosures made by one firm are useful in the valuation of other firms. The value of disclosure in our model is due to the fact that under asymmetric information firms may withdraw from value-enhancing transactions and forgo productive investments or trades. Our main results are the following:

- There may be multiple Nash equilibria to the disclosure game, where in one equilibrium firms choose to disclose no information and in another each firm discloses a relatively precise signal.
- Because of the free-rider problem that arises when the costly disclosures of one firm can be used in valuing others, the Nash equilibrium disclosure policies are often socially inefficient, especially when firm values

[26] Suppose $\rho = 0.9$, $\delta_1 = 0.8$, $\delta_2 = 0.0$, $\gamma_1 = 0.09$, and $\gamma_2 = 0.05$. Clearly, since firm 2 never benefits from information, the Nash equilibrium without any intervention has $h_2 = 0$. Thus we know from Figure 7A that $h_1^N = 1.3835$. Note that while it obtains no value from the disclosure, firm 2 is the one with the lower cost of disclosure. The socially optimal precisions are $h_1^* = 0.6514$ and $h_2^* = 1.0914$. Note that any subsidy level lower than 0.05, such that it is still costly for firm 2 to disclose, would not change the outcome. If the subsidy is set equal to 0.05, then firm 2 is indifferent to how precise its disclosure would be, as it has no cost and no benefit from the disclosure. However, the subsidy also reduces the cost of disclosure for firm 1, which is inefficient. It turns out that no matter what precision level is actually chosen by firm 2, social value is lower with the subsidy than it is in the Nash equilibrium without the subsidy. However, as in Figure 8B, it is possible to improve welfare in this example by setting a minimal precision level for the disclosure.

[27] Note that in all the examples of this section at least one Nash equilibrium exists. If a Nash equilibrium does not exist, then regulation can restore the existence of an equilibrium. Consider, for example, the parameters in Figure 4. In this case it can be shown that requiring a minimal precision of about 1.8 or more will lead to the existence of an equilibrium.

are highly correlated. Thus there is scope for disclosure regulation to improve welfare.

- Regulating disclosure by requiring a minimal precision level for the disclosure can sometimes be welfare improving, but it may be difficult or impossible to set a uniform precision level (for firms with different parameters) that will improve welfare.
- The same is true for subsidies which reduce the perceived cost of financial disclosure.
- Neither of these methods of regulation dominates the other in all situations.

While these results suggest that there is a potential value in regulating firms' disclosure policies, they also point to the difficulty in designing an appropriate and effective method for this regulation, especially when firms differ from each other in such parameters as the effective cost of the disclosure or the productive benefit that can accrue to the firm if information asymmetries are resolved. On a more positive note, suppose firms can be divided into groups by, say, industry or size, such that parameters are not too diverse while values are highly correlated within each group. If it is possible to design regulation which might be different for different groups of firms, then it is more likely that the attained social value will be higher than without regulation.

There are other possible sources of positive externalities in information disclosure which we have not considered explicitly but to which our results might apply. For example, to the extent that disclosure reduces information asymmetries, it can increase the liquidity of stocks and reduce trading costs incurred by investors trading among themselves. Obviously the arguments we have developed suggest that since the information disclosed by one firm can be used in valuing other firms, one firm's information disclosure can improve the liquidity of other firms. This is essentially a simple extension of the argument we have made. However, there is another aspect that is important. It is possible that by improving the liquidity of its own stock through disclosure (or other means) one firm, say A, favorably affects the value of other stocks. This is because the attractiveness of the stock of another firm, say firm B, may depend on how effectively investors can manage the exposure to firm B's systematic risk. If firm A lowers the cost of trading its own stock, it may increase the willingness of investors to hold firm B because the cost of adjusting a portfolio to changes in firm B's risk characteristics has decreased.[28] However, firm A will not take into account the impact that increasing the liquidity of its own stock has on the value of other stocks.

It has been argued that another reason for disclosure regulation is to establish standards for the communication of information from firms to investors.

[28] Obviously this argument assumes that investors are heterogeneous, otherwise there is no reason to trade.

However, this need for a standard does not necessarily imply that government intervention is required, since firms would tend to adopt a standard on their own if it were valuable. Alternatively, private organizations such as the Financial Accounting Standards Board (FASB) would set a standard and firms would gravitate toward it. However, if a suboptimal standard reigns, government intervention might be needed to move firms to a new and better standard. The setting of a standard, however, seems to be of secondary importance to the issue of how much information firms should be forced to disclose.

We have obtained our results in a reduced form model in which a number of strong assumptions were made concerning preferences, distributions, and the disclosure mechanism. We noted in a number of places throughout the article that many of our results are quite robust to the model specifications. Our main results regarding financial disclosure regulations imply that, while informational externalities create a scope for disclosure regulation, finding the optimal method for such regulation can be difficult. This conclusion is certainly likely to hold up in a more general model with respect to various parametric and distributional assumptions. One basic assumption of our model that should be examined further is that the choice of a precision level for the disclosure is made before the firm observes the private information about its value. This assumption makes sense if we think of the disclosure as involving an institutional structure for information release that requires some commitment and takes time to set up. It would be interesting to examine a model in which the disclosure decision can depend on the information about firm value. Preliminary analysis of such a model suggests that it is quite complex and certainly new issues arise with respect to modeling disclosure regulation and understanding its potential role.

A related issue is that our model is static, involving one period of disclosure followed by one possible transaction. There are a number of possible extensions to a more dynamic model which one can consider. First, a model in which the firm can make a number of disclosures prior to the (one) possible transaction is reducible to our model if the sequence of signals is viewed as equivalent to one signal having the same precision as the total of the sequence. There are different ways to model the game played between two or more firms in this case. We conjecture that with the same cost structure an equilibrium (without regulation) will involve no disclosures until the last period and the precisions of the disclosures in the last period are the Nash equilibrium levels that we find in our one period setting. Another dynamic case to consider is where there is one or possibly a sequence of disclosures, followed by a sequence of possible transactions. In this case the firm might have an incentive to turn down an early transaction to signal that it is a higher type, since this will potentially be valuable in future transactions if bidders increase their bid prices as a result. Finally, one could consider the case with an alternating sequence of disclosures and possible transactions. These models are clearly beyond the scope of this article, and we hope future research will shed light on some of the issues that arise.

It should also be noted that our model abstracts from many contracting and other agency issues that arise between initial owners of the firm and other investors and between shareholders and managers. Mahoney (1995), for example, argues that agency considerations can provide a rationale for certain types of disclosure regulations, and Rock (1998) suggests that a mandatory disclosure system can provide a means for credible commitment by firms to certain disclosure practices. Including such considerations in a fuller model of disclosure regulation may produce additional insights.

Finally, it would be interesting to apply some of the ideas of this article to a situation in which firms make a decision with respect to where to list their stocks; different exchanges (e.g., in different countries) have different disclosure rules. This will shed light on the types of equilibria that might emerge if harmonization of accounting standards is incomplete and firms can select among exchanges, possibly choosing among different regulation regimes. This seems to capture the current situation in European financial markets as well as more globally.

Appendix

Proof of Lemma 1 From Equation (4) we know that $x(z)$ solves

$$x(z) = -\frac{\phi(x(z)\sqrt{z})}{\Phi(x(z)\sqrt{z})\sqrt{z}} + \delta. \tag{A1}$$

If one lets $y = x(z)\sqrt{z}$ and implicitly differentiates Equation (A1) with respect to z, one obtains

$$\left(\frac{\Phi^2(y) - y\phi(y)\Phi(y) - \phi^2(y)}{\Phi^2(y)}\right)\frac{\partial y}{\partial z} = \frac{\delta}{2\sqrt{z}}. \tag{A2}$$

To establish that $x(z)\sqrt{z}$ is increasing in z it is sufficient to show that $\Phi^2(y)-y\phi(y)\Phi(y)-\phi^2(y)$ is positive for all y. To do this we will use the following lemma.

Lemma. *Let $f(x)$ be a function continuously differentiable up to the mth derivative and assume that $\lim_{x\to-\infty} f^{[k]}(x) = 0$ for $k = 0, 1, \ldots, m-1$ where $f^{[k]}(x)$ is the kth derivative of $f(x)$ and $f^{[0]}(x) = f(x)$. Then, if $f^{[m]}(x) > 0$ for all x, $f(x) > 0$ for all x.*

This lemma can be established by induction by showing that if $k \le m$ and $f^{[k]}(x) > 0$ for all x, then it follows that $f^{[k-1]}(x) > 0$ for all x. Assume to the contrary that $f^{[k-1]}(q) \le 0$ for some q. Then $f^{[k]}(x)$ must be nonpositive for some $x < q$ since $\lim_{x\to-\infty} f^{[k-1]}(x) = 0$. Obviously this contradicts the assumption that $f^{[k]}(x) > 0$ for all x. It therefore follows that $f^{[k-1]}(x) > 0$ for all x.

Now since $\Phi(y)$, $\phi(y)$, and $y\phi(y)$ all go to 0 as $y \to -\infty$, it follows that

$$\lim_{y \to -\infty} \Phi^2(y) - y\phi(y)\Phi(y) - \phi^2(y) = 0. \tag{A3}$$

Differentiating $\Phi^2(y) - y\phi(y)\Phi(y) - \phi^2(y)$ with respect to y, we obtain

$$\frac{\partial(\Phi^2(y) - y\phi(y)\Phi(y) - \phi^2(y))}{\partial y} = \phi(y)\big(\Phi(y) + y\phi(y) + y^2\Phi(y)\big). \tag{A4}$$

Note that $\lim_{y\to-\infty}(\phi(y)(\Phi(y) + y\phi(y) + y^2\Phi(y)) = 0$, $\lim_{y\to-\infty}(\Phi(y) + y\phi(y) + y^2\Phi(y)) = 0$ and that $\phi(y)(\Phi(y) + y\phi(y) + y^2\Phi(y)) > 0$ for all y if $\Phi(y) + y\phi(y) + y^2\Phi(y) > 0$ for all y.

Differentiating $\Phi(y) + y\phi(y) + y^2\Phi(y)$ with respect to y, we obtain

$$\frac{\partial(\Phi(y) + y\phi(y) + y^2\Phi(y))}{\partial y} = 2y\Phi(y) + 2\phi(y). \tag{A5}$$

Since $\lim_{y\to-\infty} y\Phi(y) = 0$, it follows that the limit of Equation (A5) is 0. Finally, differentiating one more time, we find that

$$\frac{\partial(2y\Phi(y) + 2\phi(y))}{\partial y} = 2\Phi(y), \tag{A6}$$

and since $\Phi(y) > 0$ for all y, this establishes the result using the lemma.

Proof of Proposition 1[29]**.** Recall that $B(z) = \delta\Phi\big(x(z)\sqrt{z}\big)$. To simplify the notation in this proof we define $y(z)$ to be equal to $x(z)\sqrt{z}$. Then

$$\frac{\partial B(z)}{\partial z} = \delta\frac{\partial\Phi(y(z))}{\partial z} = \delta\phi\big(y(z)\big)\frac{\partial y(z)}{\partial z}. \tag{A7}$$

We know that $y(z)$ solves

$$\phi\big(y(z)\big) = \Phi\big(y(z)\big)\big(\delta\sqrt{z} - y(z)\big). \tag{A8}$$

Implicitly differentiating Equation (A8) with respect to z and solving for $\partial y(z)/\partial z$, we obtain

$$\frac{\partial y(z)}{\partial z} = \frac{\delta\Phi\big(y(z)\big)}{2\big(\sqrt{z}\Phi\big(y(z)\big) - \delta z\phi\big(y(z)\big)\big)}. \tag{A9}$$

Thus we need to show that

$$\lim_{z\to 0}\frac{\delta^2\phi\big(y(z)\big)\Phi\big(y(z)\big)}{2\big(\sqrt{z}\Phi\big(y(z)\big) - \delta z\phi\big(y(z)\big)\big)} = 0. \tag{A10}$$

From Equation (A8) we know that

$$\delta\sqrt{z}\Phi\big(y(z)\big) = \phi\big(y(z)\big) + y(z)\Phi\big(y(z)\big), \tag{A11}$$

[29] We are deeply indebted to Larry Glosten for suggesting the following proof which greatly improved on the proof used in earlier versions of this article.

and that

$$\delta^2 z = \left(\frac{\phi\big(y(z)\big) + y(z)\Phi\big(y(z)\big)}{\Phi\big(y(z)\big)} \right)^2. \tag{A12}$$

Substituting Equations (A11) and (A12) into Equation (A10), letting $h(\cdot) = \phi(\cdot)/\Phi(\cdot)$ and simplifying, we obtain the following to show

$$\lim_{z\to 0} \frac{\delta^3 \phi\big(y(z)\big)}{2\Big(y(z) + h\big(y(z)\big) - h\big(y(z)\big)\big(y(z) + h\big(y(z)\big)\big)^2\Big)} = 0. \tag{A13}$$

Now as z goes to zero, $y(z)$ goes to $-\infty$. Moreover, it is easy to show that $h'(y) = -h(y)(y + h(y))$. Thus establishing Equation (A13) is equivalent to establishing that

$$\lim_{y\to -\infty} \frac{\delta^3 \phi(y)}{2\Big(\big(y + h(y)\big)\big(1 + h'(y)\big)\Big)} = 0. \tag{A14}$$

Using L'Hôpital's rule and the fact that $\phi'(y) = -y\phi(y)$, we see that $\lim_{y\to-\infty}(y + h(y))$ is equal to

$$\begin{aligned}\lim_{y\to -\infty} \frac{y\Phi(y) + \phi(y)}{\Phi(y)} &= \lim_{y\to -\infty} \frac{\Phi(y)}{\phi(y)} \\ &= \lim_{y\to -\infty} \frac{\phi(y)}{-y\phi(y)} = \lim_{y\to -\infty} \frac{1}{-y} = 0.\end{aligned} \tag{A15}$$

Since $(y + h(y))$ and $\Phi(y)$ both go to zero as $y \to -\infty$, it follows by L'Hôpital's rule that

$$\lim_{y\to -\infty} \frac{\delta^3 \Phi(y)}{\big(y + h(y)\big)^2} = \lim_{y\to -\infty} \frac{\delta^3 \phi(y)}{2\Big(\big(y + h(y)\big)\big(1 + h'(y)\big)\Big)}. \tag{A16}$$

Thus establishing Equation (A14) is equivalent to establishing that $\lim_{y\to-\infty} \delta^3\ \Phi(y)/(y + h(y))^2 = 0$. Using L'Hôpital's rule once again, we have

$$\begin{aligned}&\lim_{y\to -\infty} \frac{\delta^3 \Phi(y)}{\big(y + h(y)\big)^2} \\ &= \lim_{y\to -\infty} \frac{\delta^3 \Phi^3(y)}{\Phi^2(y)\big(y + h(y)\big)^2} \\ &= \lim_{y\to -\infty} \frac{3\delta^3 \Phi^2(y)\phi(y)}{2\Phi(y)\phi(y)\big(y + h(y)\big)^2 + 2\Phi^2(y)\big(y + h(y)\big)\big(1 + h'(y)\big)} \\ &= \lim_{y\to -\infty} \frac{3\delta^3 \phi(y)}{2(y + h(y))} \\ &= \lim_{y\to -\infty} \frac{3\delta^3 y\phi(y)}{2y(y + h(y))}.\end{aligned} \tag{A17}$$

The numerator of the last line of Equation (A17) goes to zero. The limit of the denominator of the last line of Equation (A17) is -1 since

$$\begin{aligned}
\lim_{y\to-\infty} y(y+h(y)) &= \lim_{y\to-\infty} \frac{y^2\Phi(y)+y\phi(y)}{\Phi(y)} \\
&= \lim_{y\to-\infty} \frac{2y\Phi(y)+\phi(y)}{\phi(y)} \\
&= 2\lim_{y\to-\infty} \frac{y\Phi(y)}{\phi(y)} + 1 \\
&= 2\lim_{y\to-\infty} \frac{y\phi(y)+\Phi(y)}{-y\phi(y)} + 1 = -1,
\end{aligned}$$

where the last equality follows from the fact that $\lim_{y\to-\infty} \Phi(y)/y\phi(y) = 0$. This establishes that the limit of $B'(z)$ as $z \to 0$ is 0. ■

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