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Mandatory Versus Voluntary Disclosures: The Cases of Financial and Real Externalities

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ABSTRACT: This paper compares the disclosures firms would seek to make voluntarily with "optimal" mandated disclosures in a single period, multi-firm model, in which there are covariances between firms' cash flows. This comparison is important because, in those circumstances in which the two types of disclosure coincide, it is possible to economize on the process of setting mandatory disclosures. The principal factors which contribute to the existence or absence of a correspondence between mandatory and voluntary disclosures are (1) the nature of the externality associated with a firm's disclosure, (2) the relation between the risk preferences of the shareholders of the firms making the disclosures and outside investors, (3) how much relative weight is placed on existing shareholders and outside investors' preferences in the social welfare function determining the optimal mandatory disclosure policy, and (4) the covariance structure between firms' cash flows.

THIS paper compares the disclosure policies that firms select voluntarily with the policies an accounting standards board or other regulatory body would mandate to maximize social welfare. This comparison is important because of the considerable resources devoted to developing accounting standards and related disclosure requirements. When an accounting standards

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board merely succeeds in mandating the disclosures which firms would adopt voluntarily, the resources used in setting the standards are wasted. By identifying a variety of situations in which mandatory and voluntary disclosures coincide, the paper provides some evidence for Beaver's (1977) proposal that promulgators of accounting standards should be obliged to provide explicit cost-benefit analyses to support the expansion of disclosure regulations.

The claim that voluntary and mandatory disclosure requirements should be presumed to be the same, unless otherwise demonstrated, stands in contrast to much of the prevailing scholarly literature on the subject. Proponents of additional mandatory disclosures argue that information about firms' financial conditions constitutes a public good which will be under-provided without regulation. They also assert that firms will tend to suppress the disclosure of unfavorable information.[1] Opponents of regulation counter by arguing that managers have incentives to disclose information about the firms they run to differentiate themselves from more poorly run enterprises. This incentive, as well as the incentive of investors to obtain trading profits through costly search, are considered to provide sufficient motives for voluntary information production and disclosure so as to ensure a properly functioning securities' market.[2]

This paper does not directly contradict either of these views. The point made is that, in presenting these externality-based arguments for or against additional disclosures, one must differentiate between the various kinds of externalities that can arise. We consider two alternative types of externalities here, "real" and "financial." A disclosure by one firm is said to create a real externality for other firms if the disclosure alters those firms' cash flows. For example, the disclosure of a firm's trade secrets generates positive real externalities for its competitors. In contrast, a disclosure by one firm generates only financial externalities on other firms if the disclosure has the potential of altering the equilibrium prices of those firms without altering the actual distributions of their cash flows. Financial externalities arise when one firm's disclosures affect only investors' *perceptions* of the distributions of other firms' cash flows. For example, disclosures by one firm in an industry may alter investors' beliefs about the profitability of other firms in the same industry, and thereby change their market values (Foster 1981).

Mandatory and voluntary disclosure policies do not always coincide when disclosures generate only financial externalities. Shareholders' attitudes toward risk, shareholders' relative weights in the social welfare function defining the optimal mandatory disclosure policy, and the covariance structure between firms' cash flows can all affect whether voluntary and mandatory disclosure policies coincide. However, there are a variety of circumstances in which these distinctly motivated disclosure policies do coincide when financial externalities alone exist.

These disclosure issues are studied using a "snap-shot" of an overlapping generations model (Samuelson 1958; Dye 1988) in which one generation of

[1] See, for example, Beaver (1977, 1981) or Gonedes and Dopuch (1974) for a summary of these efficiency arguments for and against regulated disclosures.

[2] For example, Hirshleifer (1971), Demski (1974), and Wilson (1975) illustrate the possibility of excessive information production by private parties.

shareholders is forced by life-cycle considerations to sell its firms to the next generation of shareholders before the firms' cash flows are realized. Because of this forced-sale assumption, the first generation of shareholders cannot directly share in the risk of the firms' cash flows with the second generation of shareholders. However, they indirectly share in this risk by participating in the stock market on which shares are transferred from one generation to the next. Disclosure policies matter here because disclosures affect the perceived riskiness of the securities when the exchange of shares takes place, and so disclosure policies affect risk-sharing between the two generations of shareholders.

This paper builds upon Demski (1973, 1974) who showed that (1), in general, rankings of information systems may not be complete, and (2) the selection among financial reporting systems may have redistributive consequences. These papers indicate that progress in comparing financial reporting systems depends on identifying more restricted settings where such rankings are viable. The present paper pursues this line of inquiry and reveals the following: in many contexts where comparisons of information systems are most easily made—where only financial externalities are present—mandated disclosures are superfluous, because the optimal mandated disclosures simply coincide with firms' voluntary disclosure decisions. Where comparisons of information systems are most difficult—where real externalities are present—optimal mandatory and equilibrium voluntary disclosure tend to diverge.

The paper proceeds as follows. Section I outlines the basic model. Section II provides the preliminary analysis, by studying the disclosures which "representative" market participants with common objectives make. Sections III and IV, respectively, study disclosures with financial and real externalities. Section V concludes the paper.

I. Model Description for Disclosures with Financial Externalities

The basic chronology of events corresponding to this model of disclosure requirements is summarized in the following time line:

Entrepreneurs (indexed by $i=1,\ldots,n$) own all firms. Firm i's cash flows, denoted $\tilde{z}_i$, are distributed $(\tilde{z}_1,\ldots,\tilde{z}_n)' \sim N(\tilde{\boldsymbol{\mu}},\boldsymbol{\Phi})$. Priors on $\tilde{\boldsymbol{\mu}}' \equiv (\tilde{\mu}_1,\ldots,\tilde{\mu}_n)$ are $\tilde{\boldsymbol{\mu}} \sim \boldsymbol{N}(\boldsymbol{M},\boldsymbol{\Omega})$.	Entrepreneurs choose disclosures policies (r_i), and produce information $(\tilde{x}_i)$ according to the disclosure policies they select.	Security market opens; entrepreneurs sell their firms (at prices P_i) to investors.	Investors learn realized values $(\tilde{z}_i)$ of cash flows.

The time line is constructed to capture some aspects of an environment in which the investment horizon of current shareholders is shorter than that of future shareholders. Explicitly, the initial owners of the firms, henceforth called *entrepreneurs* (or "E's" for short), are posited to sell their ownership interests in their firms, and consume their remaining wealth, before the firms' cash flows realize their values. The eventual owners of the firms, henceforth called *investors* ("I's"), purchase the firms from the entrepreneurs and engage in consumption only after the firms' cash flows realize their values. Consequently, this model is a snapshot of an overlapping generations model in which one generation of shareholders (E's) values possession of firms only because the next generation of shareholders (I's) is willing to purchase the firms from them.[3]

In the basic model, the E's subsequently sell their firms to I's after making disclosures which reveal information about the firms' cash flows. Each of the n E's and each of the k I's are assumed to be weakly risk-averse with constant absolute risk-aversion. Entrepreneur i's expected utility function is given by $u_i(w) = -(1/\beta_i)e^{-\beta_i w}$, $i=1,\ldots,n$, where w denotes i's wealth after receiving the proceeds from the sale of his firm. Investor i's utility function is given by $v_i(w) = (-1/\gamma_i)e^{-\gamma_i w}$, $i=1,\ldots,k$, where w in this case denotes i's wealth after the realization of all firms' cash flows. This difference in the interpretation of the arguments of E's' and I's' utility functions devolves from the difference in their investment horizons, as discussed above.[4]

In some cases, which are explicitly noted below, the time line is expanded to include another market on which E's can trade among themselves in advance of adopting any disclosure policies.

To allow for the possibility of financial externalities, there must be some nontrivial covariance among the firms' cash flows. We assume initially that both I's' and E's' believe that the joint distribution of firms' cash flows $\tilde{\mathbf{z}}' = (\tilde{z}_1, \ldots, \tilde{z}_n)$ is n-variate normal, with unknown mean vector $\tilde{\boldsymbol{\mu}}' = (\tilde{\mu}_1, \ldots, \tilde{\mu}_n)$ and known covariance matrix $\boldsymbol{\Phi}$, and that their prior beliefs about $\tilde{\boldsymbol{\mu}}$ are identical and summarized by another n-variate normal distribution with mean vector $\boldsymbol{M}$ and covariance matrix $\boldsymbol{\Omega}$.[5]

A disclosure by entrepreneur i is represented by an unbiased estimate $\tilde{x}_i$ of the mean of firm i's unknown cash flows $\tilde{\mu}_i$, and a disclosure policy ("DP" for short) for firm i consists of a specification of the precision (i.e., inverse of variance) r_i of this sample estimate. The entrepreneur is assumed to have no private information of his own: his choice of precision r_i and the realized sample estimate x_i are public knowledge, and he acquires no information about the realized value of $\tilde{\mu}_i$ other than through the sample estimate (the appropriateness of these assumptions is discussed below). If $r_i = \infty$, entrepreneur i must reveal $\tilde{\mu}_i$; if $r_i = 0$, entrepreneur i's disclosure reveals nothing and, hence, does not affect I's' beliefs about the distribution of cash flows.

This paper studies the choice of disclosure policies, i.e., the choice of the r_i. Once the r_i have been chosen, firms are presumed to have no control over the choice of the realized values of the x_i. Thus, the paper can be viewed as the study of information system choice. This stylized representation of a firm's disclosure process captures the importance of the precision of a firm's disclosures in revis-

[3] See Dye (1988) for other applications of overlapping generations models to accounting.

[4] We shall on occasion also consider risk-neutral E's and/or I's. Such preferences can be represented as the limit of negative exponential preferences $(-1/\beta) \times \exp(-\beta w)$, as β approaches zero.

[5] Unless otherwise noted, any vector is to be considered a column vector. Row vectors are indicated by an apostrophe. Both vectors and matrices are in boldface to distinguish them from scalars.

ing I's' perceptions of firms' cash flows. It is appropriate for studying the requirements an accounting standards board would like to impose on firms or irreversible choices (imposed by intertemporal consistency requirements) of accounting policy by an individual firm. It is not appropriate for studying voluntary earnings forecasts, where firms have control over the details of what is to be disclosed *after* receiving private information about μ_i. In such situations, signalling phenomena must be considered explicitly, whereas in our setting, signalling is irrelevant.

The assumption that E's and I's share common prior beliefs about the distribution of cash flows is adopted for two reasons. First, firms frequently commit themselves to disclosures in the future regarding information they do not possess when they make the commitment. For example, the decision to list on a particular stock exchange constitutes a commitment to make periodic disclosures about information the firm does not possess when the listing decision is made. Postulating that firms do not have superior information at the time they choose their disclosure policies is one way to model such commitments. Second, there has been substantial research documenting the disclosure policies firms would adopt if they were endowed with superior private information at the time disclosure decisions are made (e.g., Grossman 1981 and Milgrom 1981). This line of research has established that if firms cannot make false statements about their private information, then they will disclose their information fully (unless the information is proprietary) to distinguish the information they have from that possessed by other firms with worse information. We seek to distinguish the motives for disclosure in this paper from this well-known adverse selection motive.

We begin by developing an equilibrium pricing equation contingent on the disclosures firms make and the disclosure policies they adopt. Let $\tilde{\mathbf{z}}$ be the random vector describing the firms' cash flows, and let $E[\tilde{\mathbf{z}}|\boldsymbol{x},\boldsymbol{R}]$ be the vector whose ith element is the expected value of firm i's cash flows, conditional on $\boldsymbol{x}'=(x_1,\ldots,x_n)$, the vector of disclosures being drawn from a normal distribution with unknown mean vector $\tilde{\boldsymbol{\mu}}$ and known precision matrix $\boldsymbol{R}$. This matrix $\boldsymbol{R}$ summarizes the disclosure policies of all firms: the ith diagonal element of $\boldsymbol{R}$, denoted r_i, describes firm i's disclosure policy as discussed above (when there is doubt about the individual disclosure policies $\boldsymbol{r}=(r_1,\ldots,r_n)$ corresponding to the precision matrix $\boldsymbol{R}$, we shall write $\boldsymbol{R}=\boldsymbol{R}(r)$).

The theory developed below applies whether or not the off-diagonal elements of $\boldsymbol{R}$ are zero. But, since there is, in fact, no loss of generality in assuming that such off-diagonal elements are zero as long as—when these off-diagonal elements exist—they are taken to be given exogenously, we adopt the convention that $\boldsymbol{R}$ is diagonal to ease the notation in some of the proofs.[6]

[6] The economic issue at stake which makes this diagonal representation feasible is the assumption that the off-diagonal elements in $\boldsymbol{R}$ are exogenously given. This seems to be a reasonable assumption: if an off-diagonal element is nonzero, it reflects the possibility that firm i's disclosures may reveal information about $\tilde{\mu}_j$, firm $j\neq i$'s unknown mean. While firms can control, and accounting standards boards can dictate, how much information a disclosure reveals about the mean of the distribution of the cash flows of the firm making the disclosure (through specification of the r's), firms have little control (and accounting standards boards would have difficulty dictating) how much information one firm's disclosure would reveal about other firms.

To see that there is no loss of generality in taking $\boldsymbol{R}$ to be diagonal when its off-diagonal elements

Let $\mathrm{Var}(\tilde{\mathbf{z}}|\boldsymbol{x},\boldsymbol{R})$ be the (posterior) covariance matrix for the joint distribution of $\tilde{\mathbf{z}}$, conditional on $\boldsymbol{x}$ and $\boldsymbol{R}$. Then, with "$\mathbf{1}$" denoting a (column) vector of n ones and

$$\gamma \equiv 1 \Big/ \sum_{i=1}^{k} \frac{1}{\gamma_i},$$

the market-clearing prices $P_1, \ldots, P_n$ for the n firms are given by the well-known expression[7]

$$\boldsymbol{P}(\boldsymbol{x};\boldsymbol{R}) \equiv \begin{bmatrix} P_1 \\ \vdots \\ P_n \end{bmatrix} = E[\tilde{\mathbf{z}}|\boldsymbol{x},\boldsymbol{R}] - \gamma\, \mathrm{Var}(\tilde{\mathbf{z}}|\boldsymbol{x},\boldsymbol{R})\mathbf{1}. \tag{1}$$

According to standard Bayesian updating formulas (e.g., Degroot 1970)[8] $\mathrm{Var}(\tilde{\mathbf{z}}|\boldsymbol{x},\boldsymbol{R}) = \boldsymbol{\Phi} + (\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}$, so equation (1) can be rewritten as:

$$\boldsymbol{P}(\boldsymbol{x};\boldsymbol{R}) = E[\tilde{\mathbf{z}}|\boldsymbol{x},\boldsymbol{R}] - \gamma(\boldsymbol{\Phi} + (\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1})\mathbf{1}. \tag{2}$$

Equation (2) states that the price of every firm's securities depends on the expected cash flows conditional on what information I's know at the time the market opens minus two types of risk-premia: risk-premia attached to uncertainty about the firm's cash flows conditional on knowing the true mean of the cash flows of the securities (for firm i, if $\mathbf{1}_i$ denotes a column of zeros except for a one in the ith component, this risk premium is $\gamma\mathbf{1}_i'\boldsymbol{\Phi}\mathbf{1}$) and risk-premia attached to uncertainty about the actual realizations of $\tilde{\boldsymbol{\mu}}$ (for firm i, this is $\gamma\mathbf{1}_i'(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}$).

II. Disclosures with a Representative Entrepreneur and a Representative Investor[9]

Introduction

Optimal mandatory and voluntary disclosure policies solve different maximization problems. An accounting standards board, in choosing among mandatory disclosure policies, accounts for the influence one entrepreneur's disclosure policy has on (1) other entrepreneurs, and (2) all investors. In contrast,

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are zero, let $\hat{\boldsymbol{R}}$ denote the nxn matrix whose entries consist of the elements of $\boldsymbol{R}$ with 0's replacing $\boldsymbol{R}$'s diagonal elements. Also define a new matrix $\hat{\boldsymbol{\Omega}}$ by $\hat{\boldsymbol{\Omega}}^{-1} = \boldsymbol{\Omega}^{-1} + \hat{\boldsymbol{R}}$ and another nxn matrix $\boldsymbol{R}^*$ whose off-diagonal elements are zero, and whose diagonal elements are the same as the elements of $\boldsymbol{R}$. Then, it is clear by examining the pricing equation (2) below that, with $\hat{\boldsymbol{\Omega}}^{-1}$ replacing $\boldsymbol{\Omega}^{-1}$ and $\boldsymbol{R}^*$ replacing $\boldsymbol{R}$, the pricing equation remains correct, and of course, in this formulation, $\boldsymbol{R}^*$ is diagonal.

[7] This result is standard (e.g., see Stapleton and Subrahmanyam 1978).

[8] From DeGroot (1970), investors' posterior assessment of $\tilde{\mu}$ is normal with mean $(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}(\boldsymbol{\Omega}^{-1}\boldsymbol{M}+\boldsymbol{R}\boldsymbol{x})$ and precision matrix $\boldsymbol{R}+\boldsymbol{\Omega}^{-1}$. Since the distribution of $\tilde{\mathbf{z}}$ conditional on $\tilde{\mu}$ is normal with mean $\tilde{\mu}$ and covariance matrix $\boldsymbol{\Phi}$, $\tilde{\mathbf{z}} = \tilde{\mu} + \tilde{\epsilon}$, where $\tilde{\epsilon}$ and $\tilde{\mu}$ are independent and $\tilde{\epsilon}$ has mean (vector) zero and covariance matrix $\boldsymbol{\Phi}$. Hence, the posterior distribution of $\tilde{\mathbf{z}}$, conditional on $\tilde{\boldsymbol{x}} = \boldsymbol{x}$, has covariance matrix $\boldsymbol{\Phi} + (\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}$.

[9] I wish to thank one of the anonymous referees for suggesting that I begin the analysis with the study of "representatives'" disclosure policies. Several of the results in this section are taken directly from the referee's review of an earlier draft of this paper.

an individual entrepreneur will ignore both of these potential externalities in choosing his or her disclosure policy. We wish to identify the potentially distinct effects of these two sources of differences between mandatory and voluntary disclosure policies. This section begins the formal analysis of disclosure policies by studying the special case in which all entrepreneurs have engaged in efficient risk-sharing among themselves prior to choosing their disclosure policies. The principal consequence of this assumption is that all entrepreneurs will have the same attitudes toward risk subsequent to engaging in efficient risk-sharing and, consequently, problems in achieving unanimity among entrepreneurs regarding disclosure policies disappear. This unanimity allows reference to a representative entrepreneur without ambiguity.

To achieve efficient risk-sharing among the E's, we modify the original time line by introducing a market on which E's alone may trade *prior* to making any disclosures.[10] The equilibrium portfolio of entrepreneur i in this market involves receiving an interest of β/β_i in each firm, where:

$$\beta = \left[\sum_i 1/\beta_i \right]^{-1}.$$

Hence, at the time entrepreneur i decides to choose his disclosure policy, he will seek to maximize:

$$\begin{aligned} U_i(\mathbf{R}) &\equiv E[(-1/\beta_i)e^{-\beta_i(\beta/\beta_i \mathbf{1}'\mathbf{P}(\tilde{\mathbf{x}};\mathbf{R}))}] \\ &= E[(-1/\beta_i)e^{-\beta \mathbf{1}'\mathbf{P}(\tilde{\mathbf{x}};\mathbf{R})}].^{11} \end{aligned}$$

Observe that the entrepreneur's attitude toward risk is completely specified by the aggregate risk-aversion parameter β and, hence, is independent of which particular entrepreneur is being considered. For this reason, we refer to β as the risk parameter of the representative entrepreneur.

Similarly, since a market in the firms' shares opens before any investor engages in consumption in this model, an investor with risk-aversion parameter γ_i and exogenous wealth w_i winds up with fraction γ/γ_i of each firm, and consequently, obtains expected utility:

$$\begin{aligned} V_i(\mathbf{R}) &\equiv E[(-1/\gamma_i)e^{-\gamma_i(w_i + \gamma/\gamma_i \mathbf{1}'(\tilde{\mathbf{z}} - \mathbf{P}(\tilde{\mathbf{x}};\mathbf{R})))}] \\ &= (-1/\gamma_i)e^{-\gamma_i w_i} E[e^{-\gamma \mathbf{1}'(\tilde{\mathbf{z}} - \mathbf{P}(\tilde{\mathbf{x}};\mathbf{R}))}], \end{aligned} \tag{3}$$

where:

$$1/\gamma = \sum_i 1/\gamma_i.$$

As a consequence of these observations, the analysis in this section proceeds as if

[10] Alternatively, we could allow E's to write contracts among themselves to share the proceeds from the sale of their firms, thereby forming an explicit syndicate (as in Wilson 1968). The induced preferences of each entrepreneur are the same regardless of which method of risk-sharing is chosen.

[11] This result is also standard: that entrepreneur i winds up owning fraction β/β_i of each firm when a market opens in advance of any disclosures can be seen by substituting the market-clearing price into the shareholder's demand function. Alternatively, E's induced risk-aversion can be derived from syndicate theory. When entrepreneur i is a member of a syndicate, each of the optimal syndicate contracts is linear in the syndicates' aggregate wealth, x, when entrepreneurs have constant absolute risk-aversion, with i's contract of the form $s_i(x) = \beta/\beta_i x + k_i$ for some constant k_i (see Wilson 1968).

we were studying a two-person economy, with one representative entrepreneur with risk parameter β and one representative investor with risk parameter γ.

The social welfare function used to study mandatory disclosures in this section involves a weighted average of E's' and I's' expected utilities. Letting λ and ξ be the nonnegative weights attached to the representative entrepreneur and investor respectively, this social welfare function can be written as:

$$W(\boldsymbol{R}) \equiv \lambda(-1/\beta)E[e^{-\beta \mathbf{1}'\boldsymbol{P}(\boldsymbol{x};\boldsymbol{R})}] + \xi(-1/\gamma)E[-e^{-\gamma \mathbf{1}'(\boldsymbol{x}-\boldsymbol{P}(\boldsymbol{x};\boldsymbol{R}))}].^{12} \tag{4}$$

Analysis

Proposition 1 presents optimal mandatory disclosure policies as a function of the weights λ and ξ. To state this proposition, we refer to explicit expressions for the expected utilities of the representative entrepreneur and investor. These expected utility calculations make use of characteristics of only the sum of the firms' securities and cash flows. The following notation simplifies the presentation of Proposition 1, by identifying some salient characteristics of these aggregates, as well as some other parameters which will be useful in obtaining closed-form expressions for the representatives' expected utilities. Thus:

$m \equiv \mathbf{1}'\boldsymbol{M}$ is the sum of the expected values of all firms' cash flows;
$\phi = \mathbf{1}'\boldsymbol{\Phi}\mathbf{1}$ is the variance in aggregate cash flows given knowledge of the true mean $\mathbf{1}'\boldsymbol{\mu}$;
$\omega \equiv \mathbf{1}'\boldsymbol{\Omega}\mathbf{1}$ is the prior variance in the mean of the aggregate cash flows;
$\nu \equiv \mathbf{1}'(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}$ is the posterior variance in the mean of the aggregate cash flows:

$\varkappa \equiv \exp[-\beta(m-.5\beta\omega-\gamma\phi)]$;
$\delta \equiv \exp[-.5\gamma^2\phi]$.

Proposition 1: Financial Disclosures with Representative Entrepreneurs and Investors

(i) The optimal mandatory disclosure policy (DP) consists of full disclosure for each firm if, and only if, $\lambda\varkappa(.5\beta-\gamma)+.5\xi\delta\gamma \leq 0$;
(ii) the optimal mandatory DP consists of no disclosure for each firm if, and only if, $\lambda\varkappa(.5\beta-\gamma)\exp[-\beta\omega(.5\beta-\gamma)]+.5\xi\delta\gamma\exp[-.5\gamma^2\omega] \geq 0$;
(iii) the optimal mandatory DP consists of a unique finite, nonzero level of aggregate disclosure (measured by the difference $1/\nu-1/\omega$ between the posterior and prior precisions) if, and only if, each of the inequalities listed in parts (i) and (ii) above are reversed;[13]
(iv) the optimal mandatory aggregate level of disclosure is nonincreasing in the weight ξ placed on I's; the optimal mandatory disclosure is

[12] We have subsumed the investor's wealth into the parameter ξ in this formulation of the social welfare function.

[13] In contrast to the uniqueness of the individual firm's disclosure policies r_i which implement the "extreme" aggregate disclosures $\nu=0$ and $\nu=\infty$, typically the individual disclosure policies which implement "interior" aggregate disclosures $\nu\epsilon(0,\infty)$ are not unique. The reason is that there are generally multiple solutions $\boldsymbol{r}$ to the equation $\mathbf{1}'(\boldsymbol{R}(\boldsymbol{r})+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}=\nu$, as can be easily verified by example, even in the case of two entrepreneurs.

nondecreasing (nonincreasing) in the weight λ placed on E's if, and only if, $.5\beta<\gamma$ ($.5\beta>\gamma$); and

(v) if lump-sum payments are permitted between the representative entrepreneur and the representative investor, then the optimal mandatory DP evaluated at the optimal lump-sum payment entails no (full) disclosure if $\beta>\gamma$ ($\beta<\gamma$).

Abbreviated proofs appear in the Appendix. More detailed proofs are available from the author.

This proposition gives an essentially complete description of optimal mandatory disclosures when entrepreneurs have efficiently shared their risks and disclosures are costless. The implications of the proposition are easiest to understand by considering special cases:

1. The representative investor prefers no disclosure: this corresponds to the case of $\lambda=0$, so part (ii) applies.
2. (a) The representative entrepreneur prefers full (no disclosure if $.5\beta<\gamma$ ($.5\beta>\gamma$). This statement corresponds to the case of $\xi=0$, so part (i) or (ii) applies depending on whether $.5\beta<\gamma$ or $.5\beta>\gamma$.
   (b) Full disclosure uniquely maximizes the expected aggregate market value of all firms. This statement follows from 2(a) because the objective function of a risk-neutral representative entrepreneur ($\beta=0$) is to maximize the expected aggregate market value of all firms.
3. Regardless of the weights placed on E's or I's in the social welfare function, no disclosure is optimal if $.5\beta>\gamma$: this again follows from part (ii).
4. If the level of "voluntary" disclosures is determined by what the representative entrepreneur would do to maximize its expected utility, then the optimal mandatory disclosure is always weakly less than the level of voluntary disclosures: this follows immediately from part (iv).
5. When E's are risk-neutral and I's are strictly risk-averse, then (a) when there are no lump-sum payments between the representative entrepreneur and investors, full disclosure is optimal if, and only if, $.5\xi\delta\leq\lambda$. The result follows from part (i) because a risk-neutral E is approximated by letting β approach zero and, hence, $\varkappa$ converges to 1; (b) when optimal lump-sum payments are allowed, full disclosure is uniquely optimal. This follows from part (v).
6. No disclosure is always optimal when I's are risk-neutral (and E's are strictly risk-averse): this follows from $.5\beta>\gamma$ and $\varkappa$ and δ remaining positive when evaluated at $\gamma=0$.
7. Increasing investor wealth, holding all other constants fixed, (weakly) increases the optimal mandatory aggregate level of disclosure: this follows because an increase in investor wealth, *ceteris paribus*, decreases the weight ξ placed on the representative investor's preferences in the social welfare function.

Cases 1 and 2 are easy to derive and explain. To calculate an investor's expected utility, note that equation (2) yields expressions (5) and (6) below:

$$E[\mathbf{1}'(\tilde{\mathbf{z}}-\boldsymbol{P}(\boldsymbol{x};\boldsymbol{R}))|\boldsymbol{x},\boldsymbol{R}]=\gamma\mathbf{1}'(\boldsymbol{\Phi}+(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1})\mathbf{1}=\gamma(\phi+\nu) \tag{5}$$

$$\mathrm{Var}(\mathbf{1}'(\tilde{\mathbf{z}}-\boldsymbol{P}(\boldsymbol{x};\boldsymbol{R}))|\boldsymbol{x},\boldsymbol{R})=\mathrm{Var}(\mathbf{1}'\tilde{\mathbf{z}}|\boldsymbol{x},\boldsymbol{R})=\phi+\nu. \tag{6}$$

Therefore, since $\tilde{\mathbf{z}}$ given $\boldsymbol{x}$ is distributed multivariate normal, we can calculate the investor's expected utility as follows:

$$\begin{aligned}(-1/\gamma)E[\exp(-\gamma\mathbf{1}'(\tilde{\mathbf{z}}-\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})))]&=(-1/\gamma)\exp(-.5\gamma^2(\phi+\nu))\\&=(-\delta/\gamma)\exp(-.5\gamma^2\nu).\end{aligned} \tag{7}$$

Notice that I's benefit from the risk arising from uncertainty about the means of the firms' cash flows: equation (7) is increasing in the posterior variance ν. This paradoxical result occurs because the entrepreneurs are "forced" to sell all of their ownership of the risky assets at the equilibrium price and the risk premium implicit in that price exceeds the risk premium required to compensate the investors for the risk they are bearing. This can be seen formally by noting that when the aggregate mean cash flows have a posterior variance of ν, then the expected value of the market portfolio $(\mathbf{1}'E\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})=m-\gamma(\phi+\nu))$, is $\gamma\nu$ below what it would have been were there no uncertainty about the expected value of the aggregate cash flows. In contrast, the additional risk which I's bear as a result of this uncertainty has a certainty-equivalent cost of only $.5\gamma\nu$ (since $E[-\exp(-\gamma\mathbf{1}'\tilde{\mathbf{z}})|\boldsymbol{R}]=-\exp(-\gamma(\mathbf{1}'\boldsymbol{M}-.5\gamma(\phi+\nu)))$). Hence the drop in the equilibrium prices arising from increased uncertainty about the firms' cash flows outweighs I's' "costs" of bearing the associated additional risk. Consequently, if I's had exclusive control of the disclosure policies, they would insist that firms engage in no disclosure whatsoever.

To calculate the representative entrepreneur's expected utility, we use the fact that $\mathbf{1}'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})$ is normally distributed with mean $m-\gamma(\phi+\nu)$ and variance[14] $\omega-\nu$ to conclude:

$$\begin{aligned}(-1/\beta)E[e^{-\beta\mathbf{1}'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})}]&=(-1/\beta)\exp(-\beta(m-\gamma(\phi+\nu)-.5\beta(\omega-\nu))\\&=(-\varkappa/\beta)\exp(-\nu\beta(.5\beta-\gamma)).\end{aligned} \tag{8}$$

Now, it is clear from this expression that the representative entrepreneur's expected utility is increasing or decreasing in the posterior variance ν of the aggregate expected cash flows if, and only if, $.5\beta>\gamma$. The same offsetting factors identified above explain the economics underlying this inequality. On the one hand, the expected aggregate market value of the firms is declining at the rate $\gamma\nu$ in the posterior variance of the aggregate disclosures. This is, of course, undesirable to E's. But, on the other hand, increases in the variance of the disclosures *reduces* the risk E's experience in selling their firms, because the less informative the disclosures the less the prices of the firms change in reaction to those disclosures. Specifically, equation (8) shows that an entrepreneur with risk aversion parameter β obtains a certainty equivalent benefit of $.5\beta\nu$ from making

[14] There are several ways of obtaining this expression for the variance of $\mathbf{1}'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})$, i.e., the variance of $\mathbf{1}'E[\tilde{\boldsymbol{\mu}}|\tilde{\boldsymbol{x}},\boldsymbol{R}]$. One simple way is: from regression theory, $\mathbf{1}'\tilde{\boldsymbol{\mu}}=\mathbf{1}'E[\tilde{\boldsymbol{\mu}}|\tilde{\boldsymbol{x}},\boldsymbol{R}]+\tilde{\epsilon}$, where $\tilde{\epsilon}$ is a random variable whose covariance with $\mathbf{1}'E[\tilde{\boldsymbol{\mu}}|\tilde{\boldsymbol{x}},\boldsymbol{R}]$ is zero. Hence, $\mathrm{Var}(\mathbf{1}'\tilde{\boldsymbol{\mu}})=\mathrm{Var}(\mathbf{1}'E[\tilde{\boldsymbol{\mu}}|\tilde{\boldsymbol{x}},\boldsymbol{R}])+\mathrm{Var}(\tilde{\epsilon})$. Since $\mathrm{Var}(\mathbf{1}'\tilde{\boldsymbol{\mu}})=\omega$ and $\mathrm{Var}(\tilde{\epsilon})=\nu$ (as the posterior variance of the expected value of cash flows), we conclude $\mathrm{Var}(\mathbf{1}'E[\tilde{\boldsymbol{\mu}}|\tilde{\boldsymbol{x}},\boldsymbol{R}])=\omega-\nu$.

disclosures with a posterior variance of ν. Whether no disclosure or full disclosure is optimal for the E's depends upon whether the increase in the expected selling price of the firm attainable by engaging in full disclosure outweighs the accompanying disutility of the increased variance in the selling price. This explains the role of the inequality $.5\beta>\gamma$, and hence, case 2(a).[15]

The intuition for case 2(b) where disclosures are chosen to maximize the sum of the expected market values of all firms can be described succintly. The optimality of full disclosures derives from the simple fact that aggregate market value maximization is accomplished by eliminating any risk-premium associated with I's' uncertainty about the firms' expected cash flows.

Cases 3 through 7 follow directly by recognizing that the social welfare function is a weighted average of the representative entrepreneur's and representative investor's utility functions, and hence the outcome in those cases depend upon what happens in the two initial "corner" cases.

To explain what happens in part (v) of the proposition, in which optimal lump-sum payments between the representative E and the representative I are allowed, we begin with the following fact: since both representatives have constant absolute risk-aversion, the post-redistribution social welfare function is maximized by maximizing the sum of these representatives' certainty equivalents.[16] To apply this fact to the present problem, observe from equations (7) and (8) above, that the certainty equivalent of $\mathbf{1}'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})$ for the representative E and the certainty equivalent of $\mathbf{1}'(\tilde{\boldsymbol{z}}-\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})$ for the representative I are respectively given by $C_e=\nu(.5\beta-\gamma)$ and $C_i=.5\gamma\nu$, so $C_e+C_i=.5\nu(\beta-\gamma)$. Therefore, $\nu=\omega$ ($\nu=0$) is optimal if $\beta>\gamma$ ($\beta<\gamma$).

The intuition here is this. Although a lump-sum distribution cannot explicitly transfer risk across generations, one generation can offset the effects a DP has on the risk the other generation must bear with an appropriately sized fixed payment. Therefore, given the opportunity to make such transfers, the optimal DP must maximize the sum of the expected benefits (i.e., the certainty equivalents) of the disclosures.

[15] While it is not surprising that a comparison of the representative entrepreneur's and investor's risk parameters matters in this decision problem of the entrepreneurs (because these parameters reveal information about the tradeoff between expected selling price and increased risk), the existence of this knife-edge situation, as well as the relative size of the parameters which determine its location, are probably specific to the particular parameterization of the problem we have studied.

[16] Formally, we appeal to the following lemma.

Lemma: For arbitrary nonnegative weights $\{\aleph_i\}$, the value of the objective function of the program:

$$\underset{\{\ell_i\}}{\text{Max}} \quad \sum_i -\aleph_i \exp(-\zeta_i(C_i+\ell_i)) \qquad \text{subject to } \sum_i \ell_i=0,$$

when evaluated at its optimum equals $-H\times\exp\left[-\zeta\sum_i C_i\right]$,

where $\zeta\equiv 1\Big/\sum_i 1/\zeta_i$ and $H\equiv(1/\zeta)\exp\left[\sum_i(\zeta/\zeta_i)\log\zeta_i N_i\right]$.

This result is proven by elementary calculus. Here, C_i is to be interpreted as individual i's certainty-equivalent.

III. Disclosure Policies with Financial Externalities

In this section, we evaluate the individual expected utility-maximizing disclosure decisions of *undiversified* entrepreneurs in a regime where disclosures produce only financial externalities, i.e., where the disclosures at most alter outsiders' perceptions of the distributions of cash flows, but disclosures do not shift either the realized mean vector $\tilde{\mu}$ or covariance matrix $\mathbf{\Phi}$ of the joint distribution of cash flows. In this and all remaining sections, we revert to the original time line in which the initial owners of the firms, the entrepreneurs, have no opportunity to share their risks with other entrepreneurs prior to making disclosures in accordance with the disclosure policies they voluntarily select.[17]

Assume that each entrepreneur chooses that policy which maximizes his or her expected utility, while taking the disclosure policies of other firms as given. An equilibrium in voluntary disclosure policies exists if each entrepreneur's perceptions of the other entrepreneurs' policies is correct, or more precisely, in the case of two entrepreneurs, if the following definition is satisfied.

Definition 1: $\mathbf{r}^* = (r_1^*, r_2^*)$ constitutes an equilibrium set of voluntary disclosure policies (with associated precision matrix $\mathbf{R}^* = \mathbf{R}(\mathbf{r}^*)$) if, for $i = 1,2$:

$$E[-1/\beta_i \exp(-\beta_i \mathbf{1}_i' \mathbf{P}(\tilde{\mathbf{x}}; \mathbf{R}^*))] \geq E[-1/\beta_i \exp(-\beta_i \mathbf{1}_i' \mathbf{P}(\tilde{\mathbf{x}}; \mathbf{R}(\mathbf{r})))],$$

where $\mathbf{r} = (r_i, r_j^*), j \neq i$, for any $r_i \geq 0$.

This definition is simply a (pure strategy) Nash equilibrium in disclosure policies.[18]

The restriction to two entrepreneurs is made as a concession to the complexity of the mathematics in the risk-averse case. It is easy to show that the results generalize to an arbitrary number of entrepreneurs when the entrepreneurs are risk-neutral. The two-entrepreneur case permits a comparison of the attitudes of an undiversified entrepreneur toward disclosure with those of the rest of the market by interpreting one of the "entrepreneurs" as a collection of investors who have diversified positions in a large collection of firms (like the representative entrepreneur of the previous section). This case also allows determination of whether competition between E's produces different disclosures than when disclosures are imposed by fiat. The number of investors is always allowed to be arbitrary.

The next proposition is a complete description of all equilibria corresponding to virtually all parameterizations of the E's' and I's' preferences and covariance

[17] While the formal theory provides no motivation for the absence of such risk-sharing opportunities, this nondiversification may arise because of moral hazard or adverse selection concerns. As a referee has pointed out, this exogenous assumption of nondiversification is not easily made endogenous formally, because whatever motivated the entrepreneurs to be nondiversified in the first place might provide motivation for subsequent generations of the firms' owners to be nondiversified as well. But, it is clear that many firms start out as undiversified operations that subsequently grow into publicly traded firms with highly diversified owners, and so the evolution of the ownership structure assumed here is the natural one.

[18] As the notation indicates, we do not consider equilibria for randomized disclosure policies (in which the entrepreneurs select distributions of disclosure policies in anticipation that other entrepreneurs will do likewise) in this paper.

Table 1
Equilibria for Financial Externalities[1]

(d_1^∞, d_1^0) \ (d_2^∞, d_2^0)	$(-,-)$	$(-,+)$	$(+,-)$	$(+,+)$
$(-,-)$	$(0,0)$	$(0,\infty)$	$(0,0)$	$(0,\infty)$
$(-,+)$	$(\infty,0)$	$**$[2]	$(-d_2^0/d_2^\infty, -d_1^0/d_1^\infty)$	$(0,\infty)$
$(+,-)$	$(0,0)$	$(-d_2^0/d_2^\infty, -d_1^0/d_1^\infty)$	$***$	(∞,∞)
$(+,+)$	$(\infty,0)$	$(\infty,0)$	(∞,∞)	(∞,∞)

[1] The elements of the table are (r_1, r_2).
[2] For the entries marked by asterisks, there are multiple equilibria, given by

$$** = \{(0,\infty), (\infty,0), (-d_2^0/d_2^\infty, -d_1^0/d_1^\infty)\}$$
$$*** = \{(0,0), (\infty,\infty), (-d_2^0/d_2^\infty, -d_1^0/d_1^\infty)\}.$$

Except for the parameterizations indicated by asterisks, all equilibria are unique.

structures among securities (in this proposition, $\hat{\omega}_{ij}$ is the (i,j)th element of $\boldsymbol{\Omega}$ and $|\boldsymbol{\Omega}|$ is $\boldsymbol{\Omega}$'s determinant).

Proposition 2: Financial Externalities and Voluntary Disclosures

> *Let* $d_i^0 \equiv (\gamma - \beta_i/2)\hat{\omega}_{ii} + \gamma\hat{\omega}_{12}$, $d_i^\infty \equiv (\gamma - \beta_i/2) \times |\boldsymbol{\Omega}| / \hat{\omega}_{jj}$ for $j \neq i$ and $i = 1,2$. Then, every equilibrium for the case of risk-averse E's is determined by the sign of the terms d_i^0, d_i^∞ as described in Table 1.

To learn about the economics underlying Proposition 2, we start by noting that the certainty equivalent of $\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})$ for entrepreneur 1 is $CE \equiv E[\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})] - \beta_1 \mathrm{Var}(\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R}))$. After performing some algebra and dropping terms not involving the disclosure policies $\boldsymbol{R}$, this certainty equivalent can be decomposed into a mean effect and a variance effect expressed as follows:[19]

$$CE(\text{mean}) = -\gamma(\omega_{22} + r_2 - \omega_{12})/\{(\omega_{11} + r_1)(\omega_{22} + r_2) - \omega_{12}^2\},$$
$$CE(\text{variance}) = .5 \times \beta_1 \times (\omega_{22} + r_2)/\{(\omega_{11} + r_1)(\omega_{22} + r_2) - \omega_{12}^2\},$$

where ω_{ij} is the (i,j)th element of the *inverse* of $\boldsymbol{\Omega}$. Note that whether additional disclosure is beneficial to entrepreneur 1 is independent of the magnitude of entrepreneur 1's disclosure (i.e., the sign of the derivative of CE terms with respect to r_1 does not depend on r_1). Hence, entrepreneur 1 will typically engage in either full disclosure or no disclosure.[20] Now, if entrepreneur 2 has engaged in, say, no disclosure, then entrepreneur 1's optimal response is determined by com-

[19] The details are as follows: from equation (2) above, $E[\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})] = \mathbf{1}'\boldsymbol{M} - \gamma\mathbf{1}_1'(\boldsymbol{\Phi} + (\boldsymbol{R} + \boldsymbol{\Omega}^{-1})^{-1}\mathbf{1})$. To compute $\mathrm{Var}(\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R}))$, use $\mathrm{Var}(\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})) = \mathrm{Var}(\mathbf{1}_1'E[\tilde{\boldsymbol{z}}|\tilde{\boldsymbol{x}},\boldsymbol{R}]) = \mathrm{Var}(\mathbf{1}_1'(\boldsymbol{R} + \boldsymbol{\Omega}^{-1})^{-1}\boldsymbol{R}\tilde{\boldsymbol{x}})$ and the fact that the priors for $\tilde{\boldsymbol{x}}$ are multivariate-normal with mean $\boldsymbol{M}$ and covariance matrix $\boldsymbol{R}^{-1} + \boldsymbol{\Omega}$ to conclude that $\mathrm{Var}(\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})) = (\boldsymbol{R} + \boldsymbol{\Omega}^{-1})^{-1}\boldsymbol{R}(\boldsymbol{R}^{-1} + \boldsymbol{\Omega})\boldsymbol{R}(\boldsymbol{R} + \boldsymbol{\Omega}^{-1})^{-1}$. Some matrix manipulations reduce this to $\mathrm{Var}(\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})) = \boldsymbol{\Omega} - (\boldsymbol{\Omega}^{-1} + \boldsymbol{R})^{-1}$. Combining these terms we conclude $E[\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})] - .5\beta_1 \mathrm{Var}(\mathbf{1}_1'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})) = \mathbf{1}'\boldsymbol{M} - \gamma\mathbf{1}_1'(\boldsymbol{\Phi} + (\boldsymbol{R} + \boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}) - .5 \times \beta_1 \times \mathbf{1}_1'\{\boldsymbol{\Omega} - (\boldsymbol{\Omega}^{-1} + \boldsymbol{R})^{-1}\}\mathbf{1}_1$. Dropping the terms not involving $\boldsymbol{R}$ and writing out the terms of the matrix inverses in the preceding expression produces the expressions for CE(mean) and CE(variance) in the text.

[20] This explanation is informal: the proof of the proposition does not appeal to vague notions used in the text such as "typically."

paring CE(mean)+CE(variance) evaluated at $r_1=0$ and $r_1=\infty$. If entrepreneur 1 selects $r_1=\infty$, then it is clear from the expression for CE(mean) that this has the effect of raising the expected price of the firm by:

$$\gamma(\omega_{22}-\omega_{12})/(\omega_{11}\omega_{22}-\omega_{12}^2)=\gamma(\hat{\omega}_{11}+\hat{\omega}_{12})$$

above what it would have been had the entrepreneur engaged in no disclosure. Offsetting this effect, the variance in the selling price is higher with full disclosure by:

$$.5\times\beta_1\times\omega_{22}/(\omega_{11}\omega_{22}-\omega_{12}^2)=.5\times\beta_1\times\hat{\omega}_{11}$$

than it would have been with no disclosure. Combining these effects, we see that full (no) disclosure is the preferred option when:

$$\gamma(\hat{\omega}_{11}+\hat{\omega}_{12})-.5\times\beta_1\times\hat{\omega}_{11}$$

is positive (negative), i.e., when d_1^0 is positive (negative). Note that since firm 2 was postulated to engage in no disclosure, firm 2's mean is uncertain at the time investors purchase the firms, and so the risk-premium investors require to purchase firm 1 depends on the covariance between the means of the firms' cash flows.

In similar fashion, it can be shown that the net benefit to entrepreneur 1 of engaging in full disclosure rather than no disclosure when entrepreneur 2 engages in full disclosure is given by d_1^∞. Note that, in this case, the covariance between firms' 1 and 2's means has no effect on the risk premium investors who purchase firm 1 receive, because, when firm 2 engages in full disclosure, there is no uncertainty about firm 2's mean.

Thus, the net benefit to entrepreneur 1, measured in terms of certainty equivalent value, of engaging in full disclosure instead of no disclosure, is given by the "d_1" terms, appropriately indexed according to whether entrepreneur 2 engages in full disclosure ($r_2=\infty$) or no disclosure ($r_2=0$). Symmetric comments obviously apply when entrepreneurs 1 and 2 are interchanged. Equilibrium requirements (such as requiring firm 1 to want to engage in full disclosure when firm 2 believes that firm 1 will engage in full disclosure) then aid in the completion of the table.

This proposition sheds light on why stock exchanges might seek to impose mandatory disclosures beyond those which newly listed firms would supply voluntarily. To see this, adopt the interpretation that entrepreneur 1 is undiversified and about to be listed on an exchange, and "entrepreneur 2" is a label for the rest of the (diversified) market. When the two generations of shareholders are similar, the aggregate risk parameters β and γ are approximately equal and both will be near zero in "large" markets. Hence, β_1 will be much larger than γ; β_2 will be approximately equal to γ; and both d_1^∞ and d_1^0 will be negative (for future reference, we refer to parameters satisfying these conditions as the "typical" case). Therefore, by Proposition 2, entrepreneur 1 will engage in no disclosure. When the covariance $\hat{\omega}_{12}$ is positive, d_2^0 will be positive, so, by Proposition 2, the rest of the market will be in favor of full disclosure. Intuitively, the risk-premium the rest of the market would forego by not engaging in full disclosure far exceeds the variance-reducing benefits of no disclosure, whereas the reverse is true for the single undiversified entrepreneur, leading to the difference in their preferred disclosure policies.

Now switch the indices 1 and 2 in the expressions for CE(mean) and CE(variance) above and differentiate their sum with respect to r_1. When $\hat{\omega}_{12}$ is positive, the result is that "the market" would prefer entrepreneur 1 to engage in full disclosure. Since it has just been seen that the undiversified entrepreneur 1 would prefer no disclosure (under the indicated parameter values), we conclude that there is tension between the disclosures the undiversified firm would like to make, and the disclosures the rest of the market will insist that it make.

If all entrepreneurs are risk-neutral, this tension disappears because in this case, full disclosure is the only equilibrium. To see this, note that risk-neutral E's are approximated by $\beta_i=0$, $i=1,2$. Therefore, sgn $d_i^\infty=$sgn γ, and $d_i^0=\gamma(\hat{\omega}_{ii}+\hat{\omega}_{12})$, $i=1,2$. Now, since $\mathbf{\Omega}$ is positive-definite,

$$\sum_{i,j} \hat{\omega}_{ij}$$

is positive and, therefore, at least one of d_1^0, d_2^0 is positive. Consequently, there are three entries in the table consistent with the case of risk-neutral E's (both d_1^∞ and d_2^∞ positive, and at least one of d_1^0, d_2^0 positve), and for each of these entries, $(r_1, r_2)=(\infty\,\infty)$ is the unique equilibrium. Notice that, from case 2 following Proposition 1, full disclosure is also the unique DP which maximizes the aggregate expected selling price of all firms. Hence, mandatory disclosures are indeed redundant.

But the risk-neutral case is not the only instance of the redundancy of mandatory disclosures. It is possible to show that when the social welfare function which mandatory disclosure policies are designed to maximize consists of a weighted average of the various entrepreneurs' and investors' expected utilities, the unique optimal disclosure policy for each firm entails no disclosure if $\hat{\omega}_{12}<0$ and $d_i^\infty<0$, $i=1,2$. It entails full disclosure for each firm if $\hat{\omega}_{12}>0$ and $d_i^\infty>0$, $i=1,2$, and sufficiently little weight is placed on each investor's utility function in the social welfare function.[21] Since the conditions $\hat{\omega}_{12}<0$ and $d_i^\infty<0$, $i=1,2$ imply that $d_i^0<0$, $i=1,2$, it follows from Proposition 2 that the conditions which guarantee that no disclosure is the optimal mandatory disclosure policy also guarantee that no disclosure is the unique equilibrium voluntary disclosure policy. Similarly, the conditions which guarantee that full disclosure is the optimal mandatory disclosure policy also guarantee that full disclosure is the unique equilibrium voluntary disclosure policy. These observations do not prove that optimal mandatory and equilibrium voluntary disclosure policies coincide because these conditions are not an exhaustive specification of the parameter space of triples $(d_1^\infty, d_2^\infty, \hat{\omega}_{12})$. Nevertheless, they provide some evidence of how likely it is for mandatory and voluntary disclosure policies to be congruent.

[21] Very similar conditions yield similar conclusions when either E's' utilities alone, or both E's' and I's' utilities, are calculated after optimal lump-sum redistributions have occurred. A complete specification of these conditions, along with all relevant proofs, is available from the author.

IV. Disclosure Policies with Real Externalities

This section considers disclosure policies when disclosures made by one firm may affect some firm's (perhaps its own) distribution of cash flows. The proprietary effects of these "real" disclosures are shown to have the potential to create differences between mandatory and voluntary disclosures where none existed when attention was confined to disclosures with merely financial effects.

There are many ways disclosures may affect firms' cash flows. To motivate the relation between disclosures and cash flows used below, consider two examples: (1) a firm's announcement that next year's earnings are anticipated to be four times as large as this year's earnings is more likely to provoke other firms to enter its market the smaller the firm's forecast errors have been historically, and (2) the detailed disclosure of a firm's contingent liabilities is more likely to influence the outcome of a pending lawsuit than will a vaguely worded footnote disclosure which denigrates the merits of the lawsuit. In both examples (and in many others), whether a disclosure influences some firm's cash flows depends in part upon the accuracy of the disclosure. Such examples are the basis for the maintained assumption here that the amount by which the disclosure of information affects firms' subsequent expected cash flows varies with the precision of the disclosure.

Consistent with this assumption, we assume that, conditional on realization $\boldsymbol{x}$ of the random vector $\tilde{\boldsymbol{x}}$ drawn from the joint normal distribution $\boldsymbol{N}(\tilde{\mu},\boldsymbol{R}(\boldsymbol{r})^{-1})$, the posterior joint distribution of cash flows is given by $\boldsymbol{N}(\boldsymbol{G}(\boldsymbol{r},\boldsymbol{x}),\ (\boldsymbol{R}(\boldsymbol{r})+\Omega^{-1})^{-1}+\Phi)$, where $\boldsymbol{G}(\boldsymbol{r},\boldsymbol{x})$ represents the pair of functions $(G_1(\boldsymbol{r},\boldsymbol{x}),\ G_2(\boldsymbol{r},\boldsymbol{x}))$. $G_i(\boldsymbol{r},\boldsymbol{x})$ denotes the ith firm's expected cash flows, conditional on the disclosures $\boldsymbol{x}'=(x_1,x_2)$ according to the disclosure policies summarized by the matrix $\boldsymbol{R}(\boldsymbol{r})$.[22] It follows from equation (2) above that, conditional on $\tilde{\boldsymbol{x}}=\boldsymbol{x}$, the equilibrium prices of the securities are:

$$\boldsymbol{P}(\boldsymbol{x};\boldsymbol{r})\equiv\boldsymbol{G}(\boldsymbol{r},\boldsymbol{x})-\gamma((\boldsymbol{R}(\boldsymbol{r})+\Omega^{-1})^{-1}+\Phi)\mathbf{1}.$$

If $\boldsymbol{G}(\boldsymbol{r})'\equiv(G_1(\boldsymbol{r}),G_2(\boldsymbol{r}))\equiv E\boldsymbol{G}(\boldsymbol{r},\tilde{\boldsymbol{x}})'$, then the expected values of the n firms are given by:

$$E\boldsymbol{P}(\boldsymbol{x})=\boldsymbol{G}(\boldsymbol{r})-\gamma((\boldsymbol{R}(\boldsymbol{r})+\Omega^{-1})^{-1}+\Phi)\mathbf{1}.$$

Disclosures generate real externalities if any component of $\boldsymbol{G}(\boldsymbol{r})$ varies with the disclosure requirements $\boldsymbol{r}$. Among the many ways in which disclosures may exhibit real externalities, two are singled out for attention: those that have firm-specific or market-wide expected cash-flow effects.

Definition 2: There are positive private returns to additional disclosures of $\tilde{\mu}_i$ if:

$$\frac{\partial}{\partial r_i}G_i(\boldsymbol{r})\geq 0,\ \text{for all } \boldsymbol{r}\geq 0.$$

[22] Notice that, by invoking these distributional assumptions, we are assuming that the posterior variance of firms' cash flows are updated in the same way they were when disclosures generated only financial externalities.

Definition 3: There are positive market-wide returns to additional disclosures of $\tilde{\mu}_i$ if:

$$\frac{\partial}{\partial r_i} G_1(\boldsymbol{r})+G_2(\boldsymbol{r})\geq 0, \text{ for all } \boldsymbol{r}\geq 0.$$

If there are positive private returns to additional disclosures of $\tilde{\mu}_i$, then the expected value of i's cash flows increases with the precision of the disclosures about $\tilde{\mu}_i$. If there are positive market-wide returns to additional disclosures of $\tilde{\mu}_i$, then the sum of the expected cash flows of all firms increases with the precision of the disclosures of $\tilde{\mu}_i$. Negative private and market-wide returns to additional disclosures of $\tilde{\mu}_i$ are defined analogously.

Our first observation is that investors are indifferent to the presence of any kind of real externalities, holding fixed their knowledge of the posterior variance of the aggregate cash flows, i.e., holding fixed $\nu=\mathbf{1}'(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}$. This can be seen from equation (7) above. Each investor's equilibrium expected utility is independent of the mean of either any firm's cash flows or aggregate cash flows. The reason, of course, is that competition among investors bids away any benefits or costs of externalities derived from firms' disclosures. Investors are favorably affected by the presence of externalities only to the extent that the externalities induce entrepreneurs to reduce the precision of the disclosure policies they select.

Entrepreneurs are not similarly apathetic about disclosure policies. Consider first the case of an entrepreneur, say entrepreneur 1, who has diversified his or her risk prior to selecting a disclosure policy. The certainty equivalent benefit he or she derives from the disclosure policy (r_1,r_2) is:

$$(-1/\beta_1)E[e^{-\beta\mathbf{1}'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})}]=(-1/\beta_1)\exp(-\beta(\mathbf{1}'\boldsymbol{G}(\boldsymbol{r})-\gamma(\phi+\nu)-.5\beta(\omega-\nu)). \qquad (9)$$

Observe that entrepreneurs remain unanimous about disclosure policies when they share the benefits and costs of the real externalities of their disclosures in proportion to their risk tolerances. Also, in the typical case (one condition of which is $\gamma\cong\beta$, so $\gamma>.5\beta$), it is apparent from equation (9) that the presence of positive market-wide returns to additional disclosures reinforces the risk-based incentives they have to engage in full disclosure. Negative market-wide returns obviously have the opposite effect. And, the presence of private returns to additional disclosure are relevant only to the extent that they influence the market-wide returns to disclosure.

Determining the effects of the introduction of externalities on undiversified entrepreneurs is somewhat more complex. Entrepreneur 1's certainty equivalent benefit associated with the vector $\boldsymbol{r}=(r_1,r_2)$ of disclosure policies is given by:

$$(\mathbf{1}'_1\boldsymbol{G}(\boldsymbol{r})+\{-\gamma(\omega_{22}+r_2-\omega_{12})+.5\times\beta_1\times(\omega_{22}+r_2)\}\times\{(\omega_{11}+r_1)(\omega_{22}+r_2)-\omega_{12}^2)\}^{-1}.$$

In the typical case where β_1 is much larger than γ and γ is near zero, we know that from a purely risk-sharing standpoint, the entrepreneur has an incentive to engage in no disclosure whatsoever. If there are negative private returns to additional disclosure, then this merely serves to reinforce the entrepreneur's incentive to engage in no disclosure. When there are positive private returns to

Table 2
Equilibria for Real Externalities

	(d_2^∞, d_2^0)	$(-,-)$	$(-,+)$	$(+,-)$	$(+,+)$
(d_1^∞, d_1^0)					
$(-,-)$		$(0,0)$	$(0,\iota)$	$(0,0)$	$(0,\iota)$
$(-,+)$		$(\iota,0)$	?	?	$(?,\iota)$
$(+,-)$		$(0,0)$	?	?	$(?,\iota)$
$(+,+)$		$(\iota,0)$	$(\iota,?)$	$(\iota,?)$	(ι,ι)

additional disclosures by entrepreneur 1, tension exists between the mean benefits of additional disclosure and the risk-sharing costs, the net result typically being a nonzero but finite level of disclosure. For the special case in which the entrepreneurs are risk-neutral, this tension disappears and, as was the case when real externalities were presumed absent, full disclosure is the only equilibrium disclosure policy.

As a specific example of disclosures with real cash flow effects, suppose each firm experiences negative private returns to additional disclosures because there are costs of making disclosures credible. To make the disclosure more credible, the firm might have to hire a more reputable auditor to attest to the precision of the disclosures. Consistent with classical audit sampling, we shall assume that disclosure costs increase quadratically in the precision of the disclosure:

$$C(r_i) = .5 \times c \times r_i^2.$$

Table 2 is the counterpart to Table 1 in Proposition 2, for this disclosure cost specification. It summarizes what can be said about equilibrium voluntary disclosure policies, on the basis of only the signs of the terms d_i^0 and d_i^∞. In this table, "ι" indicates that the equilibrium voluntary disclosure is "interior," i.e., $0 < r < \infty$, and "?" indicates that whether the equilibrium disclosure policy is positive cannot be determined by knowing only the signs of the "d_i" terms.[23]

Full disclosure never occurs presently when there are at most finite benefits to disclosure, because there are infinite costs of engaging in full disclosure. Observe that when no disclosure was an equilibrium policy when disclosure costs were zero, the presence of disclosure costs merely reinforces the incentives not to engage in any disclosure, at least in all those cases in which the disclosure policy is determinant based on knowledge of the "d_i" terms.

Returning to the general case of real externalities, we now ask: how do the voluntary disclosure policies entrepreneurs select compare to mandatory policies? The answer depends on whether the entrepreneurs are diversified and on what the mandatory policies are designed to maximize. If each entrepreneur is diversified and mandatory policies maximize a weighted average of the entrepreneurs' expected utilities, then mandatory and voluntary disclosure policies will coincide, because of entrepreneurs' common attitudes about the desir-

[23] The proof of this result is similar to that of Proposition 2's and is not presented here.

ability of maximizing the certainty equivalent of $\mathbf{1}'\boldsymbol{P}(\tilde{\boldsymbol{x}};\boldsymbol{R})$, measured using the aggregate risk-aversion parameter β. As was true in Section III, as the social welfare function is modified to place more weight on investors' expected utilities, the optimal amount of disclosure declines.[24] These remarks are independent of the nature of the real externalities.

When entrepreneurs are undiversified, the relation between mandatory and voluntary disclosure policies depends upon how the real externalities manifest themselves. Suppose, in the "typical" parameterization, that the undiversified firm experiences negative private returns to additional disclosures and positive market-wide returns to additional disclosures. If mandatory disclosure policies maximize a weighted average of the entrepreneurs' expected utilities and sufficiently little weight is placed on the preferences of the undiversified entrepreneur, then the mandatory disclosures for the undiversified firm will strictly exceed its voluntary disclosures. In this case, the positive externalities of additional disclosures reinforce the risk-based benefits of additional disclosures identified in the previous section.[25]

One might describe the consequences of eliminating mandatory disclosure requirements here by saying that a "race to the bottom" occurs, because each undiversified firm would (immediately) reduce its disclosures if mandatory disclosure regulations were lifted. But, it is clear that a "race to the top" could emerge in which equilibrium voluntary disclosures exceed optimal mandatory disclosures, by positing that each firm experiences positive private returns to additional disclosure, but negative market-wide returns. Plausible scenarios can be constructed involving any possible combination of positive or negative private or market-wide returns to additional disclosures. Consequently, without possessing detailed *a priori* knowledge about the relation between private and market-wide returns to additional disclosures for each firm, it is difficult to surmise whether mandatory disclosures will exceed voluntary disclosures. It is easy to show that this problem arises whether or not investors are risk-averse. Hence, even if we alter the objective function of mandatory disclosure policies so that it involves only aggregate market value maximization, comparisons of mandatory and voluntary disclosures require specific information about the nature of the externalities produced by the disclosures.

V. Conclusions

This paper has compared mandatory and voluntary disclosures in a simple market economy where disclosures by one firm can alter investors' perceptions about the distributions of other firms' cash flows ("financial externalities") and possibly the actual distributions of other firms' cash flows ("real externalities"). Divergence between optimal voluntary and mandatory disclosures was shown to depend upon which form of externalities a firm's disclosures generated. The

[24] This follows because an increase in the precision of any firm's disclosures decreases the posterior variance $\mathbf{1}'(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}$.

[25] This result obtains because, in the two-firm case, if there are negative private returns and positive market-wide returns to additional disclosures for firm 1, then this necessarily implies that firm 2's expected cash flows increase with firm 1's disclosures.

existence of "information transfers" among firms (associated with financial externalities), as documented by Foster (1981) and Olsen and Dietrich (1985) was shown, in some cases, not to be sufficient cause for regulation, because the regulated disclosures would simply coincide with the disclosures firms would make voluntarily. The situation with disclosures which exhibit real externalities turned out to be much more complicated; it was difficult to make any general statement about the relation between optimal voluntary and mandatory disclosures in such cases.

Of course, categorizing a disclosure as "real" or "financial" is in practice a nontrivial exercise. The reason for postulating that disclosures can be so labeled is that it illustrates most starkly the various consequences of adopting voluntary, as opposed to mandatory, disclosures. However, it seems reasonable to assert that some disclosures are more likely to have real effects than others. Disclosures of substantial increases in earnings forecasts, of marginal cost data, of profitability of a conglomerate by line segment, all intuitively would be judged as having potential cash-flow effects. In contrast, most accountants would probably agree that disclosure of interest and depreciation expenses, and allowances for uncollectibles, are unlikely to reveal proprietary data.

These results were derived in a highly stylized model, with parametric preferences and probability distributions. Two limitations should be noted. First, the model studied here has several stages (entrepreneurs pick disclosure policies, and then make disclosures in compliance with those policies, while investors formulate their demand functions, purchase shares, and ultimately consume firms' cash flows in proportion to share purchases), but it is basically a single-period model. Multiple disclosures with multiple rounds of trading are left for future analyses. Second, the present study does not allow investors to acquire information on their own. The distributional effects of disclosure policies in the presence of information-seeking investors have not been studied.

Appendix

Proofs of Propositions

Proof of Proposition 1

The social welfare function can be written as the following concave function:

$$W(\nu)\equiv -\lambda\varkappa/\beta\times\exp(-\beta\nu(.5\beta-\gamma))-\xi\delta/\gamma\times\exp(-.5\gamma^2\nu).$$

The posterior variance ν of the expected aggregate cash flows can take on any value between zero (full disclosure) and the prior variance ω (no disclosure). Hence, full (resp., no) disclosure (in the aggregate—i.e., of $\mathbf{1}'\tilde{\mu}$) is optimal if, and only if, $W'(0)$ (resp., $W'(\omega)$) is negative (resp., positive). Computing these derivatives and appealing to the concavity of $W(\bullet)$ proves whether $\nu=0$ or $\nu=\omega$ in each of (i)–(iii). Completing the proofs of (i)–(iii) requires merely translating statements about the aggregate variance ν into statements about the disclosure policies r_i. Part (iv) follows immediately by observing how $W'(\nu)$ varies with λ and ξ. Part (v)'s proof can be reconstructed from the discussion following Proposition 1.

Proof of Proposition 2

We begin by computing entrepreneur i's expected utility corresponding to a given set of disclosure policies.

Equation (2) in the text can be written as:

$$\boldsymbol{P}(\boldsymbol{x})=(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}(\boldsymbol{\Omega}^{-1}\boldsymbol{M}+\boldsymbol{R}\boldsymbol{x})-\gamma(\boldsymbol{\Phi}+(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1})\mathbf{1}.$$

Since the prior beliefs about $\tilde{\boldsymbol{x}}$ are $\boldsymbol{N}(\boldsymbol{M},\boldsymbol{R}^{-1}+\boldsymbol{\Omega})$,

$$\begin{aligned} E[\boldsymbol{P}(\tilde{\boldsymbol{x}})] &= \boldsymbol{M}-\gamma(\boldsymbol{\Phi}+(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1})\mathbf{1}, \text{ and} \\ \operatorname{Var}(\boldsymbol{P}(\tilde{\boldsymbol{x}})) &= \operatorname{Var}((\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\boldsymbol{R}\boldsymbol{x})=(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\boldsymbol{R}(\boldsymbol{R}^{-1}+\boldsymbol{\Omega})\boldsymbol{R}(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1} \\ &= \boldsymbol{\Omega}(\boldsymbol{R}^{-1}+\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}. \end{aligned}$$

Hence, entrepreneur i's expected utility is given by:

$$\begin{aligned} E[-e^{-\beta_i \mathbf{1}_i' \boldsymbol{P}(\tilde{\boldsymbol{x}})}] = -\exp[&-\beta_i\mathbf{1}_i'\boldsymbol{M}+\beta_i\gamma\mathbf{1}_i'(\boldsymbol{\Phi}+(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1} \\ &+1/2\beta_i^2\mathbf{1}_i'\boldsymbol{\Omega}(\boldsymbol{R}^{-1}+\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}\mathbf{1}_i]. \end{aligned} \tag{A2.1}$$

There are several irrelevant constants in the above expression. In choosing among disclosure policies, it suffices for entrepreneur i to examine the disclosure policy r_i which *minimizes*:

$$q_i \equiv \gamma\mathbf{1}_i'(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}+1/2\beta_i\mathbf{1}_i'\boldsymbol{\Omega}(\boldsymbol{R}^{-1}+\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}\mathbf{1}_i. \tag{A2.2}$$

The following legend summarizes the notation we use to denote the (k,l)th element of various matrices:

Matrix	(k,l)*th element*
$\boldsymbol{R}+\boldsymbol{\Omega}^{-1}$	a_{kl}
$(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}$	a^{kl}
$\boldsymbol{R}^{-1}+\boldsymbol{\Omega}$	b_{kl}
$(\boldsymbol{R}^{-1}+\boldsymbol{\Omega})^{-1}$	b^{kl}
$\boldsymbol{\Omega}$	$\hat{\omega}_{kl}$
$\boldsymbol{\Omega}^{-1}$	ω_{kl}

From Theil (1971, 33), we conclude:

$$\frac{\partial b^{kl}}{\partial r_i}=\frac{\partial b^{kl}}{\partial b_{ii}}\cdot\frac{\partial b_{ii}}{\partial r_i}=\frac{b^{ki}b^{il}}{r_i^2}.$$

Using this fact, we get:

$$\partial/\partial r_i\mathbf{1}_j'(\boldsymbol{R}+\boldsymbol{\Omega}^{-1})^{-1}\mathbf{1}=\partial/\partial r_i\sum_k a^{jk}=-\sum_k a^{ji}a^{ik} \text{ and} \tag{A2.3}$$

$$\begin{aligned} \partial/\partial r_i\mathbf{1}_j'\boldsymbol{\Omega}(\boldsymbol{R}^{-1}+\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}\mathbf{1}_j &= \partial/\partial r_i\sum_{k,l}\hat{\omega}_{jk}b^{kl}\hat{\omega}_{lj} \\ &= \sum_{k,l}\hat{\omega}_{jk}\frac{b^{ki}b^{il}}{r_i^2}\hat{\omega}_{lj} \\ &= \left(\sum_k\frac{\hat{\omega}_{jk}b^{ki}}{r_i}\right)^2, \end{aligned} \tag{A2.4}$$

the last line following from the symmetry of $\mathbf{\Omega}$ and $(\boldsymbol{R}^{-1}+\mathbf{\Omega})^{-1}$.

Now observe:

$$\boldsymbol{R}^{-1}(\mathbf{\Omega}+\boldsymbol{R}^{-1})^{-1}\mathbf{\Omega}=[\mathbf{\Omega}^{-1}(\mathbf{\Omega}+\boldsymbol{R}^{-1})\boldsymbol{R}]^{-1}$$
$$=(\boldsymbol{R}+\mathbf{\Omega}^{-1})^{-1},$$

or, component-by-component,

$$\sum_k \frac{b^{ik}\hat{\omega}_{kj}}{r_i}=a^{ij}$$

so, by symmetry,

$$a^{ij}=\sum_k \frac{\hat{\omega}_{jk}b^{ki}}{r_i}. \tag{A2.5}$$

Therefore, by letting $j=i$ in each of expressions (A2.3) to (A2.5), we have:

$$\partial/\partial r_i\{\gamma \mathbf{1}_i'(\boldsymbol{R}+\mathbf{\Omega}^{-1})^{-1}\mathbf{1}+\beta_i/2\,\mathbf{1}_i'\mathbf{\Omega}(\boldsymbol{R}^{-1}+\mathbf{\Omega})^{-1}\mathbf{\Omega}\mathbf{1}_i\}$$

$$=-\gamma a^{ii}\sum_j a^{ij}+\beta_i/2(a^{ii})^2$$

$$=a^{ii}\left((\beta_i/2-\gamma)a^{ii}-\gamma\sum_{j\neq i}a^{ij}\right). \tag{A2.6}$$

Now, we appeal to the assumption that there are only two entrepreneurs,

$$\begin{pmatrix} a^{11}a^{12} \\ a^{21}a^{22}\end{pmatrix}=(\boldsymbol{R}+\mathbf{\Omega}^{-1})^{-1}$$

$$=\frac{1}{(\omega_{11}+r_1)(\omega_{22}+r_2)-\omega_{12}^2}\begin{pmatrix}\omega_{22}+r_2 & -\omega_{12} \\ -\omega_{21} & \omega_{11}+r_1\end{pmatrix}.$$

Thus, if $i=1$, expression (A2.6) has the same sign as:

$$(\beta_1/2-\gamma)(r_2+\omega_{22})+\gamma\omega_{12}, \tag{A2.7}$$

since $a^{ii}>0$ and $(\omega_{11}+r_1)(\omega_{22}+r_2)-\omega_{12}^2>0$ (as the determinant of the positive-definite matrix $(\mathbf{\Omega}^{-1}+\boldsymbol{R})$). Consequently, r_1 is either zero or infinity as this expression is positive or negative. Similarly, r_2 is either zero or infinity as:

$$(\beta_2/2-\gamma)(r_1+\omega_{11})+\gamma\omega_{12} \tag{A2.8}$$

is positive or negative.

The two expressions (A2.7) and (A2.8) completely determine the equilibrium of this voluntary disclosure game. We will illustrate how one verifies this in two cases; the remaining cases are proven analogously. We begin by letting $e_i=\beta_i/2-\gamma$, $g_i=(\beta_i/2-\gamma)\omega_{jj}+\gamma\omega_{12}$, for all i and $j\neq i$. With this notation, r_1 is 0 or ∞ as:

$$g_1+e_1r_2 \tag{A2.9}$$

is positive or negative; r_2 is 0 or ∞ as:

$$g_2+e_2r_1 \tag{A2.10}$$

is positive or negative.

Claim 1:
(a) $e_1,g_1>0$ and $g_2>0$ implies $(r_1,r_2)=(0,0)$ is the unique equilibrium.
(b) $e_1,g_1>0$ and $g_2<0$ implies $(r_1,r_2)=(0,\infty)$ is the unique equilibrium.

Proof: $e_1,g_1>0$ and $r_2\geq 0$ imply expression (A2.9) is positive, so $r_1=0$. Substituting $r_1=0$ into expression (A2.10), we see $r_2=0$ or ∞ as g_2 is positive or negative.

Claim 2: $e_1,e_2>0$ and $g_1,g_2<0$ imply each of: $(r_1,r_2)=(\infty,0)$, $(0,\infty)$, $(-g_2/e_2,-g_1/e_1)$ is an equilibrium, and there are no other equilibria.

Proof: We start by showing that there are no equilibria other than those listed.

Suppose $r_1=-g_2/e_2$ is part of some equilibrium (r_1,r_2).

If $r_2>-g_1/e_1$, then $g_1+e_1r_2>0$. By expression (A2.9), it follows that r_1 must be zero. This contradicts $r_1=-g_2/e_2$.

If $r_2<-g_1/e_1$, then r_1 must be ∞ by expression (A2.9), which is another contradiction. Thus, the only possible r_2 consistent with $r_1=-g_2/e_2$ is $r_2=-g_1/e_1$.

Now suppose $r_1>-g_2/e_2$ is part of an equilibrium. By expression (A2.10), it follows that $r_2=0$. But then $g_1+e_1r_2=g_1<0$, so $r_1=\infty$. So the only equilibrium (r_1,r_2) consistent with $r_1>-g_2/e_2$ is $(r_1,r_2)=(\infty,0)$.

Finally, suppose $r_1<-g_2/e_2$ is part of an equilibrium. Then $r_2=\infty$ by expression (A2.10) which implies $r_1=0$ by expression (A2.9). So $(r_1,r_2)=(0,\infty)$ is the only possible equilibrium with $r_1<-g_2/e_2$.

Verification of this claim is completed by establishing that each of the three candidate equilibria identified above are in fact equilibria. This verification procedure is straightforward and omitted.

To complete the proof of the proposition, we must convert the variables e_i,g_i, which are expressed in terms of the elements of the *inverse* of $\mathbf{\Omega}$ into the variables d_i^0,d_i^∞ used in the text, which are expressed in terms of the elements of $\mathbf{\Omega}$. Recalling our notation, it is easy to show that:

$$g_1=(\omega_{11}\omega_{22}-\omega_{12}^2)[(\beta_1/2-\gamma)\hat{\omega}_{11}-\gamma\hat{\omega}_{12}],$$

so

$$\operatorname{sgn} g_1=\operatorname{sgn}(\beta_1/2-\gamma)\hat{\omega}_{11}-\gamma\hat{\omega}_{12}\equiv -\operatorname{sgn} d_1^0.$$

Similarly, $\operatorname{sgn} g_2=-\operatorname{sgn} d_2^0$.
Finally, it is obvious that $\operatorname{sgn} e_i=-\operatorname{sgn} d_i^\infty$.

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