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Discretionary disclosure, efficiency, and signal informativeness ☆

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Abstract

This paper studies a competitive asset market characterized by an adverse selection problem. The analysis focuses on the link between the market participants' productive activities and discretionary disclosures. While informed parties' discretion over disclosure allows them to earn private gains, it leads to an inefficient allocation of resources. A more informative signal makes the informed parties better off, but reduces the uninformed parties' welfare. Nonetheless, it improves the economy's allocative efficiency. The paper also shows that when the signal quality is endogenous, the informed parties over-invest in the signal informativeness relative to the level that maximizes social welfare.

JEL classification: D61; G14; M41

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1. Introduction

Prior studies on corporate disclosures have established that firms, seeking to maximize outsiders' valuation, disclose private information only when it is

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sufficiently favorable. In Verrecchia (1983), such a partial disclosure equilibrium is sustained by firms' disclosure-related proprietary costs while in Dye (1985) and Jung and Kwon (1988), it is sustained by outsiders' uncertainty about firms' informedness. Irrespective of the sustaining forces, a noteworthy feature common to the aforementioned studies is that information has *no* intrinsic value because it pertains to the exogenous future value of a firm.[1]

In fact, in his follow-up study, Verrecchia (1990) shows that a firm faced with proprietary disclosure costs is ex ante better off if it has *no* information. The reason is that the proprietary costs are deadweight losses. Extending Dye (1985) and Jung and Kwon (1988) to a costly information acquisition setting, Pae (1999) obtains a similar result that a firm can maximize its ex ante value by pre-committing *not* to acquire any information. Given these ex ante results, Verrecchia (2001) comments: "…if…a manager continues to exercise discretion in the disclosure of information ex post, there must be some unstated, unmodeled, and/or unresolved agency problem or efficiency consideration that lurks in the background" (p. 147). Calling for a comprehensive economic theory of disclosure, he then makes the following proposal: "My suggestion for linking disclosure to efficiency, incentives, and the endogeneity of the market process is through the reduction in the information asymmetry component of the cost of capital" (p. 173).[2]

Prompted by Verrecchia's lead, the innovation in this paper is to introduce an intrinsic value of information in a discretionary disclosure model, thereby addressing efficiency issues. The model setup is as follows. There is a group of entrepreneurs in an economy. Each entrepreneur is endowed with an asset and seeks to sell it to outside investors in a competitive market. The asset can be any tradable asset, such as property, equipment, a patent, a division, or even a firm in its entirety. The future performance of the asset is jointly determined by the entrepreneurs' and investors' pre- and post-trade inputs in the asset (as explained below) and a random state of nature. The information in my model pertains to the state of nature. Such information is valuable because the input decisions can be conditioned on it, thereby improving the future performance of the asset.[3] The entrepreneurs differ in terms of their informedness; some entrepreneurs receive a private signal about the random state of nature, but the others remain uninformed about the random state. Both the informed and uninformed entrepreneurs provide efforts for their asset prior to selling it. The informed entrepreneurs disclose or withhold their private signal when

[1] Throughout this paper, the analysis is restricted to risk-neutral settings. As emphasized by Verrecchia (1990), information can have a positive value even in a pure exchange economy if firms' risks are priced.

[2] By the information asymmetry component of the cost of capital, he refers to "a transaction cost that arises from the adverse-selection problem inherent in the exchange of assets among investor agents of varying degrees of informedness" (p. 164).

[3] For example, suppose that an entrepreneur sells his firm to third-party investors. In this case, the entrepreneur's time and effort for the firm prior to the sale (say, R&D activities to develop a new product), the investors' investment in the acquired firm's operations (say, hiring managers to produce and market the product), and random factors (say, consumer preferences regarding the product, competitors' product quality, and the general conditions of the economy) jointly affect the firm's future income. Any information about the random factors is valuable since it improves the resource allocation decisions of the entrepreneur and investors.

they sell the asset. While the investors know that some entrepreneurs are informed, they do not know which individual entrepreneurs are informed. Based on the disclosure of a signal or the lack thereof, the investors decide whether or not to purchase the asset. If they acquire the asset, they make a capital investment that further enhances the future return of the asset. In this setting, the paper's main focus is on how the pre- and post-trade input decisions (i.e., the entrepreneurs' effort and the investors' capital investment) are linked to discretionary disclosure, and the efficiency implications of that link. When there is no disclosure by the informed entrepreneurs, an adverse selection problem arises and it prevents an efficient allocation of resources. While full disclosure would resolve this problem, I show that a full disclosure equilibrium does not exist, which implies that efficiency losses are inevitable.

To explain the paper's main results, it is useful to first consider a hypothetical benchmark—referred to as the first-best case—where each entrepreneur's informedness is publicly known and all informed entrepreneurs disclose their signal irrespective of its realization. There is no efficiency loss in the first-best case because the economy's allocation of resources (in terms of the entrepreneurs' effort provision and the investors' capital investment to the asset) fully incorporates the information content of the signal. A more informative signal in the first-best case leads to a Pareto improvement in welfare because the resource allocation is more accommodating to the random state of nature.

Such an efficient (first-best) outcome is not achievable in the presence of uncertainty about the informedness of individual entrepreneurs, which leads to the second-best case. Efficiency losses arise from the informed entrepreneurs' rent-seeking behavior through discretionary disclosure. In particular, when their signal is unfavorable (i.e., less than a cutoff value), the informed entrepreneurs mimic the uninformed entrepreneurs by choosing the same effort as that of the uninformed entrepreneurs and selling the asset with no disclosure. This translates into a downward revision of the investors' belief about the future prospects of the asset, which in turn induces the investors to make an inefficient investment. Consequently, the equilibrium efficiency in the case of nondisclosure is lower than that in the first-best case for two reasons. First, the resource allocation for the informed entrepreneurs' asset does not reflect the information content of the suppressed signal. Second, there is a negative externality in the sense that the equilibrium allocation for the uninformed entrepreneurs' asset is different from the first-best allocation. Even if the informed entrepreneurs' rent-seeking behavior leads to a sub-optimal allocation of resources, thereby reducing the efficiency of the economy and making the uninformed entrepreneurs worse off, their incentive to seek private gains is unavoidable in equilibrium.

The paper then examines the welfare implications of an improvement in the signal quality, which is measured by the signal's informativeness about the random state of nature. An increase in the signal informativeness makes the informed (uninformed) entrepreneurs better (worse) off ex ante. Intuitively, when the signal that might be suppressed becomes more informative, the investors given no disclosure become more conservative about the future prospects of the asset (recall that the signal is withheld only when it is unfavorable). Thus, they price the asset lower and invest less. This reduces the uninformed entrepreneurs' ex ante welfare. Nonetheless, insofar as the signal is disclosed with a positive probability and thus used for the productive activities, an increase in the signal informativeness improves the economy's allocative efficiency and enhances the informed entrepreneurs' ex ante welfare.

Finally, the model is extended to a setting in which the informed entrepreneurs choose signal informativeness at some cost. The analysis here shows that the informed entrepreneurs over-invest in the signal quality, i.e., the privately optimal level of signal informativeness exceeds the level that maximizes social welfare. This result follows because a more informative signal allows the informed entrepreneurs to gain more at the expense of the uninformed entrepreneurs' welfare and, hence, the informed entrepreneurs' private value of an improvement on the signal quality exceeds its social value.

Intuitive empirical implications emerge from the analysis. I particularly focus on the input decisions and the asset price in the case of nondisclosure. It is shown that the market penalizes nondisclosure more severely (in terms of a lower asset price), either when an entrepreneur is more likely to be informed, or when the signal (that might be suppressed) is more informative. In contrast, the investors pay more for the asset when they have a higher prior belief about the asset return. See Corollary 1 for more details.

This paper extends several studies in the disclosure literature. It extends Dye (1985) and Jung and Kwon (1988) by incorporating productive activities and examining their interaction with discretionary disclosures. It is also related to Pae (1999). The key difference is that while the signal in Pae (1999) is *post*-decision information, the signal in this paper is *pre*-decision information. This difference explains why this paper draws different conclusions about the efficiency implications of discretionary disclosures and the private vs. social preferences over the informativeness of signal.

While Lanen and Verrecchia (1987) also consider pre-decision information in a voluntary disclosure model (where a firm is potentially sold to outside investors), their model is different from the model in this paper and, thus, the two models reach different conclusions about the efficiency implication of discretionary disclosures. Specifically, the firm in Lanen and Verrecchia's model makes the most efficient operating decision if it can pre-commit to *nondisclosure* of its private signal. The driving force is that nondisclosure saves on the firm's disclosure-related costs while its operating decision is directly observable to outside investors. Thus, it is ex ante optimal for the firm to separate its operating and disclosure decisions.[4] In contrast, the productive activities in this paper (i.e., the entrepreneurs' effort provision and the

[4] The key result in Lanen and Verrecchia (1987) is that when such a separation is impossible, the firm's operating decision might diverge from the most efficient one because, depending on the magnitude of proprietary disclosure costs, the firm might use its operating decision as a noisy communication device to the market rather than directly disclosing its proprietary information.

investors' capital investment) are always intertwined with the entrepreneurs' disclosure decision, and the most efficient outcome is achieved if the entrepreneurs can make an ex ante commitment to *full disclosure* (which is the first-best case in this paper). Of course, as noted earlier, the entrepreneurs' ex post incentives to withhold unfavorable signals render such a commitment not self-enforcing, thereby leading to a sub-optimal allocation of resources in equilibrium.

Shavell (1994) too considers a setting in which a seller of an asset may reveal private information about a characteristic of the asset and a buyer invests in the asset to enhance its value. This paper differs from his study in several ways. First, not only the buyer's investment but also the seller's effort affects the future return of the asset. This feature of the model allows me to examine: (i) the link between the pre- and post-trade productive actions in conjunction with discretionary disclosure; and (ii) how those actions change with various parameters that affect the link. Second, while Shavell (1994) focuses on a perfect information case, the signal in this paper is imperfect and its informativeness varies with costs. This enables me to investigate the economic consequences of various levels of signal informativeness and derive the privately and socially optimal levels of signal informativeness.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 provides an analysis of the first-best case. Section 4 analyzes the second-best case in which the informed entrepreneurs exercise discretion over disclosure of their private signal. Section 5 examines welfare implications and extends the basic model to a setting where signal informativeness is endogenous. Section 6 concludes the paper. Appendix A provides the proofs that are not included in the main text.

2. The model

Consider a competitive market of an asset, in which all market participants have risk-neutral preferences and a zero discount rate for their future payoffs. The asset can be any tradable asset whose future performance is jointly determined by the asset traders' pre- and post-trade inputs to the asset and a random state of nature. Specifically, assume that an entrepreneur initially owns the asset and seeks to sell it in its entirety for liquidity reasons.[5] There are many such entrepreneurs, and without loss of generality, the total number of the entrepreneurs is normalized to one. Potential buyers of the asset are called investors. A vector, (e, k, x), determines the future return of the asset, where: (i) $e \geqslant 0$ is the entrepreneur's effort for the asset prior to sale; (ii) $k \geqslant 0$ is the investors' post-trade capital investment in the asset to enhance the asset return; and (iii) x is a random variable representing the true state of nature, which is realized after the decisions on e and k are made. For simplicity, assume that $x \sim \mathrm{N}(\theta, \sigma_x^2)$ where $\theta > 0$. In addition, to focus on the efficiency implications of the entrepreneurs' discretionary disclosure of private information about x (as specified below), I assume that e and k are observable.

[5] Alternatively, as in Dye (1990), one can view this model as a snapshot of an overlapping generations model, in which the asset ownership is transferred from one generation to the next through a market transaction. Under either scenario, the seller's objective is to maximize the market price of the asset.

Let $xR(e,k)$ be the gross return of an asset given (e,k,x). Then, the net return of this asset is

$$xR(e,k) - k - C(e), \tag{1}$$

where $C(e)$ is an entrepreneur's cost of effort e. In (1), C is an increasing and convex function satisfying $C(0) = C'(0) = 0$, and the return function R: $\Re^2_+ \rightarrow \Re_+$ is an increasing and concave function satisfying

$$R(0,0) = 0, \quad \lim_{e\rightarrow 0} R_e(e,k) = \lim_{k\rightarrow 0} R_k(e,k) = \infty$$

and

$$\lim_{e\rightarrow \infty} R_e(e,k) = \lim_{k\rightarrow \infty} R_k(e,k) = 0, \tag{2}$$

where subscripts to R denote partial derivatives. The second-order partial derivatives of R and C, i.e., R_{ee}, R_{ek}, R_{kk} and C'', are assumed to be continuous. Since x is a random parameter that affects the future performance of the asset, all parties in the model are interested in any information about x. In this regard, I assume that only a fraction of the entire population of the entrepreneurs has access to such information.[6]

Formally, the sequence of events is as follows. Let $q \in (0,1)$ denote the fraction of the entrepreneurs, who have *no* information about x. On the other hand, the remaining entrepreneurs (i.e., the $(1-q)$ fraction of all entrepreneurs) receive a private signal about x, which is defined by

$$y \equiv x + \varepsilon, \tag{3}$$

where $\varepsilon \sim \mathrm{N}(0, \sigma^2_\varepsilon)$ and $\mathrm{E}[x\varepsilon] = 0$, i.e., signal y is x plus pure noise. Both the informed and uninformed entrepreneurs choose effort $e \geqslant 0$ for their asset. They then seek to sell the asset to outside investors. In doing so, the informed entrepreneurs may or may not disclose signal y concurrently. As is commonplace in the literature, I assume that: (i) any disclosed information is truthful and the uninformed entrepreneurs cannot make any disclosure; and (ii) the investors do not know each entrepreneur's informedness (even though they know the distribution of the informed and uninformed entrepreneurs (i.e., q) and have rational expectations about the informed entrepreneurs' disclosure-related incentives).[7] The investors, after observing effort e and the disclosure or nondisclosure of signal y, price the asset competitively. If there is no trade of the asset, the game is over. If the investors acquire the asset at a positive price (and thus become new owners of it), they invest $k \geqslant 0$ in the asset. At the end of the period, x is realized and the investors receive the return of the asset,

[6] For simplicity, I follow Dye (1985) and Jung and Kwon (1988) by assuming a given distribution of the entrepreneurs' informedness. Pae (1999) shows that such a distribution can be made endogenous when the entrepreneurs' information acquisition costs are unknown to the outside investors.

[7] The first assumption is shared by many studies in the literature, e.g. Jovanovic (1982), Verrecchia (1983, 1990), Dye (1985), Lanen and Verrecchia (1987), Jung and Kwon (1988), Shavell (1994), and Pae (1999). The second assumption is crucial to the partial disclosure equilibrium in this paper. Otherwise, unraveling dictates full disclosure equilibrium; see Grossman (1981) and Milgrom (1981).

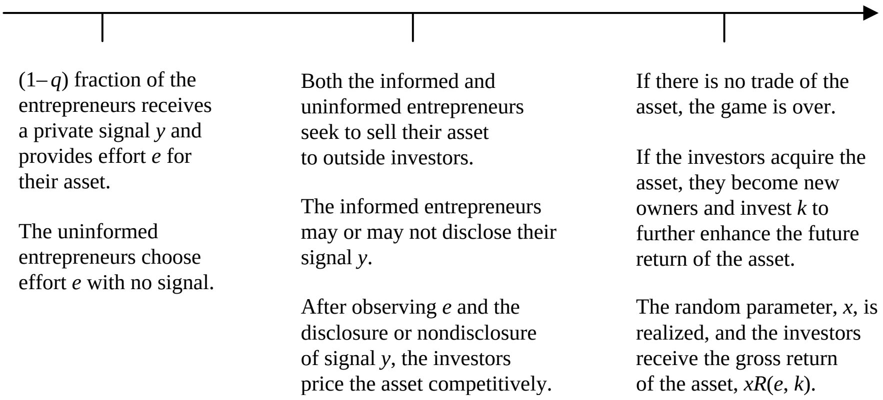

Fig. 1. Time line.

$xR(e, k)$. The model structure is common knowledge. Fig. 1 depicts the sequence of events and Appendix B summarizes the notation used in this paper.[8]

3. The first-best case

Before proceeding to the analysis of the model, I consider a *hypothetical* benchmark—referred to as the first-best case—in which a social planner (or a regulator) has perfect information about all individual entrepreneurs' informedness and she can enforce all informed entrepreneurs to reveal their signal. That is, the informed entrepreneurs have no discretion over disclosure of signal y in this benchmark. From the model description, it is clear that disclosure of y (irrespective of its realization) improves the economy's productive efficiency. The reason is that the resource allocation decision affecting the asset return, i.e., (e, k), fully reflects signal y's information about the unknown parameter x. The entrepreneurs' and investors' choice of (e, k) in the first-best case is referred to as the first-best allocation.

[8] Two remarks on the modeling choice follow. First, I rule out the possibility that the investors might have information about x from independent sources. In the presence of such information, one merely needs to redefine the informed entrepreneurs' signal y as conveying information about x on top of the investors' information, and doing so does not alter the paper's results. Second, the return function, R, allows two inputs (e, k) to have heterogeneous effects on the future performance of the asset. If these two inputs are homogeneous in this respect, the model can be simplified so that only one party makes the input decision. For example, if $R(e, k) = R(e + k)$ and $C(e) = e$, the net return of the asset given in (1) can be restated as $xR(s) - s$, where $s \equiv e + k$ is a choice variable and R is an increasing and concave function of s. In that case, it does not matter whether the entrepreneur or investors choose s before or after the transaction of the asset. The paper's main results remain unchanged in this alternative formulation of the model.

Given the assumptions on the distribution of (x, y), it is well known that the posterior mean of x given y, denoted by μ, is a convex combination of signal y and the prior mean θ, i.e.,

$$\mu \equiv \mathrm{E}[x|y] = \beta y + (1-\beta)\theta \ \text{ where } \ \beta \equiv \frac{t}{t+u} \in [0,1],\ t \equiv \frac{1}{\sigma_\varepsilon^2} \ \text{ and } u \equiv \frac{1}{\sigma_x^2}. \tag{4}$$

Note that β increases with $t \equiv 1/\sigma_\varepsilon^2 \geqslant 0$, i.e., the precision of the noise contained in signal y. This implies that, for any given y, μ places a bigger weight on signal y relative to the prior mean θ when t is higher. Throughout this paper, I call t signal y's informativeness about x and say that y is more informative about x when t is higher. As defined in (3), for any given x, signal y's distribution becomes more concentrated around x as t increases. Thus, placing a greater weight on a more informative signal is intuitive. Finally, observe that, given the one-to-one correspondence between y and $\mu = \beta y + (1-\beta)\theta$, referring to an informed entrepreneur who has signal y is equivalent to referring to an informed entrepreneur whose posterior belief is μ.

I now derive the first-best allocation by backward induction. Consider an informed entrepreneur with posterior belief μ. Suppose that this entrepreneur has incurred cost $C(e)$ to provide effort e for his asset and seeks to sell it. Since y must be disclosed here, the investors have the same posterior belief about x, μ. Given (e, μ), suppose that the investors purchase this asset at a positive price, P, and subsequently invest k in the asset. Then, their post-trade net expected payoff equals

$$-P + [\mu R(e,k) - k], \tag{5}$$

where the expression in the brackets represents the investors' net expected payoff from the asset. Suppose that $\mu \leqslant 0$ in (5). Then, since $\mu R(e,k) - k \leqslant 0$ for any (e,k), the investors have no reason to buy the asset for $P > 0$; that is, the asset cannot be sold to the investors and, consequently, there is no capital investment in the asset, i.e., $k = 0$. The informed entrepreneur's payoff given e and $\mu \leqslant 0$ thus equals[9]

$$\mu R(e,0) - C(e). \tag{6}$$

Next, suppose that $\mu > 0$ in (5). The investors' post-trade decision problem in this case is to choose an investment, k, to maximize their net expected payoff given in (5). Since P is sunk when the investors make a decision on k, their problem is to solve

$$\max_k \mu R(e,k) - k.$$

[9] Here, an implicit assumption is that the investors purchase the asset if, and only if, their net expected payoff from the asset is positive. This assumption is innocuous because the only remaining case is where the asset with a zero net expected return is priced at zero, and the equilibrium allocation in that case is the same as that in the no trade case analyzed in the paper. To be precise, suppose that the investors have acquired the asset when $\mu \leqslant 0$. Then, for any P and $\mu \leqslant 0$ in (5), the investors' optimal investment is $k = 0$ so that their net expected payoff from the asset equals $\mu R(e,0) \leqslant 0$. In the competitive market, that payoff must equal the asset price, P. This means that if the entrepreneur sells the asset, he has to pay the investors the absolute value of P. The entrepreneur can minimize that payment by choosing $e = 0$, so that $P = \mu R(0,0) = 0$ for all $\mu \leqslant 0$. Alternatively, he can choose not to sell the asset, in which case (as shown in the analysis below) his optimal effort is $e = 0$. Thus, the equilibrium allocation is $(e,k) = (0,0)$ in both cases.

Let $k^0(e, \mu)$ be the solution to the investors' optimization problem given (e, μ). Since $\mu > 0$ here, $k^0(e, \mu)$ is the value of k that solves the first-order condition:

$$\mu R_k(e, k) = 1. \tag{7}$$

It is easy to check that $k^0(e, \mu)$ increases with μ and goes to zero as $\mu \to 0$. Anticipating the optimal post-trade investment in the asset, i.e., $k^0(e, \mu)$, rational investors in the competitive market bid the asset price up to their maximized net expected return from the asset, i.e., they break even. Therefore,

$$P(e, \mu) \equiv \mu R(e, k^0(e, \mu)) - k^0(e, \mu) \tag{8}$$

must be the competitive market price of the asset given (e, μ). Using (7), one can verify that $P(e, \mu)$ increases with μ and goes to zero as $\mu \to 0$, which is intuitive. Since the entrepreneur sells the asset for $P(e, \mu)$, his payoff given effort e and $\mu > 0$ equals

$$P(e, \mu) - C(e). \tag{9}$$

Combining (6) and (9), I denote the informed entrepreneur's payoff given (e, μ) by

$$J(e, \mu) \equiv \begin{cases} \mu R(e, 0) - C(e) & \text{for } \mu \leqslant 0, \\ P(e, \mu) - C(e) & \text{for } \mu > 0, \end{cases} \tag{10}$$

where $P(e, \mu)$ is given in (8).

Going backward, I examine this informed entrepreneur's effort decision given μ and then characterize the first-best allocation given μ, which is denoted by $(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu))$. From the preceding analysis, it is clear that the optimal effort of the entrepreneur with posterior belief μ, $e^{\mathrm{F}}(\mu)$, must be the maximizer of $J(e, \mu)$ given in (10). Specifically, consider the case of $\mu \leqslant 0$. Since $J(e, \mu) = \mu R(e, 0) - C(e) \leqslant 0$ for all e and both R and C increase with e, it is obvious that the optimal effort is $e = 0$ (so that $J(0, \mu) = \mu R(0, 0) - C(0) = 0$ for all $\mu \leqslant 0$). Hence, $(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) = (0, 0)$ for all $\mu \leqslant 0$.

Next, consider $\mu > 0$. Given $J(e, \mu) = P(e, \mu) - C(e)$ in (10), the entrepreneur chooses e by solving

$$\max_e P(e, \mu) - C(e) \Leftrightarrow \max_e \mu R(e, k^0(e, \mu)) - k^0(e, \mu) - C(e), \tag{11}$$

where I use $P(e, \mu)$ given in (8). The optimal effort, as denoted by $e^{\mathrm{F}}(\mu)$, therefore equals the value of e that solves the first-order condition of (11),

$$\mu R_e(e, k^0(e, \mu)) = C'(e),$$

where I use (7). Substituting $e^{\mathrm{F}}(\mu)$ into the investors' optimal investment yields the investment in the first-best case, i.e., $k^{\mathrm{F}}(\mu) \equiv k^0(e^{\mathrm{F}}(\mu), \mu)$. However, since (7) holds for any (e, μ), it follows that the first-best allocation given μ, $(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu))$, is in fact the pair of (e, k) that solves two simultaneous equations,

$$\mu R_e(e, k) = C'(e) \quad \text{and} \quad \mu R_k(e, k) = 1. \tag{12}$$

Given the properties of R and C (see Section 2) and $\mu > 0$, (12) implies that $e^{\mathrm{F}}(\mu) > 0$ and $k^{\mathrm{F}}(\mu) > 0$, and both $e^{\mathrm{F}}(\mu)$ and $k^{\mathrm{F}}(\mu)$ converge to zero as $\mu \to 0$. Further, applying the implicit function theorem to (12) shows that both $e^{\mathrm{F}}(\mu)$ and $k^{\mathrm{F}}(\mu)$ increase with μ

if, and only if, R and C satisfy the following regularity conditions:

$$R_k R_{ek} - R_e R_{kk} > 0 \quad \text{and} \quad \mu R_e R_{ek} - R_k(\mu R_{ee} - C'') > 0. \tag{RC}$$

I hereafter assume that (RC) holds, so that a higher (positive) posterior belief μ induces the entrepreneur and investors to devote more resources to the asset (i.e., induces higher e and k), which is intuitive.[10]

In sum, combining both cases of $\mu \leqslant 0$ and $\mu > 0$, I have shown that the first-best allocation is $(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) = (0, 0)$ for all $\mu \leqslant 0$ and $(e^{\mathrm{F}}(\mu),\ k^{\mathrm{F}}(\mu))$ solves (12) for all $\mu > 0$. More compactly and equivalently,

$$(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) \in \arg\max_{e,k}\ \mu R(e,k) - k - C(e) \quad \text{for all } \mu. \tag{13}$$

Eq. (13) makes it clear that the first-best allocation is actually the same as the allocation that maximizes the net expected return of the asset for any given μ.

Now let $\pi^{\mathrm{F}}(\mu)$ denote the informed entrepreneur's equilibrium payoff given μ in the first-best case. As noted earlier, the entrepreneur's first-best effort given μ, $e^{\mathrm{F}}(\mu)$, is the maximizer of $J(e,\mu)$ stated in (10). Thus, in equilibrium, it must be true that

$$\pi^{\mathrm{F}}(\mu) = J(e^{\mathrm{F}}(\mu), \mu) \quad \text{for all } \mu.$$

However, for the subsequent analysis, it is more convenient to exploit (13) and write $\pi^{\mathrm{F}}(\mu)$ as the maximized net expected return of the asset given μ, i.e.,

$$\begin{aligned} \pi^{\mathrm{F}}(\mu) &= \max_{e,k}\ \mu R(e,k) - k - C(e) \\ &= \mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu)) \quad \text{for all } \mu. \end{aligned} \tag{14}$$

In Appendix A, I verify that $\pi^{\mathrm{F}}(\mu)$ given in (14) equals $J(e^{\mathrm{F}}(\mu), \mu)$ for all μ. Thus, I henceforth use $\pi^{\mathrm{F}}(\mu)$ as the first-best equilibrium payoff of the informed entrepreneur whose posterior belief is μ. From (13) and (14), it is evident that $\pi^{\mathrm{F}}(\mu) = 0$ for all $\mu \leqslant 0$, $\pi^{\mathrm{F}}(\mu) > 0$ for all $\mu > 0$, and $\pi^{\mathrm{F}}(\mu) \to 0$ as $\mu \to 0$ from above. In addition, by applying the envelope theorem to (14), one can check that $\pi^{\mathrm{F}}(\mu)$ is increasing and convex for all $\mu > 0$, i.e.,

$$\pi^{\mathrm{F}\prime}(\mu) = R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) > 0$$

and

$$\pi^{\mathrm{F}\prime\prime}(\mu) = R_e(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) e^{\mathrm{F}\prime}(\mu) + R_k(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) k^{\mathrm{F}\prime}(\mu) > 0.$$

As will be shown, the convexity of $\pi^{\mathrm{F}}(\mu)$ plays an important role in the subsequent analysis.

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[10] Given the earlier assumptions on R and C, a sufficient (but not necessary) condition for (RC) to hold is $R_{ek} \geqslant 0$, which is satisfied by many well-known production technologies. For example, a Cobb–Douglas production function $R(e,k) = Ae^{\gamma}k^{1-\gamma}$ where $A > 0$ and $\gamma \in (0,1)$ has this property. More generally, $R_{ek} \geqslant 0$ holds if $R(e,k)$ is either additively or multiplicatively separable so that $R(e,k) = r^1(e) + r^2(k)$ or $R(e,k) = r^1(e) \cdot r^2(k)$ where r^i is an increasing and concave function for all $i = 1, 2$. Also note that $R_{ek} \geqslant 0$ means that two inputs e and k "fit" with each other in generating the asset return. In other words, they exhibit complementarity, which is reasonable to assume in many joint production settings. For more details about this issue, see Milgrom and Roberts (1990, 1995).

Thus far, the analysis has focused on the informed entrepreneur whose posterior belief is μ. Now consider an uninformed entrepreneur, whose lack of signal is public information in the first-best case. Since the uninformed entrepreneur's posterior belief about x is the same as the prior belief, i.e., $\mu = \theta > 0$, it directly follows from the preceding analysis that: (i) the first-best allocation for the uninformed entrepreneur's asset equals $(e^{\mathrm{F}}(\theta),\ k^{\mathrm{F}}(\theta))$ characterized in (13); and (ii) the uninformed entrepreneur's payoff in the first-best case is

$$\pi^{\mathrm{F}}(\theta) = \theta R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta)) > 0, \tag{15}$$

where (14) is used and the inequality is due to $\theta > 0$.

I now define social surplus by the sum of the ex ante equilibrium payoffs of all market participants. Let Ω^{F} denote social surplus in the first-best case. As noted earlier, since the asset market is competitive, the investors' ex ante payoff must equal zero in equilibrium. Thus, aggregating all entrepreneurs' ex ante equilibrium payoffs yields Ω^{F}. To be precise, let $\Omega_{\mathrm{i}}^{\mathrm{F}}$ be an informed entrepreneur's ex ante equilibrium payoff in the first-best case. Since $\pi^{\mathrm{F}}(\mu)$ given in (14) is his equilibrium payoff given μ, $\Omega_{\mathrm{i}}^{\mathrm{F}}$ must be the expected value of $\pi^{\mathrm{F}}(\mu)$. Similarly, let $\Omega_{\mathrm{u}}^{\mathrm{F}}$ be an uninformed entrepreneur's ex ante equilibrium payoff in the first-best case, which must equal $\pi^{\mathrm{F}}(\theta)$ given in (15). That is,

$$\Omega_{\mathrm{i}}^{\mathrm{F}} \equiv \int_{-\infty}^{\infty} \pi^{\mathrm{F}}(\mu)\, \mathrm{d}\Phi(\mu|\cdot) \quad \text{and} \quad \Omega_{\mathrm{u}}^{\mathrm{F}} \equiv \pi^{\mathrm{F}}(\theta), \tag{16}$$

where $\Phi(\mu|\cdot) \equiv \Phi(\mu|\theta, \beta/u)$ is the distribution function of $\mu \sim \mathrm{N}(\theta, \beta/u)$, i.e.,

$$\Phi(\mu|\cdot) = \int_{-\infty}^{\mu} \varphi(m|\cdot)\, \mathrm{d}m$$

where

$$\varphi(\mu|\cdot) \equiv \frac{1}{\sqrt{2\pi}} \sqrt{\frac{u(t+u)}{t}} \exp\left[-\frac{1}{2}\frac{u(t+u)}{t}(\mu - \theta)^2\right]$$

is the normal density function of μ.[11] Throughout the analysis, $\Omega_{\mathrm{i}}^{\mathrm{F}}$ is assumed to be well defined as a finite number for any parameters in the model. Using the ex ante payoffs given in (16), let

$$\lambda^{\mathrm{F}} \equiv \Omega_{\mathrm{i}}^{\mathrm{F}} - \Omega_{\mathrm{u}}^{\mathrm{F}}, \tag{17}$$

i.e., λ^{F} is the difference between the informed and uninformed entrepreneurs' ex ante payoffs in the first-best case. Given (16) and (17), the social surplus in the first-best case can be expressed as

$$\Omega^{\mathrm{F}} \equiv q\Omega_{\mathrm{u}}^{\mathrm{F}} + (1-q)\Omega_{\mathrm{i}}^{\mathrm{F}} = \pi^{\mathrm{F}}(\theta) + (1-q)\lambda^{\mathrm{F}}. \tag{18}$$

Note that while λ^{F} defined in (17) is each informed entrepreneur's private value of signal y in the first-best case, it also represents each informed entrepreneur's

[11] Using (3) and (4), observe that μ is a normal random variable with $\mathrm{E}[\mu] = \beta\mathrm{E}[y] + (1-\beta)\theta = \theta$ and $Var[\mu] = \beta^2 Var[y] = \beta/u = t/[u(t+u)]$. To simplify the notation, I suppress the mean and variance of μ in the distribution and density functions of μ.

contribution to social surplus, so that $(1-q)\lambda^{\mathrm{F}}$ given in (18) denotes the aggregate social value of signal y.

Proposition 1.

(i) *Signal y is valuable, implying that the informed entrepreneur's ex ante payoff is greater than that of the uninformed entrepreneur, i.e., $\lambda^{\mathrm{F}} \equiv \Omega_{\mathrm{i}}^{\mathrm{F}} - \Omega_{\mathrm{u}}^{\mathrm{F}} > 0$.*

(ii) *An increase in the signal informativeness, t, leads to a Pareto improvement. In particular, the uninformed entrepreneur's ex ante payoff is not affected by a change in t, but the informed entrepreneur's ex ante payoff increases with t at a positive rate, i.e., $\partial\Omega_{\mathrm{u}}^{\mathrm{F}}/\partial t = 0 < \partial\Omega_{\mathrm{i}}^{\mathrm{F}}/\partial t$.*

The intuition is straightforward. For part (i), recall that the first-best allocation for the informed entrepreneur's asset, $(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu))$, always incorporates signal y's information about x through posterior belief μ. As a result, $(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu))$ is optimal for any μ in the sense of (13). In contrast, the first-best allocation for the uninformed entrepreneur's asset, $(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta))$, is optimal only when $\mu=\theta$; a measure-zero event. The informed entrepreneur is thus ex ante better off than the uninformed entrepreneur.

Part (ii) states that an increase in the informativeness of signal y, t, leads to a Pareto improvement in welfare. To explain this result, first note that the uninformed entrepreneur's ex ante payoff, $\Omega_{\mathrm{u}}^{\mathrm{F}} = \pi^{\mathrm{F}}(\theta)$, is independent of t. This is because he has no access to signal y, so that y's informativeness is irrelevant to his payoff. Second, when y becomes more informative, the informed entrepreneur's ex ante payoff becomes larger because a more informative signal allows the first-best allocation to be "fine-tuned" to the unknown parameter x through the posterior belief μ. To see the precise economic intuition behind this result, it is useful to examine t's effect on the distribution of μ. Recall that $\mu \sim \mathrm{N}(\theta, \beta/u)$ and β increases with t. An increase in t thus makes the variance of μ larger, leading to a more "diffuse" distribution of μ. Such a change in the distribution of μ benefits the informed entrepreneur ex ante because his payoff, $\pi^{\mathrm{F}}(\mu)$, is convex in μ. In fact, it can be shown that the marginal change of the informed entrepreneur's ex ante payoff induced by a change in t is given by

$$\frac{\partial\Omega_{\mathrm{i}}^{\mathrm{F}}}{\partial t} = \int_0^{\infty} T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)\,\mathrm{d}\mu > 0, \tag{19}$$

where[12]

$$T(\mu) \equiv \int_{-\infty}^{\mu} \left[\frac{\partial\Phi(m|\cdot)}{\partial t}\right] \mathrm{d}m = \frac{1}{2}\frac{1}{(t+u)^2}\,\varphi(\mu|\cdot). \tag{20}$$

[12] To derive the expressions in (19) and (20), I differentiate $\Omega_{\mathrm{i}}^{\mathrm{F}}$ given in (16) with respect to t and then integrate the resulting expression by parts twice by using the following results: (i) $\Phi_t(\mu|\cdot) = -\frac{1}{2}\big(u/t(t+u)\big)(\mu-\theta)\varphi(\mu|\cdot)$; (ii) $\varphi_\mu(\mu|\cdot) = -\big(u(t+u)/t\big)(\mu-\theta)\varphi(\mu|\cdot)$; and (iii) $\pi^{\mathrm{F}}(\mu)=0$ for all $\mu \leqslant 0$. A technical part of the derivation is to establish that

$$\lim_{\mu\to\infty}[\pi^{\mathrm{F}}(\mu)\Phi_t(\mu|\cdot)] = \lim_{\mu\to\infty}[\pi^{\mathrm{F}\prime}(\mu)T(\mu)] = 0.$$

The assumption that $\Omega_{\mathrm{i}}^{\mathrm{F}}$ is finite and the second-order derivatives of R and C are continuous plays an important role here. Details of the derivation procedure are available upon request.

To explain the above expressions, first note that $T(\mu)$ aggregates the change of μ's distribution function when t increases. Since $\varphi(\mu|\cdot)$ is a normal density function, it is clear from (20) that $T(\pm\infty) = 0$ and $T(\mu) > 0$ for all $\mu \in (-\infty, \infty)$. This property of $T(\mu)$ is intuitive since an increase in t shifts μ's probability density towards tails; i.e., an increase in t has a mean-preserving spread effect on μ's distribution in the sense of Rothschild and Stiglitz (1970). In general, $\partial\Omega_{\mathrm{i}}^{\mathrm{F}}/\partial t$ equals the aggregation of $[T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)]$ for all μ, but since $\pi^{\mathrm{F}\prime\prime}(\mu) = 0$ for $\mu \leqslant 0$ (recall that $\pi^{\mathrm{F}}(\mu) = 0$ for all $\mu \leqslant 0$), $\partial\Omega_{\mathrm{i}}^{\mathrm{F}}/\partial t$ actually equals the aggregation of $[T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)]$ over the range where $\pi^{\mathrm{F}\prime\prime}(\mu) > 0$, i.e., the integration over $\mu > 0$. This result will be useful for the comparative static analysis in Section 5.[13]

In summary, a higher-quality signal enhances the economy's allocative efficiency through a more informative decision on (e, k). Using Proposition 1 in conjunction with (18) and (19) yields that t's effect on the social surplus in the first-best case is

$$\frac{\partial\Omega^{\mathrm{F}}}{\partial t} = q\frac{\partial\Omega_{\mathrm{u}}^{\mathrm{F}}}{\partial t} + (1-q)\frac{\partial\Omega_{\mathrm{i}}^{\mathrm{F}}}{\partial t} = (1-q)\int_0^{\infty} T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)\,\mathrm{d}\mu > 0. \tag{21}$$

4. Analysis of the second-best case

In Section 3, I assumed that a social planner has perfect information about each entrepreneur's informedness and she can enforce all informed entrepreneurs to reveal their signal. I now turn to the original model—referred to as the second-best case—in which only the distribution of the entrepreneurs' informedness is known but each entrepreneur's informedness is unknown. This means that the informed entrepreneurs exercise discretion over disclosure of signal y in the second-best case. Also note that, given the one-to-one relation between y and $\mu = \beta y + (1-\beta)\theta$, "disclosure of y" is equivalent to "disclosure of μ". Thus, I use them interchangeably throughout the paper. The entrepreneurs' and investors' equilibrium decisions on (e, k) in the second-best case are referred to as the second-best allocation.

Suppose that an entrepreneur, who has provided e, seeks to sell his asset with a voluntary disclosure of μ. Since μ is disclosed, the equilibrium in this case must be the same as that in the first-best case: i.e., (i) if $\mu \leqslant 0$, the asset is not sold to the investors; and (ii) if $\mu > 0$, the equilibrium asset price given (e, μ) equals $P(e, \mu)$ stated in (8). Going backward, this entrepreneur's effort decision, e, must be sequentially rational and consistent with his subsequent disclosure decision. This implies that when an informed entrepreneur voluntarily discloses μ, the second-best allocation is the same as the first-best allocation because he chooses $e = e^{\mathrm{F}}(\mu)$, which maximizes $J(e, \mu)$ given in (10). And, as a result, the entrepreneur's payoff in the second-best case equals his payoff in the first-best case, i.e., $J(e^{\mathrm{F}}(\mu), \mu) = \pi^{\mathrm{F}}(\mu)$.

[13] Inspecting (4) actually reveals that a change in t affects μ's distribution in two ways; i.e., through the change in β and through the change in y's distribution (variance). Throughout the paper, I use μ's distribution function because doing so amounts to combining these two effects and simplifies the analysis. If one instead uses y's distribution function, t's overall effect on the informed entrepreneur's ex ante payoff can be decomposed into the above-mentioned two effects. See Appendix A after the proof of Proposition 1.

Now suppose that an entrepreneur seeks to sell his asset with no disclosure. While nondisclosure in the first-best case means that the entrepreneur is uninformed, nondisclosure in the second-best case might occur when the entrepreneur is informed but suppresses a signal. An important efficiency implication follows. Since the investors take account of the latter possibility, their decision on the asset purchase and/or capital investment in the second-best case differs from that in the first-best case. This in turn induces both the informed entrepreneurs (who suppress their signal) and the uninformed entrepreneurs to choose an effort different from their first-best effort. Taken together, the future performance of the asset given nondisclosure in the second-best case differs from that in the first-best case.

Formally, I derive the equilibrium in the case of nondisclosure as follows (see Proposition 2 for a summary). Let e be given and define ND to be the set of posterior beliefs that are not disclosed by informed entrepreneurs. For simplicity, I assume that if an informed entrepreneur is indifferent between disclosing and withholding private information, he withholds it. Let ζ be the investors' equilibrium posterior belief about x when there is no disclosure in the second-best case. Then, it must be true that

$$\zeta \equiv \mathrm{E}[x|\text{nondisclosure}] = \frac{q\theta + (1-q)\int_{\mu \in ND} \mu \, \mathrm{d}\Phi(\mu|\cdot)}{q + (1-q)Pr[\mu \in ND]}. \tag{22a}$$

Given (22a), it is easy to see that, for any e, ND cannot be an empty set in equilibrium. To show this, suppose that $ND = \varnothing$. Then, $\zeta = \theta > 0$ in (22a), which implies that the asset price given no disclosure in the second-best case equals $P(e, \theta) > 0$ given in (8). Now consider an informed entrepreneur with posterior belief μ. By suppressing μ, he sells the asset for $P(e, \theta)$ so that his payoff equals $P(e, \theta) - C(e)$. On the other hand, as shown in Section 3, his payoff equals $J(e, \mu)$ given in (10) if he reveals μ. The entrepreneur compares those two payoffs when he makes a disclosure decision. Since the effort cost, $C(e)$, is sunk, it is irrelevant to the disclosure decision. Observe that $\mu R(e, 0) \leqslant 0$ for all $\mu \leqslant 0$ and $P(e, \mu) = \mu R(e, k^0(e, \mu)) - k^0(e, \mu)$ increases with $\mu > 0$ and goes to zero as $\mu \to 0$. Hence, any informed entrepreneur with $\mu \leqslant \theta$ has an incentive to suppress μ. This contradicts the supposition that $ND = \varnothing$.

The above analysis in fact makes it clear that ND in (22a) must be $ND = (-\infty, \mu^*]$ for some μ^*, so that

$$\zeta = \frac{q\theta + (1-q)\int_{-\infty}^{\mu^*} \mu \, \mathrm{d}\Phi(\mu|\cdot)}{q + (1-q)\Phi(\mu^*|\cdot)}. \tag{22b}$$

Depending on μ^* and the other parameters, ζ can be either positive or negative. In the following analysis, I focus on the case where $\zeta > 0$ in equilibrium; i.e., the asset is traded to the investors given no disclosure.[14] I now establish a relation between ζ and μ^* and provide a characterization of μ^*. Recall from (8) that the asset price given e

[14] If $\zeta \leqslant 0$ in equilibrium, the analysis is trivial. Since the asset is not traded, the resulting equilibrium allocation equals $(e, k) = (0, 0)$. As shown in Lemma A.1 (see Appendix A), $\zeta > 0$ actually prevails in equilibrium if θ exceeds a positive constant.

and a posterior belief $\mu > 0$ equals $P(e, \mu)$. Since the investors' posterior belief about x is $\zeta > 0$ here, the asset price given (e, ζ) must equal $P(e, \zeta)$. This implies that if the informed entrepreneur with posterior belief μ suppresses μ, he earns $P(e, \zeta) - C(e)$. On the other hand, if this entrepreneur reveals μ, his payoff equals $J(e, \mu)$ given in (10). A comparison of $P(e, \zeta) - C(e)$ and $J(e, \mu)$ then shows that μ^* must satisfy $P(e, \zeta) = P(e, \mu^*)$, i.e., $\zeta = \mu^*$. Replacing ζ in the left-hand side of (22b) by μ^* and rearranging terms yield a characterization of μ^* such that

$$\mu^* = \theta - \frac{1-q}{q} \int_{-\infty}^{\mu^*} \Phi(\mu|\cdot)\, \mathrm{d}\mu. \tag{22c}$$

Lemma A.1 in Appendix A shows that μ^* is positive (so that it is consistent with $\mu^* = \zeta > 0$) when θ is greater than a positive constant. It also shows that μ^* cannot exceed θ, i.e., $\mu^* < \theta$.

In summary, any informed entrepreneur with an unfavorable posterior belief (in the sense of $\mu \leqslant \mu^*$) does not disclose it.[15] In the absence of disclosure, the investors' posterior belief about x, ζ, equals μ^*, and they acquire the asset for $P(e, \mu^*)$. As shown in (8), that price is the investors' maximized expected return from the asset given e and μ^*.

Thus far, the analysis has taken e as given. I now go backward to characterize the effort decisions of the informed entrepreneurs (who suppress μ) and the uninformed entrepreneurs. Here, an important question is whether the uninformed entrepreneurs can separate from the informed entrepreneurs by choosing a particular effort, which is observable to the investors. I show that they cannot do so; i.e., the effort decision cannot be used as a signaling device. The reason is that, in equilibrium, some informed entrepreneurs always have an incentive to choose the same effort as that of the uninformed entrepreneurs.

To explain this result, let e^* be the uninformed entrepreneurs' equilibrium effort and suppose that all informed entrepreneurs always choose an effort different from e^*. More precisely, let $e(\mu)$ be the effort chosen by an informed entrepreneur whose posterior belief is μ and suppose that $e(\mu) \neq e^*$ for all μ. The investors, after observing $e(\mu) \neq e^*$, infer that this entrepreneur is informed. If so, unraveling occurs at the asset trading stage (à la Grossman, 1981; Milgrom, 1981); in particular, there is no trading if $\mu \leqslant 0$, whereas the asset is traded to the investors at the price of $P(e(\mu), \mu)$ if $\mu > 0$, where $P(e, \mu)$ is given in (8).[16] As a result, the payoff of the informed entrepreneur with posterior belief μ equals $J(e(\mu), \mu)$ given in (10). The informed entrepreneur anticipates this payoff. Thus, if he chooses an effort $e(\mu) \neq e^*$, he must choose $e^{\mathrm{F}}(\mu)$ because $e^{\mathrm{F}}(\mu)$ maximizes $J(e(\mu), \mu)$. Consequently, this entrepreneur's payoff equals $J(e^{\mathrm{F}}(\mu), \mu) = \pi^{\mathrm{F}}(\mu)$, which is given in (14). Now consider the uninformed entrepreneurs. Since $ND = \varnothing$ at the asset trading stage due to unraveling, it follows that $\zeta = \theta$, implying that the uninformed entrepreneurs' asset is sold for

[15] Given (4), note that suppressing $\mu \leqslant \mu^*$ is equivalent to suppressing signal $y \leqslant y^*$ where y^* is implicitly defined by $\mu^* = \beta y^* + (1 - \beta)\theta$. Thus, one can equivalently say that the set of signals that are not disclosed is $(-\infty, y^*]$.

[16] This is a direct consequence of the fact that $P(e, \mu)$ is increasing in $\mu > 0$.

$P(e^*, \theta)$. This further implies that the uninformed entrepreneurs' equilibrium effort must be $e^* = e^{\mathrm{F}}(\theta)$, so that their payoff equals $\pi^{\mathrm{F}}(\theta) > 0$. However, this cannot be sustained in equilibrium. To see why, recall that $\pi^{\mathrm{F}}(\mu) \leqslant \pi^{\mathrm{F}}(\theta)$ for all $\mu \leqslant \theta$. All the informed entrepreneurs with posterior belief $\mu \leqslant \theta$ thus have an incentive to mimic the uninformed entrepreneurs (i.e., to choose $e^{\mathrm{F}}(\theta)$ and suppress μ), which contradicts the supposition that $e(\mu) \neq e^*$ for all μ.

Given that the uninformed entrepreneurs cannot separate from the informed entrepreneurs by using their effort decision, the remaining question is what effort the uninformed entrepreneurs will choose. The answer to this question follows from (8)–(10). Given $\zeta = \mu^* > 0$, note that if the uninformed entrepreneurs choose an effort e, their payoff equals

$$P(e, \mu^*) - C(e) = \mu^* R(e, k^0(e, \mu^*)) - k^0(e, \mu^*) - C(e) = J(e, \mu^*).$$

The uninformed entrepreneurs' optimal effort, e^*, must be the one that maximizes this payoff, which implies that $e^* = e^{\mathrm{F}}(\mu^*) > 0$ where the inequality is due to $\mu^* > 0$. Moreover, given $e^* = e^{\mathrm{F}}(\mu^*)$, (8) and the definition of $k^{\mathrm{F}}(\mu) \equiv k^0(e^{\mathrm{F}}(\mu), \mu)$ imply that the uninformed entrepreneurs' asset price is

$$P(e^*, \mu^*) = P(e^F(\mu^*), \mu^*) = \mu^* R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{\mathrm{F}}(\mu^*) > 0. \tag{23}$$

In other words, in the second-best case, the uninformed entrepreneurs sell their asset at the price that would prevail in the first-best case in which the investors have posterior belief $\mu^* < \theta$. Given the asset price stated in (23), it follows that the uninformed entrepreneurs' payoff in the second-best case equals

$$\begin{aligned} P(e^*, \mu^*) - C(e^*) &= P(e^{\mathrm{F}}(\mu^*), \mu^*) - C(e^{\mathrm{F}}(\mu^*)) \\ &= J(e^{\mathrm{F}}(\mu^*), \mu^*) = \pi^{\mathrm{F}}(\mu^*) > 0. \end{aligned} \tag{24}$$

Finally, it remains to check that all the informed entrepreneurs with posterior belief $\mu \leqslant \mu^*$ are better off by mimicking the uninformed entrepreneurs. By choosing $e^* = e^{\mathrm{F}}(\mu^*)$ and suppressing their posterior belief $\mu \leqslant \mu^*$, the informed entrepreneurs sell their asset at the price of $P(e^*, \mu^*)$ given in (23), in which case their payoff equals $\pi^{\mathrm{F}}(\mu^*)$ given in (24). On the other hand, as explained earlier, if they choose an effort different from e^*, their payoff equals $\pi^{\mathrm{F}}(\mu)$ given in (14). Since $\pi^{\mathrm{F}}(\mu) = 0$ for all $\mu \leqslant 0$, $\pi^{\mathrm{F}}(\mu) > 0$ for all $\mu > 0$, and $\pi^{\mathrm{F}\prime}(\mu) > 0$ for all $\mu > 0$, any informed entrepreneurs with posterior belief $\mu \leqslant \mu^*$ are better off by mimicking the uninformed entrepreneurs.

Proposition 2 below summarizes the equilibrium of the second-best case when there is no disclosure. Corollary 1 then compares the second-best allocation with the first-best allocation and provides comparative static results.

Proposition 2. *Both the uninformed entrepreneurs and the informed entrepreneurs with posterior belief* $\mu \leqslant \mu^*$ (*where* μ^* *is characterized by* (22c)) *provide effort* $e^* > 0$ *and sell their asset with no disclosure. The investors purchase the asset for* $P(e^*, \mu^*) > 0$ *characterized by* (23) *and invest* $k^* > 0$ *in the asset. Here,* $e^* = e^{\mathrm{F}}(\mu^*)$ *and* $k^* = k^{\mathrm{F}}(\mu^*)$ *are characterized by* (13).

Corollary 1.

(i) *For the asset initially owned by the uninformed entrepreneurs, the second-best allocation is less than the first-best allocation, i.e.,* $e^* < e^{\mathrm{F}}(\theta)$ *and* $k^* < k^{\mathrm{F}}(\theta)$.

(ii) *For the asset initially owned by the nondisclosing informed entrepreneurs, the second-best allocation is greater than the first-best allocation, i.e.,* $e^* \geqslant e^{\mathrm{F}}(\mu)$ *and* $k^* \geqslant k^{\mathrm{F}}(\mu)$ *for all* $\mu \leqslant \mu^*$.

(iii) *The uninformed (nondisclosing informed) entrepreneurs' equilibrium asset price is lower (higher) than the asset price in the first-best case, i.e.,* $P(e^*, \mu^*) < P(e^{\mathrm{F}}(\theta), \theta)$ *and* $P(e^*, \mu^*) \geqslant P(e^{\mathrm{F}}(\mu), \mu)$ *for all* $\mu \leqslant \mu^*$.

(iv) *The effort and investment in the case of nondisclosure increase with the fraction of the uninformed entrepreneurs and the prior mean of x, but they decrease with the signal informativeness, i.e.,* $\partial e^*/\partial q > 0$, $\partial k^*/\partial q > 0$, $\partial e^*/\partial \theta > 0$, $\partial k^*/\partial \theta > 0$, $\partial e^*/\partial t < 0$, *and* $\partial k^*/\partial t < 0$.

(v) *The asset price in the case of nondisclosure increases with the fraction of the uninformed entrepreneurs and the prior mean of x, but it decreases with the signal informativeness, i.e.,* $\partial P(e^*, \mu^*)/\partial q > 0$, $\partial P(e^*, \mu^*)/\partial \theta > 0$, *and* $\partial P(e^*, \mu^*)/\partial t < 0$.

Parts (i) and (ii) directly follow from the fact that $(e^*, k^*) = (e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*))$ where $\mu^* \in (0, \theta)$ and both $e^{\mathrm{F}}(\mu)$ and $k^{\mathrm{F}}(\mu)$ increase with $\mu > 0$. If the uninformed entrepreneurs were able to credibly reveal their lack of information, the investors' posterior belief about x would be θ. The uninformed entrepreneurs then would have chosen the first-best effort, $e^{\mathrm{F}}(\theta)$, and the investors would have made the first-best investment, $k^{\mathrm{F}}(\theta)$. However, in the presence of the informed entrepreneurs who suppress unfavorable signals and claim a lack of information, the investors' equilibrium posterior belief about x is $\zeta = \mu^*$, which is lower than θ. As such, the equilibrium effort and investment in the uninformed entrepreneurs' asset are less than the first-best allocation. In contrast, since the nondisclosing informed entrepreneurs' posterior belief is $\mu \leqslant \mu^*$, the equilibrium allocation for the nondisclosing informed entrepreneurs' asset exceeds the first-best allocation. Such a misallocation of resources is inevitable given the informed entrepreneurs' incentive to be pooled with the uninformed entrepreneurs.

To explain part (iii), recall that when the effort is optimally chosen in the first-best case (and thus the capital investment is also optimally chosen according to $k^{\mathrm{F}}(\mu) \equiv k^0(e^{\mathrm{F}}(\mu), \mu)$, the asset price equals $P(e^{\mathrm{F}}(\mu), \mu)$. Hence, an increase in μ makes the asset price higher in two ways. First, since $e^{\mathrm{F}\prime}(\mu) > 0$, an increase in μ results in a higher effort, which in turn induces the investors to pay more for the asset. Second, an increase in μ has a direct positive effect on the asset price, i.e., the investors pay more for the asset when they have a higher posterior belief about x. Part (iii) then directly follows from the fact that $P(e^*, \mu^*) = P(e^{\mathrm{F}}(\mu^*), \mu^*)$ where $\mu^* < \theta$ and the results on the equilibrium efforts stated in parts (i) and (ii).

Part (iv) states comparative static results on the equilibrium allocation, (e^*, k^*). To explain them, note that since $e^* = e^{\mathrm{F}}(\mu^*)$ and $k^* = k^{\mathrm{F}}(\mu^*)$, any parametric change affects (e^*, k^*) through the investors' equilibrium posterior belief about x given nondisclosure, i.e., μ^*. The proof shows that μ^* increases with q. The intuition is that when there is no disclosure but an entrepreneur is more likely to be uninformed (i.e., when q is high), the investors believe that nondisclosure is more likely due to the lack of information. This translates into an upward revision in the investors' posterior belief about x, i.e., μ^* increases. Since both $e^{\mathrm{F}}(\mu^*)$ and $k^{\mathrm{F}}(\mu^*)$ increase with μ^*, a higher q implies an increase in the equilibrium effort and investment. An increase of the prior mean of x, θ, has a similar positive effect on μ^*. Therefore, its qualitative effect on (e^*, k^*) is the same as that of q. In contrast to the effect of q and θ, an increase of t has a negative effect on μ^*. That is, when the signal becomes more informative, the investors revise their posterior belief about x downward. Such a downward revision is intuitive, given that the investors are aware of the possibility that a highly informative (unfavorable) signal might be withheld. Hence, an increase of t reduces both e^* and k^*.

Part (v) provides empirical implications for the asset price in the case of nondisclosure. From (23), i.e., $P(e^*, \mu^*) = P(e^{\mathrm{F}}(\mu^*), \mu^*)$, note that

$$\frac{\partial P(e^*, \mu^*)}{\partial \alpha} = \frac{\mathrm{d}P(e^{\mathrm{F}}(\mu^*), \mu^*)}{\mathrm{d}\mu^*}\frac{\partial \mu^*}{\partial \alpha} \quad \text{for any parameter } \alpha = q, \theta, t.$$

Since $P(e^F(\mu), \mu)$ increases with μ (see the discussion of part (iii)), $\partial P(\cdot)/\partial\alpha$ has the same sign as $\partial\mu/\partial\alpha$ for any α. The results then follow from the earlier results on each parameter's impact on μ^*, i.e., part (iv). First, recall that when an entrepreneur is more likely to be informed (i.e., when q is low), μ^* decreases since nondisclosure is more likely due to the suppression of an unfavorable signal. Hence, nondisclosure is more severely punished by the market in terms of a lower asset price. Next, even with no disclosure, the investors pay more for the asset if the prior belief about the unknown parameter x is higher (i.e., when θ is high). In contrast, the investors penalize an entrepreneur's failure to disclose more severely when they believe the signal that might be suppressed is highly informative (i.e., when t is high). As discussed earlier, this is intuitive since informed entrepreneurs withhold unfavorable signals.

5. Welfare analysis and signal informativeness

The analysis in Section 4 has shown that if informed entrepreneurs receive a signal that leads to an unfavorable posterior belief about x (in the sense of $\mu \leqslant \mu^*$), they always mimic uninformed entrepreneurs, taking advantage of the investors' uncertainty about individual entrepreneurs' informedness. As shown in Corollary 1, this precludes an efficient allocation of resources. In this section, I formally investigate the private and social welfare implications of the second-best allocation.[17] I then examine how a change in the informativeness of signal y, as measured by $t \equiv 1/\sigma_\varepsilon^2$, alters the entrepreneurs' ex ante payoffs and social surplus. Finally, the

[17] Since the investors always price-protect themselves in the competitive market and thereby earn a normal (zero) net expected return in equilibrium, the analysis in this section focuses on the entrepreneurs' ex ante payoffs and social surplus.

model is extended to a setting in which t is endogenously chosen by the informed entrepreneurs at some cost. After deriving a privately optimal level of signal informativeness, I compare it with the socially optimal level of signal informativeness.

Let $\Omega_{\rm u}^*$ and $\Omega_{\rm i}^*$ be the uninformed and informed entrepreneurs' ex ante payoffs, respectively, in the second-best case. From the analysis in Section 4, it is clear that

$$\Omega_{\rm u}^* = \pi^{\rm F}(\mu^*) \tag{25}$$

and

$$\begin{aligned}\Omega_{\rm i}^* &= \int_{-\infty}^{\mu^*} \pi^{\rm F}(\mu^*)\,{\rm d}\Phi(\mu|\cdot) + \int_{\mu^*}^{\infty} \pi^{\rm F}(\mu)\,{\rm d}\Phi(\mu|\cdot) \\ &= \Phi(\mu^*|\cdot)\pi^{\rm F}(\mu^*) + \int_{\mu^*}^{\infty} \pi^{\rm F}(\mu)\,{\rm d}\Phi(\mu|\cdot).\end{aligned} \tag{26}$$

As in the first-best case, let λ^* be the difference between the informed and uninformed entrepreneurs' ex ante payoffs, i.e., let λ^* be each informed entrepreneur's private value of signal y in the second-best case:

$$\lambda^* \equiv \Omega_{\rm i}^* - \Omega_{\rm u}^*. \tag{27}$$

The second-best social surplus is then given by

$$\Omega^* \equiv q\Omega_{\rm u}^* + (1-q)\Omega_{\rm i}^* = \pi^{\rm F}(\mu^*) + (1-q)\lambda^*. \tag{28}$$

Finally, define $L^* \equiv \Omega^{\rm F} - \Omega^*$ to be the efficiency loss relative to the first-best case.

Proposition 3. *Relative to the first-best case,*

(i) *the informed entrepreneurs are better off while the uninformed entrepreneurs are worse off, i.e.,* $\Omega_{\rm i}^* > \Omega_{\rm i}^{\rm F}$ *and* $\Omega_{\rm u}^* < \Omega_{\rm u}^{\rm F}$;
(ii) *the disparity between the informed and uninformed entrepreneurs' ex ante payoffs is greater, i.e.,* $\lambda^* > \lambda^{\rm F}$; *and*
(iii) *the social surplus is lower so that there is an efficiency loss, i.e.,* $L^* = \Omega^{\rm F} - \Omega^* > 0$.

The results are intuitive. For part (i), recall that the informed entrepreneurs disclose their signal if, and only if, the signal leads to a sufficiently high posterior belief about x, i.e., $\mu > \mu^*$. In that case, their payoff equals the first-best payoff. On the other hand, if $\mu \leqslant \mu^*$, they suppress the signal and earn more than their payoff in the first-best case. The informed entrepreneurs' rent-seeking behavior imposes a negative externality on the uninformed entrepreneurs in that it induces a sub-optimal allocation as shown in Corollary 1 (i). This makes the uninformed entrepreneurs worse off than they are in the first-best case.

Part (ii) follows from part (i). In particular, having private information is more attractive in the second-best case than in the first-best case because the informed entrepreneurs have an option to conceal their informedness in the second-best case while they have no such discretion in the first-best case.

Part (iii) is driven by the misallocation of resources in the case of nondisclosure. It is worth emphasizing that this result holds independently of the distribution of the entrepreneurs' informedness, i.e., q. To elaborate on this, note that the informed entrepreneurs gain at the expense of the uninformed entrepreneurs' welfare. Those gains and losses are wealth redistribution. What matters from an efficiency standpoint is the extent to which the informed entrepreneurs' signal is incorporated into (e, k) through the posterior belief about x, i.e., via μ. In the first-best case, μ is always reflected in the effort and investment decisions. In the second-best case, μ is used for those decisions only when $\mu > \mu^*$. Thus, the economy suffers from an inefficient resource allocation whenever $\mu \leqslant \mu^*$. Such efficiency losses are unavoidable since the informed entrepreneurs' incentives to seek private gains persist in equilibrium.

I next examine the welfare consequences of a change in the signal informativeness, t (see Proposition 4 for a summary). First, when the signal becomes more informative, the uninformed entrepreneurs become worse off. In particular, using (25), note that

$$\frac{\partial \Omega_{\mathrm{u}}^*}{\partial t} = \frac{\partial \pi^{\mathrm{F}}(\mu^*)}{\partial t} = \pi^{\mathrm{F}\prime}(\mu^*)\frac{\partial \mu^*}{\partial t} < 0, \tag{29}$$

where the last inequality follows because $\pi^{\mathrm{F}\prime}(\mu) > 0$ for all $\mu > 0, \mu^* > 0$, and $\partial\mu^*/\partial t < 0$ (see the discussion below Corollary 1). The intuition is clear. Ceteris paribus, a higher posterior belief about x would increase the uninformed entrepreneurs' ex ante payoff at the rate of $\pi^{\mathrm{F}\prime}(\mu^*)$. However, an increase in t induces the investors to revise downward their posterior belief about x in the case of nondisclosure. As a result, the uninformed entrepreneurs' ex ante welfare is reduced.

Second, using (26) and integrating by parts, it can be shown that

$$\begin{aligned}\frac{\partial \Omega_{\mathrm{i}}^*}{\partial t} &= \frac{\partial}{\partial t}\left[\Phi(\mu^*|\cdot)\pi^{\mathrm{F}}(\mu^*) + \int_{\mu^*}^{\infty} \pi^{\mathrm{F}}(\mu)\,\mathrm{d}\Phi(\mu|\cdot)\right] \\ &= \Phi(\mu^*|\cdot)\pi^{\mathrm{F}\prime}(\mu^*)\frac{\partial \mu^*}{\partial t} - \int_{\mu^*}^{\infty} \pi^{\mathrm{F}\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu > 0. \end{aligned} \tag{30a}$$

The first term denotes t's negative effect on the informed entrepreneur's ex ante payoff when he withholds $\mu \leqslant \mu^*$ (recall that $\pi^{\mathrm{F}\prime}(\mu^*) > 0$ and $\partial\mu^*/\partial t < 0$). The second term represents t's effect on Ω_{i}^* when he discloses $\mu > \mu^*$. Appendix A (see the proof of Proposition 4) shows that the second term is not only positive but also dominant over the first term, so that the overall effect of t on Ω_{i}^* is positive. By using (22c), Appendix A further shows that

$$\frac{\partial \Omega_{\mathrm{i}}^*}{\partial t} = \int_{\mu^*}^{\infty} T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)\,\mathrm{d}\mu - \left(\frac{q}{1-q}\right)\frac{\partial \Omega_{\mathrm{u}}^*}{\partial t} > 0, \tag{30b}$$

where $T(\mu)$ is defined in (20). This expression is comparable to (19) and makes the rent transfer between the entrepreneurs clear. The first term in (30b) is positive (since $T(\mu) > 0$ and $\pi^{\mathrm{F}\prime\prime}(\mu) > 0$ for all $\mu \geqslant \mu^* > 0$), and it represents an ex ante gain from a more diffuse distribution of μ in conjunction with the convexity of $\pi^{\mathrm{F}}(\mu)$; see the discussion of Proposition 1 (ii). To explain the second term in (30b), recall that an increase in t reduces an uninformed entrepreneur's welfare by $\partial\Omega_{\rm u}^*/\partial t$ (see (29)). Thus, the aggregate loss of the uninformed entrepreneurs amounts to $q(\partial\Omega_{\rm u}^*/\partial t)$. Such a loss translates into the informed entrepreneurs' ex ante gain. Dividing the aggregate loss by the fraction of the informed entrepreneurs (i.e., $1-q$), the second term represents each informed entrepreneur's gain from discretionary disclosure.

From the above analysis, it is evident that the disparity between the informed and uninformed entrepreneurs' ex ante payoffs (i.e., $\lambda^* \equiv \Omega_{\rm i}^* - \Omega_{\rm u}^*$) widens as t increases. While the same phenomenon occurs in the first-best case (i.e., $\lambda^{\rm F} \equiv \Omega_{\rm i}^{\rm F} - \Omega_{\rm u}^{\rm F}$ increases with t, which can be directly inferable from Proposition 1), it can be shown that the increasing rate of the disparity is higher in the second-best case, i.e., $\partial\lambda^*/\partial t > \partial\lambda^{\rm F}/\partial t$. In other words, as the signal becomes more informative, its private value in the second-best case increases faster relative to its private (and social) value in the first-best case. To explain the intuition, note that the uninformed entrepreneurs' ex ante payoff in the first-best case is not affected by a change in t (see Proposition 1). However, as shown in (29), they become worse off when the signal becomes more informative in the second-best case, because the negative externality imposed by the informed entrepreneurs' discretionary disclosure becomes more severe. The uninformed entrepreneurs' loss translates into the informed entrepreneurs' gain, so that having access to a more informative signal is more attractive from the informed entrepreneurs' perspective.

Finally, I turn to the impact of t on the second-best social surplus, i.e., $\Omega^* \equiv q\Omega_{\rm u}^* + (1-q)\Omega_{\rm i}^*$. First, an increase of t enhances social surplus, as it does in the first-best case (see (21)). More precisely, from (28), (29), and (30b),

$$\frac{\partial\Omega^*}{\partial t} = q\frac{\partial\Omega_{\rm u}^*}{\partial t} + (1-q)\frac{\partial\Omega_{\rm i}^*}{\partial t} = (1-q)\int_{\mu^*}^{\infty} T(\mu)\pi^{\rm F}{}''(\mu)\,{\rm d}\mu > 0. \tag{31}$$

Eq. (31) shows that t's impact on each informed entrepreneur's contribution to the second-best social surplus equals $\int_{\mu^*}^{\infty} T(\mu)\pi^{\rm F}{}''(\mu)\,{\rm d}\mu$, so that the aggregate contribution equals $(1-q)\int_{\mu^*}^{\infty} T(\mu)\pi^{\rm F}{}''(\mu)\,{\rm d}\mu$. The reason for (31) is clear from the fact that there is a rent transfer between the informed and uninformed entrepreneurs (so that the second term in (30*b*) vanishes in the calculation of $\partial\Omega^*/\partial t$). As a result, t's net effect on the second-best social surplus equals the ex ante gain from a more diffuse distribution of the posterior belief, μ.

Nonetheless, observe that the extent to which an increase in t improves social surplus in the second-best case is less than that in the first-best case, i.e., $\partial\Omega^*/\partial t < \partial\Omega^{\rm F}/\partial t$. By definition, this is equivalent to saying that the efficiency loss, $L^* \equiv \Omega^{\rm F} - \Omega^*$, increases with t. The economics behind this result is clear if one compares (31) with its counterpart in the first-best case, which is (21). Since t's impact on social surplus depends on the range in which $\pi^{\rm F}(\mu)$ exhibits strict convexity (i.e., $\pi^{\rm F}{}'' > 0$), and given the fact that $\mu^* > 0$, it must be true that

$$\int_{\mu^*}^{\infty} T(\mu)\pi^{\rm F}{}''(\mu)\,{\rm d}\mu < \int_{0}^{\infty} T(\mu)\pi^{\rm F}{}''(\mu)\,{\rm d}\mu.$$

In essence, the above inequality follows because the second-best allocation incorporates signal y's information about the unknown parameter x in a limited way relative to the first-best case (i.e., only when $\mu > \mu^*$). The next proposition summarizes the results of the preceding analysis.

Proposition 4. *As the signal becomes more informative,*

(i) *the informed entrepreneurs become better off while the uninformed entrepreneurs become worse off, i.e.,* $\partial\Omega_{\mathrm{i}}^*/\partial t > 0 > \partial\Omega_{\mathrm{u}}^*/\partial t$;
(ii) *the disparity between the informed and uninformed entrepreneurs' ex ante payoffs increases at a rate greater than that in the first-best case, i.e.,* $\partial\lambda^*/\partial t > \partial\lambda^{\mathrm{F}}/\partial t$; *and*
(iii) *the second-best social surplus increases, but its increasing rate is less than that in the first-best case, so that the efficiency loss increases with t, i.e.,* $\partial\Omega^*/\partial t > 0$ *and* $\partial L^*/\partial t = \partial\Omega^{\mathrm{F}}/\partial t - \partial\Omega^*/\partial t > 0$.

Proposition 4 illustrates how the private and social values of information diverge when privately informed parties exercise discretion over disclosure of a signal that always improves the economy's productive efficiency. The informed entrepreneurs' rent-seeking incentive is responsible for such a divergence. This divergence becomes larger when the signal becomes more informative, since a more informative signal intensifies the informed entrepreneurs' incentive to seek private gains.

Given Proposition 4, it is straightforward to extend the model to a setting in which signal y's quality, as measured by $t \equiv 1/\sigma_\varepsilon^2$, is endogenous. Specifically, assume that the informed entrepreneurs choose t by spending $Y(t)$, where Y is an increasing and convex function satisfying $Y(0) = Y'(0) = 0$ and $Y'(t) \to \infty$ as $t \to \infty$. A natural interpretation of $Y(t)$ is that the informed entrepreneurs incur cost $Y(t)$ to produce a signal of quality t and a higher-quality signal is costlier to produce. An implicit assumption here is that it is prohibitively costly for the uninformed entrepreneurs to produce any informative signal. Such a difference in the signal production costs might arise because individuals have different abilities to gather and process raw information, thereby generating a meaningful signal that is value-relevant to their business (like signal y in this model).[18] Note that the informed entrepreneurs always have an option to incur no cost for their signal quality. In that case, discretionary disclosure becomes a non-issue because the informed entrepreneurs' signal is completely uninformative (i.e., $t = 0$) and, as a result, the situation is equivalent to $q = 1$ (i.e., all entrepreneurs are uninformed). However, as shown below, such a degenerating case does not occur in equilibrium because the informed entrepreneurs choose t greater than zero. In fact, they invest too much in the signal quality.

Proposition 5. *The informed entrepreneurs choose* $t^* > 0$, *and it is greater than the socially optimal level of signal informativeness.*

[18] In other words, one can assume that all entrepreneurs in the model have equal access to some raw information, but only $(1 - q)$ fraction of the entrepreneurs—who are referred to as the informed entrepreneurs—have the ability to map it into a summary signal y.

The proof goes as follows. Consider an informed entrepreneur's signal quality choice problem:

$$\max_{t \geqslant 0} \Omega_{\mathrm{i}}^{*} - Y(t). \tag{32}$$

Proposition 4 (i) shows that the informed entrepreneur's marginal benefit of t is always positive, i.e., $\partial\Omega_{\mathrm{i}}^{*}/\partial t > 0$ for all t. Since $Y'(0) = 0$ and $Y'(t) \to \infty$ as $t \to \infty$, the optimization problem in (32) must have an interior solution, $t^{*} > 0$. In particular, due to (30b), the first-order condition characterizing t^{*} must be

$$\int_{\mu^{*}}^{\infty} T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)\,\mathrm{d}\mu - \left(\frac{q}{1-q}\right)\frac{\partial\Omega_{\mathrm{u}}^{*}}{\partial t} = Y'(t). \tag{33}$$

As explained earlier, the first term in left-hand side of (33) is the ex ante gain from a more diffuse distribution of μ in conjunction with the convexity of $\pi^{\mathrm{F}}(\mu)$. The second term denotes t's effect on each informed entrepreneur's private gain that comes at the expense of the uninformed entrepreneurs' welfare. Next, to derive the socially optimal level of signal informativeness, consider a social planner's problem:

$$\max_{t \geqslant 0} q\Omega_{\mathrm{u}}^{*} + (1-q)[\Omega_{\mathrm{i}}^{*} - Y(t)]. \tag{34}$$

Observe that (34) is different from (32) because the social planner must take into account the fact that the uninformed entrepreneurs' ex ante payoff is also influenced by the signal informativeness. Such a consideration is absent in (32). Let t^{s} solve (34). Then, due to (31), $t^{\mathrm{s}} > 0$ and it must be characterized by the first-order condition of (34), i.e.,

$$\int_{\mu^{*}}^{\infty} T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)\,\mathrm{d}\mu = Y'(t). \tag{35}$$

Comparing (35) with (33) directly reveals that the informed entrepreneur's marginal benefit of t exceeds the social marginal benefit of t. As a result, t^{*} must be greater than t^{s}. This completes the proof.

In essence, the informed entrepreneurs' private incentive for the signal quality is different from that of the social planner because of their private gain. The source of that private gain is the informed entrepreneurs' discretion over disclosure, which in turn comes from the investors' uncertainty about the entrepreneurs' possession of private information.

6. Summary

Prior studies in the voluntary disclosure literature have paid relatively little attention to efficiency issues (see Verrecchia, 2001). This paper addresses the efficiency implications of discretionary disclosure. Presenting a model of a competitive asset market characterized by an adverse selection problem, the paper primarily focuses on the link between the market participants' productive actions and discretionary disclosure. The paper also examines the private and social welfare implications of signal informativeness.

The informed entrepreneurs' discretion over disclosure allows them to earn more than they would get without such discretion. However, the informed entrepreneurs' private gain is detrimental not only to the uninformed entrepreneurs but also to the society as a whole. In particular, discretionary disclosure undermines the economy's productive efficiency because it leads to a sub-optimal allocation of resources, i.e., an allocation different from the first-best allocation. The resulting efficiency losses are inevitable because the informed entrepreneurs always have an incentive to seek private gains at the expense of the uninformed entrepreneurs. A more informative signal intensifies the informed entrepreneurs' rent-seeking incentive, which further increases the disparity between the informed and uninformed entrepreneurs' ex ante payoffs. Indeed, it is shown that the informed entrepreneurs over-invest in the signal quality relative to the quality that maximizes social welfare. Since the private benefit from a more informative signal exceeds the social benefit of it, such an over-investment in the signal quality is also unavoidable. The paper provides empirical implications of various parameters for the equilibrium allocation and asset price in the case of nondisclosure.

I conclude this study with an alternative interpretation of the model. Instead of having two groups of entrepreneurs that differ in terms of their informedness, suppose that all entrepreneurs are ex ante identical but they receive a private signal with a positive probability, i.e., $(1-q)$ in the present model. Then, the social surplus in the present model equals a representative entrepreneur's ex ante payoff. This implies that if an entrepreneur can pre-commit to full disclosure of his private signal, he is ex ante better off because his payoff in that case is equal to the first-best payoff. Nevertheless, no entrepreneur can make such a pre-commitment credibly because of the ex post-incentive problem. That is, once he is informed, he has an unavoidable incentive to exercise discretion over disclosure by revealing the private signal selectively.

Appendix A

Proof of $J(e^{\mathrm{F}}(\mu),\mu)=\pi^{\mathrm{F}}(\mu)$ for all μ where $J(e,\mu)$ and $\pi^{\mathrm{F}}(\mu)$ are respectively given in (10) and (14)

Consider any $\mu\leqslant 0$. Since $(e^{\mathrm{F}}(\mu),k^{\mathrm{F}}(\mu))=(0,0)$ for all $\mu\leqslant 0$,

$$J(e^{\mathrm{F}}(\mu),\mu)=\mu R(0,0)-C(0)=0$$

and

$$\pi^{\mathrm{F}}(\mu)=\mu R(e^{\mathrm{F}}(\mu),k^{\mathrm{F}}(\mu))-k^{\mathrm{F}}(\mu)-C(e^{\mathrm{F}}(\mu))=\mu R(0,0)-0-0=0.$$

Next, consider any $\mu>0$. Note that

$$\begin{aligned} J(e^{\mathrm{F}}(\mu),\mu) &= P(e^{\mathrm{F}}(\mu),\mu)-C(e^{\mathrm{F}}(\mu)) \\ &= \mu R(e^{\mathrm{F}}(\mu),k^{0}(e^{\mathrm{F}}(\mu),\mu))-k^{0}(e^{\mathrm{F}}(\mu),\mu)-C(e^{\mathrm{F}}(\mu)) \end{aligned}$$

$$\begin{aligned}&= \mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu))\\ &= \pi^{\mathrm{F}}(\mu),\end{aligned}$$

where the first equality is due to (10); the second equality follows from (8); the third equality follows from the definition of $k^{\mathrm{F}}(\mu) \equiv k^0(e^{\mathrm{F}}(\mu), \mu)$; and the last equality is due to (14). In fact, $\pi^{\mathrm{F}}(\mu)$ is the upper envelope of $J(e, \mu)$ so that, for any given μ, $\pi^{\mathrm{F}}(\mu) \geqslant J(e, \mu)$ for all e and $\pi^{\mathrm{F}}(\mu) = J(e^{\mathrm{F}}(\mu), \mu)$. The reason is that $J(e, \mu)$ is the payoff for any effort e, whereas $\pi^{\mathrm{F}}(\mu)$ is the payoff for the optimal effort $e^{\mathrm{F}}(\mu)$. ■

Proof of Proposition 1. (i) From (16) and (17),

$$\begin{aligned}\lambda^{\mathrm{F}} \equiv \Omega_{\mathrm{i}}^{\mathrm{F}} - \Omega_{\mathrm{u}}^{\mathrm{F}} &= \int_{-\infty}^{\infty} [\pi^{\mathrm{F}}(\mu) - \pi^{\mathrm{F}}(\theta)]\, \mathrm{d}\Phi(\mu|\cdot)\\ &= \int_{-\infty}^{\infty} \{[\mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu))]\\ &\quad - [\mu R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta))]\}\, \mathrm{d}\Phi(\mu|\cdot)\end{aligned}$$

where I use (14) for $\pi^{\mathrm{F}}(\mu)$ and (15) for $\pi^{\mathrm{F}}(\theta)$ along with the fact that $\mathrm{E}[\mu] = \theta$, i.e.,

$$\begin{aligned}\pi^{\mathrm{F}}(\theta) &= \theta R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta))\\ &= \int_{-\infty}^{\infty} [\mu R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta))]\, \mathrm{d}\Phi(\mu|\cdot).\end{aligned}$$

Then, $\lambda^{\mathrm{F}} > 0$ follows from (13). That is, for any given μ, $\mu R(e, k) - k - C(e)$ attains its unique maximum at $(e, k) = (e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu))$, which implies that

$$\mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu)) > \mu R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta)),\quad \forall \mu \neq \theta.$$

(ii) First, since $\Omega_{\mathrm{u}}^{\mathrm{F}} = \pi^{\mathrm{F}}(\theta)$ is independent of t, a change of t has no impact on $\Omega_{\mathrm{u}}^{\mathrm{F}}$, i.e., $\partial \Omega_{\mathrm{u}}^{\mathrm{F}} / \partial t = 0$. Next, recall that $\Omega_{\mathrm{i}}^{\mathrm{F}}$ is the expected value of $\pi^{\mathrm{F}}(\mu)$ with $\mu \sim \mathrm{N}(\theta, \beta/u)$ where $\beta = t/(t+u)$. Since $\beta_t \equiv \partial\beta/\partial t > 0$, $Var[\mu] = \beta/u$ increases with t. An increase of t thus has a mean-preserving spread effect on the distribution of μ in the sense of Rothschild and Stiglitz (1970). This result combined with the fact that $\pi^{\mathrm{F}}(\mu)$ is strictly convex in $\mu > 0$ establishes that $\partial \Omega_{\mathrm{i}}^{\mathrm{F}} / \partial t > 0$ (see Theorem 4 in Hanoch and Levy, 1969). Finally, since no one is made strictly worse off by an increase in t whereas all informed entrepreneurs are made strictly better off, an increase in t leads to a Pareto improvement. ■

Decomposition of (19)

As mentioned in footnote 13, I decompose the right-hand side of (19) into two terms by using signal y's distribution function. I also explain the sign of each term.

Let $\Psi(y|\cdot)$ be the distribution function of $y \sim \mathrm{N}(\theta, 1/u + 1/t)$. Then, similar to (16),

$$\Omega_{\mathrm{i}}^{\mathrm{F}} = \int_{-\infty}^{\infty} \pi^{\mathrm{F}}(\mu)\, \mathrm{d}\Psi(y|\cdot),$$

where $\mu = \beta y + (1-\beta)\theta$. Differentiating the above expression with respect to t and rearranging terms, one can verify that

$$\frac{\partial \Omega_{\mathrm{i}}^{\mathrm{F}}}{\partial t} = \beta_t\, Cov[y, \pi^{\mathrm{F}\prime}(\mu)] + \beta^2 \int_{y^0}^{\infty} S(y) \pi^{\mathrm{F}\prime\prime}(\mu)\, \mathrm{d}y, \tag{19$'$}$$

where $S(y) \equiv \int_{-\infty}^{y} \Psi_t(z|\cdot)\, \mathrm{d}z$ and y^0 is defined by $\beta y^0 + (1-\beta)\theta = 0$, that is, y^0 corresponds to $\mu = 0$. Since $y \sim \mathrm{N}(\theta, 1/u + 1/t)$, an increase in t reduces y's variance and, therefore, it has an *inverse* mean-preserving spread effect on y's distribution. Since μ is linear in y, $\pi^{\mathrm{F}}(\mu)$ is convex in y. Hence, the second term in (19′) is negative. Next, it can be shown that the first term, $\beta_t\, Cov[y, \pi^{\mathrm{F}\prime}(\mu)]$, is positive since $\beta_t > 0$ and $\pi^{\mathrm{F}\prime}(\mu)$ increases with $\mu > 0$ and thus with $y > y^0$. Finally, using $\Phi(\mu|\cdot) = \Psi(y|\cdot)$ and $\mu = \beta y + (1-\beta)\theta$, it can be also shown that the sum of the two terms in (19′) equals (19), which is positive. Thus, the first term is larger than the absolute value of the second term. ■

Lemma A.1. *Let* μ^* *be the constant characterized by* (22c). *Then, there exists a positive constant* $\underline{\theta}$ *such that* $\mu^* > 0$ *if, and only if,* $\theta > \underline{\theta}$.

Proof of Lemma A.1. Fix $\theta > 0$ and define a function

$$A(m) \equiv m - \theta + \frac{1-q}{q} \int_{-\infty}^{m} \Phi(\mu|\cdot)\, \mathrm{d}\mu.$$

Then, μ^* characterized by (22c) must be a solution to $A(m) = 0$. Observe that

$$A'(m) = 1 + \frac{1-q}{q} \Phi(m|\cdot) > 0, \quad \forall m,$$

$$A(0) = -\theta + \frac{1-q}{q} \int_{-\infty}^{0} \Phi(\mu|\cdot)\, \mathrm{d}\mu$$

and

$$A(\theta) = \frac{1-q}{q} \int_{-\infty}^{\theta} \Phi(\mu|\cdot)\, \mathrm{d}\mu > 0.$$

The intermediate value theorem then implies that there exists a unique constant $\mu^* \in (0, \theta)$ satisfying $A(\mu^*) = 0$ if, and only if, $A(0) < 0$.

Now let θ vary. In particular, note that

$$\frac{\partial A(0)}{\partial \theta} = -1 + \frac{1-q}{q} \int_{-\infty}^{0} \Phi_\theta(\mu|\cdot)\, \mathrm{d}\mu < 0,$$

where $\Phi_\theta(\mu|\cdot) \equiv \partial \Phi(\mu|\cdot)/\partial \theta$. The inequality follows because, given the normality of μ, an increase in θ has a first-order stochastic dominance effect on μ's distribution, i.e., $\Phi_\theta(\mu|\cdot)<0$ for all $\mu\in(-\infty,\infty)$. It remains to show that there exists a positive constant, denoted by $\underline{\theta}$, such that $A(0)<0$ for all $\theta>\underline{\theta}$ and $A(0)>0$ for all $\theta<\underline{\theta}$. Clearly, $\underline{\theta}$ is the value of θ satisfying $A(0)=0$, i.e.,

$$\theta=\frac{1-q}{q}\int_{-\infty}^{0}\Phi(\mu|\theta,\beta/u)\,\mathrm{d}\mu,$$

where I make it explicit that $\Phi(\mu|\cdot)\equiv\Phi(\mu|\theta,\beta/u)$. Denote the right-hand side of the above equation by $B(\theta)$ and note that $\underline{\theta}$ must be a positive fixed point of B, i.e., it must solve $\theta=B(\theta)$. Since $B(0)>0$, $B'(\theta)<0$ for all θ (recall that $\Phi_\theta(\mu|\cdot)<0$), and $B(\theta)\to 0$ as $\theta\to\infty$, it follows that there exists a unique positive fixed point of B, which is $\underline{\theta}$. ■

Proof of Corollary 1. (i) & (ii) See the main text.

(iii) Given $e^*=e^{\mathrm{F}}(\mu^*)$ and the results in parts (i) and (ii), it suffices to show that $P(e^*,\mu^*)$ increases with μ^*. Note that

$$\begin{aligned}\frac{\mathrm{d}P(e^*,\mu^*)}{\mathrm{d}\mu^*}&=\frac{\mathrm{d}P(e^{\mathrm{F}}(\mu^*),\mu^*)}{\mathrm{d}\mu^*}=\frac{\mathrm{d}}{\mathrm{d}\mu^*}[\mu^* R(e^{\mathrm{F}}(\mu^*),k^0(e^{\mathrm{F}}(\mu^*),\mu^*))-k^0(e^{\mathrm{F}}(\mu^*),\mu^*)]\\&=R(e^{\mathrm{F}}(\mu^*),k^{\mathrm{F}}(\mu^*))+\mu^* R_e(e^{\mathrm{F}}(\mu^*),k^{\mathrm{F}}(\mu^*))e^{\mathrm{F}\prime}(\mu^*).\end{aligned}$$

The second equality follows from using $P(e,\mu)$ in (8) evaluated at $(e,\mu)=(e^{\mathrm{F}}(\mu^*),\mu^*)$, and I use (7) and the definition of $k^{\mathrm{F}}(\mu)\equiv k^0(e^{\mathrm{F}}(\mu),\mu)$ to simplify the last expression. Since $e^{\mathrm{F}}(\mu^*)>0$ and $k^{\mathrm{F}}(\mu^*)>0$, the first term in the last expression is positive. The second term is also positive because $\mu^*>0$, $R_e(e,k)>0$ for all $e>0$ and $k>0$, and $e^{\mathrm{F}\prime}(\mu)>0$ for all $\mu>0$. Therefore, $\mathrm{d}P(e^*,\mu^*)/\mathrm{d}\mu^*>0$.

(iv) Any parameter $\alpha(=q,\theta,t)$ affects $(e^*,k^*)=(e^{\mathrm{F}}(\mu^*),k^{\mathrm{F}}(\mu^*))$ through μ^*, and both $e^{\mathrm{F}}(\mu)$ and $k^{\mathrm{F}}(\mu)$ increase with $\mu>0$. Hence,

$$\frac{\partial e^*}{\partial\alpha}=(\text{in sign})\frac{\partial k^*}{\partial\alpha}=(\text{in sign})\frac{\partial\mu^*}{\partial\alpha},\quad\forall\alpha=q,\theta,t.$$

Consider q. Viewing μ^* characterized by (22c) as a function of q, differentiate both sides of (22c) with respect to q and rearrange terms. Then,

$$\frac{\partial\mu^*}{\partial q}[q+(1-q)\Phi(\mu^*|\cdot)]=(\theta-\mu^*)+\int_{-\infty}^{\mu^*}\Phi(\mu|\cdot)\,\mathrm{d}\mu.$$

$\partial\mu^*/\partial q>0$ follows since $[q+(1-q)\Phi(\mu^*|\cdot)]>0$, $\theta>\mu^*$, and $\int_{-\infty}^{\mu^*}\Phi(\mu|\cdot)\,\mathrm{d}\mu>0$. Similarly, from (22c),

$$\frac{\partial\mu^*}{\partial\theta}[q+(1-q)\Phi(\mu^*|\cdot)]=q-(1-q)\int_{-\infty}^{\mu^*}\Phi_\theta(\mu|\cdot)\,\mathrm{d}\mu.$$

Since $\Phi_\theta(\mu|\cdot)<0$ for all $\mu\in(-\infty,\infty)$ (see the proof of Lemma A.1), the right-hand side is positive. Thus, $\partial\mu^*/\partial\theta>0$ follows. Finally, consider t's effect on μ^*. Again from (22c),

$$\frac{\partial\mu^*}{\partial t}[q+(1-q)\Phi(\mu^*|\cdot)]=-(1-q)\int_{-\infty}^{\mu^*}\Phi_t(\mu|\cdot)\,\mathrm{d}\mu.\tag{A.1}$$

From (20), $T(\mu) \equiv \int_{-\infty}^{\mu} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu > 0$ for all $\mu \in (-\infty, \infty)$. Thus, the right-hand side of (A.1) is negative. Hence, $\partial\mu^*/\partial t < 0$ follows.

(v) For any parameter $\alpha (= q, \theta, t)$,

$$\frac{\partial P(e^*, \mu^*)}{\partial \alpha} = \frac{\mathrm{d}P(e^{\mathrm{F}}(\mu^*), \mu^*)}{\mathrm{d}\mu^*}\frac{\partial \mu^*}{\partial \alpha}.$$

The proof of part (iii) has shown that $\mathrm{d}P(e^{\mathrm{F}}(\mu^*), \mu^*)/\mathrm{d}\mu^* > 0$. The comparative static results on $P(e^*, \mu^*)$ then directly follow from the results in part (iv). ■

Proof of Proposition 3. (i) From (26) and (16),

$$\Omega_{\mathrm{i}}^* - \Omega_{\mathrm{i}}^{\mathrm{F}} = \int_{-\infty}^{\mu^*} [\pi^{\mathrm{F}}(\mu^*) - \pi^{\mathrm{F}}(\mu)]\,\mathrm{d}\Phi(\mu|\cdot) > 0,$$

where the inequality follows because $\pi^{\mathrm{F}}(\mu^*) > \pi^{\mathrm{F}}(\mu)$ for all $\mu < \mu^*$. Next, from (25) and (16),

$$\Omega_{\mathrm{u}}^* - \Omega_{\mathrm{u}}^{\mathrm{F}} = \pi^{\mathrm{F}}(\mu^*) - \pi^{\mathrm{F}}(\theta) < 0,$$

where the inequality follows because $\pi^{\mathrm{F}}(\mu)$ increases with $\mu > 0$ and $0 < \mu^* < \theta$.

(ii) From the definitions of λ^* and λ^{F},

$$\lambda^* - \lambda^{\mathrm{F}} = (\Omega_{\mathrm{i}}^* - \Omega_{\mathrm{u}}^*) - (\Omega_{\mathrm{i}}^{\mathrm{F}} - \Omega_{\mathrm{u}}^{\mathrm{F}}) = (\Omega_{\mathrm{i}}^* - \Omega_{\mathrm{i}}^{\mathrm{F}}) + (\Omega_{\mathrm{u}}^{\mathrm{F}} - \Omega_{\mathrm{u}}^*) > 0,$$

where the inequality follows from part (i).

(iii) From (25), (26), and (28),

$$\begin{aligned}\Omega^* &= q[\pi^{\mathrm{F}}(\mu^*)] + (1-q)\left[\Phi(\mu^*)\pi^{\mathrm{F}}(\mu^*) + \int_{\mu^*}^{\infty} \pi^{\mathrm{F}}(\mu)\,\mathrm{d}\Phi(\mu|\cdot)\right] \\ &= [q + (1-q)\Phi(\mu^*)]\pi^{\mathrm{F}}(\mu^*) + (1-q)\int_{\mu^*}^{\infty} \pi^{\mathrm{F}}(\mu)\,\mathrm{d}\Phi(\mu|\cdot). \qquad \text{(A.2)}\end{aligned}$$

Consider the first term of (A.2). Using (14) for $\pi^{\mathrm{F}}(\mu^*)$ yields

$$\begin{aligned}[q + (1-q)\Phi(\mu^*)]\pi^{\mathrm{F}}(\mu^*) &= [q + (1-q)\Phi(\mu^*)][\mu^* R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*))] \\ &\quad - [q + (1-q)\Phi(\mu^*)][k^{\mathrm{F}}(\mu^*) + C(e^{\mathrm{F}}(\mu^*))]. \qquad \text{(A.3)}\end{aligned}$$

Replace ζ in the left-hand side of (22b) by μ^* (since $\zeta = \mu^*$), multiply both sides by $R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*))$, and rearrange terms. Then,

$$\begin{aligned}&[q + (1-q)\Phi(\mu^*)][\mu^* R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*))] \\ &\quad = \left[q\theta + (1-q)\int_{-\infty}^{\mu^*} \mu\,\mathrm{d}\Phi(\mu|\cdot)\right] R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)). \qquad \text{(A.4)}\end{aligned}$$

Substituting (A.4) into the right-hand side of (A.3) and rearranging terms yields

$$\begin{aligned}[q + (1-q)\Phi(\mu^*)]\pi^{\mathrm{F}}(\mu^*) &= q[\theta R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{\mathrm{F}}(\mu^*) - C(e^{\mathrm{F}}(\mu^*))] \\ &\quad + (1-q)\int_{-\infty}^{\mu^*} [\mu R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) \\ &\quad - k^{F}(\mu^*) - C(e^{\mathrm{F}}(\mu^*))]\,\mathrm{d}\Phi(\mu|\cdot). \qquad \text{(A.5)}\end{aligned}$$

Next, consider the second term of (A.2). From (14),

$$(1-q)\int_{\mu^*}^{\infty} \pi^{\mathrm{F}}(\mu)\,\mathrm{d}\Phi(\mu|\cdot) = (1-q)\int_{\mu^*}^{\infty} [\mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu))]\,\mathrm{d}\Phi(\mu|\cdot). \tag{A.6}$$

Combining (A.5) and (A.6) thus yields

$$\begin{aligned}\Omega^* &= q[\theta R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{\mathrm{F}}(\mu^*) - C(e^{\mathrm{F}}(\mu^*))]\\ &\quad + (1-q)\int_{-\infty}^{\mu^*} [\mu R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{F}(\mu^*) - C(e^{\mathrm{F}}(\mu^*))]\,\mathrm{d}\Phi(\mu|\cdot)\\ &\quad + (1-q)\int_{\mu^*}^{\infty} [\mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu))]\,\mathrm{d}\Phi(\mu|\cdot).\end{aligned} \tag{A.7}$$

Now use (14)–(16), and (18) to see

$$\begin{aligned}\Omega^{\mathrm{F}} &= q[\theta R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta))]\\ &\quad + (1-q)\int_{-\infty}^{\infty} [\mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu))]\,\mathrm{d}\Phi(\mu|\cdot).\end{aligned} \tag{A.8}$$

Then, from (A.7) and (A.8),

$$\begin{aligned}\Omega^{\mathrm{F}} - \Omega^* &= q\{[\theta R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta))]\\ &\quad - [\theta R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{\mathrm{F}}(\mu^*) - C(e^{\mathrm{F}}(\mu^*))]\}\\ &\quad + (1-q)\int_{-\infty}^{\mu^*} \{[\mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu))]\\ &\quad - [\mu R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{\mathrm{F}}(\mu^*) - C(e^{\mathrm{F}}(\mu^*))]\}\,\mathrm{d}\Phi(\mu|\cdot).\end{aligned} \tag{A.9}$$

As shown in (13), $\theta R(e,k) - k - C(e)$ attains its unique maximum at $(e,k) = (e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta))$, so that

$$\theta R(e^{\mathrm{F}}(\theta), k^{\mathrm{F}}(\theta)) - k^{\mathrm{F}}(\theta) - C(e^{\mathrm{F}}(\theta)) > \theta R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{\mathrm{F}}(\mu^*) - C(e^{\mathrm{F}}(\mu^*)).$$

The first term in (A.9) is thus positive. Again from (13), $\mu R(e,k) - k - C(e)$ attains its unique maximum at $(e,k) = (e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu))$, so that

$$\mu R(e^{\mathrm{F}}(\mu), k^{\mathrm{F}}(\mu)) - k^{\mathrm{F}}(\mu) - C(e^{\mathrm{F}}(\mu)) > \mu R(e^{\mathrm{F}}(\mu^*), k^{\mathrm{F}}(\mu^*)) - k^{\mathrm{F}}(\mu^*) - C(e^{\mathrm{F}}(\mu^*))\quad \forall \mu \neq \mu^*.$$

Hence, the second term in (A.9) is also positive. Consequently, (A.9) is positive and the desired result, $L^* \equiv \Omega^{\mathrm{F}} - \Omega^* > 0$, follows. ■

Proof of Proposition 4. (i) $\partial\Omega_{\mathrm{u}}^{*}/\partial t<0$ is shown in the main text. Let $\mu_t^* \equiv \partial\mu^*/\partial t$. Using Leibnitz's rule and integrating by parts, it is straightforward to verify that

$$\begin{aligned}\frac{\partial\Omega_{\mathrm{i}}^{*}}{\partial t} &= \frac{\partial}{\partial t}\left[\Phi(\mu^*|\cdot)\pi^{\mathrm{F}}(\mu^*) + \int_{\mu^*}^{\infty} \pi^{\mathrm{F}}(\mu)\,\mathrm{d}\Phi(\mu|\cdot)\right] \\ &= \Phi(\mu^*|\cdot)\pi^{\mathrm{F}\prime}(\mu^*)\mu_t^* - \int_{\mu^*}^{\infty} \pi^{\mathrm{F}\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu, \end{aligned} \tag{A.10}$$

which is given in (30a) in the main text. Next, from (A.1),

$$\Phi(\mu^*|\cdot)\mu_t^* = -\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \frac{q}{1-q}\mu_t^*. \tag{A.11}$$

Substituting (A.11) into (A.10) and using (29) yields

$$\frac{\partial\Omega_{\mathrm{i}}^{*}}{\partial t} = -\pi^{\mathrm{F}\prime}(\mu^*)\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \int_{\mu^*}^{\infty} \pi^{\mathrm{F}\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \left(\frac{q}{1-q}\right)\frac{\partial\Omega_{\mathrm{u}}^{*}}{\partial t}. \tag{A.12}$$

Consider the first and second terms in (A.12), which are equal to

$$\begin{aligned} &-\pi^{\mathrm{F}\prime}(\mu^*)\left[\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu + \int_{\mu^*}^{\infty} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \int_{\mu^*}^{\infty} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu\right] \\ &\qquad - \int_{\mu^*}^{\infty} \pi^{\mathrm{F}\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu \\ &= \pi^{\mathrm{F}\prime}(\mu^*)\int_{\mu^*}^{\infty} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \int_{\mu^*}^{\infty} \pi^{\mathrm{F}\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu, \end{aligned}$$

where the equality follows because

$$\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu + \int_{\mu^*}^{\infty} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu = \int_{-\infty}^{\infty} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu = T(\infty) = 0.$$

As a result, (A.12) reduces to

$$\frac{\partial\Omega_{\mathrm{i}}^{*}}{\partial t} = \int_{\mu^*}^{\infty} \Delta(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \left(\frac{q}{1-q}\right)\frac{\partial\Omega_{\mathrm{u}}^{*}}{\partial t} \quad \text{where } \Delta(\mu) \equiv \pi^{\mathrm{F}\prime}(\mu^*) - \pi^{\mathrm{F}\prime}(\mu). \tag{A.13}$$

Consider the first term (A.13) in conjunction with (20). Then,

$$\int_{\mu^*}^{\infty} \Delta(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu = \int_{\mu^*}^{\infty} \Delta(\mu)\,\mathrm{d}T(\mu) = -\int_{\mu^*}^{\infty} T(\mu)\Delta'(\mu)\,\mathrm{d}\mu,$$

where the last equality follows from integrating by parts and using the fact that $T(\infty) = 0 = \Delta(\mu^*)$. Since $\Delta'(\mu) = -\pi^{\mathrm{F}\prime\prime}(\mu)$, (A.13) becomes

$$\frac{\partial\Omega_{\mathrm{i}}^{*}}{\partial t} = \int_{\mu^*}^{\infty} T(\mu)\pi^{\mathrm{F}\prime\prime}(\mu)\,\mathrm{d}\mu - \left(\frac{q}{1-q}\right)\frac{\partial\Omega_{\mathrm{u}}^{*}}{\partial t},$$

which is (30b) in the main text. Since $T(\mu) > 0$ for all $\mu \in (-\infty, \infty)$ and $\pi^{F\prime\prime}(\mu) > 0$ for all $\mu \geqslant \mu^*$, the first term is positive. The second term is also positive because $\partial\Omega_u^*/\partial t < 0$. Therefore, $\partial\Omega_i^*/\partial t > 0$ holds.

(ii) Since $\partial\Omega_u^F/\partial t = 0$, it follows from the definitions of λ^* and λ^F that

$$\frac{\partial\lambda^*}{\partial t} - \frac{\partial\lambda^F}{\partial t} = \left[\frac{\partial\Omega_i^*}{\partial t} - \frac{\partial\Omega_u^*}{\partial t}\right] - \frac{\partial\Omega_i^F}{\partial t} = \left[\frac{\partial\Omega_i^*}{\partial t} - \frac{\partial\Omega_i^F}{\partial t}\right] - \frac{\partial\Omega_u^*}{\partial t}. \tag{A.14}$$

Differentiate Ω_i^F given in (16) with respect to t and use the fact that $\lim_{\mu\to\infty}[\pi^F(\mu)\Phi_t(\mu|\cdot)] = 0$ (see footnote 12). Then,

$$\frac{\partial\Omega_i^F}{\partial t} = -\int_{-\infty}^{\infty} \Phi_t(\mu|\cdot)\pi^{F\prime}(\mu)\,\mathrm{d}\mu. \tag{A.15}$$

Next, from (A.12),

$$\frac{\partial\Omega_i^*}{\partial t} = -\pi^{F\prime}(\mu^*)\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \int_{\mu^*}^{\infty} \pi^{F\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \left(\frac{q}{1-q}\right)\frac{\partial\Omega_u^*}{\partial t}.$$

Add an expression,

$$\left[\int_{-\infty}^{\mu^*} \pi^{F\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \int_{-\infty}^{\mu^*} \pi^{F\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu\right] = 0$$

to the right-hand side of the above equation and rearrange terms. Then, by using (A.15), I obtain

$$\frac{\partial\Omega_i^*}{\partial t} - \frac{\partial\Omega_i^F}{\partial t} = -\pi^{F\prime}(\mu^*)\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu + \int_{-\infty}^{\mu^*} \pi^{F\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \left(\frac{q}{1-q}\right)\frac{\partial\Omega_u^*}{\partial t}.$$

Hence, (A.14) is equivalent to

$$\begin{aligned}\frac{\partial\lambda^*}{\partial t} - \frac{\partial\lambda^F}{\partial t} = &-\pi^{F\prime}(\mu^*)\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu \\ &+ \int_{-\infty}^{\mu^*} \pi^{F\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu - \left(\frac{1}{1-q}\right)\frac{\partial\Omega_u^*}{\partial t}.\end{aligned} \tag{A.16}$$

Using (29) and (A.1), observe that

$$\left(\frac{1}{1-q}\right)\frac{\partial\Omega_u^*}{\partial t} = \left(\frac{1}{1-q}\right)\pi^{F\prime}(\mu^*)\mu_t^* = -\frac{\pi^{F\prime}(\mu^*)\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu}{q + (1-q)\Phi(\mu^*|\cdot)}.$$

Thus, (A.16) finally reduces to

$$\begin{aligned}\frac{\partial\lambda^*}{\partial t} - \frac{\partial\lambda^F}{\partial t} = &\int_{-\infty}^{\mu^*} \pi^{F\prime}(\mu)\Phi_t(\mu|\cdot)\,\mathrm{d}\mu \\ &+ [\pi^{F\prime}(\mu^*)\int_{-\infty}^{\mu^*} \Phi_t(\mu|\cdot)\,\mathrm{d}\mu]\left[-1 + \frac{1}{q + (1-q)\Phi(\mu^*|\cdot)}\right] > 0.\end{aligned}$$

To explain the last inequality, note that the first term is positive since $0<\mu^*<\theta$, $\pi^{F\prime}(\mu)=0$ for all $\mu\leqslant 0$, $\pi^{F\prime}(\mu)>0$ for all $\mu>0$, and $\Phi_t(\mu|\cdot)>0$ for all $\mu<\theta$ (see footnote 12). The second term is also positive since

$$\pi^{F\prime}(\mu^*)\int_{-\infty}^{\mu^*}\Phi_t(\mu|\cdot)\,\mathrm{d}\mu=\pi^{F\prime}(\mu^*)T(\mu^*)>0$$

and

$$-1+\frac{1}{q+(1-q)\Phi(\mu^*|\cdot)}=\frac{(1-q)[1-\Phi(\mu^*|\cdot)]}{q+(1-q)\Phi(\mu^*|\cdot)}>0.$$

(iii) See (31) in the main text for $\partial\Omega^*/\partial t>0$. Next, from (21) and (31),

$$\begin{aligned}\frac{\partial\Omega^F}{\partial t}-\frac{\partial\Omega^*}{\partial t}&=(1-q)\left[\int_0^{\infty}T(\mu)\pi^{F\prime\prime}(\mu)\,\mathrm{d}\mu-\int_{\mu^*}^{\infty}T(\mu)\pi^{F\prime\prime}(\mu)\,\mathrm{d}\mu\right]\\&=(1-q)\int_0^{\mu^*}T(\mu)\pi^{F\prime\prime}(\mu)\,\mathrm{d}\mu>0\end{aligned}$$

where the inequality holds since $T(\mu)>0$ for all $\mu\in(-\infty,\infty)$ and $\pi^{F\prime\prime}(\mu)>0$ for all $\mu>0$. ■

Appendix B

Notation

q	fraction of the entrepreneurs who have no private signal
(e,k)	pair of an entrepreneur's effort and investors' capital investment in the asset
$xR(e,k)-k-C(e)$	net return of the asset given (e,k,x), where $x\sim \mathrm{N}(\theta,1/u)$ with $u\equiv 1/\sigma_x^2$
$y\equiv x+\varepsilon$	an informed entrepreneur's private signal about x, where $\varepsilon\sim\mathrm{N}(0,1/t)$ with $t\equiv 1/\sigma_\varepsilon^2$
$\mu\equiv \mathrm{E}[x\|y]$	posterior mean of x given signal y
$k^0(e,\mu)$	the investors' optimal investment given (e,μ)
$J(e,\mu)$	the entrepreneur's payoff given (e,μ)
$P(e,\mu)\equiv \mu R(e,k^0(e,\mu))-k^0(e,\mu)$	the competitive asset price given (e,μ)
$e^F(\mu)$	the entrepreneur's optimal effort given μ
$k^F(\mu)\equiv k^0(e^F(\mu),\mu)$	the investors' optimal investment given μ
$\pi^F(\mu)$	maximized net expected return of the asset given μ
Ω_j^F	the informed ($j=$ i) and uninformed ($j=$ u) entrepreneurs' ex ante payoffs in the first-best case
$\lambda^F\equiv\Omega_i^F-\Omega_u^F$	private (and social) value of signal y in the first-best case
$\Omega^F\equiv q\Omega_u^F+(1-q)\Omega_i^F$	the first-best social surplus
ND	the set of posterior beliefs that are not disclosed by the informed entrepreneurs

ζ	the investors' posterior belief about x given nondisclosure in the second-best case
μ^*	the posterior belief about x, below which the informed entrepreneurs do not make a disclosure
Ω_j^*	the informed ($j = \mathrm{i}$) and uninformed ($j = \mathrm{u}$) entrepreneurs' ex ante payoffs in the second-best case
$\lambda^* \equiv \Omega_\mathrm{i}^* - \Omega_\mathrm{u}^*$	private value of signal y in the second-best case
$\Omega^* \equiv q\Omega_\mathrm{u}^* + (1-q)\Omega_\mathrm{i}^*$	the second-best social surplus
$L^* \equiv \Omega^\mathrm{F} - \Omega^*$	efficiency loss
$Y(t)$	the cost of producing signal y of quality t

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