Available online at www.sciencedirect.com
SCIENCE DIRECT®

ELSEVIER

Journal of Accounting and Economics 37 (2004) 261–289

JOURNAL OF Accounting & Economics

www.elsevier.com/locate/econbase

Voluntary disclosure of precision information ☆

John S. Hughes[a,*], Suil Pae[b]

[a] *The Anderson School of Management, University of California at Los Angeles, Los Angeles, CA 90095-1481, USA*

[b] *Department of Accounting, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong*

Received 29 March 2002; received in revised form 28 April 2003; accepted 16 May 2003

Abstract

This paper presents a model of an entrepreneur's acquisition and voluntary disclosure of precision information as a supplement to primary disclosure of an estimate of a tradable asset's value. Our analysis shows that equilibrium disclosure can be characterized by four regions. For estimates above (below) the prior expectation of the asset value, the entrepreneur discloses only high (low) precision information. The main idea is to enhance (diminish) confidence in estimates that improve upon (detract from) prior beliefs. We further show that the entrepreneur over-invests in the acquisition of precision information due to the option value of discretion over disclosure.

JEL classification: D61; G14; M41

Keywords: Precision information; Voluntary disclosure; Efficiency

1. Introduction

Of late, there is renewed interest among empirical accounting researchers about the role of voluntary supplemental disclosure in pricing assets. A number of empirical studies have documented selective supplemental disclosures made in conjunction with management earnings forecasts or announcements, and examined

☆ The authors appreciate helpful suggestions and comments from the seminar participants at HKUDT, David Aboody, Jing Liu, Doug Skinner (the editor), and especially Paul Fischer (the referee).

*Corresponding author. Tel.: +1-310-794-9533; fax: +1-310-267-2193.
E-mail address: john.hughes@anderson.ucla.edu (J.S. Hughes).

0165-4101/$ - see front matter
doi:10.1016/j.jacceco.2003.05.002

the influence of such disclosures on market price reactions to those forecasts or announcements.[1] This study provides an equilibrium analysis that rationalizes management incentives associated with supplemental disclosures and market reactions to those disclosures.

We present a model of voluntary disclosure of precision information in which an entrepreneur endowed with an asset seeks to sell it to competitive investors. Prior to the sale of the asset, the entrepreneur receives and discloses an estimate of the asset's value. However, there is asymmetry of information between the entrepreneur and investors with respect to the estimate's precision. In particular, the entrepreneur may or may not acquire information about the precision of the estimate at a publicly unobservable cost, and if informed of precision, she may or may not disclose her precision information along with the estimate.

Our main results regarding voluntary disclosure of precision information are as follows. High estimates relative to the prior mean of the asset value will be accompanied by voluntary disclosure of high precision, but not low precision. The intuition for this result is that when the estimate of the asset value is high, the entrepreneur seeks to induce investors to place more weight on the estimate than on the prior mean in forming a posterior expectation of the asset's value, thereby leading to a high asset price. Conversely, low estimates relative to the prior mean of the asset value will be accompanied by voluntary disclosure of low precision, but not high precision. Here, the entrepreneur's incentive is reversed; by disclosing low precision of the estimate, the entrepreneur seeks to induce investors to place more weight on the prior mean of the asset's value than on the estimate. In sum, we establish a partial disclosure equilibrium conditional on the estimate's deviation from its prior expectation; i.e., nondisclosure of low (high) precision information when the estimate is higher (lower) than the prior expectation.[2]

While we find it useful from a modeling standpoint to stylize supplemental disclosures as precision information, we could provide other interpretations to the linear functional form derived for the market's posterior expectation of the asset's value. For example, the value implications of an innovation in earnings or revenues might depend on a persistence parameter. Voluntary supplemental disclosures in that case would relate to the persistence parameter, rather than precision information as specifically depicted in our analysis.

[1] For example, a recent study by Hutton et al. (2003) examines how price reactions to management earnings forecasts are influenced by supplemental disclosures in the form of specific statements of verifiable data and "soft talk". In Section 5, we will discuss several empirical studies and how their findings relate to our model predictions.

[2] As shown in the analysis, the fact that precision information enters the formation of the posterior expectation of the asset value multiplicatively as a coefficient to the estimate's deviation from its prior mean is crucial to this result. Also note that while one could perform a transformation of variables such that the posterior expectation itself was the object of disclosure, the equilibrium nondisclosure region in that case would be a single region similar to the low-end pooling in Dye (1985) and Jung and Kwon (1988). However, this portrayal would not speak to supplemental disclosures as such, or distinguish their price effects from those of primary disclosures.

Further broadening the interpretation of our results that the entrepreneur supplements high (low) estimates only with disclosures that reinforce (negate or downplay) those estimates, we suggest that firms in general will seek to enhance (diminish) investors' confidence in good (bad) news. In other words, the notion of using supplemental disclosures to improve upon primary disclosures by inducing higher posterior beliefs strikes us as fundamental and pervasive to a variety of supplemental disclosures accompanying value-relevant accounting information including, but not limited to, earnings forecasts or announcements.

Besides characterizing equilibrium disclosure behavior, we provide an analysis of welfare consequences of voluntary rather than mandatory supplemental disclosure. Our results indicate inefficiencies from voluntary disclosure with respect to decisions by the entrepreneur to acquire precision information, as well as decisions by investors about whether to buy the asset and undertake further investment. These inefficiencies represent opportunity costs of allowing supplemental disclosures to remain voluntary, and they may in fact be unavoidable as it is typically impossible to detect whether firms have acquired precision information.

This paper is related to several studies in the voluntary disclosure literature. Investors' uncertainty about the entrepreneur's possession of private information plays a role similar to that in Dye (1985) and Jung and Kwon (1988) in sustaining partial disclosure equilibrium.[3] The efficiency issues explored in this paper are similar to those in Pae (1999, 2002). The principal novelty in our paper lies in the characterization of the disclosure of precision information contingent on the realization of the estimate, rather than of the estimate per se. Subramanyam (1996) examines how the market reacts to an estimate of firm value when investors have uncertainty about the estimate's precision. Penno (1997) examines the role of prior precision of an estimate with respect to disclosure of the estimate in the context of Dye (1985) and Jung and Kwon (1988) under the assumption that precision is common knowledge, rather than being an object of voluntary disclosure as it is in our paper. Penno (1996) considers a setting where a firm, after observing a public signal about the firm's value, privately chooses the precision of an estimate that the firm is going to issue later. He shows that a bad (good) realization of the public signal induces a choice of a high (low) precision of the estimate. The idea is that given bad (good) news, the firm would like more (less) weight to be placed by investors on the *yet* to be drawn independent estimate of the firm's value.

Section 2 presents our model. Section 3 sets a benchmark for voluntary disclosure by examining the case of mandatory disclosure. Section 4 provides a characterization of voluntary disclosure/nondisclosure regions conditional on the estimate. Section 5 generalizes the results derived in Section 4 and discusses empirical implications. Section 6 considers efficiency issues and Section 7 concludes the paper.

[3] While there is no proprietary cost of disclosing information in our model, it is well known that the presence of proprietary disclosure cost also sustains partial disclosure equilibrium (see Verrecchia, 1983).

2. Model

A risk-neutral entrepreneur is endowed with an asset that requires an up-front investment of $k > 0$ in order to generate a future return. Since the entrepreneur lacks resources to invest in the asset, she seeks to sell it to risk-neutral investors in a competitive capital market. The present value of the asset is represented by a normal random variable x, whose mean and precision are known constants denoted by $\theta \equiv E[x]$ and $h \equiv 1/Var[x]$, respectively. We assume that the unconditional expected net present value of the asset is positive, i.e.,

$$\theta > k. \tag{1}$$

Prior to the sale of the asset, the entrepreneur receives a point estimate of the asset's present value and discloses that estimate to investors.[4] Let

$$y \equiv x + \varepsilon \tag{2}$$

be the estimate of x, where ε is a normal random variable with $E[\varepsilon] = E[\varepsilon x] = 0$ and $s \equiv 1/Var[\varepsilon]$, i.e., ε is pure noise with precision s. With this definition, it is clear that when s is high, y is a more precise estimate of x. Given the normality of (x, y), it is well known that the posterior expected value of x given y and its distributional parameters equals

$$E[x|y, \beta] = \beta y + (1 - \beta)\theta, \quad \text{where } \beta \equiv \frac{s}{s+h} \in [0, 1]. \tag{3}$$

That is, the posterior expectation of x is a convex combination of the estimate y and the prior mean θ, where the weight assigned to the estimate relative to the prior mean, i.e., β, increases with s, which is intuitive. As we elaborate below, a priori, the precision s is a random variable, and the entrepreneur has discretion to acquire perfect information about s and disclose it. In this regard, observe that since s and β have a monotonic relation, there is no loss of generality if we focus on β and refer to it as precision information.[5]

While by assumption the entrepreneur is always informed of y and discloses it, she does not learn the precision information β unless she chooses to do so at a cost $c \in [0, \infty)$. We assume that the entrepreneur's information acquisition cost c is a random variable, whose realization is observable only to the entrepreneur. Although investors do not observe c, its distribution function, H, is common knowledge. Initially, we take the probability that the entrepreneur has acquired precision information as given and denote it by a constant $\lambda \in (0, 1)$; later in Section 6, we will endogenize λ by examining the entrepreneur's information acquisition decision.

Fig. 1 provides a timeline depicting the sequence of events and recapping our notation and key assumptions. At $t = 1$, the entrepreneur, endowed with

[4] Alternatively, we could assume that it is common knowledge that the entrepreneur has an estimate. The unraveling result of Grossman (1981) and Milgrom (1981) then ensures that the estimate is always disclosed.

[5] That is, even if β is a function of s, we refer to β as precision information. Also note that since s affects the posterior expected value of the asset only through β, investors are interested in β rather than s per se.

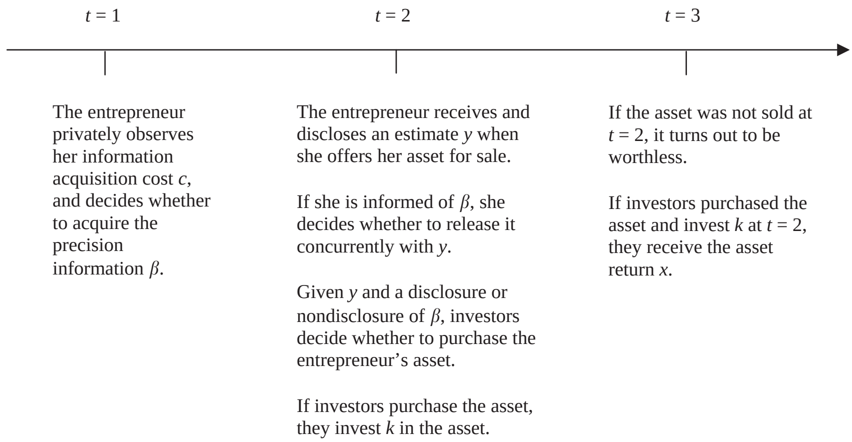

Fig. 1. Timeline.

an asset that has a prior expected present value θ, chooses whether to acquire the precision information β at a private cost c. At $t=2$, the entrepreneur receives an estimate y of the asset's present value x, and she either learns β or remains uninformed. The entrepreneur then offers the asset for sale. In doing so, she always discloses y but exercises discretion over whether to disclose β if she is informed of β. Here, we assume that any disclosed information is truthful, and the uninformed entrepreneur makes no disclosure and cannot credibly reveal her lack of precision information. We also assume that if the informed entrepreneur is indifferent between disclosing and withholding precision information, she withholds it. Based on all available information, investors form a posterior expectation of the asset's present value and decide whether to purchase the asset. Competition among investors drives the asset price to the maximum of the asset's expected net present value or zero. If investors purchase the asset, then they invest k. At $t=3$, if investors had purchased and invested k in the asset at $t=2$, the asset's present value x is realized and investors receive it. Otherwise, the asset is worthless and disappears from the investment opportunity set.

Before proceeding, we emphasize that while we choose to model voluntary disclosure of precision information in the context of asset trading, the qualitative content of our main results has applicability to other settings. For example, we can alternatively model a setting where a firm makes its disclosure decision to maximize the market's posterior expectation of the firm's value conditional on a public signal. All that is required for the equilibrium of our model to apply to this alternative setting is that the market's posterior expectation is given by a linear function of the public signal's deviation from its prior mean, and the coefficient of that deviation is the object of voluntary disclosure. We will discuss this issue and other applications fully in Section 5.

3. Mandatory disclosure

To establish a benchmark for assessing the consequences of voluntary disclosure of precision information, we begin with a *hypothetical* case, in which a social planner has perfect knowledge of the entrepreneur's possession of precision information and mandates its disclosure if the entrepreneur is informed.

3.1. Entrepreneur has acquired precision information

Suppose that the entrepreneur has acquired precision information, β, and she is forced to disclose it concurrently with an estimate y. Given (y, β), investors form a posterior expectation of x as specified in (3), implying that the expected net present value of the asset equals

$$\pi(y, \beta) \equiv E[x|y, \beta] - k = \beta y + (1 - \beta)\theta - k = \beta(y - \theta) + (\theta - k). \tag{4}$$

Since β is non-negative, the posterior expectation $\pi(y, \beta)$ is always increasing in y. Next, observe that β is a multiplier to the estimate y's deviation from its prior mean θ. Hence, $\pi(y, \beta)$ is increasing in β if, and only if, the estimate is greater than its prior mean, i.e. $(y - \theta) > 0$. Finally, note that the last term, $\theta - k$, is a positive constant due to (1).

Using the aforementioned properties of $\pi(y, \beta)$, we can partition the space of (y, β) according to whether or not the asset's expected net present value is positive. First, for any estimate $y > k$ and for any β, the asset has a positive expected net present value, i.e.,[6]

$$\pi(y, \beta) > 0 \quad \text{for all } y > k \text{ and } \beta \in [0, 1]. \tag{5}$$

Second, when $y \leqslant k$, the asset may or may not have a positive expected net present value, depending on the magnitude of β. To be explicit, let $y \leqslant k$ be given. We use $\pi(y, \beta)$ stated in (4) to define β_o such that

$$\pi(y, \beta_o) = 0.$$

Since β_o depends on the given estimate y, we denote it as a function of y, i.e.,

$$\beta_o(y) \equiv -\frac{\theta - k}{y - \theta}.$$

Clearly, $\beta_o(y)$ is an increasing and convex function satisfying

$$\beta_o(k) = 1 \quad \text{and} \quad \lim_{y \to -\infty} \beta_o(y) = 0.$$

Since $\pi(y, \beta)$ decreases with β when $y \leqslant k$ (recall that $k < \theta$), we conclude that

$$\pi(y, \beta) > 0 \text{ if, and only if,} \beta < \beta_o(y), \quad \text{where } y \leqslant k. \tag{6}$$

Fig. 2 provides an illustration, showing the locus of $\beta_o(y)$ and whether the posterior expected net present value of the asset, $\pi(y, \beta)$, is positive. The intuition is

[6] Using (4) and (1), observe that $\pi(y, \beta) = \beta(y - \theta) + (\theta - k) > \beta(k - \theta) + (\theta - k) = (\theta - k)(1 - \beta) \geqslant 0$.

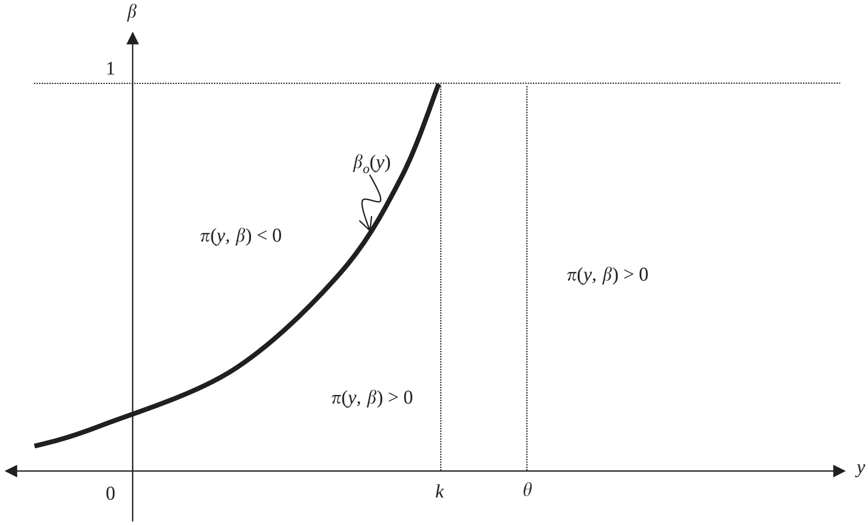

Fig. 2. Partition of (y, β) space. The locus of $\beta_o(y)$ partitions the space of (y, β) according to whether the asset has a positive or negative expected net present value.

as follows. As shown in (3), β is the weight placed on the estimate y relative to the prior mean θ in the posterior expectation of the asset's present value x, i.e., $E[x|y, \beta] = \beta y + (1 - \beta)\theta$. If $y > k$, then $E[x|y, \beta] > k$ for any β (since $\theta > k$), and hence, $\pi(y, \beta) \equiv E[x|y, \beta] - k$ is positive for any β. On the other hand, if $y \leqslant k$, then $\pi(y, \beta)$ is positive (negative) whenever β is less (greater) than $\beta_o(y)$. In sum, when (y, β) is disclosed, investors' expectation of the asset's net present value equals $\pi(y, \beta)$ and they purchase the asset and invest k in it if, and only if, $\pi(y, \beta) > 0$. These observations come into play when we take up the efficiency implications of discretionary disclosure in Section 6.

3.2. Entrepreneur has not acquired precision information

Now suppose that the entrepreneur has not acquired the precision information β, and thus, an estimate y is disclosed without β. In this case, all parties form posterior beliefs about β based on y. Let $F(\beta|y)$ be the distribution function of β conditional on y,[7] and define $\beta_m(y)$ to be the expected value of β conditional on y, i.e., $\beta_m(y) \equiv E[\beta|y] = \int_0^1 \beta \, \mathrm{d}F(\beta|y) \in [0, 1]$. Since $\pi(y, \beta)$ stated in (4) is linear in β, we can express the conditional expected net present value of the uninformed

[7] Given the monotonic relation between β and s, the distribution function $F(\beta|y)$ can be derived from the posterior distribution of s given y as follows. Suppose that s follows a particular prior distribution. We know that conditional on s, $y \sim N(\theta, h^{-1} + s^{-1})$. Hence, by using Bayes rule, one can obtain a posterior distribution of s given y. Using that posterior distribution, a transformation of variables from s to $\beta \equiv s/(s + h)$ yields the posterior distribution of β given y, which is $F(\beta|y)$.

entrepreneur's asset as

$$\int_0^1 \pi(y,\beta)\,\mathrm{d}F(\beta|y) = \pi(y,\beta_{\mathrm{m}}(y)),$$

implying that the asset is traded if, and only if, $\pi(y,\beta_{\mathrm{m}}(y))>0$.

To examine whether $\pi(y,\beta_{\mathrm{m}}(y))$ is positive or not, first consider $y>k$. In this case, we can use the result stated in (5), i.e., $\pi(y,\beta)>0$ for all $y>k$ and $\beta\in[0,1]$, thereby concluding that $\pi(y,\beta_{\mathrm{m}}(y))>0$ for all $y>k$.

Next, consider $y\leqslant k$. From (6), we know that $\pi(y,\beta_{\mathrm{m}}(y))>0$ if and only if $\beta_{\mathrm{m}}(y)<\beta_{\mathrm{o}}(y)$. Hence, we need to examine how $\beta_{\mathrm{m}}(y)$ and $\beta_{\mathrm{o}}(y)$ vary with y. Since we know that $\beta_{\mathrm{o}}(y)\equiv-(\theta-k/y-\theta)$ decreases with y and converges to zero as $y\to-\infty$, we focus on $\beta_{\mathrm{m}}(y)$. Note that given β, the estimate y follows a normal distribution with mean θ and precision $(h^{-1}+s^{-1})^{-1}=\beta h$. Using this fact, it can be shown that $\beta_{\mathrm{m}}(y)$ attains its unique maximum at $y=\theta$, and decreases with y as the estimate y diverges from the prior mean θ,[8] that is,

$$\beta'_{\mathrm{m}}(y)>(<)0 \text{ if } y<(>)\theta. \tag{7}$$

This property is intuitive. Given that the precision of the estimate y equals βh, a large deviation of y from θ must lead to an inference that βh is small on average. Since h is a constant, β must be inferred to be small on average when the estimate y deviates from its prior mean θ.

Returning to the comparison of $\beta_{\mathrm{m}}(y)$ and $\beta_{\mathrm{o}}(y)$ for $y\leqslant k$, note that $\beta_{\mathrm{m}}(k)<1=\beta_{\mathrm{o}}(k)$, where the strict inequality is due to $k<\theta$, $\beta_{\mathrm{m}}(\theta)\leqslant 1$ and (7). Although this result implies that $\beta_{\mathrm{m}}(y)<\beta_{\mathrm{o}}(y)$ for y sufficiently close to k, we cannot characterize the ordering of $\beta_{\mathrm{m}}(y)$ and $\beta_{\mathrm{o}}(y)$ for the entire range of $(-\infty,k]$. Nonetheless, for simplicity, we assume throughout the paper that $\beta_{\mathrm{m}}(y)$ and $\beta_{\mathrm{o}}(y)$ intersect only once, in which case there must be a cutoff value denoted by $y^{\mathrm{u}}\in(-\infty,k)$ such that

$$\beta_{\mathrm{m}}(y)>\beta_{\mathrm{o}}(y) \text{ if, and only if, } y<y^{\mathrm{u}}. \tag{C.1}$$

While this assumption is not essential, it eases the analysis by ensuring that $\beta_{\mathrm{m}}(y)<\beta_{\mathrm{o}}(y)$ for $y\in(y^{\mathrm{u}},k]$, which in turn implies that $\pi(y,\beta_{\mathrm{m}}(y))>0$ if and only if $y\in(y^{\mathrm{u}},k]$.[9]

Combining the results for all y, we conclude that when an estimate y is released with no disclosure of β, the posterior expected net present value of the uninformed entrepreneur's asset is positive (and thus traded to investors) if, and only if, y exceeds the cutoff value y^{u}, i.e.,

$$\pi(y,\beta_{\mathrm{m}}(y))>0 \text{ if, and only if, } y>y^{\mathrm{u}}.$$

[8] See Subramanyam (1996, p. 217).

[9] Technically speaking, (C.1) rules out two possibilities. The first is that $\beta_{\mathrm{m}}(y)<\beta_{\mathrm{o}}(y)$ for all $y\in(-\infty,k]$. In this case, the asset is traded to investors irrespective of the estimate y, which appears a less interesting case. The second case is that in the range of $(-\infty,k]$, $\beta_{\mathrm{m}}(y)$ and $\beta_{\mathrm{o}}(y)$ intersect multiple times, implying that there are mutually disjoint intervals in which $\pi(y,\beta_{\mathrm{m}}(y))>0$. Our results in the subsequent analysis can be extended to this case without any qualitative change, except that the set of y in which $\pi(y,\beta_{\mathrm{m}}(y))>0$ changes from the single interval $(y^{\mathrm{u}},k]$ to the union of those disjoint intervals.

4. Voluntary disclosure

In the previous section, we considered a hypothetical case where a social planner has perfect knowledge of the entrepreneur's possession of precision information, β, and mandates its disclosure. We now move to our model wherein it is not known whether the entrepreneur has acquired the precision information β. The entrepreneur in this case has full discretion over the disclosure of β if she has acquired it. As noted in Section 2, for now we assume an arbitrary probability $\lambda \in (0, 1)$ that the entrepreneur has acquired the precision information β.

If β is disclosed, the analysis proceeds as it did in Section 3.1. Hence, we focus on the case where y is disclosed but β is not. Recall that the uninformed entrepreneur cannot credibly communicate that she has no precision information. Thus, given no disclosure of β, investors cannot tell whether nondisclosure is because the entrepreneur lacks precision information or withholds it. Taking those two possibilities into account along with the estimate y, they form posterior beliefs about β given y, which we denote by $\zeta(y)$. Replacing β in (4) by $\zeta(y)$, we can express investors' posterior expectation of the net present value of the asset as

$$\pi(y, \zeta(y)) = E[x|y, \zeta(y)] - k = \zeta(y)(y - \theta) + \theta - k. \tag{8}$$

Note that we have a knife-edge case of $y = \theta$, in which case investors' beliefs about β have no effect on the posterior expectation, i.e.,

$$\pi(\theta, \zeta(\theta)) = \zeta(\theta)(\theta - \theta) + \theta - k = \theta - k > 0 \quad \text{for any } \zeta(\theta).$$

Thus, in characterizing the informed entrepreneur's equilibrium disclosure behavior, we focus on the two cases where the estimate y either exceeds or is exceeded by the prior expectation θ.

4.1. When the estimate is greater than the prior expectation

Fix $y > \theta$. Since $\theta > k$, we must have $y > k$, which implies that $\pi(y, \zeta(y)) > 0$ for any $\zeta(y) \in [0, 1]$. Thus, even with no disclosure of β, investors conclude that the asset has a positive expected net present value. On the other hand, notice that the informed entrepreneur could sell her asset for $\pi(y, \beta)$ by disclosing β. Since $\pi(y, \zeta(y))$ is independent of β whereas $\pi(y, \beta)$ is increasing in β (recall that $y > \theta$ here), it follows that there must be a lower bound of β, which we denote by β_L, such that the entrepreneur with precision information $\beta > \beta_L$ prefers disclosing that β rather than withholding it. In equilibrium, β_L must satisfy $\pi(y, \beta_L) = \pi(y, \zeta(y))$, or equivalently, $\beta_L = \zeta(y)$, i.e.,

$$\beta_L = \frac{(1 - \lambda)\beta_m(y) + \lambda \int_0^{\beta_L} \beta \, dF(\beta|y)}{(1 - \lambda) + \lambda F(\beta_L|y)}, \tag{9}$$

where the right-hand side of (9) indicates that $\zeta(y)$ is a weighted average of investors' conditional beliefs about β if the entrepreneur did not acquire precision information, and the conditional beliefs about β if she acquired that information but chose to withhold it. Rearranging terms in (9) yields the following characterization of β_L:

$$\beta_L = \beta_m(y) - \frac{\lambda}{1-\lambda}\int_0^{\beta_L} F(\beta|y)\,d\beta. \tag{10}$$

It can be shown that there exists a unique value of $\beta_L \in (0, \beta_m(y))$ that solves the above equation.[10] Denoting that solution by $\beta_L(y)$ establishes a unique conditional nondisclosure set $[0, \beta_L(y)]$ where $y > \theta$ and $\beta_L(y) = \zeta(y)$.

4.2. When the estimate is less than the prior expectation

As will be shown later, in contrast to the earlier cases where $y \geqslant \theta$, the asset may not be traded in this case (i.e., $\pi(y, \zeta(y)) \leqslant 0$ for some $y < \theta$).[11] For the moment, however, let us assume that $\pi(y, \zeta(y)) > 0$ and characterize the nondisclosure set of precision information consistent with this assumption. Recall that if $y < \theta$, the posterior mean of the asset's net present value $\pi(y, \beta)$ is decreasing in β whereas $\pi(y, \zeta(y))$ is independent of β. Hence, provided that $\pi(y, \zeta(y)) > 0$, the set of β suppressed by the informed entrepreneur must be $[\beta_H, 1]$ where β_H satisfies $\pi(y, \beta_H) = \pi(y, \zeta(y))$, i.e., $\beta_H = \zeta(y)$, or equivalently,

$$\beta_H = \frac{(1-\lambda)\beta_m(y) + \lambda\int_{\beta_H}^1 \beta\,dF(\beta|y)}{(1-\lambda) + \lambda[1 - F(\beta_H|y)]}. \tag{11}$$

Similar to the case of $y > \theta$, we can rearrange (11) to obtain a characterization of β_H:

$$\beta_H = \beta_m(y) + \frac{\lambda}{1-\lambda}\int_{\beta_H}^1 [1 - F(\beta|y)]\,d\beta. \tag{12}$$

It is easy to verify that there exists a unique value of $\beta_H \in (\beta_m(y), 1)$ that solves the above equation. Denoting that solution by $\beta_H(y)$ establishes a unique conditional nondisclosure set $[\beta_H(y), 1]$, for which $\beta_H(y) = \zeta(y)$. The next proposition summarizes our analysis of the informed entrepreneur's voluntary disclosure of the precision information β.

Proposition 1. *There exists a unique equilibrium that has the following characteristics.*

(i) *When the estimate exceeds the prior expectation, the informed entrepreneur withholds (discloses) precision information if it is less than or equal to (greater than) a cutoff value, i.e., when $y > \theta$, the informed entrepreneur withholds (discloses) β if $\beta \leqslant (>) \beta_L(y)$.*
(ii) *When the estimate is less than the prior expectation (and provided that the asset is traded), the informed entrepreneur withholds (discloses) precision information if it is greater than or equal to (less than) a cutoff value, i.e., when $y < \theta$, the informed entrepreneur withholds (discloses) β if $\beta \geqslant (<) \beta_H(y)$.*

[10] The proof is similar to that used by Jung and Kwon (1988, pp. 149–150) and thus omitted here.
[11] See Section 4.4 for details.

The intuition for the low-end pooling of precision information, as stated in part (i), is straightforward. Given a high estimate relative to the prior expectation (i.e., $y > \theta$), investors' posterior expectations of the asset's present value are increasing in β. Thus, if the entrepreneur is informed of high precision (in the sense of $\beta > \beta_L(y)$), she discloses that information thereby inducing investors to place a high relative weight on the high estimate. In this case, disclosure of high precision information *enhances* investors' confidence in the estimate and thus leads to a higher posterior expectation of the asset's present value than without disclosure. On the other hand, if the precision of the high estimate is low, the informed entrepreneur prefers to be indistinguishable from an entrepreneur who had not acquired precision information. In the absence of disclosure, investors take into account the informed entrepreneur's incentive to suppress low precision information. This translates into investors' posterior beliefs about β that are lower than the beliefs they would hold if they knew the entrepreneur is uninformed, i.e., $\zeta(y) = \beta_L(y) < \beta_m(y)$.

The intuition for the high-end pooling of precision information, as stated in part (ii), is exactly opposite to the case of $y > \theta$. When $y < \theta$, the entrepreneur's incentive is to *negate* or *downplay* a low estimate's effect on investors' posterior expectation of the asset's present value by inducing *less* confidence in the low estimate, which can be done by disclosing low precision information.[12] More specifically, as $\pi(y, \beta)$ decreases with β in this case, the informed entrepreneur has an incentive to withhold a high value of β (in the sense of $\beta > \beta_H(y)$) and become indistinguishable from the uninformed entrepreneur. Rational investors take account of that incentive in forming their posterior beliefs about β, implying that they infer greater precision in the absence of disclosure than if they knew the entrepreneur is uninformed, i.e., $\zeta(y) = \beta_H(y) > \beta_m(y)$.

4.3. Comparative static analysis of the nondisclosure sets

In this section, we investigate how the conditional nondisclosure sets change with the estimate y. For this purpose, we impose a regularity condition on $F(\beta|y)$:

$$\frac{\partial F(\beta|y)}{\partial y} \geqslant (\leqslant) 0 \quad \text{for all } y > (<)\theta \text{ and } \beta, \tag{C.2}$$

where the weak inequalities hold strictly for some positive measure of β. With this condition, we have the following comparative static results.[13]

[12] Recall that β is low when s is low, i.e., when there is a large measurement error in the estimate y. Hence, given the disclosure of a low β, investors discount the information content of the estimate $y(<\theta)$ and thereby form a posterior expectation of the asset's present value toward to the prior mean θ.

[13] As shown in the proof of Proposition 2 (see Appendix A), (C.2) is sufficient but not necessary for the comparative static results. Technically speaking, (C.2) strengthens (7) in a way that a further deviation of the estimate y from its prior mean θ affects the distribution of β in the sense of inverse first-order stochastic dominance.

Proposition 2. (i) *As the estimate* $y(>\theta)$ *increases, the nondisclosure set* $[0, \beta_L(y)]$ *shrinks, i.e.,* $\partial\beta_L(Y)/\partial y < 0$.

(ii) *As the estimate* $y(<\theta)$ *decreases, the nondisclosure set* $[\beta_H(y), 1]$ *expands, i.e.,* $\partial\beta_H(Y)/\partial y > 0$.

To explain part (i), observe that when $y > \theta$, (C.2) implies that an increase of y changes the distribution of β such that β is more likely to be small. In addition, recall that as indicated by the right-hand side of (9), investors' beliefs about β given no disclosure of precision information equal the weighted average of the uninformed entrepreneur's β (i.e., $\beta_m(y)$) and the conditional expected value of the informed entrepreneur's undisclosed β (i.e., $E[\beta|\beta \leqslant \beta_L, y]$). As such, if y increases but β_L remains unchanged, investors' beliefs about β are revised to be lower, thereby diminishing the positive effect of an increased estimate on the expected net present value of the asset. Given this change in investors' beliefs about β, the marginal informed entrepreneur (i.e., the informed entrepreneur with precision information $\beta = \beta_L$) is no longer indifferent between withholding and disclosing precision information; i.e., she becomes strictly better off by revealing her precision information concurrently with the increased estimate. Hence, the new cutoff value representing the upper bound of the nondisclosure set associated with the increased estimate must be strictly lower than the old cutoff value.

We next explain the intuition for why the conditional nondisclosure set $[\beta_H(y), 1]$ expands as y decreases. Recall again from (C.2) that when the estimate y becomes even lower than its prior mean θ, the distribution of β changes such that β is more likely to be small. Also note that, as indicated by the right-hand side of (11), investors' beliefs about β given nondisclosure of precision information equal the weighted average of the uninformed entrepreneur's β (i.e., $\beta_m(y)$) and the conditional expected value of the informed entrepreneur's undisclosed β (i.e., $E[\beta|\beta \geqslant \beta_H, y]$). Thus, if y decreases but β_H remains unchanged, investors' beliefs about β are revised to be lower, thereby diminishing the negative effect of a decreased estimate on the expected net present value of the asset. This implies that the informed entrepreneur who suppressed β before the decrease of y continues to do so, but the entrepreneur who disclosed β slightly below β_H before the decrease of y is motivated to suppress that β concurrent with the decrease of y. Hence, the new cutoff value representing the lower bound of the nondisclosure set associated with the decreased estimate must be strictly lower than the old cutoff value.

4.4. Analysis of asset trading

Recall that in Section 4.2, our analysis was conducted under the assumption that, given an estimate $y < \theta$ and no disclosure of β, the asset has a positive expected net present value, that is, $\pi(y, \zeta(y)) > 0$, and thus it is traded. We showed that the unique nondisclosure set is $[\beta_H(y), 1]$, where $\beta_H(y)$ is characterized by (12) and $\beta_H(y) = \zeta(y)$ holds. For this result to be valid, $\pi(y, \beta_H(y)) > 0$ must be satisfied. In this section, we investigate the set of estimates $y<\theta$ satisfying this requirement and examine the equilibrium when an estimate y leads to $\pi(y,\beta_H(y))\leqslant 0$.

First, consider $y\in(k,\theta)$. Given our earlier result in Section 3 that $\pi(y,\beta)>0$ for all $y>k$ and $\beta\in[0,1]$, it is clear that $\pi(y,\beta_H(y))>0$ for all $y\in(k,\theta)$. Second, consider $y\leqslant k$. In this case, recall the result stated in (6): For any estimate $y\leqslant k$, there is a cutoff value $\beta_o(y)$ such that $\pi(y,\beta)>0$ if, and only if, $\beta<\beta_o(y)$. Consequently, for the analysis in Section 4.2 to be valid (i.e. $\pi(y,\beta_H(y))>0$ for all $y\leqslant k$), it is required that $\beta_H(y)<\beta_o(y)$ for all $y\leqslant k$. However, given (C.1) and the fact that $\beta_H(y)>\beta_m(y)$, it is clear that this requirement cannot be satisfied. In particular, note that: (i) $\beta_H(k)<1=\beta_o(k)$; and (ii) $\beta_H(y)>\beta_o(y)$ for all $y\leqslant y^u$ where y^u is the value of the estimate y below which $\beta_m(y)>\beta_o(y)$. Hence, from the continuity of $\beta_H(y)$ and $\beta_o(y)$, it follows that those two functions must intersect at least once in the range of (y^u,k), implying that $\beta_H(y)\geqslant\beta_o(y)$ not only for all $y\leqslant y^u$ but also for some $y\in(y^u,k)$. Denoting Y_{NT} to be the set of those estimates, i.e.,

$$Y_{NT}=\{y\leqslant k|\beta_H(y)\geqslant\beta_o(y)\},$$

we conclude that when $y<\theta$, $\pi(y,\beta_H(y))>0$ holds for all $y\notin Y_{NT}$.

We now examine the equilibrium when an estimate $y\in Y_{NT}$ is released with no disclosure of precision information. Observe that in the equilibrium for this case, it must be true that the expected net present value of the asset is non-positive, i.e., $\pi(y,\zeta(y))\leqslant 0$ for all $y\in Y_{NT}$, and hence, the asset is not traded. This result follows because otherwise (i.e., $\pi(y,\zeta(y))>0$ for some $y\in Y_{NT}$) we know from the analysis in Section 4.2 that the unique nondisclosure set is $[\beta_H(y),\ 1]$ and $\zeta(y)=\beta_H(y)<\beta_o(y)$, which is a contradiction to $y\in Y_{NT}$. Since the asset is not traded in the absence of the disclosure of precision information, it follows that only the informed entrepreneur with precision information $\beta<\beta_o(y)$ discloses that β to sell her asset for $\pi(y,\beta)>0$. As a result, the unique conditional nondisclosure set for $y\in Y_{NT}$ must be $[\beta_o(y),\ 1]$.[14] It remains to establish that, given no disclosure of precision information $\beta\in[\beta_o(y),1]$, investors' beliefs about β are consistent with $\pi(y,\zeta(y))\leqslant 0$ for all $y\in Y_{NT}$, i.e., $\zeta(y)\geqslant\beta_o(y)$. Lemma A.1 in Appendix A shows that this consistency requirement is met.

Fig. 3 provides a graphical summary of our analysis. We depict the informed entrepreneur's equilibrium decision about whether to disclose the precision information β contingent on the public disclosure of the estimate y for all possible combinations of (y,β). In doing so, for ease of exposition, we consider a situation where $\beta_H(y)$ and $\beta_o(y)$ intersect only once, so that $Y_{NT}=(-\infty,y^h]$ for some $y^h\in(y^u,k)$; i.e.,[15]

$$\beta_H(y)\geqslant\beta_o(y)\ \text{if, and only if,}\ y\leqslant y^h.$$

[14] Recall that the informed entrepreneur is assumed to withhold β if she is indifferent between withholding and disclosing it. If the entrepreneur informed of $\beta\geqslant\beta_o(y)$ discloses that β, then $\pi(y,\beta)\leqslant 0$, implying no trade of the asset. Therefore, she cannot sell the asset whether she discloses $\beta\geqslant\beta_o(y)$ or suppresses it.

[15] If $\beta_H(y)$ and $\beta_o(y)$ intersect multiple times in the range of (y^u,k), then Y_{NT} is the union of the intervals in which $\beta_H(y)=\beta_o(y)$. The discussion in the main text can be adapted for that case with no qualitative change; we merely need to replace $y\leqslant y^h$ with $y\in Y_{NT}$.

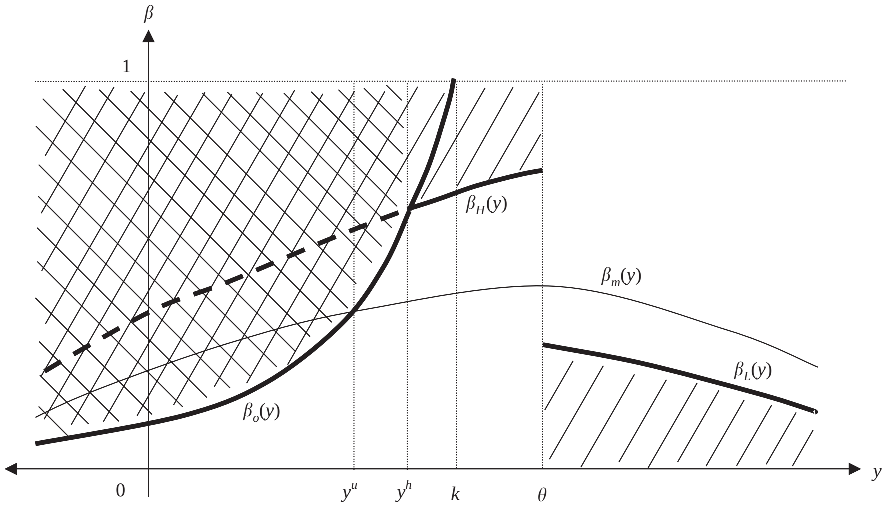

Fig. 3. Voluntary disclosure of precision information. The informed entrepreneur observing β in shaded regions withholds precision information β. When $y \leqslant y^{\mathrm{h}}$ is released with no disclosure of β (i.e., the cross-hatched region), the asset is not traded since $\pi(y, \zeta(y)) \leqslant 0$. When $y > y^{\mathrm{h}}$ is released with no disclosure of β (i.e., the single-hatched region), the asset is traded at the price of $\pi(y, \beta_j(y)) > 0$ where $j = \mathrm{H}, \mathrm{L}$.

The entrepreneur informed of β belonging to the shaded (non-shaded) area withholds (discloses) that β. When an estimate $y \leqslant y^{\mathrm{h}}$ is released, the entrepreneur informed of $\beta \geqslant \beta_{\mathrm{o}}(y)$ suppresses that β and, given no disclosure of β, the expectation of the asset's net present value is negative.[16] Next, when an estimate $y \in (y^{\mathrm{h}}, \theta)$ is released, the entrepreneur with $\beta \geqslant \beta_{\mathrm{H}}(y)$ finds it optimal to suppress that information. Given no disclosure of β, investors' posterior beliefs about β equal $\beta_{\mathrm{H}}(y)$ and the expected net present value of the asset is positive, i.e., $\pi(y, \beta_{\mathrm{H}}(y)) > 0$. When an estimate y exceeds the prior mean of the asset's present value θ, the entrepreneur informed of $\beta \leqslant \beta_{\mathrm{L}}(y)$ finds it optimal to suppress that information. Given no disclosure of β, investors' beliefs about β equal $\beta_{\mathrm{L}}(y)$ and $\pi(y, \beta_{\mathrm{L}}(y)) > 0$. In sum, given that the informed entrepreneur observing β in the non-shaded area discloses that precision information and thereby sells the asset for $\pi(y, \beta) > 0$, we have a high-end pooling of precision information when the estimate conveys bad news (i.e., lower than the prior expectation of the asset's net present value), and a low-end pooling when the estimate conveys good news (i.e., greater than the prior expectation).

Fig. 3 also illustrates how the nondisclosure sets change with the estimate y. As we discussed following Proposition 2, these changes are due to the fact that: (i) a further

[16] In Fig. 3, observe that $\beta_{\mathrm{H}}(y)$ is *dashed* for $y = y^{\mathrm{h}}$, indicating that it does *not* represent investors' posterior beliefs about β given no disclosure of β; recall that $\beta_{\mathrm{H}}(y)$ is derived under the assumption that $\pi(y, \beta_{\mathrm{H}}(y)) = \pi(y, \zeta(y)) > 0$, which is not valid for $y \leqslant y^{\mathrm{h}}$. Instead, since the nondisclosure set conditional on $y \leqslant y^{\mathrm{h}}$ equals $[\beta_{\mathrm{o}}(y), 1]$, investors' posterior beliefs about β must equal the expression given in the right-hand side of (11) with β_{H} there being replaced by $\beta_{\mathrm{o}}(y)$. As shown in Lemma A.1, that expression is greater than $\beta_{\mathrm{o}}(y)$.

deviation of y from θ alters the distribution of β such that β is more likely to be small; and (ii) a high-end (low-end) pooling of undisclosed precision information occurs when the estimate conveys bad (good) news.

5. Discussion

5.1. Generalization to other objects of voluntary disclosure

Thus far, our analysis has exclusively focused on the voluntary disclosure of precision information that is supplemental to the public disclosure of an estimate of the asset's future cash flow. However, our main results have more general applicability. For example, suppose that a firm's manager seeks to maximize investors' expectations of the firm's value, where the expectation takes the form of a linear function of a public signal and the slope coefficient to that signal is the object of the manager's voluntary disclosure. That is, consider

$$\gamma z + \alpha, \tag{13}$$

where (i) z is a random variable with zero mean and is always publicly disclosed; (ii) γ is a positive random variable that is the manager's private information and may or may not be disclosed at the manager's discretion; and (iii) α is a known constant. One interpretation of (13) is that z is an unexpected innovation in an accounting variable such as earnings or revenues and γ is a measure of the persistence of that innovation.[17]

Comparing (13) and (4) directly reveals that there is a one-to-one correspondence between (γ, z, α) and $(\beta,\ y-\theta,\ \theta-k)$. Accordingly, it follows from our analysis that when z is positive (negative), the manager would withhold low (high) values of γ. In qualitative terms, this disclosure policy might take the form of supplemental information attesting to high (low) persistence accompanying a positive (negative) innovation. Also note that if a larger value of $|z|$ is less likely to persist in the future, then the distribution of γ conditional on z exhibits the same properties as those of the distribution of β conditional on y that are assumed in our analysis. As a result, the conditional nondisclosure set of γ must change with the public signal z in the same way as the conditional nondisclosure set of β changes with the estimate y in our analysis.

5.2. Empirical implications of results

Natural candidates for empirical tests of our results include supplemental disclosures made concurrently with earnings forecasts or earnings announcements. For example, managers often provide earnings forecasts in the form of a range whose magnitude could be interpreted as a measure of dispersion inversely related to

[17] An implicit assumption here is that the manager has information superior to the market about the innovation's impact on the firm's earnings generation process, which appears reasonable.

forecast precision.[18] In light of our analysis, one can design a test of management's choice of forecast range as follows. Consistent with prior studies of management forecasts,[19] the mid-point of the forecast range could be regarded as the estimate in our model, and the average of analysts' forecasts before the release of the range forecast could be used as a proxy for the prior expectation of the estimate. The news conveyed by the range forecast would be good (bad) if the mid-point of the forecast range is greater (less) than the average of analysts' forecasts. Using the magnitude of forecast range as an inverse measure of precision, we predict that the average forecast range of bad news firms would be larger than that of good news firms.[20]

Predictions similar to those pertaining to the disclosure of earnings forecast ranges can also be offered for supplemental disclosures made concurrently with earnings announcements. Brown and Kim (1993) examine whether market reactions to earnings announcements are influenced by the presence of non-earnings disclosures. They find that for small firms,[21] market reactions to the earnings news accompanied by non-earnings disclosures are significantly more favorable than to earnings news not so accompanied, unconditional on the direction of the news. If we interpret non-earnings information as supplemental disclosures influencing investors' confidence in earnings information, this evidence is consistent with the prediction from our analysis; the primary incentive to make supplemental disclosures is to raise investors' expectations of firm value by increasing investors' confidence in earnings news.[22] A more discriminating test, however, would partition earnings news into good and bad, and seek to determine whether supplemental disclosures tend to be reinforcing good earnings news and negating bad earnings news.

A recent study by Hutton et al. (2003) examines two broad categories of supplemental disclosures in a discriminating way as discussed above. Partitioning management earnings forecasts into good and bad news, they find that some (but not all) good news earnings forecasts are accompanied by verifiable forward-looking disclosures, whereas disclosures accompanying some bad news earnings forecasts tend to be soft by comparison. They further document significantly positive market reactions to good news earnings forecasts that are accompanied by supplemental

[18] This interpretation is consistent with Baginski et al. (1993), who provide evidence that analysts' beliefs become more dispersed following management forecasts with wider ranges.

[19] These studies include Pownall et al. (1993) and Baginski et al. (1993) along with earlier work.

[20] Skinner (1994) and Bamber and Cheon (1998) documented that on average managers provide less specific (i.e., ranges rather point) forecasts when earnings news is bad than when earnings news is good, which is consistent with our prediction. However, notice that their explanation for this phenomenon differs from ours. They explained this phenomenon on the basis of managers' incentive to avoid legal liability or proprietary disclosure costs (in the sense of Verrecchia, 1983). In contrast, our explanation for the variation of forecast specificity is based on managers' incentive to influence investors' beliefs about the precision of forecast. In reality, of course, those two incentives could work together contributing to the observed phenomenon.

[21] They argue that relative to small firms, large firms have less discretion over non-earnings disclosures due to other competing information providers such as security analysts and financial press.

[22] Admittedly, how non-earnings information enhances or diminishes investors' confidence in earnings information is an open question and an interesting research topic.

disclosures, insignificant market reactions to good news earnings forecasts that are unaccompanied by such disclosures, and significant market reactions to bad news earnings forecasts irrespective of supplemental disclosures. If we interpret verifiable statements (soft talk) accompanying good (bad) news earnings forecasts as supplemental disclosures intended to convey high (low) forecast precision, their findings of higher incidence of verifiable statements accompanying good news earnings forecasts than accompanying bad news earnings forecasts, and more favorable market reactions to good news earnings forecasts so accompanied than unaccompanied, are consistent with our predictions. However, the lack of differential market reaction to bad news earnings forecasts based on whether or not they are accompanied by soft talk is difficult to reconcile with our analysis. This difficulty suggests that an interpretation of soft talk as low precision information may be questionable.[23]

6. Efficiency considerations

We now return to the model to complete the picture on voluntary disclosure of precision information by examining the entrepreneur's precision information acquisition decision and the resulting efficiency of the equilibrium for both the mandatory and voluntary disclosure cases. In this regard, recall from Section 2 that the entrepreneur's information gathering cost, c, is a private random variable and its distribution function H is common knowledge. In examining the efficiency of the equilibrium, we focus on the entrepreneur's ex ante welfare defined by her ex ante expected payoff net of the expected information acquisition cost. Since the entrepreneur is endowed with all the bargaining power in our model (i.e., she extracts all rents from investors through the selling price of the asset), ordering the entrepreneur's ex ante welfare is equivalent to ordering social welfare. Below, we state our results and discuss the underlying intuition, delegating the formal analysis and proof to Appendix B.

Proposition 3. *Given any distribution of the precision information acquisition cost,*

(i) *the entrepreneur acquires precision information more often in the voluntary disclosure case than in the mandatory disclosure case; and*
(ii) *the entrepreneur's net ex ante welfare in the voluntary disclosure case is lower than that in the mandatory disclosure case.*

The intuition is as follows. First, consider the mandatory disclosure case. After observing the realization of c, the entrepreneur's decision about whether to acquire precision information hinges on the comparison of the cost and benefit of being informed. The benefit is the difference between the equilibrium ex ante payoff she

[23] In this regard, it may be more appropriate to view soft talk as "cheap talk" in a modeling sense rather than as (hard) low precision information. While cheap talk could have value implications in some settings, there is no scope for this role in our analysis.

earns if informed and the payoff she earns if uninformed.[24] Since the precision information allows investors to make a more informative investment in the asset, it is clear that the informed entrepreneur's ex ante payoff, say, Ω_{i}, is strictly greater than that of the uninformed entrepreneur, say, Ω_{u}. Hence, the benefit from being informed of precision information, $\Delta \equiv \Omega_{\mathrm{i}} - \Omega_{\mathrm{u}}$, is positive. Given $\Delta > 0$, it follows that only the entrepreneur who observes c less than Δ will acquire precision information, implying that the probability that the entrepreneur is informed in the mandatory disclosure case equals $H(\Delta)$.[25]

Next, consider the voluntary disclosure case. The key difference from the mandatory disclosure case is that the informed entrepreneur has an option not to disclose precision information thereby pooling with the uninformed entrepreneur. This option is valuable. Specifically, notice that if the informed entrepreneur always reveals her precision information under voluntary disclosure, she can replicate her payoff under mandatory disclosure. However, the existence of nondisclosure equilibrium under voluntary disclosure implies that the informed entrepreneur is better off by withholding precision information for some realization of (y, β). This means that the informed entrepreneur's ex ante equilibrium payoff in the voluntary disclosure case, say, Ω_{i}^*, is strictly greater than her payoff in the mandatory disclosure case.

Now consider the uninformed entrepreneur. Her ex ante equilibrium payoff in the voluntary disclosure case, say, Ω_{u}^*, is strictly less than her payoff in the mandatory disclosure case. The reason is that, in contrast to the mandatory disclosure case, she is pooled with the informed entrepreneur whose asset has a lower expected net present value than the uninformed entrepreneur's asset. This makes the uninformed entrepreneur worse off relative to the mandatory disclosure case; her asset is either sold at a price lower than that in the mandatory disclosure case, or not traded even when it could be traded at a positive price in the mandatory disclosure case. Given the above discussion, it is clear that the benefit from being informed of precision information in the voluntary disclosure case, i.e., $\Delta^* \equiv \Omega_{\mathrm{i}}^* - \Omega_{\mathrm{u}}^*$, exceeds that in the mandatory disclosure case, i.e., $\Delta^* > \Delta$. Thus, the probability that the entrepreneur is informed in the voluntary disclosure case, $H(\Delta^*)$, is greater than that in the mandatory case. This explains the first part of Proposition 3.[26]

[24] The entrepreneur's ex ante payoff is defined to be the expected asset price where expectation is taken across all possible (y, β).

[25] While information gathering is not mandated under mandatory disclosure, it can be shown that the cutoff information acquisition cost below which the entrepreneur chooses to be informed, i.e., Δ, is the same as the cutoff value to which the entrepreneur would pre-commit before observing the realization of her information acquisition cost. Hence, the mandatory disclosure case is actually the same as the case where the entrepreneur makes a pre-commitment not only to the ex post disclosure of precision information when she is informed, but also to the optimal decision rule to become informed ex ante. See Appendix B for details.

[26] While commitment to an information acquisition strategy is superfluous in the mandatory disclosure case, observe that no commitment is credible in the voluntary disclosure case given that the information acquisition cost is privately observed. Part (i) of Proposition 3 is similar to Pae's (1999) result in a somewhat different setting. A notable difference is that in Pae's (1999) model information acquisition costs are entirely deadweight losses from an ex ante efficiency standpoint, whereas in our model the information is valuable due to its relevance to the investment decision of investors.

To explain the second part of Proposition 3, define the entrepreneur's net ex ante welfare by the weighted average of the informed entrepreneur's ex ante payoff and that of the uninformed entrepreneur (with the weight to the former being the probability of being informed) less the expected information acquisition cost. From the first part of Proposition 3, it is clear that there is an efficiency loss arising from the entrepreneur's over-investment in the acquisition of precision information (relative to the mandatory disclosure case). We now explain an additional efficiency loss associated with the investment in the asset due to nondisclosure of precision information in the voluntary disclosure case. There are two reasons for this additional efficiency loss. First, as noted earlier, the informed entrepreneur's nondisclosure may preclude the trade of the uninformed entrepreneur's asset that has a positive expected net present value. From the analysis in Sections 3 and 4, one can see that this case occurs when the estimate y leads to $\beta_m(y) < \beta_o(y) \leqslant \beta_H(y)$, e.g., when $y \in (y^u, y^h]$ in Fig. 3. Second, recall that the informed entrepreneur's asset is traded in the mandatory disclosure case only when its expected net present value is positive. In the voluntary disclosure case, however, her asset may be traded even when its expected net present value is negative. In particular, again from Sections 3 and 4, we know that when the estimate y leads to $\beta_o(y) > \beta_H(y)$ (e.g., when $y \in (y^h, k]$ in Fig. 3), the informed entrepreneur suppresses $\beta > \beta_o(y)$, and given no disclosure of β, the asset is traded. Hence, the efficiency loss in this case amounts to the negative expected net present value of the informed entrepreneur's asset that would not be traded in the mandatory disclosure case.[27]

Combining the over-investment in information acquisition and sub-optimal investment in the asset, we conclude that the entrepreneur's net ex ante welfare under voluntary disclosure is lower than that under mandatory disclosure. Since the ordering of the entrepreneur's net ex ante welfare is also a social welfare ordering, it follows that within the confines of our model, the policy implication of the above analysis is to mandate disclosure of precision information. The difficulty, however, is that without monitoring who acquires this information (recall that the cost of acquiring precision information is private in our model), it is hard to envision how such a policy could be enforced. Consequently, it seems likely that firms will continue to have considerable discretion over supplemental disclosures.

We conclude this section by noting that although our analysis only deals with the setting in which the entrepreneur acquires precision information before the estimate is realized, the efficiency implications of our model remain essentially unchanged if we consider an alternative setting where the acquisition decision is made after the estimate is realized. The principal difference is that since the entrepreneur chooses

[27] Note that for the other values of the estimate y, there is no efficiency loss associated with the investment in the asset due to nondisclosure of β in the voluntary disclosure case, relative to the mandatory disclosure case. When $y > k$, both the nondisclosing informed entrepreneur's asset and the uninformed entrepreneur's asset have a positive expected net present value and they are traded. On the other hand, when the estimate y leads to $\beta_H(y) > \beta_m(y) \geqslant \beta_o(y)$, (e.g., when $y \leqslant y^u$ in Fig. 3), the asset is not traded given no disclosure of β but its expected net present value is non-positive. That is, the asset owner is either the informed entrepreneur who withholds $\beta \geqslant \beta_o(y)$, or the uninformed entrepreneur whose precision information is $\beta_m(y) \geqslant \beta_o(y)$.

whether to become informed after observing the estimate y, the probability of the informed entrepreneur, λ, depends on y. Still it is easy to see that in this alternative setting the entrepreneur continues to over-invest in precision information acquisition under voluntary disclosure as compared to mandatory disclosure. The reason is that given the discretion over disclosure of precision information, the benefit from being informed in the voluntary disclosure case is no less than that in the mandatory disclosure case and is strictly greater for some y. Furthermore, inefficiencies in asset trading and subsequent investment persist. This follows from the fact that nontrivial partial disclosure equilibrium prevails thereby precluding full utilization of the precision information obtained.

7. Conclusion

This paper offers a rational explanation for selective voluntary disclosure of precision information that accompanies estimates of the value of tradable assets. The decision to disclose precision information, if acquired, is determined by an assessment of such disclosure's effect on the posterior expectation of the asset value. When an estimate of value is greater (less) than the prior expectation, asset sellers have an incentive to disclose high (low) precision information and to withhold low (high) precision information. One might design a direct test of these results, using forecast range as the inverse measure of precision.

As discussed in Section 5, although our results are derived in the context of voluntary disclosure of precision information, qualitatively similar results can be obtained in a broad array of settings, where a firm manager seeks to maximize investors' expectations of the firm's value. All we require is that investors' expectations are expressible as a linear function of a public signal (unrestricted in sign) and a slope coefficient to that signal represents the object of the manager's voluntary disclosure. A promising example for empirical testing might be where the coefficient relates to the persistence of earnings or revenue innovations.

Beyond the above generalization, we believe that our analysis captures the intuitively appealing idea of disclosing supplemental information that enhances (diminishes) confidence in good (bad) news, and not disclosing supplemental information that has the reverse effect. This characterization of incentives for and reactions to disclosures concurrent with earnings announcements/forecasts appears to have some currency as an explanation for the findings of empirical studies on supplemental disclosures.

A further dimension of this paper is the consideration of the equilibrium efficiency. Discretion over disclosure translates into an over-investment in information gathering and a sub-optimal investment in assets subsequent to trade. The option not to disclose precision information that lowers posterior beliefs about the asset value itself has a value, which would not be present if disclosure were mandatory. However, mandating disclosure of precision information seems less tenable than mandating disclosure of estimates. For example, all firms are required to report earnings, an estimate pertaining to firm value. But, in light of the wide variety of supplemental information that might also be disclosed, and uncertainty about the cost of obtaining such information, it would seem to be infeasible to mandate its disclosure. The contribution of our analysis of efficiency is to make the implicit cost of discretion transparent. Finally, it would seem worthwhile to broaden the theoretical inquiry to consider partial disclosure of multiple correlated signals that enter posterior expectations in ways other than the multiplicative form considered here. We commend this task to future research.

Appendix A

Proof of Proposition 2. (i) Viewing β_L characterized by (10) as a function of y, differentiate both sides of (10) with respect to y. Then, after rearranging the resulting terms,

$$\beta'_L(y)\left[1+\frac{\lambda}{1-\lambda}F(\beta_L(y)|y)\right] = \beta'_m(y) - \frac{\lambda}{1-\lambda}\int_0^{\beta_L} F_y(\beta|y)\,d\beta.$$

Given $y > \theta$, (7) and (C.2) imply that the right-hand side of the above equation is negative. Since

$$1+\frac{\lambda}{1-\lambda}F(\beta_L(y)|y) > 0,$$

it follows that $\beta'_L(y) < 0$.

(ii) Viewing β_H characterized by (12) as a function of y, differentiate both sides of (12) with respect to y. Then, after rearranging the resulting terms,

$$\beta'_H(y)\left[1+\frac{\lambda}{1-\lambda}[1-F(\beta_H(y)|y)]\right] = \beta'_m(y) - \frac{\lambda}{1-\lambda}\int_{\beta_H}^{1} F_H(\beta|y)\,d\beta.$$

Given $y < \theta$, (7) and (C.2) imply that the right-hand side of the above equation is positive. Since

$$1+\frac{\lambda}{1-\lambda}[1-F(\beta_H(y)|y] > 0,$$

it follows that $\beta_H{}'(y) > 0$. □

Lemma A.1. *Let $\zeta(y, \beta_o(y))$ be investors' beliefs about β when an estimate $y \leqslant k$ is released with no disclosure of $\beta \in [\beta_o(y), 1]$. Then,*

$$\zeta(y, \beta_o(y)) \geqslant \beta_o(y) \text{ if, and only if, } y \in Y_{NT} \equiv \{y \leqslant k | \beta_H(y) \geqslant \beta_o(y)\}. \tag{A.1}$$

Proof of Lemma A.1. Consider any $y \leqslant k$ and note that $\zeta(y, \beta_o(y)) \geqslant \beta_o(y)$ if, and only if,

$$\zeta(y, \beta_o(y)) = \frac{(1-\lambda)\beta_m(y) + \lambda\int_{\beta_o(y)}^1 \beta\,dF(\beta|y)}{(1-\lambda)+\lambda[1-F(\beta_o(y)|y)]} \geqslant \beta_o(y).$$

Replace $\beta_o(y)$ in the the above inequality by an arbitrary constant $b \in [0, 1]$. Then, after rearranging terms, the above inequality reduces to

$$Q(y,b) \equiv (1-\lambda)[\beta_m(y) - b] + \lambda\left[\int_b^1 (\beta - b)\,dF(\beta|y)\right] \geqslant 0 \quad \text{at } b = \beta_o(y).$$

Notice that

$$\frac{\partial Q(y,b)}{\partial b} = -1 + \lambda F(b|y) < 0.$$

In addition, from (11), we know that $\zeta(y, \beta_H(y)) = \beta_H(y)$, which is equivalent to $Q(y,b) = 0$ at $b = \beta_H(y)$. Hence, it must be true that

$$Q(y,b) \geqslant (<) 0 \quad \text{for all } b \leqslant (>) \beta_H(y). \tag{A.2}$$

We now show (A.1). First, suppose that $y \in Y_{NT}$. Then, since $\beta_o(y) \leqslant \beta_H(y)$ for any $y \in Y_{NT}$, (A.2) implies that $Q(y, \beta_o(y)) \geqslant 0$, which is equivalent to $\zeta(y, \beta_o(y)) \geqslant \beta_o(y)$. Next, we show that the converse is true, i.e., if $\zeta(y, \beta_o(y)) \geqslant \beta_o(y)$, then $y \in Y_{NT}$. Suppose that $y \notin Y_{NT}$. Then, since $\beta_o(y) > \beta_H(y)$, (A.2) implies that $Q(y, \beta_o(y)) < 0$, which is equivalent to $\zeta(y, \beta_o(y)) < \beta_o(y)$, a contradiction. Hence, it must be true that $y \in Y_{NT}$. $\square$

Appendix B. Information gathering and equilibrium efficiency

This appendix provides a formal analysis supporting the results we discussed in Section 6.

B.1. Information gathering and welfare in the mandatory disclosure case

Suppose that at $t = 1$ the entrepreneur has decided to acquire precision information. Then, at $t = 2$, she learns β and must disclose it concurrently with an estimate y. From the analysis in Section 3, we know that the entrepreneur's asset is traded to investors if, and only if, $\pi(y, \beta) > 0$. In particular, when $y \leqslant k$, the asset sold for $\pi(y, \beta) > 0$ if, and only if, $\beta < \beta_o(y)$. On the other hand, when $y > k$, the asset is sold for $\pi(y, \beta) > 0$ for any β. Thus, the informed entrepreneur's expected payoff conditional on y can be expressed as

$$\Omega_i(y) = \begin{cases} \int_0^{\beta_o(y)} \pi(y, \beta)\,dF(\beta|y) & \forall y \leqslant k, \\ \int_0^1 \pi(y, \beta)\,dF(\beta|y) & \forall y > k. \end{cases} \tag{B.1}$$

Letting G be the distribution function of y, we denote the informed entrepreneur's ex ante expected payoff as

$$\Omega_i \equiv \int_{-\infty}^{\infty} \Omega_i(y)\,dG(y). \tag{B.2}$$

Next, suppose that the entrepreneur has decided not to acquire precision information at $t=1$. In the mandatory disclosure case, the uninformed entrepreneur's lack of precision information is common knowledge. At $t=2$, she releases an estimate y with no disclosure of β, and the asset price in this case equals $\max\{0,\pi(y,\beta_{\mathrm{m}}(y))\}$. From (5), (6) and (C.1), it follows that the uninformed entrepreneur's asset is sold for $\pi(y,\beta_{\mathrm{m}}(y))$ only when $y>y^{\mathrm{u}}$. Hence, her expected payoff conditional on y is given by

$$\Omega_{\mathrm{u}}(y)=\begin{cases}0 & \forall y\leqslant y^{\mathrm{u}},\\ \pi(y,\beta_{\mathrm{m}}(y)) & \forall y>y^{\mathrm{u}},\end{cases} \tag{B.3}$$

and her ex ante expected payoff equals

$$\Omega_{\mathrm{u}}\equiv\int_{-\infty}^{\infty}\Omega_{\mathrm{u}}(y)\,\mathrm{d}G(y). \tag{B.4}$$

We now define the ex ante value of precision information as

$$\Delta\equiv\Omega_{\mathrm{i}}-\Omega_{\mathrm{u}}. \tag{B.5}$$

It can be shown that $\Delta>0$, implying that the entrepreneur is better off when she is informed of precision information relative to when she is uninformed.[28] Given that Δ is the ex ante benefit from precision information, the entrepreneur chooses to acquire precision information only when she observes the realization of her information acquisition cost c less than Δ. Hence, recalling that H is the distribution function of c, the probability that the entrepreneur becomes informed in the mandatory disclosure case must equal $H(\Delta)$.

Finally, given the preceding analysis, we define the entrepreneur's net ex ante welfare in the mandatory disclosure case by

$$W\equiv[H(\Delta)\Omega_{\mathrm{i}}+(1-H(\Delta))\Omega_{\mathrm{u}}]-\int_{0}^{\Delta}c\,\mathrm{d}H(c), \tag{B.6}$$

where the expression in the brackets denotes the entrepreneur's ex ante expected payoff given that she is informed with probability $H(\Delta)$, and $\int_0^{\Delta}c\,\mathrm{d}H(c)$ represents her expected cost of acquiring precision information. In the next section, we will compare W with the entrepreneur's ex ante welfare in the voluntary disclosure case.

Before proceeding to the next section, it is worth noting that the cutoff value of the information acquisition cost, Δ, below which the entrepreneur chooses to acquire precision information after observing the realization of c, is the same as the cutoff value to which she would pre-commit prior to knowing the realization of c. To see why such a commitment is optimal ex ante, replace Δ in the right-hand side of (B.6) with an arbitrary value δ and denote the resulting expression as $W(\delta)$. Now consider the problem of maximizing $W(\delta)$ with respect to δ, which is the problem that the entrepreneur would solve to determine the cutoff value in her commitment. The

[28] A detailed analysis is available upon request. The intuition is that when the entrepreneur has precision information and discloses it always, she in effect induces investors to invest only in a positive expected net present value asset. In contrast, the investment in the asset of the uninformed entrepreneur is based on the expected precision $\beta_{\mathrm{m}}(y)$, and thus, it is less efficient relative to the investment based on actual β.

optimal solution, δ^o, is unique and characterized by the first-order condition

$$[\Omega_i - \Omega_u] - \delta^o = 0,$$

implying $\delta^o = \Delta$ where Δ is given by (B.5). Thus, we can see that the equilibrium under mandatory disclosure is in fact the same as that in the case where the entrepreneur pre-commits not only to the ex post disclosure of precision information when she is informed, but also to a decision rule for becoming informed ex ante. Moreover, because maximizing the entrepreneur's ex ante welfare is equivalent to maximizing social welfare, a social planner would choose the same decision rule.

B.2. Information gathering and welfare in the voluntary disclosure case

When the entrepreneur has discretion over the disclosure of precision information, her equilibrium expected payoff differs from that in the mandatory disclosure case. As a result, her information acquisition decision is different. To formalize the analysis, we partition the set of estimates $y \leqslant k$ as follows:

$$Y_{NT}^{-} \equiv \{y \leqslant k | \beta_H(y) \geqslant \beta_o(y) \text{ and } \beta_m(y) \geqslant \beta_o(y)\}$$
$$Y_{NT}^{+} \equiv \{y \leqslant k | \beta_H(y) \geqslant \beta_o(y) \text{ and } \beta_m(y) < \beta_o(y)\}$$
$$Y_T \equiv \{y \leqslant k | \beta_H(y) < \beta_o(y)\}.$$

Clearly, $Y_{NT}^{-} \cup Y_{NT}^{+} \cup Y_T = (-\infty, k]$. Also note that $Y_{NT}^{-} \cup Y_{NT}^{+} = Y_{NT} \equiv \{y \leqslant k | \beta_H(y) \geqslant \beta_o(y)\}$, where Y_{NT} is the set of estimates such that when $y \in Y_{NT}$ is released with no disclosure of β, the asset is not traded (see Section 4.4). On the other hand, Y_T is the set of estimates no greater than k such that when $y \in Y_T$ is released with no disclosure of β, the asset is sold for $\pi(y, \beta_H(y)) > 0$. Relating the above definitions to the case depicted in Fig. 3, it is clear that

$$Y_{NT}^{-} = (-\infty, y^u],\ Y_{NT}^{+} = (y^u, y^h] \quad \text{and} \quad Y_T = (y^h, k].$$

Let $\Omega_i^*(y, \lambda)$ and $\Omega_u^*(y, \lambda)$ be the entrepreneur's expected payoff conditional on y if she is informed of precision information and if uninformed, respectively. For $y = \theta$, we know that precision information has no effect on the asset's expected net present value, which equals $\theta - k > 0$. Thus, it must be true that $\Omega_i^*(\theta, \lambda) = \Omega_u^*(\theta, \lambda) = \theta - k$ for any $\lambda \in (0, 1)$. For $y \neq \theta$, we have

$$\Omega_i^*(y, \lambda) = \begin{cases} \int_0^{\beta_o(y)} \pi(y, \beta)\, dF(\beta|y) & \forall y \in Y_{NT}, \\ \int_0^{\beta_H(y)} \pi(y, \beta)\, dF(\beta|y) + \int_{\beta_H(y)}^1 \pi(y, \beta_H(y))\, dF(\beta|y) & \forall y \in Y_T \cup (k, \theta), \\ \int_0^{\beta_L(y)} \pi(y, \beta_L(y))\, dF(\beta|y) + \int_{\beta_L(y)}^1 \pi(y, \beta)\, dF(\beta|y) & \forall y \in (\theta, \infty), \end{cases} \tag{B.7}$$

and

$$\Omega_u^*(y, \lambda) = \begin{cases} 0 & \forall y \in Y_{NT}, \\ \pi(y, \beta_H(y)) > 0 & \forall y \in Y_T \cup (k, \theta), \\ \pi(y, \beta_L(y)) > 0 & \forall y \in (\theta, \infty), \end{cases} \tag{B.8}$$

where $\Omega_{\rm i}^{*}(y,\lambda)$ and $\Omega_{\rm u}^{*}(y,\lambda)$ depend on λ through the cutoff functions $\beta_j(y)$, $j=$ H, L. To explain the above expressions, first consider the case where $y\in Y_{\rm NT}$. We know that the nondisclosure set in this case is $[\beta_{\rm o}(y),1]$ and the asset is not traded when there is no disclosure of β. Thus, the uninformed entrepreneur's payoff is zero, and only the entrepreneur informed of $\beta<\beta_{\rm o}(y)$ discloses that precision information and sells the asset for $\pi(y,\beta)>0$. Next, consider the case where $y\in Y_{\rm T}\cup(k,\theta)$. If the entrepreneur is informed of $\beta<\beta_{\rm H}(y)$, she discloses it and sells her asset for $\pi(y,\beta)$. Otherwise, she withholds precision information and sells the asset for $\pi(y,\beta_{\rm H}(y))$. The uninformed entrepreneur, being unable to make any disclosure of precision information, sells her asset for $\pi(y,\beta_{\rm H}(y))$. Finally, when $y>\theta$, the informed entrepreneur sells the asset with disclosure of β if, and only if, $\beta>\beta_{\rm L}(y)$. In contrast, the uninformed entrepreneur sells the asset for $\pi(y,\beta_{\rm L}(y))$.

As in the mandatory disclosure case, we denote the entrepreneur's ex ante expected payoffs when she is informed of precision information and when she is not, respectively, as

$$\Omega_{\rm i}^{*}(\lambda)\equiv\int_{-\infty}^{\infty}\Omega_{\rm i}^{*}(y,\lambda)\,{\rm d}G(y)\quad\text{and}$$
$$\Omega_{\rm u}^{*}(\lambda)\equiv\int_{-\infty}^{\infty}\Omega_{\rm u}^{*}(y,\lambda)\,{\rm d}G(y).\tag{B.9}$$

The ex ante value of precision information under voluntary disclosure thus equals

$$\Delta(\lambda)\equiv\Omega_{\rm i}^{*}(\lambda)-\Omega_{\rm u}^{*}(\lambda).\tag{B.10}$$

One can show that $\Delta^{\dagger}(\lambda)>0$ for any $\lambda\in(0,1)$, i.e., the entrepreneur is better off when she is informed of precision information relative to when she is uninformed. Now observe that since the entrepreneur chooses to acquire precision information when the information acquisition cost is less than $\Delta^{\dagger}(\lambda)$, the cutoff value of the information acquisition cost, say, δ, must satisfy $\delta=\Delta^{\dagger}(H(\delta))$, where we use the fact that $\lambda=H(\delta)$. Let δ^{*} be that cutoff value and denote $\Delta^{*}\equiv\Delta^{\dagger}(H(\delta^{*}))$. Then, the equilibrium probability that the entrepreneur becomes informed in the voluntary disclosure case equals $H(\Delta^{*})$. Finally, let W^{*} be the entrepreneur's net ex ante welfare in the voluntary disclosure case, i.e.,

$$W^{*}\equiv[H(\Delta^{*})\Omega_{\rm i}^{*}+(1-H(\Delta^{*}))\Omega_{\rm u}^{*}]-\int_{0}^{\Delta^{*}}c\,{\rm d}H(c),\tag{B.11}$$

where $\Omega_{\tau}^{*}\equiv\Omega_{\tau}^{*}(\lambda^{*})$, $\tau=i,u$, and $\lambda^{*}\equiv H(\Delta^{*})$. Given the above analysis, Proposition 3 in Section 6 can be restated as

Proposition 3. *Given any distribution of the precision information acquisition cost,*

(i) $\Delta^{*}>\Delta$; *and*
(ii) $W^{*}<W$.

Proof of Proposition 3. (i) Define $\Delta(y) \equiv \Omega_{\rm i}(y) - \Omega_{\rm u}(y)$. Using (B.1) and (B.3), we have

$$\Delta(y) = \begin{cases} \int_0^{\beta_{\rm o}(y)} \pi(y,\beta)\,{\rm d}F(\beta|y) > 0 & \forall y \in Y_{\rm NT}^{+}, \\ -\int_{\beta_{\rm o}(y)}^{1} \pi(y,\beta)\,{\rm d}F(\beta|y) > 0 & \forall y \in Y_{\rm NT}^{+} \cup Y_{\rm T}, \\ 0 & \forall y \in (k,\infty). \end{cases}$$

Next, define $\Delta^*(y) \equiv \Omega_{\rm i}^*(y,\lambda^*) - \Omega_{\rm u}^*(y,\lambda^*)$. By using (B.7) and (B.8), it is straightforward to verify that for $y \neq \theta$,

$$\Delta^*(y) = \begin{cases} \int_0^{\beta_{\rm o}(y)} \pi(y,\beta)\,{\rm d}F(\beta|y) = \Delta(y) > 0 & \forall y \in Y_{\rm NT}^{-}, \\ \int_0^{\beta_{\rm o}(y)} \pi(y,\beta)\,{\rm d}F(\beta|y) > \Delta(y) > 0 & \forall y \in Y_{\rm NT}^{+}, \\ \int_0^{\beta_{\rm H}(y)} [\pi(y,\beta) - \pi(y,\beta_{\rm H}(y))]\,{\rm d}F(\beta|y) > \Delta(y) > 0 & \forall y \in Y_{\rm T}, \\ \int_0^{\beta_{\rm H}(y)} [\pi(y,\beta) - \pi(y,\beta_{\rm H}(y))]\,{\rm d}F(\beta|y) > \Delta(y) = 0 & \forall y \in (k,\theta), \\ \int_{\beta_L(y)}^{1} [\pi(y,\beta) - \pi(y,\beta_{\rm L}(y))]\,{\rm d}F(\beta|y) > \Delta(y) = 0 & \forall y \in (\theta,\infty). \end{cases}$$

In addition, we know that $\Delta^*(\theta) = 0$ since $\Omega_{\rm i}^*(\theta,\lambda^*) = \Omega_{\rm u}^*(\theta,\lambda^*) = \theta - k$. From the above results, it follows that $\Delta^*(y) > \Delta(y)$ for all $y \notin Y_{\rm NT}^{-} \cup \{\theta\}$, which implies that

$$\Delta^* - \Delta = \int_{-\infty}^{\infty} [\Delta^*(y) - \Delta(y)]\,{\rm d}G(y) > 0.$$

(ii) Consider an arbitrary $\lambda \in (0,1)$ and define

$$\Omega(y,\lambda) \equiv \lambda\Omega_{\rm i}(y) + (1-\lambda)\Omega_{\rm u}(y), \tag{B.12}$$

where $\Omega_{\rm i}(y)$ and $\Omega_{\rm u}(y)$ are given in (B.1) and (B.3), respectively. Then, it is easy to check that

$$\Omega(y,\lambda) = \begin{cases} \lambda \int_0^{\beta_{\rm o}(y)} \pi(y,\beta)\,{\rm d}F(\beta|y) & \forall y \in Y_{\rm NT}^{-}, \\ \lambda \int_0^{\beta_{\rm o}(y)} \pi(y,\beta)\,{\rm d}F(\beta|y) + (1-\lambda)\pi(y,\beta_{\rm m}(y)) & \forall y \in Y_{\rm NT}^{+} \cup Y_{\rm T}, \\ \pi(y,\beta_{\rm m}(y)) & \forall y \in (k,\infty). \end{cases} \tag{B.13}$$

Next, define

$$\Omega^*(y,\lambda) \equiv \lambda\Omega_{\rm i}^*(y,\lambda) + (1-\lambda)\Omega_{\rm u}^*(y,\lambda), \tag{B.14}$$

where $\Omega_{\rm i}^*(y,\lambda)$ and $\Omega_{\rm u}^*(y,\lambda)$ are given in (B.7) and (B.8), respectively. For $y = \theta$, note that $\Omega^*(\theta,\lambda) = \theta - k$. Using the results in Section 4, one can verify that

$$\Omega^*(y,\lambda) = \begin{cases} \lambda\Omega_{\rm i}(y) & \forall y \in Y_{\rm NT}^{+}, \\ \lambda \int_0^1 \pi(y,\beta)\,{\rm d}F(\beta|y) + (1-\lambda)\Omega_{\rm u}(y) & \forall y \in Y_{\rm T}, \\ \Omega(y,\lambda) & \forall y \in Y_{\rm NT}^{-} \cup (k,\infty). \end{cases} \tag{B.15}$$

Therefore,

$$\Omega(y,\lambda) - \Omega^*(y,\lambda) = \begin{cases} (1-\lambda)\Omega_{\mathrm{u}}(y) > 0 & \forall y \in Y_{\mathrm{NT}}^{+}, \\ -\lambda \int_{\beta_{\mathrm{o}}(y)}^{1} \pi(y,\beta)\,\mathrm{d}F(\beta|y) > 0 & \forall y \in Y_{\mathrm{T}}, \\ 0 & \forall y \in Y_{\mathrm{NT}}^{-} \cup (k,\infty). \end{cases} \quad \text{(B.16)}$$

Thus far, we have fixed an arbitrary $\lambda \in (0,1)$. For any given λ, we can find a cutoff value δ of the information acquisition cost corresponding to λ such that $\lambda = H(\delta)$. Given this cutoff value δ, the entrepreneur's net ex ante welfare in the mandatory disclosure case equals

$$\begin{aligned} W(\delta) &\equiv \int_{-\infty}^{\infty} \Omega(y, H(\delta))\,\mathrm{d}G(y) - \int_0^{\delta} c\,\mathrm{d}H(c) \\ &= \int_{-\infty}^{\infty} [H(\delta)\Omega_{\mathrm{i}}(y) + (1-H(\delta))\Omega_{\mathrm{u}}(y)]\,\mathrm{d}G(y) - \int_0^{\delta} c\,\mathrm{d}H(c) \\ &= [\Omega_{\mathrm{u}} + H(\delta)\Delta] - \int_0^{\delta} c\,\mathrm{d}H(c), \end{aligned} \quad \text{(B.17)}$$

where $\Delta = \Omega_{\mathrm{i}} - \Omega_{\mathrm{u}}$. Since $\delta = \Delta$ in the equilibrium of the mandatory disclosure case, $W(\delta)$ evaluated at $\delta = \Delta$ must be equal to W given in (B.6). In addition, observe that

$$\Delta \in \arg\max_{\delta}\ H(\delta)\Delta - \int_0^{\delta} c\,\mathrm{d}H(c). \quad \text{(B.18)}$$

Next, for an arbitrary $\lambda = H(\delta)$, denote the entrepreneur's net ex ante welfare in the voluntary disclosure case as

$$\begin{aligned} W^*(\delta) &\equiv \int_{-\infty}^{\infty} \Omega^*(y, H(\delta))\,\mathrm{d}G(y) - \int_0^{\delta} c\,\mathrm{d}H(c) \\ &= \int_{-\infty}^{\infty} [H(\delta)\Omega_{\mathrm{i}}^*(y, H(\delta)) + (1-H(\delta))\Omega_{\mathrm{u}}^*(y, H(\delta))]\,\mathrm{d}G(y) - \int_0^{\delta} c\,\mathrm{d}H(c) \\ &= [\Omega_{\mathrm{u}}^*(H(\delta)) + H(\delta)\Delta(H(\delta))] - \int_0^{\delta} c\,\mathrm{d}H(c), \end{aligned} \quad \text{(B.19)}$$

where $\Delta(H(\delta)) = \Delta(\lambda) \equiv \int_{-\infty}^{\infty} [\Omega_{\mathrm{i}}^*(y,\lambda) - \Omega_{\mathrm{u}}^*(y,\lambda)]\,\mathrm{d}G(y)$. In the equilibrium of the voluntary disclosure case, we must have $\delta = \Delta^*$, implying that $W^*(\delta)$ evaluated at $\delta = \Delta^*$ must equal W^* given in (B.11).

We now show the desired result, $W(\Delta) > W^*(\Delta^*)$. From above analysis,

$$\begin{aligned} W(\Delta) &= \Omega_{\mathrm{u}} + H(\Delta)\Delta - \int_0^{\Delta} c\,\mathrm{d}H(c) \\ &> \Omega_{\mathrm{u}} + H(\Delta^*)\Delta - \int_0^{\Delta^*} c\,\mathrm{d}H(c) \\ &> \Omega_{\mathrm{u}}^* + H(\Delta^*)\Delta^* - \int_0^{\Delta^*} c\,\mathrm{d}H(c) = W^*(\Delta^*). \end{aligned} \quad \text{(B.20)}$$

The first inequality holds due to (B.18). To explain the second inequality, observe that

$$\Omega_{\mathrm{u}} + H(\Delta^*)\Delta = \int_{-\infty}^{\infty} \Omega(y, \lambda^*)\,\mathrm{d}G(y),$$

where $\lambda^* \equiv H(\Delta^*)$ and $\Omega(\cdot)$ is given by (B.13). On the other hand,

$$\Omega_{\mathrm{u}}^* + H(\Delta^*)\Delta^* = \int_{-\infty}^{\infty} \Omega^*(y, \lambda^*)\,\mathrm{d}G(y),$$

where $\Omega^*(\cdot)$ is given by (B.15). Hence,

$$[\Omega_{\mathrm{u}} + H(\Delta^*)\Delta] - [\Omega_{\mathrm{u}}^* + H(\Delta^*)\Delta^*] = \int_{-\infty}^{\infty} [\Omega(y, \lambda^*) - \Omega^*(y, \lambda^*)]\,\mathrm{d}G(y).$$

Now observe that we must have $\Omega(y, \lambda^*) - \Omega^*(y, \lambda^*) > 0$ for all $y \in Y_{\mathrm{NT}}^+ \cup Y_{\mathrm{T}}$ because (B.16) holds for any arbitrary λ. Hence, the right-hand side of the above equation is positive, which implies that the second inequality in (B.20) must hold. □

References

Baginski, S.P., Conrad, E.J., Hassell, J.M., 1993. The effects of management forecast precision on equity pricing and on the assessment of earnings uncertainty. The Accounting Review 68, 913–927.

Bamber, L.S., Cheon, Y.S., 1998. Discretionary management earnings forecast disclosures: antecedents and outcomes associated with forecast venue and forecast specificity choices. Journal of Accounting Research 36, 167–190.

Brown, L.D., Kim, K.J., 1993. The association between non-earnings disclosures by small firms and positive abnormal returns. The Accounting Review 68, 668–680.

Dye, R.A., 1985. Disclosure of nonproprietary information. Journal of Accounting Research 23, 123–145.

Grossman, S.J., 1981. The informational role of warranties and private disclosure about product quality. Journal of Law and Economics 24, 461–483.

Hutton, A.P., Miller, G.S., Skinner, D.J., 2003. The role of supplementary statements with management earnings forecasts. Journal of Accounting Research 41 (5), 867–890.

Jung, W.O., Kwon, Y.K., 1988. Disclosure when the market is unsure of information endowment of managers. Journal of Accounting Research 26, 146–153.

Milgrom, P., 1981. Good news and bad news: representation theorems and applications. Bell Journal of Economics 12, 380–391.

Pae, S., 1999. Acquisition and discretionary disclosure of private information and its implications for firms' productive activities. Journal of Accounting Research 37, 465–474.

Pae, S., 2002. Discretionary disclosure, efficiency, and signal informativeness. Journal of Accounting and Economics 33, 279–311.

Penno, M.C., 1996. Unobservable precision choices in financial reporting. Journal of Accounting Research 34, 141–149.

Penno, M.C., 1997. Information quality and voluntary disclosure. The Accounting Review 72, 275–284.

Pownall, G., Wasley, C., Waymire, G., 1993. The stock price effects of alternative types of management earnings forecasts. The Accounting Review 68, 896–912.

Skinner, D.J., 1994. Why firms voluntarily disclose bad news. Journal of Accounting Research 32, 39–60.

Subramanyam, K.R., 1996. Uncertain precision and price reactions to information. The Accounting Review 71, 207–219.

Verrecchia, R.E., 1983. Discretionary disclosure. Journal of Accounting and Economics 5, 179–194.