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Industry-Wide Disclosure Dynamics

RONALD A. DYE AND SRI S. SRIDHAR*

1. Introduction

Voluntary disclosures by some firms seem to provoke other firms to make related disclosures. After Citibank announced the extent to which its Third World loans were in default, many other “money center” banks did likewise.¹ After Chambers Development took a write-off for their waste disposal arrangements, many other waste management firms did too.² Several insurance companies successively disclosed their potential liability from hurricanes Hugo³ and Andrew,⁴ and the Los Angeles earthquakes.⁵ Besides having disclosures that occur in “herds,” there appears to be another characteristic common to each of these examples: the disclosures were motivated by managers’ attempt to influence the financial market’s assessment of the firms’ values, rather than the product market behavior of other firms. In this paper, we propose a theory to explain such disclosure dynamics, when each firm selects a disclosure policy to maximize its expected price at each point in time, while taking into account that other firms behave similarly.

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¹ *Washington Post* (May 20, 1987), p. A1, and *Inter News Service* (June 9, 1987).

² *Wall Street Journal* (October 21, 1992) and (March 19, 1992).

³ *National Underwriter, Property and Casualty/Risk and Benefits Management Edition* (October 30, 1989).

⁴ *Business Insurance* (October 26, 1992).

⁵ For instance, *Reuters* (October 19, 1989) and *Journal of Commerce* (February 17, 1994).

The existing literature on firms' voluntary disclosure propensities does not provide an explanation for this phenomenon. The literature that examines a firm's propensity to make value-maximizing disclosures either ignores or takes as exogenously given the effects of a firm's disclosure on other firms (see, e.g., Dye [1985], Jung and Kwon [1988], and Verrecchia [1983]). The literature that examines the interaction among several firms' disclosure policies emphasizes the product market effects of the disclosures, such as the influence of disclosures on other firms' output levels, rather than on the financial market effects of the disclosures (see, e.g., Clarke [1983], Fried [1984], Feltham, Giger, and Hughes [1992], and Darrrough [1993]).

We assume that one firm's receipt of information is positively correlated with the receipt of information by other firms in the industry. This correlation by itself does not create interdependencies among firms' disclosures. If investors know when the individual firms receive information, firms will disclose all their information to distinguish themselves from other firms with even worse information (Grossman [1981] and Milgrom [1981]). If investors do not know when firms receive information, however, one firm's disclosure causes investors to update their assessments that other firms (who have not disclosed) have received information. We show that, as long as there is a positive correlation among firms' receipt of information, this revision in investors' perceptions causes disclosures by a sufficient number of firms to increase the probability that other firms will disclose their information subsequently.

Since this herding behavior is a *multifirm* phenomenon, we seek to determine how firms' equilibrium disclosure policies vary with how much each firm knows about its competitors' information, with the number of firms, and with the history of past disclosures in the industry. To obtain such comparative statistics, we specialize the model by assuming that if any one firm in an industry receives information about itself during a period, then so do all other firms in the same industry.⁶ This assumption is descriptive of some industry-specific event that causes a new dimension of firms' production technologies/product characteristics to become especially value relevant. For example, a country's nationalization of production facilities makes the size of firms' foreign investments in the country important.

⁶ The assumption that all firms receive information during a "period" imposes restrictions on the "period's" length. A "period" must be long enough so that each firm in the relevant cohort group can obtain information about itself whenever any member of the cohort group has, and yet short enough so investors do not learn that firms in the cohort group have received information during the period.

It is plausible that there were time intervals during which manufacturers of silicon breast implants, asbestos, and other products prone to liability claims, but not investors (or the potential purchasers of these products), were aware of the risks associated with their use. The time interval in such examples constitutes the first "period" in the model developed below.

Given our assumption about the arrival of information, we show that a firm's equilibrium disclosure policy depends on whether the firm receives information only about itself (the "private knowledge" case) or receives information about other firms as well (the "industry-wide common knowledge" case). In both cases, a firm is willing to disclose only if the information it receives is above a threshold. In the private knowledge case, each individual firm is *more* inclined to disclose as the number n of firms (in the industry) that receive information increases, since the threshold level decreases in n . In contrast, in the industry-wide common knowledge case, individual firms are *less* likely to disclose as n increases, since the threshold increases in n . And yet, in both cases, the probability that *some* firm discloses its information increases with n , implying that as more firms receive information, the probability that *all* firms eventually make a disclosure increases (since once one firm discloses, investors know that all firms have received information, and then the Grossman-Milgrom result applies). This last result is intuitive: as more firms possess private information about some new value-relevant dimension, it becomes more likely that every firm will disclose information on its performance along that dimension.

Our results also have implications for how the expected prices of non-disclosing firms vary with time and the number of firms in the industry, since we demonstrate that the threshold disclosure level for each firm in each period is also the expected price the firm receives when no firm discloses. This result, in conjunction with the previously described results, implies that in the private knowledge (industry-wide common knowledge) case, the expected price of a nondisclosing firm is decreasing (increasing) in the number of firms in the industry. We further show that for both cases there are equilibria in which if no firm discloses its information immediately upon receipt, then no firm ever does so.

In addition, we establish that firms tend to release good news earlier than bad news. This occurs because the financial markets revise upward their beliefs that a nondisclosing firm has received information after observing other firms in the industry making disclosures. This forces firms with less favorable information also to disclose in subsequent periods to separate themselves from firms with even worse information.

Among the papers previously cited, this work is most closely related to Dye [1985] and Jung and Kwon [1988]. Those papers examine how *one* firm's propensity to disclose information changes as investors' perception of the probability that the firm received information changes exogenously. In contrast, we evaluate how this probability evolves endogenously, as investors learn about whether a firm has information through the disclosures or nondisclosures of other firms over time.

The next section describes the basic model. Section 3 considers the general private knowledge model where firms' receipt of information is positively, but imperfectly, correlated. Section 4 specializes the model of section 3 to the case where information arrives to the industry in "waves," i.e., if any firm receives information, then all firms do.

Section 5 motivates the industry-wide common knowledge case further and then analyzes that case. Section 6 contains concluding comments.

2. *The Basic Model*

In all the models we examine, the sequence of events is as follows: in the first period, each firm either receives value-relevant information about itself or not. No information is received after the first period.⁷ The manager of each firm that receives information independently and simultaneously decides whether to disclose that information in the first period. If a firm has information, the only options are to disclose nothing or to disclose the information. A manager of a firm with no information necessarily makes no disclosure; however, a firm cannot credibly state that it has no information. After the (non)disclosures are witnessed, investors establish a period 1 price for each firm, equal to the expected value of its cash flows conditional on available information and their conjectures about the firms' disclosure policies (as described below).

The managers of firms that received but did not disclose information in the first period have another opportunity to do so in the second period. These managers again make their disclosure decisions independently and simultaneously, this time predicated on the first-period disclosures. Then, investors establish a second-period price for each firm's shares equal to the expected value of its cash flows, based on all information released over the two periods. There are no disclosures or other actions that occur after the second period.

In each period, each manager is assumed to maximize the expected price of his firm for that period, taking as given his information, his conjectures about the disclosure policies of the managers of other firms, and (in the case of period 2 disclosure policies) the history of past disclosures. An equilibrium exists when each manager's and all investors' conjectures about other managers' disclosure policies are correct. The equilibrium is described formally in Definition 1 below.

In our model, the information in question is the realization of a random variable \tilde{x}_i , which represents the expected value of firm i 's cash flows. Thus, if firm i 's manager discloses x_i , the value of firm i is x_i . If the manager does not disclose any information, the firm's value is the expected value of \tilde{x}_i , conditional on the manager's conjectured disclosure policy and all current and past disclosures of all firms.

In general, we assume that whether any firm $i \in \{1, 2, \dots, n\}$ learns the realization of the random variable \tilde{x}_i is positively correlated with whether the other $n - 1$ firms do. To represent this correlation, we assume a pair of events, " E " and " NE ," occurring with respective probabilities $p \in (0, 1)$ and $1 - p$, that affect whether any firm receives information. Neither managers nor investors learn whether event E itself has

⁷ This imposes restrictions on the length of a period, as discussed in the previous footnote.

occurred.⁸ Given E (resp., NE) occurs, the probability $q \in (0, 1)$ (resp., $\hat{q} \in [0, q]$) that each firm receives information is independent of that of every other firm.

We assume that cash flows \tilde{x}_i and \tilde{x}_j of firms i and j are independent of each other for $i \neq j$;⁹ the support of \tilde{x}_i is the interval $[\underline{x}_i, \bar{x}_i]$, with $\bar{x}_i < \bar{x}_j$. $F_i(\cdot)$ and $f_i(\cdot)$ denote the cumulative distribution and density functions of \tilde{x}_i ; $f_i(\cdot)$ is assumed to be continuous.

3. The General Private Knowledge Model

In the general private knowledge model, if firm i 's manager learns anything, he learns only the realization of \tilde{x}_i .

A manager who receives no information faces no disclosure decision. If a manager receives but does not disclose his information, then his firm's expected price is independent of the information. In contrast, if the manager discloses x_i , then the price of the firm is x_i . These remarks apply to both periods. Accordingly, in period 1 there exists a cutoff x_{i1c}^n such that, if the manager of firm i has information x_i , he will disclose it in period 1 if and only if $x_i \geq x_{i1c}^n$. (The superscript n refers to the number of firms.) Similar logic applies to second-period disclosures, except that investors' posterior probability that firm i has received information, given that it did not disclose in the first period, is affected by the number k of first-period disclosers. Accordingly, firm i 's second-period cutoff $x_{i2c}^n(k)$ varies with k , and its manager discloses x_i , when known, in the second period provided $x_{i1c}^n > x_i \geq x_{i2c}^n(k)$. A firm's disclosure policy is completely described by these cutoffs.

To define an equilibrium, we need some additional notation. Fix a collection of cutoffs $\{x_{i1c}^n\}_i, \{x_{i2c}^n(k)\}_i$. For these cutoffs, let $\Pr_i(\text{info} | k)$ denote investors' posterior probability that firm i has information given that k firms other than i disclose information in period 1. Likewise, let $\Pr_i(\text{info} | k, l)$ denote investors' posterior probability that firm i has information, given that k firms other than i disclose in period 1 and an additional l firms other than i disclose in period 2. Finally, let $\Pr_i(k | \text{info})$ and $\Pr_i(l | k, \text{info})$ denote firm i 's perception of the probability that k firms other than i will disclose information in period 1, and given that, an additional l firms other than i will disclose information in period 2,

⁸ This is done to simplify the analytics: if firms learned whether event E occurred, they might adopt different disclosure policies predicated on this outcome. We do not invoke this condition when $q = 1$ and $\hat{q} = 0$, because in that case, learning the realization of x_i is equivalent to learning that event E occurred.

⁹ Note that the independence of \tilde{x}_i and \tilde{x}_j for $i \neq j$ precludes any common "industry effects" regarding the firms' realized cash flows. Such industry effects can be incorporated into the analysis easily, however, as long as the industry effect is itself publicly observable. Specifically, if for each firm i in the industry, its cash flows can be written as the sum $\tilde{x}_i + \bar{\eta}$, where $\bar{\eta}$ is the common publicly observable industry effect, then all of the analysis in the text applies without modification.

when firm i has information. (Obviously, these conditional probabilities vary with the cutoffs, but to reduce notational clutter, this dependence is not explicitly expressed.)

DEFINITION 1. An equilibrium in the general private knowledge model is a collection of cutoffs $\{x_{ic}^n\}_{i,t}$ and prices $\{P_{it}^{ND}\}_{i,t}$ such that:

- (a) if the manager of firm i learns x_i , then he discloses x_i in period 1 if and only if $x_i \geq x_{i1c}^n$.
- (b) if the manager of firm i learns x_i and k firms other than i make a disclosure in period 1, then the manager discloses x_i in period 2 if and only if $x_i \geq x_{i2c}^n(k)$.
- (c) if k firms other than i , but not i , disclose information in period 1, then the price of firm i in period 1 is $P_{i1}^{ND}(k)$, where:

$$P_{i1}^{ND}(k) = \Pr_i(\text{info} \mid k) E[\tilde{x}_i \mid \tilde{x}_i < x_{i1c}^n] + \Pr_i(\text{no info} \mid k) E[\tilde{x}_i].$$

- (d) If k firms other than i disclose information in period 1, and l additional firms other than i disclose information in period 2, but firm i discloses no information, then the price of firm i in period 2 is $P_{i2}^{ND}(k, l)$, where:

$$P_{i2}^{ND}(k, l) = \Pr_i(\text{info} \mid k, l) E[\tilde{x}_i \mid \tilde{x}_i < x_{ic}^n(k)] + \Pr_i(\text{no info} \mid k, l) E[\tilde{x}_i],$$

where: $x_{ic}^n(k) = \text{Min}\{x_{i1c}^n, x_{i2c}^n(k)\}$.

$$(e) \quad x_{i1c}^n = \sum_{k=0}^{n-1} \Pr_i(k \mid \text{info}) P_{i1}^{ND}(k).$$

$$(f) \quad x_{i2c}^n(k) = \sum_{l=0}^{n-1-k} \Pr_i(l \mid k, \text{info}) P_{i2}^{ND}(k, l)$$

for every $k \in \{0, 1, \dots, n-1\}$.

Parts (a) and (b) state that the disclosure regions are “upper-tailed,” consistent with the description of disclosure policies preceding the formal definition of the equilibrium. Parts (c) and (d) state that, if a firm makes no disclosures in a period, then the firm’s equilibrium price in that period is a weighted average of its price assuming outsiders are certain the firm received no information ($E[\tilde{x}_i]$) and the price assuming outsiders knew that the manager had information and chose not to disclose it (in period 1, this is $E[\tilde{x}_i \mid \tilde{x}_i < x_{i1c}^n]$; in period 2, this is $E[\tilde{x}_i \mid \tilde{x}_i < x_{ic}^n(k)]$).

To understand part (e), note that the cutoff x_{i1c}^n must be such that if the manager of firm i learns $\tilde{x}_i = x_{i1c}^n$, then he must be indifferent between disclosure and no disclosure. If he discloses, the firm’s price is x_{i1c}^n , and if he does not disclose, the price is $P_{i1}^{ND}(k)$ if k firms other than i make a disclosure. The manager sets the cutoff at the firm’s expected price given no disclosure, where expectations are taken with respect to the number of other firms that may disclose in period 1. Part (f) is the second-period counterpart to part (e).

There may be multiple sets of equilibrium cutoffs, some of which are more plausible than others. If $\{x_{i1c}^n\}_i$ and $\{\hat{x}_{i1c}^n\}_i$ are both equilibrium period 1 cutoffs, with $\hat{x}_{i1c}^n \leq x_{i1c}^n$ for all i , we argue that $\{x_{i1c}^n\}_i$ is the more plausible equilibrium. The argument runs as follows: as Definition 1 (e) shows, the cutoff for any firm is equal to its expected price given no disclosure. Now, suppose a manager could influence investors' perceptions of which equilibrium was in force by taking some action. Which would he choose? If he discloses, his firm's price is independent of investors' perceptions of his disclosure policy (i.e., the cutoff chosen). But, if the manager's realization of \tilde{x}_i were such that he preferred not to disclose—and he could affect investors' perceptions before any other firm disclosed—obviously he would prefer that investors believe that the adopted equilibrium is the one which maximizes his firm's expected price, given that he makes no disclosure. This equilibrium is the one with the higher cutoff. Now, even if there were no action which would signal which equilibrium had been implemented, investors (aware of every manager's preference for this equilibrium) would expect firms to adopt the equilibrium with the higher cutoffs. This same logic applies to equilibrium period 2 cutoffs.

Appealing to the preceding logic, we state some of our results below in terms of *undominated cutoffs*: a set of equilibrium period one cutoffs $\{x_{i1c}^n\}_i$ is called undominated if there is no other set of equilibrium cutoffs $\{x_{i1c}^{n'}\}_i$ for which $x_{i1c}^{n'} \geq x_{i1c}^n$ for all i , with a strict inequality holding for at least one i . Undominated cutoffs for period 2 are defined analogously.

THEOREM 1. In the general private knowledge model with $1 > q > \hat{q} \geq 0$, if $\{\tilde{x}_i\}_i$ is independently and identically distributed for each $i \in \{1, 2, \dots, n\}$, and if $x_i - E[\tilde{x}_i \mid \tilde{x}_i \leq x_i]$ is weakly increasing in x_i for every i , then for any equilibrium $\{x_{itc}^n\}_{i,t}$, $\{P_{it}^{ND}\}_{i,t}$, the following hold:

- (a) $P_{i1}^{ND}(k)$ is strictly decreasing in k for every i ; and, for any undominated period 2 cutoffs,
- (b) there exists a $k^* \in \{1, \dots, n-1\}$ such that:
 - (b.i) if the number k of firms that disclosed in period 1 is less than or equal to k^* , then no firm that received, but did not disclose, information in period 1 will disclose its information in period 2 and $P_{i2}^{ND}(k, 0) = P_{i1}^{ND}(k)$;
 - (b.ii) if the number k of firms that disclosed information in period 1 exceeds k^* , then there is a positive probability that some firm that did not disclose information in period 1 will disclose its information in period 2, and $P_{i2}^{ND}(k, l)$ is strictly decreasing in l for any such k .

See Appendix A for proof.

The assumptions of the theorem are standard, apart from the condition that requires the left-tailed conditional expectation $E[\tilde{x}_i | \tilde{x}_i \leq x_i]$ to increase at a rate no faster than the upper-bound x_i . This condition is satisfied by many common distributions and has been used elsewhere to characterize disclosure policies.¹⁰

All parts of this theorem rely on the following statistical fact: as more firms disclose, investors' posterior assessment that event E occurred increases, and hence, since $q > \hat{q}$, the likelihood increases that a nondisclosing firm has information. This fact explains Theorem 1(a): while investors are justifiably skeptical about whether a firm has received information when it makes no disclosure, the statistical fact implies even more skepticism as the number of period 1 disclosers increases. Hence, $P_{i1}^{ND}(k)$ strictly decreases in k . This statistical fact is also responsible for the "herding behavior" exhibited in Theorem 1(b). "Herding" captures the increasing propensity of a firm to disclose its information as the number of other firms that have previously disclosed their information increases. This herding behavior occurs for firm i whenever the realized value of \tilde{x}_i falls in the interval $[x_{i2c}^n(k), x_{i1c}^n)$. The theorem indicates that a firm with such information will make no disclosures in the first period, but will disclose in the second period if and only if $k > k^*$ other firms disclose in the first period.

4. The Perfectly Correlated Private Value Model

In the remainder of the paper, we assume that $q = 1$ and $\hat{q} = 0$ in the private value model, i.e., that either all firms in an industry receive information or none of them does. This setting—in which information arrives to all firms in an industry at the same time, but investors may not be aware of its arrival—appears to be fairly common. For example, some chemical firms presumably knew about industry-wide practices of improper waste disposal before they became public knowledge.

Theorem 2 below contains the main result of this section. (In the statement of this theorem, we write P_{it}^{ND} in place of $P_{it}^{ND}(k = 0)$ for $t = 1, 2$, and x_{i2c}^n in place of $x_{i2c}^n(k = 0)$, and we do not mention explicitly $\{x_{i2c}^n(k)\}_i$ for $k \geq 1$, since for all $k \geq 1$, $x_{i2c}^n(k) = x_i$.)

THEOREM 2. For any $n \geq 2$, let $\{x_{itc}^n\}_{i,t}$ be an equilibrium set of disclosure policies in the perfectly correlated private knowledge model, and let $\pi_1^n = \prod_j F_j(x_{j1c}^n)$ be the probability that no firm discloses information in period 1, given that all firms receive information.

- (a) If $\{x_{itc}^{n+1}\}_{i,t}$ constitute equilibrium cutoffs when there are $n + 1$ firms, then any undominated equilibrium cutoffs with n firms

¹⁰ See, for example, Dye [1986]. It can be shown that this condition holds for a non-negative random variable whenever its density is nonincreasing on its support. We use this condition only to ensure symmetry of equilibrium cutoffs across all firms.

$\{x_{itc}^n\}_{i,t}$ are such that $x_{itc}^{n+1} \leq x_{itc}^n$ for every $i = 1, \dots, n$ and $t = 1, 2$ and $\pi_1^{n+1} \leq \pi_1^n$ (if $\pi_1^{n+1} > 0$, then the preceding inequalities are strict, i.e., $x_{itc}^{n+1} < x_{itc}^n$ and $\pi_1^{n+1} < \pi_1^n$).

- (b) For any equilibrium period 1 cutoffs $\{x_{i1c}^n\}_i$, any undominated equilibrium cutoffs for period 2, $\{x_{i2c}^n\}_i$, are such that if no firm discloses in period 1, then no firm discloses in period 2, i.e., $x_{i2c}^n > x_{i1c}^n$ for each i . In this equilibrium, $x_{i2c}^n = P_{i2}^{ND} = P_{i1}^{ND}$.

See Appendix A for proof.

Theorem 2 (a) implies that as the number of firms that may possess information increases, each firm is more likely to disclose its information whenever it has any. There are two offsetting factors that contribute to this result: first, if none of the $n - 1$ other firms discloses, then the benefits to the remaining firm of not disclosing increase as n increases, because investors are more likely to attribute the lack of disclosure to the lack of information the greater the number of firms that have not disclosed. Hence, investors discount the price of each nondisclosing firm less than they would have had the number n of nondisclosing firms been smaller (the discount arises because $E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n] < E[\tilde{x}_i]$). Second, offsetting this first effect, as the number of firms increases—holding the disclosure policies of these other firms fixed—it is more likely that there will be some firm j for which $\tilde{x}_j \geq x_{j1c}^n$. If firm i fails to disclose, but some other firm does disclose its information, then firm i will wind up with the lower price $E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]$ rather than the price P_{i1}^{ND} . Avoiding this outcome provides the manager of firm i with an incentive to lower the threshold cutoff. What the theorem shows is that the latter effect always outweighs the former, at least for undominated equilibria: as the number of firms in the industry increases, each firm discloses its information more often. This further implies that, as the number of firms with information increases, the probability that some firm will disclose its information also increases. Note that Theorem 2(a) holds without imposing any assumptions on the distributions generating the random variables \tilde{x}_i .

Theorem 2 (b) establishes that if no firm releases its information immediately, then no firm ever does. This interpretation follows because $x_{i2c}^n > x_{i1c}^n$ implies that if a manager did not disclose his information in period 1, then $\tilde{x}_i < x_{i1c}^n$ and so a fortiori $\tilde{x}_i < x_{i2c}^n$.¹¹ For some parametrizations of the model, there are no equilibria with disclosures in the second period, because full disclosure in the first period is the only equilibrium. This is true, for example, if \tilde{x}_i is uniformly distributed on $[\underline{x}_i, \bar{x}_i]$, $i = 1, 2, \dots, n \geq 2$ or if $\{\tilde{x}_i\}_{i=1}^n$ is a family of independently and identically distributed (*iid*) exponential random variables with mean $1/\lambda > 0$,

¹¹ Also, this result demonstrates that the equilibrium of the model is stationary in that more periods could be added and the cutoffs $x_{itc}^n = x_{i2c}^n$ and $P_{it}^{ND} = P_{i1}^{ND}$ for every $t > 2$ and every i would constitute an equilibrium of the extended model.

$n > 2$, and $p \in (0, 1)$.¹² But, there are other distributions that do not always lead to full disclosure in the first period; e.g., if $\{\tilde{x}_i\}_i$ is *iid* with common density $f(x) = (\alpha + 1)x^\alpha$, $x \in [0, 1]$, and $\alpha + 2 \geq (\alpha + 1)n > 0$.¹³

5. The Industry-Wide Common Knowledge Model

In this section, firms' *receipt* of information is perfectly correlated, as in the previous section and, in addition, when firms receive information, they learn about other firms' information as well as their own. That is, in this case, each firm either jointly receives or does not receive information about the entire vector (x_1, \dots, x_n) when any firm learns information. This assumption may be applicable to manufacturers who conduct tests of the toxicity/suitability/etc., of its competitors' products at the same time it tests its own products.

Taking the disclosure decisions of other firms as given, it is clear that there are cutoffs x_{i1c}^n and x_{i2c}^n that describe each firm's disclosure policy here as well. Note, however, that if firm i knows that some firm $j \neq i$ intends to disclose its information in the first period, then firm i must also do so, by Grossman's [1981] and Milgrom's [1981] adverse selection argument. This implies that firm i faces a nontrivial disclosure decision in period 1 only when it knows that no other firm intends to disclose in period 1. Consequently, the cutoff x_{i1c}^n is both the *threshold* below which no disclosure by firm i will occur, given that no other firm intends to disclose in the first period, *and* the period 1 *price* of firm i if no firm discloses in period 1. The latter interpretation follows because, if i knows that no other firm will make a disclosure, i discloses x_i if and only if the price x_i given disclosure exceeds the price with no disclosure, so the price with no disclosure is the cutoff.

Furthermore, it is clear that if no firm makes a disclosure in period 1, then no firm will make a disclosure in period 2, since there is nothing firms learn either about themselves or about other firms as time passes.¹⁴ Consequently, period 2 is superfluous in this model.

DEFINITION 2. An equilibrium in the industry-wide common knowledge model consists of a collection of cutoffs $\{x_{i1c}^n\}_i$, prices $\{P_{i1}^{ND}\}_i$, and a probability π_1^n such that:

¹² Proofs are available from the authors.

¹³ This last claim is proved at the end of the proof of Theorem 2. As demonstrated by this example, it is possible that no firm discloses in the first period even when an arbitrarily large numbers of firms may possess information. The reason is that, although $\alpha +$

1 is restricted to be positive, α can be negative, so $\frac{\alpha+2}{\alpha+1} = 1 + \frac{1}{\alpha+1}$ can be large and so the inequality in the text can hold for very large n .

¹⁴ Thus, in this case, either all firms disclose their information upon receipt or none does. In contrast, in the previous settings, some firms' disclosures may follow (in fact, be triggered by) disclosures of other firms.

- (a) if the manager of firm i learns $x = (x_1, \dots, x_n)$ and $x_j < x_{j1c}^n$ for all $j \neq i$, then the manager of firm i discloses x_i if and only if $x_i \geq x_{i1c}^n$;
- (b) $\pi_1^n \equiv \prod_{i=1}^n F_i(x_{i1c}^n)$ is the probability no firm discloses information, given that firms receive information;
- (c) if no disclosure occurs by any firm, then the price of firm i is $P_{i1}^{ND} = [(1-p)E[\tilde{x}_i] + p\pi_1^n E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]] / (1-p + p\pi_1^n)$;
- (d) $x_{i1c}^n = P_{i1}^{ND}$, for each $i = 1, \dots, n$.

The following theorem characterizes some aspects of an equilibrium for this case and indicates how the equilibrium changes with the number of firms that may receive information.¹⁵

THEOREM 3. Let $\{x_{i1c}^n\}_i$, $\{P_{i1}^{ND}\}_i$, and π_1^n be an equilibrium in the industry-wide common knowledge model when there are $n \geq 2$ firms. If $x_i - E[\tilde{x}_i | \tilde{x}_i \leq x_i]$ is weakly increasing in x_i for every i , then:

- (a) $x_{i1c}^{n+1} > x_{i1c}^n$ for all $i = 1, \dots, n$; and
- (b) the probability $1 - \prod_{i=1}^n F_i(x_{i1c}^n)$ of a disclosure, given that firms have received information, is increasing in n .

See Appendix A for proof.

Theorem 3 (a) indicates that the threshold level x_{i1c}^n for each firm is increasing in the number n of firms in the industry. The explanation is related to that given for Theorem 2 (a) above; as the number of firms increases, the failure of any firm to disclose increases outsiders' perceptions that no firm has any information. This, in turn, reduces the discount investors impose on nondisclosing firms, thus increasing the firm's benefit from not disclosing. In contrast to Theorem 2(a), however, there is no offsetting benefit to disclosing information more frequently, so the threshold disclosure level increases. Hence, the thresholds move in opposite directions as n increases in the two cases; that is, $x_{i1c}^{n+1} < x_{i1c}^n$ in the private knowledge model as opposed to $x_{i1c}^{n+1} > x_{i1c}^n$ in the industry-wide common knowledge model.

Interestingly, this increased propensity of each firm to withhold information as the number of firms increases is more than offset by the influence of more disclosing firms. Specifically, as Theorem 3(b) shows, as the number of firms increases, the overall probability that some firm (and hence, all firms) will disclose information increases.

6. Conclusion

We view firms' voluntary disclosures as being motivated primarily by their effects on investor perceptions of firms' values. We attribute the interactions among corporate disclosures to the influence that one firm's disclosure has on the market's perception that other firms in the same

¹⁵ This theorem does not make use of the concept of undominated equilibrium, since the results which follow hold for all equilibria of this model.

industry have received firm-specific, value-relevant information that they have not yet disclosed. We demonstrate how such a perspective can explain herding behavior in disclosures, and how firms' disclosure policies can vary depending on what each firm knows about other firms' information.

We also consider what happens when all firms receive information at the same time. If the information each firm receives is private only to that firm, we show that the firm is *more* inclined to disclose (and hence that its expected price with no disclosure decreases) as the number of firms that receive information increases. In contrast, if the information a firm receives is known by all other firms in the industry, then a firm is *less* inclined to disclose its information (and hence its expected price with no disclosure increases) as the number of firms potentially endowed with information increases. In both cases, we establish that the probability some firm will disclose its information increases with the number of firms that potentially receive information.

APPENDIX A

Proofs of Theorems

Proof of Theorem 1

Rewrite the pricing equation and the equilibrium cutoff from Definition 1 (c) and 1 (e) as:

$$P_{i1}^{ND}(k) = E[\tilde{x}_i] + \Pr_i(\text{info} | k) (E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n] - E[\tilde{x}_i]) \quad (A1)$$

$$x_{i1c}^n = \sum_{k=0}^{n-1} \Pr_i(k | \text{info}) P_{i1}^{ND}(k) = E[\tilde{x}_i] + (E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n] - E[\tilde{x}_i]) \sum_{k=0}^{n-1} \Pr_i(k | \text{info}) \Pr_i(\text{info} | k). \quad (A2)$$

Before proving the central parts of Theorem 1, we need some subsidiary calculations. We first show that $x_{i1c}^n \in (\underline{x}_i, E[\tilde{x}_i])$. Suppose that $x_{i1c}^n = \underline{x}_i$. Then $\Pr_i(\text{info} | k) = 0$ for all k . By (A2) $x_{i1c}^n = E[\tilde{x}_i]$. This is a contradiction.

Next suppose $x_{i1c}^n = \bar{x}_i$. Then, by (A1), $P_{i1}^{ND}(k) = E[\tilde{x}_i]$ for all k , so by (A2), $x_{i1c}^n = E[\tilde{x}_i]$. This is another contradiction.

Since $x_{i1c}^n < \bar{x}_i$ for all i :

$$E[\tilde{x}_i] > E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n] \text{ for all } i. \quad (A3)$$

We now show $x_{i1c}^n < E[\tilde{x}_i]$. Suppose $x_{i1c}^n \geq E[\tilde{x}_i]$. Then, (A2) and (A3) imply $\sum_{k=0}^{n-1} \Pr_i(k | \text{info}) \Pr_i(\text{info} | k) \leq 0$. Since $x_{j1c}^n < \bar{x}_j$ for all j , $\Pr_i(k | \text{info}) > 0$ for all k , and since $x_{i1c}^n > \underline{x}_i$, $\Pr_i(\text{info} | k) > 0$ for all k , so this last sum is positive, yielding another contradiction. Hence, $x_{i1c}^n < E[\tilde{x}_i]$.

Since $\Pr_i(\text{info} | k) > 0$ for all k , (A1) and (A3) imply $P_{i1}^{ND}(k) < E[\tilde{x}_i]$.

By (A1), (A3), and the easily provable fact that $\Pr_i(\text{info} | k)$ is strictly increasing in k ,¹⁶ it follows that $P_{i1}^{ND}(k)$ is strictly decreasing in k , prov-

¹⁶ The proof is straightforward but lengthy. It is available from the authors.

ing Theorem 1 (a). Similarly, since $\Pr_i(\text{info} \mid k, l)$ is easily shown to be strictly increasing in l for each k , and since $P_{i2}^{ND}(k, l)$ is expressible as $P_{i2}^{ND}(k, l) = E[\tilde{x}_i] + \Pr_i(\text{info} \mid k, l) (E[\tilde{x}_i \mid \tilde{x}_i < x_{ic}^n(k)] - E[\tilde{x}_i])$, we conclude $P_{i2}^{ND}(k, l)$ is strictly decreasing in l .

Note, in analogy with (A1), that we can express $x_{i2c}^n(k)$ as:

$$\begin{aligned} x_{i2c}^n(k) &= \sum_{l=0}^{n-k-1} \Pr_i(l \mid k, \text{info}) P_{i2}^{ND}(k, l) \\ &= E[\tilde{x}_i] + (E[\tilde{x}_i \mid \tilde{x}_i < x_{ic}^n(k)] \\ &\quad - E[\tilde{x}_i]) \sum_{l=0}^{n-k-1} \Pr_i(l \mid k, \text{info}) \Pr_i(\text{info} \mid k, l). \end{aligned} \quad (\text{A4})$$

LEMMA A1. If $\{\tilde{x}_i\}_i$ is *iid* and $x_i - E[\tilde{x}_i \mid \tilde{x}_i < x_i]$ is weakly increasing in i , then the cutoffs $\{x_{i1c}^n\}_i$, $\{x_{i2c}^n(k)\}_{i,k}$ are symmetric, i.e., $x_{i1c}^n = x_{j1c}^n$ and $x_{i2c}^n(k) = x_{j2c}^n(k)$ for all i, j, k . (The proof is available from the authors.)

We are now in a position to prove Theorem 1 (b). Notice from (A2) that x_{i1c}^n is a convex combination of the first-period prices $P_{i1}^{ND}(k)$, $k = 0, \dots, n-1$. Therefore, since $P_{i1}^{ND}(k)$ declines in k , there exists k^* , $0 < k^* < n$ such that for all $\hat{k} > k^* > k'$:

$$P_{i1}^{ND}(\hat{k}) < P_{i1}^{ND}(k^*) \leq x_{i1c}^n < P_{i1}^{ND}(k'). \quad (\text{A5})$$

We next claim that $x_{i2c}^n(\hat{k}) < x_{i1c}^n$. Suppose $x_{i2c}^n(\hat{k}) \geq x_{i1c}^n$. Since the equilibrium is symmetric, this last inequality holds for every firm i . Hence, no firm that did not disclose in period 1 will disclose in period 2, i.e., $x_i < x_{i1c}^n$ implies $x_i < x_{i2c}^n(\hat{k})$, or equivalently, $\Pr_i(l=0 \mid \hat{k}, \text{info}) = 1$, so (A4) reduces to $x_{i2c}^n(\hat{k}) = P_{i2}^{ND}(\hat{k}, l=0) = P_{i1}^{ND}(\hat{k})$. (The last equality follows since, if investors believe $x_{i2c}^n(\hat{k}) \geq x_{i1c}^n$, then they will believe that no firm that did not disclose in period 1 will disclose in period 2, and hence the period 1 and 2 prices must be the same.) This contradicts (A5), proving the claim.

Thus, the manager of firm i whose information \tilde{x}_i falls in the interval $[x_{i2c}^n(\hat{k}), x_{i1c}^n]$ will disclose this information in the second period if $k^* + 1$ or more firms disclose in the first period.

To prove the first half of Theorem 1 (b), pick $k' < k^*$ and note that (A5) establishes $P_{i1}^{ND}(k') > x_{i1c}^n$. If each firm i believes that every firm $j \neq i$ uses the period 2 cutoff $x_{j2c}^n(k') = P_{j1}^{ND}(k')$, and investors believe that all firms (including i) use the cutoffs $x_{j2c}^n(k') = P_{j1}^{ND}(k')$, then every firm will believe that those firms that did not disclose their information in period 1 will not disclose their information in period 2. Thus, firm i 's expected price with no disclosure in period 2 is the price $P_{i1}^{ND}(k') \equiv P_{i2}^{ND}(k', 0)$ (given these beliefs about other firms), and so firm i itself will not disclose x_i in period 2 if it made no disclosure in period 1.

If $\{\hat{x}_{i2c}^n(k')\}_i$ is another set of equilibrium cutoffs such that $\hat{x}_{i2c}^n(k') \geq x_{i2c}^n(k')$, then it is clear from the preceding that $\hat{x}_{i2c}^n(k') = P_{i1}^{ND}(k')$. It follows that the cutoffs $\{x_{i2c}^n(k')\}_i$ are undominated. ■

Proof of Theorem 2

We begin by showing how the conditions comprising an equilibrium can be compressed to a set of $n + 1$ equations. In the case of the perfectly

correlated receipt of information, $P_{i1}^{ND}(k) = E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]$ for all $k \geq 1$, since if any firm discloses information, investors will know that all firms have received information. Setting $P_{i1}^{ND} \equiv P_{i1}^{ND}(k=0)$ and letting $\pi_1 \equiv \prod_j F_j(x_{j1c}^n)$ be the probability that no firm discloses information in the first period, given (all) firms receive information, the equations describing the first period cutoffs are:

$$x_{i1c}^n = (\pi_1 / F_i(x_{i1c}^n)) \times P_{i1}^{ND} + (1 - \pi_1 / F_i(x_{i1c}^n)) \times E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n], \\ i = 1, 2, \dots, n.$$

Let $F_{i1} \equiv F_i(x_{i1c}^n)$. Substitute P_{i1}^{ND} into the preceding expression and multiply both sides by $1 - p + p\pi_1$ to get:

$$(1 - p + p\pi_1)x_{i1c}^n = (\pi_1 / F_{i1})((1 - p)E[\tilde{x}_i] \\ + p\pi_1 E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]) + (1 - \pi_1 / F_{i1})E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n](1 - p + p\pi_1).$$

Upon rearrangement, this can be written as:

$$(1 - p + p\pi_1)(x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]) = \\ (\pi_1 / F_{i1})(1 - p)(E[\tilde{x}_i] - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n])$$

or equivalently, using integration by parts to express:

$$F_{i1} \times (x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]) \text{ as } \int_{x_1}^{x_{i1c}^n} F_i(x) dx, \\ \varphi_i(x_{i1c}^n) \equiv \int_{x_i}^{x_{i1c}^n} F_i(x) dx / \{E[\tilde{x}_i] - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]\} \\ = \frac{\pi_1(1 - p)}{1 - p + p\pi_1}, i = 1, 2, \dots, n. \quad (A6)$$

The system of equations (A6), along with the probability $\pi_1 = \prod_j F_j(x_{j1c}^n)$, describe equilibrium period 1 cutoffs.

LEMMA A2.

- (a) $x_{i1c}^n < P_{i1}^{ND} \leq E[\tilde{x}_i]$, and if $\pi_1^n \equiv \prod_{i=1}^n F_i(x_{i1c}^n) > 0$,
then $x_{i1c}^n < P_{i1}^{ND} < E[\tilde{x}_i]$.
- (b) $x_{i2c}^n \leq P_{i2}^{ND} < E[\tilde{x}_i]$ and if $\pi_1^n \pi_2^n \equiv \prod_{i=1}^n F_i(\text{Min}\{x_{i1c}^n, x_{i2c}^n\}) > 0$,
then $x_{i2c}^n < P_{i2}^{ND} < E[\tilde{x}_i]$.

(This proof is similar to the proof of related results proved in Theorem 1 and is not presented here.)

Next notice that $\varphi_i(\cdot)$ is continuous and strictly increasing on $[x_i, \bar{x}_i]$, that $\varphi_i(x_i) = 0$, and $\varphi_i(\bar{x}_i) = \infty$. Therefore, $\varphi_i^{-1}(\cdot)$ is continuous and strictly increasing on $[0, \infty)$, with range $[x_i, \bar{x}_i]$. Therefore, the function:

$$\Gamma_n: [0, 1] \rightarrow [0, 1] \text{ defined by } \Gamma_n(0) = 0,$$

$$\Gamma_n(\pi_1) = \prod_{i=1}^n F_i \left(\Phi_i^{-1} \left(\frac{1-p}{\frac{1-p}{\pi_1} + p} \right) \right), \pi_1 \neq 0,$$

is continuous and strictly increasing on $[0,1]$, with range contained in the interval $[0,1]$.

If $\Gamma_n(\cdot)$ has a fixed point, say π_1^n , then $x_{i1c}^n \equiv \Phi_i^{-1} \left(\frac{(1-p)\pi_1^n}{1-p+p\pi_1^n} \right)$ constitute equilibrium first-period cutoffs, and conversely, any equilibrium cutoffs define a fixed point of $\Gamma_n(\cdot)$.

LEMMA A3. If $\pi \in [0,1]$ and $\Gamma_n(\pi) \geq \pi$, then $\Gamma_n(\cdot)$ has a fixed point π_n with:

$$\pi_n \geq \pi.$$

Proof. Pick any $\pi \in [0,1]$, and construct the sequence (π^k) as follows:

$$\pi^0 \equiv \pi, \pi^1 = \Gamma_n(\pi^0), \text{ and in general, } \pi^{k+1} = \Gamma_n(\pi^k).$$

Since $\pi^1 \geq \pi^0$ and $\Gamma_n(\cdot)$ is strictly increasing, $\pi^2 = \Gamma_n(\pi^1) \geq \Gamma_n(\pi^0) = \pi^1$.

In this way, we conclude $\pi^{k+1} \geq \pi^k$, i.e., the sequence $\{\pi^k\}$ is increasing. It follows, since each $\pi^k \in [0,1]$, that this sequence has a limit, say π_n (note subscript). By the continuity of $\Gamma_n(\cdot)$, we conclude that π_n is a fixed point of $\Gamma_n(\cdot)$, as:

$$\Gamma_n(\pi_n) = \Gamma_n(\lim_{k \rightarrow \infty} \pi^k) = \lim_{k \rightarrow \infty} \Gamma_n(\pi^k) = \lim_{k \rightarrow \infty} \pi^{k+1} = \pi_n$$

This proves the lemma.

To prove Theorem 2(a), let π_{n+1} be a fixed point of $\Gamma_{n+1}(\cdot)$.

Define π^k iteratively as in the proof of Lemma A3 above, using $\pi_{n+1} \equiv \pi^0$ as the starting value, i.e., $\pi^1 = \Gamma_n(\pi^0)$, $\pi^2 = \Gamma_n(\pi^1)$, etc. By definition:

$$\pi^1 \equiv \Gamma_n(\pi_{n+1}) = \prod_{i=1}^n F_i \left(\Phi_i^{-1} \left(\frac{1-p}{\frac{1-p}{\pi_{n+1}} + p} \right) \right) \geq \prod_{i=1}^{n+1} F_i \left(\Phi_i^{-1} \left(\frac{1-p}{\frac{1-p}{\pi_{n+1}} + p} \right) \right) =$$

$$\Gamma_{n+1}(\pi_{n+1}) = \pi_{n+1} = \pi^0.$$

(The inequality is strict unless $\pi_{n+1} = 0$.) Thus, the sequence $\{\pi^k\}$ is increasing. It follows, as in the proof of Lemma A3, that the limit, say π_n , of this sequence is a fixed point of $\Gamma_n(\cdot)$. We conclude:

$$\pi_n = \lim_{k \rightarrow \infty} \pi^k \geq \pi^1 \geq \pi^0 = \pi_{n+1}$$

with the inequalities being strict if $\pi_{n+1} > 0$.

Let $x_{i1c}^n \equiv \Phi_i^{-1} \left(\frac{\pi_n(1-p)}{1-p+p\pi_n} \right)$ be the period 1 cutoff for firm i associated

with π_n . Suppose that $\{\hat{x}_{i1c}^n\}_i$ is *another* set of equilibrium period 1 cutoffs with associated probability $\hat{\pi}_n$. Observe that, since $\Phi_i(\cdot)$ is strictly increasing, $\hat{x}_{i1c}^n > x_{i1c}^n$ for some i if and only if $\hat{\pi}_n > \pi_n$ (and

therefore, $\hat{x}_{i1c}^n > x_{i1c}^n$ for some i if and only if $\hat{x}_{j1c}^n > x_{j1c}^n$ for all $j \in \{1, \dots, n\}$. Therefore, if $\{x_{i1c}^{n*}\}_i$ is a set of *undominated* equilibrium period 1 cutoffs, and π_n^* is the associated probability, $\pi_n^* \geq \pi_n \geq \pi_{n+1}$ (and, as noted previously, the last inequality is strict if $\pi_{n+1} > 0$), and $x_{i1c}^{n*} \geq x_{i1c}^{n+1}$ for every i (again, the inequality is strict if $\pi_{n+1} > 0$). This proves Theorem 2(a).

The proof of Theorem 2(b) is similar to the proof of Theorem 1b(i): one argues, as in that proof, that if firm i believes every firm $j \neq i$ uses $P_{j1}^{ND} \equiv x_{j2c}^n$ as their period 2 cutoffs, then i will itself use $P_{i1}^{ND} = x_{i2c}^n$ as its period 2 cutoff. The details are not repeated.

Proof That There Are Less Than Full Disclosure Equilibria

Suppose $\{\tilde{x}_i\}_i$ is iid with common density $f(x) = (\alpha + 1)x^\alpha$, $x \in [0, 1]$, and $\alpha + 2 \geq (\alpha + 1)n > 0$. It is easy to verify:

$$E[\tilde{x} | \tilde{x} < x] = \frac{\alpha + 1}{\alpha + 2} x; F(x) = x^{\alpha+1}; \varphi_i(x) = \frac{x^{\alpha+2}}{(\alpha + 1)(1 - x)},$$

for $x \in [0, 1]$. Appealing to (A6), it is easy to check that, if $\{x_{i1c}^n\}_i$ and equilibrium cutoffs, then $x_{i1c}^n = x_{j1c}^n$, i.e., that the equilibrium is symmetric. Thus, the set of equilibrium equations (A6) reduces to the single equation:

$$\frac{x^{\alpha+2}}{(\alpha + 1)(1 - x)} = \frac{(1 - p)x^{n(\alpha+1)}}{1 - p + px^{n(\alpha+1)}} \quad (A7)$$

We now show that (A7) has a solution $x \in (0, 1)$. After cross-multiplying and setting $k = \frac{p}{1-p}$, (A7) can be rewritten as:

$$x^{\alpha+2 - (\alpha+1)n} + kx^{\alpha+2} = (\alpha + 1)(1 - x). \quad (A8)$$

Notice that, if $\alpha + 2 - (\alpha + 1)n > 0$, then:

$$LHS(A8) = 0, RHS(A8) = \alpha + 1, \text{ at } x = 0$$

$$LHS(A8) = 1 + k, RHS(A8) = 0, \text{ at } x = 1.$$

Since both sides of (A8) are continuous, it follows that there exists $x \in (0, 1)$, solving (A8).

Proof of Theorem 3

The equations for an equilibrium can be written as:

$$x_{i1c}^n = \frac{(1 - p)E[\tilde{x}_i] + p\prod_{j=1}^n F_j(x_{j1c}^n)E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]}{1 - p + p\prod_{j=1}^n F_j(x_{j1c}^n)}, \quad i = 1, 2, \dots, n,$$

or as:

$$(1 - p)(E[\tilde{x}_i] - x_{i1c}^n) = p \prod_{j=1}^n F_j(x_{j1c}^n)(x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]), \quad (A9)$$

or as:

$$\Psi_i(x_{i1c}^n) = \frac{1-p}{p\pi_1}, i = 1, 2, \dots, n$$

where:

$$\Psi_i(x_i) = (x_i - E[\tilde{x}_i | \tilde{x}_i < x_i]) / (E[\tilde{x}_i] - x_i) \text{ and } \pi_1 = \prod_{j=1}^n F_j(x_{j1c}^n).$$

The existence of a solution to this system of equations is equivalent to the existence of a fixed point of the map $\zeta_n : [0,1] \rightarrow [0,1]$ defined by:

$$\zeta_n(\pi) = \prod_{i=1}^n F_i \left(\Psi_i \left(\frac{1-p}{p\pi_1} \right) \right), \text{ for } \pi_1 \neq 0; \zeta_n(0) = \prod_{i=1}^n F_i(E[\tilde{x}_i]).$$

Notice that, if $F_i(x_{i1c}^n) = 0$ for any i , then the equilibrium equations (A9) would imply that $E[\tilde{x}_i] = x_{i1c}^n$, and hence that $F_i(x_{i1c}^n) = \Pr(x_i \leq E[\tilde{x}_i]) = 0$. This is impossible. Thus, $F_i(x_{i1c}^n) > 0$ for every i . This fact, along with (A9) also implies $x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i \leq x_{i1c}^n] > 0$ and $E[\tilde{x}_i] > x_{i1c}^n$ for every i .

For further reference, notice that $F_i(x_{i1c}^n) = 1$ is also impossible, since $\Pr(\tilde{x}_i \leq x_{i1c}^n) < \Pr(\tilde{x}_i \leq E[\tilde{x}_i]) < 1$.

Suppose $n > m$, and for every $i = 1, \dots, m$, $x_{i1c}^n \leq x_{i1c}^m$. By applying (A9) twice, we get for all $i \in (1, \dots, m)$:

$$\begin{aligned} (1-p)(E[\tilde{x}_i] - x_{i1c}^n) &\geq (1-p)(E[\tilde{x}_i] - x_{i1c}^m) = \\ p \prod_{j=1}^m F_j(x_{j1c}^m)(x_{i1c}^m - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^m]) &\geq \\ p \prod_{j=1}^m F_j(x_{j1c}^n)(x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]) &> \\ p \left[\prod_{j=m+1}^n F_j(x_{j1c}^n) \right] \left[\prod_{j=1}^m F_j(x_{j1c}^n) \right] (x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]) &= \\ p \left[\prod_{j=1}^n F_j(x_{j1c}^n) \right] (x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]) &= (1-p)(E[\tilde{x}_i] - x_{i1c}^n). \end{aligned}$$

This is a contradiction. Therefore, for some $i \in \{1, \dots, m\}$, $x_{i1c}^n > x_{i1c}^m$. For this i :

$$\begin{aligned} p \prod_{j=1}^n F_j(x_{j1c}^n)(x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]) &= \\ (1-p)(E[\tilde{x}_i] - x_{i1c}^n) &< (1-p)(E[\tilde{x}_i] - x_{i1c}^m) = \\ p \prod_{j=1}^m F_j(x_{j1c}^m)(x_{i1c}^m - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^m]) &). \end{aligned}$$

Since $x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n] \geq x_{i1c}^m - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^m]$, it follows that $\prod_{j=1}^n F_j(x_{j1c}^n) < \prod_{j=1}^m F_j(x_{j1c}^m)$. This inequality proves Theorem 3 (b).

To prove that x_{i1c}^n is increasing in n for every i , note that by taking the ratio of equation (A9) for firm i to this same equation for firm j , we get for all $i, j \in \{1, \dots, n\}$:

$$\frac{E[\tilde{x}_i] - x_{i1c}^n}{E[\tilde{x}_j] - x_{j1c}^n} = \frac{x_{i1c}^n - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^n]}{x_{j1c}^n - E[\tilde{x}_j | \tilde{x}_j < x_{j1c}^n]} \quad (A10)$$

If $x_{i1c}^{n+1} > x_{i1c}^n$ and $x_{j1c}^{n+1} \leq x_{j1c}^n$, then, since $x_{kc} - E[\tilde{x}_k | \tilde{x}_k < x_{kc}]$ is increasing in x_{kc} for both $k = i$ and $k = j$, we conclude:

$$\frac{x_{i1c}^{n+1} - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^{n+1}]}{x_{j1c}^{n+1} - E[\tilde{x}_j | \tilde{x}_j < x_{j1c}^{n+1}]} \quad (A11)$$

is no smaller than $RHS(A10)$; at the same time, $LHS(A10)$ is strictly larger than:

$$\frac{E[\tilde{x}_i] - x_{i1c}^{n+1}}{E[\tilde{x}_j] - x_{j1c}^{n+1}} \quad (A12)$$

This contradicts the fact that (A11) and (A12) are equal. Thus we know that the cutoff points for all firms move in the same direction as n changes. Next suppose that $x_{i1c}^{n+1} < x_{i1c}^n$ and $x_{j1c}^{n+1} < x_{j1c}^n$, for all $j \neq i$. With $n + 1$ firms in the industry, the pricing equation corresponding to (A9) is:

$$(1 - p)(E[\tilde{x}_i] - x_{i1c}^{n+1}) = p \prod_{j=1}^{n+1} F_j(x_{j1c}^{n+1})(x_{i1c}^{n+1} - E[\tilde{x}_i | \tilde{x}_i < x_{i1c}^{n+1}]). \quad (A13)$$

Now, $LHS(A9) < LHS(A13) = RHS(A13) < RHS(A9)$, thus yielding a contradiction. Hence, we have $x_{i1c}^{n+1} > x_{i1c}^n$ for all $i = 1, \dots, n$.¹⁷ ■

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¹⁷ It is easy to show that for the special case of $n = 2$, it is unnecessary to invoke the assumption that $x_i - E[x_i | x_i \leq x_i]$ is increasing in x_i to obtain this result.