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Discretionary Risk Disclosures

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ABSTRACT: We model managers' equilibrium strategies for voluntarily disclosing information about their firm's risk. We consider a multifirm setting in which the variance of each firm's future cash flow is uncertain. A manager can disclose, at a cost, this variance before offering the firm for sale in a competitive stock market with risk-averse investors. In our partial disclosure equilibrium, managers voluntarily disclose if their firm has a low variance of future cash flows, but withhold the information if their firm has highly variable future cash flows. We establish how the manager's discretionary risk disclosure affects the firm's share price, expected stock returns, and beta, within the framework of the Capital Asset Pricing Model. We show that whereas one manager's discretionary disclosure of his firm's risk does not affect other firms' share prices, it does affect the other firms' betas. Also, we demonstrate that a disclosing firm has lower risk premium and beta *ex post* than a nondisclosing firm. Finally, we show that *ex ante,* the expected risk premium and expected beta of each firm are higher under a mandatory risk disclosure regime than in the partial disclosure equilibrium that arises under a voluntary disclosure regime.

Keywords: *disclosures; risk; asset pricing.*

I. INTRODUCTION

Theoretical research on the disclosure of financial accounting information focuses on disclosures about financial statement items, such as asset values, earnings, operating cash flows, or management forecasts of earnings. This focus was natural as, until recently, virtually all mandated disclosures related to financial statement items. However, a recent change in financial disclosure requirements, Securities and Exchange Commission (SEC) Financial Reporting Release No. 48 (FRR No. 48), requires firms to disclose information about risk. We model the effect of additional disclosures of risk information beyond the disclosures required by FRR No. 48. Our study is the first attempt to model managers' voluntary risk disclosures.

Our research objective is to investigate how a manager chooses his disclosure strategy when he has discretion over whether he informs investors about the variance of his firm's

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cash flows. In our model, each firm's future cash flows depend on two sources of risk: one market-wide, the other firm-specific. Since disclosure of sensitivity to market risk is mandated by FRR No. 48, we assume that investors know the variance of the market-wide cash flow factor. However, we assume that initially investors do not know the variance of firm-specific cash flows. Each risk-neutral manager can convey the information about his firm's firm-specific cash flow by disclosing, at a cost, his firm's cash flow variance before offering the firm for sale in a competitive market of risk-averse investors.

This paper contributes to the literature in two ways. First, we identify the equilibrium disclosure strategy in a general exchange economy where each manager has the discretion to issue a costly disclosure about his firm's risk. We show that managers of firms with low variance of future cash flows voluntarily disclose their (favorable) risk information, whereas managers of firms with high variance of future cash flows do not disclose.

This first contribution extends results on a manager's discretionary disclosure of earnings forecasts (Verrecchia 1983, 1990) to a manager's discretionary disclosure of *risk*. Analogous to the results in Verrecchia (1990), we identify factors that affect the manager's choice of the threshold level of the firm's cash flow variance, above which the manager will not make a risk disclosure. We show that, *ceteris paribus*, a firm that discloses risk has a higher share price than one that does not, but, relative to a voluntary disclosure regime, imposing mandatory full disclosure of firm risk will lower the *ex ante* share price of each firm. Firm value falls because mandating risk disclosure forces firms that would not disclose in a voluntary regime to incur disclosure costs.

Our second contribution stems from analyzing multiple firms and relating their risk disclosure decisions to stock returns. We establish that even when some of the managers choose not to disclose, the expected returns behave in a manner consistent with the Capital Asset Pricing Model (CAPM). Under the CAPM, a firm's risk premium is the expected excess return on its stock over the riskless bond, which is proportional to the risk premium on the market, that is, firm risk premium = beta $\times$ market risk premium. This allows us to separate the effect of investors' uncertainty about firms' risk premia into two components. The first, direct effect, from Barry and Brown (1985), is that an increase in investors' uncertainty about the variance of the firms' future cash flows leads to a higher risk premium. In our setting, a second, indirect effect emerges. Because greater investor uncertainty increases the range of variances under which a manager will make voluntary disclosures about his firm's risk, this second indirect effect lowers the risk premium. When the latter indirect effect dominates the former direct effect, greater investor uncertainty about the firm's variance of future cash flows will *decrease* the risk premium. Which effect dominates remains an empirical question.

Our second contribution also enables us to investigate how a manager's voluntary disclosure about his firm's risk affects the CAPM parameters of his own and other firms. First, we find that a firm's beta and risk premium both increase if the manager discloses a high variance, and they increase further in the absence of disclosure. Second, we find that the risk premium of each firm's stock return depends only on that firm's own risk disclosures. Third, we specify how the beta of each firm and the risk premium on the return on the market portfolio depend on whether other firm managers disclose and what variance they disclose. In particular, we show that disclosure by one manager will raise the betas of other firms in the economy, provided the firms have positive betas, while leaving the share prices of these other firms unaffected.

Recent empirical research concludes that FRR No. 48 disclosures are useful to investors (e.g., Rajgopal 1999; Linsmeier et al. 2002). Our model provides a theoretical framework for how fully rational investors may use voluntary disclosures about the firm-specific variance of future firm cash flows, which are beyond the risk disclosures required under FRR No. 48. Our findings provide several empirically testable hypotheses. For example, our results suggest that firms are more likely to voluntarily disclose information about risk in industries where investors are more uncertain about the level of firm-specific risk, such as nondiversified research and development intensive firms. Our work also suggests that a firm will have a lower risk premium and a lower beta if its manager discloses risk information than if the manager does not disclose. In addition, we predict a lower market risk premium and a higher beta for a firm if other firms disclose, but predict that a firm's risk premium remains unaffected by the disclosure of other firms. Further, we predict that expected betas will increase if regulators expand disclosure requirements beyond FRR No. 48 to require new disclosures of firm-specific variance. Collectively, our results suggest that when conducting or interpreting results from event studies, researchers should recognize that abnormal returns based on the market model may be biased if betas have changed in response to risk disclosures by the firm itself or by other firms, or in response to changes in disclosure requirements.

II. INSTITUTIONAL BACKGROUND AND RELATED RESEARCH

Risk Disclosure and the Financial Reporting Environment

Risk disclosures are an increasingly important part of financial reporting. Significant losses from firms' use of financial instruments in the 1990s prompted investors, regulators, auditors, and other private bodies to call for expanded disclosures on risks of financial instruments.[1] The SEC concluded that financial disclosures relating to risk were often incomplete and, in some circumstances, misleading. To address this problem, in January 1997, the SEC issued Financial Reporting Release No. 48, *Disclosure of Accounting Policies for Derivative Financial Instruments and Derivative Commodity Instruments and Disclosure of Quantitative Information About Market Risk Inherent in Derivative Financial Instruments, Other Financial Instruments and Derivative Commodity Instruments* (FRR No. 48). While not the only regulation to require disclosure of risk-related measures, FRR No. 48 is the current standard for risk-related disclosure.[2]

FRR No. 48 requires firms to disclose quantitative and qualitative information about market risk exposures. Jorion's (2002) empirical evidence that the variance of banks' trading revenues is related to their risk disclosures suggests that these disclosures are informative about the firm's risk.[3] Consequently, we assume that the FRR No. 48 disclosures provide information about the sensitivity of the firm's cash flows to market-wide risk factors, so that this information is common knowledge. FRR No. 48 disclosure requirements pertain only to market risk related to financial instruments and derivatives. As a consequence,

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[1] Included in this list calling for additional disclosures were the American Institute of Certified Public Accountant (AICPA), the Association for Investment Management and Research (AIMR), and the Financial Executives Institute (FEI), see Linsmeier and Pearson (1997) for a survey discussing the losses from financial instruments and the calls for additional disclosures.

[2] Issued in June 1998, SFAS No. 133, *Accounting for Derivative Instruments and Hedging Activities*, encourages, but does not require, firms to disclose quantitative information about market risks of financial instruments (para. 15D). In September 1998, the U.K. Accounting Standards Board issued Financial Reporting Standard No. 13 about risk disclosure. Also, Bank of International Settlements regulates financial institutions worldwide based on Value-at-Risk, which is one of the types of quantitative risk disclosures under FRR No. 48.

[3] Moreover, while one may question whether the FRR No. 48 disclosures effectively communicate market-wide risk information, Hodder et al. (2001) generally concede that financial statement users interpret these disclosures as doing so.

investors may not know the sensitivity of the firm's other cash flows to market-wide risk. Despite this caveat, we believe that FRR No. 48 disclosures suggest that investors are more likely to have common knowledge about market risk than about other firm-specific risk. Hence, we assume that investors use the voluntary disclosures to make inferences concerning firm-specific risk.

Risk-Disclosure-Related Research

To clarify how our study of managers' voluntary risk disclosures contributes to the literature, we review the existing research and explain why these models fail to address our research question.

First, Barry and Brown (1985) and Coles et al. (1995), among others, investigate the effect of investors' uncertainty about the variance of cash flows on market values, stock returns, and betas. They establish that a firm's beta increases in the investors' uncertainty about the variance of a firm's cash flows. Neither of these papers, however, allows firm managers to disclose the variance of their firms' cash flows, whereas managers' risk disclosure decisions and the effect of these decisions on share prices are the focus of our study.

Second, recent accounting studies explore the effect of the managers' disclosure choices on the precision of earnings. For example, Dye (1990) and Penno (1996) investigate a manager who chooses the precision of the earnings reported, and Kirschenheiter and Melumad (2002) consider a setting where managers' accrual decisions affect investors' inferences about the precision of earnings. These papers do not allow the manager to either disclose or refrain from disclosing the precision. Hence, none of these studies consider the effect of a manager's discretionary risk disclosure.

Third, recent research on hedge accounting, including DeMarzo and Duffie (1995), Melumad et al. (1999), and Kanodia et al. (2000), evaluates managers' optimal hedging decisions when each manager must disclose his firm's risk relative to the benchmark where the manager cannot disclose his firm's risk. Because these studies do not allow the manager to choose whether to report or withhold his private information regarding the firm's risk exposure, they do not bear on discretionary risk disclosure. In contrast, we evaluate mandatory disclosure of risk relative to our alternative benchmark of voluntary disclosures, while abstracting from the manager's decision to hedge.

In summary, no model currently exists that explains how a manager decides whether to voluntarily disclose the variance of his firm's future cash flow that he privately observes. To address this question, we first derive the share prices in a general exchange economy with risk-averse investors where managers can make such a disclosure choice. We then use these share prices to develop the CAPM and investigate the relation between the CAPM parameters (e.g., the beta and the excess return on the market) and the managers' disclosure decisions.

III. MODEL

Consider the following single period model of a capital market with I investors who trade in J firms' risky stock and a riskless bond. At time 0, each manager privately observes the variance of his firm's risky cash flows. The managers then simultaneously decide whether to truthfully disclose their firm's variance to all investors or to refrain from disclosing. If the manager of firm j discloses, then he incurs a disclosure cost of $C_j > 0$, which reduces the firm's cash flows at time 2. At time 1, investors trade in the J risky stocks and the riskless bond after observing what disclosure, if any, each manager made. At time 2, investors consume and the economy ends. Figure 1 illustrates the timeline. The following detailed assumptions are common knowledge to all investors and managers.

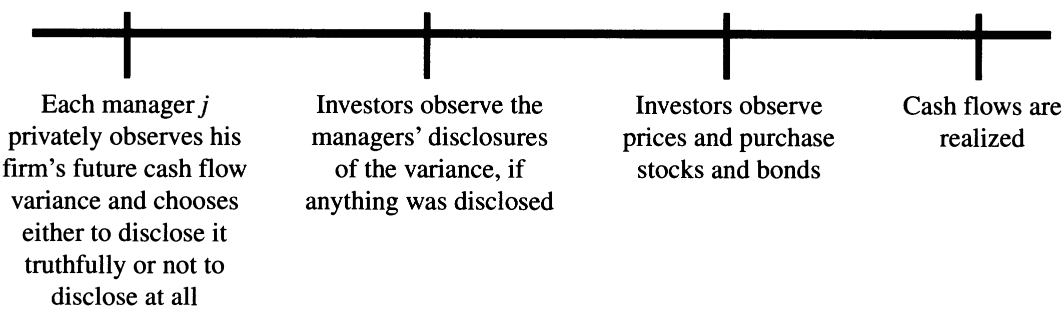
Assumption A1 (Assumption on Cash Flows): The exchange economy includes J firms indexed by $j = 1,...,J$. Firm j has a risky investment project in place that pays X_j at time 2. The lower case, x_j, denotes the realization of the random variable X_j. We assume a single market-wide factor model describes cash flow uncertainty, that is, $X_j = \mu_j + \gamma_j F + \varepsilon_j$, where μ_j is the expected cash flow, F denotes the market-wide cash flow factor, γ_j is the factor loading, or sensitivity of firm j's cash flows to the market-wide factor, and the firm-specific cash flow is ε_j for $j = 1,...,J$. Since the randomness in the cash flow of firm j depends only on these two factors, we refer to F and ε_j as the market-wide risk factor and firm-specific risk factor for firm j, respectively. The total supply of the jth risky asset is one share, with share price of P_j.

Assumption A2 (Assumptions on Distributions): Assume the market-wide factor F is distributed $N(0, \sigma^2)$, and firm-specific cash flow is distributed $\varepsilon_j | v_j \sim N(0, v_j)$, where the variance v_j is the realization of a continuous, non-negative random variable V_j and is drawn from a probability distribution $f_j(v_j)$. Hence, the conditional distribution of a firm's future cash flows is $X_j | v_j \sim N(\mu_j, \gamma_j^2\sigma^2 + v_j)$.

Assumption A3 (Assumption on Managers): At time 0, firm j's manager privately observes the realized variance, v_j. He then chooses whether to truthfully disclose v_j, incurring cost $C_j > 0$ if he does disclose and no cost if he does not disclose.[4] Manager j's disclosure strategy is characterized by $N_j \subseteq \Re^+$, the set of values of the variance that the manager j will not disclose. Let $\underline{N}$ denote the vector of disclosure strategies for all J managers and let $\underline{N}_{-j}$ denote the $(J - 1)$-dimensional vector of all firms' disclosure strategies except firm j. Each manager holds the same beliefs about other managers' disclosure strategies, as do the investors.

Assumption A4 (Assumption on Investors): There are I individuals in the market indexed by $i = 1,...,I$ who have constant absolute risk aversion, $a_i > 0$. Each investor

FIGURE 1
Timeline



[4] We assume that the disclosure costs are exogenous, and we do not model the investment decision. Endogenizing the disclosure costs, as in Gigler (1994) among others, and explicitly modeling the managers' investment decisions would represent an extension of our model. Also, while we assume any disclosure the manager makes is entirely truthful, Stocken's (2000) arguments could support the truthful risk disclosures in our setting.

allocates his initial wealth of W_i^0 between purchasing S_{ij} shares of the jth stock and investing B_i in riskless bonds. Investors take as given the share price of each firm, P_j, and the return on the bond, R_f. Let $\hat{N}_j \subseteq \Re^+$ denote the set of values of the variance in the firm's future cash flows that the investors believe is manager j's nondisclosure set, where the circumflex (or "hat") over the N_j denotes investors' inferences concerning the disclosure strategy that the manager has adopted rather than the manager's actual disclosure strategy. Let $\underline{\hat{N}}$ denote the J-dimensional vector of these inferred disclosure strategies. If investors expect full disclosure but a manager does not disclose, then investors infer the worst; that is, the investors believe that the manager observed but did not disclose the highest possible variance. Finally, consistent with A1 and A3, let $P_j = P_j(v_j, N_j|\underline{N}_{-j}, \underline{\hat{N}})$ denote price of firm j's shares.[5]

We maintain Assumptions A1–A4 throughout the paper. In particular, A2 denotes the marginal effect of the single market-wide factor, F, on firm j's cash flows, as γ_j. Since FRR No. 48 requires that firms disclose their exposures to market risk, we assume that each γ_j is common knowledge. In addition, we assume that the variance in underlying risk factors is observable. Hence, we assume that the variance of the market-wide factor, σ^2, is also common knowledge. Assumption A3 specifies that the manager observes the variance of the firm-specific cash flow risk, but the investors do not. Thus, we assume that the manager must decide whether to disclose the variance of the firm-specific cash flows. Firm j's disclosure of the variance of total future cash flows then becomes equivalent to the disclosure of firm-specific variance, v_j.

Building upon the beliefs and strategies of the managers and investors discussed above, we next specify their payoffs and define our equilibrium concept. Investor i allocates his initial wealth, W_i^0, between the riskless bond and the risky stocks. Each investor trades the bond and the shares in the risky assets to maximize his terminal wealth, W_i, given his information about the variance of the asset's return. Using $1_{\{v_j \notin N_j\}}$ as the indicator function if the manager discloses, we have:

$$W_i = B_i R_f + \sum_{j=1}^{J} S_{ij} \, (X_j - 1_{\{v_j \notin N_j\}} C_j).$$

Hence, each investor's maximization problem is:

$$\max_{\{S_{ij}\}_{j=1}^{J},\, B_i} \qquad E\lfloor -e^{-a_i W_i} \rfloor$$

subject to:

$$B_i + \sum_{j=1}^{J} S_{ij} P_j \leq W_i^0$$

where the investor takes the prices of all assets as given. For each firm j, the price clears

[5] This is abbreviated notation because, in equilibrium, the share price of firm j depends on whether its manager discloses. Only when the manager of firm j discloses do investors observe the variance and condition the share price on the realized value of the variance, v_j. Further, while we allow for the possibility that firm value depends on whether other managers disclose and what disclosures they make, the disclosure made by competing firms do not affect the equilibrium price of firm j.

the market for shares; that is, $\Sigma_{i=1}^{I} S_{ij} = 1$. The equilibrium share price of each firm, P_j, is set by the investors' demand for the firm's stock, which in turn depends on what, if anything, the manager of firm j disclosed and how investors interpret no disclosure.

A *credible* equilibrium to this game is defined as follows:

a) Each manager chooses his disclosure strategy to maximize the value of his firm j, net of any disclosure cost, given investors' inferences about the managers' disclosure strategies. That is, $P_j(v_j, N_j|\underline{N}_{-j}, \hat{\underline{N}}) \geq P_j(v_j, N_j'|\underline{N}_{-j}, \hat{\underline{N}})$ for each j, each $v_j \geq 0$ and for all $N_j' \subseteq \Re^+$.
b) In equilibrium, $\underline{N} = \hat{\underline{N}}$ (investors correctly anticipate the managers' disclosure strategies).

We refer to the endogenous vector of disclosure strategies, $\underline{N}$, that simultaneously solve both Conditions (a) and (b) as a credible equilibrium.

IV. CONSEQUENCES OF RISK DISCLOSURES

We present our results in two subsections. First, we establish how a manager's optimal risk disclosures affect his firm's share prices. Second, we establish how a manager's risk disclosures affect his firm's and other firms' stock returns. All proofs appear in the Appendix.

Effect of Managers' Risk Disclosures on Share Prices

We begin by characterizing the disclosure strategies that arise in a credible voluntary disclosure equilibrium in Lemma 1:

Lemma 1: If equilibrium share prices are monotonically decreasing in the disclosed variance of the firm's cash flows, then the nondisclosure region is determined by a threshold level of variance where the manager discloses only if he (privately) observes that the variance of his firm's future cash flow is below this level (i.e., the nondisclosure region is upper-tailed).

Lemma 1 states that if the equilibrium share prices decrease in the disclosed variance of the firm's cash flows, then managers withhold unfavorable news and disclose favorable news about this variance.[6] Hence, Lemma 1 allows us to express firm j's disclosure strategy, N_j, by its disclosure threshold, y_j.[7]

Lemma 1 describes the credible disclosure strategy based on the presumption that share prices decrease in variances. Next, in Lemma 2, we show that share prices in this economy exhibit this property whether the manager discloses or withholds his private information about the variance of his firm-specific cash flows.

Lemma 2: Let $a = (\Sigma_{i=1}^{I} a_i^{-1})^{-1}$ measure investors' aggregate risk aversion and let $\bar{\gamma} = \Sigma_{j=1}^{J} \gamma_j$ be the sum of the factor loadings. Given any vector of inferred disclosure strategies, $\hat{\underline{N}}$, the share price of firm j is given as follows:

[6] Lemma 1 generalizes to all types of disclosures, not just risk disclosures. We are grateful to an anonymous referee for suggesting this generalization.

[7] Lemma 1 shows that credible disclosure strategies will be one of three types: no disclosure, full disclosure, or partial disclosure strategies, where the manager discloses the variance if it is less than some threshold level, but does not disclose his firm's variance if it exceeds this threshold. To focus on how risk disclosure regulation affects the relation between the manager's disclosure strategy and his firm's share price and the firm's cost of capital, the discussion in the body of the paper concentrates on partial disclosure equilibria.

a) If manager j discloses his private information about the variance in the firm-specific cash flow of his firm ($v_j \notin N_j$), then the price of his firm is:

$$P_j(v_j, N_j|\underline{N}_{-j}, \underline{\hat{N}}) = \{\mu_j - C_j - a(\gamma_j\sigma^2\overline{\gamma} + v_j)\}R_f^{-1}.$$

b) If manager j withholds his private information about the variance in the firm-specific cash flow of his firm ($v_j \in N_j$), then there exists a scalar, $\delta_j(\hat{N}_j)$, such that the price of his firm is:

$$P_j(v_j, N_j|\underline{N}_{-j}, \underline{\hat{N}}) = \{\mu_j - a(\gamma_j\sigma^2\overline{\gamma} + \delta_j(\hat{N}_j)E\lfloor V_j|v_j \in \hat{N}_j\rfloor)\}R_f^{-1}.$$

Four aspects of Lemma 2 are noteworthy. First, the price of the firm given disclosure decreases in the variance of the firm's cash flows. Consequently, Lemma 1 implies that we can describe each manager's nondisclosure set by a disclosure threshold for each firm j, y_j, such that the manager does not disclose the observed variance, v_j, if and only if $v_j \geq y_j$. We therefore can write part (b) of Lemma 2 as:

$$P_j(v_j, N_j|\underline{N}_{-j}, \underline{\hat{N}}) = \{\mu_j - a(\gamma_j\sigma^2\overline{\gamma} + \delta_j(y_j)E[V_j|V_j \geq y_j])\}R_f^{-1}.$$

Second, comparing the price equations (a) and (b) of Lemma 2 reveals similarities. When the manager discloses the variance, share prices separate the effect of the mean and the variance of total cash flows, with the variance being multiplied by the aggregate risk aversion, a, as in Wilson (1968). When the manager discloses, the total cash flow variance equals the sum of: (1) the market-wide risk portion, $\gamma_j\sigma^2\overline{\gamma}$, plus (2) the disclosed variance, v_j, of the firm-specific cash flow term. When the manager refrains from disclosing the firm-specific cash flow variance, the product $H(y_j) \equiv \delta(y_i)E\lfloor V_j|V_j \geq y_j\rfloor$ replaces the variance of the firm-specific cash flow term in the price equation. This product consists of the expected variance in the firm's cash flows given no disclosure, $E\lfloor V_j|V_j \geq y_j\rfloor$, multiplied by the new risk adjustment factor $\delta_j(y_j) \geq 1$. This scalar risk adjustment factor exceeds unity because no disclosure implies additional uncertainty, so that investors in turn require a lower price.

Third, the size of the economy affects the pricing of the firm's shares. In the limiting case where there are infinitely many firms in the economy, the effect of firm-specific cash flow risk becomes negligible and does not affect share prices. When that happens, the role of discretionary disclosure of firm-specific variance disappears. However, our analysis applies to large, but finite, economies. This leads to our fourth and final point.

Lemma 2 shows us that the market sets share prices as if it adds the effects of the market-wide and firm-specific risks together. The manager chooses whether to disclose based on the difference in the share price when he discloses the variance vs. when he does not. Since the market-wide risk factor enters share price additively whether or not the manager discloses, it does not affect his disclosure decision. This fact proves useful in analyzing the manager's equilibrium disclosure strategy.

We now present the existence result regarding the credible partial disclosure equilibrium in Theorem 1.

Theorem 1: For some intermediate level of disclosure costs there exists a credible partial disclosure equilibrium.

Theorem 1 shows that, given Assumptions A1–A4, a partial disclosure equilibrium exists. However, Theorem 1 does not ensure uniqueness. To rule out an infinite multiplicity of partial disclosure equilibria, we introduce the following additional assumption.

Assumption A5 (Assumption of a Non-Constant Risk Adjustment): The risk adjustment associated with nondisclosure, $H(y_j) \equiv \delta(y_j)E\lfloor V_j | V_j \geq y_j \rfloor$, is differentiable and its derivative differs from one for all y_j in the support of V_j, almost surely.

In our model, Assumption A5 does not hold trivially because it fails to hold when investors believe the variance has an exponential distribution. In that case, there exists a single disclosure cost level that supports a partial disclosure equilibrium and further, every threshold level supports an equilibrium at that cost. We therefore require Assumption A5 to perform comparative statics on the partial disclosure equilibrium.

Throughout the remainder of the paper, we maintain Assumptions A1–A5 and intermediate disclosure cost levels for all firms. These assumptions enable us to use the credible equilibrium of Theorem 1 as our voluntary disclosure benchmark. To gain further intuition regarding the managers' voluntary disclosure decision in the partial disclosure equilibrium, we present comparative statics for two examples of a single-firm economy. (Proofs for these two examples are available from the authors.)

Example 1: Assume that investors' prior beliefs are that the cash flow variance is uniformly distributed. Then partial disclosure is the unique credible equilibrium for sufficiently low disclosure costs, and the resulting disclosure threshold is increasing in the mean and increasing in the standard deviation of investors' prior distributions over the variances. Further, an increase in the standard deviation of investors' prior distribution over the variance increases the probability that the manager discloses and decreases the firm's share price if the manager does not disclose.

This example illustrates the effect of changes in investors' prior distribution regarding the variance of the firm's cash flows. Increasing the standard deviation of the investors' prior beliefs about the variance increases the equilibrium disclosure threshold, y, and this in turn implies that disclosure is more likely. As discussed further below (see Corollary 3), this result extends earlier results in Barry and Brown (1985).

Example 2: Assume that investors believe that the cash flow variance is Gamma distributed with parameters 2 and λ, and that investors' aggregate risk aversion is low such that $a^2 < 2\lambda$. Then partial disclosure arises as an equilibrium for intermediate disclosure costs. Furthermore, the associated disclosure threshold is decreasing in the disclosure costs, increasing in investors' risk aversion, and decreasing in λ.

The comparative static result on disclosure costs is qualitatively similar to the comparative static results reported for discretionary disclosures of management forecasts in Verrecchia (1990): higher disclosure costs lead to a higher disclosure threshold or lower likelihood of disclosure. We also demonstrate that an increase in investors' risk aversion leads to a higher disclosure threshold or higher likelihood of disclosure. To interpret the last comparative static result in Example 2, recall that increasing λ decreases both the mean and the standard deviation of investors' prior beliefs about the variance, V.[8] Consistent with Verrecchia (1990), we measure initial information asymmetry between investors and the manager by the standard deviation of investors' prior beliefs about the variance. Decreasing

[8] This is so because the prior mean and standard deviations are $(2/\lambda)$ and $(\sqrt{2}/\lambda)$, respectively.

the standard deviation of investors' prior beliefs reduces the pressure from investors for disclosure and, hence, a lower disclosure threshold. Also, in this example, the partial disclosure threshold is chosen relative to the mean, so decreasing the mean decreases the threshold. Simultaneous decreases in both the mean and the standard deviation of investors' prior beliefs about the variance of the firm's future cash flows lead to a lower disclosure threshold, again similar to Verrecchia (1990).[9]

We next compare the effect of voluntary risk disclosure on share prices from two different perspectives. First, in a partial disclosure equilibrium, we establish how discretionary risk disclosure affects share prices. This is an *ex post* comparison of the share price after the variance has been realized. Second, we compare a firm's *ex ante* share price in a voluntary risk disclosure regime to its *ex ante* share price in a mandatory risk disclosure regime.

Corollary 1: In the partial disclosure equilibrium, a firm has higher share price if the manager discloses than if he does not.

Corollary 1 holds because disclosing firms have more favorable news. Corollary 1 adopts an *ex post* perspective, so we cannot use this result to infer how different disclosure decisions affect the share price of a single firm. To do this, we next adopt an *ex ante* perspective and address the question of which disclosure regime, mandatory or voluntary, produces higher share price for each firm. This comparison is given in the following corollary.

Corollary 2: Consider the benchmark of a voluntary risk disclosure regime. First, imposing a mandatory full risk disclosure regime will lower the *ex ante* expected share price of the firm. Second, imposing a mandatory no-disclosure regime will increase the *ex ante* expected share price of the firm.

The intuition behind Corollary 2 is as follows. First, under voluntary disclosure, the manager will choose not to disclose for some realizations of the variance. This implies that no disclosure results in higher firm value for those realizations than does disclosing. Hence, imposing mandatory risk disclosures reduces the *ex post* share prices for nondisclosers. Second, the result comparing voluntary disclosure to forbidden disclosure is similar to Verrecchia's (1983) result on disclosures of a signal about the firm's liquidating value. *A priori*, both managers and investors base their expected variance on their prior beliefs about variances, regardless of the regime in effect. Mandating no disclosure leaves the expected variance unchanged, but ensures that the firm will not incur the disclosure costs, which, in turn, increases share prices. Corollary 2 contributes to the literature by comparing voluntary to mandatory risk disclosure regimes in a general exchange economy with risk-averse investors and endogenous share prices.

Effect of Managers' Risk Disclosures on Market Returns

The equilibrium prices derived in connection with Theorem 1 enable us to extend the literature further by analyzing the CAPM in a partial disclosure equilibrium. Our investigation of the relation between risk disclosure decisions and the CAPM requires us to specify

[9] We mention for completeness that the manager never discloses if disclosure costs are sufficiently high. Further, full disclosure can arise in Example 2, and this is the only equilibrium when the disclosure costs are sufficiently low.

a return measure. We define returns using a gross measure for the stocks between time 1 and time 2, that is, the return on the stock of firm j is:

$$R_j = \frac{(X_j - 1_{\{v_j \notin N_j\}} C_j)}{P_j(v_j, N_j | \underline{N}_{-j}, \underline{\hat{N}})}.$$

Similarly, we denote the gross return on the market portfolio as $R_m = W_m / W_m^0$, where W_m^0 and W_m are the initial and ending wealth in the economy, respectively. In our next result, we verify that the CAPM holds even when investors are uncertain about the firm-specific variance, and a manager's disclosure could resolve this uncertainty.

Theorem 2: The CAPM holds in each credible partial disclosure equilibrium, that is, the risk premium for each firm j can be expressed as:

$$E\lfloor R_j \rfloor - R_f = \beta_j (E[R_m] - R_f)$$

$$\text{where} \quad \beta_j = \frac{COV\lfloor R_j, R_m \rfloor}{VAR[R_m]}.$$

Theorem 2 enables us to analyze the effect of discretionary risk disclosure on CAPM parameters. To facilitate this presentation, we introduce one final assumption.

Assumption A6 (Assumption of Positive Share Prices): The following are positive: the share price of firm j under mandatory disclosure, the factor loading for firm j, or $\gamma_j > 0$, and the sum of the factor loadings, or $\overline{\gamma} > 0$.

Besides ensuring that the price of firm j is positive for each observable variance, Assumption A6 guarantees that the market risk premium is positive, that is, $E[R_m] - R_f > 0$. We next establish how a manager's disclosure decision affects the market risk premium and his own firm's CAPM beta and risk premium.

Corollary 3: Assume that manager j discloses the variance v_j. The market risk premium increases in the disclosed variance, that is, $\frac{\partial}{\partial v_j}(E[R_m] - R_f) > 0$. The higher the variance disclosed by the manager j, the higher the beta of firm j and the higher the risk premium of firm j, that is, $\frac{\partial \beta_j}{\partial v_j} > 0$ and $\frac{\partial}{\partial v_j}(E[R_j] - R_f) > 0$, respectively.

First, since the total market risk increases with the variance of firm j, the market premium must also increase. Second, given disclosure by manager j, that firm's beta increases with the disclosed variance because a higher disclosed variance implies that investors know that firm j's cash flow contributes a higher fraction to the total risk in the market. Mathematically, the risk premium of firm j equals the product of the market risk premium and firm j's beta. Hence, combining these two effects, the risk premium of firm j must also increase in the disclosed variance. It also follows that, *ceteris paribus*, a firm has lower beta and lower risk premium if its manager discloses risk than if its manager withholds the risk information.

Corollary 3 also suggests an alternative hypothesis regarding the relation between the initial level of investor uncertainty about the variance in a firm's cash flows and that firm's beta. Barry and Brown (1985) show that higher dispersion in investors' prior beliefs about the variance of the firm's future cash flow leads to a higher risk premium. In our setting, however, higher dispersion in prior beliefs has a second indirect effect: higher dispersion in investors' prior beliefs about the variance could lead to a (disproportionately) higher disclosure threshold (i.e., managers make more frequent risk disclosures). Thus, managers of firms whose investors face excessive estimation uncertainty may disclose so much more frequently that the net effect of greater investor prior uncertainty about the firm's risk premium actually *decreases* beta—the opposite to Barry and Brown's (1985) prediction.

Corollary 3 shows that manager j's disclosure decision can affect his firm's beta and risk premium. We now consider whether other managers' disclosure decisions affect the beta and the risk premium of firm j. We answer these questions in the following Corollary.

Corollary 4: Assume $\gamma_k > 0$ and that manager k discloses his firm-specific variance, v_k. The beta of all other firms is decreasing in v_k, or $\frac{\partial \beta_j}{\partial v_k} < 0$ for $k \neq j$, but the risk premium of these other firms is unaffected by manager k's disclosure, or $\frac{\partial}{\partial v_k}(E[R_j] - R_f) = 0$ for $k \neq j$.

An increase in the variance disclosed by manager k decreases the beta of firm j because the fraction of the total market risk contributed by firm k has increased. This further implies that, *ceteris paribus*, firm j's beta is lower when manager k does not disclose (because firm k's variance is so high that it is in the nondisclosure region) than when manager k discloses.

However, manager k's disclosures do not affect the risk premium of firm j. Recall that the risk premium of firm j equals the product of the market risk premium and firm j's beta. The decrease in the beta of firm j caused by the disclosure by firm k of a relatively high variance, v_k, offsets the increase in the market risk premium caused by the increased risk, leaving the product of the market risk premium and the beta of firm j unchanged. The share prices in Lemma 2 provide the intuition for this result. The share price of firm j does not depend on the disclosures made by other managers, so neither does the expected return of firm j. Consequently, a manager's risk disclosure decision does not affect the risk premium of the other firms in the economy. In summary, our results suggest that, in our model, one firm's risk disclosures affect the betas, but not the share prices or risk premia, of the other firms in the economy.

Building on the analysis of the effect of managers' voluntary risk disclosure decisions on CAPM parameters, we next compare these parameters across risk disclosure regimes. More specifically, we compare the *ex ante* expected beta of firms in a voluntary risk disclosure regime to the *ex ante* expected beta of those same firms if they were required to make risk disclosures.

Corollary 5: *Ex ante*, before the firm's cash flow variance is realized, the expected risk premium and the expected beta are higher under mandatory risk disclosure than under a voluntary risk disclosure regime. Further, the expected risk premium and the expected beta are higher under a voluntary risk disclosure regime than under a no-disclosure regime.

Corollary 5 states that the expected risk premium for each firm will be higher under mandatory full disclosure of firm risk than under either voluntary risk disclosures or mandated nondisclosure. Thus, a firm's cost of capital, as measured by its expected risk premium, will be weakly higher in a mandatory disclosure regime than in a voluntary risk disclosure regime. To understand the intuition for this result, consider what managers cannot do when disclosure becomes mandatory. In a voluntary regime, managers with unfavorable news can choose to withhold their private information. Investors incur a cost in terms of an additional risk premium, but this cost is less than investors' direct costs if the manager had been forced to disclose. Under mandatory full disclosure of firm risk, managers must incur the higher cost of disclosing unfavorable news, driving share prices down and driving firm risk premia up in equilibrium. Moving from a voluntary to a mandatory regime appears to redistribute wealth from current to future shareholders, but the net welfare effect will depend on the level and the nature of the disclosure costs incurred and any other unmodeled potential benefits of disclosure.

V. SUMMARY

Recent regulation requiring firms to disclose the sensitivity of certain of their future cash flows to market-wide risks highlights the need for a theoretical analysis of managers' risk disclosure decisions. Our research objective is to describe a manager's risk disclosure strategies and to assess the possible effects of requiring disclosure of a second aspect of risk—firm-specific risk. We consider a multifirm setting in which the manager of each firm privately observes the variance of his firm's future cash flows, and then maximizes firm value by choosing to either disclose truthfully at a cost or else refrain from disclosing. After observing all managers' discretionary disclosure decisions, risk-averse investors trade, and their demand for shares determines the share price of each firm.

We endogenously derive the price of each firm's shares. We show that these prices depend on market-wide risk, but that the manager's disclosure decision does not. Hence, in our setting, each manager's discretionary risk disclosure decision rests solely on the firm-specific cash flow risk his firm faces and is independent of other managers' disclosure decisions. From an *ex post* perspective, we find that in a voluntary risk disclosure regime, firms have higher share prices if their managers disclose the firm's risk than if they do not. From an *ex ante* perspective, however, imposing mandatory risk disclosure requirements lowers the firms' share prices relative to share prices in a discretionary disclosure regime. Requiring firms to make risk disclosures that they would not make voluntarily forces firms to incur additional disclosure costs that decrease firm value.

We verify that the Capital Asset Pricing Model holds in our setting when managers use a partial disclosure strategy, and we establish how a manager's disclosure strategy choice affects his firm's beta and risk premium. For example, we find that a firm's beta, its risk premium, and the market risk premium are all higher if a firm fails to disclose the variance of its firm-specific cash flows. Also, one firm's beta is higher if other firms' managers disclose than if they do not. In addition, a firm's expected risk premium and its expected beta are both higher in a mandatory risk disclosure regime than in a voluntary disclosure regime.

Our results offer empirically testable hypotheses. First, when investors are more uncertain about the level of the variance in the firm's future cash flows prior to disclosure, we predict that voluntary risk disclosure is more likely. Second, for a firm with positive beta, we predict that the risk premium on its stock, its beta, and the risk premium on the market increase as the disclosed variance increases, and increase further under nondisclosure. Third, for firms with positive betas, we predict that a higher variance disclosed by one firm lowers the betas for other firms. Finally, we predict that an increase in the mandatory disclosure requirements will increase the firms' average beta and risk premium.

Our analysis has a number of limitations, all of which we view as raising questions for future research. First, we presume that the disclosures mandated by FRR No. 48 ensure that managers fully disclose the sensitivity of their firms' cash flows to market-wide risk factors. However, FRR No. 48 requires firms to disclose the sensitivity to market risk factors of cash flows related only to financial instruments and derivatives. Thus, a question for future research is: Does the manager's discretionary disclosure decision change if investors do not know the sensitivity of all of the firm's cash flows to market-wide risk? Second, while FRR No. 48 mandates disclosure, it allows for a choice among three formats. How do managers choose among these formats? Third, the benefits from disclosure are limited in our setting because we do not consider the possible interaction between risk disclosure and managers' real investment decisions. How might a manager's discretionary disclosures about his firm's risk affect the firm's investment decisions? Fourth, we assume that investors are price takers. If we assume some investors are informed traders, does the manager's risk disclosure affect total order flows and bid-ask spreads? Fifth, we do not consider other types of disclosures. Do a manager's discretionary disclosures of earnings forecasts interact with his decision to disclose about risk? All of these questions remain open and deserve further exploration.

APPENDIX

Proof of Lemma 1

Manager j's discretionary disclosure decision is characterized by $P_j(\hat{N}_j) \leq P_j(v_j)$ for all $v_j \notin N_j$ and $P_j(\hat{N}_j) \geq P_j(v_j)$ for all $v_j \in N_j$. It suffices to show that Lemma 1 holds for each firm individually, so we drop the j subscript and consider one manager's disclosure decision. The remainder of this proof is by contradiction. Assume N is not upper-tailed; there must be variances $v_1 < v_2$ where $v_1 \in N$ and $v_2 \notin N$. The contradiction arises because we assume price is monotonically decreasing in the disclosed variance, so $v_1 < v_2$ implies:

$$P(V \in \hat{N}) \geq P(V = v_1) > P(V = v_2) \geq P(V \in \hat{N}).$$

If price is increasing in the disclosure, then, by an analogous argument, the nondisclosure region is lower-tailed. This completes the proof of Lemma 1.

Proof of Lemma 2

Below, we follow Huang and Litzenberger (1988, Chapter 4.15). Since each investor's budget constraint is binding, we can substitute out the number of bonds through $B_i = W_i^0 - \Sigma_{j=1}^J S_{ij}P_j$. Hence:

$$\begin{aligned} W_i &= (W_i^0 - \sum_{j=1}^J S_{ij}P_j)\, R_f + \sum_{j=1}^J S_{ij}(X_j - 1_{\{v_j \notin N_j\}}C_j) \\ &= W_i^0 R_f + \sum_{j=1}^J S_{ij}(X_j - 1_{\{v_j \notin N_j\}}C_j - P_j R_f) \end{aligned}$$

and we can write the investor's problem as:

$$\max_{\{S_{ij}\}_{j=1}^{J}} E[-e^{-a_i(\Sigma_{j=1}^{J} S_{ij}(X_j - 1_{\{v_j \notin N_j\}} C_j - P_j R_f) + W_i^0)}]$$
$$= -e^{-a_i W_i^0 R_f} E[e^{-a_i \Sigma_{j=1}^{J} S_{ij}(\gamma_j F + \varepsilon_j)}] e^{-a_i \Sigma_{j=1}^{J} S_{ij}(\mu_j - 1\{v_j \notin N_j\} C_j - P_j R_f)}$$

where expectations are taken conditional on investors' beliefs and whether investors know the firm-specific variance for each stock. Evaluating the expectations term yields:

$$E[e^{-a_i \Sigma_{j=1}^{J} S_{ij}(\gamma_j F + \varepsilon_j)}] = E[e^{-a_i(\Sigma_{j=1}^{J} S_{ij}\gamma_j)F}] \prod_{j=1}^{J} E[e^{-a_i S_{ij} \varepsilon_j}]$$
$$= e^{(a_i^2/2)(\Sigma_{j=1}^{J} S_{ij}\gamma_j)^2 \sigma^2} \prod_{j=1}^{J} E[e^{[-(a_i S_{ij})^2/2]V_j}].$$

To express each investor's problem in certainty equivalents we change signs, take logs, and divide through by $(-a_i)$:

$$\max_{\{S_{ij}\}_{j=1}^{J}} W_i^0 + \sum_{j=1}^{J} S_{ij}\,(\mu_j - 1_{\{v_j \notin N_j\}} C_j - P_j R_f)$$
$$- \frac{a_i}{2} \left(\sum_{j=1}^{J} S_{ij}\gamma_j \right)^2 \sigma^2 - a_i^{-1} \sum_{j=1}^{J} \ln(E[e^{[-(a_i S_{ij})^2/2]V_j}]).$$

The first-order condition for an interior maximum for S_{ij} is:

$$\gamma_j \left(\sum_{k=1}^{J} S_{ik}\gamma_k \right) \sigma^2 + S_{ij} H_{ij}\,(\underline{\hat{N}}, S_{ij}) = a_i^{-1}(\mu_j - 1_{\{v_j \notin N_j\}} C - P_j R_f)$$

where $H_{ij}(\underline{\hat{N}}, S_{ij}) = v_j$ if manager j discloses v_j, and otherwise:

$$H_{ij}(\underline{\hat{N}}, S_{ij}) = \frac{E[V_j e^{[-(a_i S_{ij})^2/2]V_j} \mid V_j \in \hat{N}_j, \underline{\hat{N}}_{-j}]}{E[e^{[-(a_i S_{ij})^2/2]V_j} \mid V_j \in \hat{N}_j, \underline{\hat{N}}_{-j}]}$$

where expectations are taken conditional on the disclosure strategies of other managers.

We conjecture, and subsequently verify, that there exists an equilibrium in which $H_{ij}(\underline{\hat{N}}, S_{ij}^*) = H_j(\hat{N}_j)$ when evaluated at the equilibrium demand for stock, S_{ij}^*. We can write all J first-order conditions for investor i's demand for stocks in vector form as:

$$\underline{\underline{M}}\,\underline{S_i} = a_i^{-1}(\underline{m} - \underline{P} R_f)$$

where:

$$\underline{S_i} = (S_{i1}, .., S_{iJ})$$
$$\underline{m} = (\mu_1 - 1_{\{v_1 \notin N_1\}} C, .., \mu_j - 1_{\{v_j \notin N_j\}} C)$$
$$\underline{P} = (P_1, .., P_j)$$

and the elements of the $J \times J$ matrix $\underline{\underline{M}}$ are:

$$M_{jj} = \gamma_j^2\sigma^2 + H_j(\hat{N}_j)$$

and:

$$M_{kj} = \gamma_k\gamma_j\sigma^2 \quad \text{for } k \neq j.$$

Since $H_j(\hat{N}_j) > 0$, the matrix $\underline{\underline{M}}$ is invertible. Hence:

$$\underline{S_i} = \underline{\underline{M}}^{-1}a_i^{-1}(\underline{m} - \underline{P}R_f).$$

Summing over all investors on both sides, we find the aggregate demand for stock j:

$$\sum_{i=1}^{I} \underline{S_i} = \sum_{i=1}^{I} \underline{\underline{M}}^{-1}a_i^{-1}(\underline{m} - \underline{P}R_f).$$

Since each stock is normalized to be in net supply of 1:

$$\underline{1} = \underline{\underline{M}}^{-1}\left(\sum_{i=1}^{I} a_i^{-1}\right)(\underline{m} - \underline{P}R_f)$$

where the left-hand side is a vector of 1s. This reduces to:

$$\underline{\underline{M}}\,\underline{1} = a^{-1}(\underline{m} - \underline{P}R_f)$$

where $a^{-1} = (\Sigma_{i=1}^{I} a_i^{-1})$. Rearranging terms, the equilibrium share prices follow:

$$\underline{P} = [\underline{m} - a\underline{\underline{M}}\,\underline{1}]R_f^{-1}.$$

To verify the conjecture, substitute the equilibrium share prices into the individual investor's demand function:

$$\underline{S_i^*} = \underline{\underline{M}}^{-1}a_i^{-1}(\underline{m} - [\underline{m} - a\underline{\underline{M}}\,\underline{1}]) = \underline{\underline{M}}^{-1}a_i^{-1}a\underline{\underline{M}}\,\underline{1} = a_i^{-1}a\underline{1}.$$

Since $a_iS_{ij}^* = a$ is the same for all investors and each firm j, the conjecture that $H_{ij}(\underline{\hat{N}}, S_{ij}^*) = H_j(\hat{N}_j)$ follows. Defining the scalar as in Gron et al. (2002):

$$\delta_j(\hat{N}_j) \equiv \frac{H_j(\hat{N}_j)}{E[V_j|V_j \in \hat{N}_j]} = \frac{E[V_je^{(-a^2/2)V_j}|V_j \in \hat{N}_j]}{E[e^{(-a^2/2)V_j}|V_j \in \hat{N}_j]E[V_j|V_j \in \hat{N}_j]} \geq 1$$

completes the proof of Lemma 2.

Proof of Theorem 1

Before proving the theorem, we introduce additional notation to simplify the presentation. Define $D(y)$ as the difference between the price if y is disclosed and the price if y is not disclosed, i.e.:

$$D(y) \equiv P(V|V = y) - P(V|V \geq y) = \frac{\mu - C - ay}{R_f} - \frac{\mu - a\delta(y)E[V|V \geq y]}{R_f}.$$

The argument in $D(y)$ refers to both the disclosed variance as well as investors' inferred disclosure threshold. Clearly with a continuous random variable with a connected support, $D(y)$ will be a continuous function. Also, it is clear that the manager wishes to disclose y if $D(y) > 0$. Further, if we define $k = a^2/2$ and $H(y) = \delta(y)E[V|V \geq y]$, then:

$$H(y) = \frac{E[Ve^{kV}|V \geq y]}{E[e^{kV}|V \geq y]} = \frac{\int_y^{\bar{V}} Ve^{kV} f(V|V \geq y)dV}{\int_y^{\bar{V}} e^{kV} f(V|V \geq y)dV} = \frac{\int_y^{\bar{V}} Ve^{kV} f(V)dV}{\int_y^{\bar{V}} e^{kV} f(V)dV}.$$

If H is differentiable, it follows that:

$$H'(y) \equiv \frac{\partial H}{\partial y}(y) = \frac{e^{ky} f(y)}{\left\{\int_y^{\bar{V}} e^{kV} f(V)dV\right\}^2} \left\{\int_y^{\bar{V}} (V - y)e^{kV} f(V)dV\right\} \geq 0.$$

Since $H(y)$ is increasing, there are three cases to consider: $H'(y) > 1$, $0 < H'(y) < 1$, and $H'(y) = 1$.

Case 1: Suppose $H'(y) > 1$ on some interval $[y_L, y_U]$ and assume $H(y_L) - y_L < C/a < H(y_U) - y_U$. Then $D(y_L) < 0 < D(y_U)$. In this case, the partial disclosure determined by threshold y^* is credible, where y^* solves the equation $C/a = H(y^*) - y^*$. Further, this equilibrium is unique on the set of partial disclosure strategies if $H'(y) - 1$ is strictly positive, because in that case $D(y) = 0$ holds only at $y = y^*$.

Case 2: Suppose $0 < H'(y) < 1$ on some interval $[y_L, y_U]$ and assume disclosure costs solve the condition: $H(y_L) - y_L > C/a > H(y_U) - y_U$. Then $D(y_U) < 0 < D(y_L)$. As in case 1, the partial disclosure strategy determined by threshold y^* is credible, where y^* solves the equation $C/a = H(y^*) - y^*$. Further, this equilibrium is unique on the set of partial disclosure strategies if $(H'(y) - 1)$ is strictly negative, since then $D(y) = 0$ holds only at $y = y^*$.

Case 3: Suppose Assumption A5 does not hold, so that $H'(y) = 1$ for all y. In this case $H(y) - y = H(0)$ for all y. Further full disclosure (or no disclosure) is the unique credible equilibrium as $H(0) > C/a$ (or $H(0) < C/a$). In the case that $H(0) = C/a$, all possible strategies support a credible equilibrium, since the manager is indifferent among all his reporting choices.

This completes the proof of Theorem 1.

Proof of Corollary 1

The result that disclosure results in a higher market price is immediate from Lemma 2, since we have shown that the no-disclosure set is upper-tailed.

Proof of Corollary 2

The result that the share price of the firm is lower under a mandatory disclosure than a partial disclosure regime follows from the fact that the variance term is additively separable in the prices. We add the following additional notation to that used in the proof of theorem 1. Define $ED(y)$ as the difference between the *ex ante* expected share price of the firm under mandatory full disclosure vs. partial disclosure with threshold, y, i.e.:

$$\begin{aligned}
ED(y) &\equiv E[P(V)|V \geq 0] - (\Pr\{V|V \leq y\}E[P(V)|V \leq y] \\
&\quad - \Pr\{V|V \geq y\}E[P(V|V \geq y)|V \geq y]) \\
&= \frac{\mu - C - aE[V]}{R_F} - F(y)\left(\frac{\mu - C - aE[V|V \leq y]}{R_F}\right) \\
&\quad - (1 - F(y))\left(\frac{\mu - a\delta(y)E[V|V \geq y]}{R_F}\right) \\
&= (1 - F(y))(a/R_F)\{(\delta(y) - 1)E[V|V \geq y] - C/a\}.
\end{aligned}$$

While the difference $(\delta(y) - 1)$ is strictly positive if the threshold, y, is set less than the maximum variance, the fact that a credible partial disclosure equilibrium exists with threshold y ensures that $ED(y) < 0$. Hence, the expected share price under mandatory full disclosure is lower than it is under the credible partial disclosure regime. An analogous argument can be used to show that share price under mandatory no disclosure is higher than it is under the voluntary regime, completing the proof of Corollary 2.

Proof of Theorem 2

Two-fund separation follows by recalling that Lemma 2 established that $S_{ij}^* = a_i^{-1}a$. Define $\bar{\gamma} = \Sigma_{j=1}^J \gamma_j$ and $\bar{\Omega} = \Sigma_{j=1}^J \Omega_j$ where:

$$\Omega_j = \gamma_j\sigma^2\bar{\gamma} + 1_{\{V_j \leq y_j\}}v_j + 1_{\{V_j \geq y_j\}}\delta_j(y_j)E[V_j \geq y_j].$$

The expected excess return on stock j is:

$$E[R_j] - R_f = E\left[\frac{X_j - 1_{\{V_j \leq y_j\}}C_j}{P_j}\right] - R_f = \frac{\{\mu_j - 1_{\{V_j \leq y_j\}}C_j - P_jR_f\}}{P_j} = \frac{a\Omega_j}{P_j}$$

Define the return on the market as $R_m = W_m/W_m^0$ where the economy's terminal wealth is:

$$\begin{aligned}
W_m &= \sum_{i=1}^{I} W_i = \sum_{i=1}^{I}\left(W_i^0 - \sum_{j=1}^{J} S_{ij}^*P_j\right)R_f + \sum_{j=1}^{J}\sum_{i=1}^{I} S_{ij}^*\,(X_j - 1_{\{V_j \leq y_j\}}C_j) \\
&= W_m^0R_f + \sum_{j=1}^{J}(X_j - 1_{\{V_j \leq y_j\}}C_j - P_jR_f)
\end{aligned}$$

and the initial wealth of the economy is $W_m^0 = \Sigma_{i=1}^I W_i^0$. The expected excess return on the market is:

$$E[R_m] - R_f = E\left[\frac{W_m - W_m^0 R_f}{W_m^0}\right] = \frac{\sum_{j=1}^{J} (\mu_j - 1_{\{V_j \le y_j\}} C_j - P_j R_f)}{W_m^0} - \frac{\sum_{j=1}^{J} a\Omega_j}{W_m^0} = \frac{a\overline{\Omega}}{W_m^0}$$

To complete the proof of Theorem 2, substitute:

$$\beta_j = \frac{COV[R_j, R_m]}{VAR[R_m]} = \frac{W_m^0}{P_j}\frac{\Omega_j}{\overline{\Omega}}$$

into the CAPM equation in Theorem 2.

Lemma 3: Under Assumption A6, the expected excess return on stock j is increasing and convex in the variance under mandatory disclosure.

Proof of Lemma 3

If manager j discloses, the return on stock j is:

$$R_j(v_j) = \frac{X_j - C_j}{P_j(v_j)} = \frac{\mu_j + \gamma_j F + \varepsilon_j - C_j}{(\mu_j - C_j - a\gamma_j\sigma^2\overline{\gamma} - av_j)} R_f,$$

and the corresponding expected excess return on stock j is:

$$E[R_j(v_j) - R_f] = \frac{a\gamma_j\sigma^2\overline{\gamma} + av_j}{(\mu_j - C_j - a\gamma_j\sigma^2\overline{\gamma} - av_j)} R_f.$$

By differentiation, we find that:

$$\frac{\partial}{\partial v_j} E[R_j(v_j) - R_f] = \frac{a(\mu_j - C_j - a\gamma_j\sigma^2\overline{\gamma} - av_j) + a(a\gamma_j\sigma^2\overline{\gamma} + av_j)}{(\mu_j - C_j - a\gamma_j\sigma^2\overline{\gamma} - av_j)^2} R_f > 0$$

and:

$$\frac{\partial^2}{\partial v_j^2} E[R_j(v_j) - R_f] = \frac{a^2[\mu_j - C_j]R_f}{[P_j(v_j)]^2} < 0.$$

Proof of Corollary 3

$\frac{\partial}{\partial v_j}(E[R_j] - R_f) > 0$ follows from Lemma 3. Note that both $\frac{\partial P_j}{\partial v_j} = -aR_f^{-1}$ and $\frac{\partial \Omega_j}{\partial v_j} = \frac{\partial \overline{\Omega}}{\partial v_j} = 1$ are positive when the manager discloses. Differentiation of the expected excess return on the market:

$$\frac{\partial}{\partial v_j}(E[R_m] - R_f) = \frac{a}{W_m^0}\left(\frac{\partial \Omega}{\partial v_j}\right)$$

and the beta from the proof of Theorem 2 yield the result.

Proof of Corollary 4

It follows immediately from $\frac{\partial P_j}{\partial v_k} = \frac{\partial \Omega_j}{\partial v_k} = 0$ and $\frac{\partial \overline{\Omega}}{\partial v_k} = 1$ that $\frac{\partial}{\partial v_k}(E[R_j] - R_f) = 0$ and $\frac{\partial \beta_j}{\partial v_k} = \frac{W_m^0 \Omega_j}{P_j}(-\overline{\Omega}^{-2})$ when the manager discloses.

Proof of Corollary 5

We show first that the expected excess stock return and the firm's beta are higher under the mandatory full disclosure regime than under voluntary disclosure, then we show how it extends to the comparison between forbidden disclosure and voluntary disclosure. From the proof of Corollary 3, the expected excess return on stock j is:

$$E[R_j(v_j)] - R_f = \frac{aR_f}{P_j(v_j)}(\gamma_j\sigma^2\overline{\gamma} + v_j)$$

if manager j discloses, and:

$$E[R_j(y_j)] - R_f = \frac{aR_f}{P_j(y_j)}(\gamma_j\sigma^2\overline{\gamma} + \delta(y_j)E[V_j | V_j \geq y_j])$$

if manager j does not disclose. As noted we have $P_j(v_j) > P_j(y_j)$ and $v_j < E\lfloor V_j | V_j \geq y_j \rfloor$, implying that disclosure results in lower excess expected return than nondisclosure in the voluntary regime. Based on Lemma 3, we can apply Jensen's inequality to $R_j(v_j)$ and find that:

$$\begin{aligned} E[R_j(V_j) | V_j \geq y_j] - R_f &= aE\left[\frac{\gamma_j\sigma^2\overline{\gamma} + V_j}{\mu_j - C_j - a\gamma_j\sigma^2\overline{\gamma} - aV_j} \middle| V_j \geq y_j\right] R_f \\ &\geq a \frac{\gamma_j\sigma^2\overline{\gamma} + E[V_j | V_j \geq y_j]}{(\mu_j - C_j - a\gamma_j\sigma^2\overline{\gamma} - aE[V_j | V_j \geq y_j])} R_f \\ &= aE\left[\frac{\gamma_j\sigma^2\overline{\gamma} + V_j}{(\mu_j - C_j - a\gamma_j\sigma^2\overline{\gamma} - aE[V_j | V_j \geq y_j])} \middle| V_j \geq y_j\right] R_f \\ &= E[R_j(E[V_j | V_j \geq y_j])] - R_f \end{aligned}$$

Applying Law of Iterated Expectations:

$$\begin{aligned} &E[R_j(V_j) - R_f | \textit{Full disclose}] \\ &\quad = \Pr\{V_j \leq y_j\}E[R_j(V_j) - R_f | V_j \leq y_j] + \Pr\{V_j \geq y_j\}E[R_j(V_j) - R_f | V_j \geq y_j] \\ &\quad \geq \Pr\{V_j \leq y_j\}E[R_j(V_j) - R_f | V_j \leq y_j] + \Pr\{V_j \geq y_j\}(R_j(E[V_j | V_j \geq y_j]) - R_f) \\ &\quad = E[R_j(V_j) - R_f | \textit{Voluntarily disclose}] \end{aligned}$$

as claimed in the first part of the Corollary. To compare voluntary to no disclosure equilibrium, an analogous proof applies; this completes the proof of Corollary 5.

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