Discretionary Disclosure and Efficiency of Entrepreneurial Investment*

JOHN S. HUGHES, *University of California, Los Angeles*

SUIL PAE, *Sungkyunkwan University SKK GSB*

1. Introduction

Entrepreneurial enterprises pose challenges for efficient investment decisions. Up-front investments by entrepreneurs prior to seeking the capital necessary for project implementation may or may not be publicly observable. Subsequently, entrepreneurs may receive private information about the prospects for project success, disclosure of which to capital providers is discretionary to a significant extent. The option to abandon a project as a means to avoid further losses when unable to raise capital adds to the complexity of characterizing equilibria in such settings. Although some prior studies have examined up-front investments, discretionary disclosure, and real options attached to entrepreneurship in isolation or in some pairwise combinations, the equilibria derived in those studies offer limited insights into entrepreneurs' decisions in the real world where all three aspects are present. For example, models examining disclosure and project abandonment without considering up-front investments assume necessarily an exogenous future cash flow distribution. Comparative statics on equilibrium disclosure and abandonment in such models regarding a change in the cash flow distribution fail to capture the effects of implied changes in disclosure and abandonment on that distribution through up-front investments which, in turn, affect disclosure and abandonment, and so on.[1] In the same vein, models considering only up-front investments and disclosure fail to predict investment decisions by entrepreneurs who take into account their option to later abandon unprofitable projects when making up-front investments. In sum, the insights and predictions about entrepreneurial activities are sensitive to the limitations of models that do not incorporate all three interrelated features—up-front investments, discretionary disclosure, and real options—that are likely to be encountered by entrepreneurs in the real world.

This paper presents a model of a start-up firm owned by an entrepreneur that integrates the aforementioned features. We focus on the efficiency implications of discretionary disclosure for ex ante investment and ex post project abandonment, and the effects of changes in the information environment on the equilibrium efficiency. Of particular interest are the effects of relaxing the assumption that ex ante investment is observable. Among the interesting results, we show that the prospects of overinvestment and a decrease in investment implied by an increase in information quality in the case of observable

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1. That is, such models ignore a fixed-point problem that arises when up-front investments affecting the cash flow distribution are endogenous. Similarly, comparative statics of the other parameters (holding the cash flow distribution constant) on disclosure and abandonment are problematic as well: up-front investments, if endogenous, would change in response to changes in the other parameters and thus affect disclosure and abandonment, and so on.

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investment vanish if investment is unobservable.[2] If investment is observable and its marginal cost is low, information is ex ante valueless and, thus, an increase in information quality or likelihood of being privately informed has no effect on ex ante investment and welfare. If investment is unobservable, greater likelihood of receiving higher-quality information improves the welfare by alleviating underinvestment, irrespective of the cost of investment. These predictions may be amenable to empirical testing in an initial public offering (IPO) setting by partitioning firms on the basis of whether up-front investments are observable (e.g., investments in research and development) or unobservable (e.g., investments in the form of human capital or otherwise not amenable to public disclosure). From a regulatory standpoint, also interesting is our result that when investment is observable, policies restricting discretion over disclosure may be accompanied by a welfare-enhancing decline in investment.

To be precise, in our model, the entrepreneur after making an ex ante investment that determines the prior mean of the distribution of a project's future cash flow may or may not receive a private signal about the project's future payoff. The entrepreneur with a signal then decides whether or not to disclose it to prospective buyers of the firm, referred to as investors, who can supply capital necessary to implement the project. If there are no takers for the firm upon disclosure or the lack thereof, the firm is not traded and its project is abandoned. Given this structure, a low-end pooling equilibrium arises at the disclosure stage of the game.[3] We define efficiency losses against a full-disclosure benchmark, wherein the informed entrepreneur always discloses her private signal and the ex ante investment is observable.[4]

Even when the ex ante investment is observable, discretion over disclosure results in efficiency losses due to overinvestment (underinvestment) if the marginal cost of investment is low (intermediate). A large ex ante investment, that is, a high prior mean of the project's future payoff, implies that not only the uninformed entrepreneur's firm but also the informed entrepreneur's firm making no disclosure that would not be traded in the benchmark is traded, notwithstanding rational inferences by investors from the lack of disclosure. Concomitantly, investors' requirement on the minimum signal to purchase the firm making disclosure is less stringent than the same requirement in the benchmark. Taken together, ceteris paribus, both imply that the entrepreneur's ex ante incentive for investment is stronger than that in the benchmark. Thus, if the marginal cost of investment is low, the entrepreneur overinvests.

Next, an intermediate ex ante investment implies the following at the disclosure stage: the uninformed entrepreneur's firm that would be traded in the benchmark is not traded, and the requirement on the minimum signal for trade of the firm making disclosure is more stringent than the same requirement in the benchmark. This increases the possibility that the initial investment may go wasted, which weakens the entrepreneur's ex ante incentive for investment relative to that in the benchmark. Such a weakened incentive combined with an intermediate marginal cost of investment engenders underinvestment. In sum, discretionary disclosure per se can lead to an efficiency loss in the form of an ex ante

2. The case of unobservable investment may be interpreted as a reduced-form representation of an agency problem emanating from hidden effort.

3. As in earlier studies (e.g., Dye 1985; Jung and Kwon 1988), the friction that precludes complete unraveling (á la Grossman 1981; Milgrom 1981) in our model is the possibility that the entrepreneur may have not received a signal and lacks the means of credibly communicating nonreceipt. As in those studies, we set aside distortions of signals by assuming truthful disclosure.

4. In this benchmark, the informed entrepreneur's firm is traded if and only if a signal indicates a positive expected payoff of the project, whereas the uninformed entrepreneur's firm is traded with no disclosure if and only if its project has a positive expected payoff. Ex ante, without knowing whether or not she will be informed, the entrepreneur makes an efficient investment that maximizes the expected firm value less the cost of investment.

overinvestment or underinvestment, which in turn results in an inefficient project abandonment decision ex post.[5]

Greater likelihood of receiving a private signal and higher precision of that signal can be viewed as reflecting the effects of advances in information technology. These primitives affect the equilibrium investment depending on their effects on the marginal benefit of investment, which is equal to the ex ante probability of trade. In the case of overinvestment, they have no impact on the equilibrium investment because the firm is traded with or without disclosure, that is, the ex ante probability of trade equals one. In fact, information is ex ante valueless in this case (although it affects the firm value ex post through discretionary disclosure by the entrepreneur) and, thus, greater likelihood of receiving a private signal or higher precision of that signal does not affect the ex ante welfare. In the case of underinvestment, by contrast, only the informed entrepreneur's firm disclosing a signal above a minimum signal is traded. Hence, greater likelihood of receiving a signal increases the ex ante probability of trade, implying a positive effect on the equilibrium investment. This alleviates underinvestment and thereby improves welfare. Higher precision of signal may reduce the equilibrium investment if the investment is greater than the capital required for implementing the project. This is because in such a case, the minimum signal requirement increases with the signal precision, which lowers the ex ante probability of trade and, hence, the marginal benefit of investment. However, the direct effect of higher precision on the expected firm value leads to an increase in the ex ante welfare.

Adding moral hazard to the entrepreneur's investment introduces further efficiency losses. Investors, who cannot observe the ex ante investment determining the prior mean of the firm's future cash flow, price the firm with or without disclosure on the basis of a conjecture of investment that the entrepreneur cannot influence. This dampens the incentive to invest. The consequences are underinvestment occurring in the observable investment case is exacerbated and, interestingly, overinvestment arising in the observable investment case changes to underinvestment. Since the equilibrium investment is smaller in the unobservable investment case compared with that in the observable investment case, investors' requirement on the minimum signal for their purchase of a firm making disclosure is more stringent. In addition, a firm making no disclosure may not be traded, or may be traded at a price lower than the price in the observable investment case. Unlike the comparative statics in the observable investment case, greater likelihood of receiving a signal or higher precision of signal always leads to a more efficient ex post project abandonment decision, which motivates the entrepreneur to invest more ex ante. Thus, underinvestment is alleviated and ex ante welfare is unconditionally improved.

To the best of our knowledge, this study is the first to simultaneously consider the following typical stages of the development of entrepreneurial firms: (i) ex ante investment affecting the distribution of future cash flow; (ii) discretionary disclosure of a private signal about the cash flow; and (iii) ex post project abandonment. As noted earlier, while several studies in the literature have examined each stage in isolation or in some pairwise combinations, how they all interact and consequent efficiency implications have been unexplored heretofore. Our analysis shows that the effects of parametric changes on equilibrium are subtle and less transparent than in earlier studies that do not incorporate all of these aspects.

Pae (1999, 2004) considers the ex ante investment and discretionary disclosure, assuming that the ex ante investment is observable and there is no option to abandon a project

5. If the marginal cost of investment is sufficiently high, there is no efficiency loss. Both in the benchmark and discretionary disclosure cases, the equilibrium investment is so small that only the informed entrepreneur's firm disclosing a signal that indicates a positive expected return of the project is traded. See section 5 for details.

ex post. Under these assumptions, discretion over disclosure has no efficiency implication for investment in his model. By contrast, in our model, the presence of an abandonment option can induce overinvestment or underinvestment if investment is observable, and overinvestment changes to underinvestment if investment is unobservable.[6] Although Hughes and Pae (2004) allow an option to abandon projects under discretionary disclosure, the object of disclosure is precision information and there is no ex ante investment. Recently, Cheynel (2013) constructively imbeds an abandonment option in an asset-pricing model wherein investors, individually endowed with a project, receive private information over which they have discretion to disclose. Similar to our model, either overinvestment or underinvestment arises in the sense that some projects are financed (not financed) that should be (not be) abandoned.[7] However, in her model, there is no ex ante investment, which is paramount in our study. In this respect, it is worth emphasizing that the value of an abandonment option in our model is derived not only from avoidance of further losses ex post, but also from its impact on investment ex ante.[8] A series of related studies consider staged investments by capital suppliers as a device for resolving a variety of inefficient decisions of entrepreneurs or agency problems: project continuation–abandonment in Admati and Pfleiderer (1994); theft in Bergmann and Hege (1998); renegotiation in Neher (1999); and moral hazard and risk in Wang and Zhou (2004).[9] However, none of these studies examines the consequences of discretionary disclosure for investment and project abandonment decisions.

The next section presents the model. Sections 3 and 4 contain our analyses of ex ante investment and ex post abandonment decisions under discretionary disclosure when ex ante investment is unobservable and observable, respectively. Section 5 addresses efficiency issues. Section 6 concludes the paper. All proofs are provided in the Appendix.

2. Model

We consider a three-stage game of a start-up company that is to be sold through an IPO in a competitive capital market. All parties are risk neutral, and the risk-free interest rate is normalized at zero. At the first stage, an entrepreneur who owns a firm entirely makes an investment to create a project. Initially, we consider a case in which the entrepreneur invests her human capital (including time and effort) for the project and this investment is unobservable to outsiders. Later in section 4, we will consider an alternative case in which the investment is observable; for example, the investment represents expenditures on R&D activities, whose information is publicly available in IPO prospectuses.

The entrepreneur's investment at the first stage alone, however, is not enough to generate a future cash flow from the project, which is the firm's sole cash flow denoted by a random variable x. In order to implement the project and thereby generate x, a fixed

6. Pae (2002) considers discretionary disclosure and observable investment made after information may have been received. However, the prior mean of a productivity parameter affecting the postsignal investment is assumed to be large enough to ensure trade in the absence of disclosure, which renders the implicit abandonment option ineffective.
7. In her model, the friction that prevents unraveling of private information is identical to that in Dye (1985), and the tension that determines financing of a project is whether the effect of an increase in aggregate consumption from the project's cash flow is greater or lesser than the effect of a decrease in aggregate risk.
8. Kanodia, Singh and Spero (2005) show that noise in accounting measurement of an investment can actually be value-enhancing. Our model differs from theirs in that the accounting signal in their model is always released publicly, and there is no way to alter the investment payoff ex post (e.g., by abandoning the project). Their focus is on the optimal imprecision of the accounting signal from an ex ante perspective in reducing signaling costs.
9. There is an element of staged financing in our model in the form of investors' potential supply of additional capital necessary for project implementation, conditional on the entrepreneur's disclosure or nondisclosure of information about the project's future payoff.

amount of capital $k > 0$ must be provided by outside investors, who make that decision at the second stage with or without the entrepreneur's disclosure of private information, as explained below. If the project is supplemented by the required capital k, the entrepreneur's investment determines the project's mean cash flow, which is denoted by $\theta \geq 0$. Without loss of generality, suppressing the relation between θ and investment, we directly represent the entrepreneur's investment by θ and use it interchangeably for the project's mean cash flow and investment throughout the paper. Let $c(\theta)$ be the entrepreneur's cost of investment to achieve a mean return of θ for the project, where c is a strictly increasing convex function that satisfies $c(0) = c'(0) = 0$. We also assume $c'''(\theta) > 0$ for all $\theta > 0$ to ensure a globally unique optimum to the entrepreneur's investment problem. The project, if implemented, generates a cash flow x that follows a normal distribution

$$x \sim N(\theta, h^{-1}), \tag{1}$$

where $h \equiv \text{Var}[x]^{-1}$. If the project is not implemented, the entrepreneur's initial investment θ is wasted in the sense that there is no cash flow from the project.

At the second stage, the entrepreneur has liquidity needs and seeks to sell her firm to outside investors, who can provide the capital k required to launch the project. Prior to offering the firm for sale, the entrepreneur receives private information about the project's future cash flow x with a probability $\lambda \in (0, 1)$. This information is represented by a signal

$$y \equiv x + \varepsilon, \tag{2}$$

where $\varepsilon \sim N(0, q^{-1})$ is independent of x and its precision $q \equiv \text{Var}[\varepsilon]^{-1}$ is referred to as the quality of signal y. Similar to Dye (1985) and Jung and Kwon (1988), we assume that when selling the firm after receiving or not receiving y, (i) the informed entrepreneur has discretion over disclosure of y and disclosure is truthful, and (ii) the uninformed entrepreneur cannot credibly reveal that she has no private signal. After disclosure or nondisclosure of y, the firm may be traded to investors, in which case they supply k to implement the project, and the cash flow x is realized at the third stage according to (1). If no trade occurs, the project is abandoned and the game is over. The game structure is common knowledge. Figure 1 provides a time-line.[10]

3. Equilibrium when the ex ante investment is unobservable

This section provides an analysis of the case where the entrepreneur invests unobservable resources, for example, human capital. We use backward induction to derive equilibrium. Suppose that the entrepreneur has invested $\theta \geq 0$ at the first stage, and consider the second stage where she might receive a private signal y and seeks to sell her firm by

10. Three remarks on our modeling choices are in order. First, the entrepreneur's liquidity constraint to sell the firm at the second stage is a modeling convenience to justify that when revealing private information selectively, her objective is to maximize the firm value. Ex ante, when making an investment decision without knowing whether or not she will be privately informed, her objective is to maximize the expected firm value net of the cost of investment. Admittedly, rather than selling the firm, the entrepreneur may finance the project in other ways, for example, by a debt contract, but derivation of an optimal financing mechanism is beyond the scope of this paper. Second, whether or not the firm is traded to investors is the same as whether or not the project is continued, that is, investors will not purchase the firm at a positive price if they are to abandon the project subsequently. This feature of the model allows us to focus on the consequences of discretionary disclosure for investment efficiencies in a simple manner and to suppress other issues such as investors' additional information gathering. Third, our model can be extended to a setting in which a positive exit value in the case of project abandonment exists as a function of the ex ante investment θ. Provided that the effect of θ on the project's expected cash flow conditional on y is greater than its effect on the exit value, there is no qualitative change in our results (except that when θ is observable, an additional marginal benefit of investment comes from the presence of the exit value).

Figure 1 Time-line

Stage 1	Stage 2	Stage 3
• The entrepreneur chooses θ, which is the mean of the future cash flow of the firm.	• The entrepreneur receives a signal y with a probability λ. • The informed entrepreneur decides whether or not to disclose y, whereas the uninformed entrepreneur makes no disclosure. • The firm may be traded to investors, who provide the capital k required for implementing the project.	• If the firm is traded at the second stage, its cash flow x is realized.

disclosing or suppressing y. We assume that the informed entrepreneur withholds y if she is indifferent between disclosing and withholding it. Since the actual investment θ is unobservable, investors' pricing of the firm with or without disclosure of y is based on their conjecture of θ, which we denote as $\theta^c \geq 0$.

Suppose that y is disclosed. Given y and θ^c, competitive investors purchase the firm and supply k to implement the project if, and only if, the posterior expectation of the project's future payoff x is greater than k. Given the normality of (x, y) whose conjectured mean is θ^c, the net posterior expectation equals

$$\pi(y, \theta^c) \equiv E[x|y, \theta^c] - k = [\beta y + (1-\beta)\theta^c] - k, \tag{3}$$

where the weight β placed on y relative to θ^c is given by

$$\beta \equiv \frac{q}{h+q} \in (0, 1). \tag{4}$$

Note that $\pi(y, \theta^c)$ increases with y for any given θ^c. Let y^o solve $\pi(y^o, \theta^c) = 0$, that is,

$$y^o = (1-\beta^{-1})\theta^c + \beta^{-1}k = \theta^c + (k-\theta^c)\beta^{-1}. \tag{5}$$

Hereafter, we refer to y^o as the cutoff signal, above which the firm disclosing y is traded to investors at the price of $\pi(y, \theta^c) > 0$ and its project is implemented. If $y \leq y^o$, the firm is not traded and the project is abandoned.

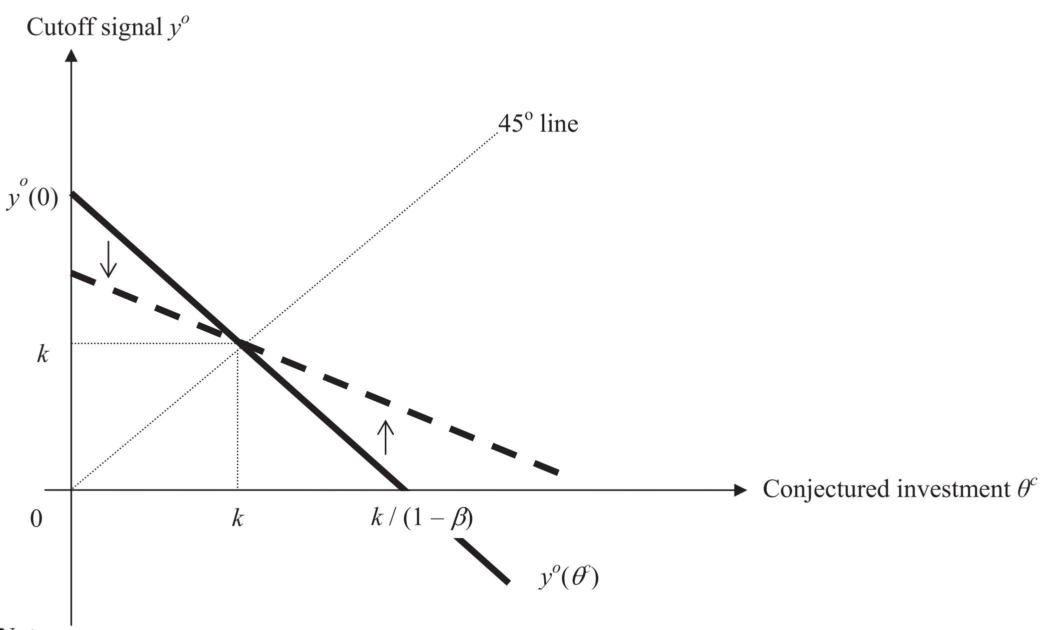
Two comparative static properties of the cutoff signal y^o will be useful throughout the analysis. First, y^o decreases with θ^c. Intuitively, when investors have a higher conjecture of the prior mean of the firm's future cash flow, they impose a less stringent requirement on the minimum signal to purchase the firm. As a result, the firm that is not traded with a disclosure of y when θ^c is small can be traded with a disclosure of the same signal y when θ^c is large. Also note that if θ^c is less (greater) than k, then y^o is greater (less) than k. This implies that the cutoff signal y^o is a good (bad) signal in the sense that it exceeds (falls short of) θ^c, which is the conjectured mean of y. Second, because β increases with q, the cutoff signal y^o decreases (increases) with q when θ^c is less (greater) than k. For intuition, observe that as q increases, investors place greater weight β on y in forming their posterior expectation $\pi(y, \theta^c)$. When θ^c is less than k, y^o is a good signal as noted earlier. This implies that the firm value $\pi(y, \theta^c)$ at $y = y^o$ becomes positive as q increases. Hence, a new cutoff signal corresponding to a higher q must be lower than before. The converse is true when θ^c is greater than k because y^o is a bad signal. Figure 2 depicts the effect of an improvement in the signal quality q on the cutoff signal y^o, where the dashed line represents y^o for a higher value of q.

Next, suppose that no signal is disclosed. Investors, who cannot distinguish nondisclosure by the informed entrepreneur and that by the uninformed entrepreneur, purchase the firm if, and only if, their net posterior expected payoff from the project is positive. Since $\pi(y, \theta^c)$ increases with y and the firm value in the absence of disclosure is a non-negative constant, investors conjecture that the set of undisclosed signals takes the form of an interval $ND = (-\infty, y^b]$, that is, a low-end pooling, where y^b is referred to as the boundary signal of the informed entrepreneur's nondisclosure set ND. Given $ND = (-\infty, y^b]$ and a conjecture θ^c for the prior mean of y, investors' net posterior expectation of the project's future cash flow is equal to

$$\mu^b \equiv E[x|ND, \theta^c] - k = \frac{(1-\lambda)(\theta^c - k) + \lambda \int_{-\infty}^{y^b} \pi(y, \theta^c)\phi(y|\theta^c)\mathrm{d}y}{(1-\lambda) + \lambda\Phi(y^b|\theta^c)}, \tag{6}$$

where we make investors' conjecture θ^c explicit in the density and distribution functions of y that are denoted by ϕ and Φ, respectively. Below, taking the conjecture θ^c as given, we characterize the boundary signal y^b that is consistent with the outcome of the second-stage game in the absence of disclosure, which depends on whether μ^b given in (6) is positive or not.

Figure 2 The cutoff signal y^o as a function of θ^c



Notes:
The effect of an increase in the signal quality q on $y^o(\theta^c)$ is represented by arrows, so that the dashed line corresponds to a new cutoff signal for an increased q.

First, suppose that $\mu^b > 0$, in which case investors purchase the firm making no disclosure at the price of μ^b. Since this price is a positive constant and the price of a firm disclosing y, that is, $\pi(y, \theta^c)$, increases with y, consistency with investors' conjecture of the nondisclosure set $ND = (-\infty, y^b]$ requires that the boundary signal y^b must equate μ^b with

$$\pi(y^b, \theta^c) = E[x|y^b, \theta^c] - k = \beta y^b + (1-\beta)\theta^c - k. \tag{7}$$

Solving $\mu^b = \pi(y^b, \theta^c)$ yields the following characterization of y^b:

$$y^b = \theta^c - \frac{\lambda}{1-\lambda}\int_{-\infty}^{y^b} \Phi(y|\theta^c)dy. \tag{8}$$

It can be shown that y^b increases with θ^c. Also note that, to be consistent with $\mu^b > 0$, the boundary signal y^b given in (8) must be strictly greater than the cutoff signal y^o given in (5).[11]

Second, suppose that μ^b given in (6) is nonpositive, that is, no trading occurs in the absence of disclosure. In this case, for any given θ^c, the boundary signal y^b of the nondisclosure set $ND = (-\infty, y^b]$ must be equal to the cutoff signal y^o. If $y^b > y^o$, the entrepreneur with a signal belonging to the interval (y^o, y^b) would disclose it to sell the firm at a positive price: recall that $\pi(y, \theta^c) > 0$ for all $y > y^o$. If $y^b < y^o$, the entrepreneur with a signal belonging to the interval (y^b, y^o) withholds that signal because the firm is not traded irrespective of her disclosure decision.[12] In sum, if no trade occurs given no disclosure, then $y^b = y^o$ must hold and hence $ND = (-\infty, y^o]$.

So far, we have taken θ^c as given to characterize the boundary signals (y^b or y^o) consistent with two possible outcomes in the second stage, that is, trade or no trade. It remains to identify conditions under which the given θ^c is consistent with the boundary signal for each outcome.

Lemma 1. *There exists a unique value of investment, denoted by $\theta^o > 0$, at which the boundary signal y^b characterized by* (8) *is equal to the cutoff signal y^o given by* (5). *In addition,*

$$\mu^o \equiv \frac{(1-\lambda)(\theta^c - k) + \lambda\int_{-\infty}^{y^o}\pi(y,\theta^c)\phi(y|\theta^c)dy}{(1-\lambda) + \lambda\Phi(y^o|\theta^c)} \begin{cases} = 0 \; for \; \theta^c = \theta^o \\ < 0 \;\; for \; all \; \theta^c < \theta^o. \end{cases} \tag{9}$$

For any given conjecture θ^c, Lemma 1 offers a simple criterion in the form of a cutoff value of investment θ^o such that whether or not investors buy the firm making no disclosure depends on whether or not their conjecture θ^c exceeds the cutoff value θ^o. If $\theta^c > \theta^o$, then $ND = (-\infty, y^b]$ and the boundary signal y^b characterized in (8) is consistent with a positive net posterior expected return of the project, that is, $\mu^b > 0$, so that the firm is traded. If $\theta^c \leq \theta^o$, the boundary signal of the nondisclosure set is equal to the cutoff signal y^o, that is, $ND = (-\infty, y^o]$, and no trade occurs. This nondisclosure set is consistent with no trade and project abandonment if, and only if, investors' net posterior expectation of the project's return, which is given by the expression μ^o defined in (9), is nonpositive. Lemma 1 shows that $\mu^o \leq 0$ for all $\theta^c \leq \theta^o$. In fact, since $y^o = y^b$ at $\theta^c = \theta^o$, we must have: (i) $\mu^b = \mu^o = 0$ at $\theta^c = \theta^o$; and (ii) $\mu^o < 0$ for any given $\theta^c < \theta^o$. Last, Lemma 1 implies that the cutoff value θ^o must be greater than the required capital k.[13] The next proposition summarizes our analysis of the second-stage game for a given conjecture θ^c.

11. For any given θ^c, recall that $\pi(y, \theta^c)$ increases with y and $\pi(y^o, \theta^c) = 0$ by the definition of y^o. Hence, if $y^b \leq y^o$, then $\pi(y^b, \theta^c) \leq 0$ must hold, which is a contradiction to $\pi(y^b, \theta^c) = \mu^b > 0$.
12. This is due to our assumption that the entrepreneur withholds a signal if she is indifferent between withholding and disclosing it. However, note that the presence of a disclosure cost—no matter how small it is—would eliminate such indifference.
13. If $\theta^o \leq k$, then $\mu^o(\theta^c = \theta^o) < 0$, which is a contradiction to Lemma 1.

Proposition 1. *Let θ^c be given. When a signal y is disclosed, investors buy the firm at the price of $\pi(y, \theta^c)$ if, and only if, y exceeds the cutoff signal y^o, where y^o is given by* (5). *When no signal is disclosed, there exists a unique cutoff value $\theta^o > k$ such that:*

(i) *if $\theta^c \leq \theta^o$, then $ND = (-\infty, y^o]$ and the firm is not traded*;
(ii) *if $\theta^c > \theta^o$, then $ND = (-\infty, y^b]$ and the firm is traded at the price of $\pi(y^b, \theta^c) > 0$, where y^b given by* (8) *is greater than y^o.*

We now step back to the first stage to examine the entrepreneur's ex ante investment θ. First, suppose that the entrepreneur anticipates at the first stage that investors' conjecture θ^c at the second stage is less than or equal to the cutoff value θ^o.[14] Based on Proposition 1, this means that she expects her firm to be traded only when she receives and discloses a signal y exceeding y^o at the second stage with its price being equal to $\pi(y, \theta^c)$, and the project will be abandoned otherwise. As a result, for an arbitrary investment θ and a conjecture $\theta^c \leq \theta^o$, the informed entrepreneur's expected firm value equals

$$V(\theta, \theta^c) \equiv \int_{y^o}^{\infty} \pi(y, \theta^c)\phi(y|\theta)\mathrm{d}y, \tag{10}$$

where $\phi(y|\theta)$ is the normal density of $y \equiv x + \varepsilon$ with mean θ and precision p, that is,

$$p \equiv \mathrm{Var}[y]^{-1} = \left[\frac{1}{h} + \frac{1}{q}\right]^{-1} = \frac{hq}{h+q}. \tag{11}$$

Taking $\theta^c \leq \theta^o$ as given, the entrepreneur chooses θ to maximize her ex ante welfare,

$$W_1(\theta, \theta^c) = \lambda V(\theta, \theta^c) - c(\theta), \tag{12}$$

where $\lambda V(\theta, \theta^c)$ is the ex ante expected firm value.

Lemma 2. *Let $\Phi(y|\theta)$ be the distribution function of y given θ. Then,*

$$V_\theta \equiv \frac{\partial V(\theta, \theta^c)}{\partial \theta} = \beta[1 - \Phi(y^o|\theta)] > 0. \tag{13}$$

In addition, when y^o stated in (5) *is evaluated at $\theta^c = \theta$ and substituted into* (13), *we have*

$$\lim_{\theta \to 0} V_\theta > 0, \quad V_{\theta\theta} \equiv \frac{\partial V_\theta}{\partial \theta} > 0, \quad \textit{and} \quad \lim_{\theta \to \infty} V_\theta = \beta.$$

Since y^o and $\pi(y, \theta^c)$ appearing in the informed entrepreneur's expected firm value $V(\theta, \theta^c)$ stated in (10) are determined by the conjecture θ^c, the investment θ affects $V(\theta, \theta^c)$ only through the mean of y. Lemma 2 shows that the marginal effect of θ on $V(\theta, \theta^c)$ is equal to β times the probability that the informed entrepreneur's firm is traded to investors, that is, $[1 - \Phi(y^o|\theta)]$. Lemma 2 also shows that, when $\theta^c = \theta$, V_θ is strictly positive even for the smallest investment, increases with θ, and is bounded above by β even for an arbitrarily large investment.[15]

14. To be precise, since the entrepreneur does not know investors' conjecture θ^c, we need to use a different notation for her conjecture of investors' conjecture. However, to save notation, our subsequent analysis abuses notation slightly in the sense that we use θ^c to represent both conjectures that are to be self-fulfilling in equilibrium.

15. Equation (13) can be related to "delta (Δ)" in the option theory, which refers to the derivative of a call option value with respect to its underlying stock price. Note that if there were no abandonment option, the informed entrepreneur's expected firm value would be $S \equiv E[\pi(y, \theta^c)]$, where the expectation is taken for all y. Since $S = \beta\theta + (1 - \beta)\theta^c - k$, we have a relation that $\partial V/\partial\theta = (\partial V/\partial S)\cdot(\partial S/\partial\theta) = \Delta\cdot\beta$, that is, $\Delta = \beta^{-1}(\partial V/\partial\theta)$. We thank an anonymous referee for drawing our attention to this matter.

It follows from Lemma 2 that the marginal benefit of investment θ in the entrepreneur's problem to maximize $W_1(\theta, \theta^c)$ is given by:

$$MB_1(\theta, \theta^c) = \lambda\beta[1 - \Phi(y^o|\theta)] \quad \text{for } \theta^c \leq \theta^o, \tag{14}$$

where MB_1 depends on θ^c through the cutoff signal y^o. Equating (14) with the marginal cost, $c'(\theta)$, yields the optimality condition for the entrepreneur's investment problem given $\theta^c \leq \theta^o$.

Second, suppose that the entrepreneur anticipates that investors' conjecture θ^c is greater than the cutoff value θ^o. According to Proposition 1 again, the entrepreneur believes that her firm will be always traded in this case: (i) if she is uninformed, she makes no disclosure and sells the firm at the price of $\pi(y^b, \theta^c)$; (ii) if she is informed, the firm is traded at the price of $\pi(y, \theta^c)$ with disclosure of $y > y^b$ and at the price of $\pi(y^b, \theta^c)$ with no disclosure of $y \leq y^b$. Taking $\theta^c > \theta^o$ as given, the entrepreneur chooses θ to maximize her ex ante welfare,

$$W_2(\theta, \theta^c) = \left\{(1-\lambda)\pi(y^b, \theta^c) + \lambda\left[\Phi(y^b|\theta)\pi(y^b, \theta^c) + \int_{y^b}^{\infty} \pi(y, \theta^c)\phi(y|\theta)\mathrm{d}y\right]\right\} - c(\theta), \tag{15}$$

where the expression in the curly bracket is the ex ante expected firm value.[16] Differentiating this value with respect to θ yields the marginal benefit of investment θ when $\theta^c > \theta^o$:

$$MB_2(\theta, \theta^c) = \lambda\beta[1 - \Phi(y^b|\theta)] \quad \text{for } \theta^c > \theta^o, \tag{16}$$

where MB_2 depends on θ^c through the boundary signal y^b. As in the case of $\theta^c \leq \theta^o$, we obtain the optimality condition for the entrepreneur's problem by equating (16) with $c'(\theta)$.[17]

The last step is to impose $\theta^c = \theta$ on the entrepreneur's optimality conditions because the conjectured investment θ^c must be the same as the actual investment θ in equilibrium. Let

$$MB_i(\theta) \equiv MB_i(\theta, \theta) \quad \text{for } i = 1, 2, \tag{17}$$

where $MB_i(\cdot, \cdot)$, $i = 1,2$, is given by (14) and (16), respectively. We also define a constant

$$m^o \equiv MB_1(\theta^o) = MB_2(\theta^o). \tag{18}$$

In words, m^o is the marginal benefit of investment evaluated at $\theta^c = \theta = \theta^o$, where the last equality in (18) follows from Lemma 1: recall that $y^o = y^b$ at $\theta^c = \theta = \theta^o$. Below, we summarize our analysis of the equilibrium investment and provide its comparative static properties.

Proposition 2. *Let θ^{**} be the equilibrium investment when investment is unobservable.*

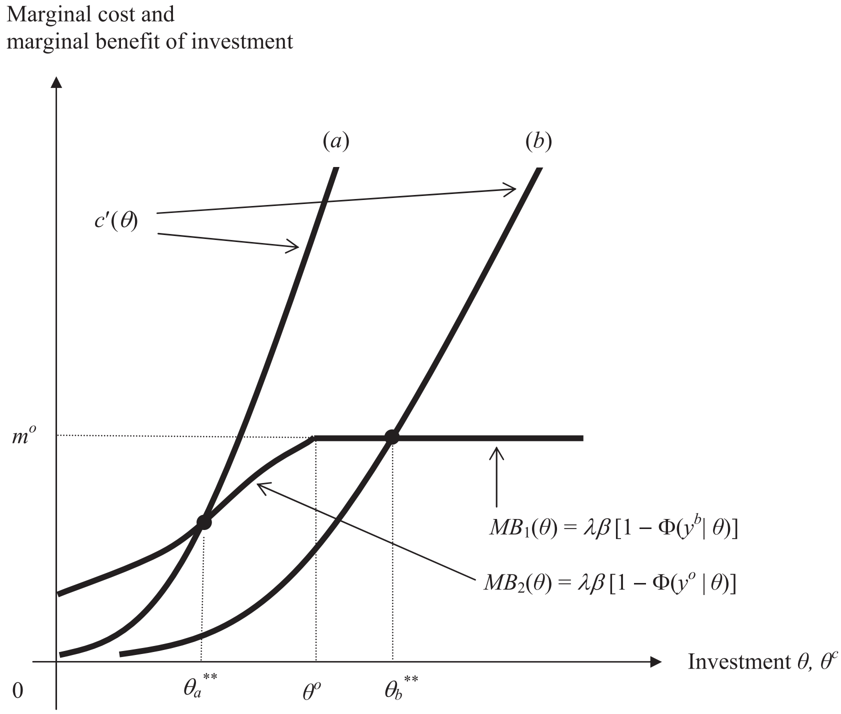
(i) *If $c'(\theta^o) \geq m^o$, then θ^{**} is less than or equal to θ^o and is characterized by*

$$MB_1(\theta) = \lambda\beta[1 - \Phi(y^o|\theta)] = c'(\theta). \tag{19}$$

16. Different from the ex ante expected firm value $\lambda V(\theta, \theta^c)$ in the case of $\theta^c \leq \theta$, (15) includes the firm value $\pi(y^b, \theta^c)$ that the entrepreneur obtains when she is uninformed or when she is informed of a signal and withholds it. Also note that since $ND = (-\infty, y^b]$ in this case, the lower bound of integration is y^b.

17. It is evident in (14) and (16) that $\beta > 0$ is necessary for the entrepreneur to make a positive investment θ. If either $h = \infty$ or $q = 0$, then $\beta \equiv q/(h + q) = 0$, implying that the project's expected payoff, $\pi(y, \theta^c)$, is independent of y and thus a zero investment would be optimal. It is noteworthy that the limiting case of $h = \infty$ is similar to the minimum investment result (Proposition 1) in Kanodia et al. (2005) that holds for any noisy accounting measurement of investment. Specifically, with $h = \infty$, we have $x = \theta$ and hence $y \equiv x + \varepsilon = \theta + \varepsilon$, and a zero investment would be made for any $q > 0$. We thank an anonymous referee for this point.

Figure 3 Determination of the equilibrium investment when θ is unobservable



Notes:
The firm making no disclosure is not traded in case (*a*), but is traded in case (*b*).

(ii) *If* $c'(\theta^o) < m^o$, *then* θ^{**} *is greater than* θ^o *and is characterized by*

$$MB_2(\theta) = \lambda\beta[1 - \Phi(y^b|\theta)] = c'(\theta). \tag{20}$$

Irrespective of whether θ^{**} *satisfies* (19) or (20), *it increases with* λ *and* q.

Figure 3 depicts the determination of the equilibrium investment θ^{**}. For $\theta \leq \theta^o$, Lemma 2 implies that $MB_1(\theta)$ increases with θ. For $\theta > \theta^o$, $MB_2(\theta)$ remains constant as θ increases because θ's effect on the boundary signal y^b offsets its effect on the mean of y.[18] These properties of the marginal benefit along with Lemma 2 and the conditions on $c(\theta)$ imply that an equilibrium investment θ^{**} satisfying either (19) or (20) always exists. Whether θ^{**} exceeds or falls short of the cutoff value θ^o hinges on whether the marginal cost evaluated at $\theta = \theta^o$ is greater or lesser than the marginal benefit m^o defined in (18). Figure 3 depicts two cases (*a*) and (*b*), where the equilibrium investment θ^{**} is, respectively, given by θ_a^{**} and θ_b^{**}. From Proposition 1 with θ^c being replaced by θ^{**}, whether a firm making no disclosure is traded depends on whether θ^{**} exceeds θ^o: the firm is not traded in case (*a*), whereas it is in case (*b*).

Given the second-order condition for the entrepreneur's maximization problem, the effects of the likelihood of being informed, λ, and the signal quality, q, on the equilibrium investment θ^{**} are determined by their effects on the marginal benefit. Consider part (i), for

18. After imposing $\theta^c = \theta$ on the boundary signal y^b characterized by (8), one can verify that $\partial y^b/\partial\theta = 1$. Using this result along with $\partial\Phi/\partial y^b = -\partial\Phi/\partial\theta$, we have $\partial\Phi(y^b|\theta)/\partial\theta = (\partial\Phi/\partial y^b) \cdot (\partial y^b/\partial\theta) + \partial\Phi/\partial\theta = 0$.

which $MB_1(\theta) = \lambda\beta[1 - \Phi(y^o|\theta)]$. An increase in λ has a positive effect on $MB_1(\theta)$, motivating the entrepreneur to invest more ex ante. An increase in q has three effects on $MB_1(\theta)$. First, a higher q increases β and hence the marginal benefit. The second and third effects are through the probability that the informed entrepreneur's firm is traded, that is, $1 - \Phi(y^o|\theta)$: recall that q affects the cutoff signal y^o (through β) and y's precision p stated in (11) increases with q. The net effect of the second and third effects depends on whether the equilibrium investment $\theta^{**} \leq \theta^o$ is less or greater than the required capital k.[19] If θ^{**} is less than k, an increase in q decreases y^o and thus $\Phi(y^o|\theta)$, implying a positive effect on $MB_1(\theta)$. Next, holding $y^o > \theta^{**}$ constant, increased precision lowers the upper-tail density of y, which reduces the probability that y exceeds y^o, that is, a negative effect on $MB_1(\theta)$. We show in the proof that when $\theta^{**} < k$, the second positive effect dominates the third negative effect. Hence, the marginal benefit increases with q, implying an increase in the equilibrium investment θ^{**}. Alternatively, if θ^{**} is greater than k, the second and third effects of q discussed above are reversed, so that their net effect is negative.[20] The proof shows that this negative effect of q on the probability of trade is dominated by the first positive direct effect of q on $MB_1(\theta)$, which leads to an increase in the equilibrium investment θ^{**}.

For part (ii), an increase in λ has two positive effects on the marginal benefit, which is $MB_2(\theta) = \lambda\beta[1 - \Phi(y^b|\theta)]$; one is its direct effect and the other is through a decrease in the boundary signal y^b. An increase in q has three effects on $MB_2(\theta)$. The first effect through an increase in β is positive. The second effect through the boundary signal y^b is negative because y^b increases with q. The third effect through y's precision p is positive: greater precision increases $[1 - \Phi(y^b|\theta)]$ because $y^b < \theta$ for any θ. It can be verified that the second and third effects offset each other, so that the net effect of q on $MB_2(\theta)$ is positive.

Combining parts (i) and (ii) shows that, irrespective of whether or not the equilibrium investment θ^{**} exceeds the cutoff value θ^o, the entrepreneur invests more when she is more likely to be informed or when the signal is more informative about the firm's future cash flow.

4. Equilibrium when the ex ante investment is observable

This section assumes that the entrepreneur's ex ante investment is observable. We make this change for two reasons. First, outside investors (or angels and venture capitalists who also often supply capital for start-ups) may have information about firms' up-front investments that affect future cash flows. For example, start-up firms invest in R&D activities before they go for IPOs. While these firms may have a superior private information about the future payoff of R&D investments subsequently (such as y), the information about R&D investments per se is available from IPO prospectuses. Second, later in section 5, we will compare the equilibrium in the observable investment case with that in the unobservable investment case derived in the previous section, and explore the efficiency implications of those equilibria relative to the allocation that would prevail in the absence of discretion over disclosure.

When investment is observable, the outcome of the second-stage disclosure game is no longer based on the conjectured investment θ^c. Given disclosure or nondisclosure of y, investors decide whether or not to purchase the firm on the basis of an actual investment θ, and the entrepreneur anticipates that decision. Hence, to obtain the equilibrium outcome of the second-stage game, we only need to replace θ^c appearing in Lemma 1 and Proposition 1 by θ.

Turning to the entrepreneur's investment at the first stage, suppose that she chooses θ less than or equal to the cutoff value θ^o. In this case, Proposition 1 (with θ^c being replaced by θ) implies that the firm is traded only when the entrepreneur is subsequently informed and discloses a signal y exceeding y^o. Let $V(\theta)$ be the expected firm value in this case, that is,

19. Recall the ordering that $k < \theta^o$, which implies that $\theta^{**}(\leq \theta^o)$ can be either lesser or greater than k.
20. That is, when $\theta^{**} > k$, the effect of q on $[1 - \Phi(y^o|\theta)]$ through y^o is negative and its effect through the precision p is positive. The former effect dominates the latter effect.

$$V(\theta) \equiv V(\theta, \theta), \tag{21}$$

where $V(\theta, \theta)$ is given by (10). The entrepreneur's ex ante welfare is given by

$$W_1^+(\theta) = \lambda V(\theta) - c(\theta). \tag{22}$$

Next, suppose that the entrepreneur chooses θ greater than θ^o. According to Proposition 1 again, the ex ante expected firm value in this case is given by

$$(1-\lambda)\pi(y^b, \theta) + \lambda\left[\Phi(y^b|\theta)\pi(y^b, \theta) + \int_{y^b}^{\infty} \pi(y, \theta)\phi(y|\theta)\mathrm{d}y\right] = \theta - k,$$

where the equality is obtained after simplifying the expression on the left-hand side by using (8) with θ^c being replaced by θ. Thus, the entrepreneur's ex ante welfare is equal to

$$W_2^+(\theta) = \theta - k - c(\theta). \tag{23}$$

Combining the two cases discussed above, the entrepreneur's problem is to choose θ that maximizes her ex ante welfare:

$$W^+(\theta) = \begin{cases} W_1^+(\theta) = \lambda V(\theta) - c(\theta) & \text{for all } \theta \leq \theta^o \\ W_2^+(\theta) = \theta - k - c(\theta) & \text{for all } \theta > \theta^o, \end{cases} \tag{24}$$

which is a continuous function of θ because $\lambda V(\theta) = \theta - k$ at $\theta = \theta^o$.[21] Given Lemma 2 and our assumptions on $c(\theta)$, an equilibrium investment must be strictly positive and finite. For $\theta \leq \theta^o$, the ex ante investment θ is more effective in increasing the expected firm value $V(\theta)$ than in the unobservable investment case in the sense that its marginal effect is greater. This can be verified from the fact that when $\theta = \theta^c$, (13) in Lemma 2 changes to:

$$V_\theta = [1 - \Phi(y^o|\theta)], \tag{25}$$

which is greater than V_θ in (13): $\beta \in (0, 1)$ does not appear in (25). The reason is that an increase in θ not only enhances the mean of y but also increases the firm value $\pi(y, \theta)$ when y exceeding y^o is disclosed.[22] If θ is unobservable, the entrepreneur does not consider this effect. Note that V_θ given in (25) preserves monotonicity and has a property that $V_\theta \to 1$ as $\theta \to \infty$. For $\theta > \theta^o$ (i.e., when trade always occurs with or without disclosure), a unit increase in θ has a full effect on the ex ante expected firm value. In sum, the marginal benefit of θ for $W^+(\theta)$ stated in (24) is:

$$MB^+(\theta) = \begin{cases} MB_1^+(\theta) = \lambda[1 - \Phi(y^o|\theta)] & \text{for } \theta \leq \theta^o \\ MB_2^+(\theta) = 1 & \text{for } \theta > \theta^o, \end{cases} \tag{26}$$

which is discontinuous at $\theta = \theta^o$ because $MB_1^+(\theta) < 1 = MB_2^+(\theta)$. Similar to the unobservable investment case, we define a constant:

$$m^+ \equiv MB_1^+(\theta^o). \tag{27}$$

The following proposition is a counterpart of Proposition 2.

Proposition 3. *Let θ^* be the equilibrium investment when the investment is observable.*

(i) *If $c'(\theta^o) \geq 1$, then θ^* is strictly less than the cutoff value θ^o and is characterized by*

$$\lambda[1 - \Phi(y^o|\theta)] = c'(\theta). \tag{28}$$

21. This result is shown in the appendix as a part of the proof of Lemma 1.
22. While θ also affects the cutoff signal y^o when θ is observable, this effect vanishes because $\pi(y, \theta) = 0$ at $y = y^o$.

The equilibrium investment θ^* *increases with* λ, *but decreases (increases) with* q *if* θ^* *is greater (less) than the required capital* k.

(ii) *If* $c'(\theta^o) \leq m^+$, *then* θ^* *is strictly greater than the cutoff value* θ^o *and is characterized by*

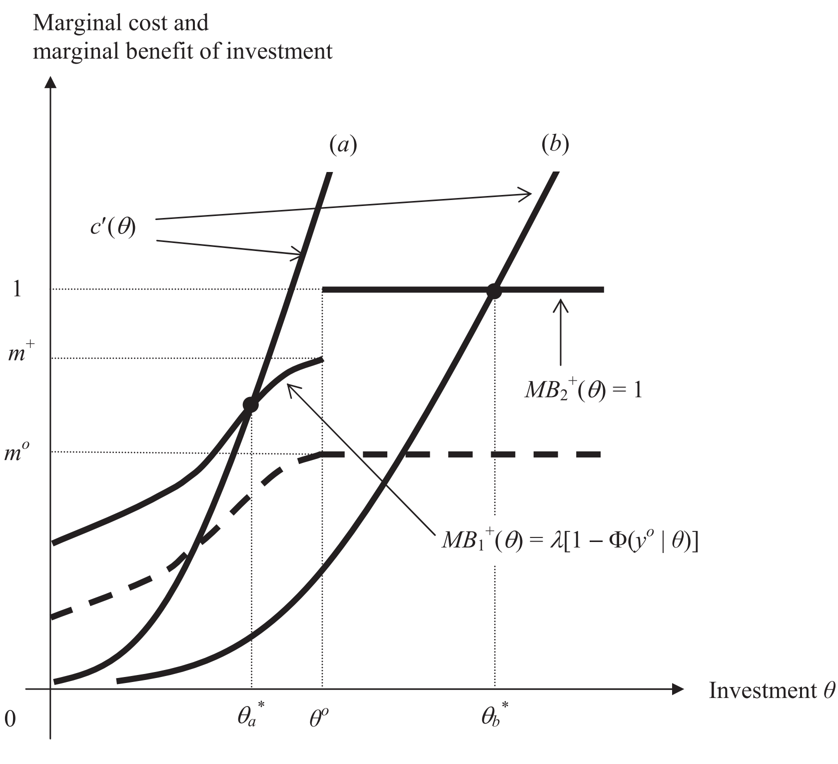
$$1 = c'(\theta). \tag{29}$$

An increase in λ *or* q *has no impact on the equilibrium investment* θ^*.

(iii) *If* $m^+ < c'(\theta^o) < 1$, *then* θ^* *satisfies either* (28) *or* (29). *The effects of* λ *and* q *on* θ^* *satisfying* (28) *or* (29) *are the same as those in part* (i) *or* (ii), *respectively*.

Figure 4 depicts the marginal benefit $MB_i^+(\theta)$, $i = 1, 2$, and the marginal cost $c'(\theta)$. If the marginal cost increases so fast as to satisfy $c'(\theta^o) \geq 1$, the equilibrium investment θ^* is less than the cutoff value θ^o, for example, θ_a^* satisfying (28) prevails. If the marginal cost increases slowly in the sense that $c'(\theta^o) < m^+$, then θ^* is greater than θ^o, for example, θ_b^* satisfying (29) prevails. The only remaining case is where the marginal cost is intermediate in the sense of $m^+ < c'(\theta^o) < 1$. Given the monotonicity of $c'(\theta)$ and discontinuity of $MB^+(\theta)$ at $\theta = \theta^o$, it is evident in Figure 4 that the marginal cost and benefit intersect two times, below and above θ^o, implying that $W^+(\theta)$ stated in (24) has two local optima satisfying (28) and (29). The equilibrium investment θ^* must be one of them. In all three cases, given an equilibrium investment θ^*, one can use Proposition 1 to note that a firm revealing y greater than the cutoff signal $y^o(\theta^*)$ is traded at the price of $\pi(y, \theta^*)$, and a firm making

Figure 4 Determination of the equilibrium investment when θ is observable



Notes:
The firm making no disclosure is not traded in case (*a*), but is traded in case (*b*).

no disclosure is traded at the price of $\pi(y^b(\theta^*), \theta^*)$ if and only if θ^* exceeds θ^o, where y^o and y^b are given by (5) and (8), respectively, with θ^c being replaced by θ^*.

When the equilibrium investment θ^* is characterized by (28), θ^* increases with λ because of its positive effect on the marginal benefit, $MB_1^+(\theta) = \lambda[1 - \Phi(y^o|\theta)]$. For the effect of q, recall our explanation for part (i) of Proposition 2: if $\theta > k$, then q affects the probability of trade, $1 - \Phi(y^o|\theta)$, negatively through y^o and positively through the precision p, and the effect through y^o is dominant over the effect through p; if $\theta < k$, the net effect is reversed. Thus, if θ^* exceeds (falls short of) k, a more informative signal decreases (increases) the marginal benefit and hence the equilibrium investment θ^*. For intuition, recall that since $\theta^* \leq \theta^o$ in this case, only the firm disclosing a signal y greater than y^o is traded. Although project abandonment subsequent to the realizations of y less than y^o allows the entrepreneur to avoid further expected loss beyond the initial investment, the possibility of ex post abandonment of project translates into an implicit cost to the entrepreneur when she makes an ex ante investment without knowing whether or not she will receive a signal. When $\theta^* > k$, a higher q increases the likelihood of that possibility, that is, decreases the probability of trade, $1 - \Phi(y^o|\theta)$, implying a greater implicit cost. This motivates the entrepreneur to economize on investment, leading to a smaller θ^*. The converse is true when θ^* is less than k: an increased probability of trade implied by a higher q reduces the implicit cost, leading to a greater ex ante investment θ^*.

The equilibrium investment θ^* satisfying (29) is independent of the likelihood of being informed and the signal quality because they do not affect the marginal benefit of investment, which is the ex ante probability of trade and equals one in this case: recall that since $\theta^* > \theta^o$, the firm is always traded with or without disclosure. In fact, note that θ^* satisfying (29) is the same as the investment that would be made if there were no information: there would be no disclosure issue and the ex ante welfare would be equal to $\theta - k - c(\theta)$, implying an optimal condition the same as (29). This means that the signal y is ex ante valueless in this case although its ex post affects the firm value through the informed entrepreneur's discretion over disclosure. This also explains why a change in λ or q has no effect on the ex ante equilibrium investment θ^*.

We conclude this section with a remark from an empirical standpoint. Comparative statics in Propositions 2 and 3 reveal that the effects of advances in information technology, captured by an increase in λ or q, on up-front investments critically hinge on whether the investments are observable or not. Of particular interest is that one may observe declines in investments which, after controlling for the other factors in our model, may be attributable to equilibrium responses of firms to improved quality of private information (as indicated in part (ii) of Proposition 3). Further, note that discontinuous changes in investments are also possible. This may be attributable to firms that have intermediate marginal cost of investment (in the sense of part (iii) of Proposition 3) and change their investments between the levels satisfying (28) or (29) in response to changes in information quality and likelihood of its receipt.[23]

23. Observe that if $m^+ < c'(\theta^o) < 1$ holds and thus the equilibrium investment θ^* is one of the two local optima of $W^+(\theta)$, changes in λ or q can lead to a discontinuous change in θ^*. To illustrate, first note that θ^o must satisfy $\lambda V(\theta^o) - (\theta^o - k) = 0$, which is shown in the proof of Lemma 1. By differentiating this equation with respect to λ, it can be verified that θ^o increases with λ. Next, consider the case in which $m^+ < c(\theta^o) < 1$ holds and let θ_1 and θ_2 solve (28) and (29), respectively. We know that θ_1 is a continuously increasing function of λ, and $\theta_1 < \theta^o < \theta_2$ holds for any given λ in this case. Now, suppose that we have $\theta^* = \theta_2$ in this case for some λ, say, $\lambda = \lambda'$. As λ increases from λ', θ_2 remains unchanged but θ^o increases and becomes equal to θ_2 at some λ, say, $\lambda = \lambda'' > \lambda'$. Since $\theta^o = \theta_2$, we have $c'(\theta^o) = 1$, and part (i) of Proposition 3 implies that $\theta^* = \theta_1(<\theta^o = \theta_2)$ at $\lambda = \lambda''$. Thus, the change in θ^* from θ_2 satisfying (29) to a value of θ_1 satisfying (28) occurs in a discontinuous manner when λ varies in the interval $[\lambda', \lambda'']$. One can use a similar argument to see the discontinuity of θ^* at some value of q.

5. Efficiencies

This section examines the efficiency implications of the equilibria derived in sections 3 and 5. Substituting the equilibrium investment θ^{**} into $W_1(\theta, \theta^c)$ in (12) and $W_2(\theta, \theta^c)$ in (15) yields the equilibrium ex ante welfare when the ex ante investment is unobservable:

$$W(\theta^{**}) \equiv \begin{cases} W_1(\theta^{**}) \equiv W_1(\theta^{**}, \theta^{**}) = \lambda V(\theta^{**}) - c(\theta^{**}) & \text{when } \theta^{**} \leq \theta^o \\ W_2(\theta^{**}) \equiv W_2(\theta^{**}, \theta^{**}) = \theta^{**} - k - c(\theta^{**}) & \text{when } \theta^{**} > \theta^o, \end{cases} \tag{30}$$

where $W_2(\theta, \theta^c)$ is now greatly simplified because $\theta = \theta^c = \theta^{**}$. Similarly, substituting θ^* into (24) yields the equilibrium ex ante welfare when the ex ante investment is observable:

$$W^+(\theta^*) = \begin{cases} W_1^+(\theta^*) = \lambda V(\theta^*) - c(\theta^*) & \text{when } \theta^* \leq \theta^o \\ W_2^+(\theta^*) = \theta^* - k - c(\theta^*) & \text{when } \theta^* > \theta^o. \end{cases} \tag{31}$$

Since $W(\theta)$ and $W^+(\theta)$ have the same functional form and θ^* maximizes $W^+(\theta)$, it follows that $W(\theta^{**}) < W^+(\theta^*)$. In particular, the entrepreneur is worse off because she invests less when θ is unobservable, that is, $\theta^{**} < \theta^*$.

We return to Figure 4 to elaborate on the reason for a smaller ex ante investment and its consequences for ex post project abandonment in the unobservable investment case compared to the observable investment case. The dashed curve is the marginal benefit of θ when θ is unobservable, that is, $MB_i(\theta)$, $i = 1, 2$, stated in (19) and (20). Since $MB_i^+(\theta) > MB_i(\theta)$ for all θ and $i = 1, 2$, the entrepreneur invests more when θ is observable than when it is not. Combining this result, $\theta^* > \theta^{**}$, with the fact that y^o decreases with θ, we obtain $y^o(\theta^*) < y^o(\theta^{**})$. In words, the minimum signal that a firm has to disclose to induce trade is lower when θ is observable than when it is unobservable. Intuitively, investors place a less stringent requirement on the minimum signal for their purchase of the firm when the prior mean of the project's future cash flow is higher. As a consequence, if a signal y satisfying $y^o(\theta^*) < y < y^o(\theta^{**})$ arrives, the entrepreneur is able to sell the firm in the observable investment case, but not in the unobservable investment case. If a signal y exceeding $y^o(\theta^{**})$ arrives, she sells her firm in both cases, but the firm value is higher in the observable investment case; recall that $\pi(y, \theta)$ increases with θ. The entrepreneur also suffers from privacy of investment in the case of no disclosure. In particular, when the observable investment θ^* satisfies (29), then $\theta^* > \theta^o$ and we have two possibilities. If $\theta^* > \theta^o > \theta^{**}$, a firm making no disclosure is traded only in the observable investment case. Alternatively, if $\theta^* > \theta^{**} > \theta^o$, a firm making no disclosure is traded in both cases, but its price is higher in the observable investment case.[24] Below, we summarize our welfare analysis and provide comparative static results on the ex ante welfare.

PROPOSITION 4. *The entrepreneur is worse off when the ex ante investment θ is unobservable, relative to when it is observable.*

(i) *If θ is observable, an increase in λ* or *q has a positive effect on the ex ante welfare when the equilibrium investment θ^* is characterized by* (28), *but has no effect when θ^* is characterized by* (29).

(ii) *If θ is unobservable, an increase in λ* or *q has a positive effect on the ex ante welfare.*

24. This is because: (i) $\pi(y, \theta)$ increases with y and θ; (ii) the firm value given no disclosure and $\theta \geq \theta^o$ equals $\pi(y^b, \theta)$; (iii) $y^b(\theta)$ increases with θ; and (iv) $\theta^* > \theta^{**}$. Note that if θ^* satisfies (28), there is no trade in both cases in the absence of disclosure because $\theta^{**} < \theta^* < \theta^o$. Thus, privacy of investment has no negative payoff consequence for the entrepreneur making no disclosure at the second stage, relative to the observable investment case.

Consider part (i) for which the equilibrium ex ante welfare equals $W^{+}(\theta^{*})$ stated in (31). Since there is no indirect effect of parameters through θ^{*} on $W^{+}(\theta^{*})$,[25] we only need to examine direct effects. When θ^{*} is characterized by (28), the direct effect of λ on $W^{+}(\theta^{*}) = W_1^{+}(\theta^{*})$ equals $V(\theta^{*}) > 0$. The direct effect of q on $W_1^{+}(\theta^{*})$ is through the informed entrepreneur's expected firm value $V(\theta)$ given by (21). In this regard, it is useful to rewrite (21) and (10) to include the signals for which the project is abandoned:

$$V(\theta) = \int_{y^o}^{\infty} \pi(y,\theta)\phi(y|\theta)\mathrm{d}y = \int_{-\infty}^{\infty} v(y,\theta)\phi(y|\theta)\mathrm{d}y, \tag{10$'$}$$

where $v(y, \theta)$ is defined to be zero for all $y \leq y^o$ and to be equal to $\pi(y, \theta)$ for all $y > y^o$. Note that $V(\theta)$ stated in (10′) is the expected value of $v(y, \theta)$, which is a piece-wise linear convex function of y. An increase in q has two effects on $V(\theta)$. The first effect is the aggregation of changes in $\pi(y, \theta)$ for all $y > y^o$, which is positive. Second, because y's precision increases with q and $V(\theta)$ is the expected value of a convex function $v(y, \theta)$, a greater q has a negative effect on $V(\theta)$. The proof shows that the former dominates the latter. Thus, the net effect of q on $V(\theta)$ is positive, implying that $W^{+}(\theta^{*}) = W_1^{+}(\theta^{*})$ increases with q. When the equilibrium investment θ^{*} satisfies (29), $W^{+}(\theta^{*}) = W_2^{+}(\theta^{*})$ and, thus, there is no direct effect of λ or q on the ex ante welfare.

For part (ii), we need to consider both direct and indirect effects. Since $W(\theta^{**})$ given in (30) has the same functional form as that of $W^{+}(\theta^{*})$ given in (31), the direct effects of λ and q on $W(\theta^{**})$ are the same as those in the observable investment case. Next, recall that the equilibrium unobservable investment θ^{**} is always less than the equilibrium observable investment θ^{*} that maximizes $W^{+}(\theta) = W(\theta)$. Thus, an increase in λ or q leading to an increase in θ^{**} (as shown in Proposition 2) has a positive indirect effect on $W(\theta^{**})$. Combining both the direct and indirect effects establishes that the ex ante welfare $W(\theta^{**})$ increases with λ and q unconditionally.

To elaborate on the efficiency implications of discretionary disclosure per se for ex ante investment and ex post project abandonment decisions, it is instructive to consider a benchmark in which θ is observable and the entrepreneur, if informed, always discloses y. In this "full disclosure" benchmark, there is no information asymmetry because it is common knowledge that only the uninformed entrepreneur makes no disclosure. The benchmark can be regarded as a setting in which regulatory bodies can detect with a positive probability any omission of material information (such as y) and impose sanctions strict enough to induce the informed entrepreneur to reveal private information. Setting aside enforcement costs, the concerns of regulators in restricting discretion over disclosure should be "real effects" on decisions affecting cash flows. In this respect, comparing the benchmark equilibrium with the equilibria derived in sections 3 and 5 provides insights into the real effects on investment and abandonment decisions.

Equilibrium of the full-disclosure benchmark is readily available from our analysis in section 4. For any given θ, since only the uninformed entrepreneur makes no disclosure, the outcome of the second-stage game is as follows: (i) if $\theta \leq k$, only the informed entrepreneur's firm disclosing y greater than $y^o(\theta)$ is traded; and (ii) if $\theta > k$, the informed entrepreneur's firm disclosing y greater than $y^o(\theta)$ and the uninformed entrepreneur's firm are traded. Turning to the first stage, the benchmark ex ante welfare is equal to

$$W^f(\theta) = (1-\lambda)\max\{0, \theta - k\} + \lambda V(\theta) - c(\theta), \tag{32}$$

25. Recall that θ^{*} characterized by either (28) or (29) maximizes $W^{+}(\theta)$. Hence, the envelope theorem applies here.

for which the marginal benefit of θ is given by

$$MB^f(\theta) = \begin{cases} MB_1^f(\theta) \equiv \lambda V_\theta(\theta) = \lambda[1 - \Phi(y^o|\theta)] & \text{for } \theta \leq k \\ MB_2^f(\theta) \equiv (1-\lambda) + \lambda V_\theta(\theta) = (1-\lambda) + \lambda[1 - \Phi(y^o|\theta)] & \text{for } \theta > k. \end{cases} \quad (33)$$

$MB^f(\theta)$ is discontinuous at $\theta = k$. This contrasts with the fact that $MB_i^+(\theta)$ stated in (26) is discontinuous at $\theta = \theta^o > k$, where the inequality is implied by Lemma 1. The equilibrium investment under full disclosure, denoted by θ^f, is obtained by equating the marginal benefit $MB^f(\theta)$ with the marginal cost $c'(\theta)$. Comparing $MB_i^f(\theta)$ with $MB_i^+(\theta)$ reveals that:

$$\begin{aligned} &MB_1^f(\theta) = MB_1^+(\theta) \quad \text{for } \theta \leq k \\ &MB_2^f(\theta) > MB_1^+(\theta) \quad \text{for } k < \theta \leq \theta^o \\ &MB_2^f(\theta) < MB_2^+(\theta) \quad \text{for } \theta > \theta^o. \end{aligned} \quad (34)$$

Thus, for any given cost of investment, we have: (i) if the observable investment θ^* is less than or equal to k, then $\theta^f = \theta^*$; (ii) if $\theta^* \in (k, \theta^o]$ characterized by (28) prevails, then $\theta^f > \theta^*$; and (iii) if $\theta^* > \theta^o$ characterized by (29) prevails, then $\theta^f < \theta^*$. It follows that although investment is observable, discretionary disclosure generates efficiency losses relative to the full-disclosure benchmark (except for the case of $\theta^* \leq k$) since underinvestment (overinvestment) arises when θ^* is less (greater) than the cutoff value θ^o. Also note from our earlier analysis that such a suboptimal ex ante investment leads to inefficiencies in project abandonment ex post. In the case of underinvestment (i.e., when $k < \theta^* < \theta^f < \theta^o$), the uninformed entrepreneur's project that would be implemented under full disclosure is abandoned, and investors' requirement on the minimum signal to purchase the informed entrepreneur's project is more stringent than the same requirement under full disclosure because $y^o(\theta^*) > y^o(\theta^f)$. In the case of overinvestment (i.e., when $\theta^o < \theta^f < \theta^*$), there is no efficiency loss associated with the implementation of the uninformed entrepreneur's project. However, the informed entrepreneur's project that would be abandoned under full disclosure is implemented with no disclosure, and the minimum signal requirement to implement a project with disclosure is less stringent than the same requirement under full disclosure because $y^o(\theta^*) < y^o(\theta^f)$.

Last, combining the earlier result that the unobservable investment θ^{**} is always less than the observable investment θ^* with the ordering of θ^* and θ^f explained above, we obtain the following results. If $\theta^* \leq k$, privacy of θ leads to an underinvestment although the observable investment is efficient, that is, $\theta^{**} < \theta^* = \theta^f$. If $\theta^* \in (k, \theta^o]$, an exacerbated underinvestment problem arises in the unobservable investment case, that is, $\theta^{**} < \theta^* < \theta^f$. If $\theta^* > \theta^o$, the incentive for investment in the unobservable investment case is so much dampened that an underinvestment occurs, which contrasts with an overinvestment in the observable investment case, that is, $\theta^{**} < \theta^f < \theta^*$.

To summarize, even when investment is observable, discretionary disclosure per se may create efficiency losses by inducing the entrepreneur to invest more or less than the investment under full disclosure. If investment is private under discretionary disclosure, a further efficiency loss arises due to underinvestment. Distortions in ex ante investment result in suboptimal project abandonment ex post, irrespective of whether the investment is observable or not.

6. Conclusion

This paper integrates the following features commonplace in entrepreneurial enterprises: up-front investments in projects, discretionary disclosure of private information about project payoffs, and options to abandon unprofitable projects. These features are inseparable. Up-front investments determining the prior mean of a project's future cash flow affect disclosure and project abandonment (trade) decisions through the impact on investors' posterior expectation of the future cash flow. These effects in turn influence the entrepreneurs' up-front investment decision, and so on, implying that a simultaneous consideration of all three features is necessary. In this respect, the assessment of investment efficiency for start-ups in this study is more comprehensive than has appeared in the literature where these features have been only partially examined.

Privacy of resources such as effort constituting up-front investments by entrepreneurs, along with discretionary disclosure of subsequent information about project payoffs, contributes to inefficiencies in the form of underinvestment. This contrasts with possible overinvestment when up-front investments are publicly observable. Further, inefficiencies arise in subsequent trade or abandonment; projects that should be traded may be abandoned, or those that should be abandoned may be traded. Greater likelihood of being privately informed or improved quality of private information always enhances ex ante welfare in the case of unobservable investment, but this need not be true in the case of observable investment where information could be ex ante valueless due to its lack of impact on the ex ante expected firm value.

Portraying potential consequences for entrepreneurial investments and subsequent trade or abandonment of project, our comparative statics contribute to a foundation for empirical tests. A case in point is the disclosure reforms of the Sarbanes–Oxley Act of 2002 and a coincident expansion of the list of material items that have to be reported in an accelerated manner in SEC Form 8-K filing (Lerman and Livnat 2010), both of which may have limited discretion over disclosure by expanding liability and sanctions for withholding value-relevant information. Several studies have found declines in capital expenditures upon implementation of these measures and attributed them to various reasons: avoidance of compliance costs (Gao, Wu, and Zimmerman 2009), less willingness to take risks (Bargeron, Lehn, and Zutter 2010), and application of higher discount rates to future cash flows (Kang, Liu, and Qi 2010). Holding aside these explanations, our analysis also suggests that when investments are publicly observable, policies removing discretion over disclosure can induce declines in investments that improve welfare by reducing overinvestment. We further note that advances in information technology tend to allow more frequent access to more informative signals. Future empirical research might address differential effects of these changes in the information environment on entrepreneurial investment, project abandonment, and frequency of IPOs across firms that differ in terms of whether up-front investments are publicly observable or private.

Appendix

Proof of Lemma 1. Viewing y^b characterized by (8) as a function of θ^c, note that $y^b < 0$ at $\theta^c = 0$. Next, differentiating both sides of (8) with respect to θ^c yields

$$\frac{\partial y^b}{\partial \theta^c} = 1 - \frac{\lambda}{1-\lambda}\left[\Phi(y^b|\theta^c)\frac{\partial y^b}{\partial \theta^c} + \int_{-\infty}^{y^b} \frac{\partial \Phi(y|\theta^c)}{\partial \theta^c}\, dy\right].$$

Using the fact that $\frac{\partial \Phi(y|\theta^c)}{\partial \theta^c} = -\phi(y|\theta^c)$, we have $\frac{\partial y^b}{\partial \theta^c} = 1$. Since y^o given in (5) is positive at $\theta^c = 0$ and decreases with θ^c, there exists a unique value of $\theta^c > 0$ at which $y^b = y^o$. That value is the cutoff value of investment denoted by θ^o.

To prove (9), recall that $\mu^b = \pi(y^b, \theta^c)$ and $\pi(y^o, \theta^c) = 0$. Hence, the fact that $y^b(\theta^o) = y^o(\theta^o)$ implies that $\mu^b = \mu^o = 0$ at $\theta^c = \theta^o$. Next, define $A(\theta^c)$ and $B(\theta^c)$ to be the numerator and denominator of μ^o, respectively. Since $\mu^o(\theta^o) = 0$ and $B(\theta^c) > 0$ for all θ^c, we have $A(\theta^o) = 0$. To show $\mu^o < 0$ for all $\theta^c < \theta^o$, it suffices to show that $A(\theta^c) < 0$ for all $\theta^c < \theta^o$. Since $A(\theta^o) = 0$, we only need to show that $A(\theta^c)$ increases with θ^c. Using the fact that $E[\pi(y, \theta^c)] = \theta^c - k$, note that

$$A(\theta^c) \equiv (1-\lambda)(\theta^c - k) + \lambda \int_{-\infty}^{y^o} \pi(y,\theta^c)\phi(y|\theta^c)\mathrm{d}y = (\theta^c - k) - \lambda \int_{y^o}^{\infty} \pi(y,\theta^c)\phi(y|\theta^c)dy. \tag{A1}$$

Consider the last term of (A1) and observe that

$$\frac{\partial}{\partial\theta^c}\left[\int_{y^o}^{\infty} \pi(y,\theta^c)\phi(y|\theta^c)dy\right] = \left[\int_{y^o}^{\infty} \frac{\partial\pi}{\partial\theta^c}\phi dy\right] + \left[\int_{y^o}^{\infty} \pi\frac{\partial\phi}{\partial\theta^c}dy\right] - \pi(y^o,\theta^c)\phi(y^o|\theta^c)(1-\beta^{-1}), \tag{A2}$$

where the last term vanishes because $\pi(y^o, \theta^c) = 0$. Using (3) simplifies the first term of (A2):

$$\int_{y^o}^{\infty} \frac{\partial\pi}{\partial\theta^c}\phi dy = (1-\beta)\int_{y^o}^{\infty} \phi(y|\theta^c)dy = (1-\beta)[1-\Phi(y^o|\theta^c)]. \tag{A3}$$

For the second term of (A2), we integrate by parts to obtain

$$\begin{aligned}\int_{y^o}^{\infty} \pi\left(\frac{\partial\phi}{\partial\theta^c}\right)dy &= \int_{y^o}^{\infty} \pi\left(\frac{\partial\Phi_{\theta^c}}{\partial y}\right)dy = \int_{y^o}^{\infty} \pi\mathrm{d}(\Phi_{\theta^c}) = [\pi\Phi_{\theta^c}]_{y^o}^{\infty} - \int_{y^o}^{\infty} \Phi_{\theta^c} d\pi \\ &= 0 + \beta\int_{y^o}^{\infty} \phi(y|\theta^c)dy = \beta[1-\Phi(y^o|\theta^c)],\end{aligned} \tag{A4}$$

where the subscript denotes partial derivative and we use the following facts:

$$\frac{\partial\phi(y|\theta^c)}{\partial\theta^c} = \frac{\partial}{\partial\theta^c}\left(\frac{\partial\Phi(y|\theta^c)}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial\Phi(y|\theta^c)}{\partial\theta^c}\right) = \frac{\partial}{\partial y}(\Phi_{\theta^c}),$$

$$\Phi_{\theta^c} \equiv \frac{\partial\Phi(y|\theta^c)}{\partial\theta^c} = -\phi(y|\theta^c), \pi(y^o,\theta^c) = 0, \frac{\partial\pi}{\partial y} = \beta, \textit{ and } \lim_{y\to\infty}[\pi(y,\theta^c)\phi(y|\theta^c)] = 0.$$

Combining (A3) and (A4) reduces (A2) to

$$\frac{\partial}{\partial\theta^c}\left[\int_{y^o}^{\infty} \pi(y,\theta^c)\phi(y|\theta^c)dy\right] = 1-\Phi(y^o|\theta^c). \tag{A5}$$

Hence, we obtain

$$A'(\theta^c) = 1-\lambda[1-\Phi(y^o|\theta^c)] > 0. \tag{A6}$$

This establishes that $A(\theta^c) < 0$ for all $\theta^c < \theta^o$, implying that $\mu^o(\theta^c) < 0$ for all $\theta^c < \theta^o$. ∎

Proof of Lemma 2. Differentiating V stated in (10) with respect to θ yields

$$\frac{\partial V}{\partial\theta} = \int_{y^o}^{\infty} \pi(y,\theta^c)\frac{\partial\phi(y|\theta)}{\partial\theta}dy = \beta[1-\Phi(y^o|\theta)], \tag{A7}$$

where the last equality is obtained from (A4) by replacing θ^c that appears in the density ϕ with θ and using θ as the variable of differentiation.

Below, we treat y^o appearing in (A7) as a function of θ by replacing θ^c in (5) with θ. First, using the facts that θ is the mean of y and $\Phi_\theta = -\phi$, we have

$$\frac{\partial^2 V}{\partial\theta^2} = \beta\frac{\partial}{\partial\theta}[1-\Phi(y^o|\theta)] = \beta\left[-\frac{\partial\Phi}{\partial y^o}\frac{\partial y^o}{\partial\theta} - \frac{\partial\Phi}{\partial\theta}\right] = \phi(y^o|\theta) > 0.$$

Since $y^o \to k\beta^{-1} > 0$ when $\theta \to 0$,

$$\lim_{\theta\to 0}\frac{\partial V}{\partial\theta} = \beta\lim_{\theta\to 0}[1-\Phi(y^o|\theta)] = \beta\left[1-\Phi(k\beta^{-1}|0)\right] > 0.$$

When $\theta\to\infty$, we have $y^o\to-\infty$ and $\phi\to 0$. Thus,

$$\lim_{\theta\to\infty}\frac{\partial V}{\partial\theta} = \beta\lim_{\theta\to\infty}[1-\Phi(y^o|\theta)] = \beta\left[1-\lim_{\theta\to\infty}\int_{-\infty}^{y^o}\phi(y|\theta)dy\right] = \beta[1-0] = \beta.$$

■

Proof of Proposition 2. Since the derivation of (19) and (20) is explained in the main text, we only prove the comparative static results. To recognize explicitly the direct and indirect effects of a parameter $t = \lambda$, q on the marginal benefit of θ, we rewrite (19) and (20) as

$$MB_i(\theta^{**}(t),t) = c'(\theta^{**}(t)) \quad \text{for } i = 1,2. \tag{A8}$$

Differentiating (A8) with respect to t yields:

$$\left[\frac{\partial MB_i}{\partial\theta} - c''(\theta)\right]\frac{\partial\theta^{**}}{\partial t} = -\frac{\partial MB_i}{\partial t}.$$

The expression inside the bracket is strictly negative due to the second-order condition. Hence,

$$\frac{\partial\theta^{**}}{\partial t} \overset{sign}{=} \frac{\partial MB_i}{\partial t} \quad \text{for } t = \lambda \text{ and } q.$$

(i) We have

$$MB_1 = \lambda\beta[1-\Phi(y^o|\theta)].$$

First, consider $t = \lambda$ and note that

$$\frac{\partial MB_1}{\partial\lambda} = \beta[1-\Phi(y^o|\theta)] > 0.$$

Second, consider $t = q$. Using (4), (5), (11) and the fact that

$$\frac{\partial\Phi(y|\theta)}{\partial p} = \frac{p^{-1}}{2}(y-\theta)\phi(y|\theta), \tag{A9}$$

note that

$$\frac{\partial}{\partial q}[1-\Phi(y^o|\theta)] = -\left[\frac{\partial\Phi}{\partial y^o}\frac{\partial y^o}{\partial\beta}\frac{\partial\beta}{\partial q} + \frac{\partial\Phi}{\partial p}\frac{\partial p}{\partial q}\right] = -\phi(y^o|\theta)(\theta-k)\frac{h}{2q^2}. \tag{A10}$$

Hence,

$$\begin{aligned}\frac{\partial MB_1}{\partial q} &= \lambda\left\{\frac{\partial\beta}{\partial q}[1-\Phi(y^o|\theta)] + \beta\frac{\partial}{\partial q}[1-\Phi(y^o|\theta)]\right\}\\ &= \lambda\left\{\frac{h}{(h+q)^2}[1-\Phi(y^o|\theta)] - \beta\phi(y^o|\theta)(\theta-k)\frac{h}{2q^2}\right\}\\ &= \frac{\lambda h[1-\Phi(y^o|\theta)]}{(h+q)^2}\left\{1-\frac{\phi(y^o|\theta)}{1-\Phi(y^o|\theta)}(\theta-y^o)\frac{1}{2}\right\},\end{aligned} \tag{A11}$$

where we use (A10) and the fact that $\theta - k = \beta(\theta - y^o)$ as implied by (5) with θ^c being replaced by θ, and simplify the expression by using the definition of β stated in (4). It suffices to show that the expression inside the last curly bracket in (A11) is positive for any given $\theta \leq \theta^o$.

We consider two cases. First, when $\theta \leq k$, we have $\theta \leq y^o$. Hence, $\partial MB_1/\partial q > 0$ holds for (A11). Second, when $\theta > k$, we have $\theta > y^o$. Since y follows a normal distribution with mean θ and precision p, it follows that $\phi(y|\theta) = [g(z)/\sigma_y]$ and $\Phi(y|\theta) = G(z)$, where z is a standard normal random variable, g and G are the density and distribution functions of z, respectively, and σ_y is the standard deviation of y. By defining

$$z^o = \frac{y^o - \theta}{\sigma_y}, \tag{A12}$$

we restate the expression inside the last curly bracket in (A11) as

$$1 - \frac{\phi(y^o|\theta)}{1 - \Phi(y^o|\theta)}(\theta - y^o)\frac{1}{2} = 1 + H(z^o)(z^o/2) \tag{A13}$$

where

$$H(z) = \frac{g(z)}{1 - G(z)} > 0$$

is the hazard function for the standard normal random variable z. It remains to show that (A13) is positive. From Greene (1997, 952), we have a result that

$$0 < [H(z)]^2 - H(z)z < 1 \quad \text{for any } z.$$

Thus,

$$[H(z^o)]^2 < 1 + H(z^o)z^o \Rightarrow 1 + H(z^o)z^o > 0. \tag{A14}$$

Since we have $\theta > y^o$ here, z^o defined in (A12) is a negative number. Using this result along with (A14) establishes that

$$1 + H(z^o)(z^o/2) > 1 + H(z^o)z^o > 0.$$

Hence, (A13) is positive, which establishes that $\partial MB_1/\partial q > 0$ when $\theta > k$.

(ii) We have

$$MB_2 = \lambda\beta[1 - \Phi(y^b|\theta)].$$

First,

$$\frac{\partial MB_2}{\partial \lambda} = \beta[1 - \Phi(y^b|\theta)] - \lambda\beta[\partial\Phi/\partial y^b][\partial y^b/\partial\lambda] > 0,$$

where $[\partial y^b/\partial\lambda] < 0$ can be easily verified by differentiating both sides of (8) with respect to λ.

Second,

$$\frac{\partial MB_2}{\partial q} = \lambda\frac{\partial\beta}{\partial q}[1 - \Phi(y^b|\theta)] - \lambda\beta\frac{\partial}{\partial q}\Phi(y^b|\theta). \tag{A15}$$

The first term is positive since β increases with q. Below, we will show that the second term equals zero. Using (A9) for the second term of (A15) yields

$$\frac{\partial}{\partial q}\Phi(y^b|\theta) = \left[\frac{\partial\Phi}{\partial y^b}\frac{\partial y^b}{\partial p} + \frac{\partial\Phi}{\partial p}\right]\frac{\partial p}{\partial q} = \left[\phi(y^b|\theta)\frac{\partial y^b}{\partial p} + \frac{p^{-1}}{2}(y^b - \theta)\phi(y^b|\theta)\right]\frac{\partial p}{\partial q}. \tag{A16}$$

Since $\partial p/\partial q > 0$, it remains to show that the expression inside the bracket in (A16) vanishes. Differentiating both sides of (8) with respect to p and using the fact that

$$\int_{-\infty}^{y^o}\left[\frac{\partial\Phi(y|\theta)}{\partial p}\right]dy = \frac{-p^{-2}}{2}\phi(y^o|\theta), \tag{A17}$$

we obtain

$$\frac{\partial y^b}{\partial p} = \frac{\frac{\lambda}{1-\lambda}(p^{-2}/2)\phi(y^b|\theta)}{1+\frac{\lambda}{1-\lambda}\Phi(y^b|\theta)}. \tag{A18}$$

Substituting (A18) into the expression inside the bracket in (A16) yields

$$\frac{(p^{-1}/2)\phi(y^b|\theta)}{1+\frac{\lambda}{1-\lambda}\Phi(y^b|\theta)}\left\{\frac{\lambda}{1-\lambda}p^{-1}\phi(y^b|\theta) + (y^b-\theta)\left[1+\frac{\lambda}{1-\lambda}\Phi(y^b|\theta)\right]\right\}.$$

We now show that the expression in the curly bracket is zero. Let Z be that expression. Since

$$\phi(y^b|\theta) = \int_{-\infty}^{y^b}\phi_y(y|\theta)dy = \int_{-\infty}^{y^b}\phi(y|\theta)p(\theta-y)dy,$$

we have

$$\begin{aligned}\frac{\lambda}{1-\lambda}p^{-1}\phi(y^b|\theta) &= \frac{\lambda}{1-\lambda}\int_{-\infty}^{y^b}\phi(y|\theta)(\theta-y)dy = \frac{\lambda}{1-\lambda}\int_{-\infty}^{y^b}(\theta-y)d\Phi(y|\theta)\\ &= \frac{\lambda}{1-\lambda}\left[(\theta-y^b)\Phi(y^b|\theta) + \int_{-\infty}^{y^b}\Phi(y|\theta)dy\right],\end{aligned}$$

where we integrate by parts to obtain the last expression. Using this result yields that

$$\begin{aligned}Z &= \frac{\lambda}{1-\lambda}p^{-1}\phi(y^b|\theta) + (y^b-\theta)\left[1+\frac{\lambda}{1-\lambda}\Phi(y^b|\theta)\right]\\ &= \frac{\lambda}{1-\lambda}(\theta-y^b)\Phi(y^b|\theta) + \frac{\lambda}{1-\lambda}\int_{-\infty}^{y^b}\Phi(y|\theta)dy + (y^b-\theta) + \frac{\lambda}{1-\lambda}(y^b-\theta)\Phi(y^b|\theta)\\ &= \frac{\lambda}{1-\lambda}\int_{-\infty}^{y^b}\Phi(y|\theta)dy + (y^b-\theta) = 0,\end{aligned}$$

where the last equality holds because y^b is characterized by (8) with θ^c being replaced by θ. This completes the proof that the second term in (A15) is zero. Hence, $\partial MB_2/\partial q > 0$ holds. ■

PROOF OF PROPOSITION 3. *Since the derivation of (28) and (29) is clear from the explanations in the main text, we only prove the comparative static results. Following the logic in the proof of Proposition 2, we only need to examine the effects of* λ *and* q *on the marginal benefit of* θ.

(i) We have $MB_1^+ = \lambda[1 - \Phi(y^o|\theta)]$ in this case, so that a higher value of λ increases MB_1^+. Next, using (A10), we have

$$\frac{\partial MB_1^+}{\partial q} = \lambda\frac{\partial}{\partial q}[1-\Phi(y^o|\theta)] = -\lambda\left[\frac{\partial\Phi}{\partial y^o}\frac{\partial y^o}{\partial\beta}\frac{\partial\beta}{\partial q}+\frac{\partial\Phi}{\partial p}\frac{\partial p}{\partial q}\right] = -\lambda\phi(y^o|\theta)(\theta-k)\frac{h}{2q^2}. \tag{A19}$$

Hence, an increase in q has a negative (positive) effect on MB_1^+ if θ is greater (less) than k.
(ii) Since $MB_2^+ = 1$ in this case, the result follows immediately.
(iii) Given the discontinuity of $MB^+(\theta)$ at $\theta = \theta^o$ and the monotonicity of $c'(\theta)$, it follows immediately that θ^* must satisfy either (28) or (29) if $m^+ < c'(\theta^o) < 1$ holds. As a result, in either case, the comparative statics must be the same as those stated in part (i) or (ii). ∎

Proof of Proposition 4.
(i) Suppose that θ^* is characterized by (28), so that $W^+(\theta^*) = W_1^+(\theta^*)$. Note that

$$\frac{dW_1^+}{d\lambda} = \frac{\partial W_1^+}{\partial\theta}\frac{\partial\theta^*}{\partial\lambda}+\frac{\partial W_1^+}{\partial\lambda} = \left(\lambda\frac{\partial V}{\partial\theta}-c'(\theta)\right)\frac{\partial\theta^*}{\partial\lambda}+V(\theta^*) = V(\theta^*) > 0, \tag{A20}$$

where we use the fact that θ^* maximizes $W_1^+(\theta^*)$, that is,

$$\lambda\frac{\partial V}{\partial\theta}-c'(\theta) = 0$$

evaluated at $\theta = \theta^*$. Next, using the same fact, we have

$$\frac{dW_1^+}{dq} = \frac{\partial W_1^+}{\partial\theta}\frac{\partial\theta^*}{\partial q}+\frac{\partial W_1^+}{\partial q} = \left(\lambda\frac{\partial V}{\partial\theta}-c'(\theta)\right)\frac{\partial\theta^*}{\partial q}+\lambda\frac{\partial V}{\partial q} = \lambda\frac{\partial V}{\partial q}. \tag{A21}$$

Using the Leibnitz rule and the fact that $\pi(y^o,\theta) = 0$, we have

$$\frac{\partial V}{\partial q} = \frac{\partial}{\partial q}\left[\int_{y^o}^{\infty}\pi(y,\theta)\phi(y|\theta)dy\right] = \int_{y^o}^{\infty}\frac{\partial\pi}{\partial\beta}\frac{\partial\beta}{\partial q}\phi(y|\theta)dy+\int_{y^o}^{\infty}\pi\frac{\partial\phi}{\partial p}\frac{\partial p}{\partial q}dy. \tag{A22}$$

For the first term in (A22), note that

$$\begin{aligned}\int_{y^o}^{\infty}\frac{\partial\pi}{\partial\beta}\frac{\partial\beta}{\partial q}\phi(y|\theta)dy &= \frac{\partial\beta}{\partial q}\int_{y^o}^{\infty}(y-\theta)\phi(y|\theta)dy\\ &= -\frac{\partial\beta}{\partial q}\int_{-\infty}^{y^o}(y-\theta)\phi(y|\theta)dy = \frac{1}{(h+q)q}\phi(y^o|\theta),\end{aligned} \tag{A23}$$

where we use the following facts along with (4) and (11):

$$\int_{-\infty}^{\infty}(y-\theta)\phi(y|\theta)dy = 0 \tag{A24}$$

$$\int_{-\infty}^{y^o}(y-\theta)\phi(y|\theta)dy = -p^{-1}\phi(y^o|\theta). \tag{A25}$$

Next, consider the second term in (A22). Similar to the derivation of (A4), we integrate by parts and use (A9), (A17) and (A24) to obtain

$$\begin{aligned}\int_{y^o}^{\infty}\pi\frac{\partial\phi}{\partial p}\frac{\partial p}{\partial q}dy &= \left(\frac{\partial p}{\partial q}\right)\left[\int_{y^o}^{\infty}\pi\frac{\partial}{\partial p}\left(\frac{\partial\Phi}{\partial y}\right)dy\right] = \left(\frac{\partial p}{\partial q}\right)\left[\int_{y^o}^{\infty}\pi\left(\frac{\partial\Phi_p}{\partial y}\right)dy\right] \\ &= \left(\frac{\partial p}{\partial q}\right)\left[\int_{y^o}^{\infty}\pi\mathrm{d}\left(\Phi_p\right)\right] = \left(\frac{\partial p}{\partial q}\right)\left(\left[\pi\Phi_p\right]_{y^o}^{\infty}-\int_{y^o}^{\infty}\Phi_p d\pi\right) \\ &= \left(\frac{\partial p}{\partial q}\right)\beta\left[\frac{-p^{-2}}{2}\phi(y^o|\theta)\right] = \frac{-1}{2(h+q)q}\phi(y^o|\theta),\end{aligned} \tag{A26}$$

where we also use the definitions of β and p stated in (4) and (11) to simplify the expression. Combining (A22), (A23), and (A26) yields

$$\frac{\partial V}{\partial q} = \frac{1}{2(h+q)q}\phi(y^o|\theta) > 0. \tag{A27}$$

Hence, (A21) is positive.

Next, suppose that θ^* is characterized by (29), so that $W^+(\theta^*) = W_2^+(\theta^*)$. In this case,

$$\frac{dW_2^+}{d\lambda} = \frac{\partial W_2^+}{\partial\theta}\frac{\partial\theta^*}{\partial\lambda} + \frac{\partial W_1^+}{\partial\lambda} = [1-c'(\theta)]\frac{\partial\theta^*}{\partial\lambda} = 0 \tag{A28}$$

$$\frac{dW_2^+}{dq} = \frac{\partial W_2^+}{\partial\theta}\frac{\partial\theta^*}{\partial q} + \frac{\partial W_2^+}{\partial q} = [1-c'(\theta)]\frac{\partial\theta^*}{\partial q} = 0, \tag{A29}$$

where the last equalities hold because of (29).

(ii) Suppose that θ^{**} is characterized by (19), so that $W(\theta^{**})$=$W_1(\theta^{**})$. Note that

$$\frac{dW_1}{d\lambda} = \frac{\partial W_1}{\partial\theta}\frac{\partial\theta^{**}}{\partial\lambda} + \frac{\partial W_1}{\partial\lambda} = \left(\lambda\frac{\partial V}{\partial\theta} - c'(\theta)\right)\frac{\partial\theta^{**}}{\partial\lambda} + V(\theta^{**}). \tag{A30}$$

Since $W_1(\theta) = W_1^+(\theta)$ is maximized at $\theta = \theta^*$ and $\theta^{**} < \theta^*$, it follows that

$$\lambda\frac{\partial V}{\partial\theta} - c'(\theta) > 0 \tag{A31}$$

evaluated at $\theta = \theta^{**}$. Combining this result with $\partial\theta^{**}/\partial\lambda > 0$ (see part (i) of Proposition 2) establishes that

$$\left(\lambda\frac{\partial V}{\partial\theta} - c'(\theta)\right)\frac{\partial\theta^{**}}{\partial q} > 0.$$

Hence, (A30) is positive. Similarly, using $\partial\theta^{**}/\partial q > 0$ (see part (i) of Proposition 2), we have

$$\frac{dW_1}{dq} = \frac{\partial W_1}{\partial\theta}\frac{\partial\theta^{**}}{\partial q} + \frac{\partial W_1}{\partial q} = \left(\lambda\frac{\partial V}{\partial\theta} - c'(\theta)\right)\frac{\partial\theta^{**}}{\partial q} + \lambda\frac{\partial V}{\partial q} > 0. \tag{A32}$$

Next, suppose that θ^{**} is characterized by (20), so that $W(\theta^{**}) = W_2(\theta^{**})$. In this case,

$$\frac{dW_2}{d\lambda} = \frac{\partial W_2}{\partial\theta}\frac{\partial\theta^{**}}{\partial\lambda} + \frac{\partial W_2}{\partial\lambda} = [1-c'(\theta)]\frac{\partial\theta^{**}}{\partial\lambda}. \tag{A33}$$

Since $W_2(\theta) = W_2^{+}(\theta)$ is maximized at $\theta = \theta^*$ and $\theta^{**} < \theta^*$, it follows that

$$1 - c'(\theta) > 0$$

evaluated at $\theta = \theta^{**}$. Combining this result with $\partial\theta^{**}/\partial\lambda > 0$ (see part (ii) of Proposition 2) establishes that (A33) is positive. Similarly, using $\partial\theta^{**}/\partial q > 0$ (see part (ii) of Proposition 2), we obtain

$$\frac{dW_2}{dq} = \frac{\partial W_2}{\partial\theta}\frac{\partial\theta^{**}}{\partial q} + \frac{\partial W_2}{\partial q} = [1 - c'(\theta)]\frac{\partial\theta^{**}}{\partial q} > 0.$$

■

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