

Competition, Comparative Performance, and Market Transparency[†]

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We study how competition affects market transparency, taking into account that comparative performance is assessed via tournaments and contests. Extending Dye (1985) to a multi-firm setting in which top performers are rewarded, we show that increased competition usually makes disclosure less likely, which lowers market transparency and may decrease per capita welfare. This result appears to be robust to several model variations and as such, has implications for market regulation. (JEL D82, D83, L77, L25)

Early work on disclosure theory suggests that market forces are sufficient to induce full disclosure. Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) argue that in the absence of market frictions, adverse selection prompts those with good news to distinguish themselves from others by disclosing their information. This reduces the expected prospects for the remaining market and induces a cascade in which everyone discloses their information.

Subsequent work challenges this. Full disclosure may not occur because disclosure is costly (e.g., Verrecchia 1983; Fishman and Hagerty 1990), some market participants are unsophisticated (e.g., Fishman and Hagerty 2003), or there is uncertainty whether asymmetric information exists in the market (Dye 1985; Matthews and Postlewaite 1985; Jung and Kwon 1988; Shin 2003; Acharya, DeMarzo, and Kremer 2011). In the face of such market frictions, adverse selection may prevent market transparency.

All of the prior literature, however, focuses only on absolute performance and ignores comparative performance. That is, when one entity outperforms another, there are added spoils that go to the victor, and this needs to be taken into consideration. Comparative performance is important in any tournament or contest (e.g., Rosen 1981; Lazear and Rosen 1981), especially when scarce resources are

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allocated or when reputation adds value. As we describe in the paper, this arises in many economic arenas: financial reporting, money management, academic research, job applications, and marriage markets. The key insight is that when people choose whether to reveal information, they know that their comparative performance will be evaluated in addition to their absolute performance.

In this paper, we study how competition affects market transparency, taking into account that comparative performance matters. Our primary contribution is that we show that increased competition usually makes disclosure *less* likely, which lowers market transparency and may decrease per capita welfare. Especially in a tournament setting, we cannot rely on the invisible hand to induce informational efficiency. This has implications for market regulation.

We build on the model of Dye (1985), where incomplete disclosure results from investors' uncertainty as to whether or not a firm possesses relevant information.¹² In our variant, a finite number of firms compete in the market. All firms experience a random shock that changes their fundamental value. Each firm may or may not observe the precise value of their shock. Firms that make an observation simultaneously choose whether to announce it publicly, while firms with no new information have nothing to reveal. The firm with the best announcement gets a fixed prize from the market, which represents the rank-based remuneration previously described.

In the unique Nash equilibrium of the game, each firm with new information applies the same threshold in deciding whether or not to reveal its news. If the observed shock value is above this threshold, the firm announces it and competes for the prize. If the observed shock is lower, however, the firm conceals its information. The presence of uninformed firms lends plausible deniability to informed firms wishing to conceal a bad observation. As such, competing for the prize has an opportunity cost: firms who disclose give up their chance to pool with other firms. Given this, rational investors use Bayesian learning to adjust the market price of firms that do not release any news.

Because the probability of winning the prize drops when more firms compete, the benefit of making announcements decreases with competition. Increasing competition makes pooling with other firms more attractive compared to the benefit of vying for the prize, which leads to decreased information revelation and lower market transparency. In the limit, when the market is perfectly competitive, transparency is minimized because the individual probability of winning the prize goes to zero.

Although it may not be terribly surprising that the influence of a single fixed prize decreases with the number of firms eligible to win it, we find the result to be robust to alternate prize forms: progressive reward systems (i.e., prizes are awarded to runners up), prizes that change in size as a result of competition, prizes awarded based on percentile, and sequential disclosure. Most notably, perfect competition leads to minimal market transparency in all of the model variations we analyze except one: when

¹ Going forward for clarity, we always refer to the solicitor who holds private information as the "firm" and the solicited agent as the "investor," while keeping in mind the breadth of economic applications noted earlier.

² Disclosure is costly in Dye (1985) because firms give up their opportunity to pool with others. Our use of this framework does not make our analysis special, however. Competition would still have the same effects that we demonstrate in any other form of costly discretionary disclosure noted above.

the prize grows exponentially with competition ad infinitum. Given that this is rather unlikely to occur in reality, we view this result with considerable generality.

In much of our analysis, the distribution that the firms draw their value from does not vary with competition. We relax this assumption to add a reduced-form model of product market competition, in addition to the considerations analyzed so far.³ Now, the market share for each firm shrinks with competition, which makes the ratio of the prize to firm revenues grow with competition. For a small oligopoly, we show that this causes added competition to increase disclosure. However once additional firms are added, the effect of competition decreases market transparency. Intuitively, when a small number of firms are present, disclosure is a mechanism to prove oneself. But as the probability of winning the prize shrinks with competition, the incentive to hoard information grows too large.

Our work not only adds to the previous literature on disclosure and transparency, but also contributes to the work on tournaments and contests. Previous papers have focused on whether tournaments optimally solve moral hazard problems (e.g., Lazear and Rosen 1981; Green and Stokey 1983; Nalebuff and Stiglitz 1983a, 1983b; Moldovanu and Sela 2001). These papers weigh the merits of using relative performance measures in settings in which performance is correlated. We add to this literature by considering adverse selection instead and assessing the effect of competition while taking the tournament mechanism as primitive. Indeed, as implied by Nalebuff and Stiglitz (1983a), competition disrupts the incentives to perform. In their setting, it imposes too much risk on participants. In ours, competition induces firms to hoard information and not vie for the prize.

Finally, our analysis has normative implications. We conclude that if transparency is considered a good, policymakers cannot simply depend on competition to induce transparency. They need to carefully consider the type of competition that takes place in markets before deciding whether regulation is necessary. When comparative performance matters, competition for remuneration may make disclosure less attractive, which may lower efficiency in the market. In this light, competition should *not* be viewed as a panacea to assure information disclosure and self-regulation by market participants.

The rest of the paper is organized as follows. Section I introduces our base model, characterizes the equilibrium, and addresses whether our results are robust to other prize structures. In Section II, we add product market competition to the model. Section III concludes. Proofs of all propositions are deferred to the Mathematical Appendix. At the conclusion of the Mathematical Appendix, we explore the potential welfare implications of our results.

³This adds to the previous literature on product market competition and discretionary disclosure. See Wagenhofer (1990), Darrough and Stoughton (1990), Feltham and Xie (1992), Darrough (1993), Dye and Sridhar (1995), Clinch and Verrecchia (1997), Pae (2002), Stivers (2004), and Board (2009).

I. Discretionary Disclosure

A. Base Model

Consider that N risk-neutral firms compete in a one-period game of discretionary disclosure. Each firm $j \in \{1, \dots, N\}$ experiences a random change \tilde{x}_j , which is distributed according to a twice continuously differentiable function $F(x)$ on \mathbb{R} . We assume that $f(x) > 0$ for all $x \in \mathbb{R}$ and $E[\tilde{x}_j] = 0$.⁴ Realizations are independent and identically distributed for each firm. For now, $F(x)$ does not depend on N , but we relax this assumption in Section II.

Each firm j observes the true realization of \tilde{x}_j with probability p , and observes nothing otherwise. As such, the parameter p measures the degree of asymmetric information and may be interpreted as a measure of strong form market (in-)efficiency. The probability p is given exogenously and we assume that firms cannot alter its value.⁵

Any firm j that observes \tilde{x}_j may either conceal its value (C_j), or may credibly disclose it to investors (D_j). For a given realization $\tilde{x}_j = x$, we denote the change in firm value following these actions as $u_j^C(x)$ and $u_j^D(x)$. Following Dye (1985) and Jung and Kwon (1988), firms that do not observe \tilde{x}_j , are not permitted to fabricate one.⁶ That is, we assume that investors can freely verify and penalize false claims of \tilde{x}_j , but cannot determine whether a non-disclosing firm is in fact concealing information.

Each informed firm j determines its action using a disclosure policy $\sigma_j : \mathbb{R} \rightarrow [0, 1]$, where the firm discloses with probability $\sigma_j(x)$ and conceals its information with probability $1 - \sigma_j(x)$. As such, firms may choose non-deterministic strategies, although we will show shortly that deterministic strategies are optimal, almost surely. Let $\sigma \equiv \{\sigma_1, \dots, \sigma_N\}$ and $\sigma_{-j} \equiv \{\sigma_1, \dots, \sigma_{j-1}, \sigma_{j+1}, \dots, \sigma_N\}$. All informed firms act simultaneously to maximize firm value and do not know which of their competitors are also informed at the time of their decision. We later extend our model and consider a sequential game of disclosure in Section ID.

Investors are competitive, risk-neutral, and have rational expectations about firm behavior. As such, investors price each firm according to $u_j^C(x)$ and $u_j^D(x)$. Let P_j be the event that a firm j does not disclose new information. This may occur because the firm is legitimately uninformed or because it is concealing a bad realization (i.e., pooling with uninformed firms). Investors use Bayesian inference to calculate the firm's change in value, which is expressed as $u_j^C(x) = E[\tilde{x}_j | P_j, \sigma, p]$.

After all disclosures have been made, investors award a fixed prize ϕ to the firm with the highest disclosed value. Given this, firm j 's expected value of revealing x is

$$(1) \quad u_j^D(x) \equiv x + \phi W_j(x, p, \sigma_{-j}),$$

⁴ As will become clear, the assumption that $E[\tilde{x}_j] = 0$ is without loss of generality. For any distribution in which $E[\tilde{x}_j] \neq 0$, rational investors would update their valuations of the firms to take this into consideration. Therefore, by setting $E[\tilde{x}_j] = 0$, we are considering the news that investors cannot readily predict before any announcements are made.

⁵ See Matthews and Postlewaite (1985) for treatment of the monopoly case in which the firm chooses p endogenously (i.e., chooses the quality of its information).

⁶ This assumption is very common in the disclosure literature. See Matthews and Postlewaite (1985, 329) for a good motivation of this assumption.

where $W_j(x, p, \sigma_{-j})$ is the probability that firm j has the highest disclosure. This value depends on the probability p of competing firms being informed, and on the strategies σ_{-j} they employ.

B. Applications

Before characterizing the equilibrium of the game, it is instructive to note some of the model's economic applications. The model is relevant when superior relative performance is rewarded and rankings drive remuneration. In the context of Dye (1985), investors not only reward firms for their disclosed performance, but also take into account the future decisions by third parties who do business with each firm. When third parties allocate scarce resources to top-performing firms, this relaxes their budget constraints and increases each firm's opportunity set. Indeed, empirical evidence confirms that top firms enjoy superior access to human capital (Gatewood, Gowan, and Lautenschlager 1993), venture capital (Hsu 2004), and media attention and market coverage (Hendricks and Singhal 1996). In the context of our model, investors not only update their estimate of a firm's current operations \tilde{x}_j , but also take into account the added value ϕ a firm generates from third parties when it outperforms their competition. Stock prices reflect both components of value.

Our model also applies to other economic arenas, especially when reputation is an important driver of value. For example, academic job market candidates who outperform their competitors and receive better initial job placement, often get the prize of more exposure in the future. Likewise, scientists who win grants find it easier to obtain future funding. Disclosing one's true value in these settings not only conveys information to others, but also allows the information sender to better compete for future spoils.

This consideration is particularly important when information about objective performance is difficult to obtain or interpret, such as in the money management industry. Here, the prizes are convex investor flows that funds enjoy when they outperform competitors (Brown, Harlow, and Starks 1996; Berk and Green 2004; Del Guercio and Tkac 2008). The importance of such gains is magnified in the hedge fund industry where funds are restricted by law to marketing their services to a small set of qualified investors. Because of this, there is a paucity of public performance data and disclosure is largely discretionary via for-profit databases (Ackerman, McEnally, and Ravenscraft 1999; Malkiel and Saha 2005; Stulz 2007). Our model fits the strategic behavior in this industry particularly well. Funds that disclose performance compete for investor flows, whereas funds that do not disclose may do so for reasons unrelated to poor performance: they may be at their investor cap, may not be seeking new capital, or may be unable to make useful disclosure about valuations because some assets are illiquid.

Finally, our model is relevant when individual agents interact. For example, consider a marriage market in which young men court a particular woman, at the same time uncertainty about their future career prospects is being resolved. To see how our model applies, suppose that N men court a single woman, who chooses the

suitor with the best financial future x .⁷ This x could represent the man's salary after an important review, the companies that have made him job offers, or any other verifiable proxy of future success. With probability p , the man is already privately informed of x and may choose whether to disclose it or not. Disclosure affects both a man's social standing among peers $\theta(x)$, and also makes him eligible for the "prize" ϕ of marrying the woman. If the suitor does not disclose x , he is not eligible to win ϕ because the woman is not able to evaluate him as a marriage prospect. Disclosing yields

$$(2) \quad E[u^D(x)] = \theta(x) + \phi \Pr(x \text{ highest among suitors}),$$

and not disclosing yields

$$(3) \quad E[u^C(x)] = (1 - p)E[\theta(x)] + pE[\theta(x)|x \text{ concealed}].$$

These payoff functions match those in our model, except for the monotonic transformation θ . As such, the suitor's disclosure decision is isomorphic to the firm's disclosure decision in our extension of the Dye (1985) model.

C. Equilibrium Disclosure

The following proposition characterizes the game's unique Nash equilibrium.

PROPOSITION 1: *There exists a unique and non-trivial Nash equilibrium, in which every firm discloses according to a common threshold t^* defined implicitly by*

$$(4) \quad t^* + \phi(1 - p + pF(t^*))^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} xf(x) dx.$$

Further, the threshold t^ lies below the unconditional mean of \tilde{x}_j , i.e., $t^* < 0$.*

The proof of the proposition, which is detailed in the Appendix, proceeds in three steps. First, we show that each firm acts according to a **disclosure threshold** $t_j < 0$, in which each informed firm simply compares the expected utility it can obtain by revealing information to what it obtains by pooling. The threshold for each firm j is defined implicitly by

$$(5) \quad u_j^D(t_j) = u_j^C.$$

Second, we show that every firm uses the same disclosure threshold, which we denote as t^* . As such, there cannot be an equilibrium in which some firms are more

⁷In reality, there may be other dimensions that are compared, but we include a single one here for purpose of example.

“honest” than others. This is not too surprising, since all firms draw the observations from identical and independent distributions. Third, we show that the common disclosure threshold t^* is in fact unique.

The expression in (4) implicitly defines the unique threshold for the game. The left side is the utility that a firm enjoys if it observes $\tilde{x}_j = t^*$ and reveals its information. In this case, the firm immediately receives its own value $\tilde{x}_j = t^*$, and can also win the prize ϕ if its disclosure is the highest. But since competing firms never reveal values below t^* , any other disclosing firm will have a higher value almost surely. Firm j can win, therefore, only if all other firms pool and it is the only one to disclose. Each competing firm pools if either it is uninformed or it is informed with an observation below the threshold. These events occur with probabilities $(1 - p)$ and $pF(t^*)$, respectively.

The right side is the utility a firm obtains by concealing its observation, which equals a pooling firm’s expected change in value. Such a firm could be uninformed and have a zero expected value for its observation, or could be hiding an observation lower than the threshold. The weighted average of these possibilities yields the right side of equation (4).

It is important to note that the threshold t^* is lower than the distribution mean, which we’ve assumed to be zero. The average \tilde{x}_j for an uninformed firm is simply the distribution mean, and because firms disclose their best observations, rational investors expect the average concealed observation to be negative. The weighted average assigned to pooling firms must therefore be below the distribution mean. Since disclosure yields strictly greater expected utility than the value disclosed, no firm will ever conceal an above-average observation. So if firm j is indifferent between revealing and concealing a value $x_j = t^*$, then $t^* < 0$.

Given the existence of a unique t^* , we could use the expression in (4) and the Implicit Function Theorem to predict how disclosure behavior responds to exogenous parameter changes. However, a more empirically relevant characterization would describe the **disclosure frequency** with which firms opt to reveal private information. That is, what is the ex ante probability that a firm, if it observes its value change, will choose to share its observation with investors?

To address this, we define ω^* to be the ex ante probability that an informed firm discloses in equilibrium. By construction, $\omega^* = 1 - F(t^*)$, which implies that if a firm lowers its threshold, it discloses more of its realized values, and vice-versa. We also denote $\hat{\omega}$ as the equilibrium disclosure frequency when $\phi = 0$, which corresponds to the equilibrium condition derived in Dye (1985) and Jung and Kwon (1988) when there is no strategic interaction. When $\phi > 0$,

$$(6) \quad \omega^* \geq \hat{\omega} \quad \forall \phi \geq 0, \quad \forall N < \infty.$$

As such, $\hat{\omega}$ is a lower bound for ω^* over all ϕ and N . In fact, it is the largest possible lower bound. The lower bound is defined implicitly by

$$(7) \quad F^{-1}(1 - \hat{\omega}) = \frac{p \int_{\hat{\omega}}^1 t(\Omega) d\Omega}{1 - p\hat{\omega}}.$$

PROPOSITION 2: *Equilibrium disclosure frequency is decreasing in N and increasing in ϕ . As $N \rightarrow \infty$, ω^* converges to $\hat{\omega}$.*

According to Proposition 2, when firms disclose competitively in a tournament-like setting, increased competition reduces disclosure. Increased competition drives firms to hoard their informational advantage over investors. The result has immediate application in many economic settings, especially in the financial sector (e.g., money management), where disclosure is critical and where top-performing firms enjoy large rewards.

Mathematically, the cause of the competition effect is straightforward. As more firms enter the market, each firm's chance of making the highest disclosure diminishes exponentially. Since the disclosure decision is a trade-off between the desire to win the prize and the desire to conceal bad signals, additional firm entry tips the balance in favor of concealing. In the sections that follow, we will show this effect to be robust to other types of prizes and prize structures.

Proposition 2 also states simply that firms will be more inclined to disclose when the prize they can win is large. Again, a larger prize tips the balance between the opportunity to pool with other firms and to compete openly for the prize. This concept is also robust to our alternative model specifications.

The comparative statics in p turn out to be trickier. Jung and Kwon (1988) consider the special case where $N = 1$ and $\phi = 0$, and find disclosure to be strictly increasing in p . We are able to confirm this result by computing our comparative statics with $\phi = 0$. But when there is a prize, the situation becomes more complicated.

With a prize, if p were to increase and firms do not adjust their disclosure strategies, there would be two sources of change in firm utility. First, the increase in asymmetric information would increase the Bayesian probability of a firm having inside information. Rational investors would respond by reducing their assessment u^C of pooling firms. Second, the increase in p means competing firms are more likely to be informed. Since being informed is a prerequisite to disclosing, the increase in p makes any given disclosing firm less likely to win the prize by default. Mathematically, a higher p decreases $W_j(x)$, which implies a lower expected utility of disclosure.

These two effects of increased p work against each other. To determine whether ω^* will increase or decrease, we need to know which of these effect impact firm utility more. If the reduction in $u^C(\omega^*)$ is larger than the reduction in $u^D(\omega^*)$, then disclosure becomes more appealing. Firms will then respond to an increase in p by disclosing more frequently. Conversely, if the reduction in $u^D(\omega^*)$ dominates, then firms respond with less frequent disclosure.

If the prize value ϕ is small or zero, then the reduction in $W_j(x)$ is unimportant, so the reduction in $u^C(\omega^*)$ dominates, and equilibrium disclosure increases. This echoes the Jung and Kwon (1988) result. The same result follows when N is very large, in which case the probability of winning the prize is low from the outset. In contrast, when ϕ is large and N is modest, the reduction in $W_j(x)$ is critical. The second effect dominates, so overall the incentive to disclose is reduced more than the incentive to pool. Consequently, firms pool more often, reducing the equilibrium disclosure frequency.

D. Other Prize Structures

Now, we consider other prize structures to determine whether competition's effect of reducing disclosure is robust to alternative model specifications. In what follows, we continue to assume that $F(x)$ does not depend on N .

Increasing/Decreasing Prize Values in N .—Consider that the prize depends on N , which we denote as ϕ_N . A case might be made for either increasing or decreasing prize values. Prize value might decrease when additional firms enter because investor attention is diluted over a larger population of firms. More commonly, though, the prize might shrink because of increasing competition for a scarce resource. It is straightforward to see, based on Proposition 2, that a prize that decreases in N only strengthens our result. If the addition of further competitors causes an exogenous reduction of prize value (i.e., lower ϕ), then equilibrium disclosure falls even faster than if the prize remained constant.

Arguing that prizes increase with competition is more challenging, but may exist in developing industries. Prizes that increase with competition may overcome the competitive effect of disclosure, and are more likely to do so when N is small. But the following Proposition shows that unless the prize grows exponentially by a factor of at least $1/(1 - p\hat{\omega})$, disclosure will eventually decrease once N reaches some critical value.

PROPOSITION 3: *If ϕ_N increases with N and*

$$(8) \quad \lim_{N \rightarrow \infty} \frac{\phi_{N+1}}{\phi_N} < \frac{1}{1 - p\hat{\omega}},$$

then there exists some $\bar{N} \in \mathbb{R}$ such that $N > \bar{N}$ implies that $\omega_{N+1}^ < \omega_N^*$.*

To gain intuition for this result, consider the case in which prize value per firm remains constant:

$$(9) \quad \phi_N \equiv N\phi_1.$$

In this case,

$$(10) \quad \lim_{N \rightarrow \infty} \frac{\phi_{N+1}}{\phi_N} = 1 < \frac{1}{1 - p\hat{\omega}},$$

so the condition in (8) is satisfied, and disclosure decreases with competition for large N .

Intuitively, the chance of winning the prize declines exponentially in N , so unless the prize grows forever at the same exponential rate, the expected winnings will eventually decline in N . Realistically, however, one must ask how a prize that continues to increase exponentially with firm entry could arise. The value of high status may well increase exponentially as the number of competing firms increases from,

say, $N = 1$ to $N' = 10$. But it is difficult to believe the same exponential increase could continue from $N = 10$ to $N' = 50$. We conjecture that exponentially increasing status prizes are uncommon at best, and may never occur in industries with a large number of firms.

Multiple Prizes.—Consider that a finite number of progressive prizes K are awarded to the top firms.

DEFINITION 1: *A disclosure game with a **progressive prize structure** is one in which the firms that make the K highest disclosures each win a prize. The firm that makes the k th highest disclosure wins ϕ_k . The prizes are positive and strictly monotonic,*

$$(11) \quad \phi_1 > \phi_2 > \cdots > \phi_K > 0.$$

Compared to a model with a single prize of $\phi = \phi_1$, the addition of prizes for runners-up naturally induces greater disclosure. But although the change to a progressive prize structure may increase disclosure for any particular N , our central result remains unchanged:

PROPOSITION 4: *Under any particular progressive prize structure, equilibrium disclosure frequency strictly decreases as competition increases. That is, $\omega_{N+1}^* < \omega_N^*$ for any N .*

This result justifies our simplification in working with a single prize ϕ . Although additional prizes may change the quantitative predictions of equilibrium disclosure, the qualitative comparative statics remain unchanged. The chance of winning a lesser prize decreases with competition just as the chance of winning a single prize does. Competition therefore reduces disclosure in this setup as well.

Now, consider that prizes are awarded based on a firm's percentile. For example, each firm in the top 20 percent of the N firms could be awarded a prize, so that the $N/5$ highest disclosures each receive an additional ϕ . This variation introduces some complications that prevent us from showing the claim from the main model, "equilibrium disclosure ω_N^* is strictly decreasing in N ." Because the number of prizes is discrete, it cannot increase in exact proportion with N . For example, when 20 percent of the firms receive a prize, a single prize is awarded when $N = 5, 6, 7, 8, 9$, and we numerically find that $\omega_5^* > \omega_6^* > \cdots > \omega_9^*$. But for $N = 10$, we suddenly award a second prize, which can mean that $\omega_9^* < \omega_{10}^*$. We must therefore content ourselves with the result that disclosure decreases to its minimum possible frequency under perfect competition.

PROPOSITION 5: *Suppose that for any N , a fixed fraction λ of the competing firms win the prize ϕ . Further, suppose that $\lambda \leq p\hat{\omega}$. Then, disclosure converges to its lower bound in the perfectly competitive limit:*

$$(12) \quad \omega_N^* \rightarrow \hat{\omega} \text{ as } N \rightarrow \infty.$$

According to Proposition 5, as the market becomes perfectly competitive, disclosure is minimized. It should be noted that the condition that $\lambda < p\hat{\omega}$ is weak in the sense that it allows for a large number of firms to receive prizes. If $\lambda = p\hat{\omega}$ when $N \rightarrow \infty$, this would mean that all firms that observed a value above \hat{t} would receive a prize. Therefore, we limit the fraction of prizes ($\lambda < p\hat{\omega}$) to keep the analysis realistic and economically interesting.

Sequential Disclosure.—We complete this section with a simple sequential disclosure model to further check the robustness of our finding that disclosure is minimized under perfect competition. Suppose that firms are randomly ordered, and each in turn observes its shock value \tilde{x} with probability p , then chooses whether to disclose.

Since each firm makes a unique, history-dependent decision, we no longer have a single symmetric, deterministic disclosure threshold. Rather, each firm has a random disclosure threshold that depends upon the disclosures of the preceding firms and on the number of firms remaining to act. Let ν_j be the ex ante probability that the j th firm to act will disclose if they are informed. The average of these probabilities is the analogue of the disclosure frequency in the main model,

$$(13) \quad \bar{\nu}_N = \frac{1}{N} \sum_{j=1}^N \nu_j.$$

PROPOSITION 6 (Sequential Disclosure): *In an equilibrium with N firms,*

- (i) *The ex ante probability that the j th firm discloses converges to the minimum with perfect competition:*

$$(14) \quad \lim_{N \rightarrow \infty} \nu_j = \hat{\omega}.$$

- (ii) *The ex ante probability that a randomly selected informed firm discloses also converges to the minimum with perfect competition:*

$$(15) \quad \lim_{N \rightarrow \infty} \bar{\nu}_N = \hat{\omega}.$$

According to Proposition 6, in the perfectly competitive limit, every individual j th firm discloses with frequency $\hat{\omega}$, the minimum possible. We can also show the slightly stronger claim that the average frequency of disclosure over all N firms converges to the minimum $\hat{\omega}$.

II. Disclosure with Concurrent Product Market Competition

In this section, we analyze firms that compete directly in the product market, as well as for prizes based on their disclosures. The distribution $F(x)$ varies with N so that

firm revenue decreases with the entry of additional firms. Accordingly, firm signals have relatively less direct importance to firm price and the prize has relatively more.

A. Equal Shares Competition

To capture this effect simply, we assume that the distribution of value signals becomes compressed with the entry of additional firms. When N firms compete, we exchange the original distribution of signals $x \sim F$ for a compressed distribution $x_N \sim F_N$, so that whenever a firm would have drawn a signal x in the original model, they instead draw a scaled-down event x/N in the new model.

The new distribution is defined as

$$(16) \quad F_N(x) \equiv F(Nx).$$

An increase in N has the effect of shifting the distribution of news events while leaving the support unchanged. For example, if $x = \$10k$ had been a ninetieth percentile result with $N = 5$, $x = \$1k$ would be the new ninetieth percentile with $N = 50$. Increasing N scales down expectations while preserving the concavity and any other peculiarities of the value distribution.

We refer to this as “equal shares competition” for earnings, but wish to stress that this is not intended as a substitute for other models of competition. The goal here is simply to show how the value-scaling effect of competition affects disclosure. In Section IIB, we consider more general models of competition.

In Section I, we found that as N increases, the incentive to disclose falls as the probability of winning the prize decreases. With product market competition, though, potential revenue declines as well, which reduces the incentive to pool. These two effects oppose one another. Which effect dominates depends upon the number of competing firms.

For what values of N , then, does competition reduce disclosure? If we were to find the necessary number of firms to be in the millions, for example, then our point here would only be academic and not of practical import. To gain a sense of how many firms is “enough,” consider the following proposition.

PROPOSITION 7: *Above some threshold $\bar{N} = 1/p\hat{\omega}$, the equilibrium disclosure frequency ω_N^* decreases monotonically in N and converges to $\hat{\omega}$.*

To appreciate Proposition 7, suppose the distribution $\tilde{x} \sim F$ is symmetric and define \hat{t} as the threshold such that $1 - F(\hat{t}) = \hat{\omega}$. The fact that $\hat{t} < E[x]$ implies that

$$(17) \quad \hat{\omega} = 1 - F(\hat{t}) > 1 - F(E[x]) = 0.5.$$

Then the sufficient condition becomes $N > 2/p$. If, for example, firm information arrives with probability 0.5, then $N = 2/(0.5) = 4$ firms is enough competition that further entry will only reduce disclosure. The higher p is, the fewer firms that are

required to assure that further competition decreases disclosure. We conjecture that in many industries (e.g., financial sectors), there are already enough competitors present so that disclosure responds negatively to additional competition.

Proposition 7 also shows that ω_N^* actually converges to $\hat{\omega}$ under perfect competition, while industry profits converge to zero. Product prices decrease to their lowest possible values, which maximizes social welfare. However, perfect competition in disclosure induces firms to retain their maximum degree of asymmetric information. Thus, while perfect competition drives product prices to their most socially efficient level, it drives firm prices to their least informationally efficient. To better appreciate this, consider the following example.

EXAMPLE 1: Consider the disclosure game where \tilde{x} is Gaussian with $\mu = 0$, $\sigma = 5$, and $\phi = 1$, $p = 0.3$ and product market competition characterized by $F_N(x) = F(Nx)$. Figure 1 shows how the equilibrium disclosure changes with the number of competing firms. Disclosure initially increases, then decreases asymptotically to the lower limit $\hat{\omega} \approx 0.556$. Note that it only takes about 5 firms for increased competition to reduce disclosure, even though p is relatively low at 0.3.

Although the above condition of $N > 1/(p\hat{\omega})$ is mild enough, the condition is indeed only sufficient for competition to decrease disclosure, not necessary. Typically, an even smaller number of firms will suffice. We therefore derive the constraint on N that is both necessary and sufficient for further entry to reduce disclosure.

Consider the position of a firm j that draws the threshold value, $x_j = F^{-1}(1 - \omega^*)$. With N firms competing for the prize, firm j is indifferent between disclosing and herding. If a $(N + 1)$ th competitor enters, and firm j observes the same x_j , how do the firm's prospects change? Should it disclose, the entry reduces its expected prize winnings by a factor of $(1 - p\omega)$ because

$$(18) \quad \phi W(\omega; N) = \phi(1 - p\omega)^{N-1}$$

is exponentially decreasing in N . But the other terms, $F_N^{-1}(1 - \omega)$ and $u_N^C(\omega)$, decline by a factor of $N/(N + 1)$, as demonstrated in Lemma 5 in the Appendix. As N rises, then, this linear effect diminishes in significance compared to the exponential effect on the expected prize value. Intuitively, it seems that there is a critical number of firms at which additional competition makes herding more attractive than competing for the prize.

PROPOSITION 8: Disclosure frequency decreases with firm entry if and only if the number of competing firms exceeds some threshold:

$$(19) \quad N > 1 - \frac{p\omega_N^*}{p\omega_N^*} \Leftrightarrow \omega_{N+1}^* < \omega_N^*.$$

According to Proposition 8, if N exceeds the threshold specified by the relative probabilities of disclosing and pooling, then the exponential effect overwhelms the

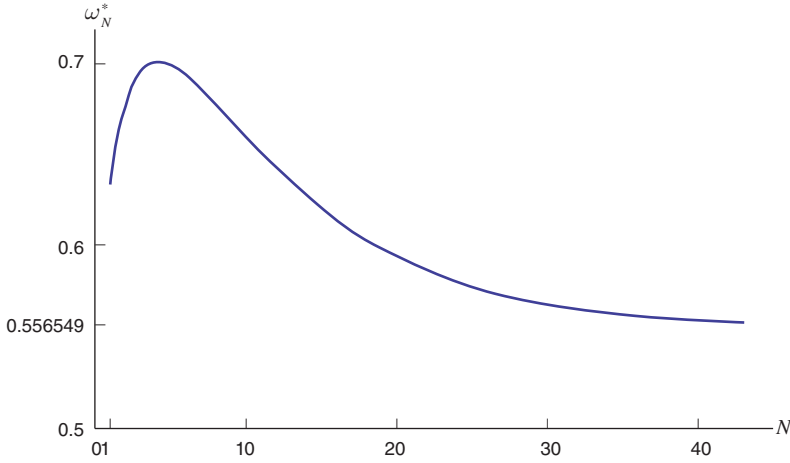


FIGURE 1. DISCLOSURE GAME WITH PRODUCT MARKET COMPETITION

Notes: Disclosure game with product market competition. The random variable is $\tilde{x} \sim N(0, 5)$ and $\phi = 1$ and $p = 0.3$. The product market competition characterized by $F_N(x) = F(Nx)$. The curve shows how the equilibrium disclosure changes with the number of competing firms. Disclosure initially increases, then decreases asymptotically to the lower limit $\hat{\omega} \approx 0.556$.

linear effect. So, the net effect of firm entry is a reduction in the incentive to disclose, which results in $\omega_{N+1}^* < \omega_N^*$. Note, however, that the threshold for N established by Proposition 8 is changing with N . That is, as N increases, ω_N^* varies, and so the probability ratio in equation (19) may also increase. Therefore, although this Proposition details the necessary and sufficient condition for N , it does not provide a tighter unconditional bound than in Proposition 7.

Economically, Propositions 7 and 8 imply that when the number of firms is small, further competition increases disclosure because the benefits of the prize are large compared to the share of industry revenues that each firm receives. However, as the number of firms rises, the benefits of revealing information rapidly drop compared to sharing industry revenues with more firms, and disclosure becomes less likely.

B. Generalized Product Market Competition

Now suppose that we define the distribution as a function of N by

$$(20) \quad F_N(x) \equiv F\left(\frac{x}{\alpha_N}\right),$$

for some decreasing sequence $\{\alpha_N\}$. By construction, if α_N decreases rapidly, then firm entry has a dramatic effect on the revenue of competing firms. If α_N decreases more slowly, then the effect is less pronounced. This formulation embeds the previous set up in which $F_N(x) = F(Nx)$.

PROPOSITION 9: *If, under generalized competition with $F_N(x) = F(x/\alpha_N)$,*

$$(21) \quad \lim_{N \rightarrow \infty} \frac{\alpha_{N+1}}{\alpha_N} > 1 - p\hat{\omega},$$

then there exists some $\bar{N} \in \mathbb{R}$ such that $N > \bar{N} \Rightarrow \omega_{N+1}^ < \omega_N^*$.*

The proof follows nearly the same structure as the proof of Proposition 8. Note, however, that in this case, we need an additional restriction on the sequence $\{\alpha_N\}$ in order to complete the proof. Roughly stated, the requirement above is that competition not reduce firm value too “quickly” as additional firms enter.

Thus, the question becomes one of whether the per-firm revenue can decrease ad-infinitum at such a rate with the entry of additional firms. Although one can posit such a model, exponentially decreasing revenue is not a common feature of micro-economic models of competition.

EXAMPLE 2: *Consider a Cournot competition with linear pricing. In such a model, per-firm earnings (and hence firm value) declines as N grows:*

$$(22) \quad \pi_N = \frac{\pi_1}{N^2}.$$

Therefore, $\alpha_N = 1/N^2$. This sequence satisfies the criterion in Proposition 9 because

$$(23) \quad \lim_{N \rightarrow \infty} \frac{\alpha_{N+1}}{\alpha_N} = \lim_{N \rightarrow \infty} \frac{N^2}{(N+1)^2} = 1 > 1 - p\hat{\omega}.$$

So under linear Cournot competition, disclosure does indeed decline with competition for large N .

III. Concluding Remarks

The primary result in this paper is that increased competition often reduces disclosure when tournament-like competition is present. We show this both in a parsimonious model, as well as in more sophisticated extensions. The fundamental idea, that firm entry makes attaining top status more difficult, is straightforward. But the exponential relationship between the number of competing firms and the probability of winning the prize is mathematically powerful. The result is a robustness that makes our central result widely generalizable.

If transparency is considered a good, then our analysis has straightforward welfare implications. However, competition’s effect on welfare depends on the goal of screening in the market. For example, suppose that it is only necessary to identify the top performer(s) in the market. Then, even if competition causes more firms to hoard information, welfare rises because a higher number of firms makes it more likely that the best firm is identified. Indeed, this is proved analytically in Monopolist Model Example A and Proposition 10 at the conclusion of the Mathematical Appendix.

However, if welfare depends on the behavior of the marginal firm, competition may decrease welfare. Monopolist Model Example B and Proposition 11 in the Mathematical Appendix provide a situation in which welfare depends on screening all potential counterparties in the market. In such case, as competition increases, the marginal firm chooses not to disclose their information, which worsens the efficiency of screening and causes per capita welfare to drop.

It is important to note that we do not assert that tournaments represent optimal screening mechanisms. But they do exist in many venues. In such cases, we cannot always appeal to the invisible hand to make markets transparent. While competition in product markets often has a favorable effect on prices, driving firms to lower and more socially efficient prices, it can have the opposite effect on disclosure. In the asymptotic limit of perfect competition, prices converge to their most efficient values, but disclosure falls to its least efficient.

In the end, our analysis implies that policymakers should consider the type of competition that takes place in markets when deciding whether to regulate them. Competition may not always cure market ailments, and may even exacerbate them.

MATHEMATICAL APPENDIX

PROOF OF PROPOSITION 1:

We establish Lemmas 1, 2, and 3, to determine the game's unique subgame-perfect Nash equilibrium. Lemma 1 establishes that all firms use a threshold strategy to determine whether to disclose information when they have it. Lemma 2 shows that all firms use a common threshold. Finally, Lemma 3 shows that this common threshold is unique.

LEMMA 1: *In any subgame-perfect Nash equilibrium, each firm acts according to a disclosure threshold $t_j < 0$,*

$$(A1) \quad \sigma_j(x_j) = \begin{cases} 1 & \text{for } x_j > t_j \\ 0 & \text{for } x_j < t_j. \end{cases}$$

The threshold is implicitly defined by the condition that a firm observing $x_j = t_j$ be indifferent between disclosing and concealing,

$$(A2) \quad u_j^D(t_j) = u_j^C.$$

PROOF OF LEMMA 1:

Suppose firm j observes the event x . In a subgame-perfect Nash equilibrium, the firm must disclose optimally given the value of x . That is, it discloses when $u_j^D(x) > u_j^C$ and conceal x when $u_j^C > u_j^D(x)$.

If the firm discloses, then it is eligible to with the prize ϕ . So its new market valuation is x , plus an additional ϕ if no competing firm makes a higher disclosure,

$$(A3) \quad u_j^D(x) = x + \phi W_j(x),$$

where $W_j(x)$ is the probability that no competing firm discloses a higher value than x :

$$(A4) \quad W_j(x) = \prod_{k \neq j} (1 - P(I_k)P(x_k > x) \cap D_j)$$

$$(A5) \quad = \prod_{k \neq j} \left(1 - p \int_x^\infty \sigma_k(z)f(z) dz \right).$$

Note that $u_j^D(x)$ is differentiable, and therefore continuous. Furthermore, for any x ,

$$(A6) \quad u_j^D(x) \leq x + \phi \quad \text{and} \quad x \leq u_j^D(x).$$

Evaluating at $x = u_j^C - \phi$ and $x = u_j^C$, these inequalities yield

$$(A7) \quad u_j^D(u_j^C - \phi) \leq u_j^C \quad \text{and} \quad u_j^C \leq u_j^D(u_j^C).$$

So if firm j observes $x_j = u_j^C - \phi$, then disclosure yields a lower expected utility than u_j^C ; and if it observes $x_j = u_j^C$, then disclosure yields a higher expected utility than u_j^C . Because $u_j^D(x)$ is continuous, the Intermediate Value Theorem assures us there is a potential observation $t_j \in [u_j^C - \phi, u_j^C]$ for which

$$(A8) \quad u_j^D(t_j) = u_j^C.$$

This t_j is the disclosure threshold for firm j , where the firm is indifferent between disclosing and pooling. Since $u_j^D(x)$ is strictly monotonic in x , we further obtain

$$(A9) \quad x > t_j \Rightarrow u_j^D(x) > u_j^C$$

$$(A10) \quad x < t_j \Rightarrow u_j^D(x) < u_j^C.$$

The subgame-optimal response of firm j is therefore to disclose any values above the threshold t_j and to conceal any values below, as desired.

Now to show that $t_j < 0$, we derive the value u_j^C that investors assign if the firm conceals its observation. In a rational expectations equilibrium, the beliefs of the investors with respect to the strategy must be consistent with the strategy actually used,

$$(A11) \quad u_j^C = E[x|P_j] = \frac{P(U_j)E[x|U_j] + P(I_j \cap C_j)E[x|I_j \cap C_j]}{P(U_j) + P(I_j)P(C_j|I_j)}$$

$$(A12) \quad = \frac{(1 - p) \cdot 0 + pP(x < t_j)E[x|x < t_j]}{(1 - p) + pP(x < t_j)}$$

$$(A13) \quad = \frac{pF(t_j)}{1 - p + pF(t_j)}E[x|x < t_j]$$

$$(A14) \quad < \frac{pF(t_j)}{1 - p + pF(t_j)}E[x] = 0,$$

and therefore,

$$(A15) \quad u_j^C < 0 < u_j^D(0).$$

Because $u_j^D(x)$ is monotonically increasing in x , the threshold t_j must be below zero. That is, all average or better values of x will be disclosed in equilibrium.

LEMMA 2: *Every firm uses the same disclosure threshold, defined as t^* .*

PROOF OF LEMMA 2:

Write equation (A15) as an integral, then apply integration by parts,

$$(A16) \quad u_j^C = \frac{p}{1 - p + pF(t_j)} \int_{-\infty}^{t_j} xf(x) dx$$

$$(A17) \quad = \frac{p}{1 - p + pF(t_j)} \left([xF(x)]_{-\infty}^{t_j} - \int_{-\infty}^{t_j} F(x) dx \right)$$

$$(A18) \quad = \frac{p}{1 - p + pF(t_j)} \left(t_j F(t_j) - \int_{-\infty}^{t_j} F(x) dx \right).$$

Using this expression, some algebraic manipulation transforms $u_j^D(t_j) = u_j^C$ into

$$(A19) \quad \phi W_j(t_j)(1 - p + pF(t_j)) = (1 - p)(-t_j) - p \int_{-\infty}^{t_j} F(x) dx.$$

Now suppose for contradiction that a non-symmetric equilibrium exists. That is, suppose an equilibrium exists in which firms j and k use different thresholds. Without loss of generality, assume that $t_k < t_j$. Equation (A19) holds for firm k as well as for j . Subtracting these yields

$$(A20) \quad \phi(W_j(t_j)(1 - p + pF(t_j)) - W_k(t_k)(1 - p + pF(t_k)))$$

$$(A21) \quad = (1 - p)(-t_j + t_k) - p \int_{t_k}^{t_j} F(x) dx < 0,$$

and therefore

$$(A22) \quad W_j(t_j)(1 - p + pF(t_j)) < W_k(t_k)(1 - p + pF(t_k)).$$

But we can obtain a contradiction by deriving the opposite inequality. We simplify equation (A4) with the assumption that all firms use threshold strategies, then evaluate at t_j :

$$(A23) \quad W_j(t_j) = \prod_{i \neq j} \left(1 - p \int_{t_j}^{\infty} \sigma_i(z) f(z) dz \right)$$

$$(A24) \quad = \prod_{i \neq j} (1 - p + pF(\max(t_i, t_j)))$$

$$(A25) \quad = (1 - p + pF(\max(t_j, t_k))) \prod_{i \neq j, k} (1 - p + pF(\max(t_i, t_j))).$$

The same holds for firm k , so we obtain

$$(A26) \quad W_k(t_k) = (1 - p + pF(\max(t_j, t_k))) \prod_{i \neq j, k} (1 - p + pF(\max(t_i, t_k))).$$

Since $t_j > t_k$, these equations show that $W_j(t_j) > W_k(t_k)$. Therefore,

$$(A27) \quad W_j(t_j)(1 - p + pF(t_j)) > W_k(t_k)(1 - p + pF(t_k)).$$

This directly contradicts equation (A22), so the hypothesized asymmetric equilibrium cannot exist.

LEMMA 3: *The common disclosure threshold t^* is unique.*

PROOF OF LEMMA 3:

Suppose for contradiction there exist two distinct equilibrium thresholds t^* and t^{**} . Without loss of generality, assume $t^* < t^{**}$. Equation (A19) holds at both thresholds. Subtracting, we obtain

$$(A28) \quad \phi(W(t^*)(1 - p + pF(t^*)) - W(t^{**})(1 - p + pF(t^{**})))$$

$$(A29) \quad = (1 - p)(t^{**} - t^*) + p \int_{t^{**}}^{t^*} F(x) dx < 0,$$

and therefore,

$$(A30) \quad W(t^*)(1 - p + pF(t^*)) < W(t^{**})(1 - p + pF(t^{**})).$$

We now obtain a contraction by deriving the opposite inequality. Since strategies are symmetric, $t_i = t_j$ in equation (A23), so the equation simplifies to

$$(A31) \quad W(t^*) = (1 - p + pF(t^*))^{N-1}.$$

And the same holds for the other equilibrium threshold,

$$(A32) \quad W(t^{**}) = (1 - p + pF(t^{**}))^{N-1}.$$

Because $t^* > t^{**}$, these equations show that $W(t^*) > W(t^{**})$. Therefore,

$$(A33) \quad W(t^*)(1 - p + pF(t^*)) > W(t^{**})(1 - p + pF(t^{**})),$$

directly contradicting equation (A30). By this contradiction, we conclude that a second distinct equilibrium threshold t^{**} cannot exist.

Taken together, Lemmas 1, 2, and 3 show that all firms use a common and unique disclosure threshold defined implicitly by

$$(A34) \quad u_j^D(t^*) = u^C(t^*).$$

We expand this equivalence using $u_j^D(t^*) = t^* + \phi W(t^*)$, equation (A31), and equation (A16) to obtain the desired expression

$$(A35) \quad t^* + \phi(1 - p + pF(t^*))^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} xf(x) dx.$$

Finally, we find $t^* < 0$ by the same argument that shows $t_j < 0$ in Lemma 1.

DEFINITION 2: For any disclosure frequency ω , define the corresponding disclosure threshold by $t(\omega)$. That is,

$$(A36) \quad t(\omega) \equiv F^{-1}(1 - \omega).$$

DEFINITION 3: Define $B(\omega)$ as the benefit of disclosing the threshold value relative to concealing, assuming that all firms disclose with frequency ω ,

$$(A37) \quad B(\omega) \equiv u^D(\omega) - u^C(\omega),$$

where

$$(A38) \quad u^D(\omega) \equiv E[u_j^D | x_j = t(\omega)] = t(\omega) + \phi(1 - p\omega)^{N-1}$$

$$(A39) \quad u^C(\omega) \equiv E[x_j | P_j, t_j = t(\omega)] = \frac{p}{1 - p\omega} \int_{\omega}^1 t(\Omega) d\Omega.$$

Note that this definition does not require that ω be the equilibrium frequency, which we denote distinctly by ω^* .

COROLLARY 1: *The equilibrium condition equation (4) in Proposition 1 can be rewritten in terms of the equilibrium disclosure frequency ω^* as*

$$(A40) \quad B(\omega^*) = 0.$$

PROOF OF COROLLARY 1:

From Definition 2 we get $F(1 - \omega^*) = t^*$. Applying this to equation (4) yields,

$$(A41) \quad t^* + \phi(1 - p + pF(t^*))^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} xf(x) dx,$$

$$(A42) \quad t(\omega^*) + \phi(1 - p\omega^*)^{N-1} = \frac{p}{1 - p\omega^*} \int_{-\infty}^{t^*} xf(x) dx.$$

Defining the integral substitution $\Omega = 1 - F(x)$ yields

$$(A43) \quad \int_{-\infty}^{t^*} xf(x) dx = \int_{1-F(t^*)}^{1-F(-\infty)} F^{-1}(1 - \Omega)(-d\Omega) = \int_{\omega^*}^1 F^{-1}(1 - \Omega) d\Omega.$$

Applying this to the right-hand side of the previous equation yields

$$(A44) \quad t(\omega^*) + \phi(1 - p\omega^*)^{N-1} = \frac{p}{1 - p\omega^*} \int_{\omega^*}^1 F^{-1}(1 - \Omega) d\Omega$$

$$(A45) \quad u^D(\omega^*) = u^C(\omega^*)$$

$$(A46) \quad B(\omega^*) = 0.$$

LEMMA 4: *$B(\omega)$ is strictly decreasing for all $\omega > \omega^*$.*

PROOF OF LEMMA 4:

First write $B(\omega) \equiv u^D(\omega) - u^C(\omega)$ explicitly as

$$(A47) \quad B(\omega, \phi, p, N) = t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{p}{1 - p\omega} \int_{\omega}^1 t(\Omega) d\Omega.$$

Note that

$$(A48) \quad \frac{\partial}{\partial \omega} t(\omega) = \frac{\partial}{\partial \omega} F^{-1}(1 - \omega) = \frac{-1}{f(F^{-1}(1 - \omega))} < 0,$$

so the first term is decreasing in ω . Clearly the second term is also decreasing in ω . In the third term,

$$(A49) \quad \frac{\partial}{\partial \omega} \left(-\frac{p \int_{\omega}^1 t(\Omega) d\Omega}{1 - p\omega} \right) = \frac{-p^2 \int_{\omega}^1 t(\Omega) d\Omega + pt(\omega)(1 - p\omega)}{(1 - p\omega)^2}$$

and the integrand $t(\Omega)$ is decreasing in Ω , so

$$(A50) \quad \dots < \frac{-p^2(1 - \omega)t(\omega) + pt(\omega)(1 - p\omega)}{(1 - p\omega)^2}$$

$$(A51) \quad = \frac{-p^2 + p^2\omega + p - p^2\omega}{(1 - p\omega)^2} t(\omega)$$

$$(A52) \quad = \frac{p(1 - p)}{(1 - p\omega)^2} t(\omega).$$

By our assumption that $\omega > \hat{\omega}$, we know that $t(\omega) < \hat{t} < E[\tilde{x}] = 0$, and so the derivative of the third term is also negative. Thus, $B(\omega)$ is strictly decreasing in ω for all $\omega > \hat{\omega}$.

PROOF OF PROPOSITION 2:

First write $B(\omega) \equiv u^D(\omega) - u^C(\omega)$ explicitly as

$$(A53) \quad B(\omega, \phi, p, N) = t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{p}{1 - p\omega} \int_{\omega}^1 t(\Omega) d\Omega.$$

For any set of parameter values (ϕ, p, N) , the equilibrium disclosure frequency is uniquely defined by $B(\omega^*, \phi, p, N) = 0$. Because B is differentiable with respect to each of its parameters, the Implicit Function Theorem tells us how the equilibrium frequency changes with the parameter values. For each parameter $\theta \in \{\phi, p, N\}$, the IFT gives

$$(A54) \quad \frac{\partial \omega^*}{\partial \theta} \equiv \frac{\partial \omega}{\partial \theta} \Big|_{B=0} = - \frac{\frac{\partial B}{\partial \theta} \Big|_{B=0}}{\frac{\partial B}{\partial \omega} \Big|_{B=0}}.$$

Lemma 4 tells us that $\frac{\partial B}{\partial \omega} < 0$ for all $\omega > \hat{\omega}$. Differentiating with respect to the other model parameters yields

$$(A55) \quad \frac{\partial B}{\partial \phi} = (1 - p\omega)^{N-1} > 0$$

$$(A56) \quad \frac{\partial B}{\partial N} = \phi(1 - p\omega)^{N-1} \ln(1 - p\omega) < 0$$

$$(A57) \quad \frac{\partial B}{\partial p} = -\omega\phi(N - 1)(1 - p\omega)^{N-2} - \frac{1}{(1 - p\omega)^2} \int_{\omega}^1 t(\Omega) d\Omega.$$

Note that $\int_{\omega}^1 t(\Omega) d\Omega < 0$ is the expected value of x for a non-disclosing firm, which is negative. So the second term of $\frac{\partial B}{\partial p}$ is positive, while the first is negative. Which term dominates depends on the parameter values.

Applying the Implicit Function Theorem yields the desired comparative statics:

$$(A58) \quad \frac{\partial B}{\partial \phi} > 0 \quad \text{so} \quad \frac{\partial \omega^*}{\partial \phi} = -\frac{\partial B / \partial \phi}{\partial B / \partial \omega} > 0,$$

$$(A59) \quad \frac{\partial B}{\partial N} < 0 \quad \text{so} \quad \frac{\partial \omega^*}{\partial N} = -\frac{\partial B / \partial N}{\partial B / \partial \omega} > 0.$$

As shown already, ω is decreasing in N . Since any monotonic bounded sequence of real numbers converges, and since we know $\omega_N^* > \hat{\omega}$ for all N , ω_N^* converges as $N \rightarrow \infty$. Let us refer to its limit as

$$(A60) \quad \omega_{\infty} = \lim_{N \rightarrow \infty} \omega_N^*.$$

The function $B(\cdot)$ is continuous in ω_N^* and N , and $B_N(\omega_N^*) = 0$ for all N . The sequence $\{B_N(\omega_N^*)\}$ therefore converges to zero as well:

$$(A61) \quad 0 = \lim_{N \rightarrow \infty} B_N(\omega_N^*)$$

$$(A62) \quad = \lim_{N \rightarrow \infty} t(\omega_N^*) + \lim_{N \rightarrow \infty} \phi(1 - p\omega_N^*)^N - \lim_{N \rightarrow \infty} \frac{p \int_{\omega_N^*}^1 t(\Omega) d\Omega}{1 - p\omega_N^*}$$

$$(A63) \quad = t(\omega_{\infty}) + 0 - \frac{p \int_{\omega_{\infty}}^1 t(\Omega) d\Omega}{1 - p\omega_{\infty}},$$

That is,

$$(A64) \quad t(\omega_{\infty}) = \frac{p \int_{\omega_{\infty}}^1 t(\Omega) d\Omega}{1 - p\omega_{\infty}},$$

and therefore $\omega_{\infty} = \hat{\omega}$.

PROOF OF PROPOSITON 3:

We consider the base model with prizes ϕ_N that increase with N according to some sequence $\{\phi_N\}$. Then the benefit of disclosing relative to concealing is a function of N ,

$$(A65) \quad B_N(\omega) = t(\omega) + \phi_N(1 - p\omega)^{N-1} - u^C(\omega).$$

The same holds for $(N + 1)$ firms, so we can subtract the two equations to obtain

$$(A66) \quad B_{N+1}(\omega) - B_N(\omega) = \phi_{N+1}(1 - p\omega)^N - \phi_N(1 - p\omega)^{N-1}$$

$$(A67) \quad = \phi_N(1 - p\omega)^{N-1} \left(\frac{\phi_{N+1}}{\phi_N} (1 - p\omega) - 1 \right).$$

Under our assumption that $\lim_{N \rightarrow \infty} \frac{\phi_{N+1}}{\phi_N} < 1/(1 - p\hat{\omega})$, there exists some \bar{N} such that

$$(A68) \quad N > \bar{N} \Rightarrow \frac{\phi_{N+1}}{\phi_N} < \frac{1}{1 - p\hat{\omega}},$$

so evaluating equation (A66) at $\omega = \omega_N^*$ for any $N > \bar{N}$ yields

$$(A69) \quad B_{N+1}(\omega_N^*) - 0 < \phi_N(1 - p\omega_N^*)^{N-1} \left(\frac{\phi_{N+1}}{\phi_N}(1 - p\hat{\omega}) - 1 \right) < 0.$$

By Lemma 4, we obtain the desired $\omega_{N+1}^* < \omega_N^*$.

PROOF OF PROPOSITION 4:

Define a firm's "rank" according to the firm's place among realized disclosures by competing firms. That is, if there are $k - 1$ higher disclosures, the firm has rank k and receives ϕ_k . A disclosing firm's rank is therefore a stochastic function of its disclosed value. We define $\tilde{r}(\omega)$ accordingly:

$$(A70) \quad \hat{r}(\omega) = \text{rank of a firm that discloses } x = F^{-1}(1 - \omega).$$

Using this notation, we would write the expected utility of disclosure in the base model as

$$(A71) \quad u^D(\omega) = t(\omega) + \phi W(\omega) = t(\omega) + \phi P(\tilde{r}(\omega) = 1).$$

With prizes for the top K firms, the expected payout becomes

$$(A72) \quad u^D(\omega) = t(\omega) + \sum_{k=1}^K \phi_k P(\tilde{r}(\omega) = k).$$

We wish to show that this value is decreasing in N . Unfortunately, we cannot claim that $P(\hat{r}(\omega) = k)$ is decreasing in N without some further restrictions. Although the chance of having *at least* the k th-highest disclosure is strictly decreasing in N , the chance of having *exactly* the k th-highest disclosure may be increasing in N , at least for certain parameter values. We therefore rearrange the sum in order to write it in terms we know to be unconditionally decreasing in N ,

$$(A73) \quad u^D(\omega) = t(\omega) + \sum_{k=1}^K \phi_k (P(\tilde{r}(\omega) \leq k) - P(\tilde{r}(\omega) \leq k - 1))$$

$$(A74) \quad = t(\omega) + \sum_{k=1}^K (\phi_k - \phi_{k+1}) P(\tilde{r}(\omega) \leq k).$$

Note that $P(\tilde{r}(\omega) \leq k)$, the probability of having at least the k th-highest disclosure, is strictly decreasing in N . Since prizes are strictly decreasing in rank, we also have $(\phi_k - \phi_{k+1}) > 0$. Therefore, $u^D(\omega)$ is unconditionally decreasing in N . We conclude that disclosure frequency decreases in N under a progressive prize structure.

PROOF OF PROPOSITION 5:

Let t_N^* be the equilibrium disclosure threshold with N firms. Suppose that a firm j observes and discloses exactly $x_j = t_N^*$. Then the probability q that any other given opponent observes a higher value is given by

$$(A75) \quad q \equiv p(1 - F(t_N^*)) = p\omega_N^*.$$

Any such realization above the threshold will certainly be disclosed, so the number of firms who disclose values higher than t_N^* is a binomial random variable $\tilde{S} \sim B(N, q)$. The probability that firm j wins a prize is bounded by the probability that fewer than λN other firms disclose values higher than \hat{t} . That is,

$$(A76) \quad W_j(t_N^*) \leq P(\tilde{S} \leq \lambda N - 1).$$

This probability is the weight of a left tail of the binomial distribution of \tilde{S} . We may bound it using Hoeffding's inequality (Hoeffding 1963), which states that the sum \tilde{s} , of any N random variables, has the probabilistic bound

$$(A77) \quad P(|\tilde{s} - E[\tilde{s}]| \geq c) \leq 2 \exp \left(\frac{-2c^2}{\sum_{i=1}^N (b_i - a_i)^2} \right),$$

where the i th random variable is contained by the interval $[a_i, b_i]$. In our application, \tilde{S} is the sum of $(N - 1)$ identically-distributed Bernoulli trials with success probability q , so

$$(A78) \quad a_i = 0, \quad b_i = 1, \quad E[\tilde{S}] = q(N - 1).$$

We first transform our probability into the same form as Hoeffding's inequality,

$$(A79) \quad W_j(t_N^*) = P(\tilde{S} \leq \lambda N - 1)$$

$$(A80) \quad = P(\tilde{S} - E[\tilde{S}] \leq \lambda N - 1 - q(N - 1))$$

$$(A81) \quad \leq P(|\tilde{S} - E[\tilde{S}]| \geq (q - \lambda)N - q + 1).$$

We then can apply the (A77) with $c = (q - \lambda)N - q + 1$ to obtain

$$(A82) \quad W_j(t_N^*) \leq 2 \exp \left(- \frac{2((q - \lambda)N - q + 1)^2}{N} \right).$$

Note that firms will always disclose values above \hat{t} , so any equilibrium threshold t_N^* must be below \hat{t} . We therefore have

$$(A83) \quad q = p(1 - F(t_N^*)) > p(1 - F(\hat{t})) = p\hat{\omega} > \lambda.$$

This ensures that as $N \rightarrow \infty$, the exponential in (A82) goes to $-\infty$ and the right hand side goes to zero for any sequence of thresholds $\{t_N^*\}$. Since firms optimally respond to $W = 0$ by concealing all realizations below \hat{t} , the disclosure frequency converges to $\hat{\omega}$, as desired.

PROOF OF PROPOSITION 6:

By our definition of \hat{t} , any firm with a realization $x_j > \hat{t}$ discloses even if they have no chance of winning the prize. This establishes a lower bound for both limits:

$$(A84) \quad \lim_{N \rightarrow \infty} \nu_j \geq \hat{\omega} \quad \text{and} \quad \lim_{N \rightarrow \infty} \bar{\nu}_N \geq \hat{\omega}.$$

- (i) Suppose firm j discloses a lower value, $x_j < \hat{t}$. If any of the $N - j$ firms yet to act observes a value above \hat{t} , they will certainly disclose it. The probability that firm j wins the prize is therefore bounded above by

$$(A85) \quad W_j(x_j) \leq W_j(\hat{t}) = (1 - p(1 - F(\hat{t})))^{N-j} = (1 - p\hat{\omega})^{N-j}.$$

So as $N \rightarrow \infty$, the probability of winning the prize converges to zero. In this limit, so firm j will optimally conceal any values below \hat{t} , disclosing no more frequently than $\hat{\omega}$. Together with (A84), this establishes the desired result.

- (ii) Again, note that a firm that realizes $x_j < \hat{t}$ will not disclose unless it has a positive probability of winning the prize. Specifically, it will not disclose if any preceding firm has already disclosed a value above \hat{t} . That is, the probability of disclosing a value below \hat{t} cannot possibly be larger than the probability that no preceding firm i has disclosed $x_i > \hat{t}$. This allows us to place a very loose upper bound on ν_j :

$$(A86) \quad \nu_j = P(\tilde{x}_j < \hat{t}) \cdot P(D_j | x_j < \hat{t}) + P(D_j | x_j > \hat{t}) \cdot P(\hat{x}_j > \hat{t})$$

$$(A87) \quad \leq (1 - \hat{\omega}) \cdot \prod_{i=1}^{j-1} P(U_i \text{ or } x_i < \hat{t}) + \hat{\omega} \cdot 1$$

$$(A88) \quad = (1 - \hat{\omega})(1 - p\omega)^{j-1} + \hat{\omega}.$$

Averaging over all j yields

$$(A89) \quad \bar{\nu}_N \leq \frac{1}{N} \sum_{j=1}^N ((1 - \hat{\omega})(1 - p\hat{\omega})^{j-1} + \hat{\omega})$$

$$(A90) \quad = (1 - \hat{\omega}) \frac{1}{N} \left(\frac{1 - (1 - p\hat{\omega})^N}{1 - (1 - p\hat{\omega})} \right) + \hat{\omega}$$

$$(A91) \quad = \frac{1 - \hat{\omega}}{p\hat{\omega}} \left(\frac{1 - (1 - p\hat{\omega})^N}{N} \right) + \hat{\omega}.$$

As $N \rightarrow \infty$, the first term vanishes, so $\lim_{N \rightarrow \infty} \bar{\nu}_N \leq \hat{\omega}$. Together with (A84), this yields the desired result.

LEMMA 5: *Under equal shares competition, the signal that corresponds to a given probability ω , previously written as $t(\omega)$ becomes*

$$(A92) \quad t_N(\omega) = \frac{1}{N} t(\omega).$$

Similarly,

$$(A93) \quad u_N^C(\omega) = \frac{1}{N} u^C(\omega)$$

$$(A94) \quad u_N^D(\omega) = \frac{1}{N} t(\omega) + \phi(1 - p\omega)^{N-1}.$$

PROOF OF LEMMA 5:

Under the definition,

$$(A95) \quad F_N(x) = F(Nx),$$

we find, for any $p \in [0, 1]$, that

$$(A96) \quad p = F_N(F_N^{-1}(p)) \equiv F(NF_N^{-1}(p)),$$

which can be rearranged to

$$(A97) \quad F_N^{-1}(p) = \frac{1}{N} F^{-1}(p),$$

so for $p = 1 - \omega$, we have

$$(A98) \quad F_N^{-1}(1 - \omega) = \frac{1}{N} F^{-1}(1 - \omega),$$

and therefore

$$(A99) \quad t_N(\omega) = \frac{1}{N} t(\omega).$$

Using this first result, the others follow quickly

$$(A100) \quad u_N^D(\omega) \equiv t_N(\omega) + \phi(1 - p\omega)^{N-1}$$

$$(A101) \quad = \frac{1}{N} t(\omega) + \phi(1 - p\omega)^{N-1}$$

$$(A102) \quad u_N^C(\omega) \equiv \frac{p}{1 - p\omega} \int_{\omega}^1 t_N(\Omega) d\Omega$$

$$(A103) \quad = \frac{p}{1 - p\omega} \int_{\omega}^1 \frac{t(\Omega)}{N} d\Omega$$

$$(A104) \quad = \frac{1}{N} u^C(\omega).$$

PROOF OF PROPOSITION 7:

Suppose that $N > 1/p\hat{\omega}$. Since the addition of a prize can only increase equilibrium disclosure,

$$(A105) \quad N > \frac{1}{p\hat{\omega}} \geq \frac{1}{p\omega_N^*} > \frac{1 - p\omega_N^*}{p\omega_N^*}.$$

By Proposition 8, which is proven later in the Appendix, this implies that $\omega_N^* > \omega_{N+1}^*$. Since the same logic holds for all larger N , the sequence $\omega_N^*, \omega_{N+1}^*, \omega_{N+2}^*, \dots$ is monotonically decreasing. Since the sequence is also bounded below by $\hat{\omega}$, it must have a limit, which we will refer to as

$$(A106) \quad \omega_{\infty} = \lim_{N \rightarrow \infty} \omega_N^*.$$

The function $B(\cdot)$ is continuous in ω_N^* and N , and $B_N(\omega_N^*) = 0$ for all N . The sequence $\{B_N(\omega_N^*)\}$ therefore converges to zero as well:

$$(A107) \quad 0 = \lim_{N \rightarrow \infty} B_N(\omega_N^*)$$

$$(A108) \quad = \lim_{N \rightarrow \infty} t(\omega_N^*) + \lim_{N \rightarrow \infty} \phi(1 - p\omega_N^*)^N - \lim_{N \rightarrow \infty} \frac{p \int_{\omega_N^*}^1 t(\Omega) d\Omega}{1 - p\omega_N^*}$$

$$(A109) \quad = t(\omega_{\infty}) + 0 - \frac{p \int_{\omega_{\infty}}^1 t(\Omega) d\Omega}{1 - p\omega_{\infty}}$$

That is,

$$(A110) \quad t(\omega_{\infty}) = \frac{p \int_{\omega_{\infty}}^1 t(\Omega) d\Omega}{1 - p\omega_{\infty}}.$$

This the same function which implicitly defines $\hat{\omega}$, so $\omega_\infty = \hat{\omega}$, as desired.

PROOF PROPOSITION 8:

Applying Lemma 5 to the definition of $B(\omega)$ under equal shares competition yields

$$(A111) \quad B_N(\omega) = u_N^D(\omega) - u_N^C(\omega)$$

$$(A112) \quad = \frac{1}{N} t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{1}{N} u^C(\omega),$$

and therefore,

$$(A113) \quad NB_N(\omega) = t(\omega) + N\phi(1 - p\omega)^{N-1} - u^C(\omega).$$

The same holds for $N + 1$. That is,

$$(A114) \quad (N + 1)B_{N+1}(\omega) = t(\omega) + (N + 1)\phi(1 - p\omega)^N - u^C(\omega).$$

Subtracting equation (A113) from equation (A114) yields

$$(A115) \quad (N + 1)B_{N+1}(\omega) - NB_N(\omega)$$

$$(A116) \quad = (N + 1)\phi(1 - p\omega)^N - N\phi(1 - p\omega)^{N-1}$$

$$(A117) \quad = \phi(1 - p\omega)^{N-1}((1 - p\omega) - Np\omega).$$

If we evaluate the expression at $\omega = \omega_N^*$, then $B_N(\omega_N^*) = 0$, so equation (A115) reduces to

$$(A118) \quad B_{N+1}(\omega_N^*) = \frac{\phi(1 - p\omega_N^*)^{N-1}}{N + 1} ((1 - p\omega_N^*) - Np\omega_N^*).$$

Focusing on the sign of the term in parenthesis, we find

$$(A119) \quad N > \frac{1 - p\omega_N^*}{p\omega_N^*} \Rightarrow B_{N+1}(\omega_N^*) < 0 \Rightarrow \omega_{N+1}^* < \omega_N^*,$$

where the second implication is due to Lemma 4. That is, disclosure at the frequency ω_N^* gives $B < 0$, so the marginal disclosure loses value. The equilibrium frequency ω_{N+1}^* must be lower. This shows that the entry of the $(N + 1)$ th firm reduces disclosure when N is large. When N is smaller than the threshold, the inequalities in equation (A119) are reversed, as shown by the same logic. This completes the equivalence.

PROOF OF PROPOSITION 9:

Under generalized competition with N firms, we have

$$(A120) \quad B_N(\omega) = t_N(\omega) + \phi(1 - p\omega)^{N-1} - u_N^C(\omega)$$

$$(A121) \quad = \alpha_N t(\omega) + \phi(1 - p\omega)^{N-1} - \alpha_N u^C(\omega),$$

and therefore,

$$(A122) \quad \frac{1}{\alpha_N} B_N(\omega) = t(\omega) + \frac{1}{\alpha_N} \phi(1 - p\omega)^{N-1} - u^C(\omega).$$

The same holds for $N + 1$, so we can subtract the two equations to obtain

$$(A123) \quad \frac{1}{\alpha_{N+1}} B_{N+1}(\omega) - \frac{1}{\alpha_N} B_N(\omega)$$

$$(A124) \quad = \frac{1}{\alpha_{N+1}} \phi(1 - p\omega)^N - \frac{1}{\alpha_N} \phi(1 - p\omega)^{N-1}$$

$$(A125) \quad = \frac{1}{\alpha_{N+1}} \phi(1 - p\omega)^{N-1} \left((1 - p\omega) - \frac{\alpha_{N+1}}{\alpha_N} \right).$$

Evaluating at ω_N^* and rearranging terms yields

$$(A126) \quad B_{N+1}(\omega_N^*) = (1 - p\omega_N^*)^{N-1} \phi \left((1 - p\omega_N^*) - \frac{\alpha_{N+1}}{\alpha_N} \right).$$

Under our assumption that $\lim_{N \rightarrow \infty} \alpha_{N+1}/\alpha_N > 1 - p\hat{\omega}$, there exists some \bar{N} such that

$$(A127) \quad N > \bar{N} \Rightarrow \frac{\alpha_{N+1}}{\alpha_N} > 1 - p\hat{\omega}.$$

So for $N > \bar{N}$, we obtain

$$(A128) \quad B_{N+1}(\omega_N^*) = (1 - p\omega_N^*)^{N-1} \phi \left((1 - p\omega_N^*) - \frac{\alpha_{N+1}}{\alpha_N} \right)$$

$$(A129) \quad < (1 - p\omega_N^*)^{N-1} \phi((1 - p\omega_N^*) - (1 - p\hat{\omega}))$$

$$(A130) \quad = (1 - p\omega_N^*)^{N-1} \phi(\hat{\omega} - \omega_N^*) < 0.$$

By Lemma 4, we conclude that $\omega_{N+1}^* < \omega_N^*$, as desired.

I. Monopolist Model Examples

A. Example

Consider that a risk-neutral monopolist sorts through N firms to choose a single firm to do business with. The monopolist wishes to select the firm with the best realization of x_j , but faces the problem that when N or p is small, no firm may disclose their information at all.

In this example, increasing competition is a boon to the monopolist and increases aggregate welfare, as the probability of at least one firm disclosing increases with N . We formalize this in the following proposition.

PROPOSITION 10: *Define Z_N as the ex ante probability that zero of the N competing firms disclose. Then,*

(i) Z_N is decreasing in N ;

(ii) $\lim_{N \rightarrow \infty} Z_N = 0$.

PROOF PROPOSITION 10:

For part 1, note that for all N and corresponding thresholds t_N^* , the threshold condition says

$$(A131) \quad 0 = t_N^* + \phi(1 - p + pF(t_N^*))^{N-1} + \frac{p}{1 - p + pF(t_N^*)} \int_{-\infty}^{t_N^*} xf(x) dx.$$

Multiplying by $(1 - p + pF(t_N^*))$ yields

$$(A132) \quad 0 = t_N^*(1 - p + pF(t_N^*)) + \phi(1 - p + pF(t_N^*))^N - p \int_{-\infty}^{t_N^*} xf(x) dx$$

$$(A133) \quad = t_N^*(1 - p + pF(t_N^*)) + \phi Z_N - p \int_{-\infty}^{t_N^*} xf(x) dx$$

and therefore,

$$(A134) \quad \phi Z_N = -t_N^*(1 - p + pF(t_N^*)) + p \int_{-\infty}^{t_N^*} xf(x) dx.$$

Differentiating with respect to N yields

$$(A135) \quad \phi \frac{dZ_N}{dN} = -\frac{\partial t_N^*}{\partial N} (1 - p + pF(t_N^*)) - t_N^* p f(t_N^*) \frac{\partial t_N^*}{\partial N} + p \frac{\partial}{\partial N} \left(\int_{-\infty}^{t_N^*} xf(x) dx \right) \frac{\partial t_N^*}{\partial N}$$

$$(A136) \quad = -\frac{\partial t_N^*}{\partial N} (1 - p + pF(t_N^*)).$$

Since $\frac{\partial t_N^*}{\partial N} > 0$, we have $\frac{\partial Z_N}{\partial N} < 0$, so Z_N is decreasing in N as desired.

For part 2, note that the probability of any single firm failing to disclose converges to a number less than 1:

$$(A137) \quad \lim_{N \rightarrow \infty} (1 - p + pF(t_N^*)) = 1 - p + pF(t_\infty) < 1.$$

So the probability of zero firms disclosing converges to zero as $N \rightarrow \infty$,

$$(A138) \quad \lim_{N \rightarrow \infty} Z_N = \lim_{N \rightarrow \infty} (1 - p + pF(t_\infty))^N = 0.$$

B. Example

Now suppose instead that the monopolist is unconstrained in how many firms she can choose and that it is efficient for the monopolist to choose firms with a realization of \tilde{x} above a threshold \underline{x} , and not do business with firms below \underline{x} . Denote the net gain from the partnership as G ; because the screening party is monopolistic, we assume that she extracts G from the relationship.

Consider first that the prize is equal to zero. If $\underline{x} \geq \hat{t}$, the monopolist can always efficiently screen counterparties because $\hat{t} \geq t^*$. However, when $\underline{x} < \hat{t}$, there may exist a region of inefficiency. In this case, for any $\tilde{x} \in [\underline{x}, t^*]$, an informed firm withholds their information, despite being qualified. For a given threshold t^* , the ex ante expected welfare loss from the forgone opportunity is

$$(A139) \quad \mathcal{L} = pN[F(t^*) - F(\underline{x})]G.$$

The difference $F(t^*) - F(\underline{x})$ is the expected fraction of firms that falls in the interval $[\underline{x}, t^*]$. Of this group of firms, a fraction p will be informed. Therefore, of the N potential counterparties, the monopolist expects $pN[F(t^*) - F(\underline{x})]$ of them to be qualified but not identified.

Now, consider that the monopolist can encourage additional disclosure by offering a prize ϕ . We assume that ϕ is a transfer between the monopolist and the firm with the highest disclosure; therefore, the size of ϕ does not affect aggregate welfare in and of itself. Again by construction, it is clear that no prize is needed when $\underline{x} \geq \hat{t}$. However, when $\underline{x} < \hat{t}$, the monopolist's goal is to set the optimal prize ϕ to minimize the forgone opportunity. Specifically, the monopolist desires to lower the firms' threshold t^* closer to \underline{x} . As such, we denote the threshold as a function of ϕ , $t(\phi)$. Obviously, there is no additional benefit to increasing the prize once $t(\phi) = \underline{x}$. Hence, we define $\underline{\phi}$ to be the prize that leads to that equality. The monopolist's problem for a given N is

$$(A140) \quad \min_{\phi \in \mathbb{R}^+} pN[F(t(\phi)) - F(\underline{x})]G + \phi(1 - Z_N).$$

PROPOSITION 11: *The optimal prize for the monopolist to offer is*

$$(A141) \quad \phi^* = \begin{cases} 0 & \text{if } N < \frac{1 - W^N}{GpF'W^{N-1}} \\ \min\left(GW - \frac{1 - W^N}{NpF'W^{N-2}}, \underline{\phi}\right) & \text{if } N \geq \frac{1 - W^N}{GpF'W^{N-1}}, \end{cases}$$

where $W = (1 - p + pF(t))$ and $F' = \partial F / \partial t$.

PROOF OF PROPOSITION 11:

We begin solving the monopolist's problem by adopting the following shorthand notation, $W \equiv (1 - p + pF(t))$ and $F' \equiv \partial F / \partial t$. Using first-order conditions with respect to ϕ we obtain,

$$(A142) \quad 0 = \frac{\partial}{\partial \phi} [\mathcal{L} + \phi(1 - Z_N)]$$

$$(A143) \quad = \frac{\partial}{\partial \phi} [pN[F(t(\phi)) - F(\underline{x})]G + \phi(1 - Z_N)].$$

Recalling that $Z_N = W^N$,

$$(A144) \quad 0 = pNF'G \frac{\partial t}{\partial \phi} + (1 - W^N) - \phi NW^{N-1} pF' \frac{\partial t}{\partial \phi}.$$

Recall from Proposition 2 we obtained $\partial t / \partial \phi$ from the Implicit Function Theorem using the net benefit of disclosure formula B . That is,

$$(A145) \quad - \frac{\partial B / \partial \phi}{\partial B / \partial t} \Big|_{B=0} = \frac{\partial t}{\partial \phi}.$$

Therefore, we can rewrite equation (A144) as,

$$(A146) = -pNF'G \left(\frac{\partial B / \partial \phi}{\partial B / \partial t} \Big|_{B=0} \right) + (1 - W^N) + \phi NW^{N-1} pF' \left(\frac{\partial B / \partial \phi}{\partial B / \partial t} \Big|_{B=0} \right).$$

A rearrangement yields

$$(A147) \quad 0 = -pNF'G \frac{\partial B}{\partial \phi} \Big|_{B=0} + \phi NW^{N-1} pF' \frac{\partial B}{\partial \phi} \Big|_{B=0} + (1 - W^N) \frac{\partial B}{\partial t} \Big|_{B=0}$$

$$(A148) \quad = -pNF'W^{N-1}G + \phi NW^{2(N-1)} pF' \\ + (1 - W^N) \left[1 + \phi(N - 1)pFW^{N-2} + \frac{p^2 F' \int_0^t x dF}{W^2} - \frac{ptF'}{W} \right]$$

$$(A149) \quad = -pNF'W^{N-1}G + \phi NW^{2(N-1)}pF' \\ + (1 - W^N) \left[1 + \phi(N - 1)pF'W^{N-2} + \frac{pF'}{W}E[\tilde{x}_j|ND] - \frac{pF'}{W}t \right]$$

$$(A150) \quad = -pNF'W^{N-1}G + \phi NW^{2(N-1)}pF' \\ + (1 - W^N)[1 + \phi(N - 1)pF'W^{N-2} + pF'\phi W^{N-2}]$$

$$(A151) \quad = -pNF'W^{N-1}G + \phi NW^{2(N-1)}pF' + (1 - W^N)[1 + \phi NpF'W^{N-2}].$$

Solving for the optimal prize we obtain,

$$(A152) \quad \phi = GW - \frac{1 - W^N}{NpF'W^{N-2}}.$$

However, if $\phi > \underline{\phi}$, then it is a dominant strategy for the monopolist to offer $\underline{\phi}$, because incentivizing firms to disclose below \underline{x} yields no additional benefit. This yields that the optimal prize is equal to

$$(A153) \quad \phi^* = \min \left(GW - \frac{1 - W^N}{NpF'W^{N-2}}, \underline{\phi} \right).$$

The prize that the monopolist can offer is positive so long as

$$(A154) \quad N \geq \frac{1 - W^N}{GpF'W^{N-1}}.$$

The monopolist is restricted to $\phi \in \Re^+$, so when

$$(A155) \quad N < \frac{1 - W^N}{GpF'W^{N-1}},$$

the prize is equal to zero.

The condition in (B11) can be appreciated as follows. First, it always holds that $\phi^* < G$. That is, the monopolist never has to give up the entire surplus that she earns from working with the most qualified counterparty. Second, when the gain is low (small G), the probability of being informed is low (small p), and the distribution is relatively flat locally around the threshold (low F'), the monopolist optimally avoids paying the cost of offering a prize. In such cases, efficiency is minimized.

The effect of competition on incentives to offer the prize may be nonmonotonic for some N , but is strictly decreasing once N reaches a threshold. To see this, consider

the limit when $N \rightarrow \infty$: it is optimal for the lender to offer no prize. Mathematically, we can rewrite the condition in (B11) and compute

$$(A156) \quad \lim_{N \rightarrow \infty} \frac{NGpF'W^{N-1}}{1 - W^N} = 0 < 1.$$

Economically, when $N \rightarrow \infty$, the prize becomes ineffective: firms minimize disclosure because their chances to win the prize tends to zero. In turn, the monopolist optimally chooses not to offer a prize and efficiency is minimized.

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