Let W denote an $N \times K$ matrix of factor portfolio weights of K portfolios. Let μ denote the vector of risk premia of the N assets, and let Σ denote the covariance matrix. Let $\tilde{R}$ denote the vector of excess returns.

The risk premia of the K factors are $\hat{\mu} := W'\mu$. The covariance matrix of the K factors is $\widehat{\Sigma} = W'\Sigma W$. We can find a frontier portfolio of the factors $\pi \in \mathbb{R}^k$ by solving:

$$\min E[(\pi' W' \tilde{R} - 1)^2]/2 \,,$$

which is equivalent to

$$\min \frac{1}{2}\pi' W' \mathsf{E}[\tilde{R}\tilde{R}'] W\pi - \hat{\mu}'\pi \quad \Leftrightarrow \quad \min \frac{1}{2}\pi'(\widehat{\Sigma} + \hat{\mu}\hat{\mu}')\pi - \hat{\mu}'\pi \,.$$

The FOC is

$$\widehat{\Sigma}\pi + \hat{\mu}\hat{\mu}'\pi = \hat{\mu} \quad \Leftrightarrow \quad \hat{\Sigma}\pi = (1 - \hat{\mu}'\pi)\hat{\mu} \quad \Leftrightarrow \quad \pi = (1 - \hat{\mu}'\pi)\widehat{\Sigma}^{-1}\hat{\mu} \,.$$

Multiply by $\hat{\mu}'$ to obtain

$$\hat{\mu}'\pi = (1 - \hat{\mu}'\pi)\hat{\mu}'\widehat{\Sigma}^{-1}\hat{\mu}$$

so

$$\hat{\mu}'\pi = \frac{\hat{\mu}'\widehat{\Sigma}^{-1}\hat{\mu}}{1 + \hat{\mu}'\widehat{\Sigma}^{-1}\hat{\mu}} \,.$$

Substituting this gives us

$$\pi = \frac{1}{1 + \hat{\mu}'\widehat{\Sigma}^{-1}\hat{\mu}}\widehat{\Sigma}^{-1}\hat{\mu} \,.$$

This means that π is a scalar multiple of the tangency portfolio, and it is guaranteed to be efficient, because $\hat{\mu}'\pi > 0$.