Final Exam
BUSI 521: Asset Pricing Theory
ECON 505: Financial Economics I
Spring 2022

This is a 3 hour exam. It is closed book and closed notes. Please write your student number on each page of your answers and staple them together.

(1) Assume there are n risky assets and a risk-free asset in a single-period model. Derive a formula for the maximum possible Sharpe ratio.
(2) Consider an investor with log utility in a continuous-time model with a single state variable X. Show that if the investor's value function is $J(w, x) = a \log w + f(x)$ for a constant a and function f, then f must satisfy a certain ODE. Derive the ODE. Also, derive the optimal portfolio for the investor.
(3) Assume the price S of a non-dividend-paying is a geometric Brownian motion. Assume there is a constant risk-free rate. Consider a security that pays the following at a given date T:

$$\begin{cases} K_1 & \text{if } S_T < K_1 \\ S_T & \text{if } K_1 \leq S_T \leq K_2 \\ K_2 & \text{if } S_T > K_2 \end{cases}$$

for constants $K_1 < K_2$. Derive a PDE that the value of the security must satisfy. Also, explain how you would price this security using an SDF process. Give as much detail as you can about pricing using an SDF process, starting with describing the SDF process.