Evaluating Past Returns

More Machine Learning

Kerry Back, JGSB, Rice University

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Outline

1. Evaluating without benchmarking - Sharpe ratios and drawdowns
2. Naive benchmarking
3. Benchmarking - alphas and information ratios
4. Attribution analysis - alphas and betas and information ratios

Data

- Monthly FMAGX returns from Yahoo Finance (FMAGX = Fidelity Magellan)
- Market return from French's data library
- Fama-French factors and momentum from French's data library

Monthly returns

In [1]:

```python
import yfinance as yf

ticker = "FMAGX"
price = yf.download(ticker, start=1970)["Adj Close"]
price_monthly = price.resample("M").last()
price_monthly.index = price_monthly.index.to_period("M")
return_monthly = price_monthly.pct_change().dropna()
```

```
[*********************100%%**********************]  1 of 1 completed
```

Monthly risk-free rates from French’s data library

In [2]:

```
from pandas_datareader import DataReader as pdr
fama_french = pdr("F-F_Research_Data_5_Factors_2x3", "famafrench", start=1970
rf = fama_french["RF"]
```

Evaluating without benchmarking

Sharpe ratio

In [3]:

```
import numpy as np

rprem = 12 * (return_monthly - rf).mean()
stdev = np.sqrt(12) * return_monthly.std()
sharpe = rprem / stdev

print(f"Annualized Sharpe ratio is {sharpe:.2%}")
```

```
Annualized Sharpe ratio is 46.21%
```

Drawdowns

- A drawdown is how much you've lost since the previous peak value.
- It's another way to represent risk.
- We'll use the daily price data.

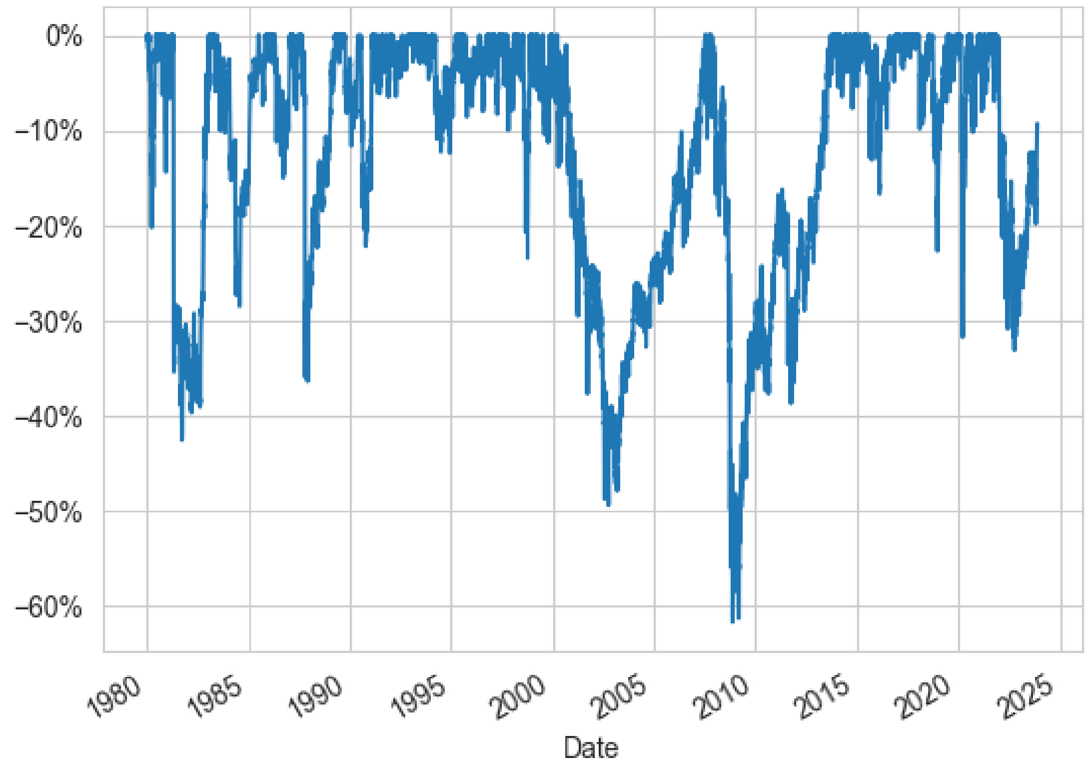
In [4]:

```
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick

import seaborn as sns
sns.set_style("whitegrid")
colors = sns.color_palette()
```

In [5]:

```
fig, ax = plt.subplots()
price_max = price.expanding().max()
drawdown = 100 * (price - price_max) / price_max
drawdown.plot(ax=ax)
ax.yaxis.set_major_formatter(mtick.PercentFormatter())
plt.show()
```



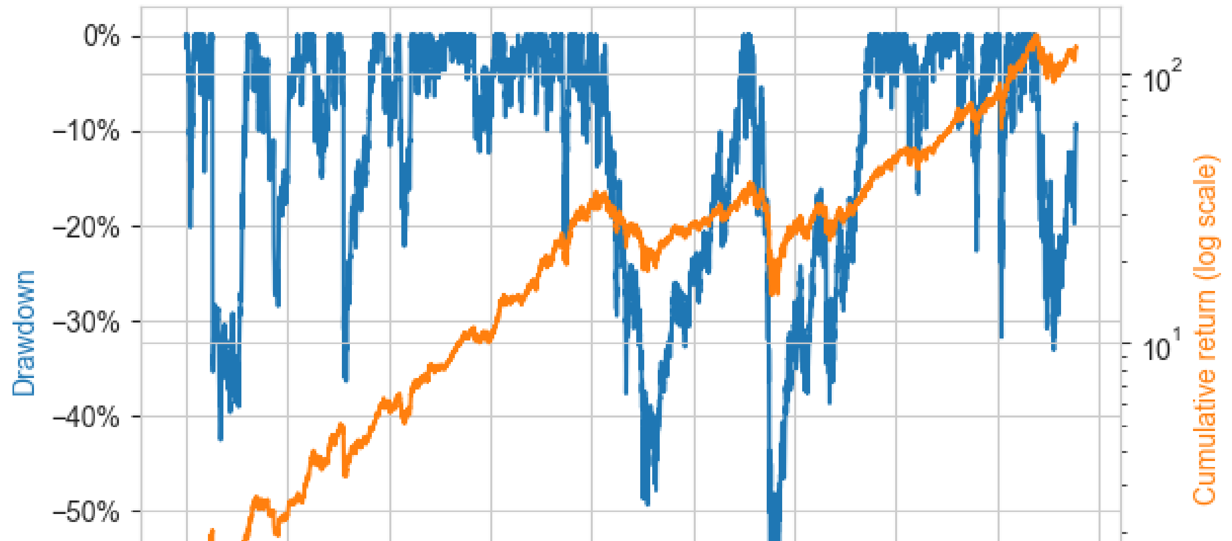
In [6]:
```
fig, ax1 = plt.subplots()

ax2 = ax1.twinx()
ax2.set_yscale("log")
drawdown.plot(ax=ax1)

cumulative_return = price / price.iloc[0]
cumulative_return.plot(ax=ax2, c=colors[1])

ax1.set_xlabel('Date')
ax1.set_ylabel('Drawdown', color=colors[0])
ax2.set_ylabel('Cumulative return (log scale)', color=colors[1])

ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
plt.show()
```



Naive Benchmarking

- Did you beat the benchmark?
- Compute returns in excess of the benchmark and the mean excess return.
- How risky are these excess returns?
- The standard deviation of return in excess of the benchmark is called tracking error.
- Reward to risk ratio = mean excess return / tracking error
- Naive = "don't adjust for beta"

Market return

In [7]:

```
mkt = fama_french["Mkt-RF"] + fama_french["RF"]
```

Mean, risk (tracking error) and reward-to-risk

In [8]:

```
mean = 12 * (return_monthly - mkt).mean()
track_error = np.sqrt(12) * (return_monthly - mkt).std()
reward_to_risk = mean / stdev

print(f"annualized mean return in excess of market is {mean:.2%}")
print(f"annualized tracking error is {track_error:.2%}")
print(f"annualized reward-to-risk ratio is {reward_to_risk:.2%}")
```

```
annualized mean return in excess of market is 0.37%
annualized tracking error is 7.29%
annualized reward-to-risk ratio is 2.02%
```

Benchmarking

Alpha

- Run the regression

$$r - r_f = \alpha + \beta(r_b - rf) + \varepsilon$$

- where $r_b =$ benchmark return
- Rearrange:

$$r - \left[\beta\bar{r}_b + (1-\beta)r_f\right] = \alpha + \varepsilon$$

- So, $\alpha + \varepsilon$ is the excess return over a beta-adjusted benchmark. It is called the active return.
- The beta-adjusted benchmark $\beta\bar{r}_b + (1-\beta)r_f$ has the same beta as r.
- α is the mean active return.

Alpha and mean-variance efficiency

- We can improve on a benchmark by adding some of another return r if and only if its alpha relative to the benchmark is positive.
- We can improve by shorting r if its alpha is negative.
- Cannot improve on benchmark $\Leftrightarrow \alpha = 0$

Information ratio

- The risk of the active return is the risk of the regression residual ε
- Reward to risk ratio $\alpha / \text{std dev of } \varepsilon$ is called the **information ratio.**
- Information ratio is the most important statistic for evaluating performance relative to a benchmark.

Code

- Use statsmodels ols function to run regressions in python.
- Define model and fit.
- Fitted object has .summary() method, .params attribute and others.
- Residual standard deviation is square root of .mse_resid
- We'll use the market as the benchmark

In [9]:

```python
import pandas as pd
import statsmodels.formula.api as smf

df = pd.concat((return_monthly, mkt, rf), axis=1).dropna()
df.columns = ["ret", "mkt", "rf"]
df[["ret_rf", "mkt_rf"]] = df[["ret", "mkt"]].subtract(df.rf, axis=0)

result = smf.ols("ret_rf ~ mkt_rf", df).fit()
```

In [10]:
```
result.summary()
```

Out[10]:

OLS Regression Results

Dep. Variable:	ret_rf	**R-squared:**	0.847
Model:	OLS	**Adj. R-squared:**	0.847
Method:	Least Squares	**F-statistic:**	2892.
Date:	Mon, 27 Nov 2023	**Prob (F-statistic):**	5.26e-215
Time:	13:54:10	**Log-Likelihood:**	1287.2
No. Observations:	524	**AIC:**	-2570.
Df Residuals:	522	**BIC:**	-2562.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-0.0002	0.001	-0.227	0.820	-0.002	0.002
mkt_rf	1.0765	0.020	53.774	0.000	1.037	1.116

Omnibus:	642.619	**Durbin-Watson:**	2.069
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	118829.986
Skew:	-5.621	**Prob(JB):**	0.00
Kurtosis:	75.912	**Cond. No.**	22.0

In [11]:

```python
alpha = 12 * result.params["Intercept"]
resid_stdev = np.sqrt(12 * result.mse_resid)
info_ratio = alpha / resid_stdev

print(f"The annualized information ratio is {info_ratio:.2%}")
```

```
The annualized information ratio is -3.48%
```

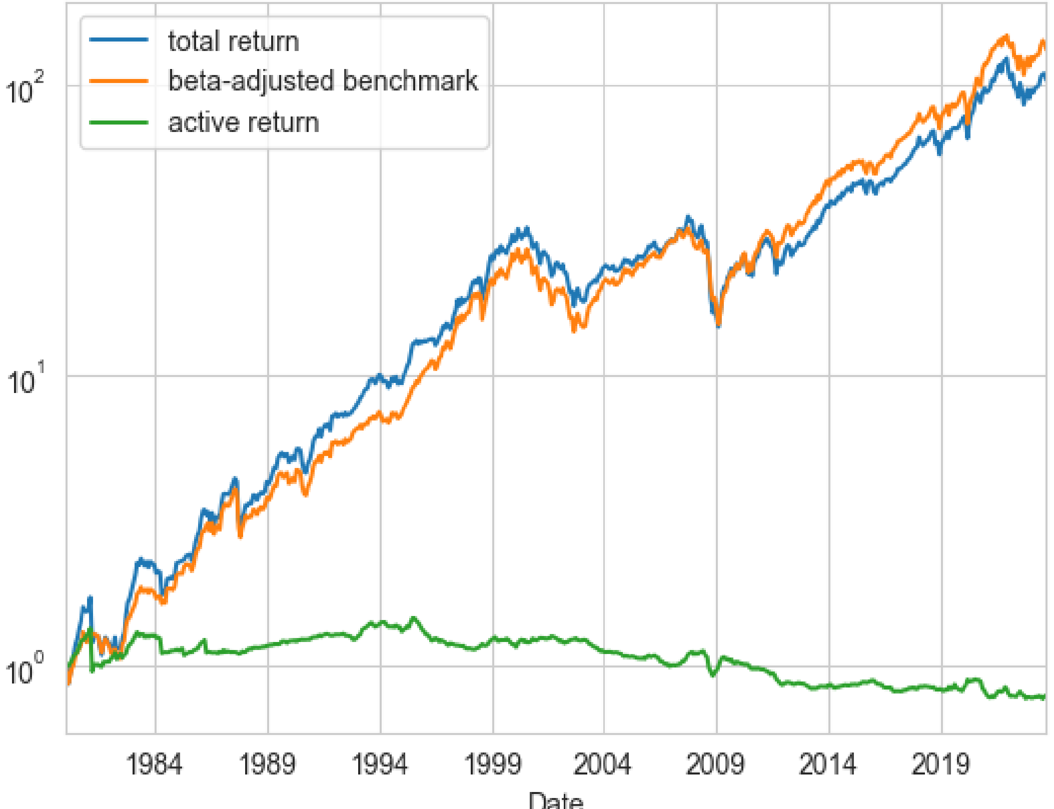
Plotting

- Plot the compound returns $(1+r_1)(1+r_2)\cdots(r+r_n)$
- Plot the compounded beta-adjusted benchmark returns
- Plot the compounded active returns
- Gives a visual of whether returns were earned from the benchmark or from active return.

In [12]:

```
beta = result.params["mkt_rf"]
beta_adjusted_bmark = beta*df.mkt + (1-beta)*df.rf
active = df.ret - beta_adjusted_bmark

(1+df.ret).cumprod().plot(label="total return", logy=True)
(1+beta_adjusted_bmark).cumprod().plot(label="beta-adjusted benchmark", logy=
(1+active).cumprod().plot(label="active return", logy=True)
plt.legend()
plt.show()
```



Attribution Analysis

Factors and attribution

- It is generally agreed that there are portfolio strategies ("factors" or "styles") that earn risk premia that are not explained by the CAPM.
    - Value, momentum, profitability, ...
- An institution evaluating a manager's results will look to see if any common factors are responsible for the results by running regressions on the benchmark and factors.
- In other words, we ask whether the returns can be attributed to common factors.

Alphas and information ratios again

- For simplicity, consider a single factor or style with return r_s. Suppose it is a long-minus-short return.
- We run the regression

$$r - r_f = \alpha + \beta_1(r_b - r_f) + \beta_2 r_s + \varepsilon$$

- We can rearrange as

$$r - \left[\beta_1 r_b + (1 - \beta_1) r_f + \beta_2 r_s\right] = \alpha + \varepsilon$$

- The return in square braces is a beta and factor adjusted benchmark.
- The alpha and the information ratio have the same meaning as before, except that now we are also adjusting for factor exposure.

Data

- Fama-French factors
  - SMB = small minus big (size factor)
  - HML = high book-to-market minus low book-to-market (value factor)
  - CMA = conservative minus agressive (investment rate factor)
  - RMW = robust minus weak (profitability factor)
- Momentum
  - UMD = up minus down
- All from French's data library

In [13]:

```
fama_french.head(3)
```

Out[13]:

Date	Mkt-RF	SMB	HML	RMW	CMA	RF
1970-01	-0.0810	0.0312	0.0313	-0.0172	0.0384	0.0060
1970-02	0.0513	-0.0276	0.0393	-0.0229	0.0276	0.0062
1970-03	-0.0106	-0.0241	0.0399	-0.0100	0.0429	0.0057

In [14]:

```
umd = pdr("F-F_Momentum_Factor", "famafrench", start=1970)[0]/100
umd.columns = ["UMD"]
umd.head(3)
```

Out[14]:

	UMD
Date	
1970-01	0.0060
1970-02	0.0023
1970-03	-0.0036

In [15]:
```
data = pd.concat((fama_french, umd, df), axis=1).dropna()
result = smf.ols("ret_rf ~ mkt_rf + SMB + HML + RMW + CMA + UMD", data).fit()
result.summary()
```

Out[15]:

OLS Regression Results

Dep. Variable:	ret_rf	**R-squared:**	0.854
Model:	OLS	**Adj. R-squared:**	0.852
Method:	Least Squares	**F-statistic:**	502.5
Date:	Mon, 27 Nov 2023	**Prob (F-statistic):**	4.68e-212
Time:	13:54:11	**Log-Likelihood:**	1298.6
No. Observations:	524	**AIC:**	-2583.
Df Residuals:	517	**BIC:**	-2553.
Df Model:	6		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-9.048e-05	0.001	-0.096	0.924	-0.002	0.002
mkt_rf	1.0729	0.022	48.365	0.000	1.029	1.116
SMB	-0.0638	0.034	-1.860	0.063	-0.131	0.004
HML	-0.0509	0.041	-1.229	0.220	-0.132	0.030
RMW	0.0332	0.043	0.778	0.437	-0.051	0.117