

# **Forming and Evaluating Portfolios**

BUSI 722: Data-Driven Finance II

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# From Predictions to Portfolio Weights

# The Setup

From Session 4, we have one-step-ahead predicted return ranks for every stock in every month.

How do we translate these predictions into **portfolio weights**?

We need a function  $w_i = g(\text{rank}_i)$  that maps each stock's predicted rank to a portfolio weight.

- The function should be **monotone**: higher predicted rank  $\Rightarrow$  larger weight.
- There are many valid choices, from simple to sophisticated.

# Sort-Based Portfolios

The simplest approach: sort stocks by predicted rank and equal-weight within groups.

## Long-only (top decile):

- Buy the top 10% of stocks, equal-weighted.
- Weight =  $1/n$  for stocks in the top decile, 0 otherwise.

## Long-short (top minus bottom):

- Buy the top 10%, short the bottom 10%, equal-weighted within each.
- Weight =  $+1/n$  for the top decile,  $-1/n$  for the bottom decile, 0 otherwise.

The weight function is a **step function** of the predicted rank.

# Sorting Is a Monotone Weight Function

Whether we go long the top decile or long-short top versus bottom, the weights are **monotone** in the predicted rank:

- A stock with a higher predicted rank gets a weight that is at least as large as a stock with a lower predicted rank.

**Sorting and equal-weighting within groups is just one particular monotone function mapping predictions to weights. There are many others.**

# Pros and Cons of Sorting

## Advantages:

- Simple, transparent, widely used in academic research.
- Robust to outliers in predictions — extreme predictions within a group get the same weight.

## Disadvantages:

- **Throws away information:** a stock barely in the top decile gets the same weight as the top-ranked stock.
- Sensitive to the choice of breakpoints (quintiles? deciles?).
- Discontinuous: small rank changes can move a stock between groups, causing unnecessary turnover.

# Smooth Weight Functions

# Weights as a Function of Normalized Rank

Scale predicted ranks to  $[0, 1]$ :

$$u_i = \frac{\text{rank}_i}{n}$$

Then set portfolio weights as a monotone function of  $u_i$ .

- This uses the **entire cross section** of predictions, not just the extremes.
- No arbitrary breakpoints.
- Different function shapes express different views about how aggressively to concentrate in top-ranked stocks.



# Linear Weights

The simplest smooth weight function:

$$w_i \propto u_i - \bar{u}$$

where  $\bar{u} = 0.5$  is the median rank.

- Stocks above the median get positive weights; below get negative. Dollar-neutral by construction.
- Every stock's weight is proportional to its **distance from the median rank**.
- Equal spacing: the difference in weight between any two adjacent-ranked stocks is the same.

# Power Functions

Apply a power transformation to the normalized rank:

$$w_i \propto u_i^p - c$$

where  $c$  is chosen so that weights sum to zero.

- $p = 1$ : linear weights (equal spacing).
- $p > 1$  (convex): **concentrates** weight in the top-ranked stocks. Higher  $p$  = more aggressive.
- $0 < p < 1$  (concave): **spreads** weight more evenly. Less concentration at the top.

The parameter  $p$  controls the trade-off between **conviction** (concentrating in the best predictions) and **diversification** (spreading weight across many stocks).

# Exponential Tilts

Apply an exponential transformation:

$$w_i \propto e^{\lambda u_i} - c$$

- $\lambda > 0$ : tilts toward top-ranked stocks; larger  $\lambda$  means more aggressive concentration.
- $\lambda \rightarrow 0$ : converges to equal weights.
- $\lambda \rightarrow \infty$ : concentrates all weight in the top-ranked stock.

## Exponential Tilts and Softmax

The exponential tilt is related to the **softmax** function used in neural networks. With a temperature parameter  $T = 1/\lambda$ , the softmax assigns:

$$w_i = \frac{e^{u_i/T}}{\sum_j e^{u_j/T}}$$

This produces long-only weights that sum to one.

## Comparing Weight Functions

Function	Shape	Character
Step (sort-based)	flat within groups	simple, discontinuous
Linear	straight line	smooth, moderate
Power ( $p > 1$ )	convex	concentrates at the top
Power ( $p < 1$ )	concave	more diversified
Exponential	convex	aggressively concentrates
Softmax	convex, long-only	all weights positive

All of these are **monotone**: a higher predicted rank always means a larger weight. They differ in how much they **differentiate** between stocks near the top versus the middle.

# Long-Only Considerations

# Why Long-Only?

Most investors cannot short stocks. Even those who can face:

- Short-selling costs (borrowing fees, margin requirements).
- Short squeezes and recall risk.
- Regulatory constraints (many funds are long-only by mandate).

## Long-only weight functions:

- Softmax:  $w_i = e^{\lambda u_i} / \sum_j e^{\lambda u_j}$
- Truncated: set  $w_i = 0$  for stocks below a rank threshold, then equal-weight or rank-weight the rest.
- Score-tilted market cap:  $w_i \propto \text{mcap}_i \cdot g(u_i)$

## Score-Tilted Market-Cap Weights

Start with market-cap weights and tilt toward higher-ranked stocks:

$$w_i \propto \text{mcap}_i \times g(u_i)$$

where  $g$  is a monotone increasing function (e.g.,  $g(u) = u^p$  or  $g(u) = e^{\lambda u}$ ).

- Stays close to the investable market-cap benchmark; large stocks retain significant weight even with low ranks.
- More realistic for large portfolios — avoids overweighting micro-caps.
- The tilt function  $g$  controls how much the portfolio deviates from the benchmark.



# Evaluating Portfolio Returns

# Portfolio Returns

Once we choose a weight function, we compute the portfolio return each month:

$$r_{p,t} = \sum_{i=1}^{n_t} w_{i,t} r_{i,t}$$

- $w_{i,t}$  = weight of stock  $i$  in month  $t$  (from the weight function applied to predicted ranks).
- $r_{i,t}$  = realized return of stock  $i$  in month  $t$ .
- This gives us a time series of monthly portfolio returns:  $r_{p,1}, r_{p,2}, \dots, r_{p,T}$ .

# Sharpe Ratio

The **Sharpe ratio** measures risk-adjusted excess return:

$$\text{Sharpe} = \frac{\bar{r}_p - r_f}{\sigma_p}$$

- $\bar{r}_p$  = mean monthly portfolio return;  $r_f$  = risk-free rate.
- $\sigma_p$  = standard deviation of monthly portfolio returns.
- Annualize by multiplying by  $\sqrt{12}$ .

A higher Sharpe ratio means more return per unit of risk. For a long-short portfolio,  $r_f$  is often set to zero (the portfolio is self-financing).

# CAPM Alpha

Regress the portfolio's excess return on the market's excess return:

$$r_{p,t} - r_{f,t} = \alpha + \beta(r_{m,t} - r_{f,t}) + \varepsilon_t$$

- $\alpha$  = the portfolio's **average return not explained by market exposure**.
- $\beta$  = the portfolio's sensitivity to the market.
- $\varepsilon_t$  = residual (idiosyncratic) return.

A positive  $\alpha$  means the portfolio earns more than its market beta would predict. This is what active management aims to deliver.

## Is Alpha Statistically Significant?

The regression also gives us a standard error for  $\alpha$ , and hence a  $t$ -statistic:

$$t = \frac{\hat{\alpha}}{\text{SE}(\hat{\alpha})}$$

- $|t| > 2$  is a rough rule for significance at the 5% level.
- With monthly data over 10 years (120 observations), we need economically large alphas to achieve significance.
- **Always report the  $t$ -statistic**, not just the alpha estimate.

## CAPM Information Ratio

The **information ratio** measures alpha relative to the volatility of the residuals:

$$IR = \frac{\hat{\alpha}}{\sigma(\varepsilon)}$$

- $\hat{\alpha}$  = estimated CAPM alpha.
- $\sigma(\varepsilon)$  = standard deviation of the regression residuals (“tracking error” relative to the CAPM benchmark).
- The IR measures how **consistently** the portfolio delivers alpha.

## Sharpe vs. Information Ratio

- The Sharpe ratio measures total risk-adjusted return.
- The information ratio measures **skill** — alpha per unit of idiosyncratic risk.
- Annualize by multiplying by  $\sqrt{12}$ .

## Comparing the Three Metrics

Metric	Numerator	Denominator
Sharpe ratio	excess return $\bar{r}_p - r_f$	total volatility $\sigma_p$
CAPM alpha	$\hat{\alpha}$	(reported with $t$ -stat)
Information ratio	$\hat{\alpha}$	residual volatility $\sigma(\varepsilon)$

- The Sharpe ratio evaluates the portfolio as a **standalone investment**.
- Alpha and the IR evaluate it as an **addition to the market portfolio**.
- A portfolio can have a high Sharpe ratio but zero alpha (if it just loads on market risk), or a high IR but low Sharpe ratio (if alpha is consistent but small).



## Putting It Together

1. Generate one-step-ahead predicted ranks and choose a weight function (sort-based, linear, power, exponential, softmax, or score-tilted).
2. Compute monthly portfolio returns.
3. Evaluate: Sharpe ratio, CAPM alpha (with  $t$ -stat), and information ratio. Session 6 extends to Fama-French factors and other considerations.