Trading Costs, Risk Forecasts

BUSI 722: Data-Driven Finance II

Kerry Back

Transaction Costs

Categories of Transaction Costs

- **Commissions:** fees paid to the broker per trade. Near zero at most brokerages today, but not negligible for institutional-sized orders.
- **Bid-ask spread:** the cost of immediacy. You buy at the ask and sell at the bid. Half the spread is a rough per-trade cost estimate.
- **Market impact:** large orders move the price against you. The more you trade, the worse the price you get. This is typically the dominant cost for institutional strategies.
- **Opportunity cost:** trades you cancel or delay because the market moved before you could execute.

Market Impact

Market impact grows with trade size relative to the stock's liquidity:

$$\text{impact} \;\approx\; c \times \sqrt{\frac{\text{shares traded}}{\text{average daily volume}}}$$

- The square-root law is a widely used approximation.
- Impact is larger for small, illiquid stocks.
- This creates a **capacity constraint**: strategies that trade heavily in small stocks face rapidly increasing costs as the fund grows.

Estimating Transaction Costs

A simple model for backtesting:

$$\text{cost}_t = \sum_{i=1}^{n} c_i \times |w_{i,t} - w_{i,t-1}|$$

- $|w_{i,t} - w_{i,t-1}|$ = the absolute change in weight (turnover) for stock i.
- c_i = estimated one-way cost for stock i (e.g., half the bid-ask spread plus estimated market impact).
- Common simplification: use a flat cost per unit of turnover (e.g., 5–20 basis points one-way).

Short-Selling Costs

What Does Shorting Cost?

To short a stock, you borrow shares and sell them. This incurs:

- **Borrow fee:** an annualized fee paid to the lender (typically 0.3–1% for easy-to-borrow stocks, but can be 10–50%+ for hard-to-borrow names).
- **Margin requirements:** you must post collateral, tying up capital.
- **Recall risk:** the lender can recall the shares at any time, forcing you to close the position.
- **Short-sale constraints:** some stocks are simply unavailable to borrow.

Implications for Long-Short Strategies

- ML models often find that the **short side is more profitable** than the long side — but also more expensive to trade.
- Short-selling costs disproportionately affect small, illiquid, and distressed stocks — exactly where many ML signals are strongest.
- A strategy that looks great in a frictionless backtest may be mediocre or unprofitable after accounting for borrow fees.
- **Always evaluate long-only and long-short separately** to see where the value is coming from.

Net-of-Cost Evaluation

Net-of-Cost Portfolio Returns

The net portfolio return in month t:

$$r_{p,t}^{\text{net}} = r_{p,t}^{\text{gross}} - \text{transaction costs}_t - \text{borrow costs}_t$$

- Recompute Sharpe ratios, CAPM alphas, and information ratios using $r_{p,t}^{\text{net}}$.
- This is the **real test** of a strategy. Many academic strategies fail it.

Avramov, Cheng, and Metzker (*Management Science*, 2023):

- ML profitability concentrates in **hard-to-trade** stocks (microcaps, distressed firms).
- Excluding these stocks or adding realistic trading costs significantly reduces profitability.

Rebalancing & Weighting

Rebalancing Frequency

How often should we retrain the model and rebalance the portfolio?

Frequency	**Pro**	**Con**
Monthly	tracks signal closely	high turnover, high cost
Quarterly	moderate turnover	signal may be stale
Annually	low turnover	very stale signal

- The optimal frequency depends on the **decay rate of the signal** versus the **cost of trading**.
- Momentum decays quickly — monthly rebalancing is typical.
- Value signals are slow-moving — quarterly may suffice.
- A hybrid approach: retrain monthly but only trade when the change in target weight exceeds a threshold (**buffer rules**).

Turnover

Turnover measures the fraction of the portfolio that changes each period:

$$\text{turnover}_t = \sum_{i=1}^{n} \left| w_{i,t} - w_{i,t-1}^{+} \right|$$

where $w_{i,t-1}^{+}$ is the weight of stock i at the end of the previous month (after returns).

- A turnover of 1.0 means the entire portfolio is replaced.
- A long-short decile portfolio rebalanced monthly can have turnover > 1.0.
- Smooth weight functions (linear, power) typically generate **lower turnover** than sort-based step functions, because small changes in predicted rank produce small changes in weight.

Value Weighting to Reduce Small-Cap Exposure

Equal-weighted portfolios tilt heavily toward small stocks, which are:

- harder and more expensive to trade,
- where many ML signals are strongest (possibly spuriously),
- not investable at scale.

Solutions:

- **Value-weight within groups:** sort into deciles by predicted rank, then value-weight (by market cap) within each decile.
- **Score-tilted market-cap weights** (from Session 5): $w_i \propto \text{mcap}_i \times g(u_i)$. Stays close to the market-cap benchmark.
- **Market-cap filters:** exclude micro-caps or nano-caps entirely. Evaluate only on stocks you could actually trade.

Risk Analysis

Fama-French Factor Analysis

Extend the CAPM regression from Session 5 to the Fama-French five-factor model:

$$r_{p,t} - r_{f,t} = \alpha + \beta_1 \mathrm{MKT}_t + \beta_2 \mathrm{SMB}_t + \beta_3 \mathrm{HML}_t + \beta_4 \mathrm{RMW}_t + \beta_5 \mathrm{CMA}_t + \varepsilon_t$$

- **SMB** (Small Minus Big): size factor
- **HML** (High Minus Low): value factor
- **RMW** (Robust Minus Weak): profitability factor
- **CMA** (Conservative Minus Aggressive): investment factor

The β coefficients reveal **which risks** the portfolio is taking on. The α measures return not explained by these known risk factors.

Interpreting Factor Loadings

- A large positive β_{SMB} means the strategy tilts toward small stocks — its returns are partly compensation for size risk.
- A large positive β_{HML} means it loads on value — part of the return comes from the value premium.
- If α shrinks when adding factors, the strategy is **repackaging known risk premia**, not generating new alpha.
- If α remains large and significant, the strategy captures something beyond the five factors.

Can also add the **momentum factor** (UMD). Download all factors from Kenneth French's website using `pandas-datareader`.

Alternative Portfolio Construction Methods

Mean-Variance Optimization

Markowitz (1952): choose weights $\mathbf{w}$ to maximize

$$\mathbf{w}'\boldsymbol{\mu} - \frac{\gamma}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$$

The **tangency portfolio** maximizes the Sharpe ratio:

$$\mathbf{w}^* \propto \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$$

Pros:

- Theoretically optimal given correct inputs.
- Balances return and risk.

Cons:

Minimum-Variance Portfolio

Set $\boldsymbol{\mu} = \mathbf{0}$ and solve only for the lowest-risk portfolio:

$$\mathbf{w}^* = \arg\min_{\mathbf{w}} \ \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \quad \text{s.t.} \quad \mathbf{w}'\mathbf{1} = 1$$

Pros:

- Requires **only the covariance matrix** — no return forecasts needed.
- Covariances are more stable and easier to estimate than expected returns.
- Historically competitive Sharpe ratios despite ignoring returns.

Cons:

- Concentrates in low-volatility stocks (can be extreme without constraints).
- Still sensitive to estimation error in $\boldsymbol{\Sigma}$ — use shrinkage estimators (Ledoit-Wolf).

Risk Parity

Equalize each asset's **contribution to total portfolio risk**:

$$w_i \times (\mathbf{\Sigma w})_i = \frac{1}{n}\mathbf{w}'\mathbf{\Sigma w} \quad \text{for all } i$$

A simple approximation: $w_i \propto 1/\sigma_i$ (inverse-volatility weighting).

Pros:

- More diversified than minimum variance — no single asset dominates risk.
- No return forecasts needed.
- Robust to estimation error; widely used in practice (e.g., Bridgewater's All Weather).

Cons:

Comparing Portfolio Construction Methods

	Tangency	Min-Variance	Risk Parity
Uses return forecasts	yes	no	no
Uses covariance	yes	yes	yes (or vol only)
Diversification	low	low	high
Estimation sensitivity	very high	moderate	low
Out-of-sample Sharpe	often poor	competitive	competitive

- **If you trust your return forecasts:** tangency portfolio, but regularize heavily (shrinkage, constraints).
- **If you don't trust return forecasts:** minimum variance or risk parity.
- **Hybrid:** use predicted ranks for stock selection (which stocks to hold), and risk-based methods for position sizing (how much of each).

Predicting Volatility and Correlations

All risk-based methods require a covariance matrix $\mathbf{\Sigma}$. How do we estimate it?

- **Sample covariance:** the default, but noisy with many assets.
- **Shrinkage** (Ledoit-Wolf): blend the sample covariance with a structured target. Reduces estimation noise substantially.
- **Factor models** (Barra-style): $\mathbf{\Sigma} = \mathbf{B}\mathbf{\Lambda}\mathbf{B}' + \mathbf{D}$. Reduces dimensionality by modeling common factors.
- **GARCH / DCC:** capture time-varying volatility and correlations.
- **Exponential weighting:** weight recent observations more heavily. Simple and effective.

Drawdowns

What Is a Drawdown?

A **drawdown** is the decline from a portfolio's peak value to a subsequent trough:

$$\text{drawdown}_t = \frac{V_t - V_{\max,t}}{V_{\max,t}}$$

where $V_{\max,t} = \max_{s \le t} V_s$ is the running maximum of portfolio value.

- Always negative (or zero at a new high).
- The **maximum drawdown** is the largest peak-to-trough decline over the sample.
- Drawdown **duration** measures how long it takes to recover to the previous peak.

Why Drawdowns Matter

- The Sharpe ratio treats upside and downside volatility equally. Investors don't.
- A strategy with a high Sharpe ratio but a 60% maximum drawdown is psychologically and practically devastating — investors redeem, managers get fired.
- Drawdowns capture **path-dependent risk** that summary statistics miss.

Reporting:

- Plot the **underwater chart**: drawdown over time. Shows how often and how deeply the portfolio falls below its peak.
- Report maximum drawdown and maximum drawdown duration alongside Sharpe ratios and alphas.

Drawdowns and Strategy Evaluation

- Compare drawdowns across different weight functions, rebalancing frequencies, and cost assumptions.
- A strategy that looks great on Sharpe ratio but has deeper drawdowns than a simpler approach may not be worth the complexity.
- Long-short strategies can have severe drawdowns during short squeezes (e.g., January 2021).
- Value-weighted portfolios typically have **shallower drawdowns** than equal-weighted portfolios (less small-cap exposure).

Key takeaway: always look at the full return path, not just summary statistics. The underwater chart is one of the most important diagnostic plots for any quantitative strategy.