

## **AUTOCORRELATION**

- Correlation of a variable with its own lagged value
- First order autocorrelation = first lag
- Second order autocorrelation = second lag

### **EXAMPLE**

Time	Variable
1	2.1
2	4.5
3	5.3
4	3.2
5	1.6

### **EXAMPLE**

Time	Variable	Lag
1	2.1	
2	4.5	2.1
3	5.3	4.5
4	3.2	5.3
5	1.6	3.2

#### **EXAMPLE**

Time	Variable	Lag
1	2.1	
2	4.5	2.1
3	5.3	4.5
4	3.2	5.3
5	1.6	3.2

Correlation of a variable with its lag is first-order autocorrelation.

#### **SECOND-ORDER AUTOCORRELATION**

Time	Variable	2nd Lag
1	2.1	
2	4.5	
3	5.3	2.1
4	3.2	4.5
5	1.6	5.3

Correlation of a variable with its 2nd lag is second-order autocorrelation.

#### **AUTOCORRELATION FUNCTION**

- The autocorrelation function (acf) shows autocorrelation at multiple lags.
- Lag 0 is usually presented too, even though the correlation at lag 0 is always 100%.
- The band is a confidence interval under the null hypothesis that the autocorrelation is zero.
- Estimates outside the band are statistically significant.

#### **FAMA-FRENCH FACTORS**

- Ask Julius to use pandas datareader to get the monthly Fama-French factors for the maximum history available.
- Ask Julius to plot the acf's of Mkt-RF, HML, and SMB.

# AUTOREGRESSIONS

#### FIRST-ORDER AUTOREGRESSION

A first-order autoregression is the equation

$$x_t = \alpha + \beta x_{t-1} + \varepsilon_t$$

- Positive  $\beta \Leftrightarrow$  positive first-order autocorrelation.
- Positive beta ⇒ momentum.
- Negative beta ⇒ reversals.

#### HIGHER ORDER AUTOREGRESSIONS

A pth order autoregression is the equation

$$x_t = \alpha + eta_1 x_{t-1} + \cdots + eta_p x_{t-p} + arepsilon_t$$

- There are standard methods for choosing the optimal p, trading off goodness of fit and parsimony.
- Ask Julius to fit an AR(p) to HML and find the optimal p.

## **LEVELS OR CHANGES?**

#### WHAT SHOULD WE TRY TO FORECAST?

- Price of AAPL?
- Change in price of AAPL?
- Percent change in price of AAPL (return)?
- A forecast of changes or percent changes implies a forecast of the price and vice versa, so only need to forecast one of them directly.
- What variable should we use in an autoregression?

#### **STATIONARITY**

- One issue is that autocorrelation or autoregression estimates are reliable only for stationary variables.
- Prices grow over time (unstationary).
- Changes in prices become larger over time (in absolute value)
- Returns are the right thing to use in an autoregression.

#### **OTHER VARIABLES**

- Autoregression for crude oil price or change in crude price or percent change in crude price?
- Autoregression for interest rate or change in interest rate or percent change in interest rate?

#### AUTOREGRESSION AND $\Delta x$

Rearrange

$$x_t = \alpha + \beta x_{t-1} + \varepsilon_t$$

as

$$\Delta x_t = lpha + (eta - 1) x_{t-1} + arepsilon_t$$

$$\Delta x_t = (eta-1)\left(x_{t-1} - rac{lpha}{1-eta}
ight) + arepsilon_t$$

#### **MEAN REVERSION**

• An AR(1) is

$$\Delta x_t = (eta-1)\left(x_{t-1} - rac{lpha}{1-eta}
ight) + arepsilon_t$$

- $\beta < 1$  implies reregression towards the mean.
- The mean is  $\alpha/(1-\beta)$ .
- $\beta > 1$  implies nonstationary.

#### **SIMULATIONS**

- Ask Julius to simulate the process x\_t = 1 + 0.5\*x\_{t-1} +
  e\_t by drawing 1,000 standard normals for e\_t. Ask Julius
  to plot the process.
- Repeat for  $x_t = 1 + 2^*x_{t-1} + e_t$

#### **AUTOCORRELATIONS**

- Autocorrelation = correlation of a variable with its lagged value
- First-order autocorrelation =  $corr(x_t, x_{t+1})$
- $p ext{-th order autocorrelation} = \operatorname{corr}(x_t, x_{t+p})$ 
  - E.g., p=12 in monthly data or p=4 in quarterly data because of seasonality

#### **AUTOCORRELATION OF HML**

- Ask Julius to use pandas-datareader to download the monthly Fama-French factors from Ken French's data library.
- Ask Julius to produce a plot of the autocorrelation function (acf) for HML.

#### **AUTOREGRESSIONS**

 An autoregression is a regression of a variable on its own lags:

$$x_t = \alpha + \beta_1 x_{t-1} + \cdots + \beta_p x_{t-p} + \varepsilon_t$$

- Ask Julius to fit an AR(1) for HML.
- Ask Julius to use the AR(1) model to forecast HML for the next 12 months and to plot the last 2 years and the forecasts.

#### **MORE EXAMPLES**

Ask Julius to plot the ACF, fit an AR model, and forecast the following:

- Percent change in crude price: Ask Julius to use pandasdatareader to download crude oil prices starting in 1980 from FRED, to downsample to end-of-month, and to compute percent changes.
- Change in Treasury yields: Ask Julius to use pandasdatareader to download 10-year Treasury yields starting in 1980 from FRED, to downsample to monthly, and to compute changes.

#### **VECTOR AUTOREGRESSION**

- Forecast related variables based on their mutual lags
- Example: does this month's SMB return forecast next month's HML return?
- Ask Julius to run a VAR(1) on the Fama-French factors and provide a summary of the results.