Module 2: Portfolio Optimization and Company Valuation

MGMT 675: Generative AI for Finance

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Learning Objectives

1. Construct optimal portfolios with real-world constraints using AI
2. Build two-stage DCF models driven by a small set of assumptions
3. Perform one-way and two-way sensitivity analysis
4. Run Monte Carlo simulations to quantify valuation uncertainty

These are core tools of investment management and corporate finance. AI handles the mathematics and code. **You focus on the assumptions and the interpretation.**

Portfolio Optimization

The Problem

Given expected returns, risks (standard deviations), and correlations for multiple assets, plus a risk-free rate:

- What is the **best portfolio**?
- What does the set of all possible portfolios look like?
- How do real-world constraints change the answer?

Mean-variance analysis provides the framework. AI handles the computation.

Key Concepts

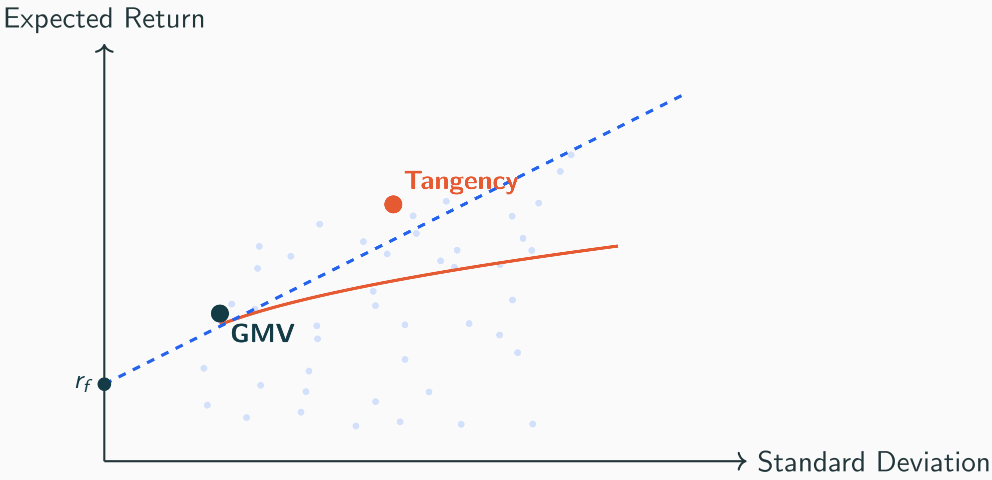
Portfolios

- **Tangency portfolio:** the risky portfolio with the highest Sharpe ratio (excess return per unit of risk)
- **Global minimum variance (GMV):** the risky portfolio with the lowest possible risk
- **Efficient frontier:** the set of risky portfolios with the highest return for each level of risk

Capital Allocation Line

- The straight line from the risk-free rate through the tangency portfolio
- Represents all combinations of the risk-free asset and the tangency portfolio
- The best attainable risk-return tradeoff

Visualizing the Opportunity Set



The cloud shows random portfolios. The orange curve is the efficient frontier. The dashed line

Adding Real-World Constraints

Common constraints

- No short sales (all weights ≥ 0)
- Maximum position (e.g., $\leq 40\%$ per asset)
- Minimum position (e.g., $\geq 2\%$ if held)
- Margin requirement (sum of absolute values ≤ 2)

What happens

- The efficient frontier shifts **inward** (lower returns and/or higher risk)
- The tangency portfolio changes
- The Sharpe ratio cannot improve with constraints—it can only stay the same or get worse
- The constrained frontier is still useful: it reflects what you can actually implement

Solution Methods

- **Solver (numerical optimization)**
  - Maximize Sharpe ratio (tangency portfolio)
  - Minimize risk for a target return (efficient frontier)
  - Minimize risk (global minimum variance portfolio)
- **Analytic/algebraic solution**
  - Solve systems of linear equations for tangency, GMV, and frontier
  - Available only when there are no constraints

Python solver options: `scipy.minimize`, `cvxopt`, `cvxpy`—AI chooses and writes the code.

Ask Claude

Discuss the advantages and disadvantages of these solver options for mean-variance analysis with inequality constraints.

Exercise: Portfolio Cloud

Consider three assets with the following characteristics:

	Asset A	**Asset B**	**Asset C**
Expected return	8%	12%	15%
Standard deviation	14%	20%	26%

Correlations: $\rho_{AB} = 0.3, \quad \rho_{AC} = 0.1, \quad \rho_{BC} = 0.5$

Assume a risk-free rate of 4%. Ask Claude to:

1. Generate 10,000 random portfolios (random weights summing to 1, no short sales) and plot each portfolio's expected return vs. standard deviation.
2. Overlay the efficient frontier and the capital allocation line on the same plot

Company Valuation: DCF

The Big Picture

Discounted cash flow (DCF) valuation estimates the value of a company's operations by forecasting future free cash flows and discounting them to the present.

- A small set of **assumptions** drives the entire model
- AI builds the pro forma financial statements and computes free cash flow
- You focus on **choosing the right assumptions** and **interpreting the output**

The Assumption Set

A pro forma model is driven by a small number of assumptions. Everything else is computed from these.

Assumption	**Driver**
Sales growth rate	% per year
COGS	% of sales
SG&A	Base amount + % of sales
Net working capital (NWC)	% of sales
PP&E	% of sales
Net other operating assets	% of sales
Depreciation	% of PP&E
Tax rate	% of pre-tax income

Capital expenditures are not assumed directly. **Cap ex = target PP&E − prior PP&E + depreciation**, i.e., whatever is needed to hit the PP&E target after

Pro Forma Income Statement

Line Item	**How It's Computed**
Sales	Prior sales $\times$ (1 + growth rate)
COGS	COGS% $\times$ sales
Gross profit	Sales − COGS
SG&A	Base + SG&A% $\times$ sales
Depreciation	Depr% $\times$ PP&E
EBIT	Gross profit − SG&A − depreciation
Taxes	Tax rate $\times$ EBIT
NOPAT	EBIT − taxes

We use **NOPAT** (net operating profit after tax) rather than net income because DCF values the *operations* of the firm, independent of how they are financed.

Pro Forma Balance Sheet and Cap Ex

Balance Sheet Items

- **NWC** = NWC% × sales
- **PP&E** = PP&E% × sales
- **Net other OA** = Net other OA% × sales
- Change in each = current − prior year

Capital Expenditures

- Target $\text{PP\&E}_t = \text{PP\&E\%} \times \text{sales}_t$
- $\text{Depreciation}_t = \text{depr\%} \times \text{PP\&E}_{t-1}$
- **$\text{Cap ex}_t = \text{PP\&E}_t - \text{PP\&E}_{t-1} + \text{depr}_t$**

Free Cash Flow to the Firm

$$\text{FCF} = \text{NOPAT} + \text{Depreciation} - \text{Cap Ex} - \Delta\text{NWC} - \Delta\text{Net Other OA}$$

Two-Stage DCF

Two-Stage DCF: Overview

Stage 1
Explicit Forecast
(e.g., 5–10 years)

Stage 2
Terminal Value
(perpetuity)

Enterprise Value
= PV(Stage 1) + PV(Stage 2)

- **Stage 1**: Project FCF each year from pro forma assumptions
- **Stage 2**: Terminal value = FCF_{T+1} / (WACC − g), where g is the long-run growth rate
- **Equity value** = Enterprise value − net debt (discount at the WACC)

Terminal Value

The terminal value typically accounts for **60–80%** of total enterprise value. Getting it right matters more than the explicit forecast.

Growing Perpetuity

- TV = FCF_{T+1} / (WACC $- g$)
- FCF_{T+1} = year T FCF $\times (1 + g)$
- g = long-run nominal growth (typically 2–3%)
- Requires $g <$ WACC

Exit Multiple

- TV = EBITDA_T $\times$ exit multiple
- Multiple from comparable firms
- Common in practice (M&A, PE)
- Cross-check against perpetuity method

Sensitivity Analysis

Sensitivity Tables

A sensitivity table shows how the output (e.g., equity value per share) changes as you vary one or two key inputs.

One-Way Table

- Vary a single input (e.g., WACC from 8% to 12%)
- Hold everything else constant
- Shows which inputs matter most

Two-Way Table

- Vary two inputs simultaneously
- Classic: WACC vs. terminal growth rate
- Also useful: sales growth vs. COGS margin
- Reveals interaction effects

Ask Claude to produce sensitivity tables in Excel (with formulas) or as formatted output. In Excel, the Data Table feature automates two-way tables.

Monte Carlo Simulation

Why Simulate?

- Sensitivity tables vary one or two inputs; in reality, **all assumptions are uncertain simultaneously**
- Monte Carlo simulation draws random values for each assumption, computes FCF and enterprise value, and repeats thousands of times
- The result: a **distribution** of enterprise values, not a single point estimate

Simulation Setup

Assign a probability distribution to each uncertain assumption, then sample and compute.

Assumption	Distribution	Example
Sales growth	Normal	$\mu = 8\%$, $\sigma = 3\%$
COGS %	Normal	$\mu = 60\%$, $\sigma = 2\%$
Terminal growth	Uniform	1.5% to 3.5%
WACC	Normal	$\mu = 10\%$, $\sigma = 1\%$

- Run 10,000 simulations $\rightarrow$ 10,000 enterprise values
- Report mean, median, 10th/90th percentiles
- Plot a histogram and identify which assumptions drive the most variance (tornado chart)

Building These Analyses with AI

Multiple Ways to Build with AI

Platform	What to Ask
Chat	"Build a two-stage DCF model for a company with these assumptions ... Generate an Excel file with pro formas, FCF, and a sensitivity table."
Cowork	Point Claude at a folder with data files. "Read the financial data, build pro formas, run a DCF, and save the results to Excel."
Code	"Build a DCF model in Python. Fetch Apple's financials from FMP, estimate assumptions from historical data, and run a Monte Carlo simulation."
Excel add-in	Open a blank workbook. "Build a two-stage DCF with pro forma statements, a sensitivity table for WACC vs. terminal growth, and a tornado chart."

Key difference: Chat and Cowork generate spreadsheets via Python (no internet). Code mode can **fetch live data** from APIs before building the model.

The same analysis that takes hours with traditional tools takes minutes with AI.

Describing the problem clearly—choosing the right assumptions, interpreting the output—is the hard part. The computation is delegated entirely.

Exercises

Exercise 1: Tangency Portfolio

- Download data for 5 ETFs + T-bill rate
- Ask AI to compute the tangency portfolio: portfolio weights, expected return, standard deviation, and Sharpe ratio
- Plot the efficient frontier and capital allocation line
- Add a no-short-sales constraint and compare: how do the weights, Sharpe ratio, and frontier change?
- Submit the data, plots, and a short paragraph on how constraints changed the result

Exercise 2: Portfolio Cloud Artifact

Using the three-asset data from earlier:

	Asset A	**Asset B**	**Asset C**
Expected return	8%	12%	15%
Standard deviation	14%	20%	26%

Correlations: $\rho_{AB} = 0.3, \quad \rho_{AC} = 0.1, \quad \rho_{BC} = 0.5; \quad r_f = 4\%$

Ask Claude to build an **interactive artifact** that generates 10,000 random portfolios, overlays the efficient frontier, and marks the tangency and GMV portfolios. Publish and submit the link.

Exercise 3: DCF Model

Build a two-stage DCF for a hypothetical company:

- Current sales: \$500M; sales growth: 10% (years 1–5), 3% terminal
- COGS: 58% of sales; SG&A: \$20M + 12% of sales
- NWC: 15% of sales; PP&E: 40% of sales; net other OA: 5% of sales; depreciation: 10% of PP&E
- Tax rate: 25%; WACC: 9%; net debt: \$200M; shares: 50M

Produce an Excel workbook with pro forma income statement, balance sheet items, FCF calculation, enterprise value, per-share value, and a two-way sensitivity table (WACC vs. terminal growth rate).

Exercise 4: DCF with Live Data

- In Code mode, ask Claude to fetch a real company's financials from Financial Modeling Prep (or Alpha Vantage)
- Estimate each assumption from historical patterns (trailing averages)
- Build a 5-year pro forma and run a two-stage DCF
- Compare your AI-generated valuation to the current market cap
- Submit the code output + a short memo explaining the comparison

Note: This exercise requires Code mode (internet access for the API call).

Exercise 5: Monte Carlo Simulation

- Take your DCF model (from exercise 3 or 4)
- Assign distributions to 4 key assumptions:
  - Sales growth, COGS %, terminal growth, WACC
  - Choose normal or uniform distributions with reasonable parameters
- Run 10,000 simulations
- Report: mean, median, 10th and 90th percentiles
- Plot: histogram of enterprise values + tornado chart
- Discuss: which assumption drives the most variation?