

OPTIMAL PORTFOLIOS THE EASY WAY

WE DON'T NEED A NUMERICAL SOLVER

- We can find the tangency portfolio without using any type of numerical solver (if short sales are allowed).
- We don't need cvxopt.
- We do need to solve some equations, but that is fairly easy.
- Can do similar for GMV portfolio and frontier.

EQUATIONS TO SOLVE

Solve

$$egin{pmatrix} \operatorname{var}_1 & \cdots & \operatorname{cov}_{1n} \ dots & dots & dots \ \operatorname{cov}_{1n} & \cdots & \operatorname{var}_n \end{pmatrix} egin{pmatrix} w_1 \ dots \ w_n \end{pmatrix} = egin{pmatrix} \mu_1 - r_f \ dots \ \mu_n - r_f \end{pmatrix}$$

- The solution minimizes (1/2) * variance minus risk premium. It is on the capital allocation line.
- Divide by sum of weights to get tangency portfolio.

SOLVE IN EXCEL

- The equations are solved in Excel with the MINVERSE and MMULT functions.
- MMULT(RPREM, MINVERSE(COV))
- RPREM should be in a row
- Result will be in a row

GLOBAL MINIMUM VARIANCE PORTFOLIO

Solve

$$egin{pmatrix} \operatorname{var}_1 & \cdots & \operatorname{cov}_{1n} \ dots & dots & dots \ \operatorname{cov}_{1n} & \cdots & \operatorname{var}_n \end{pmatrix} egin{pmatrix} w_1 \ dots \ w_n \end{pmatrix} = egin{pmatrix} 1 \ dots \ 1 \end{pmatrix}$$

Divide by sum of weights

FRONTIER (HYPERBOLA)

Solve

$$egin{pmatrix} \operatorname{var}_1 & \cdots & \operatorname{cov}_{1n} \ dots & dots & dots \ \operatorname{cov}_{1n} & \cdots & \operatorname{var}_n \end{pmatrix} egin{pmatrix} w_1 \ dots \ w_n \end{pmatrix} = egin{pmatrix} \mu_1 \ dots \ \mu_n \end{pmatrix}$$

- Divide by sum of weights
- Put some weight x on the GMV portfolio and 1-x on this portfolio. Vary x and trace out frontier.

EXCEL EXAMPLE

- portfolios.xlsx
- U.S., developed, and emerging from Applied Finance

JULIUS EXAMPLE

- Ask Julius to use yfinance to get Yahoo adjusted closing prices for
 - SPY = S&P 500
 - VBR = Vanguard small-cap value
 - IEF = Treasury bonds
 - UUP = U.S. dollar bullish
- Ask Julius to downsample prices to end-of-month and compute monthly returns as percent changes in the downsampled prices.

- Ask Julius to compute means and covariance matrix as numpy arrays.
- Tell Julius the risk-free rate is 5% and ask Julius to compute the risk premia.
- Ask Julius to multiply the risk premia by the inverse of the covariance matrix.
- Ask Julius to divide the result by the sum of its elements.