MGMT 675

AI-ASSISTED FINANCIAL ANALYSIS

RICE | BUSINESS
Jones Graduate School of Business

OPTIMAL PORTFOLIOS

EXAMPLES

- From Applied:
  - U.S., U.K., France, Germany, Japan
  - U.S., Developed, Emerging
- ETFs from Yahoo Finance

GOAL SEEK IN PYTHON

- There are several choices in python for optimizing functions.
- The qp function from cvxopt is a very good choice for portfolio optimization.
  - cvxopt = convex optimization
  - qp = quadratic programming

MATRIX MULTIPLICATION

PRIMER/REVIEW OF MATRIX MULTIPLICATION

- Excel's MMULT function
- Example:

$$\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 13 \\ 16 \end{pmatrix}$$

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- (2×2) * (2×1) = (2×1)
- height = height of 1st matrix, width = width of 2nd matrix

ANOTHER EXAMPLE

- (3×2) * (2×2) = (3×2)

$$\begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 13 & 10 \\ 16 & 12 \\ 19 & 14 \end{pmatrix}$$

ANOTHER EXAMPLE

- $(3 \times 2) * (2 \times 2) = (3 \times 2)$

$$\begin{pmatrix} 1 & 2 \\ \color{red}{2} & \color{red}{2} \\ 3 & 2 \end{pmatrix} \begin{pmatrix} \color{red}{3} & 2 \\ \color{red}{5} & 4 \end{pmatrix} = \begin{pmatrix} 13 & 10 \\ \color{red}{16} & 12 \\ 19 & 14 \end{pmatrix}$$

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CVXOPT

MATRICES IN CVXOPT: PORTFOLIO VARIANCE

- $w^\top \mathrm{Cov}\, w =$ portfolio variance =

$$\begin{pmatrix} w_1 & \cdots & w_n \end{pmatrix} \begin{pmatrix} \mathrm{var}_1 & \cdots & \mathrm{cov}_{1n} \\ \vdots & \vdots & \vdots \\ \mathrm{cov}_{1n} & \cdots & \mathrm{var}_n \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$

- $(1 \times n)$ * $(n \times n)$ * $(n \times 1)$ = (1×1)
- $P =$ covariance matrix in cvxopt notation

MATRICES IN CVXOPT: EQUALITY CONSTRAINTS

- Weights sum to 1 and expected return = target is expressed as:

$$\begin{pmatrix} 1 & \cdots & 1 \\ \mu_1 & \cdots & \mu_n \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} = \begin{pmatrix} 1 \\ \mu_{\text{target}} \end{pmatrix}$$

- $Aw = b$ in cvxopt notation (Julius should supply A and b)

MATRICES IN CVXOPT: INEQUALITY CONSTRAINTS

- No short sales is equivalent to:

$$\begin{pmatrix} -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -1 \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} \leq \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

- $Gw \leq h$ in cvxopt notation (Julius should supply G and h)

MATRICES IN CVXOPT: RISK PREMIUM

- Risk premium of a portfolio is

$$\begin{pmatrix} \mu_1 - r_f & \cdots & \mu_n - r_f \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$

- $q^\top w$ in cvxopt notation (Julius should supply q)

FRONTIER AND TANGENCY PORTFOLIOS

FRONTIER PORTFOLIOS W/O RISK-FREE ASSET

- Frontier $\sim$ hyperbola in mean/std dev space
- Minimize variance subject to expected return = target and weights sum to 1
- Vary target expected return and trace out frontier

TANGENCY PORTFOLIO

- Include risk-free asset
- Minimize variance minus risk premium
- Produces portfolio on capital allocation line
- Divide by sum of weights to get tangency portfolio

EXAMPLE 1: U.S., U.K., FRANCE, GERMANY, AND JAPAN

DATA

- Download international.xlsx and international_corrs.xlsx from the course website.
- Upload the files to Julius. Ask Julius to read them.
- Ask Julius to convert the expected returns, standard deviations, and correlation matrix into separate numpy arrays.
- Ask Julius to compute the covariance matrix from the standard deviations and correlation matrix.

FRONTIER PORTFOLIOS (W/O RISK-FREE ASSET)

- Ask Julius to use cvxopt to minimize variance subject to achieving a target expected return.
- Tell Julius whether you want to allow short sales. To allow short sales, tell Julius there are no inequality constraints.
- Ask Julius to repeat for a range of target expected returns and to plot the expected returns and standard deviations.

TANGENCY PORTFOLIO

- Give Julius a number for the risk-free rate.
- Ask Julius to minimize the variance minus the risk premium.
- Tell Julius whether you want to allow short sales.

- Ask Julius to divide by the sum of the weights to compute the tangency portfolio.
- Ask Julius to include the tangency portfolio and the capital allocation line on the previous plot.

EXAMPLE 2: U.S., DEVELOPED, AND EMERGING

U.S., DEVELOPED, AND EMERGING

- Start a new chat.
- Upload us_developed_emerging_rets.xlsx and ask Julius to read it.
- Ask Julius to compute the sample means, sample standard deviations and sample correlation matrix as numpy arrays.
- Repeat the frontier and tangency portfolio calculations.

EXAMPLE 3: ETFS

ETFS FROM YAHOO

- Example: ask Julius to use yfinance to get Yahoo adjusted closing prices for
  - SPY = S&P 500
  - VBR = Vanguard small-cap value
  - IEF = Treasury bonds
  - UUP = U.S. dollar bullish
- Ask Julius to downsample prices to end-of-month and compute monthly returns as percent changes in the downsampled prices.

- Ask Julius to compute means, standard deviations, and correlation matrix as numpy arrays.
- Ask Julius to find frontier of risky assets and tangency portfolio as before.