



#### **TOPICS**

- Frontier portfolios of risky assets (minimize risk subject to achieving a target expected return)
- Tangency portfolio (maximize Sharpe ratio)
- Estimate means, standard deviations, and correlations from historical returns

#### **GOAL SEEK IN PYTHON**

- There are several choices in python for optimizing functions.
- The qp function from cvxopt is a very good choice for portfolio optimization.
  - cvxopt = convex optimization
  - qp = quadratic programming

#### **EXAMPLES**

- From Applied:
  - U.S., U.K., France, Germany, Japan
  - U.S., Developed, Emerging
- ETFs from Yahoo Finance

# U.S., U.K., FRANCE, GERMANY, AND JAPAN

#### DATA

- Download international.xlsx and international\_corrs.xlsx from the course website.
- Upload the files to Julius. Ask Julius to read them.
- Ask Julius to convert the means, standard deviations, and correlation matrix into numpy arrays.
- Ask Julius to compute the covariance matrix as a numpy array.

### FRONTIER PORTFOLIOS (W/O RISK-FREE ASSET)

- Ask Julius to use cvxopt to minimize variance subject to achieving a target expected return.
  - If allowing short sales, tell Julius there are no inequality constraints.
  - If excluding short sales, tell Julius to not allow short sales.
- Ask Julius to repeat for a range of target expected returns and to plot the expected returns and standard deviations.

## DIGRESSION ON CYXOPT

#### FRONTIER PORTFOLIOS ALLOWING SHORT SALES

- minimize (1/2)x'Px subject to Ax = b
- P = covariance matrix. x'Px is portfolio variance
- A = array with two rows. First row = np.ones(n). Second row = asset expected returns.
- b = np.array([1, targ])
- Ax = b means weights sum to 1 and expected return = targ.
- Julius should figure all of this out.

#### FRONTIER PORTFOLIOS EXCLUDING SHORT SALES

- minimize (1/2)x'Px subject to Ax=b and  $Gx\leq h$
- $\bullet$  P, A, and b as before
- G = np.eye(n) and h = np.zeros(n)
- $Gx \leq h$  means weights are nonnegative (no shorts)
- Again, Julius should figure this out.

## BACK TO EXAMPLE

#### **TANGENCY PORTFOLIO**

- Give Julius a number for the risk-free rate.
- Ask Julius to minimize the variance minus the risk premium.
  - If allowing short sales, tell Julius there are no inequality constraints.
  - If excluding short sales, tell Julius to not allow short sales.

- Ask Julius to divide by the sum of the weights to compute the tangency portfolio.
- Ask Julius to include the tangency portfolio and the capital market line on the previous plot.

## ANOTHER DIGRESSION ON CYXOPT

#### **CVXOPT FOR TANGENCY PORTFOLIO**

- Minimize x'Px q'x
  - P = 2 times covariance matrix (but the 2 is not important)
  - $\blacksquare q = \text{risk premia}$
- No equality constraints
- If no short sales, then G=- np.eye(n) and h= np.zeros(n)
- Then divide by the sum of weights.

## US, DEVELOPED, AND EMERGING

- Start a new chat.
- Upload us\_developed\_emerging\_rets.xlsx and ask Julius to read it.
- Ask Julius to compute the sample means, sample standard deviations and sample correlation matrix as numpy arrays.
- Repeat the frontier and tangency portfolio calculations.

## DATA FROM YAHOO

#### **ETFS FROM YAHOO**

- Example: ask Julius to use yfinance to get Yahoo adjusted closing prices for
  - SPY = S&P 500
  - VBR = Vanguard small-cap value
  - IEF = Treasury bonds
  - UUP = U.S. dollar bullish
- Ask Julius to downsample prices to end-of-month and compute monthly returns as percent changes in the downsampled prices.

- Ask Julius to compute means, standard deviations, and correlation matrix as numpy arrays.
- Ask Julius to find frontier of risky assets and tangency portfolio as before.