Portfolio Optimization

MGMT 675, AI-Assisted Financial Analysis

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Outline

- Two risky assets
- Two risky assets and risk-free asset
- More risky assets
- Without short sales
- Google Colab
- Estimating parameters from historical data

Two Risky Assets

- With w_1 as the weight on the first asset and $w_2 = 1 - w_1$ as the weight on the second,
  - Portfolio expected return is

$$w_1\mu_1 + w_w\mu_2$$

  - Portfolio standard deviation is

$$\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho}$$

- Tell Julius expected returns, risks, and correlation.
- Ask Julius to plot the expected returns and standard deviations of portfolios.

With a Risk-Free Asset

- With a risk-free asset, the efficient portfolios are those that plot on the line through the risk-free rate and the tangency portfolio (called the capital market line or capital allocation line).
- Ask Julius:
    - Suppose there is a risk-free asset earning whatever percent. Calculate the tangency portfolio.
    - Add the capital allocation line to the plot.

More Risky Assets

- Can use python cvxopt library as a solver.
- Input expected returns, standard deviations, and correlation matrix.
- Tell Julius directly or upload files (we'll upload correlation matrix).
- Ask Julius to create the expected returns, standard deviations, and correlation matrix as numpy arrays.
- Ask Julius to compute the covariance matrix.

Example

- Tell Julius that GLD, IEF, LQD, SPY, and UUP have expected returns of 7.7%, 3.5%, 4.4%, 10.5%, and 1.7% and standard deviations of 17.1%, 6.8%, 8.5%, 15.9%, and 7.9%.
- Ask Julius to read the correlation matrix from correlations.csv.
- Ask Julius to convert the expected returns and standard deviations and the correlation matrix into numpy arrays.
- Ask Julius to compute the covariance matrix as a numpy array.

More Prompts

- Use the cvxopt library to find the minimum variance portfolio that satisfies two linear equations: the expected return is equal to 8% and the weights sum to 1. There are no inequality constraints.
- Repeat this for expected returns varying between 3% and 15%. Plot the expected returns and standard deviations of the minimum risk portfolios.
- Add the expected returns and risks of the five assets to the plot. Label the assets as GLD, IEF, LQD, SPY, and UUP.

Tangency Portfolio

- The Sharpe ratio is a somewhat (not too much but a little) complicated function of the portfolio weights.
- A simpler function is the risk premium minus the variance.
- One way to find a tangency portfolio is to minimize the variance minus the risk premium.
- Then divide by the sum of the weights, so the revised weights sum to 1. This is the tangency portfolio.

- Give Julius a number for the risk-free rate.
- Ask Julius to find the portfolio that minimizes the variance minus the risk premium. Tell Julius that there are no equality constraints and no inequality constraints.
- Ask Julius to add the tangency portfolio to the plot, and include the capital allocation line in the plot.

Excluding Short Sales

- Portfolio optimization is often done for strategic asset allocation: choosing weights on asset classes to be held for a long time.
- People generally don't want to short sell in this circumstance.
- To exclude short sales, we can repeat everything telling Julius to add constraints that the weights be nonnegative.
- Let's just do the tangency portfolio again.
    - Ask Julius to find the portfolio that minimizes the variance minus the risk premium when the weights have to be nonnegative.
    - Tell Julius that there are no equality constraints.
    - Ask Julius to scale the portfolio by dividing by the sum of the weights.

Google Colab

- Go to https://colab.research.google.com/ and sign in.
- Open a new notebook.
- Copy and paste code from Julius into the notebook cells.
- Run the notebook.
- Save the notebook to your Google Drive.

Estimate Parameters from Historical Data

- Can upload our own data or get online data.
- Example: get Yahoo adjusted closing prices and compute returns for
    - SPY = S&P 500
    - VBR = Vanguard small-cap value
    - IEFA = iShares EAFE
    - IEMG = iShares emerging markets

- Ask Julius to use yfinance to get adjusted closing prices for SPY, VBR, IEFA, and IEMG from Yahoo.
- Ask Julius to drop rows with missing data and downsample the data to end-of-month.
- Ask Julius to compute monthly returns as percent changes in the downsampled prices.
- Ask Julius to compute the means and covariance matrix as numpy arrays.
- Ask Julius to compute the tangency portfolio using cvxopt.
    - Give Julius a number for the monthly risk-free rate.
    - Ask Julius to find the portfolio that minimizes the variance minus the risk premium.
    - Tell Julius that there are no equality constraints and no inequality constraints.
    - Ask Julius to scale the portfolio by dividing by the sum of the weights.

Colab again

- Copy and paste the code into a new Colab notebook and save it to Drive.