

# Mean-Variance Optimization

MGMT 675: AI-Assisted Financial Analysis



- Finding the capital allocation line by solving equations
- Finding the hyperbola of efficient portfolios of only risky assets
- Excel example
- Julius workflow

# **Solving Equations for Mean-Variance Optimization**

---

# Mean-variance optimization

- The following are essentially equivalent procedures:
  1. Maximize Sharpe ratio = risk premium / risk
  2. Maximize expected return subject to not exceeding a risk limit
  3. Minimize risk subject to achieving a target expected return
  4. Maximize expected return - penalty parameter times variance
- We can run a solver or solve equations to get the solutions.

- Suppose  $y = x^2 - 2x + 5$  and you want to find the value of  $x$  that minimizes  $y$ .
- The minimum occurs at the bottom of the curve, where the slope is zero.
  - The slope is the derivative, and the derivative is  $dy/dx = 2x - 2$ .
  - So, the slope is zero when  $2x - 2 = 0$ , which implies  $x = 1$ .
- Bottom line: can solve an equation to find the minimum.

# Equations that solve mean-variance problems

- $w_i$  = weight,  $\bar{r}_i$  = expected return,  $r_f$  = risk-free rate,  $\sigma_i^2$  = variance,  $\sigma_{ik}$  = covariance.
- 3 asset example: 3 equations in 3 unknowns  $w_1, w_2, w_3$

$$\sigma_1^2 w_1 + \sigma_{12} w_2 + \sigma_{13} w_3 = \bar{r}_1 - r_f$$

$$\sigma_{21} w_1 + \sigma_2^2 w_2 + \sigma_{23} w_3 = \bar{r}_2 - r_f$$

$$\sigma_{31} w_1 + \sigma_{32} w_2 + \sigma_3^2 w_3 = \bar{r}_3 - r_f$$

# Matrix multiplication

This is not essential, but it will help to understand code.

- Three equations are represented as

$$\Sigma w = \bar{r} - r_f$$

where  $\Sigma = 3 \times 3$  array of covariances and variances,  
 $w = (w_1, w_2, w_3)$  and  $\bar{r} - r_f = (\bar{r}_1 - r_f, \bar{r}_2 - r_f, \bar{r}_3 - r_f)$

- Solution is represented as

$$w = \Sigma^{-1}(\bar{r} - r_f)$$

# Tangency and other portfolios

- Given solution  $(w_1, w_2, w_3)$  of the equations, divide by sum of  $w_i$  to get tangency portfolio
- Given tangency portfolio,
  - Given risk limit (standard deviation), optimal portfolio satisfying the risk limit is

$$\frac{\text{risk limit}}{\text{std dev of tangency portfolio}} \times \text{tangency portfolio.}$$

- Given target expected return, optimal portfolio achieving the target is

$$\frac{\text{target risk premium}}{\text{risk prem of tangency portfolio}} \times \text{tangency portfolio}$$



## Excel Example

---

# Excel Spreadsheet Example

Excel Example linked on Schedule page

based on Applied Finance Topic5.1\_SharpeRatioExamples.xlsx

## **Risky Assets Only**

---

# Finding the Markowitz Bullet (Hyperbola)

- We can also solve equations to find
  - The global minimum variance portfolio
  - Another portfolio of risky assets on the hyperbola
- And all portfolios on the hyperbola are combinations of those two portfolios.
- However, Julius will probably run a solver if you ask it to find a portfolio of risky assets with weights summing to 1 that minimizes variance subject to a target expected return.

# Julius Workflow

---

# Julius Workflow

- User prompt:
  - Asset names, expected returns, and covariances
  - Risk-free rate
- Julius prompt:
  - Compute the tangency portfolio and output to the user
  - Compute the expected return and standard deviation of the tangency portfolio and output to the user
- Optional additional prompt for Julius:
  - Compute the minimum risk portfolio with weights that sum to 1 (fully invested in risky assets) for various expected return targets.
  - Plot the means and standard deviations of the minimum risk portfolios.
  - Show the tangency portfolio and capital allocation line on the plot.
  - Display the figure and return a jpeg of it.

# Checking Julius' Work

- Run the workflow with data from Topic5.1\_SharpeRatioExamples.xlsx from Applied Finance (linked on our Schedule page for Day 1).
- Check that the tangency portfolio is correct.
- If it is correct, copy the code generated by Julius and paste it into the Julius prompt cell in the workflow. Recommend that Julius use the code.