Fitting spatial stochastic frontier models in Stata

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Abstract. In this article, we introduce a new command `xtsfsp` for fitting spatial stochastic frontier models in Stata. Over the last decades, an important theoretical progress of stochastic frontier models is the incorporation of various types of spatial components. Models with the ability to account for spatial dependence and spillovers have been developed for efficiency and productivity analysis, drawing extensive attention from industry and academia. Due to the unavailability of the statistical packages, the empirical applications of the new stochastic frontier models appear to be lagging. The `xtsfsp` command provides a routine for estimating the spatial stochastic frontier models in the style of Orea and Álvarez (2019) and Galli (2022), enabling users to handle different sources of spatial dependence. In the presented article, we introduce the spatial stochastic frontier models, describe the syntax and options of the new command, and provide several examples to illustrate its usage.

Keywords: stochastic frontier models, SFA, spatial dependence, technical efficiency, spillovers

1 Introduction

Producers might fail in optimizing their production activities, causing deviation from the maximum output or the minimum cost. Economic researchers proposed the concept of technical efficiency, which measures how well a producer is utilizing its resources to produce goods or services. A technically efficient organization makes the maximum outputs given the amount of inputs or uses the minimum amount of inputs to produce a given level of output. On the contrary, technically inefficient organization produce fewer outputs given the same inputs or uses more inputs than necessary to produce the same output. Technical efficiency is important because it allows organizations or economies to achieve their goals with the least amount of resources possible, which can lead to cost savings and increased profitability.

Aigner et al. (1977) and Meeusen and van Den Broeck (1977) introduced stochastic frontier models for evaluating technical efficiency. The essential concept behind these models is to divide the observed output of a production process into two components, namely the "frontier" output, signifying the maximum feasible output, given the inputs utilized in the production process, and the "residual" output, denoting the inefficiency in the production process. Following these initial works, stochastic frontier models gained extensive use as a tool for scrutinizing productivity and efficiency.

Methodologically, econometricians have expanded the horizons of stochastic frontier models in various directions. To name a few, Battese and Coelli (1995) incorporated the determinants of inefficiency. Wang (2003) developed the stochastic frontier model with scaling properties to capture the shape of the distribution of inefficiency. Greene (2005) extended the stochastic models with the random effects and the "true" fixed effects. Belotti and Ilardi (2018), Chen et al. (2014), and Wang and Ho (2010) circumvented the "incidental parameters problem" in the fixed effects stochastic frontier model through model transformation. Karakaplan and Kutlu (2017) developed an endogenous stochastic frontier model to control for the endogeneity in the frontier or inefficiency.

In recent years, stochastic frontier models have undergone further extension to account for spatial dependence and spatial spillover effects. Glass et al. (2016) constructed a spatial Durbin stochastic model considering both global and local spatial dependence. Kutlu et al. (2020) proposed a spatial stochastic frontier model with endogenous frontier and environmental variables. Glass et al. (2016) and Kutlu et al. (2020) combine the concepts of spatial econometrics and stochastic frontier analysis by including the spatial lag of the dependent variable. On the other hand, Orea and Álvarez (2019) developed a new stochastic frontier model with spatial correlation in both noise and inefficiency terms. Galli (2022) integrated the two different modeling ideas to specify four different sources of spatial dependence fully.

With the increasing demand in the last decades to analyze technical efficiency, Stata provides official commands `frontier` and `xtfrontier` for cross-sectional and panel stochastic model estimation, respectively. Belotti et al. (2013) developed `sfcross` and `sfpanel` commands accommodating more different distribution assumptions and allowing fixed-effect and random-effect models with the consideration of heteroscedasticity. Karakaplan (2017) introduced the `sfkk` command for estimating endogenous stochastic frontier models. Karakaplan (2018) supplemented the `xtsfkk` command for fitting the endogenous panel stochastic frontier model. Fé and Hofler (2020) provided the `sfcount` for fitting fit count-data stochastic frontier models. Lian et al. (2023) developed the `sftt` command for fitting two-tier stochastic frontier models. Kumbhakar et al. (2015) provides a practitioner's guide to stochastic frontier analysis with a suite of Stata commands (including `sfmodel`, `sfpan`, `sf_fixeff`, and `sfprim`).

In this article, we introduce `xtsfsp`, a new command for fitting spatial stochastic frontier models in the style of Orea and Álvarez (2019) and Galli (2022). The proposed `xtsfsp` command not only allows getting more accurate inefficiency scores (see e.g. Orea et al. 2018) but also examining relevant economic issues that a non-spatial stochastic frontier model tends to overlook. For instance, in microdata applications, the new command can be used to test whether the production/cost function can be viewed as a purely deterministic (engineering) process where the firm controls all the inputs (see e.g. Druska and Horrace 2004). A distinctive feature of the `xtsfsp` command is that it allows estimating a stochastic frontier model with cross-sectional correlation in the inefficiency term, a specification that is useful in applications where some firms benefit from best practices implemented in adjacent firms due to, for instance, agglomeration economies, knowledge spillovers, technology diffusion or R&D spillovers. This could especially be the case if (local) firms belong to communitarian networks (e.g. cooperatives) or common technicians (consultants) are advising all local firms. In practice, the proposed `xtsfsp` command can be useful to capture a kind of behavioral correlation. For instance, when firms tend to "keep an eye" on the decisions of other peer firms trying to overcome the limitations caused by the lack of information or they simply emulate each other. It is finally germane to mention that the `xtsfsp` command also allows capturing cross-sectional effects that might be caused by non-spatial factors (e.g., the regulation environment) if we define appropriately the so-called weight (W) matrix. A proper definition of the W matrix might, for instance, allow us to examine the existence of knowledge spillovers from supplier and user firms.

As Orea and Álvarez (2019) point out, the proposed `xtsfsp` command can be implemented using macro-level data (e.g. data of countries, regions or industries) due to the abundant evidence of important feedback processes between neighboring or non-distant regions justify the use of SAR and Durbin frontier functions in macrodata applications. The spatial weight matrix specification commonly adopted in regional economics is based on geographical distance. However, as aforementioned, the weight matrix can be defined using a non-spatial criterion. In this sense, Liu and Sickles (2023) state that the mode of production in the world economy is characterized by the division of global value chains (GVCs) and, hence, the spatial weight matrix should be constructed using the economic distance between industries within/across national economies. In this case, the proposed `xtsfsp` command can be used to estimate spatial SAR and Durbin frontier functions in order to examine the diffusion of knowledge and technology among the participants in the international production network. It is also makes sense to estimate a stochastic frontier model with cross-sectional correlation in the inefficiency term using macrodata if we change the interpretation of the estimated correlation. In these applications, the spatial correlation in the inefficiency term likely captures barriers and distortions to the efficient allocation of resources across firms that are common to several regions, such as regulation, labor market trends or common institutions (see e.g. Orea et al. 2023).

The remainder of this article unfolds as follows: Section 2 provides a brief description of the models in Orea and Álvarez (2019) and Galli (2022); Section 3 explain the syntax and options of xtsfsp; Section 4 present simulated data examples to illustrate the usage of the command; and Section 5 concludes the article.

2 The model

In this section, we briefly describe the spatial stochastic frontier models developed by Orea and Álvarez (2019) and Galli (2022). The exposition here is only introductory. Please refer to the cited papers for more technical details.

Based on the transposed version of Wang and Ho (2010) model, Orea and Álvarez

(2019) proposed a spatial stochastic frontier model which accommodates spatially-correlated inefficiency and noise terms. The model is formulated as in Eqs.(1)-(3), for $i = 1, ..., N$ and $t = 1, .., T$:

$$Y_{it} = X'_{it}\beta + \tilde{v}_{it} - s\tilde{u}_{it} \tag{1}$$

$$\tilde{v}_{it} = v_{it} + \gamma W_i^{vt}\tilde{v}_{.t} \tag{2}$$

$$\tilde{u}_{it} = u_{it} + \tau W_i^{ut}\tilde{u}_{.t} \tag{3}$$

Eq.(1) describes the stochastic frontier function where Y_{it} is the dependent variable and X_{it} is a $k \times 1$ vector of variables shaping the frontier; $s = 1$ for the production function and $s = -1$ for the cost function [1]; $\tilde{v}_{it}$ and $\tilde{u}_{it}$ represent idiosyncratic noise and inefficiency, respectively. In Eqs.(2) and (3), $W_i^{vt} = (W_{i1}^{vt}, ..., W_{iN}^{vt})$ and $W_i^{vt} = (W_{i1}^{vt}, ..., W_{iN}^{vt})$ are two known $1 \times N$ cross-sectional weight vectors depicting the structure of the cross-sectional relationship for idiosyncratic noise and inefficiency terms, respectively; $\tilde{v}_{.t} = (\tilde{v}_{1t}, ..., \tilde{v}_{Nt})'$ and $\tilde{u}_{.t} = (\tilde{u}_{1t}, ..., \tilde{u}_{Nt})'$; v_{it} is a random variable following the distribution $N(0, \sigma^2_{v,it})$ and $u_{it} = h(Z'_{it}\delta)u^*_t$. $h(Z'_{it}\delta) = \sqrt{exp(Z'_{it}\delta)}$ is the scaling function where Z_{it} is a $l \times 1$ vector of variables affecting individuals' inefficiency and u^*_t is a non-negative random variable following the distribution $N^+(0, 1)$. Different from the original setting in Orea and Álvarez (2019), we assume $\sigma^2_{v,it} = exp(D'_{it}\eta)$ to account for idiosyncratic error variance which depends on a vector of variables D_{it}[2]. Furthermore, we enforce the variance of u^*_t to be equal to one, thus allowing the term Z_{it} to include a constant. Using matrix notation, we can rewrite Eqs.(2) and (3) as

$$\tilde{v}_{.t} = (I_N - \gamma W^{vt})^{-1}v_{.t} \tag{4}$$

$$\tilde{u}_{.t} = (I_N - \tau W^{ut})^{-1}h(Z_{.t}\delta)u^*_t = \tilde{h}_{.t}u^*_t \tag{5}$$

where $Z_{.t} = (Z_{1t}, ..., Z_{Nt})'$;$\tilde{h}_{.t} = (I_N - \tau W^{ut})^{-1}h(Z_{.t}\delta)$.

The above model captures the spatial correlation of the random error and inefficiency terms with the spatial autoregressive (SAR) process [3]. Referring to Wang and Ho (2010), we can obtain the following log-likelihood function for each period t:

$$\begin{aligned} \ln L_t = &- \frac{N}{2}\ln(2\pi) - \frac{1}{2}\ln|\Pi_t| - \frac{1}{2}\tilde{\varepsilon}_{.t}\Pi^{-1}\tilde{\varepsilon}_{.t} \\ &+ \frac{1}{2}\left(\frac{\mu^2_*}{\sigma^2_*}\right) + \ln\left[\sigma_*\Phi\left(\frac{\mu_*}{\sigma_*}\right)\right] - \ln\left(\frac{1}{2}\right) \end{aligned} \tag{6}$$

[1] The real output in the production function falls short of the potential output due to inefficiency, indicated by $s = -1$ and an output loss represented by $-u_{it}$. Conversely, in the cost function, the real cost exceeds the potential cost due to inefficiency, denoted by $s = 1$, with u_{it} signifying the additional cost.

[2] If $D_{it} = 1$, the error term is conditional homoscedasticity.

[3] Orea and Álvarez (2019) also considered a specification of the spatial moving average process.

where $\Pi_t = (I_N - \rho W^{yt})^{-1} diag(\sigma^2_{v,.t})[(I_N - \rho W^{yt})^{-1}]'$, $diag(\sigma^2_{v,.t})$ represents a diagonal matrix with $\sigma^2_{v,.t} = (\sigma^2_{v,1t}, ..., \sigma^2_{v,Nt})'$ as the diagonal elements; $\tilde{\varepsilon}_{.t} = (\tilde{\varepsilon}_{1t}, ..., \tilde{\varepsilon}_{Nt})', \tilde{\varepsilon}_{it} = s(Y_{it} - X'_{it}\beta)$, and

$$\mu_* = \frac{-\tilde{\varepsilon}'_{.t}\Pi_t^{-1}\tilde{h}_{.t}}{\tilde{h}'_{.t}\Pi_t^{-1}\tilde{h}_{.t} + 1} \tag{7}$$

$$\sigma^2_* = \frac{1}{\tilde{h}'_{.t}\Pi_t^{-1}\tilde{h}_{.t} + 1} \tag{8}$$

Galli (2022) further incorporated the spatial lags of the dependent variable and the input variables into Orea and Álvarez (2019) model, which additionally measures global and local spatial spillovers affecting the frontier function. The model is expressed as

$$Y_{it} = \rho W_i^{yt} Y_{.t} + X'_{it}\beta + W_i^{xt} X_{.t}\theta + \tilde{v}_{it} + s\tilde{u}_{it} \tag{9}$$

where $W_i^{yt} = (W_{i1}^{yt}, ..., W_{iN}^{yt})$ and $W_i^{xt} = (W_{i1}^{xt}, ..., W_{iN}^{xt})$ are two known $1 \times N$ cross-sectional weight vectors [4]; $Y_{.t} = (Y_{1t}, ..., Y_{Nt})'$; $X_{.t} = (X_{1t}, ..., X_{Nt})'$. This model gives rise to the following log-likelihood function for each period t:

$$\begin{aligned} \ln L_t =& ln|I_N - \rho W^{yt}| - \frac{N}{2}\ln(2\pi) - \frac{1}{2}\ln|\Pi_t| - \frac{1}{2}\tilde{\varepsilon}_{.t}\Pi^{-1}\tilde{\varepsilon}_{.t} \\ &+ \frac{1}{2}\left(\frac{\mu_*^2}{\sigma_*^2}\right) + \ln\left[\sigma_*\Phi\left(\frac{\mu_*}{\sigma_*}\right)\right] - \ln\left(\frac{1}{2}\right) \end{aligned} \tag{10}$$

where $\tilde{\varepsilon}_{.t} = (\tilde{\varepsilon}_{1t}, ..., \tilde{\varepsilon}_{Nt})', \tilde{\varepsilon}_{it} = s(Y_{it} - X'_{it}\beta - \rho W_i^{yt} Y_{.t} - W_i^{xt} X_{.t}\theta)$.

Summing the time-specific log-likelihood functions over all periods yields the overall likelihood function for the whole sample, i.e., $lnL = \sum_{t=1}^{T} lnL_t$. Then, numerically maximize the overall log-likelihood function to obtain consistent estimates of the parameters in the above models. Specifically, we use Stata `ml model` routine with the `method-d0` evaluator to program the `xtsfsp` command. Following Gude et al. (2018), we parameterize ρ, γ, and τ as Eq.(11) to ensure the standard regularity condition for the spatial autoregressive models.

$$\begin{aligned} \eta &= \left(\frac{1}{r_{\min}}\right)(1-p) + \left(\frac{1}{r_{\max}}\right)p \\ 0 \le p &= \frac{\exp(\delta_0)}{1+\exp(\delta_0)} \le 1 \end{aligned} \tag{11}$$

where η stands for one of ρ, γ, and τ; $r_{\min}$ and $r_{\max}$ are respectively the minimum and maximum eigenvalues of the corresponding spatial weight matrix.

In summary, Galli (2022) provided a fully comprehensive specification of four different types of spatial dependence: global spillovers of dependent variable Y_{it}, local spillovers of input variables X_{it}, cross-sectional correlation of idiosyncratic noise v_{it} and inefficiency u_{it}. We term this full model "yxuv-SAR". Some restrictions can be imposed on the specific parameters to generate the following models (summarized in Table 1), which can be estimated by the `xtsfsp` command.

[4] We index W_i^{yt}, W_i^{xt}, W_i^{ut}, and W_i^{vt} with superscript yt, xt, ut, and vt, respectively. This indicates the spatial weight matrix can be time-varying and different across various spatial components

Table 1: Specific models with restricted parameters

	yuv	xuv	yv	yu	y	xuv	xv	xu	uv	u	v
ρ		0				0	0	0	0	0	0
θ	0		0	0	0				0	0	0
γ				0	0			0		0	
τ			0		0		0				0

Note: 0 indicates the parameter is restricted to be zero such that the corresponding spatial component is removed.

3 The xtsfsp command

`xtsfsp` estimates spatial stochastic frontier models in the style of Orea and Álvarez (2019) and Galli (2022).

3.1 Syntax

Estimation syntax

`xtsfsp` *depvar* [*indepvars*], `uhet(`*varlist*[`,noconstant`]`)`
[`vhet(`*varlist*[`,noconstant`]`)` `cost` `noconstant` `wy(`*wyspec*`)` `wx(`*wxspec*`)`
`wu(`*wuspec*`)` `wv(`*wvspec*`)` `normalize(`*norm_method*`)` `wxvars(`*varlist*`)`
`initial(`*matname*`)` `mlmodel(`*model_options*`)` `mlsearch(`*search_options*`)` `mlplot`
`mlmax(`*maximize_options*`)` `nolog` `mldisplay(`*display_options*`)` `level(`#`)`
`te(`*newvar*`)` `genwxvars` `delmissing` `constraints(`*constraints*`)`]

Version syntax

`xtsfsp , version`

Replay syntax

`xtsfsp` [, `level(`#`)`]

3.2 Options

`uhet(`*varlist*`[,noconstant])` specifies explanatory variables for scaling function depending on a linear combination of *varlist*. Use nonconstant to suppresses constant term.

`vhet(`*varlist*`[,noconstant])` specifies explanatory variables for idiosyncratic error variance function depending on a linear combination of *varlist*. Use nonconstant to suppresses constant term.

`noconstant` suppresses constant term.

`cost` specifies the frontier as a cost function. By default, the production function is assumed.

`wy(`*wyspec*`)` specifies the spatial weight matrix for lagged dependent variable. The expression is wy(W_1 [$W_2 ... W_T$] [,*mata array*]). By default, the weight matrices are spmatrix objects created by Stata Offical command spmatrix. mata declares weight matrices are Mata matrices. If one weight matrix is specified, it assumes a time-constant weight matrix. For time-varying cases, T weight matrices should be specified in time order. Alternatively, using array to declare weight matrices are stored in an array. If only one matrix is stored in the specified array, the time-constant weight matrix is assumed. Otherwise, the keys of the array specify time information, and the values store time-specific weight matrices.

wx(*wxspec*) specifies the spatial weight matrix for lagged independent variable. The expression is the same as wy(*wyspec*).

wu(*wuspec*) specifies the spatial weight matrix for lagged independent variable. The expression is the same as wy(*wyspec*).

wv(*wvspec*) specifies the spatial weight matrix for lagged independent variable. The expression is the same as wy(*wyspec*).

normalize(*norm_method*) specifies one of the four available normalization techniques: row, col, minmax, and spectral.

wxvars(*varlist*) specifies spatially lagged independent variables.

initial(*matname*) specifies the initial values of the estimated parameters with matrix *matname*.

mlmodel(*model_options*) specifies the ml model options.

mlsearch(*search_options*) specifies the ml search options.

mlplot specifies using ml plot to search better initial values of spatial dependence parameters.

mlmax(*maximize_options*) specifies the ml maximize options.

nolog suppresses the display of the criterion function iteration log.

mldisplay(*display_options*) specifies the ml display options.

level(#) sets confidence level; default is level(95).

te(*newvarname*) specifies a new variable name to store the estimates of technical efficiency.

genwxvars generates the spatial Durbin terms. It is activated only when wxvars(*varlist*) is specified.

delmissing allows estimation when missing values are present by removing the corresponding units from spatial matrix.

constraints(*constraints)* specifies linear constraints for the estimated model.

3.3 Dependency of xtsfsp

xtsfsp depends on the moremata package contributed by Jann (2005). If not already installed, you can install it by typing ssc install moremata.

4 Examples

The xtsfsp command described above offers great flexibility, allowing for various specifications of spatial dependence. Specifically, it enables the specification of different combinations of spatial components, with the option to have different and time-varying spatial weight matrices. Furthermore, it allows for the specification of conditional heteroscedasticity of random errors. In this section, we present four examples that demonstrate the usage of the xtsfsp command.

4.1 yxuv-SAR model with time-invariant spatial weight matrices

Referring to Galli (2022), we first consider the *yxuv*-SAR model specified by the following data-generating process (DGP 1) with $i = 1, ..., 300$ and $t = 1, .., 20$,

$$y_{it} = 0.3W_i y_{.t} + 2x_{it} + 0.5W_i x_{.t} + \tilde{v}_{it} - \tilde{u}_{it} \tag{12}$$

where $\tilde{v}_{it}$ and $\tilde{u}_{it}$ are defined as in Eqs.(2) and (3) with $\gamma = 0.3$, $\tau = 0.3$, $Z_{it} = (z_{it}, 1)'$,$\delta = (2, ln(0.2))'$, $D_{it} = 1$ and $\eta = ln(0.2)$. All the spatial matrices for the four spatial components are the same and time-invariant, created from a binary contiguity spatial weight matrix. We generate the exogenous variables X_{it} and z_{it} from the standard normal distribution, respectively. With the sample generated by DGP 1, we can fit the model in the following syntax.

```
. use xtsfsp_ex1.dta

. xtset id t

Panel variable: id (strongly balanced)
 Time variable: t, 1 to 20
         Delta: 1 unit

. * importing spatial weight matrix from xtsfsp_wmat1.mmat
. mata mata matuse xtsfsp_w1.mmat,replace
(loading w1[300,300])

. * fitting the model
. xtsfsp y x, uhet(z) wu(w1,mata) wy(w1,mata) wv(w1,mata) wx(w1,mata) wxvars(x) te(te) nolog

Spatial frontier model(yxuv-SAR)                     Number of obs =      6,000
                                                     Wald chi2(2)  = 113733.00
Log likelihood = -3953.6034                          Prob > chi2   =     0.0000

------------------------------------------------------------------------------
           y | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
frontier     |
           x |   2.006736   .0082953   241.91   0.000     1.990477    2.022994
         W_x |   .5457545   .0709898     7.69   0.000     .4066171    .6848919
       _cons |   1.008833   .0460347    21.91   0.000     .9186065    1.099059
-------------+----------------------------------------------------------------
    /lnsigv2 |  -1.566886   .0184019   -85.15   0.000    -1.602953   -1.530819
-------------+----------------------------------------------------------------
uhet         |
           z |   .9858524   .0385881    25.55   0.000      .910221    1.061484
       _cons |  -1.533232   .3361919    -4.56   0.000    -2.192156   -.8743081
-------------+----------------------------------------------------------------
Wy           |
       _cons |     .57773    .062202     9.29   0.000     .4558163    .6996436
-------------+----------------------------------------------------------------
Wv           |
       _cons |   .6066382   .0743834     8.16   0.000     .4608494     .752427
-------------+----------------------------------------------------------------
Wu           |
       _cons |   .6584973   .0938095     7.02   0.000      .474634    .8423606
-------------+----------------------------------------------------------------
         rho |   .2810617   .0286408     9.81   0.000     .2240201    .3361839
```

```
       gamma |   .2943177    .033966     8.67   0.000     .2264087    .3593786
         tau |   .3178137    .042162     7.54   0.000     .2329366    .3978845
------------------------------------------------------------------------------
Note: Wy:_cons, Wv:_cons and Wu:_cons are the transformed parameters;
      rho, gamma and tau are their origin metrics in spatial components, respectively.
      W_(x) represent Spatial Durbin terms W(x)
```

The output shows that the command fits six equations with `ml model`. The frontier equation has three explanatory variables x_{it}, $W_i x_{.t}$ and constant. The scaling function uhet() has two explanatory variables Z_{it} and constant. The equation /lnsigv2 is constructed for the variance parameter σ_v^2 which is transformed by the function $exp(\cdot)$. Three Equations (Wy, Wu, and Wv) handle the spatial dependence parameters ρ, τ, and γ, which are parameterized as Eq.(11). We directly include the spatial Durbin term $W_i x_{.t}$ in the frontier equation (represented as W_x) such that we do not need to fit a separate equation. The bottom of the table reports the transformed parameters in the original metric.

4.2 xuv-SAR model with different spatial weight matrices

We consider a restricted model xuv-SAR with different spatial weight matrices, one of which is time-varying, and the others are time-constant. The model is described as DGP 2:

$$y_{it} = 1 + 2x_{it} + 0.5W_i^{xt} x_{it} + \tilde{v}_{it} + \tilde{u}_{it}, i = 1, .., 300; t = 1, .., 10 \tag{13}$$

where the other parameters are set the same as the DGP 1 except for $W_i^{ut} = W_i^u$, $W_i^{vt} = W_i^v$, $\delta = (4, ln(0.2))'$, and W_i^{xt} is time-varying. Different from DGP 1, which set the production function frontier, DGP 2 specifies a cost function. The estimation of the model is shown as follows.

```
. * importing spatial weight matrices from xtsfsp_w2.mmat
. mata mata matuse xtsfsp_w2.mmat,replace
(loading w1[300,300], w10[300,300], w2[300,300], w3[300,300], w4[300,300], w5[300,300], w6[300,300], w7[300,300

. local w w1 w2 w3 w4 w5 w6 w7 w8 w9 w10

. use xtsfsp_ex2.dta

. xtset id t

Panel variable: id (strongly balanced)
 Time variable: t, 1 to 10
         Delta: 1 unit

. * initial values for estimated parameters
. mat b=(2,0.5,1,-1.5,4,-1.5,0.6,0.6)

. * fitting the model
. xtsfsp y x, cost uhet(z) wu(w2,mata) wv(w1,mata) wxvars(x) wx(`w´,mata) init(b) genwxvars nolog

Spatial frontier model(xuv-SAR)                     Number of obs =    3,000
                                                    Wald chi2(2)  = 57441.23
Log likelihood = -1911.966                          Prob > chi2   =   0.0000

------------------------------------------------------------------------------
           y | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
frontier     |
           x |   1.995755   .0083855   238.00   0.000      1.97932     2.01219
         W_x |   .5069274   .0221354    22.90   0.000     .4635428    .5503121
       _cons |   .9917396   .0127299    77.91   0.000     .9667894     1.01669
```

```
    /lnsigv2 | -1.614693   .0260288   -62.03   0.000    -1.665708   -1.563677
-------------+----------------------------------------------------------------
uhet         |
           z |  3.999532    .002155  1855.90   0.000     3.995308    4.003755
       _cons | -1.699077   .4472546    -3.80   0.000     -2.57568   -.8224746
-------------+----------------------------------------------------------------
Wv           |
       _cons |  .5877882   .0584159    10.06   0.000     .4732951    .7022812
-------------+----------------------------------------------------------------
Wu           |
       _cons |  .6214134   .0008532   728.34   0.000     .6197412    .6230857
-------------+----------------------------------------------------------------
         tau |  .3010498   .0003879   776.12   0.000     .3002893    .3018098
       gamma |  .2856864   .0268209    10.65   0.000     .2323035     .337353
------------------------------------------------------------------------------
Note: Wv:_cons and Wu:_cons are the transformed parameters;
      gamma and tau are their origin metrics in spatial components, respectively.
      W_(x) represent Spatial Durbin terms W(x)
```

In the second example, we use `cost` option to specify the type of frontier. The matrix `b` is utilized as the initial value for maximum likelihood estimation. The likelihood function of spatial stochastic frontier models is intricate and typically challenging to obtain optimal global solutions. Therefore, having good initial values would be beneficial for fitting spatial stochastic models. To acquire initial values for spatially-correlated parameters, practitioners can initially fit non-spatial stochastic models using the `fronteir` and `sfpanel` commands to obtain the initial values of the parameters involved in the frontier and the scaling function. Subsequently, the `mlplot` option can be employed to search for better initial values.

To demonstrate the usage of the *delmissing* option, two observations of y_{it} are replaced with missing values, and the aforementioned codes are re-run, resulting in the error message "*missing values found. use delmissing to remove the units from the spmatrix*". The inclusion of the *delmissing* option addresses this issue, and the generated variable __e_sample__ records the regression sample.

```
. * replace some observations of y to be missing
. replace y=. if _n==1 | _n==100
(2 real changes made, 2 to missing)

. local w w1 w2 w3 w4 w5 w6 w7 w8 w9 w10

. * estimation is aborted
. xtsfsp y x, cost uhet(z) wu(w2,mata) wv(w1,mata) wxvars(x) wx(`w´,mata) init(b) nolog
missing values found. use delmissing to remove the units from the spmatrix
invalid syntax

  . * re-estimation with delmissing option
. local w w1 w2 w3 w4 w5 w6 w7 w8 w9 w10

. xtsfsp y x, cost uhet(z) wu(w2,mata) wv(w1,mata) wxvars(x) wx(`w´,mata) init(b) delmissing nolog
missing values found. The corresponding units are deleted from the spmatrix
                       (...omitted outputs...)
```

4.3 uv-SAR model with conditional heteroscedasticity of random errors

We set up a restricted model uv-SAR with time-varying spatial weight matrices and conditional heteroscedasticity of random errors. The DGP 3 is described as

$$\begin{aligned} &y_{it} = 1 + 2x_{it} + \tilde{v}_{it} - \tilde{u}_{it}, i = 1,..,300; t = 1,..,10 \\ &\sigma^2_{v,it} = exp(1 + d_{it}) \\ &h(Z'_{it}\delta) = \sqrt{exp(1 + z_{it})} \end{aligned} \tag{14}$$

where the other parameters are set the same as the DGP 1 except for $W_i^{ut} = W_i^{vt} = W_i^t$. The following syntax estimates the model alongside the results.

```
. * importing spatial weight matrices from xtsfsp_w3.mmat
. mata mata matuse xtsfsp_w3,replace
(loading w1[300,300], w10[300,300], w2[300,300], w3[300,300], w4[300,300], w5[300,300], w6[300,300], w7[300,300

. local w w1 w2 w3 w4 w5 w6 w7 w8 w9 w10

. use xtsfsp_ex3.dta

. xtset id t

Panel variable: id (strongly balanced)
 Time variable: t, 1 to 10
         Delta: 1 unit

. xtsfsp y x, wu(`w´,mata) wv(`w´,mata) uhet(z) vhet(d) nolog

Spatial frontier model(uv-SAR)                        Number of obs =   3,000
                                                       Wald chi2(1)  = 7472.38
Log likelihood = -5803.0516                            Prob > chi2   =  0.0000

------------------------------------------------------------------------------
           y | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
frontier     |
           x |   1.988108   .0229991    86.44   0.000     1.943031    2.033185
       _cons |   .8997916   .0737763    12.20   0.000     .7551927    1.044391
-------------+----------------------------------------------------------------
lnsigv2      |
           d |   .9897965   .0259185    38.19   0.000     .9389972    1.040596
       _cons |   .9950927   .0259519    38.34   0.000      .944228    1.045957
-------------+----------------------------------------------------------------
uhet         |
           z |   1.019351   .0458701    22.22   0.000     .9294473    1.109255
       _cons |   1.122574   .4629594     2.42   0.015     .2151904    2.029958
-------------+----------------------------------------------------------------
Wv           |
       _cons |   .6171649   .0431991    14.29   0.000     .5324961    .7018337
-------------+----------------------------------------------------------------
Wu           |
       _cons |   .6471479   .0703691     9.20   0.000      .509227    .7850688
-------------+----------------------------------------------------------------
         tau |    .299117   .0196647    15.21   0.000     .2601042    .3371547
       gamma |   .3127036   .0317402     9.85   0.000     .2492256    .3735057
------------------------------------------------------------------------------
Note: Wv:_cons and Wu:_cons are the transformed parameters
      gamma and tau are their origin metrics in spatial components, respectively.
```

4.4 Example with real data

We use the real data to exemplify how to conduct the empirical studies with the models and the command described above.The raw data on the provinces of China, including GDP (denoted as Y), investment, labor force (denoted as L), the ratio of government expenditure to GDP (denoted as fiscal), the ratio of FDI to GDP (denoted as fdi), and trade as a percentage of GDP (denoted as trade), is collected from the CSMAR database.All nominal variables are adjusted to constant prices in 1997. The capital stocks for the provinces are estimated using the perpetual inventory method. The production function is approximated by the translog function, and the inefficiency term's scaling function is assumed to be determined by the ratio of government expenditure to GDP, the ratio of FDI to GDP, and trade as a percentage of GDP.

```
. * Translate shapefile to Stata format
. spshape2dta province
.
. use province
. drop if _ID == 26 | _ID>31
(4 observations deleted)
. spset
      Sp dataset: province.dta
Linked shapefile: province_shp.dta
            Data: Cross sectional
 Spatial-unit ID: _ID
     Coordinates: _CX, _CY (planar)
. * Create spatial contiguity matrix
. spmatrix create contiguity w_con, normalize(none)
  weighting matrix in w_con contains 1 island
. * Obtain spatial matrix as Mata matrix wm from w_con
. spmatrix matafromsp wm id = w_con
. * Match the iland (_ID = 21) with the nearest province(_ID =19)
. mata: wm[19,21]=1
. mata: wm[21,19]=1
. * Create spmatrix w_con from Mata matrix wm and rwo-normalized the matrix
. spmatrix spfrommata w_con= wm id, normalize(row) replace
. * Obtain the new spatial matrix as Mata matrix wm from w_con
. spmatrix matafromsp wm id = w_con
.
. use chnempirical.dta,clear
. * Generate varables for the translog function
. qui translog Y K L , time(year) norm
. global x  lnK lnL _t lnK_lnL _t_lnK _t_lnL _t_2 lnK_2 lnL_2
. global z fiscal trade fdi
. * Fit the model with frontier command
. frontier lnY $x,uhet($z) nolog
                (...omitted outputs...)
. * Predict the inefficiency term and efficiency scores
. predict double uhat, u
. gen double te0 = exp(-u)
. * Store the estimated parameters
. mat b0=e(b)
```

```
.
. * Fit the model with xtsfsp command
. xtset _ID year

Panel variable: _ID (strongly balanced)
 Time variable: year, 1997 to 2017
         Delta: 1 unit

. mat b1 = b0,0.6,0.6,0.6

. xtsfsp lnY $x, uhet($z) wy(wm,mata) wu(wm,mata) wv(wm,mata) init(b1) te(tesp1) nolog

Spatial frontier model(yuv-SAR)                       Number of obs =      630
                                                      Wald chi2(9)  = 16704.73
Log likelihood = 328.51183                            Prob > chi2   =   0.0000

------------------------------------------------------------------------------
         lnY | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
frontier     |
         lnK |   .6720836   .0259441    25.91   0.000     .6212341    .7229331
         lnL |   .3390601   .0185192    18.31   0.000      .302763    .3753571
          _t |   .0141493   .0051647     2.74   0.006     .0040267    .0242719
     lnK_lnL |   .0075504    .051805     0.15   0.884    -.0939855    .1090864
      _t_lnK |   .0425662   .0115137     3.70   0.000     .0199998    .0651326
      _t_lnL |  -.0034571   .0079979    -0.43   0.666    -.0191327    .0122186
        _t_2 |  -.0069112   .0013048    -5.30   0.000    -.0094686   -.0043539
       lnK_2 |  -.0875718   .0344022    -2.55   0.011     -.154999   -.0201446
       lnL_2 |  -.0388746   .0377404    -1.03   0.303    -.1128443    .0350952
       _cons |   .4968023    .043107    11.52   0.000     .4123142    .5812903
-------------+----------------------------------------------------------------
    /lnsigv2 |  -4.025468   .0581939   -69.17   0.000    -4.139526    -3.91141
-------------+----------------------------------------------------------------
uhet         |
      fiscal |   .0098788   .0056888     1.74   0.082     -.001271    .0210287
       trade |  -.0911438   .0213384    -4.27   0.000    -.1329663   -.0493212
         fdi |  -.0185831   .0100212    -1.85   0.064    -.0382243    .0010582
       _cons |  -1.963808   .3894655    -5.04   0.000    -2.727146   -1.200469
-------------+----------------------------------------------------------------
Wy           |
       _cons |  -.0467557   .0362548    -1.29   0.197    -.1178138    .0243023
-------------+----------------------------------------------------------------
Wv           |
       _cons |  -.0671769   .1600686    -0.42   0.675    -.3809057    .2465518
-------------+----------------------------------------------------------------
Wu           |
       _cons |   1.280422   .2367245     5.41   0.000       .81645    1.744393
-------------+----------------------------------------------------------------
         rho |  -.0233713   .0181157    -1.29   0.197     -.058833    .0121493
       gamma |  -.0335725   .0799361    -0.42   0.674    -.1881642     .122643
         tau |   .5649866   .0805643     7.01   0.000     .3869258    .7024182
------------------------------------------------------------------------------
Note: Wy:_cons, Wv:_cons and Wu:_cons are the transformed parameters;
      rho, gamma and tau are their origin metrics in spatial components, respectively.

. scalar loglikehood1 =  e(ll)

. mat b1 = b0,0.6

. xtsfsp lnY $x, uhet($z)  wu(wm,mata)  init(b1) te(tesp2)  nolog

Spatial frontier model:u-SAR                          Number of obs =      630
                                                      Wald chi2(9)  = 16174.91
Log likelihood = 327.60816                            Prob > chi2   =   0.0000

------------------------------------------------------------------------------
             |
```

```
         lnY | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
frontier     |
         lnK |    .672366   .0264662    25.40   0.000     .6204932    .7242388
         lnL |   .3392661   .0184417    18.40   0.000     .3031209    .3754113
          _t |   .0114801    .004803     2.39   0.017     .0020664    .0208937
     lnK_lnL |   .0128618   .0467126     0.28   0.783    -.0786932    .1044169
      _t_lnK |   .0422561   .0108054     3.91   0.000     .0210779    .0634344
      _t_lnL |  -.0027543   .0074782    -0.37   0.713    -.0174113    .0119027
        _t_2 |  -.0066348   .0012629    -5.25   0.000      -.00911   -.0041595
       lnK_2 |  -.0915034   .0326073    -2.81   0.005    -.1554126   -.0275942
       lnL_2 |  -.0395255   .0328292    -1.20   0.229    -.1038696    .0248185
       _cons |   .4644181   .0315665    14.71   0.000     .4025489    .5262874
-------------+----------------------------------------------------------------
    /lnsigv2 |  -4.013496   .0575493   -69.74   0.000     -4.12629   -3.900701
-------------+----------------------------------------------------------------
uhet         |
      fiscal |   .0097757      .0055     1.78   0.076     -.001004    .0205555
       trade |  -.0869182   .0204711    -4.25   0.000    -.1270408   -.0467956
         fdi |  -.0133135   .0090702    -1.47   0.142    -.0310908    .0044638
       _cons |  -1.986602   .3822161    -5.20   0.000    -2.735732   -1.237472
-------------+----------------------------------------------------------------
Wu           |
       _cons |    1.07423   .1922999     5.59   0.000     .6973292    1.451131
-------------+----------------------------------------------------------------
         tau |    .490752   .0729815     6.72   0.000     .3351572    .6202829
------------------------------------------------------------------------------
Note: Wu:_cons is the transformed parameter;
      tau is the origin metrics in spatial components.

. scalar loglikehood2 =  e(ll)

. local lrtest = -2*(loglikehood2-loglikehood1)

. local pvalue = 1- chi2(2,`lrtest´)

. display "Likelihood-ratio test: LR chi2(2) = `lrtest´, Prob > chi2 = `pvalue´"
Likelihood-ratio test: LR chi2(2) = 1.807357699424301, Prob > chi2 = .4050766989379054

. 
. * Plot the density of estimates of technical efficiency from different models
. twoway (kdensity te0, color(black) lpattern(solid)) ///
>        (kdensity tesp1,color(red) lpattern(dash))   ///
>            (kdensity tesp2,color(blue) lpattern(longdash)),  ///
>         legend(pos(10) ring(0) label(1 Non-spatial Stoc. Frontier) ///
>         label(2 Spatial Stoc. Frontier:yuv) label(3 Spatial Stoc. Frontier:u)) ///
>         xtitle("Technical Efficiency") ytitle("Density")
```

To construct the spatial weight matrix, we use the geographic data (province.shp and province.dbf). The Stata command `shshape2dta` is used to convert the data. Units with heavily missing values are dropped. Based on the generated province.dta, the spatial contiguity matrix is constructed using Stata Official spmatrix routines [5]. As _ID 26 represents an island, we define its contiguity unit as the nearest one. Consequently, we extract the spatial weight matrix as wm in the Mata environment from the spmatrix object w_con and assign the elements wm[19,21] and wm[21,19] a value of one. Furthermore, the spmatrix routine is used to standardize the matrix. The spmatrix object w_con can be used for the `xtsfsp` command. Alternatively, the new w_con can be placed

[5] Alternatively, the community-contributed command `spwmatrix` by (Jeanty 2010) can be used to create a spatial weight matrix in Stata

into the Mata environment as a matrix wm.

We initially fit a non-spatial stochastic frontier model using the Stata Official command `frontier`. We predict the efficiency scores and store them in the new variable te0. The estimates of the parameters are extracted to serve as initial values in the spatial stochastic frontier models. We then consider three sources of spatial dependence, known as the *yuv-SAR* model. The estimated results show that the spatial correlation coefficients ρ and γ are statistically insignificant at the 10% level. Therefore, a restricted model with spatial dependence in the inefficient term is considered. Since the restricted model is nested within the previous model, a likelihood ratio test is performed for model selection. The results indicate that the null hypothesis, stating that the restricted model cannot be rejected, is supported. The significance of τ suggests positive spatial spillovers of technical efficiency. The empirical results reveal that trade and FDI increase technical efficiency, while government intervention, as proxy by the ratio of government expenditure to GDP, decreases technical efficiency in the provinces of China. Figure 1 illustrates the distribution of efficiency scores estimated by different models. The non-spatial stochastic frontier model significantly overestimates technical efficiency. The spatial stochastic models *yuv-SAR* and *u-SAR* provide similar estimates of technical efficiency.

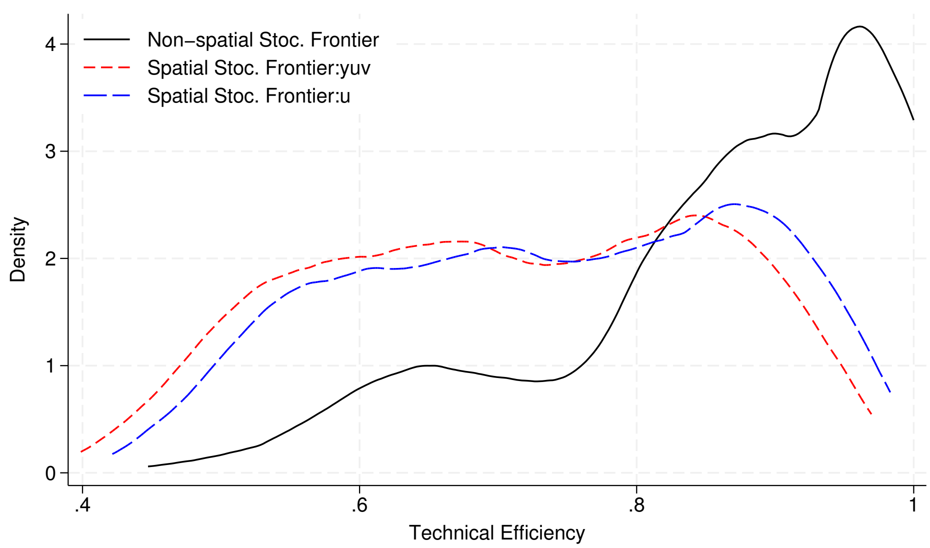

Figure 1: Distribution of efficiency scores

5 Conclusion

Geospatial units are not isolated or separated but interconnected. For instance, economic trade, social activities, and cultural exchange between different regions mutually influence each other. This spatial interdependence poses a challenge to traditional econometric methods, which generally assume cross-sectional independence. Spatial econometrics has been developed to address spatial correlation. This article presents a community-contributed command that facilitates the fitting of spatial stochastic frontier models, accounting for different sources of spatial dependence (Galli 2022; Orea and Álvarez 2019). We hope that this developed command can provide convenience to practitioners and reduce the complexity of model applications, thereby promoting robust empirical research.

However, there are certain limitations that should be acknowledged. Firstly, despite the flexibility of spatial stochastic frontier models and the introduced command, they rely on full parameterization of the spatial structure, frontier function, and distribution of random errors and inefficiency terms, following the spirit of stochastic frontier models and spatial econometrics. Model misspecification can be a potential concern, and theoretical works exploring relaxed parametric assumptions would be valuable. Secondly, these models require prior information on the spatial weight matrices. Different choices of spatial weight matrices may lead to varying results. The internalization of spatial weight matrices in spatial stochastic frontier models remains an open issue. Thirdly, the numerical computation for maximum likelihood estimation (MLE) of spatial stochastic frontier models is highly complex. When dealing with large spatial weight matrices, especially when considering time-varying spatial weight matrices, the estimation process becomes computationally intensive as the matrices need to be repeatedly inverted. Additionally, the case of time-varying spatial weight matrices with large dimensions may be memory intensive.

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