Landscape Evolution Modeling

Landscape evolution modeling is a branch of the Earth Sciences that studies how the shape, or geomorphology, of Earth's surface changes over time due to tectonic and erosive forces, which act to increase or decrease elevation locally.

First consider tectonic forces that act to increase or decrease Earth's surface elevation. Where two continents collide, Earth's surface can be forced up, such as is presently occurring in the Himalaya Mountains, where the Indian and Asian continents are colliding; the present rate of uplift is about 1 cm/year in the location that is rising the fastest. Conversely, where a tectonic plate is being stretched apart and thinned, Earth's elevation will decrease; an example of this is the East African Rift valley. Large scale changes in elevation can also occur from the relative motion along fault zones, which can result in some sections of crust being uplifted and others dropped down, either through abrupt change during a large earthquake, or due to slow but continuous creep along the fault. Another cause of uplift is post-glacial rebound, where the Earth's crust was formerly pushed down into the mantle due to the weight of over a kilometer of ice during the last ice age, and removal of the ice mass at the end of the ice age happened faster than the crust and mantle can rebound (i.e spring back up like silly putty), resulting in a slow uplift of the surface over time, such as is happening in the northern parts of North America. While these forces are generally very slow with elevation change rates on the order of mm or cm per year, over tens or hundreds of thousands of years they can results in large elevation changes.

Erosion occurs from weathering of rock into regolith, which is unconsolidated rocky material. Over long time scales weathering occurs due to the chemical reactions of groundwater and rain water with the minerals composing a rock, but over shorter time scales weathering is accelerated by freezing and thawing cycles that use the expansion of freezing water as a slow but persistent lever that forces open cracks in otherwise solid rocks. Wind, water and glacier ice then act under the force of gravity to transport sediment generally in a downhill direction. Mass can also be transported due to slow creep.

In the sections below we will take an introductory look at how to use landscape evolution modeling to study how uplift and erosion can shape a landscape over time.

Conservation of Mass

A fundamental component landscape evolution modeling is the conservation of mass, which is expressed by the equation:

$$\frac{\partial z}{\partial t} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y}\right) \tag{1}$$

where z is the local elevation, t is time and q is a volume flux through a lateral interface (with units m^2 per unit time). This equation states that the rate at which the elevation increases or decreases is equal to the negative of the divergence of the volumetric flux of that quantity.

Uplift

Perhaps the simplest model of landscape evolution over time concerns the regional or local uplift due to tectonic forces. This can be expressed mathematically as:

$$\frac{\partial z}{\partial t} = u \tag{2}$$

where u is the rate of uplift in m per unit time. Note that in general $u = u(x, y)$ and so $z = z(x, y)$. For simple problems u may be constant over time, whereas more specific cases may consider $u = u(x, y, t)$, meaning that the uplift rate may vary in both space and time.

For the simple case where u is constant in time, z after some time Δt is simply:

$$z = z_0 + u\Delta t, \tag{3}$$

where z_0 is the original elevation.

Colluvial Erosion and Deposition

Over long time scales, the flux of material that erodes due to colluvial processes such as creep (which do not involve mass transport by water) has been observed empirically to be described by a linear dependence on the slope:

$$q_x = -\kappa \frac{\partial z}{\partial x} \tag{4}$$

$$q_y = -\kappa \frac{\partial z}{\partial y} \tag{5}$$

Conveniently, this results in a diffusion equation that can be solved to model colluvial processes:

$$\frac{\partial z}{\partial t} = \kappa \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) \tag{6}$$

The figure below shows a 1D example of this type of erosion and deposition computed using the FTCS finite difference method with $\kappa = 1000$ m^2/year.

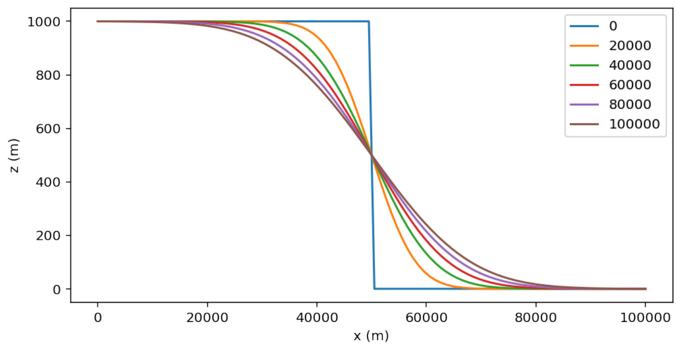

Figure 1: Example of diffusion modeling of colluvial erosion and deposition of a 1000 m cliff over a time span of $100,000$ years. The top side of the cliff is slowly eroded over time, while material is deposited at the base of the cliff.

Erosion and Deposition by Channel Flow

Channel flow refers to how precipitation on Earth's surface flows in the downhill direction and generally results in water streams merging together into increasingly larger channels with distance downhill. For example, think of a network of small streams at high elevations in a watershed that gradually merge together to form a larger stream, which eventually could merge with other streams and form a river. The ability of a stream or river to transport sediment generally depends on the speed of the river, with fast moving rivers being able to erode more quickly than others. Similarly, wide and deep rivers with a large flux of water can transport more sediment than narrow and shallow rivers. A fast moving river can have sufficient energy to carry suspended sediments and during flash floods or in steep rapids, can even move large rocks and boulders. As the speed of a river slows, sediments carried in the water will be deposited on the river bottom.

Here we will use a simple model for the effects of channel flow on surface elevation. We start with the concept of sediment flux, denoted as Q, which is the total volume flux of sediment being carried by the water. Along a flow path (increasing in size from rivulet to stream to river...), the change in surface elevation from the water flow is described as

$$\frac{\partial z}{\partial t} = -\frac{\partial Q}{\partial l} \tag{7}$$

where ∂l denotes an infinitesimal increment of distance along the flow path. Thus as Q increases, meaning the river picks up more sediment due to erosion, the surface elevation decreases due to the loss of material. Conversely, when Q decreases, sediment is deposited and the elevation increases.

Next we introduce the concept of carrying capacity Q_e, which is the maximum flux that can be supported by a given discharge of water. We can then restate the formula above as

$$\frac{\partial z}{\partial t} = -\frac{Q_e - Q}{L_e} \tag{8}$$

where L_e is some characteristic scale length over which the deposition or erosion occurs. There are two scenarios. When $Q > Q_e$ deposition occurs and the elevation increase according to:

$$\frac{\partial z}{\partial t} = \frac{Q - Q_e}{L} \tag{9}$$

Conversely, when $Q < Q_e$, meaning the flowing water can still hold more sediment, then erosion occurs. Again according to the equation

$$\frac{\partial z}{\partial t} = \frac{Q - Q_e}{L}. \tag{10}$$

In class we will implement the channel flow equation in 1D using a finite time step. To do this, we divide up the lateral dimension x into a series of n cells at locations x_i with spacing Δx. In each cell, we assume a precipitation rate v in units of m/year. Thus the total precipitation per time step is $v\Delta t$. Integrating this over the length of a cell, the total amount of precipitation p in a time step is then:

$$p_i = v_i \Delta t \Delta x \tag{11}$$

The precipitation rate v can be a constant in simple models while more complex models might have v vary over time and space, for example precipitation could be made larger in mountains to account for orographic uplift. In our 1D model, we will assume the topography slopes down from left to right without any local depressions. Thus the water will flow from left to right. The total amount of water flowing across a cell is then the sum of the locally precipitated water p_i, plus the water that flows in from the neighboring cell on the left q_{i-1}:

$$q_i = p_i + q_{i-1} \tag{12}$$

Thus when modeling this, we will start at the highest point and follow the water downhill and gradually build up the total amount of water in q; this is analogous to how rivers generally increase in water volume as you head downstream.

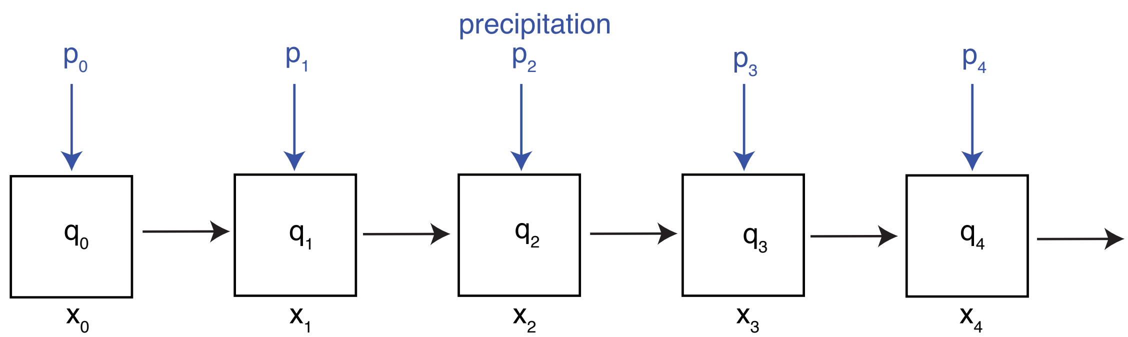

Figure 2: Simplified model for the flow of water over the landscape surface. In a given cell at position x_i the total amount of water q_i is the sum of the water q_{i-1} that flows in from the adjacent cell on the uphill side plus the local precipitation p_i. Thus $q_i = q_{i-1} + p_i$. The water in q_i then flows to the next cell downhill at location x_{i+1}. For a sequence of n cells and constant rate of precipitation p, the last cell q_n will have a total water flux of $q_n = np$.

Now we need to consider the sediment carrying capacity of the water stream Q_e. We will assume the sediment load generated locally in the cell is

$$Q_{e,i} = \kappa_r s_i q_i \tag{13}$$

where κ_r is a scaling constant and s_i is the local slope in the downward direction:

$$s_i = \left| \frac{z_{i+1} - z_i}{x_{i+1} - x_i} \right| \tag{14}$$

Thus steeper slows can carry a larger sediment load, and the larger the water flux q_i, the larger the sediment carrying capacity.

So what about the sediment load Q_i? We start out a given time step by initializing the sediment load to be zero everywhere, ie. $Q_i = 0$ for all $i = 0$ to $n - 1$. We then follow the flow of water from the highest point to the lowest point, adding precipitation to q_i and updating the sediment load in Q_i. Then we compare Q_i to the carrying capacity $Q_{e,i}$ and either deposit or erode sediment from cell i.

If $Q_i > Q_{e,i}$ the water is carrying more sediment than it can hold and so deposition occurs according to:

$$z_i = z_i + \frac{Q_i - Q_{e,i}}{\Delta x} \tag{15}$$

We then set

$$Q_i = Q_{e,i} \tag{16}$$

Conversely, when $Q_i < Q_{e,i}$ the water can hold more sediment and so we allow it to start eroding that cell with the updated elevation:

$$z_i = z_i + \frac{Q_i - Q_{e,i}}{\Delta x} \frac{\Delta x}{L_e} \tag{17}$$

where $Q_i - Q_{e,i}$ is negative and thus z will locally decrease. The scaling factor $\frac{\Delta x}{L_e}$ on the right is used to slowly increment the erosion, rather than having it occur all at once in a single cell. The sediment load is then increased according to:

$$Q_i = Q_i + (Q_{e,i} - Q_i) \frac{\Delta x}{L_e} \tag{18}$$

For both the deposition and erosion cases, we then pass q_i and Q_i to the next cell in the downhill direction (i.e. the water flows one step over).

Once we have swept from highest cell to the lowest, the time step is completed and we start a new time step, step with q and Q initialized to zero.

2D Channel Flow

For more realistic 2D scenarios, the channel flow equations become a bit more complex to implement. First, the water is generally assumed to flow from one cell to another down the direction of steepest slope. Thus you need to compute the slope everywhere in the model and know which direction is the steepest. Also, the flow is generally driven from the highest elevation to the lowest, so that the water discharge from precipitation and the load of sediments is integrated from cell to cell as the water flows downhill. Thus, while you already know how to model diffusion in 2D so that colluvial erosions should be easy to model in Python, channel flow modeling will require quite a bit more complexity in your code. For the assignment, I'll supply the codes that compute some of these complex operations.

We will also now couple the equations, so that we solve for the combined effects of uplift, diffusion and channel flow in 2D:

$$\frac{\partial z}{\partial t} = u + \kappa \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) + \text{channel\_flow}(x, y) \tag{19}$$

The channel flow equations are slightly modified for 2D modeling. We now index Q in 2D using i, j indexes for position so that $Q_{ij} = Q(x_i, y_j)$.

When $Q_{ij} > Q_{e,ij}$ the water is carrying more sediment than it can hold and so deposition occurs according to:

$$z_{ij} = z_{ij} + \frac{Q_{ij} - Q_{e,ij}}{A_{ij}} \tag{20}$$

where A_{ij} is the area of the 2D cell ij. We then set:

$$Q_{ij} = Q_{e,ij} \tag{21}$$

When $Q_{ij} < Q_{e,ijj}$ the water can hold more sediment and so we allow it to start eroding that cell with the updated elevation:

$$z_{ij} = z_i ij + \frac{Q_{ij} - Q_{e,ij}}{A_{ij}} \frac{L_{ij}}{L_e} \tag{22}$$

where $Q_{ij} - Q_{e,ij}$ is negative and thus z will locally decrease. L_{ij} is the length of the flow path to the neighboring cell. The sediment load is then increased according to:

$$Q_{ij} = Q_{ij} + (Q_{e,jj} - Q_{ij}) \frac{L_{ij}}{L_e} \tag{23}$$

For both the deposition and erosion cases, we then pass q_{ij} and Q_{ij} to the next cell in the downhill direction (i.e. the water flows one step over).

The general algorithm starts at the highest elevation, computing q_{ij}, Q_{ij} and updating its elevation z_{ij}. q_{ij}, Q_{ij} are then passed to neighboring cell in the steepest direction downhill. The algorithm then moves to the cell that is the next highest point and does the same thing. This proceeds iteratively until all cells have been updated. At that point the time step is complete and a new time step begins at the cell that is the new highest point.

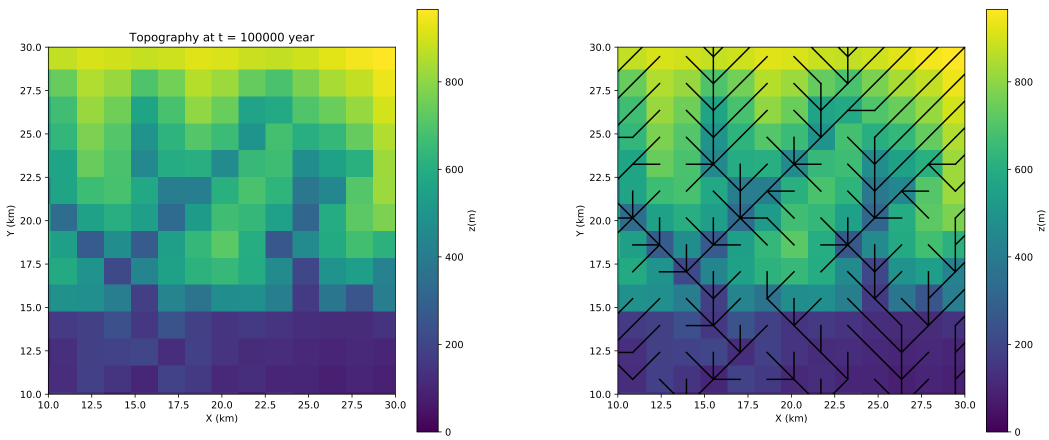

Figure 3: Example of an evolved topographic surface (left) along with the computed channelized water flow paths (right). Black lines show how water flows from one cell to the lowest neighboring cell (i.e. it flows in the steepest direct).