Homework 2-2
World Map Visualization

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1 Visualizing the whole world

Using the intersection of the prime meridian and the equator as the center point, a world map is visualized using the mercator projection, as seen in figure 1. In this figure, the economic levels of the different countries is also visualized, ranging from the most developed (level 1) until the least developed (level 7). The size of each city is also visualized as blue circles on the map, with radius as a function of the city's size.

Since the y-value coordinates approaches infinity as the latitude goes towards the poles (-90°and 90°) a bound on the latitude of $[-80, 85]$ is used. The southern bound is further away from its pole than the northern because of the size of Antarctica, which would take up too much of the map projection if the lower bound was any lower. To see the non-linear way the y-values are projected as a function of the latitude, a grid system of equally spaced latitudes and longitudes is projected onto figure 2. Here we see that even though the grid is equally spaced out in terms of latitude, when projected using the mercator formula, the spacing in terms of the y-values increases the closer to the poles one gets.

2 Projection types

The shape of the Earth can be approximated with a sphere and any point on the surface of the earth can then be expressed with spherical coordinates (latitude, longitude) with a fixed radius. There are different ways of projecting this sphere onto a flat surface and in this section, we will explore some of the more commonly used methods, limiting the latitude to the range $[10, 80]$

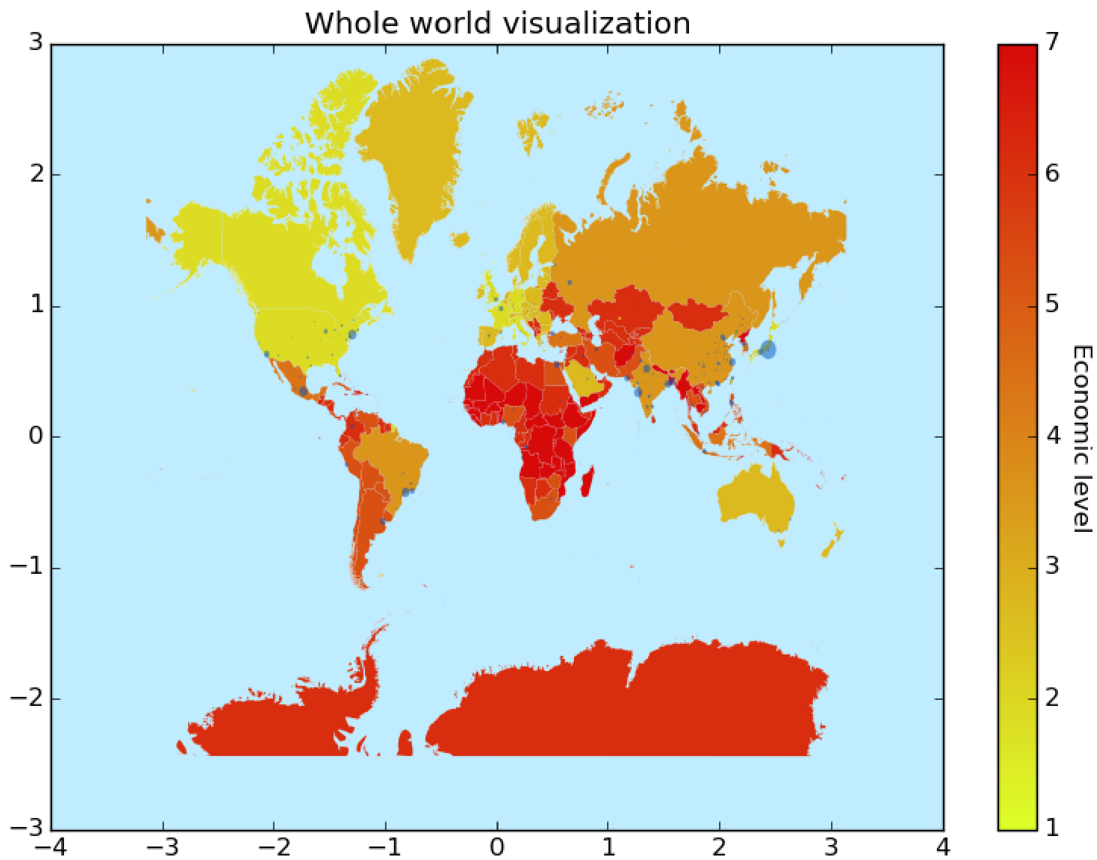

Figure 1: Whole world visualization using the mercator projection

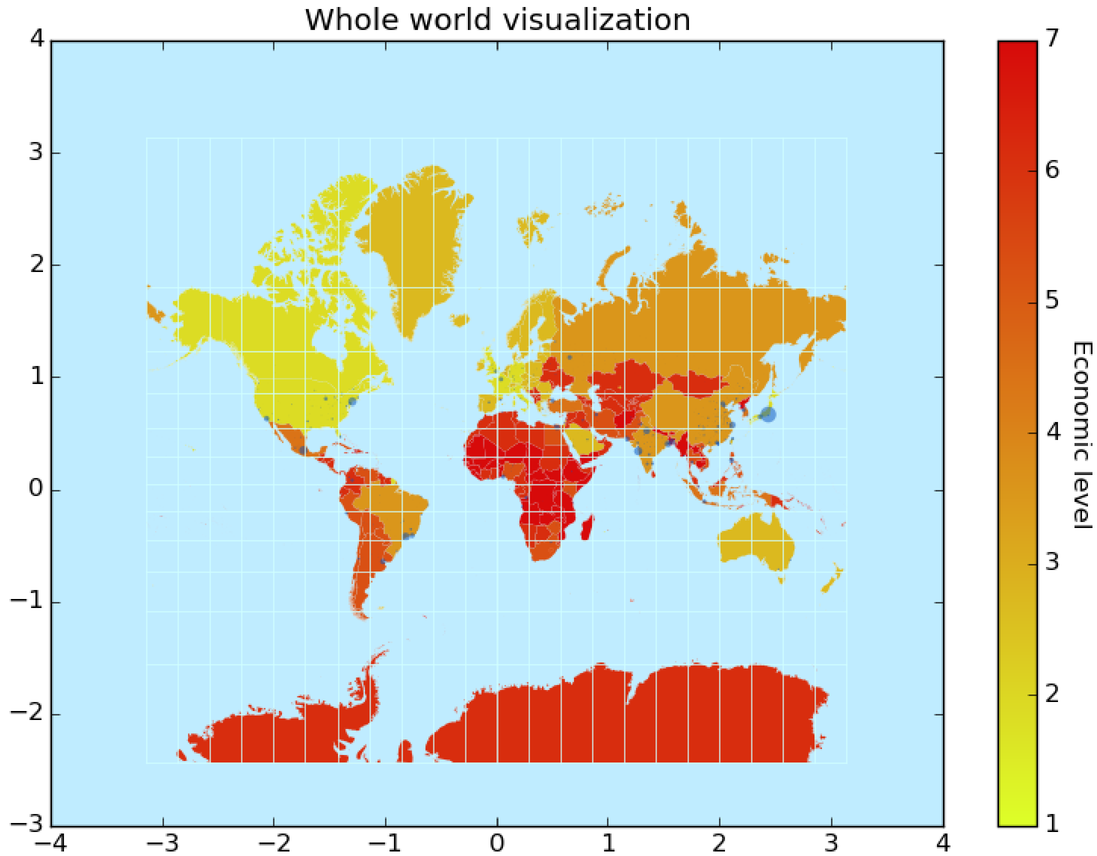

Figure 2: Whole world visualization using the mercator projection, with grid system

Figure 3: The emblem of the United Nations is produced by the azimuthal equidistant projection

2.1 Azimuthal equidistant projection

As the name suggests, this is an azimuthal map projection which has the property that distances from a central point are proportionally preserved. Thus, the resulting map projection will be a circle. Another feature of the azimuthal equidistant projection is that any straight line that passes through the center point will be a part of a greater circle. One interesting trivia is that the United Nations' emblem is the world map produced by the azimuthal equidistant projection with a center at the north pole, which can be seen in figure 3. The world map produced by my program using this projection formula can be seen in figure 4

2.2 Sinusoidal projection

The sinusoidal projection is also known as the Sanson-Flamsteed or the Mercator equal-area projection. As the last name suggests, this projection preserves the proportional area of the landparts, but as a result, also distorts the shapes. Each of the meridians (latitude lines from one pole to the other with constant longitude) except the central meridian is half of a sine wave. This is where the name sinusoidal projection comes from. The world map produced by my program using this projection formula can be seen in figure 5

2.3 Mercator projection

The Mercator projection is a cylindrical map projection, meaning that meridians are mapped to equally spaced vertical lines and circles of latitudes (circles around Earth with the same latitude) are mapped to horizontal lines.

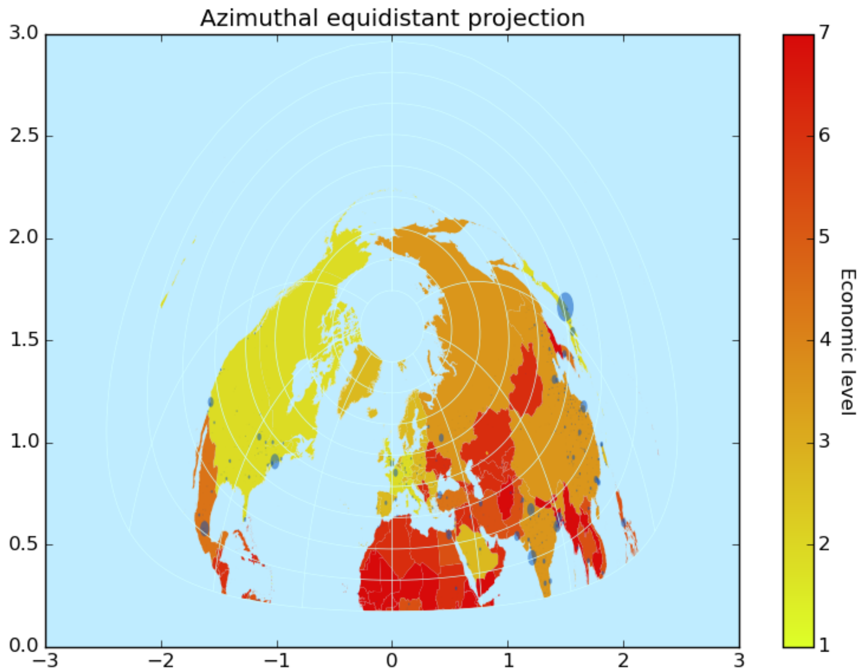

Figure 4: The azimuthal equidistant projection of the Earth within the latitude range of 10 to 80

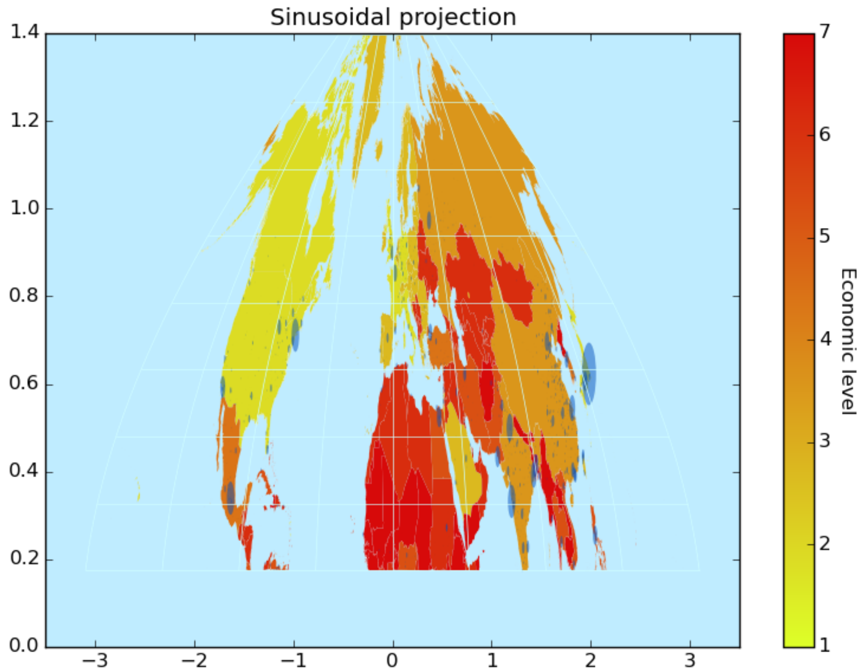

Figure 5: The sinusoidal, also known as the SansonFlamsteed or the Mercator equal-area projection of the Earth within the latitude range of 10 to 80

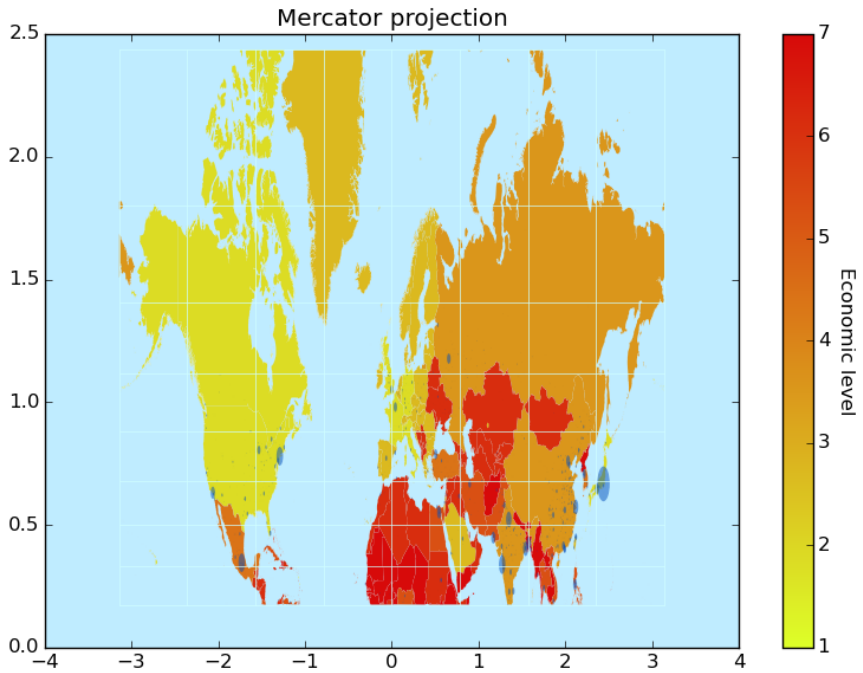

Figure 6: The mercator projection of the Earth within the latitude range of 10 to 80

This can be visualized as the spherical Earth enclosed in a cylinder where the cylinder's axis coincides with the Earth's axis of rotation. The cylinder is then wrapped around the Earth, projected onto, and finally unrolled.

One property of this projection is that the scale increases as the latitude moves away from the equator and approaches infinity as the latitude goes towards the poles. To deal with this, cut-off latitudes are used when using the mercator projection (-80 and 85 in my implementation). The world map produced by my program using this projection formula can be seen in figure 6, and also figure 1.

2.4 Area of Japan and Taiwan

The area of Japan and Taiwan as seen in the maps of figures 4 (azimuthal equidistant), 5 (sinusoidal), and 6 (mercator) can be seen in table 1.

As previously mentioned, the sinusoidal projection preserves the area of the map projection and we therefore use this as the reference.

The azimuthal projection induces distortions of shapes the further away from the center point (which is the intersection of the prime meridian and

	Area (pixels)	
Projection	Japan	Taiwan
Azimuthal equidistant	269	22
Sinusoidal	183	17
Mercator	143	10

Table 1: Area of Japan and Taiwan in the different map projections

the equator in figure 4). Since Japan and Taiwan are far away from this point, their shapes are quite distorted.

The mercator projection induces distortion increasingly as one approaches the poles. For example, although Greenland is only one-eighth the size of South America, Greenland appears to be larger in the Mercator projection, as can be seen in 2. In the case of Japan and Taiwan, they are not very close to the poles and therefore their shapes are not very distorted.

The real area of Japan is $377,944\text{km}^2$ and the area of Taiwan is $36,193\text{km}^2$.

3 Geodesic line

The shortest path between two points on a sphere follows the shortest great-circle arc between these points. A great-circle is produced by the intersection of the sphere and a plane that passes through the center of the sphere and the two points. This shortest path is also called the geodesic line. This line can be calculated using Vincenty's inverse formula. The geodesic line between Beijing and Los Angeles as visualized by the different projections can be seen in figures 8 (azimuthal equidistant), 9 (sinusoidal), and 10 (mercator).

A common map projection used when wanting to visualize the geodesic line is the azimuthal equidistant projection with center at the start location. As discussed in section 2.1, any straight line that passes through the center point of the projection map will be a part of a greater circle. Thus, the geodesic line between the center point and any point on the map will be straight. Applying this to our problem, figure 7 shows that an azimuthal equidistant map with center around Beijing will indeed plot the geodesic line to Los Angeles as a straight line.

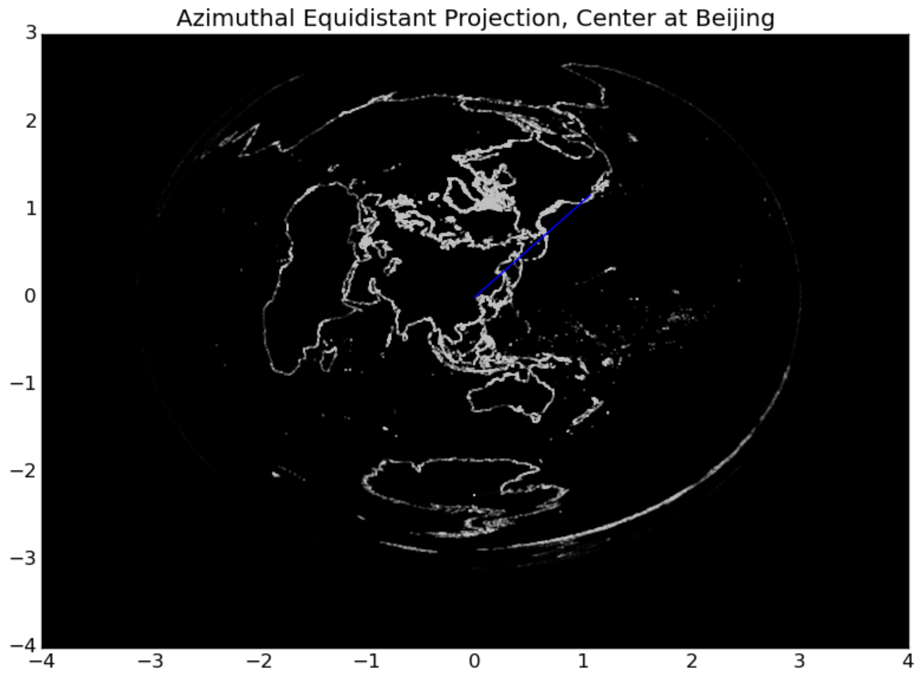

Figure 7: The geodesic line between Beijing and Los Angeles in the azimuthal equidistant projection of the Earth, with center point in Beijing

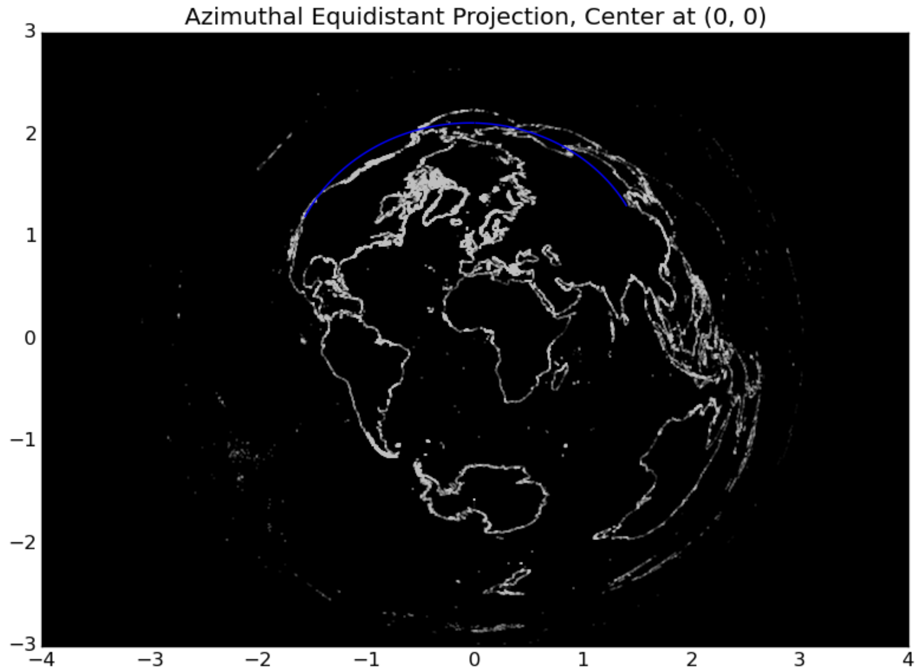

Figure 8: The geodesic line between Beijing and Los Angeles in the azimuthal equidistant projection of the Earth, with center point at the intersection of the equator and the central meridian

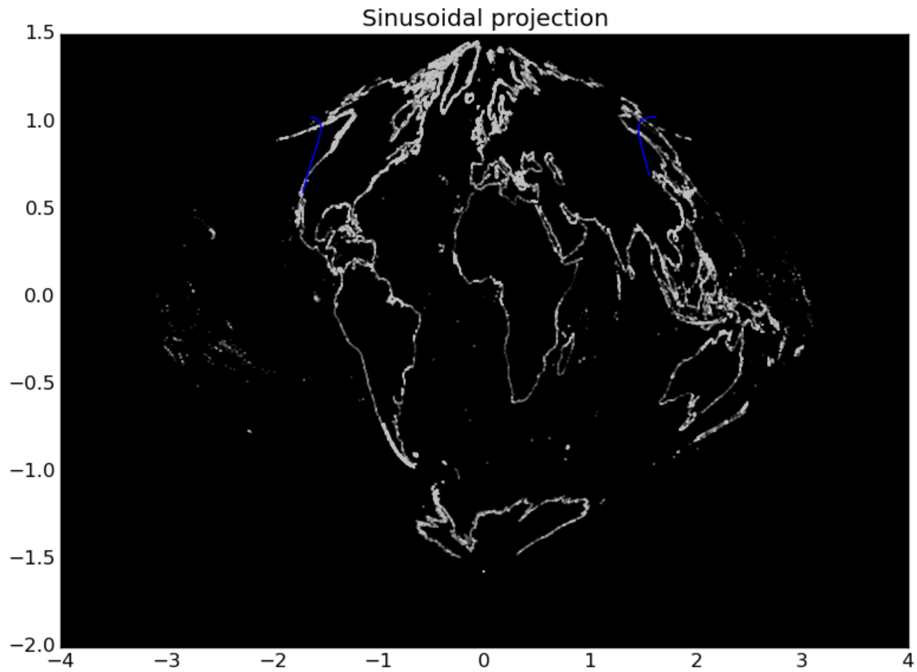

Figure 9: The geodesic line between Beijing and Los Angeles in the sinusoidal projection of the Earth

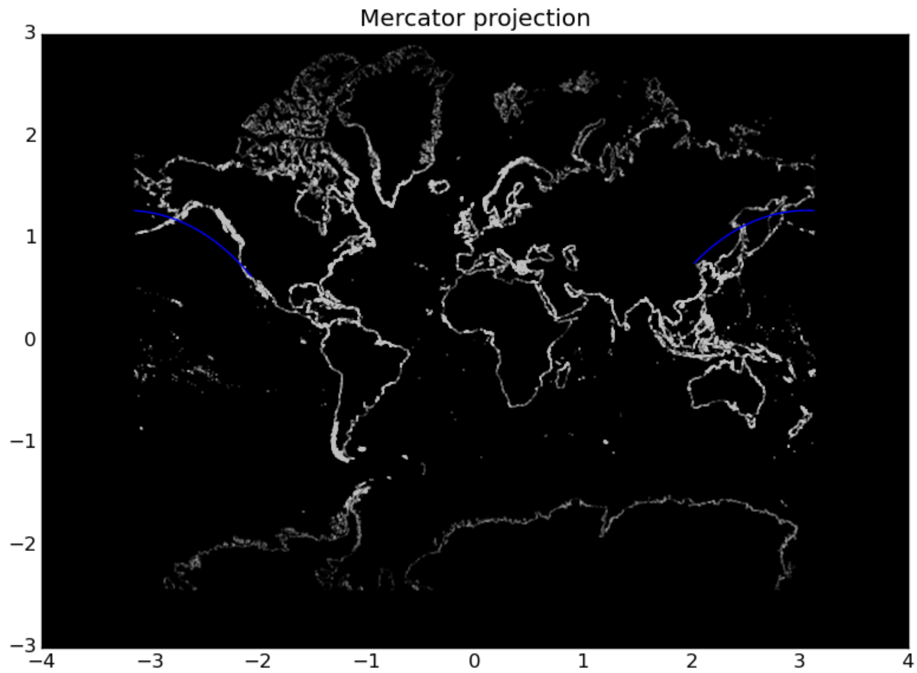

Figure 10: The geodesic line between Beijing and Los Angeles in the mercator projection of the Earth